Detection and Estimation Theory (EE623A)
MATLAB Assignment 1
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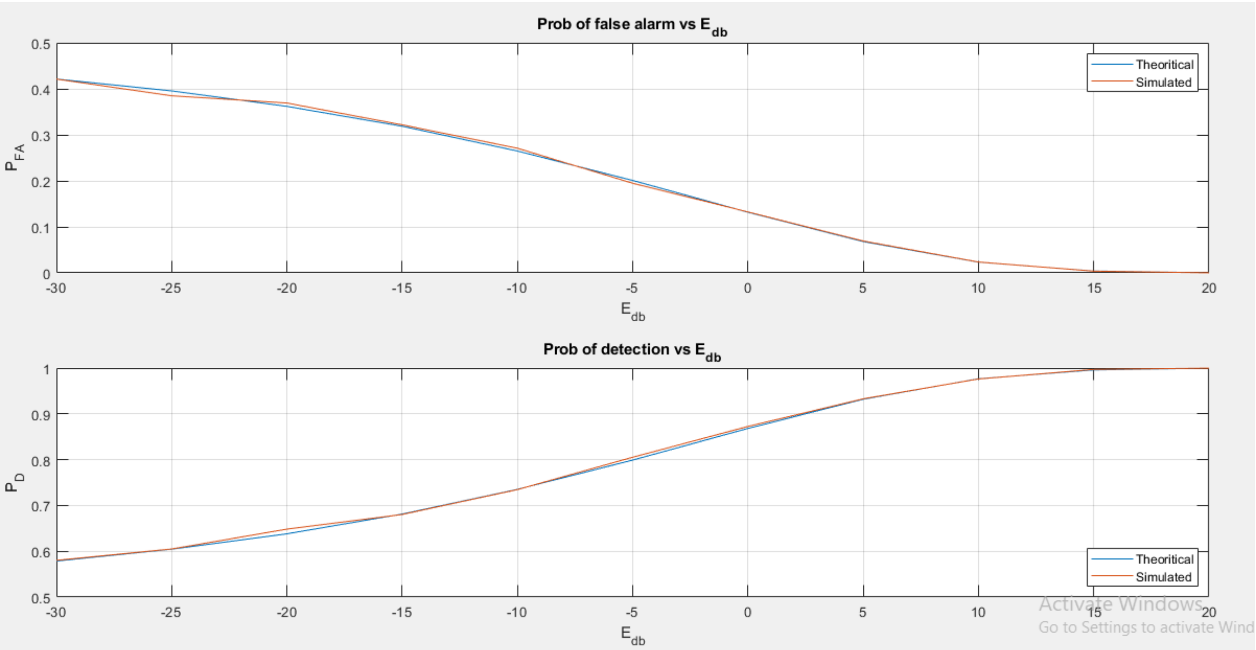
1.(a)

Code:

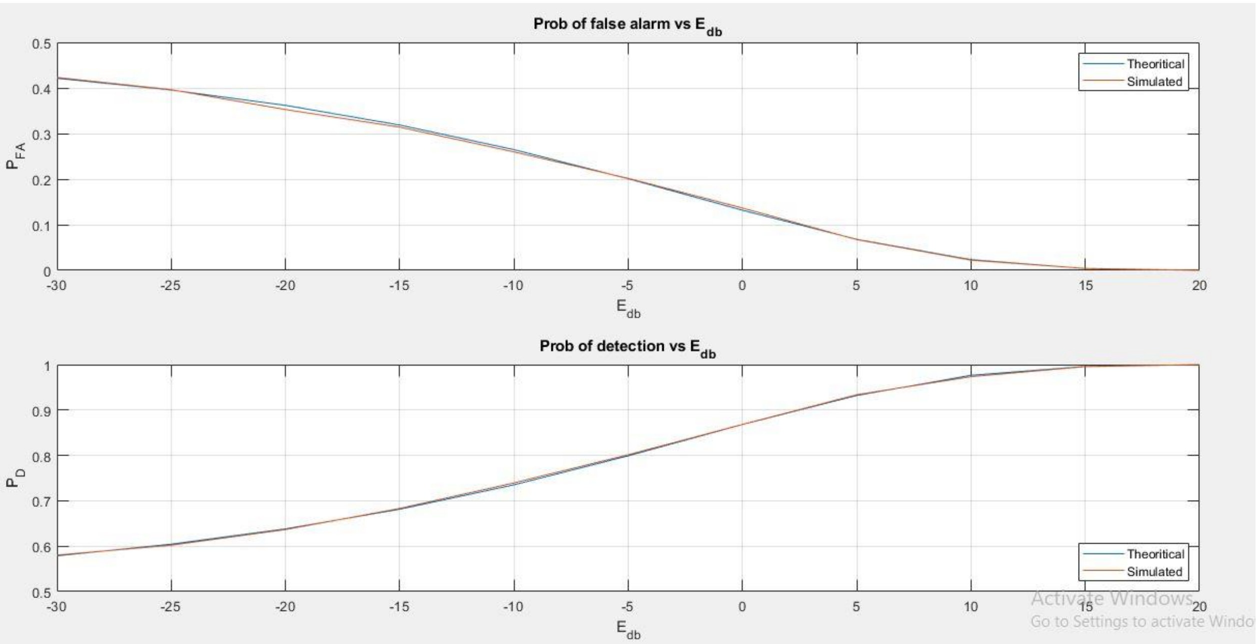
```
clear all;
clc;

edb = [-30:5:20];
e = power(10,edb/20); %converting edb to normal scale

neta = sqrt(e)/2; %η = (mo + m1)/2; given mo=0,m1=√E

Pfa_th = qfunc(sqrt(5*e)/2); %theoritical calculation of Pfa
Pd_th = qfunc(-sqrt(5*e)/2); %theoritical calculation of Pd

Pfa_sim=zeros(1,length(edb));
Pd_sim=zeros(1,length(edb));

for i=1:length(edb)
  possiblec_FA = 0;
  possiblec_D = 0;
  for j=1:10000    %10000 iterations per point
    %genrating 5 samples under H0 and checking sufficient statistic
    if mean(normrnd(0,1,[1,5])) >= neta(i)
      possiblec_FA=possiblec_FA+1;
    end
    %genrating 5 samples under H1 and checking sufficient statistic
    if mean(normrnd(sqrt(e(i)),1,[1,5])) >= neta(i)
      possiblec_D = possiblec_D+1;
    end
  end
  Pfa_sim(i)=possiblec_FA/10000; %iterations satisfied/total number of iterations
  Pd_sim(i)=possiblec_D/10000; %iterations satisfied/total number of iterations
end
%Plotting
tiledlayout(2,1)

ax1=nexttile;
plot(ax1,edb,Pfa_th)
hold on;
plot(ax1,edb,Pfa_sim)
title(ax1,'Prob of false alarm vs E_d_b')
xlabel(ax1,'E_d_b')
ylabel(ax1,'P_F_A')
legend(ax1,{'Theoritical','Simulated'},Location="northeast")
grid on;
```

```
ax2=nexttile;
plot(ax2,edb,Pd_th)
hold on;
plot(ax2,edb,Pd_sim)
title(ax2,'Prob of detection vs E_d_b')
xlabel(ax2,'E_d_b')
ylabel(ax2,'P_D')
legend(ax2,{'Theoritical','Simulated'},Location="southeast")
grid on;
```

Plot/Figure shown below when program has been executed once: -



Plot/Figure shown below when program has been executed again: -



Theory and Observations:

$$S = \frac{1}{N}\sum_{k=1}^{N} Y_k \gtrless_{H_0}^{H_1} \eta$$

$$\eta = \frac{m_0 + m_1}{2} = \frac{\sqrt{E}}{2}$$

This implies that τ has been considered 1 in $\eta = \frac{\sqrt{E}}{2} + \frac{\sigma^2 \ln(\tau)}{N\sqrt{E}}$

We have $d = \frac{\sqrt{NE}}{\sigma} = \sqrt{5E}$ here

$$P_F = Q\left(\frac{d}{2}\right) = Q\left(\frac{\sqrt{5E}}{2}\right)$$

$$P_D = 1 - Q\left(\frac{d}{2}\right) = Q\left(-\frac{d}{2}\right) = Q\left(-\frac{\sqrt{5E}}{2}\right)$$

P_F and P_D have been plotted against E and we can ***observe*** that:

- E is increasing, then P_F is decreasing and it finally goes to 0.
- E is increasing, then P_D is increasing and it finally goes to 1.
- P_D=1- P_F. Implies as P_F decreases, corresponding P_D decreases.
- If we were to plot ROC, the (P_F,P_D) point moves North-West (desirable) as E increases. Therefore, the highest value of E is preferrable over others.
- Simulated plots, generated using 10000 iterations per point, seem to closely follow theoretical plots.

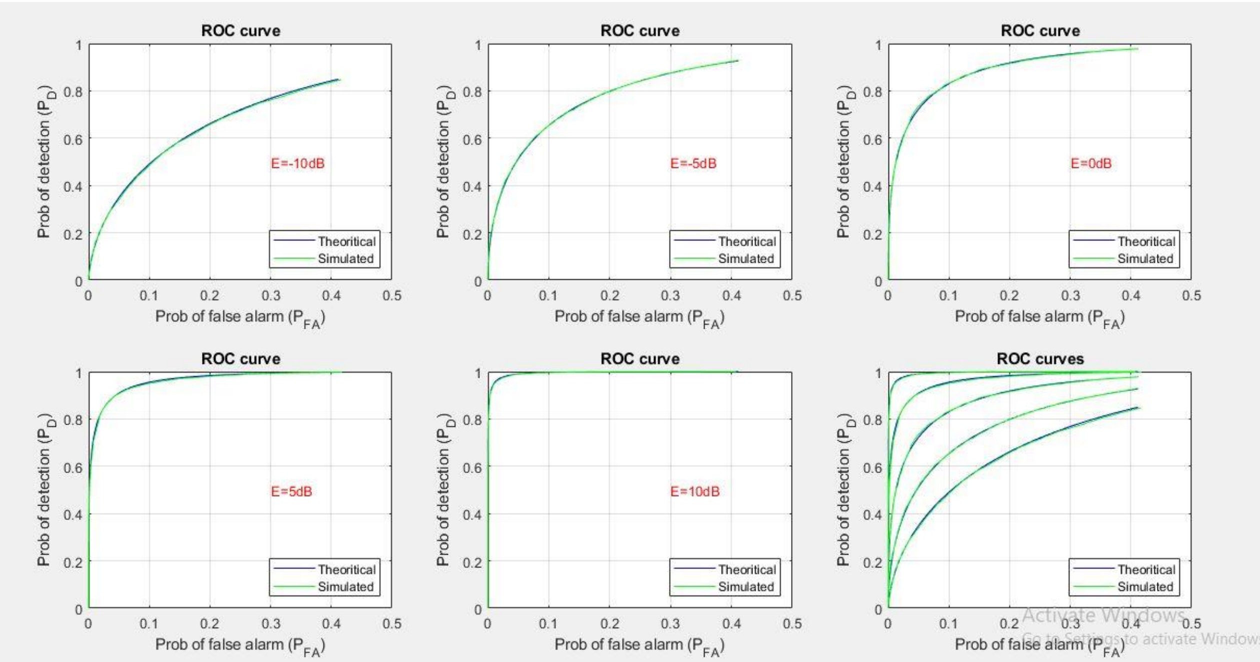
1.(b)

Code:

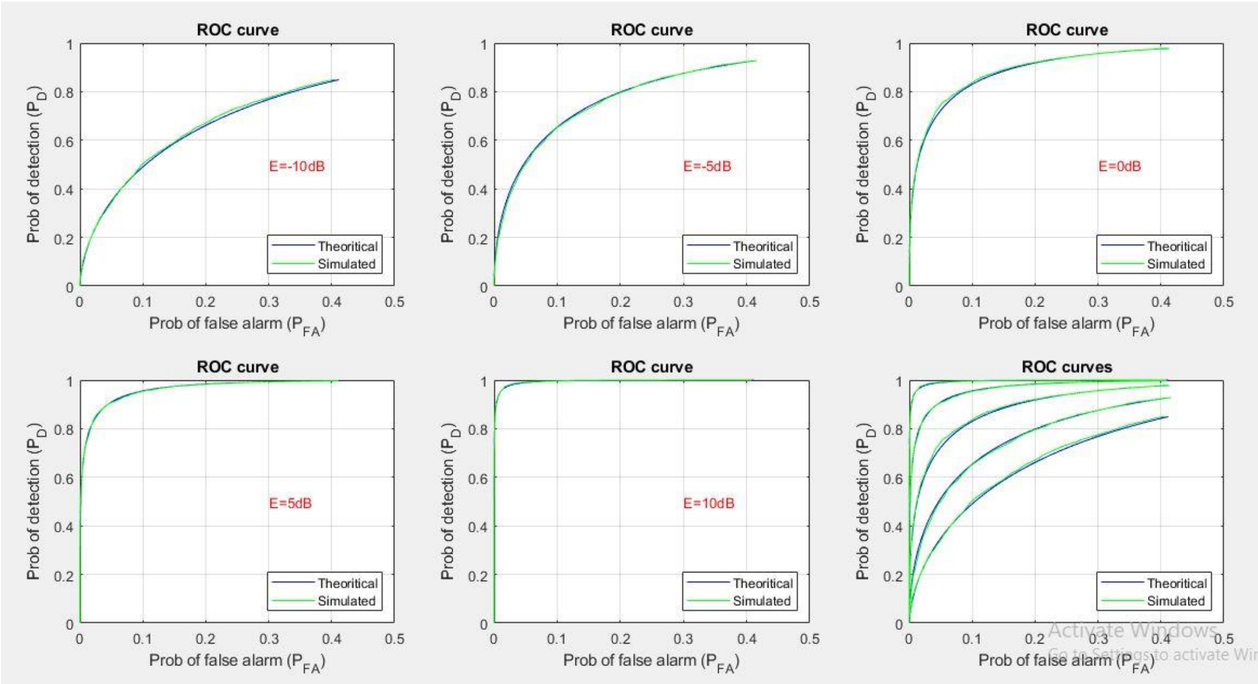
```
clear all;
clc;

edb = [-10:5:10];
e = power(10,edb/20);%converting edb to normal scale
neta = [0.1:0.01:20];%varying η
for i=1:length(edb)
   Pfa_th = zeros(1,length(neta));
   Pd_th = zeros(1,length(neta));

   Pfa_sim = zeros(1,length(neta));
   Pd_sim = zeros(1,length(neta));

   %generating 10000 iterations of 5 samples under H0 and H1 respectively
   h0 = normrnd(0,1,[10000,5]);
   h1 = normrnd(sqrt(e(i)),1,[10000,5]);
   %calculating sufficient statistic
   s0 = mean(h0,2);
   s1 = mean(h1,2);

   for j=1:length(neta)
      Pfa_th(j) = qfunc(neta(j)*sqrt(5)); %theoritical calculation of Pfa
      Pd_th(j) = qfunc((neta(j)-sqrt(e(i)))*sqrt(5)); %theoritical calculation of Pd

      possiblec_FA = 0;
      possiblec_D = 0;
      for k=1:10000   %checking sufficient statistic
         if s0(k) >= neta(j)
            possiblec_FA=possiblec_FA+1;
         end
         if s1(k) >= neta(j)
            possiblec_D = possiblec_D+1;
         end
      end
      Pfa_sim(j)=possiblec_FA/10000; %iterations satisfied/total number of iterations
      Pd_sim(j)=possiblec_D/10000; %iterations satisfied/total number of iterations

   end
   %Plotting
   subplot(2,3,i)
   plot(Pfa_th,Pd_th,'b',Pfa_sim,Pd_sim,'g')
   str = strcat('E=',num2str(edb(i)),'dB');
   text(0.3,0.5,str,'Color','red')
   title('ROC curve')
   xlabel('Prob of false alarm (P_F_A)')
   ylabel('Prob of detection (P_D)')
   legend({'Theoritical','Simulated'},Location="southeast")
   grid on;
```

```
  subplot(2,3,6)
  plot(Pfa_th,Pd_th,'b',Pfa_sim,Pd_sim,'g')
  title('ROC curves')
  xlabel('Prob of false alarm (P_F_A)')
  ylabel('Prob of detection (P_D)')
  legend({'Theoritical','Simulated'},Location="southeast")
  hold on;
  grid on;
end
```

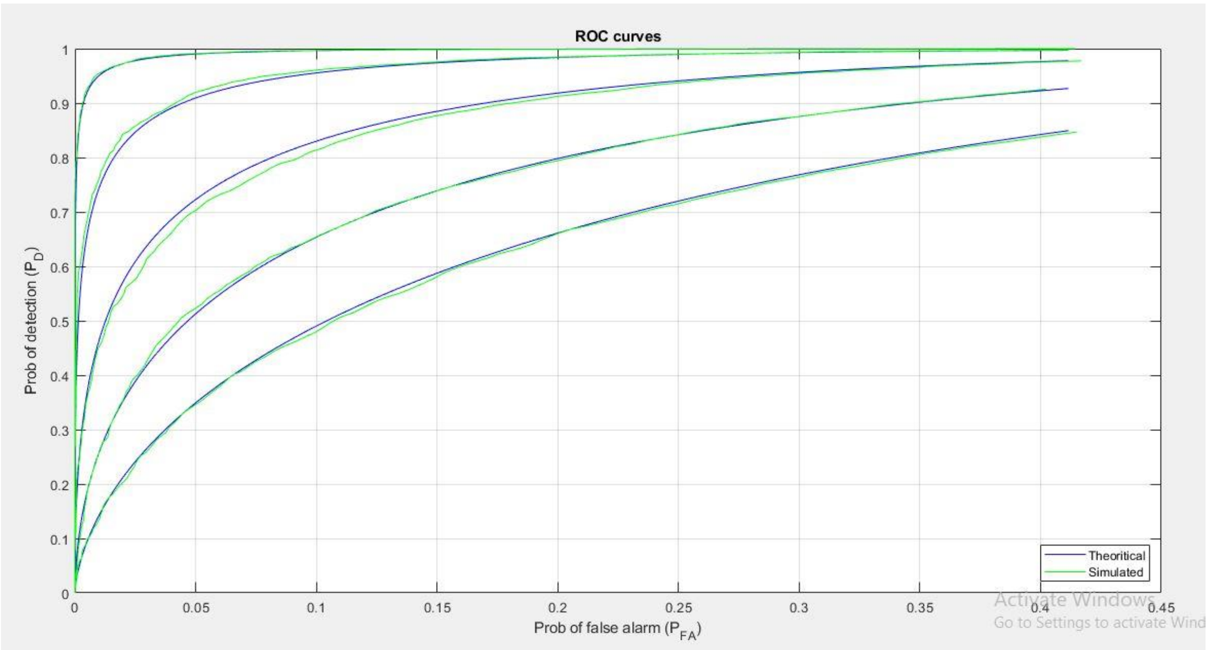
Plot/Figure shown below when program has been executed once: -



Plot/Figure shown below when program has been executed again: -



Enlarged view of ROCs together plot shown below-



Theory and Observations:

$$P_D = \int_{\eta}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2/_N}} exp\left\{-\frac{\left(s-\sqrt{E}\right)^2}{2\sigma^2/_N}\right\} ds$$

$$=> P_D = \int_{\eta}^{\infty} \frac{1}{\sqrt{2\pi/_5}} exp\left\{-\frac{\left(s-\sqrt{E}\right)^2}{2/_5}\right\} ds$$

Let $\sqrt{5}\left(s-\sqrt{E}\right) = t$, we get-

$$=> P_D = \int_{\sqrt{5}(\eta-\sqrt{E})}^{\infty} \frac{1}{\sqrt{2\pi}} exp\left\{-\frac{t^2}{2}\right\} dt$$

$$=> P_D = Q\left(\sqrt{5}\left(\eta-\sqrt{E}\right)\right)$$

$$P_F = \int_{\eta}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2/_N}} exp\left\{-\frac{s^2}{2\sigma^2/_N}\right\} ds$$

$$P_F = \int_{\eta}^{\infty} \frac{1}{\sqrt{2\pi/_5}} exp\left\{-\frac{s^2}{2/_5}\right\} ds$$

Let $s\sqrt{5} = t$, we get-

$$=> P_F = \int_{\eta\sqrt{5}}^{\infty} \frac{1}{\sqrt{2\pi}} exp\left\{-\frac{t^2}{2}\right\} dt$$

$$=> P_F = Q\left(\eta\sqrt{5}\right)$$

ROC have been plotted for different E and we can ***observe*** that:

- For a given E, as η increases, both P_F and P_D decrease.
- $P_D = Q\left(-\sqrt{5E} + Q^{-1}(P_F)\right)$.
- As E increases, the ROC curve becomes more concave towards North-West.
- Therefore, the ROC with the highest E is preferrable as it gives better performance over others. And in this ROC curve, optimum η must be chosen for best (P_F,P_D) pair.
- Simulated ROCs, generated using 10000 iterations per point, seem to closely follow theoretical ROCs.