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Single-step triangular splitting method for a class of complex symmetric linear systems

XXX · LLL · GGG · KKK

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Abstract In this paper, we recast a broad class of complex symmetric linear systems into block two-by-two real equivalent formulation and establish a single-step triangular splitting (SSTS) method to the real form. Then, we investigate the convergence properties of this method and determine its optimal iteration parameter as well as corresponding optimal convergence factor. In addition, a accelerated variant of the SSTS (named after ASSTS) method is presented, which generally improves the convergence rate of the STSS method. Finally, some numerical experiments are given to validate the theoretical results and test the performance of the SSTS and the ASSTS methods.

Keywords Complex symmetry · Two-by-two · Symmetric positive definite · SSTS · Preconditioner · Convergence
Mathematics Subject Classifications (2010) 65F10 · 65F50

1 Introduction

We consider the iterative solution of large and sparse complex symmetric linear systems

$$Au = b \text{ with } A = W + iT \in \mathbb{C}^{n\times n} \text{ and } u, b \in \mathbb{C}^n, \tag{1.1}$$

XXX
E-mail: XXX@163.com
School of Mathematical Sciences, **** University, PR China

✉ LLL
E-mail: LLL@***.edu.cn
School of Mathematical Sciences, **** University, PR China

GGG
School of Mathematical Sciences, **** University, PR China

KKK
School of Mathematical Sciences, **** University, PR China

where $W, T \in \mathbb{R}^{n\times n}$ are symmetric matrices with at least one of them being positive definite. Without loss of generality, throughout the paper, we assume that W is symmetric positive definite. The linear systems arises frequently in many scientific and engineering applications. For instance, diffuse optimal tomography [1], molecular scattering [6], structural dynamics [4], FFT-based solution of certain time-dependent PDEs [3] and so on. More examples and additional practical background can be found in Ref.[2].

The paper is organized as follows. In Section 2, we give a program of establishing the SSTS method for solving the complex symmetric system of linear equations (1.1). In Section 3, we discuss the convergence of the SSTS method and give a way of choosing the optimal parameter as well as optimal convergence factor for this method. In section **??**, we introduce an efficient predictioner to accelerate the convergence rate of the SSTS method and discuss its convergence. In Section 4, some numerical examples are presented, and the numerical results verify the efficiency of these methods. Finally, the paper is concluded in Section **??**.

2 The SSTS method

In this section, we firstly propose the SSTS method to solve the block two-by-two linear systems (**??**). For the matrix $\mathcal{A}$, we use the same splitting form in [5]

3 Convergence discussion for the SSTS method

In this section, we turn to study the convergence properties of the SSTS method. Firstly, two useful lemmas are introduced to support our theories.

Based on the above lemmas, we have the following main results about the SSTS method for the block two-by-two linear systems (**??**).

4 Numerical experiments

In this section, we illustrate the feasibility and efficiency of the SSTS and the ASSTS methods for solving complex symmetric system of linear equations (1.1). Meantime, we compare their numerical results including iteration steps (denoted as IT), elapsed CPU time in seconds (denoted as CPU) and relative residual error (denoted as RES) with those of the MHSS, the SBTS, the SSTS and the ASSTS methods. The numerical experiment are performed in MATLAB[version 9.0.0.341360 (R2016a)] with machine precision 10^{-16}.

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