Homomorphisms and isomorphisms

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The notion of an *isomorphism* is that two groups have the same gorup-theoretic structure (any property that can be derived from the axioms of the group holds for both groups). Let $(G, *)$ and $(H, \cdot)$ be groups. A map $\phi : G \to H$ such that $\phi(x*y) = \phi(x) \cdot \phi(y)$ for all $x, y \in G$ is a homomorphism. For this map to be considered an ismorphism, it must be bijective. The symbol $\cong$ represent the equivalence isomorphic relation. Since $\cong$ is an equivalence relation in the set $\mathfrak{G}$ of all groups, it follows that there are equivalence clases that are isomorphic. This is important for the classification of groups using isomorphisms.

Also, one can show isomorphic relationships between sets by looking at their group presentations. Suppose for a group G with generators $\{r_1, \ldots, r_m\}$ and group H with some subset $\{s_1, \ldots, s_m\}$ that every relation in G is fulfilled in H when r_i is changed to s_i. Then there is a unique homomorphism $\varphi : G \to H$ such that $r_i \to s_i$. Furthermore, if $\{s_1, \ldots, s_m\}$ is the set of generators for H, then φ is surjective. Also, if $|G| = |H|$, then φ is injective and so an isomorphism.

1 Excercises