Week 7

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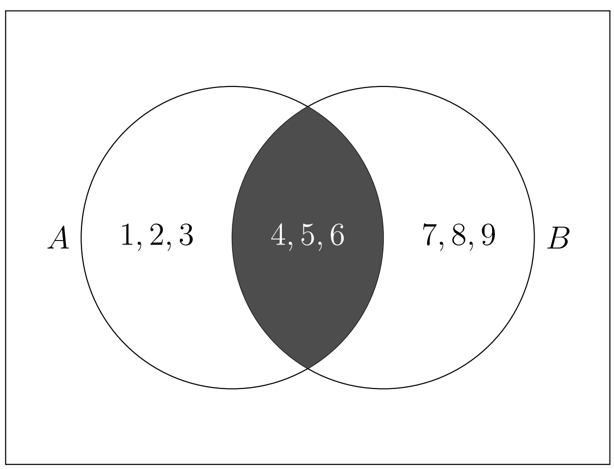
Section 3: Set Operations

September 02, 2021

Problem 23. Give examples of two sets A and B such that $|A-B| = |A\cap B| = |B-A| = 3$. Draw the accompanying Venn diagram.

Solution . Because $|A - B| = 3$, it follows that there must be 3 elements in A which do not belong to B. On the other hand, $|B - A| = 3$ implies that there are 3 elements in B such that they do not belong to A. Finally, there must be 3 elements that belong to both A and B since $|A \cap B| = 3$. Two sets A and B that fulfill this conditions are
$A = \{1, 2, 3, 4, 5, 6\}$
$B = \{4, 5, 6, 7, 8, 9\}$
The set $A - B = \{1, 2, 3\}$, $A \cap B = \{4, 5, 6\}$, and $B - A = \{7, 8, 9\}$. Therefore, $|A - B| = |A \cap B| = |B - A| = 3$.

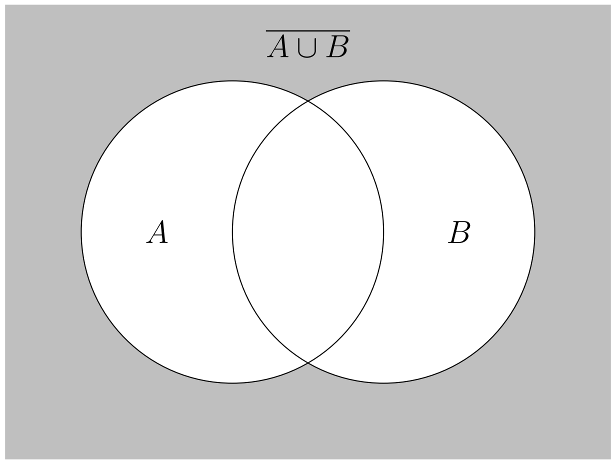


Problem 24. Give examples of three sets A, B and C such that $B \neq C$ but $B - A = C - A$

Solution . Since $B - A = C - A$, it follows that the elements of C and B that do not belong to A must be the same. Also, $B \cap A \neq C \cap A$ so that $B \neq C$. Let
$A = \{3\}$
$B = \{3, 5\}$
$C = \{5\}$
Then,$B - A = \{5\}$, and $C - A = \{5\}$. Thus, $B \neq C$ but $B - A = C - A$

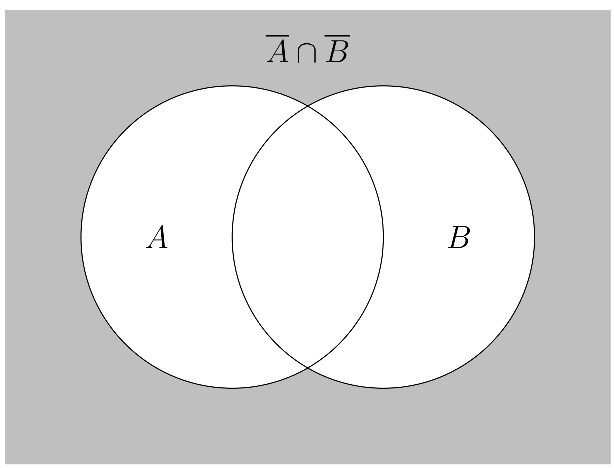
Problem 26. Let U be a universal set and let A and B be two subsets of U. Draw a Venn diagram for each of the following sets.
(a) $\overline{A \cup B}$

Solution a. $\overline{A \cup B} = \{x : x \in U \text{ and } x \notin A \cup B\}$.
Note that $A \cup B = \{x : x \in A \text{ or } x \in B\}$



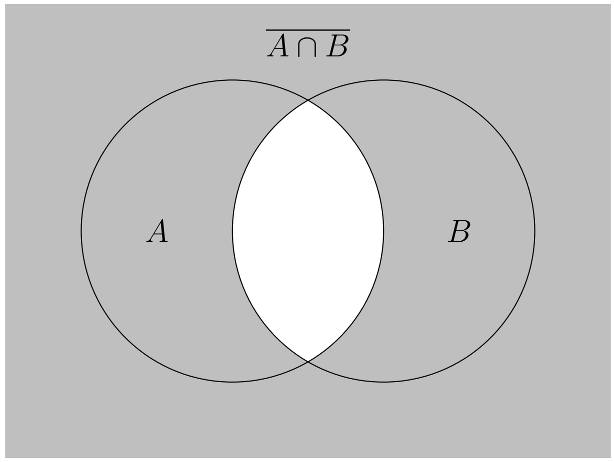
(b) $\overline{A} \cap \overline{B}$

Solution b. $\overline{A} \cap \overline{B} = \{x : x \in \overline{A} \text{ and } x \in \overline{B}\}$
Note that $\overline{A} = \{x : x \in U \text{ and } x \notin A\}$ and $\overline{B} = \{x : x \in U \text{ and } x \notin B\}$. Therefore, $\overline{A} \cap \overline{B}$ is the set of all $x \in U$ such that they don't belong to $A \cup B$.



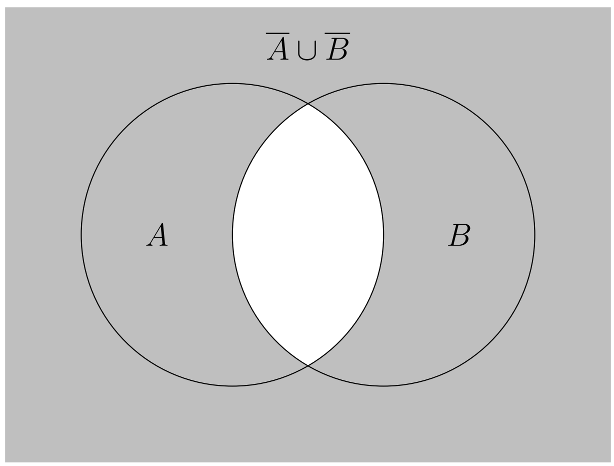
It can be seen that, $\overline{A \cup B} = \overline{A} \cap \overline{B}$.
(c) $\overline{A \cap B}$

Solution c. $\overline{A \cap B} = \{x : x \in U \text{ and } x \notin A \cap B\}$.



(d) $\overline{A} \cup \overline{B}$

Solution d. $\overline{A} \cup \overline{B} = \{x : x \in \overline{A} \text{ or } x \in \overline{B}\}$.
Thus, the set $\overline{A} \cup \overline{B}$ contains all $x \in U$ that don't belong to A or don't belong to B.



It can be seen that, $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

Problem 32. Give an example of four different subsets A, B, C and D of $\{1, 2, 3, 4\}$ such that all intersections of two subsets are different.

Solution . The subset A can be the set $\{1, 2, 3, 4\}$ since every set is a subset of itself. Each subset B, C and D takes one different number from A so that their intersections with A differ and they are not equal. Then, both B and C can take the number that only A contains, now they got an intersection. Lastly, D takes one element x of either B or C such that $x \notin B \cap C$.
Let $A = \{1, 2, 3, 4\}$
$B = \{1, 4\}$
$C = \{2, 4\}$
$D = \{3, 1\}$
Then, $A \cap B = \{1, 4\}$, $A \cap C = \{2, 4\}$, $A \cap D = \{1, 3\}$, $B \cap C = \{4\}$, $B \cap D = \{1\}$, $C \cap D = \emptyset$

Problem 33. Give an example of two nonempty sets A and B such that $\{A\cup B, A\cap B, A-B, B-A\}$ is the power set of some set.

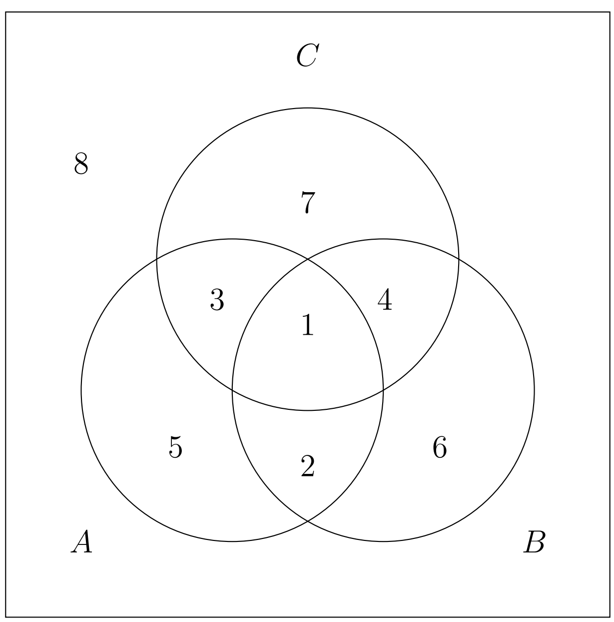
Solution . Let D be a set and $\mathcal{P}(D) = \{A\cup B, A\cap B, A-B, B-A\}$ be its power set. Because $|\mathcal{P}(D)| = 2^{|D|} = 4$, it follows that $|D| = 2$. Therefore, $D = \{n, m\}$ for some elements n, m and $\mathcal{P}(D) = \{\emptyset, \{n\}, \{m\}, \{n, m\}\}$. The cardinality $|A\cup B|$ must be greater than that of the other sets $A\cap B$, $A-B$ and $B-A$ since they are subsets of $A\cup B$. Thus, $A\cup B = \{n, m\} = D$. It is possible that $A = \{n\}$ and $B = \{m\}$.
Let,
$A = \{1\}$
$B = \{2\}$
$D = \{1, 2\}$
Then, $\mathcal{P}(D) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$ and $A\cup B = \{1, 2\}$, $A-B = \{1\}$, $B-A = \{2\}$, $A\cap B = \emptyset$.

Problem 34. Give an example of two subsets A and B of $\{1, 2, 3\}$ such that all of the following sets are different: $A\cup B$, $A\cup\overline{B}$, $\overline{A}\cup B$, $\overline{A}\cup\overline{B}$, $A\cap B$, $A\cap\overline{B}$, $\overline{A}\cap B$, $\overline{A}\cap\overline{B}$.

Solution . under construction

Problem 35. Give examples of a universal set U and sets A, B and C such that each of the following sets contains exactly one element: $A\cap B\cap C, (A\cap B)-C, (A\cap C)-B, (B\cap C)-A, A-(B\cup C), B-(A\cup C), C-(A\cup B), \overline{A\cup B\cup C}$. Draw the accompanying Venn diagram.

Solution . Using a Venn Diagram will facilitate the process of coming up with a solution for this problem since each of the sets that must contain one element represent a section of the following Venn diagram and they don't intersect each other.



Let, $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{1, 2, 3, 5\}$

$B = \{1, 2, 4, 6\}$
$C = \{1, 3, 4, 7\}$
Then, $A \cap B \cap C = \{1\}$, $(A \cap B) - C = \{2\}$, $(A \cap C) - B = \{3\}$, $(B \cap C) - A = \{4\}$, $A - (B \cup C) = \{5\}$, $B - (A \cup C) = \{6\}$, $C - (A \cup B) = \{7\}$, $\overline{A \cup B \cup C} = \{8\}$.