Week 8

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Section 6: Cartesian Products of Sets

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Problem 62. Describe the graph of the circle whose equation is $x^2 + y^2 = 4$ as a subset of $\mathbb{R} \times \mathbb{R}$.

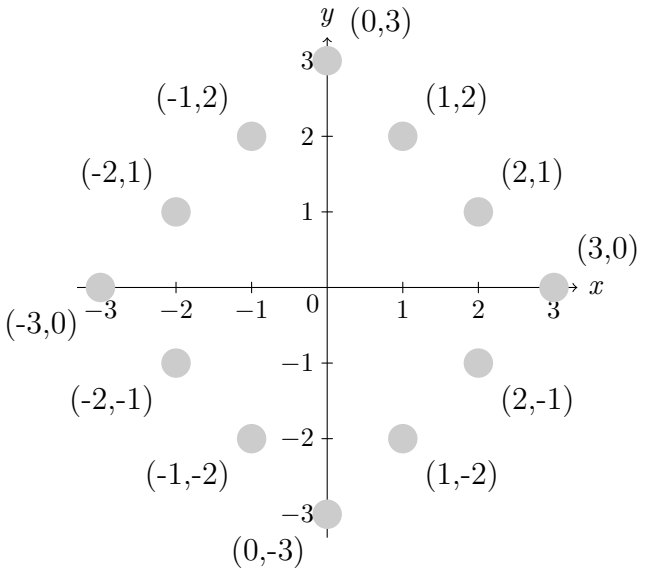
Solution . The graph of the equation $x^2 + y^2 = 4$ is the set $C = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 4\}$. The set $C \subset \mathbb{R}^2$.

Problem 63. List the elements of the set $S = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : |x| + |y| = 3\}$. Plot the corresponding points in the Euclidean xy-plane.

Solution . The set

$$\begin{aligned} S &= \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : |x| + |y| = 3\} \\ &= \{(1, 2), (-1, 2), (1, -2), (-1, -2), (3, 0), (-3, 0), (2, 1), (-2, 1), (2, -1), (-2, -1), (0, 3), (0, -3)\} \end{aligned}$$

The next figure shows the plot of these points in the Euclidean xy-plane.



Problem 64. For $A = \{1, 2\}$ and $B = \{1\}$, determine $\mathcal{P}(A \times B)$.

Solution . The set $A \times B = \{(1,1),(2,1)\}$. Since $|A \times B| = 2$, it follows that $|\mathcal{P}(A \times B)| = 2^2 = 4$. The set $\mathcal{P}(A \times B) = \{\emptyset, \{(1,1)\}, \{(2,1)\}, A \times B\}$.

Problem 65. For $A = \{x \in \mathbb{R} : |x-1| \leq 2\}$ and $B = \{y \in \mathbb{R} : |y-4| \leq 2\}$, give a geometric description of the points in the xy-plane belonging to $A \times B$.

Solution . The sets $A = [-1,3]$ and $B = [2,6]$. Since the cartesian product $A \times B = \{(a,b) : a \in [-1,3] \text{ and } b \in [2,6]\}$, it follows that the points in $A \times B$ are found on and inside the square bounded by $x = -1$, $x = 3$, $y = 2$ and $y = 6$ (Note that each side is of length 4 units).

Problem 66. For $A = \{a \in \mathbb{R} : |a| \leq 1\}$ and $B = \{b \in \mathbb{R} : |b| = 1\}$, give a geometric description of the points in the xy-plane belonging to $(A \times B) \cup (B \times A)$

Solution . The sets $A = [-1,1]$ and $B = \{-1,1\}$. Each point in $A \times B = [-1,1] \times \{-1,1\}$ lies on one of the two horizontal parallel lines $y = -1$ and $y = 1$ with $x \in [-1,1]$. Also, each point in $B \times A = \{-1,1\} \times [-1,1]$ lies on one of the two vertical parallel lines $x = -1$ and $x = 1$ with $y \in [-1,1]$. Thus, all the points belonging to $(A \times B) \cup (B \times A)$ lie just on (not inside) the square bounded by $x = -1$, $x = 1$, $y = -1$ and $y = 1$.