Section 8.3: Equivalence Relations

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We know that a function is a special kind of relation and for nonempty sets A and B, that the set of all posible relations is $\mathcal{P}(A \times B)$. One may ask how many of those subsets are functions. In other words, we are looking for the set of all functions from A to B denoted by $B^A = \{f : f : A \to B\}$. Its symbolical representation aludes to its cardinality, namly, $|B^A| = |B|^{|A|}$. This is so since for each function $f : A \to B$, every $a \in A$ must be paired with only one $b \in B$, and so each $a \in A$ can be paired with $|B|$ possible choices in an independly manner. It is like obtaining all possible combinations of $|B|$ repetible elements in $|A|$ ordered places. Namely,

$$
\begin{array}{llllll}
1 & 2 & 3 & 4 & \dots & |A| \\
b_1 & b_1 & b_1 & b_1 & \dots & b_1 \\
b_2 & b_1 & b_1 & b_1 & \dots & b_1 \\
b_1 & b_2 & b_1 & b_1 & \dots & b_1 \\
b_2 & b_2 & b_1 & b_1 & \dots & b_1 \\
\vdots & & & & & \\
b_{|B|} & b_{|B|} & b_{|B|} & b_{|B|} & \dots & b_{|B|-2} \\
b_{|B|} & b_{|B|} & b_{|B|} & b_{|B|} & \dots & b_{|B|-1} \\
b_{|B|} & b_{|B|} & b_{|B|} & b_{|B|} & \dots & b_{|B|}
\end{array}
$$

Problem 13. Let $A = \{1, 2, 3\}$ and $B = \{x, y\}$. Determine B^A.

Solution

$$
\begin{aligned}
B^A &= \{f : f : A \to B\} \\
&= \{f_{xxx}, f_{yxx}, f_{xyx}, f_{yyx}, f_{xxy}, f_{yxy}, f_{xyy}, f_{yyy}\},
\end{aligned}
$$

where $f_{abc} = \{(1, a), (2, b), (3, c)\}$.

Problem 16. (a) Give an example of two sets A and B such that $|B^A| = 8$

Solution It suffices to have a set B with 2 elements and a set A with 3. For instance, $B = \{a, b\}$ and $A = \{1, 2, 3\}$.However this is not necessary. An alternate example is $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{0\}$.

(b) Give an example of an element in B^A for the sets A and B given in (a).

Solution One example is $\{(1, a), (2, a), (3, a)\}$.

Problem 17. (a) For nonempty sets A, B and C, what is a possible interpretation of th notation C^{B^A}?

Solution One possible interpretation is that C^{B^A} is the set of all functions from B^A to C, namely, $C^{B^A} = \left\{f : f : B^A \to C\right\}$, where $B^A = \{g : g : A \to B\}$. Thus, $\{(g_1, c_1), (g_2, c_1), (g_3, c_1), \ldots, (g_k, c_1)\} \in C^{B^A}$.

(b) According to the definition given in (a), determine C^{B^A} for $A = \{0, 1\}$, $B = \{a, b\}$ and $C = \{x, y\}$.