常用极限

$\lim\limits_{x\to 0}\frac{\sin x}{x}=1$ $\lim\limits_{x\to\infty}(1+\frac{1}{x})^x=e$ $\lim\limits_{n\to\infty}a^n=0,|a|<1$ $\lim\limits_{n\to\infty}\sqrt[n]{a}=1,a>0$ $\lim\limits_{n\to\infty}\sqrt[n]{n}=1$

$\lim\limits_{n\to\infty}\sqrt[n]{a_1^n+a_2^n+\cdots+a_n^n}=A=\max\{a_i\}$ 夹逼 $\sqrt[n]{A^n}\leq I\leq\sqrt[n]{nA^n}$

$\lim\limits_{x\to 0^+}x^a\ln^b x=0$，$a>0$，$b$任意，　任意正整数$k$，$\lim\limits_{x\to 0}\frac{e^{-\frac{1}{x^2}}}{x^k}=0$

常用等价无穷小（$\lim\frac{\alpha}{\beta}=1$）替换（本质是泰勒公式）当$x\to 0$（或$f(x)\to 0$）时

$x\sim\sin x\sim\tan x\sim\arcsin x\sim\arctan x\sim\ln(1+x)\sim e^x-1$ $a^x-1\sim x\cdot\ln a$

$\sqrt[n]{1+x}-1\sim\frac{x}{n}$ $(1+x)^\alpha-1\sim\alpha x$ 若$\alpha(x)\to 0,\alpha(x)\beta(x)\to 0$，则$(1+\alpha(x))^{\beta(x)}-1\sim\alpha(x)\beta(x)$

$x-\ln(1+x)\sim\frac{x^2}{2}$

$1-\cos x\sim\frac{x^2}{2}$ $1-\cos^\alpha x\sim\frac{\alpha}{2}x^2$ $\tan x-\sin x=\tan(1-\cos x)\sim\frac{x^3}{2}$

$x-\sin x\sim\frac{x^3}{6}$ $x-\arcsin x\sim-\frac{x^3}{6}$ $x-\tan x\sim-\frac{x^3}{3}$ $x-\arctan x\sim\frac{x^3}{3}$

x^a+x^b $(a<b)$是x的a阶无穷小，即$x^a+x^b\sim x^a$，由定义$\lim\limits_{x\to 0}\frac{x^a+x^b}{x^k}=A$，则$k=a$

加减关系在一定条件下可替换

① 若$\alpha\sim\alpha_1$，$\beta\sim\beta_1$，$\lim\frac{\alpha_1}{\beta_1}=A\neq 1$，则$\alpha-\beta\sim\alpha_1-\beta_1$

② 若$\alpha\sim\alpha_1$，$\beta\sim\beta_1$，$\lim\frac{\alpha_1}{\beta_1}=A\neq -1$，则$\alpha+\beta\sim\alpha_1+\beta_1$

常用无穷大量的比较（可由洛必达法则证明）

当$n\to\infty$时，$\ln^\alpha n\ll n^\beta\ll a^n\ll n!\ll n^n$，其中$\alpha>0,\beta>0,a>1$

常用麦克劳林公式 $n=0,1,2,\cdots$　　$\ln$、$\tan$、$\arctan$的分母没有阶乘

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+o(x^n)$$

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots+(-1)^n\frac{x^{n+1}}{n+1}+o(x^{n+1})$$

$$(1+x)^\alpha=1+\alpha x+\frac{\alpha(\alpha-1)}{2!}x^2+\cdots+\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n+o(x^n)$$

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+o(x^{2n+1})$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots+(-1)^n\frac{x^{2n}}{(2n)!}+o(x^{2n})$$

$$\cos^\alpha x=1-\frac{\alpha}{2!}x^2+\frac{\alpha^2}{4!}x^4-\cdots+(-1)^n\frac{\alpha^n x^{2n}}{(2n)!}+o(x^{2n})$$

$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$

$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)} + o(x^{2n+1})$

基本初等函数的导数公式

$C' = 0$　　$(x^a)' = ax^{a-1}$　　$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$　　$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$(a^x)' = a^x \ln a$　　$(e^x)' = e^x$　　$(\log_a x)' = \frac{1}{x\ln a}$　　$(\ln|x|)' = \frac{1}{x}$

$(\sin x)' = \cos x$　　$(\cos x)' = -\sin x$　　$(\tan x)' = \sec^2 x$　　$(\cot x)' = -\csc^2 x$

$(\sec x)' = \sec x \cdot \tan x$　　$(\csc x)' = -\csc x \cdot \cot x$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$　　$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

$(\arctan x)' = \frac{1}{1+x^2}$　　$(\text{arccot}\, x)' = -\frac{1}{1+x^2}$

高阶导数 常用公式法、归纳法、泰勒公式法

$(f \cdot g)^{(n)} = C_n^0 f^{(n)} g + C_n^1 f^{(n-1)} g^{(1)} + \cdots + C_n^{n-1} f^{(1)} g^{(n-1)} + C_n^n f g^{(n)} = \sum_{k=0}^{n} f^{(n-k)} g^{(k)}$ 常试构造 $R_m(x) f(x)$

$(\sin(ax+b))^{(n)} = a^n \cdot \sin\left(ax+b+\frac{n\pi}{2}\right)$　　$(\cos(ax+b))^{(n)} = a^n \cdot \cos\left(ax+b+\frac{n\pi}{2}\right)$

$(\ln(1+x))^{(n)} = (-1)^{n-1} \frac{(n-1)!}{(1+x)^n}$　　$\left(\frac{1}{ax+b}\right)^{(n)} = \frac{(-1)^n n!\, a^n}{(ax+b)^{n+1}}$

$(\ln(ax+b))^{(n)} = a\left(\frac{1}{ax+b}\right)^{(n-1)} = (-1)^{n-1} \frac{(n-1)!\, a^n}{(ax+b)^n}$

$[(ax+b)^t]^{(n)} = a^n (ax+b)^{t-n} \cdot t(t-1)\cdots(t-n+1)$，当 $n \geq t+1$，有 $I^{(n)} = 0$

指数求导法 $(x^x)' = (e^{x\ln x})' = e^{x\ln x} \cdot (x\ln x)' = x^x \cdot (\ln x + 1)$

不定积分基本公式

arcsin 与 arctan 的内部为 $\frac{x}{a}$，arctan 与 $\ln\left|\frac{x-a}{x+a}\right|$ 的系数为 $\frac{1}{a}$ 和 $\frac{1}{2a}$

$\int k\,dx = kx + C$　　$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$　　$\int \frac{1}{x} dx = \ln|x| + C$

$\int \frac{1}{x^2} dx = -\frac{1}{x} + C$　　$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$

$\int a^x dx = \frac{a^x}{\ln a} + C \quad (a>0, a \neq 1)$　　$\int e^x dx = e^x + C$

$\int \sin x\,dx = -\cos x + C$　　$\int \cos x\,dx = \sin x + C$

$\int \tan x\,dx = -\ln|\cos x| + C$　　$\int \cot x\,dx = \ln|\sin x| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$　　$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \cdot \tan x\,dx = \sec x + C$

$\int \csc x \cdot \cot x\,dx = -\csc x + C$

$\int \arcsin x\,dx = x\arcsin x + \sqrt{1-x^2} + C$

$\int \arccos x\,dx = x\arccos x + \sqrt{1-x^2} + C$

$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + C$

$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a} + C \quad (a>0)$

$\int \frac{1}{1+x^2}dx = \arctan x + C$

$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\frac{x}{a} + C \quad (a\neq 0)$

$\int \frac{1}{\sqrt{x^2\pm a^2}}dx = \ln\left|x+\sqrt{x^2\pm a^2}\right| + C$

$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

$\int \sqrt{a^2-x^2}\,dx = \frac{a^2}{2}\arcsin\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} + C$

常见凑微分

$\int f(ax^n+b)x^{n-1}dx = \frac{1}{an}\int f(ax^n+b)\,d(ax^n+b)$

$-\int f(\frac{1}{x})\frac{1}{x^2}dx = \int f(\frac{1}{x})d\frac{1}{x}$

$\int \frac{f(\sqrt{x})}{2\sqrt{x}}dx = \int f(\sqrt{x})d\sqrt{x}$

$\int f(x)\frac{x}{\sqrt{x^2+1}}dx = \int f(x)\,d\sqrt{x^2+1}$

$\int (1-\frac{1}{x^2})f(x+\frac{1}{x})dx = \int f(x+\frac{1}{x})d(x+\frac{1}{x})$

$\int (1+\frac{1}{x^2})f(x-\frac{1}{x})dx = \int f(x-\frac{1}{x})d(x-\frac{1}{x})$

$\int \frac{f(\ln x)}{x}dx = \int f(\ln x)d(\ln x)$

$\int f(x\ln x)(\ln x+1)dx = \int f(x\ln x)d(x\ln x)$

$\int f(x)(1-\ln x)dx = \int x^2 f(x)d(\frac{\ln x}{x})$

常见不可积函数(原函数不是初等函数)

$\int e^{ax^2}dx \quad (a\neq 0)$

$\int \frac{e^x}{x}dx$

$\int \sin^\alpha x\,dx$ (α不是整数)

$\int \sin x^2 dx$

$\int \cos x^2 dx$

$\int \frac{\sin x}{x}dx$

$\int \frac{\cos x}{x}dx$

$\int \frac{x^n}{\ln x}dx \quad (n\neq -1)$

$\int \frac{\ln x}{x+a}dx \quad (a\neq 0)$

$\int \frac{1}{\sqrt{x^4+a}}dx \quad (a\neq 0)$

$\int \frac{1}{\sqrt{1+k\sin^2 x}}dx \quad (k\neq 0, k\neq -1)$

$\int \sqrt{1+k\sin^2 x}\,dx \quad (k\neq 0, k\neq -1)$