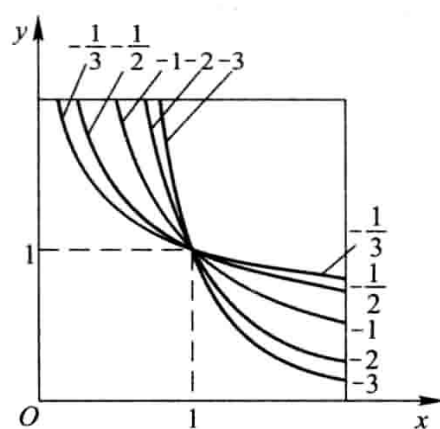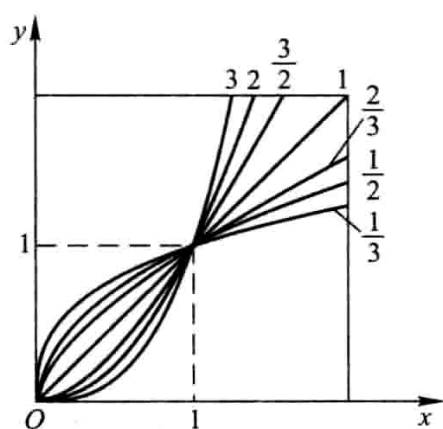# 附录Ⅱ 基本初等函数的图形
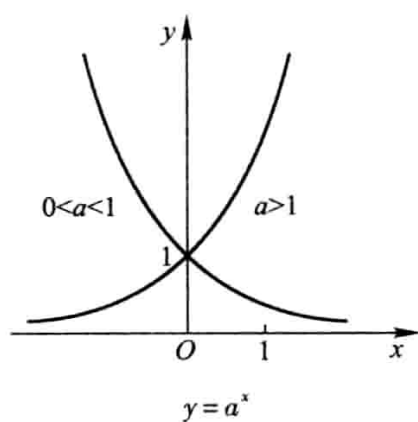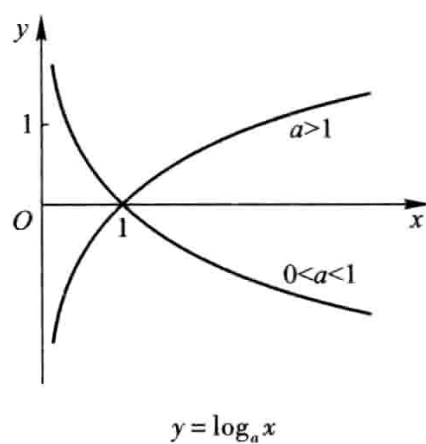
幂函数

$$y = x^\mu$$

指数函数

$$y = a^x$$

对数函数

$$y = \log_a x$$

三角函数

$$y = \sin x$$

$y = \cos\ x$
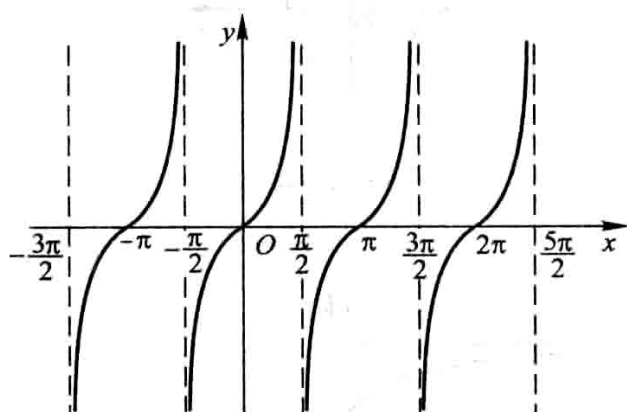
$y = \tan\ x$
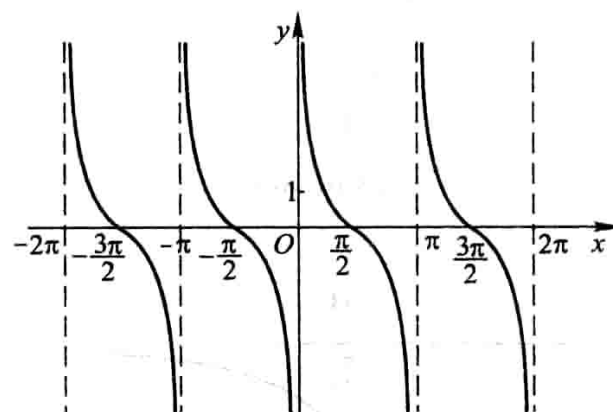
$y = \cot\ x$
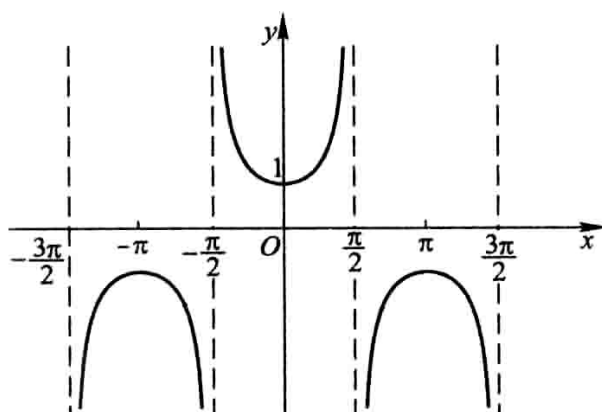
$y = \sec\ x$
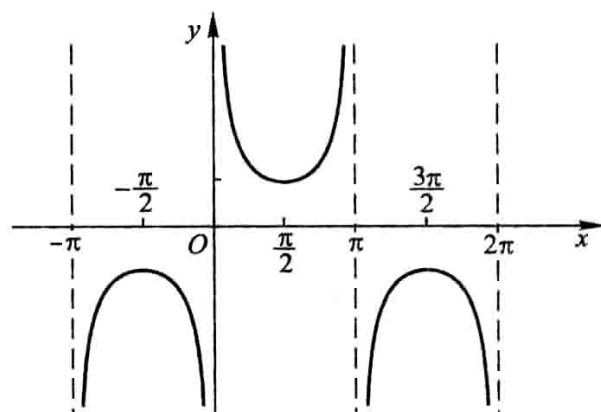
$y = \csc\ x$

反三角函数
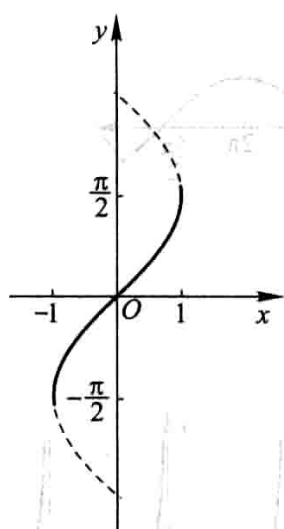
$y = \arcsin x$
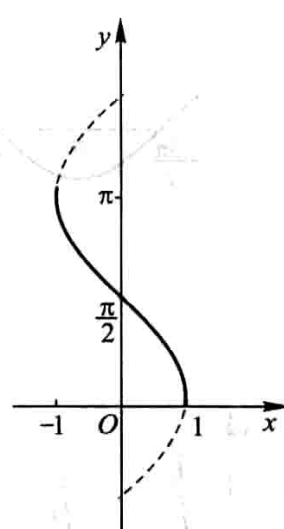
$y = \arccos x$
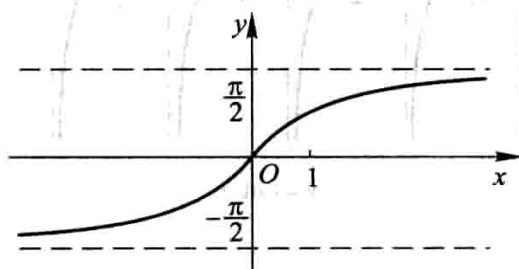
$y = \arctan x$
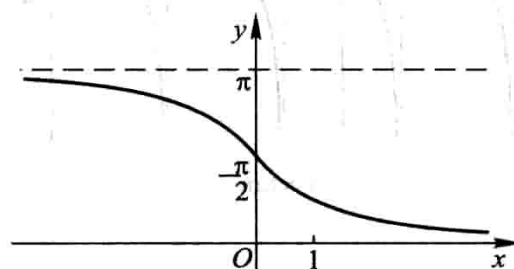
$y = \text{arccot } x$

附录 Ⅲ 几种常用的曲线

（1）三次抛物线
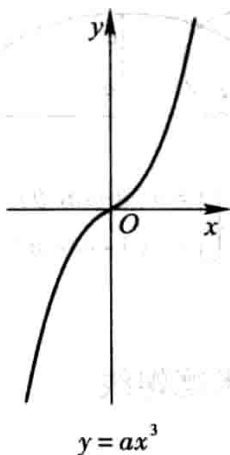
$$y = ax^3$$

（2）半立方抛物线

$$y^2 = ax^3$$

（3）概率曲线

$$y = e^{-x^2}$$

（4）箕舌线

$$y = \frac{8a^3}{x^2 + 4a^2}$$

（5）蔓叶线

$$y^2(2a - x) = x^3$$

（6）笛卡儿叶形线

$$x^3 + y^3 - 3axy = 0$$
$$x = \frac{3at}{1 + t^3}, y = \frac{3at^2}{1 + t^3}$$
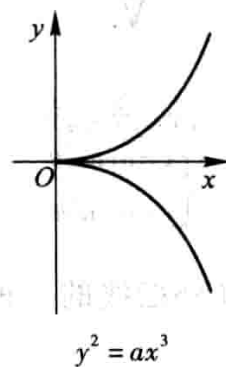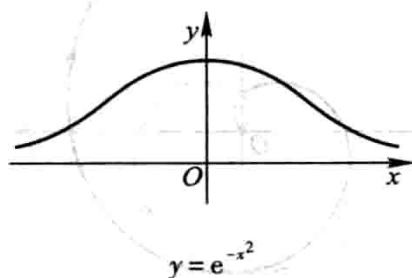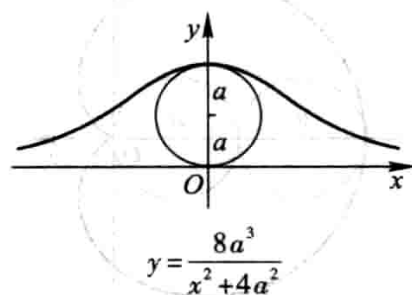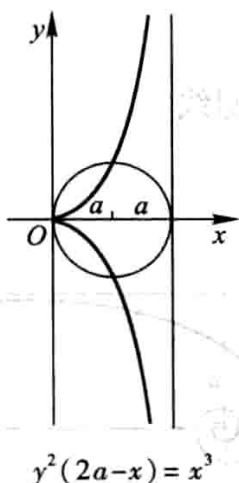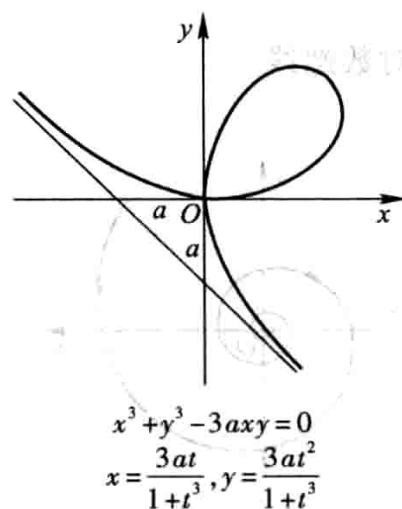
（7）星形线（内摆线的一种）　　　（8）摆线
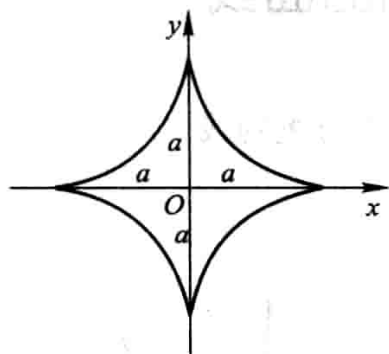
$$x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$$

$$\begin{cases} x=a\cos^3\theta \\ y=a\sin^3\theta \end{cases}$$
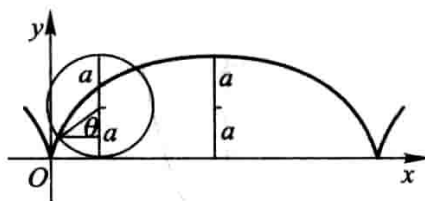
$$\begin{cases} x=a(\theta-\sin\theta) \\ y=a(1-\cos\theta) \end{cases}$$

（9）心形线（外摆线的一种）　　　（10）阿基米德螺线

$$x^2+y^2+ax=a\sqrt{x^2+y^2}$$

$$\rho=a(1-\cos\theta)$$ $\rho=a(1+\cos\theta)$ 向右

$$\rho=a\theta$$

（11）对数螺线　　　（12）双曲螺线

$$\rho=e^{a\theta}$$
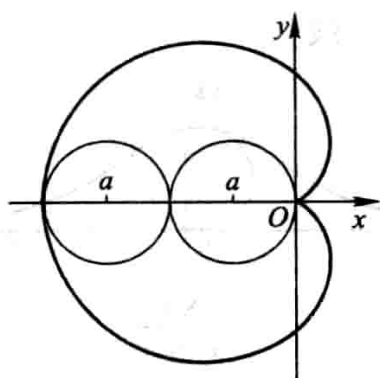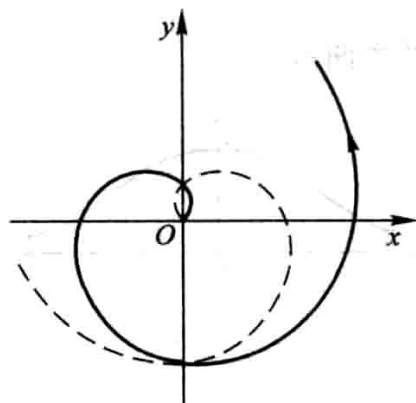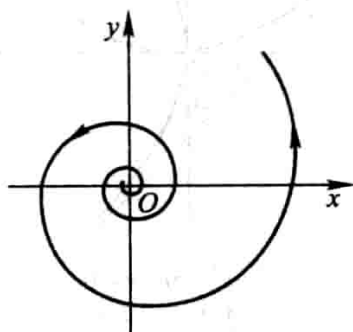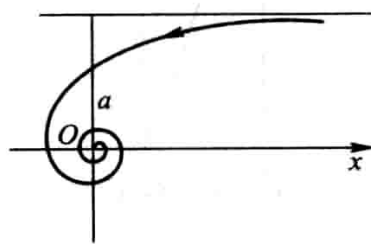
$$\rho\theta=a$$
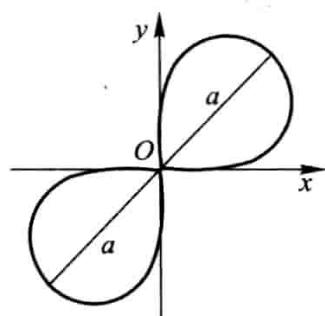
（13）伯努利双纽线

$$(x^2+y^2)^2=2a^2xy$$

$$\rho^2=a^2\sin 2\theta$$

（14）伯努利双纽线
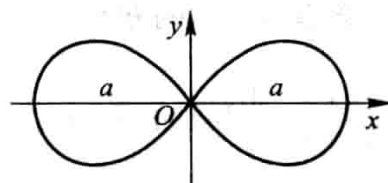
$$(x^2+y^2)^2=a^2(x^2-y^2)$$

$$\rho^2=a^2\cos 2\theta$$

（15）三叶玫瑰线
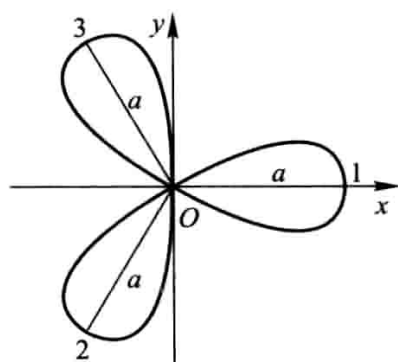
$$\rho=a\cos 3\theta$$

（16）三叶玫瑰线
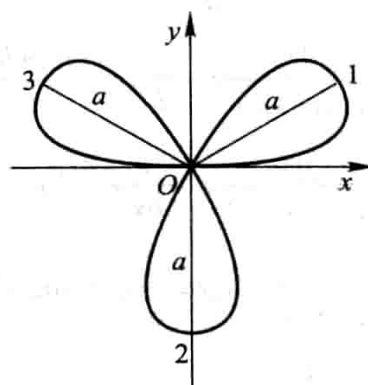
$$\rho=a\sin 3\theta$$

（17）四叶玫瑰线

$$\rho=a\sin 2\theta$$

（18）四叶玫瑰线

$$\rho=a\cos 2\theta$$
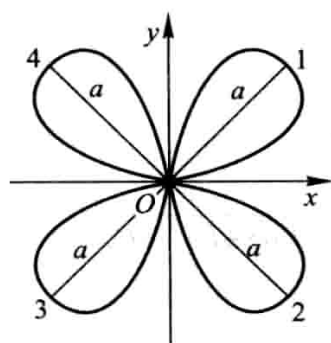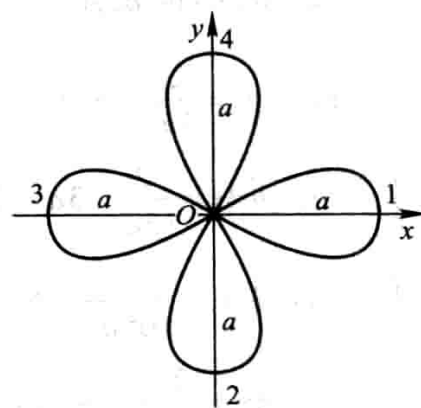
附录Ⅳ 积 分 表

（一）含有 $ax+b$ 的积分

1. $\displaystyle\int \frac{\mathrm{d}x}{ax+b} = \frac{1}{a}\ln|ax+b| + C.$

2. $\displaystyle\int (ax+b)^{\mu}\mathrm{d}x = \frac{1}{a(\mu+1)}(ax+b)^{\mu+1} + C \ (\mu \neq -1).$

3. $\displaystyle\int \frac{x}{ax+b}\mathrm{d}x = \frac{1}{a^2}(ax+b-b\ln|ax+b|) + C.$

4. $\displaystyle\int \frac{x^2}{ax+b}\mathrm{d}x = \frac{1}{a^3}\left[\frac{1}{2}(ax+b)^2 - 2b(ax+b) + b^2\ln|ax+b|\right] + C.$

5. $\displaystyle\int \frac{\mathrm{d}x}{x(ax+b)} = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right| + C.$

6. $\displaystyle\int \frac{\mathrm{d}x}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right| + C.$

7. $\displaystyle\int \frac{x}{(ax+b)^2}\mathrm{d}x = \frac{1}{a^2}\left(\ln|ax+b| + \frac{b}{ax+b}\right) + C.$

8. $\displaystyle\int \frac{x^2}{(ax+b)^2}\mathrm{d}x = \frac{1}{a^3}\left(ax+b-2b\ln|ax+b| - \frac{b^2}{ax+b}\right) + C.$

9. $\displaystyle\int \frac{\mathrm{d}x}{x(ax+b)^2} = \frac{1}{b(ax+b)} - \frac{1}{b^2}\ln\left|\frac{ax+b}{x}\right| + C.$

（二）含有 $\sqrt{ax+b}$ 的积分

10. $\displaystyle\int \sqrt{ax+b}\,\mathrm{d}x = \frac{2}{3a}\sqrt{(ax+b)^3} + C.$

11. $\displaystyle\int x\sqrt{ax+b}\,\mathrm{d}x = \frac{2}{15a^2}(3ax-2b)\sqrt{(ax+b)^3} + C.$

12. $\displaystyle\int x^2\sqrt{ax+b}\,\mathrm{d}x = \frac{2}{105a^3}(15a^2x^2-12abx+8b^2)\sqrt{(ax+b)^3} + C.$

13. $\displaystyle\int \frac{x}{\sqrt{ax+b}}\mathrm{d}x = \frac{2}{3a^2}(ax-2b)\sqrt{ax+b} + C.$

14. $\displaystyle\int \frac{x^2}{\sqrt{ax+b}}\mathrm{d}x = \frac{2}{15a^3}(3a^2x^2-4abx+8b^2)\sqrt{ax+b} + C.$

15. $\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{ax+b}} = \begin{cases} \dfrac{1}{\sqrt{b}}\ln\left|\dfrac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right|+C & (b>0), \\[4mm] \dfrac{2}{\sqrt{-b}}\arctan\sqrt{\dfrac{ax+b}{-b}}+C & (b<0). \end{cases}$

16. $\displaystyle\int \frac{\mathrm{d}x}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{\mathrm{d}x}{x\sqrt{ax+b}}.$

17. $\displaystyle\int \frac{\sqrt{ax+b}}{x}\mathrm{d}x = 2\sqrt{ax+b} + b\int \frac{\mathrm{d}x}{x\sqrt{ax+b}}.$

18. $\displaystyle\int \frac{\sqrt{ax+b}}{x^2}\mathrm{d}x = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{\mathrm{d}x}{x\sqrt{ax+b}}.$

（三）含有 $x^2 \pm a^2$ 的积分

19. $\displaystyle\int \frac{\mathrm{d}x}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a}+C.$

20. $\displaystyle\int \frac{\mathrm{d}x}{(x^2+a^2)^n} = \frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{2n-3}{2(n-1)a^2}\int \frac{\mathrm{d}x}{(x^2+a^2)^{n-1}}.$

21. $\displaystyle\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C.$

（四）含有 ax^2+b $(a>0)$ 的积分

22. $\displaystyle\int \frac{\mathrm{d}x}{ax^2+b} = \begin{cases} \dfrac{1}{\sqrt{ab}}\arctan\sqrt{\dfrac{a}{b}}x+C & (b>0), \\[4mm] \dfrac{1}{2\sqrt{-ab}}\ln\left|\dfrac{\sqrt{a}x-\sqrt{-b}}{\sqrt{a}x+\sqrt{-b}}\right|+C & (b<0). \end{cases}$

23. $\displaystyle\int \frac{x}{ax^2+b}\mathrm{d}x = \frac{1}{2a}\ln|ax^2+b|+C.$

24. $\displaystyle\int \frac{x^2}{ax^2+b}\mathrm{d}x = \frac{x}{a} - \frac{b}{a}\int \frac{\mathrm{d}x}{ax^2+b}.$

25. $\displaystyle\int \frac{\mathrm{d}x}{x(ax^2+b)} = \frac{1}{2b}\ln\frac{x^2}{|ax^2+b|}+C.$

26. $\displaystyle\int \frac{\mathrm{d}x}{x^2(ax^2+b)} = -\frac{1}{bx} - \frac{a}{b}\int \frac{\mathrm{d}x}{ax^2+b}.$

27. $\displaystyle\int \frac{\mathrm{d}x}{x^3(ax^2+b)} = \frac{a}{2b^2}\ln\frac{|ax^2+b|}{x^2} - \frac{1}{2bx^2}+C.$

28. $\displaystyle\int \frac{\mathrm{d}x}{(ax^2+b)^2} = \frac{x}{2b(ax^2+b)} + \frac{1}{2b}\int \frac{\mathrm{d}x}{ax^2+b}.$

（五）含有 ax^2+bx+c （$a>0$）的积分

29. $\displaystyle\int \frac{\mathrm{d}x}{ax^2+bx+c} = \begin{cases} \dfrac{2}{\sqrt{4ac-b^2}}\arctan\dfrac{2ax+b}{\sqrt{4ac-b^2}} + C & (b^2<4ac), \\[4mm] \dfrac{1}{\sqrt{b^2-4ac}}\ln\left|\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| + C & (b^2>4ac). \end{cases}$

30. $\displaystyle\int \frac{x}{ax^2+bx+c}\mathrm{d}x = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{2a}\int \frac{\mathrm{d}x}{ax^2+bx+c}.$

（六）含有 $\sqrt{x^2+a^2}$ （$a>0$）的积分

31. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \operatorname{arsh}\frac{x}{a} + C_1 = \ln(x+\sqrt{x^2+a^2}) + C.$

32. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{(x^2+a^2)^3}} = \frac{x}{a^2\sqrt{x^2+a^2}} + C.$

33. $\displaystyle\int \frac{x}{\sqrt{x^2+a^2}}\mathrm{d}x = \sqrt{x^2+a^2} + C.$

34. $\displaystyle\int \frac{x}{\sqrt{(x^2+a^2)^3}}\mathrm{d}x = -\frac{1}{\sqrt{x^2+a^2}} + C.$

35. $\displaystyle\int \frac{x^2}{\sqrt{x^2+a^2}}\mathrm{d}x = \frac{x}{2}\sqrt{x^2+a^2} - \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2}) + C.$

36. $\displaystyle\int \frac{x^2}{\sqrt{(x^2+a^2)^3}}\mathrm{d}x = -\frac{x}{\sqrt{x^2+a^2}} + \ln(x+\sqrt{x^2+a^2}) + C.$

37. $\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{x^2+a^2}} = \frac{1}{a}\ln\frac{\sqrt{x^2+a^2}-a}{|x|} + C.$

38. $\displaystyle\int \frac{\mathrm{d}x}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2 x} + C.$

39. $\displaystyle\int \sqrt{x^2+a^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2}) + C.$

40. $\displaystyle\int \sqrt{(x^2+a^2)^3}\,\mathrm{d}x = \frac{x}{8}(2x^2+5a^2)\sqrt{x^2+a^2} + \frac{3}{8}a^4\ln(x+\sqrt{x^2+a^2}) + C.$

41. $\displaystyle\int x\sqrt{x^2+a^2}\,\mathrm{d}x = \frac{1}{3}\sqrt{(x^2+a^2)^3} + C.$

42. $\int x^2 \sqrt{x^2+a^2}\, dx = \dfrac{x}{8}\left(2x^2+a^2\right)\sqrt{x^2+a^2} - \dfrac{a^4}{8}\ln\left(x+\sqrt{x^2+a^2}\right)+C.$

43. $\int \dfrac{\sqrt{x^2+a^2}}{x}\, dx = \sqrt{x^2+a^2}+a\ln\dfrac{\sqrt{x^2+a^2}-a}{|x|}+C.$

44. $\int \dfrac{\sqrt{x^2+a^2}}{x^2}\, dx = -\dfrac{\sqrt{x^2+a^2}}{x}+\ln\left(x+\sqrt{x^2+a^2}\right)+C.$

（七）含有 $\sqrt{x^2-a^2}$ （$a>0$）的积分

45. $\int \dfrac{dx}{\sqrt{x^2-a^2}} = \dfrac{x}{|x|}\,\mathrm{arch}\,\dfrac{|x|}{a}+C_1 = \ln\left|x+\sqrt{x^2-a^2}\right|+C.$

46. $\int \dfrac{dx}{\sqrt{(x^2-a^2)^3}} = -\dfrac{x}{a^2\sqrt{x^2-a^2}}+C.$

47. $\int \dfrac{x}{\sqrt{x^2-a^2}}\, dx = \sqrt{x^2-a^2}+C.$

48. $\int \dfrac{x}{\sqrt{(x^2-a^2)^3}}\, dx = -\dfrac{1}{\sqrt{x^2-a^2}}+C.$

49. $\int \dfrac{x^2}{\sqrt{x^2-a^2}}\, dx = \dfrac{x}{2}\sqrt{x^2-a^2}+\dfrac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|+C.$

50. $\int \dfrac{x^2}{\sqrt{(x^2-a^2)^3}}\, dx = -\dfrac{x}{\sqrt{x^2-a^2}}+\ln\left|x+\sqrt{x^2-a^2}\right|+C.$

51. $\int \dfrac{dx}{x\sqrt{x^2-a^2}} = \dfrac{1}{a}\arccos\dfrac{a}{|x|}+C.$

52. $\int \dfrac{dx}{x^2\sqrt{x^2-a^2}} = \dfrac{\sqrt{x^2-a^2}}{a^2 x}+C.$

53. $\int \sqrt{x^2-a^2}\, dx = \dfrac{x}{2}\sqrt{x^2-a^2}-\dfrac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|+C.$

54. $\int \sqrt{(x^2-a^2)^3}\, dx = \dfrac{x}{8}\left(2x^2-5a^2\right)\sqrt{x^2-a^2}+\dfrac{3}{8}a^4\ln\left|x+\sqrt{x^2-a^2}\right|+C.$

55. $\int x\sqrt{x^2-a^2}\, dx = \dfrac{1}{3}\sqrt{(x^2-a^2)^3}+C.$

56. $\int x^2\sqrt{x^2-a^2}\, dx = \dfrac{x}{8}\left(2x^2-a^2\right)\sqrt{x^2-a^2}-\dfrac{a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right|+C.$

57. $\int \dfrac{\sqrt{x^2-a^2}}{x}\, dx = \sqrt{x^2-a^2}-a\arccos\dfrac{a}{|x|}+C.$

58. $\displaystyle \int \frac{\sqrt{x^2-a^2}}{x^2} dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln |x+\sqrt{x^2-a^2}| + C.$

（八）含有 $\sqrt{a^2-x^2}$ （$a>0$）的积分

59. $\displaystyle \int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C.$

60. $\displaystyle \int \frac{dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{a^2\sqrt{a^2-x^2}} + C.$

61. $\displaystyle \int \frac{x}{\sqrt{a^2-x^2}} dx = -\sqrt{a^2-x^2} + C.$

62. $\displaystyle \int \frac{x}{\sqrt{(a^2-x^2)^3}} dx = \frac{1}{\sqrt{a^2-x^2}} + C.$

63. $\displaystyle \int \frac{x^2}{\sqrt{a^2-x^2}} dx = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin \frac{x}{a} + C.$

64. $\displaystyle \int \frac{x^2}{\sqrt{(a^2-x^2)^3}} dx = \frac{x}{\sqrt{a^2-x^2}} - \arcsin \frac{x}{a} + C.$

65. $\displaystyle \int \frac{dx}{x\sqrt{a^2-x^2}} = \frac{1}{a}\ln \frac{a-\sqrt{a^2-x^2}}{|x|} + C.$

66. $\displaystyle \int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2 x} + C.$

67. $\displaystyle \int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin \frac{x}{a} + C.$

68. $\displaystyle \int \sqrt{(a^2-x^2)^3}\, dx = \frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2} + \frac{3}{8}a^4\arcsin \frac{x}{a} + C.$

69. $\displaystyle \int x\sqrt{a^2-x^2}\, dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3} + C.$

70. $\displaystyle \int x^2\sqrt{a^2-x^2}\, dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2} + \frac{a^4}{8}\arcsin \frac{x}{a} + C.$

71. $\displaystyle \int \frac{\sqrt{a^2-x^2}}{x} dx = \sqrt{a^2-x^2} + a\ln \frac{a-\sqrt{a^2-x^2}}{|x|} + C.$

72. $\displaystyle \int \frac{\sqrt{a^2-x^2}}{x^2} dx = -\frac{\sqrt{a^2-x^2}}{x} - \arcsin \frac{x}{a} + C.$

（九）含有 $\sqrt{\pm ax^2+bx+c}$ （$a>0$）的积分

73. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}|+C.$

74. $\displaystyle\int \sqrt{ax^2+bx+c}\,\mathrm{d}x = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c}\ +$

$$\frac{4ac-b^2}{8\sqrt{a^3}}\ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}|+C.$$

75. $\displaystyle\int \frac{x}{\sqrt{ax^2+bx+c}}\,\mathrm{d}x = \frac{1}{a}\sqrt{ax^2+bx+c}\ -$

$$\frac{b}{2\sqrt{a^3}}\ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}|+C.$$

76. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{c+bx-ax^2}} = \frac{1}{\sqrt{a}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}}+C.$

77. $\displaystyle\int \sqrt{c+bx-ax^2}\,\mathrm{d}x = \frac{2ax-b}{4a}\sqrt{c+bx-ax^2}\ +$

$$\frac{b^2+4ac}{8\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}}+C.$$

78. $\displaystyle\int \frac{x}{\sqrt{c+bx-ax^2}}\,\mathrm{d}x = -\frac{1}{a}\sqrt{c+bx-ax^2}+\frac{b}{2\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}}+C.$

（十）含有 $\sqrt{\pm\dfrac{x-a}{x-b}}$ 或 $\sqrt{(x-a)(b-x)}$ 的积分

79. $\displaystyle\int \sqrt{\frac{x-a}{x-b}}\,\mathrm{d}x = (x-b)\sqrt{\frac{x-a}{x-b}}+(b-a)\ln(\sqrt{|x-a|}+\sqrt{|x-b|})+C.$

80. $\displaystyle\int \sqrt{\frac{x-a}{b-x}}\,\mathrm{d}x = (x-b)\sqrt{\frac{x-a}{b-x}}+(b-a)\arcsin\sqrt{\frac{x-a}{b-a}}+C.$

81. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{(x-a)(b-x)}} = 2\arcsin\sqrt{\frac{x-a}{b-a}}+C$ （$a<b$）.

82. $\displaystyle\int \sqrt{(x-a)(b-x)}\,\mathrm{d}x = \frac{2x-a-b}{4}\sqrt{(x-a)(b-x)}\ +$

$$\frac{(b-a)^2}{4}\arcsin\sqrt{\frac{x-a}{b-a}}+C \ （a<b）.$$

（十一）含有三角函数的积分

83. $\displaystyle\int \sin x\,\mathrm{d}x = -\cos x+C.$

84. $\displaystyle\int \cos x\mathrm{d}x = \sin x + C.$

85. $\displaystyle\int \tan x\mathrm{d}x = -\ln|\cos x| + C.$

86. $\displaystyle\int \cot x\mathrm{d}x = \ln|\sin x| + C.$

87. $\displaystyle\int \sec x\mathrm{d}x = \ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + C = \ln|\sec x + \tan x| + C.$

88. $\displaystyle\int \csc x\mathrm{d}x = \ln\left|\tan\frac{x}{2}\right| + C = \ln|\csc x - \cot x| + C.$

89. $\displaystyle\int \sec^2 x\mathrm{d}x = \tan x + C.$

90. $\displaystyle\int \csc^2 x\mathrm{d}x = -\cot x + C.$

91. $\displaystyle\int \sec x\tan x\mathrm{d}x = \sec x + C.$

92. $\displaystyle\int \csc x\cot x\mathrm{d}x = -\csc x + C.$

93. $\displaystyle\int \sin^2 x\mathrm{d}x = \frac{x}{2} - \frac{1}{4}\sin 2x + C.$

94. $\displaystyle\int \cos^2 x\mathrm{d}x = \frac{x}{2} + \frac{1}{4}\sin 2x + C.$

95. $\displaystyle\int \sin^n x\mathrm{d}x = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\mathrm{d}x.$

96. $\displaystyle\int \cos^n x\mathrm{d}x = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\mathrm{d}x.$

97. $\displaystyle\int \frac{\mathrm{d}x}{\sin^n x} = -\frac{1}{n-1}\cdot\frac{\cos x}{\sin^{n-1}x} + \frac{n-2}{n-1}\int \frac{\mathrm{d}x}{\sin^{n-2}x}.$

98. $\displaystyle\int \frac{\mathrm{d}x}{\cos^n x} = \frac{1}{n-1}\cdot\frac{\sin x}{\cos^{n-1}x} + \frac{n-2}{n-1}\int \frac{\mathrm{d}x}{\cos^{n-2}x}.$

99. $\displaystyle\int \cos^m x\sin^n x\mathrm{d}x = \frac{1}{m+n}\cos^{m-1}x\sin^{n+1}x + \frac{m-1}{m+n}\int \cos^{m-2}x\sin^n x\mathrm{d}x$

$\displaystyle\qquad\qquad = -\frac{1}{m+n}\cos^{m+1}x\sin^{n-1}x + \frac{n-1}{m+n}\int \cos^m x\sin^{n-2}x\mathrm{d}x.$

100. $\displaystyle\int \sin ax\cos bx\mathrm{d}x = -\frac{1}{2(a+b)}\cos(a+b)x - \frac{1}{2(a-b)}\cos(a-b)x + C.$

101. $\displaystyle\int \sin ax\sin bx\mathrm{d}x = -\frac{1}{2(a+b)}\sin(a+b)x + \frac{1}{2(a-b)}\sin(a-b)x + C.$

102. $\displaystyle\int \cos ax\cos bx\mathrm{d}x = \frac{1}{2(a+b)}\sin (a+b)x + \frac{1}{2(a-b)}\sin (a-b)x + C.$

103. $\displaystyle\int \frac{\mathrm{d}x}{a+b\sin x} = \frac{2}{\sqrt{a^2-b^2}}\arctan \frac{a\tan \dfrac{x}{2}+b}{\sqrt{a^2-b^2}} + C \ (a^2>b^2).$

104. $\displaystyle\int \frac{\mathrm{d}x}{a+b\sin x} = \frac{1}{\sqrt{b^2-a^2}}\ln \left| \frac{a\tan \dfrac{x}{2}+b-\sqrt{b^2-a^2}}{a\tan \dfrac{x}{2}+b+\sqrt{b^2-a^2}} \right| + C \ (a^2<b^2).$

105. $\displaystyle\int \frac{\mathrm{d}x}{a+b\cos x} = \frac{2}{a+b}\sqrt{\frac{a+b}{a-b}}\arctan \left(\sqrt{\frac{a-b}{a+b}}\tan \frac{x}{2} \right) + C \ (a^2>b^2).$

106. $\displaystyle\int \frac{\mathrm{d}x}{a+b\cos x} = \frac{1}{a+b}\sqrt{\frac{a+b}{b-a}}\ln \left| \frac{\tan \dfrac{x}{2}+\sqrt{\dfrac{a+b}{b-a}}}{\tan \dfrac{x}{2}-\sqrt{\dfrac{a+b}{b-a}}} \right| + C \ (a^2<b^2).$

107. $\displaystyle\int \frac{\mathrm{d}x}{a^2\cos^2 x+b^2\sin^2 x} = \frac{1}{ab}\arctan \left(\frac{b}{a}\tan x \right) + C.$

108. $\displaystyle\int \frac{\mathrm{d}x}{a^2\cos^2 x-b^2\sin^2 x} = \frac{1}{2ab}\ln \left| \frac{b\tan x+a}{b\tan x-a} \right| + C.$

109. $\displaystyle\int x\sin ax\mathrm{d}x = \frac{1}{a^2}\sin ax - \frac{1}{a}x\cos ax + C.$

110. $\displaystyle\int x^2\sin ax\mathrm{d}x = -\frac{1}{a}x^2\cos ax + \frac{2}{a^2}x\sin ax + \frac{2}{a^3}\cos ax + C.$

111. $\displaystyle\int x\cos ax\mathrm{d}x = \frac{1}{a^2}\cos ax + \frac{1}{a}x\sin ax + C.$

112. $\displaystyle\int x^2\cos ax\mathrm{d}x = \frac{1}{a}x^2\sin ax + \frac{2}{a^2}x\cos ax - \frac{2}{a^3}\sin ax + C.$

（十二）含有反三角函数的积分（其中 $a>0$）

113. $\displaystyle\int \arcsin \frac{x}{a}\mathrm{d}x = x\arcsin \frac{x}{a} + \sqrt{a^2-x^2} + C.$

114. $\displaystyle\int x\arcsin \frac{x}{a}\mathrm{d}x = \left(\frac{x^2}{2}-\frac{a^2}{4} \right)\arcsin \frac{x}{a} + \frac{x}{4}\sqrt{a^2-x^2} + C.$

115. $\displaystyle\int x^2\arcsin \frac{x}{a}\mathrm{d}x = \frac{x^3}{3}\arcsin \frac{x}{a} + \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2} + C.$

116. $\displaystyle\int \arccos \frac{x}{a}\mathrm{d}x = x\arccos \frac{x}{a} - \sqrt{a^2-x^2} + C.$

117. $\displaystyle\int x\arccos\frac{x}{a}\mathrm{d}x=\left(\frac{x^2}{2}-\frac{a^2}{4}\right)\arccos\frac{x}{a}-\frac{x}{4}\sqrt{a^2-x^2}+C.$

118. $\displaystyle\int x^2\arccos\frac{x}{a}\mathrm{d}x=\frac{x^3}{3}\arccos\frac{x}{a}-\frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2}+C.$

119. $\displaystyle\int\arctan\frac{x}{a}\mathrm{d}x=x\arctan\frac{x}{a}-\frac{a}{2}\ln(a^2+x^2)+C.$

120. $\displaystyle\int x\arctan\frac{x}{a}\mathrm{d}x=\frac{1}{2}(a^2+x^2)\arctan\frac{x}{a}-\frac{a}{2}x+C.$

121. $\displaystyle\int x^2\arctan\frac{x}{a}\mathrm{d}x=\frac{x^3}{3}\arctan\frac{x}{a}-\frac{a}{6}x^2+\frac{a^3}{6}\ln(a^2+x^2)+C.$

（十三）含有指数函数的积分

122. $\displaystyle\int a^x\mathrm{d}x=\frac{1}{\ln a}a^x+C.$

123. $\displaystyle\int\mathrm{e}^{ax}\mathrm{d}x=\frac{1}{a}\mathrm{e}^{ax}+C.$

124. $\displaystyle\int x\mathrm{e}^{ax}\mathrm{d}x=\frac{1}{a^2}(ax-1)\mathrm{e}^{ax}+C.$

125. $\displaystyle\int x^n\mathrm{e}^{ax}\mathrm{d}x=\frac{1}{a}x^n\mathrm{e}^{ax}-\frac{n}{a}\int x^{n-1}\mathrm{e}^{ax}\mathrm{d}x.$

126. $\displaystyle\int xa^x\mathrm{d}x=\frac{x}{\ln a}a^x-\frac{1}{(\ln a)^2}a^x+C.$

127. $\displaystyle\int x^na^x\mathrm{d}x=\frac{1}{\ln a}x^na^x-\frac{n}{\ln a}\int x^{n-1}a^x\mathrm{d}x.$

128. $\displaystyle\int\mathrm{e}^{ax}\sin bx\mathrm{d}x=\frac{1}{a^2+b^2}\mathrm{e}^{ax}(a\sin bx-b\cos bx)+C.$

129. $\displaystyle\int\mathrm{e}^{ax}\cos bx\mathrm{d}x=\frac{1}{a^2+b^2}\mathrm{e}^{ax}(b\sin bx+a\cos bx)+C.$

130. $\displaystyle\int\mathrm{e}^{ax}\sin^n bx\mathrm{d}x=\frac{1}{a^2+b^2n^2}\mathrm{e}^{ax}\sin^{n-1}bx(a\sin bx-nb\cos bx)+$
$\displaystyle\qquad\frac{n(n-1)b^2}{a^2+b^2n^2}\int\mathrm{e}^{ax}\sin^{n-2}bx\mathrm{d}x.$

131. $\displaystyle\int\mathrm{e}^{ax}\cos^n bx\mathrm{d}x=\frac{1}{a^2+b^2n^2}\mathrm{e}^{ax}\cos^{n-1}bx(a\cos bx+nb\sin bx)+$
$\displaystyle\qquad\frac{n(n-1)b^2}{a^2+b^2n^2}\int\mathrm{e}^{ax}\cos^{n-2}bx\mathrm{d}x.$

（十四）含有对数函数的积分

132. $\displaystyle\int \ln x\mathrm{d}x = x\ln x - x + C.$

133. $\displaystyle\int \frac{\mathrm{d}x}{x\ln x} = \ln|\ln x| + C.$

134. $\displaystyle\int x^n\ln x\mathrm{d}x = \frac{1}{n+1}x^{n+1}\left(\ln x - \frac{1}{n+1}\right) + C.$

135. $\displaystyle\int (\ln x)^n\mathrm{d}x = x(\ln x)^n - n\int (\ln x)^{n-1}\mathrm{d}x.$

136. $\displaystyle\int x^m(\ln x)^n\mathrm{d}x = \frac{1}{m+1}x^{m+1}(\ln x)^n - \frac{n}{m+1}\int x^m(\ln x)^{n-1}\mathrm{d}x.$

（十五）含有双曲函数的积分

137. $\displaystyle\int \operatorname{sh} x\mathrm{d}x = \operatorname{ch} x + C.$

138. $\displaystyle\int \operatorname{ch} x\mathrm{d}x = \operatorname{sh} x + C.$

139. $\displaystyle\int \operatorname{th} x\mathrm{d}x = \ln \operatorname{ch} x + C.$

140. $\displaystyle\int \operatorname{sh}^2 x\mathrm{d}x = -\frac{x}{2} + \frac{1}{4}\operatorname{sh} 2x + C.$

141. $\displaystyle\int \operatorname{ch}^2 x\mathrm{d}x = \frac{x}{2} + \frac{1}{4}\operatorname{sh} 2x + C.$

（十六）定积分

142. $\displaystyle\int_{-\pi}^{\pi} \cos nx\mathrm{d}x = \int_{-\pi}^{\pi} \sin nx\mathrm{d}x = 0.$

143. $\displaystyle\int_{-\pi}^{\pi} \cos mx\sin nx\mathrm{d}x = 0.$

144. $\displaystyle\int_{-\pi}^{\pi} \cos mx\cos nx\mathrm{d}x = \begin{cases} 0, & m \neq n, \\ \pi, & m = n. \end{cases}$

145. $\displaystyle\int_{-\pi}^{\pi} \sin mx\sin nx\mathrm{d}x = \begin{cases} 0, & m \neq n, \\ \pi, & m = n. \end{cases}$

146. $\displaystyle\int_{0}^{\pi} \sin mx\sin nx\mathrm{d}x = \int_{0}^{\pi} \cos mx\cos nx\mathrm{d}x = \begin{cases} 0, & m \neq n, \\ \dfrac{\pi}{2}, & m = n. \end{cases}$

147. $I_n = \displaystyle\int_0^{\frac{\pi}{2}} \sin^n x \, \mathrm{d}x = \int_0^{\frac{\pi}{2}} \cos^n x \, \mathrm{d}x$,

$I_n = \dfrac{n-1}{n} I_{n-2}$

$$= \begin{cases} \dfrac{n-1}{n} \cdot \dfrac{n-3}{n-2} \cdot \cdots \cdot \dfrac{4}{5} \cdot \dfrac{2}{3} \ (n \text{ 为大于 1 的正奇数}), I_1 = 1, \\ \dfrac{n-1}{n} \cdot \dfrac{n-3}{n-2} \cdot \cdots \cdot \dfrac{3}{4} \cdot \dfrac{1}{2} \cdot \dfrac{\pi}{2} \ (n \text{ 为正偶数}), I_0 = \dfrac{\pi}{2}. \end{cases}$$