Section 9.3: One-To-One and Onto Functions

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We have seen that a function from A to B is a relation that fulfills the following condition:

$$a = b \implies f(a) = f(b).$$

Furthermore, functions can posses to important properties. A function $f : A \to B$ is said to be **One-to-One** if every image is unique to its respective $x \in A$, namely,

$$\begin{gathered} f(a) = f(b) \implies a = b \\ \equiv a \neq b \implies f(a) \neq f(b). \end{gathered}$$

Obviously, for this to be true, B must contain at least the same number of elements as A, namely, $|A| \leq |B|$. On the other hand, the function f is said to be **Onto** if every element in B is the image of some element of A, namely,

$$b \in B \implies \exists a \in A, f(a) = b.$$

Hence, $f(A) = B$. Clearly, $|B| \leq |A|$, otherwise, there would be not enough elements of A to cover all elements of B. Then, if a function is both one-to-one and onto, then $|A| = |B|$.

Problem 20. A function $f : \mathbb{Z} \to \mathbb{Z}$ is defined by $f(n) = 2n + 1$. Determine whether f is injective, surjective.

Solution First we show that it is injective. Consider two $f(a) = f(b)$ for some $a, b \in \mathbb{Z}$. Then, $2a + 1 = 2b + 1$. Substracting 1 to both sides, we get $2a = 2b$. Dividing by 2, we obtain $a = b$. However, it is not surjective. Consider any even integer r and so there is no integer n such that $f(n) = 2n + 1 = r$.