Section 8.3: Equivalence Relations

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May 30, 2022

This chapter reviews some properties that we realized and proved in the problems of **Section 8.3**. However, there's something worth noting. Let R be some relation on some nonempty set A. I previously showed that the union of the equivalence classes by R is A and they all are pairwise disjoint. Nevertheless, I didn't ponder on it much to realize what this meant, namely, that the set of these distinct equivalence classes is a partition of A!!!! This was proven by the authors by just showing that each $x \in A$ belongs to exaclty one equivalence class by R.

Problem 36. Give an example of an equivalence relation R on the set $A = \{v, w, x, y, z\}$ such that there are exactly three distinct equivalence classes. What are the equivalence classes for your example?

Solution 36. Consider the parition $P = \{\{v\}, \{w\}, \{x, y, z\}\}$ of A. By **Theorem 4**, the relation R definded by $a\ R\ b$ if $a, b \in X$ for some $X \in P$ is an equivalence relation. Hence, the distinct equivalance classe are

$$\begin{aligned}
a_1 &= \{x, y, z\} \\
a_2 &= \{w\} \\
a_3 &= \{v\}
\end{aligned}$$

Problem 37. A relation R is defined on $\mathbb{N}$ by $a\ R\ b$ if $a^2 + b^2$ is even. Prove that R is an equivalence relation. Determine the distinct equivalence classes.

Proof. We first prove that R is an equivalence relation. Consider some positive integer c. Then, $c^2 + c^2 = 2c^2$. Since c^2 is an integer, it follows that $2c^2$ is even and so $c\ R\ c$. Hence, R is reflexive. $\square$