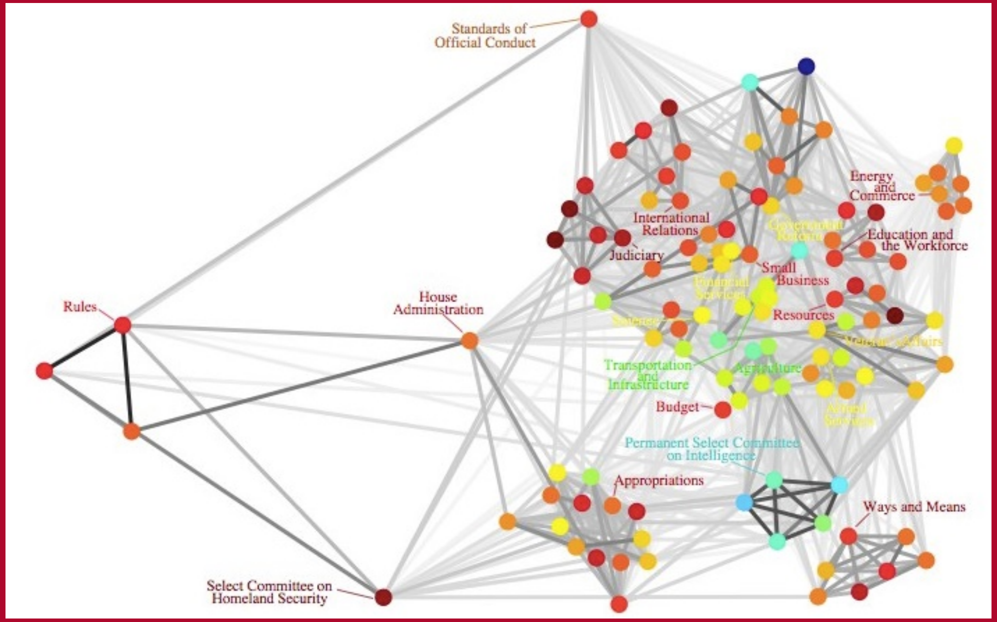
Automatic Control Theory

Chapter 3



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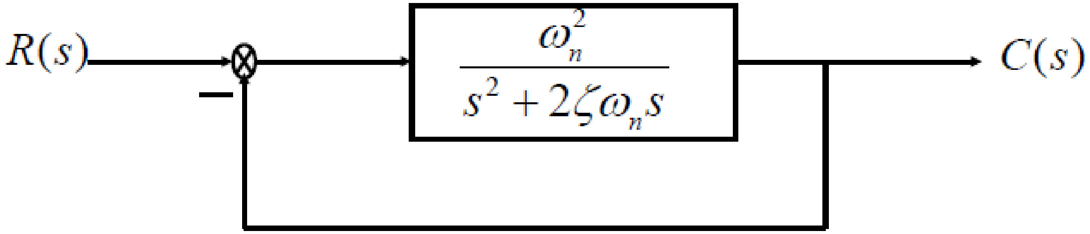
Main contents

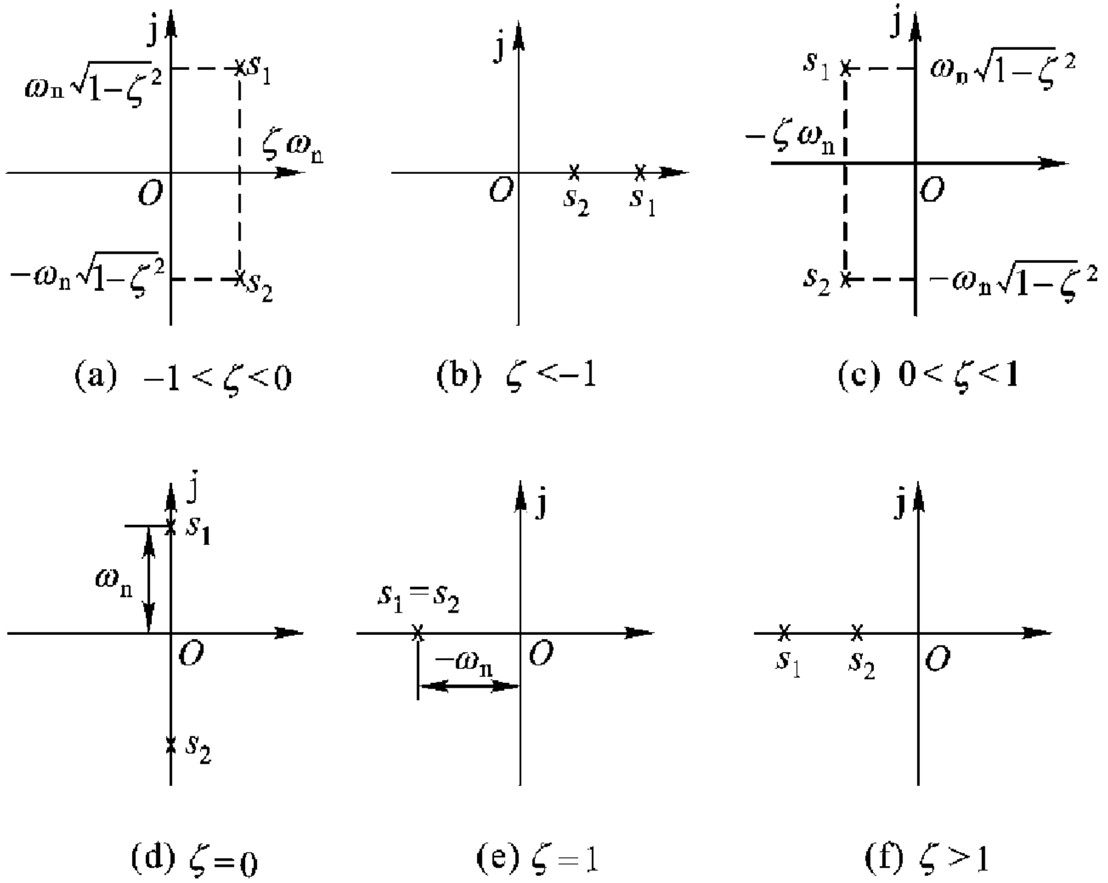
1. Typical test signals for the time response of control systems.
2. The unit-step response and time-domain specifications.
3. Time response of first-order and second-order systems.
4. Improvement performance of second systems.
5. Condition for a feedback system to be stable
6. Routh-Hurwitz criterion
7. The steady-state error of feedback control system.

The performance of feedback control systems

Review



$$T(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

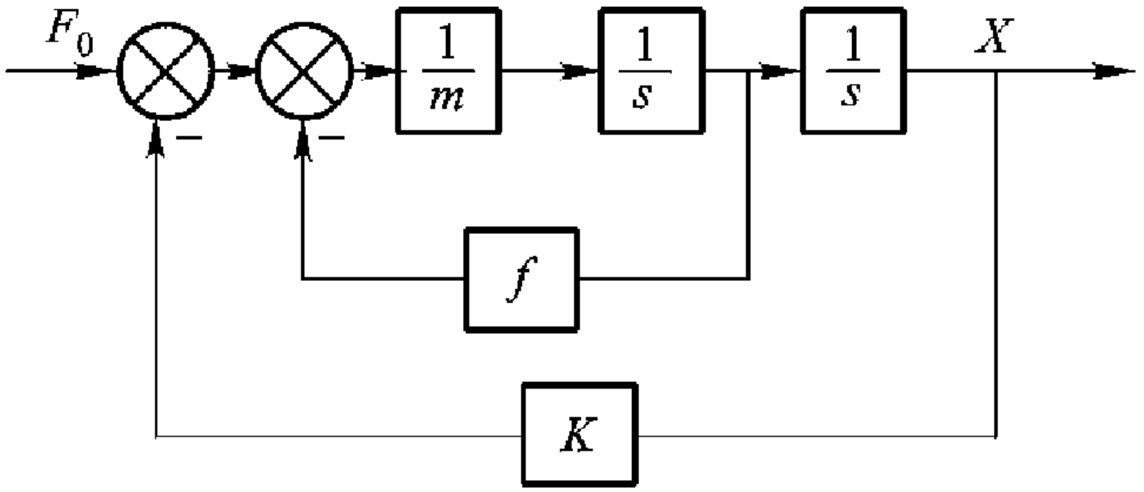


what is next

Improvement performance of second systems

Improvement performance of a second system

Example



m : mass

K : elasticity coefficient

f : friction ratio

Try to get best damping ratio

Improvement performance of a second system

Solution

$$\Phi(s)=\frac{X(s)}{F_0(s)}=\frac{1}{ms^2+fs+K}=\frac{\frac{1}{m}}{s^2+\frac{f}{m}s+\frac{K}{m}}$$

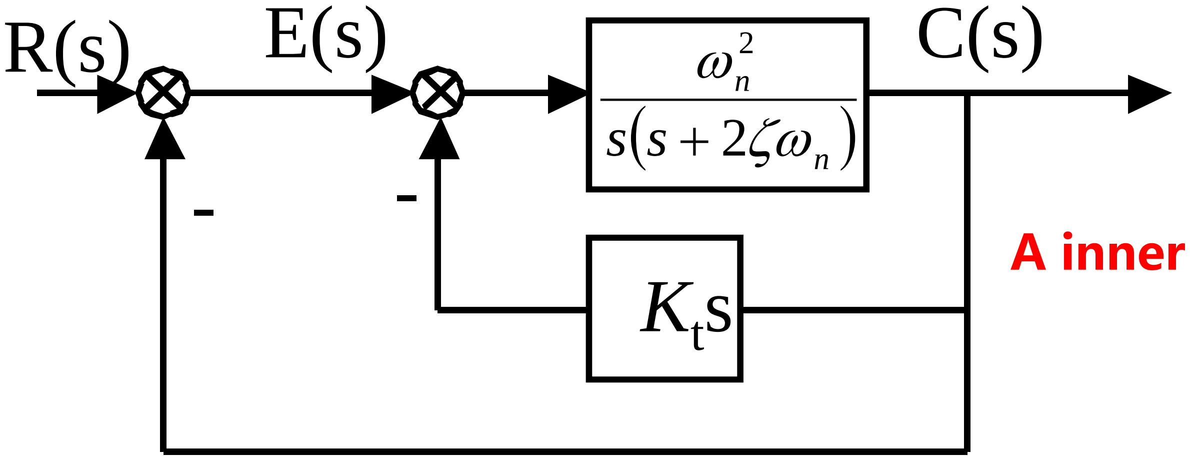
$$\omega_n=\sqrt{K/m}\qquad \zeta=\frac{f}{2\sqrt{Km}}$$

$$\frac{f}{2\sqrt{Km}}=0.707\qquad f=1.414\sqrt{Km}$$

Improvement performance of a second system

(1). Velocity feedback control of output

Open loop gain is reduced



A inner feedback is introduced

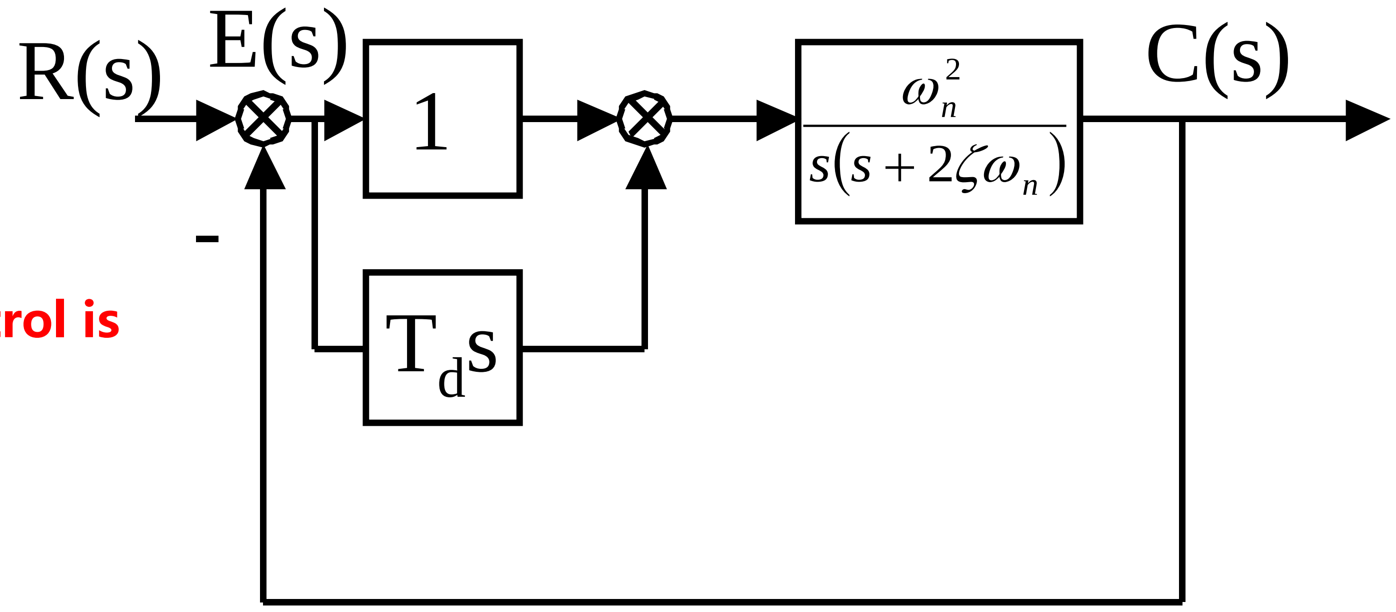
$$\Phi(s)=\frac{C(s)}{R(s)}=\frac{\omega_n^2}{s^2+2(\zeta+K_t\omega_n/2)\omega_n s+\omega_n^2}=\frac{\omega_n^2}{s^2+2\zeta_t\omega_n s+\omega_n^2}$$

where

$$\zeta_t=\zeta+\frac{1}{2}K_t\omega_n$$

Improvement performance of a second system

(2).The proportion-differential control of error signal



A new zero is generated

anticipatory control is introduced

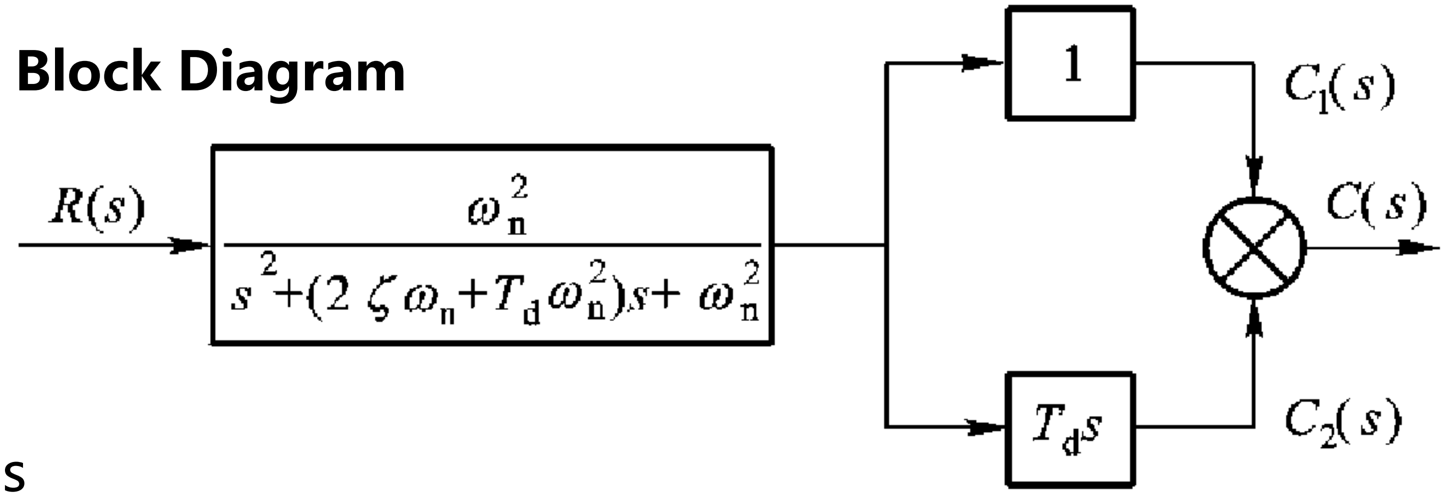
$$\Phi(s)=\frac{C(s)}{R(s)}=\frac{(T_d s+1)\omega_n^2}{s^2+2\zeta\omega_n s+(T_d s+1)\omega_n^2}=\frac{(T_d s+1)\omega_n^2}{s^2+2\zeta_d\omega_n s+\omega_n^2}$$

where

$$\zeta_d=\zeta+\frac{1}{2}T_d\omega_n$$

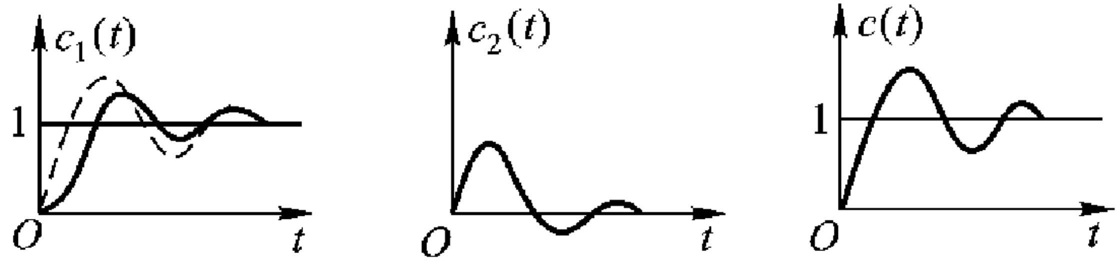
(2).The proportion-differential control of error signal

Equivalent Block Diagram



T_d increases

$$\zeta_d = \zeta + \frac{1}{2} T_d \omega_n$$

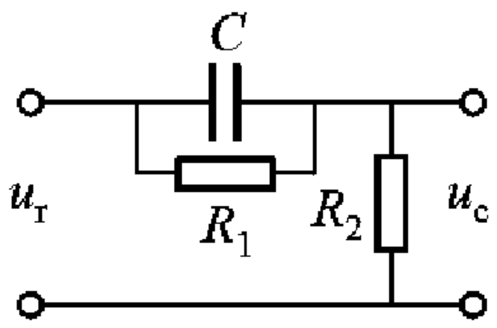


(a) (b) (c)

Improvement performance of a second system

(2).The proportion-differential control of error signal

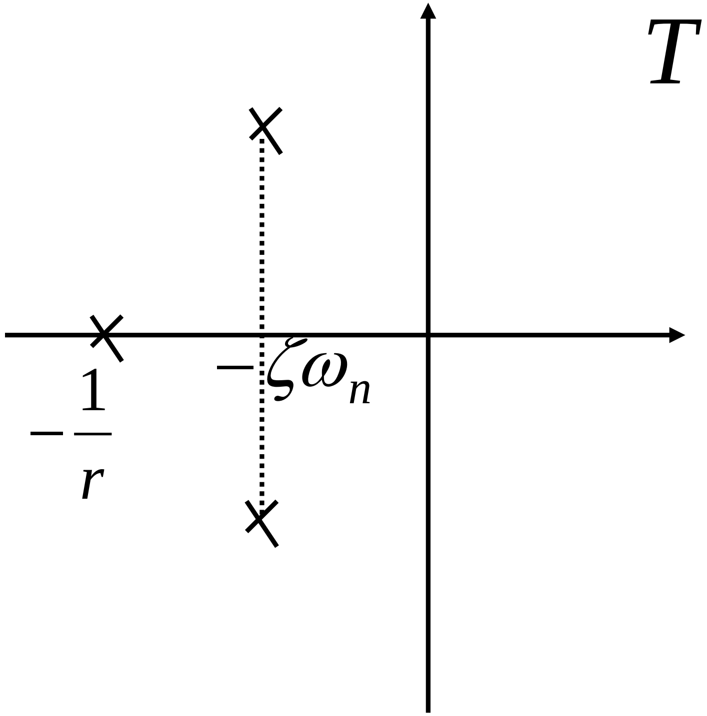
Implemented by RC circuit



$$\frac{U_c(s)}{U_r(s)} = \frac{\alpha(T_d s + 1)}{\alpha T_d s + 1} \qquad \alpha = \frac{R_2}{R_1 + R_2} < 1, \ T_d = R_1 C$$

$$\frac{U_c(s)}{U_r(s)} \approx \alpha(T_d s + 1) \qquad \text{for} \quad \alpha \ll 1 \quad \alpha T_d \ll T_d$$

Effects of third pole on the second-order system response



$$T(s) = \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(rs+1)}$$

$$\left|\frac{1}{r}\right| \geq 10\left|\zeta\omega_n\right|$$ r相较很远的时候 影响很小 可忽略 退化为2阶系统

$$s_{1,2} = -\zeta\omega_n \pm j\sqrt{1-\zeta^2}\ \omega_n$$

The response of a third-order system can be approximated by the second-order system !

More details in Chapter 4 The root locus method !

Improvement performance of a second system

核心

- **Poles distribution for different Damping ratio**

续

- **Condition for a feedback system to be stable**