

Week 1

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Section 4: The Implication

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“At times, the desire becomes almost overpowering, in its intensity. It is not mere curiosity, that prompts me; but more as though some unexplained influence were at work”

— William Hope Hodgson, *The House on the Borderland*

Problem 19. Consider the statements P : *17 is even.* and Q : *19 is prime.* Write each of the following statements in words and indicate whether it is true or false.

(A) $\sim P$: 17 is odd; **TRUE**

Proof. The real integer 17 is odd, thus the statement P is false. The negation of this statement will have the opposite truth value, *true*. \square

(B) $P \vee Q$: 17 is even or 19 is prime; **TRUE**

Proof. The real integer 19 is prime, thus Q is true. Because this statement is the disjunction of P and Q , it is only needed for one of them to be true for the statement to be true. \square

(C) $P \wedge Q$: 17 is even and 19 is prime; **FALSE**

Proof. We already know that the statement Q is true and P is false. This statement is a conjunction of both Q and P , and since one of them is false, the statement is false. \square

(D) $P \implies Q$: if 17 is even, then 19 is prime; **TRUE**

Proof. According to this statement P implies Q , so the only way for it to be false is when P is true and Q is false. We know that P is false, which means that this statement is true, because it tells nothing about what would happen when P is false. \square

Problem 20. For statements P and Q , construct a truth table for $(P \implies Q) \implies (\sim P)$

P	Q	$P \implies Q$	$\sim P$	$(P \implies Q) \implies (\sim P)$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Problem 21. Consider the statements $P : \sqrt{2} \text{ is rational.}$ and $Q : 22/7 \text{ is rational.}$ Write each of the following statements in words and indicate whether it is true or false.

(A) $P \implies Q$: if $\sqrt{2}$ is rational, then $22/7$ is rational ; **TRUE**

Proof. We know that $\sqrt{2}$ is irrational (we don't prove it here, because it is out of the scope of this exercise), thus the statement P is false. Since the statement we declare is an implication, it does not say anything about the truth value of Q when P is false. Therefore it did not "lie". \square

(B) $Q \implies P$: if $22/7$ is rational, then $\sqrt{2}$ is rational ; **FALSE**

Proof. The fraction $22/7$ is the quotient of two integers and the denominator is a natural non-zero number, which means that it is rational. The statement Q is true and P is false, therefore the implication is not true. \square

(C) $(\sim P) \implies (\sim Q)$: if $\sqrt{2}$ is irrational, then $22/7$ is irrational ; **FALSE**

Proof. The first statement $\sim P$ is true because $\sqrt{2}$ is irrational, but $\sim Q$ is not since $22/7$ is rational. Therefore, $\sim P$ does not imply $\sim Q$. \square

(D) $(\sim Q) \implies (\sim P)$: if $22/7$ is irrational, then $\sqrt{2}$ is irrational ; **TRUE**

Proof. This statement is similar to the previous one, but the places of the negations are switched. $\sim Q$ is false, therefore we can not prove the falseness of this implication because it tells nothing about the case when $\sim Q$ is false. \square

Problem 22. Consider the statements:

$$P : \sqrt{2} \text{ is irrational.} \quad Q : \frac{2}{3} \text{ is rational.} \quad R : \sqrt{3} \text{ is rational.}$$

Write each of the following statements in words and indicate whether the statement is true or false.