Section 8.2: Properties of relations

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This chapter mentioned three properties of interested for some relation R on a single set A. Since most of these properties involve implications with universal quantifiers, the easiest way to check wether a relation has certain property is by looking for specific examples for which the implication in question is false.

(a) **Reflexive Property:** if $x \in A$, then $(x, x) \in R$. (x is related to itself)

(b) **Symmetric Property:** $\forall x, y \in A$, if $x \ R \ y$, then $y \ R \ x$ (x is related to y and viceversa). Note that for the relation R to not be symmetric, it must be true that $x \ R \ y$ and $y \not R \ x$. For this to happen, it is necessary that $x \neq y$.

(c) **Transitive Property:** $\forall x, y, z \in A$, if $x \ R \ y$ and $y \ R \ z$, then $x \ R \ z$. Note that for the relation R to not be symmetric, it must be true that $x \ R \ y$, $y \ R \ z$ and $x \not R \ z$. For this to happen, it is necessary that $x \neq y$ and $z \neq y$.