Section 9.4: Bijective Functions

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As it was mentioned in the previous section, for finite sets A and B, $|A| \geq |B|$ is a necessary and sufficient condition for an onto function $f : A \to B$ to exist. The same can be said for $|A| \leq |B|$ and some one-to-one function $g : A \to B$. Since we are talking about positive integers, it must be true that $|A| = |B|$ is a necessary and sufficient condition for an onto and one-to-one function $\varphi : A \to B$ to exist, knwon as a bijective function.
In fact, for finite sets B and C such that $|B| = |C| = n$, there are $n!$ distinct bijective functions from B to C. Namely, every bijective function is a permutation of the elements of $|C|$ for n spaces. Furthermore, for any function f from B to C, f is onto if and only if f is one-to-one. All this makes sense for finite sets, we must make sure to pair all elements of C with the constriction of assigning one unique element to every element of B. However, this intuition does not work for analyzing the cases with infinite ones.
Let A, B be sets. So far, we defined the function $f : A \to B$ as a relation from A to B such that

(a) $x \in A \implies \exists b \in B, (a, b) \in f$

(b) $(a, b), (a, c) \in f \implies b = c$

If a relation satisfies (b), then it is called **well-defined**.
Lastly, the identity function i_S on ANY nonempty set S defined by $i_S(n) = n$ for all $n \in S$ is bijective.