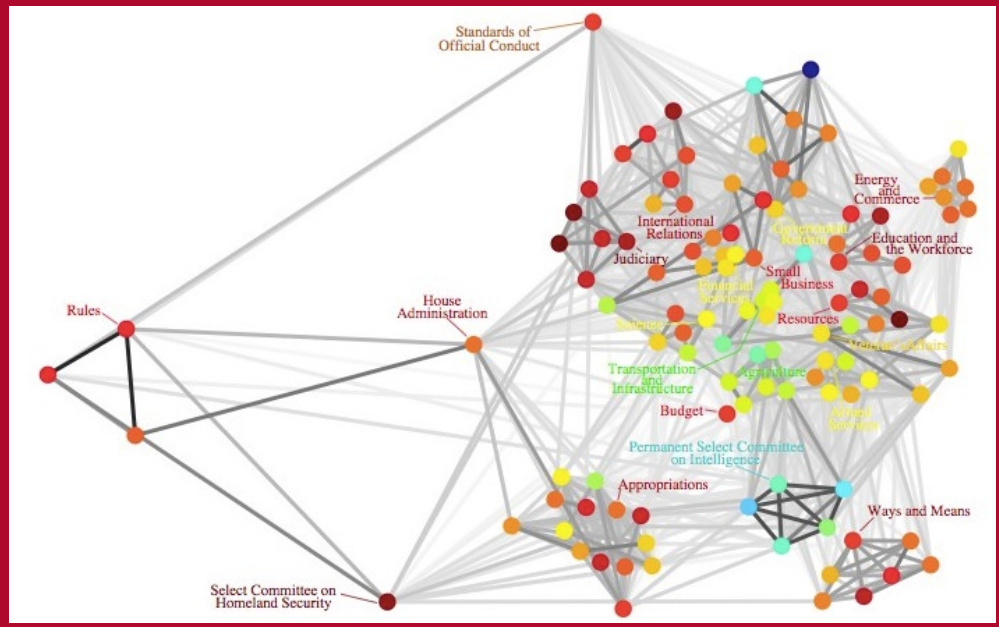
# Automatic Control Theory

## Chapter 3

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# The performance of feedback control systems

## Main contents

1.Typical test signals for the time response of control systems.

2.The unit-step response and time-domain specifications.

3.Time response of first-order and second-order systems.

4. Improvement performance of  second systems.

5. Condition for a feedback system to be stable

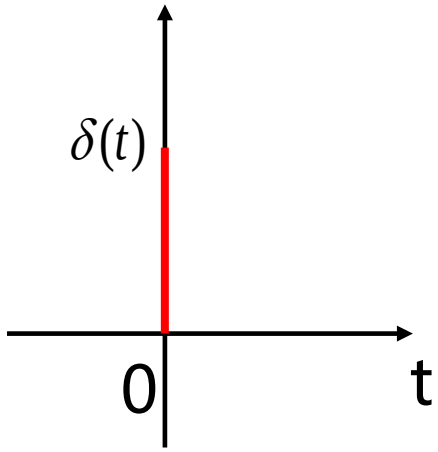
6. Routh-Hurwitz criterion

7.  The steady-state error of feedback control system.

# Typical test signals for the time response of control systems.

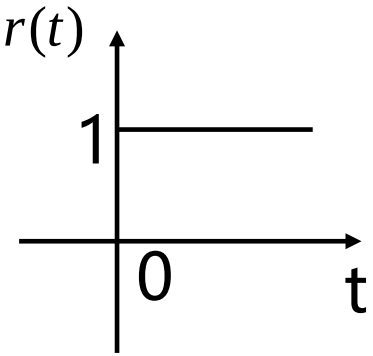
## 1. Unit impulse function

$$\delta(t)=\begin{cases}\infty & t=0\\ 0 & t\neq 0\end{cases}\qquad \int_{-\infty}^{+\infty}\delta(t)dt=1\qquad L\{\delta(t)\}=1$$




## 2. Unit step function

$$r(t)=1(t)=\begin{cases}1 & t\geq 0\\ 0 & t<0\end{cases}\qquad L\{1(t)\}=\frac{1}{s}$$

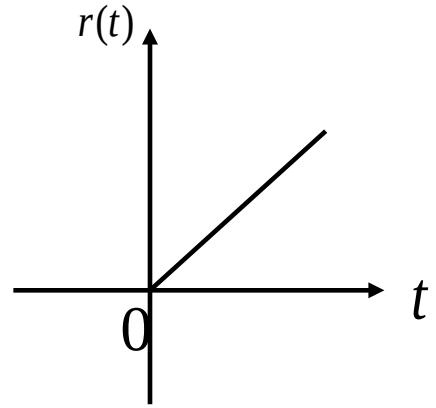


# Typical test signals for the time response of control systems.

## 3. Unit ramp function

$$r(t) = t \cdot 1(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

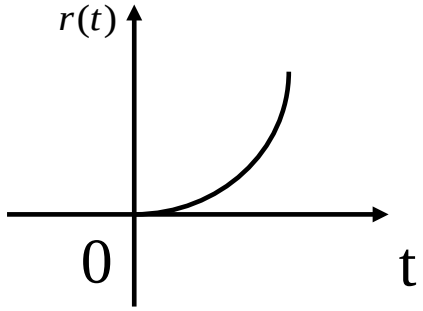
$$L\{t \cdot 1(t)\} = \frac{1}{s^2}$$




## 4. Unit parabolic function

$$r(t) = \frac{1}{2}t^2 \cdot 1(t) = \begin{cases} \frac{1}{2}t^2 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{\frac{1}{2}t^2 \cdot 1(t)\} = \frac{1}{s^3}$$



# The unit-step response and time-domain specifications

## Significances

Transition state (过渡态)：初始到接近最终状态的响应过程

Steady-state（稳态）： t 趋于无穷时的输出状态

**Unit-step response**:

The response of a control system when the input is a unit-step function.

跟踪与复现阶跃作用对于系统来说是较为严格的工作条件

# The unit-step response and time-domain specifications

## Time-domain specifications

1. 超调量（Percent overshoot）$\sigma\%$:

指响应超出稳态值的最大偏离量与稳态值之比。

$$\sigma\% = \frac{y_{\max} - y_{ss}}{y_{ss}} \times 100\%$$

# The unit-step response and time-domain specifications

2. 上升时间（rise time）$t_r$：指单位阶跃响应曲线，从稳态值得10%上升到 90%所需要的时间（也有指从零上升到稳态之所需要的时间）。

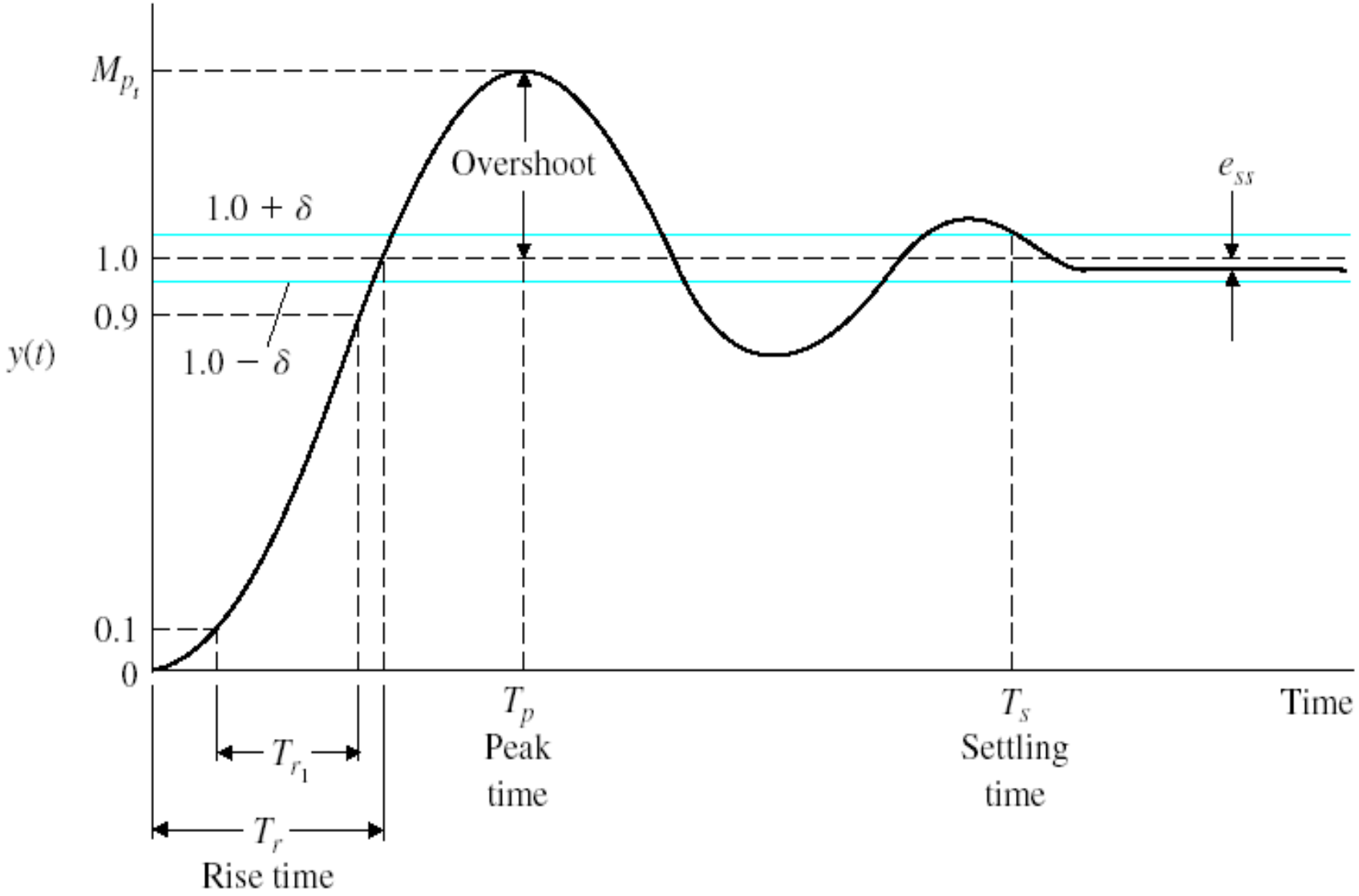
3. 调节时间（settling time）$t_s$：在单位阶跃响应曲线的稳态值附近，取$\pm 5\%$（或$\pm\ 2\%$）作为误差带，响应曲线达到并不在超出该误差带的最小时间。

4. 延迟时间(delay time)$t_d$：指响应曲线第一次达到其终值一半所需的时间。

5. 峰值时间(Peak time) $t_p$：指响应超过其终值到达第一个峰值所需的时间。

6. 稳态误差(steady-state error）$e_{ss}$:期望值与稳态值之差，　$e_{ss}=\lim_{s\to 0} sE(s)$

# The unit-step response and time-domain specifications



# Time response of a first-order system

Assume the transfer function of a first-order system is

$$T(s) = \frac{1}{Ts+1}$$

The unit step response for this system

$$c(t) = L^{-1}[\frac{1}{Ts+1} \cdot \frac{1}{s}] = 1 - e^{-1/T\, t}$$

特殊时间!

when

$t = T$ $\quad c(t) = 0.632$

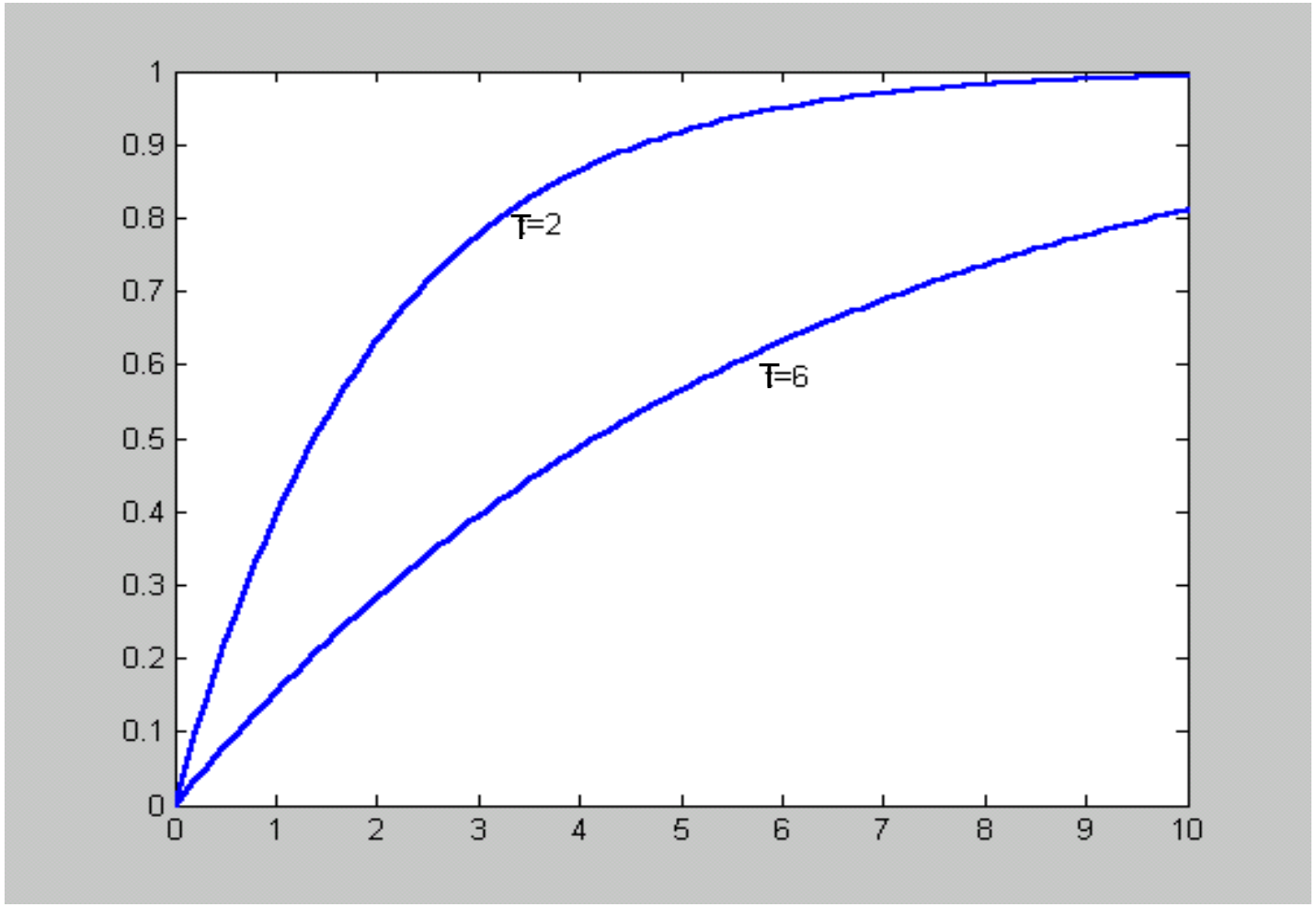
$t = 3T$ $\quad c(t) = 0.95$ for 5% steady-state error

$t = 4T$ $\quad c(t) = 0.982$ for 2% steady-state error

考试的时候要标明5% 还是2%
才能写是3T 4T

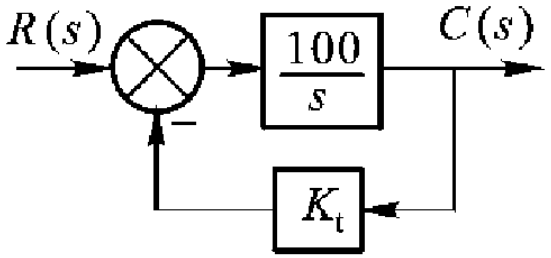
# Time response of a first-order system

Response due to unit step signal



# Time response of a first-order system

## Example




Try to get:

- Time response of a first-order system
- Settling time $t_s$
- The feedback gain $K_t$ if $t_s$ is less than 0.1 s

# Time response of a first-order system

Solution

$$\Phi(s)=\frac{C(s)}{R(s)}=\frac{\frac{100}{s}}{1+\frac{100}{s}\times K_t}=\frac{\frac{1}{K_t}}{\frac{1}{100K_t}\cdot s+1} \qquad T=\frac{1}{100K_t}\text{s}$$

$$h(t)=\frac{1}{K_t}(1-\mathrm{e}^{-\frac{1}{T}t})=\frac{1}{K_t}(1-\mathrm{e}^{-100K_t\cdot t})$$

$$t_s=3T=\frac{0.03}{K_t}(\text{s})$$

$$\frac{0.03}{K_t}\leqslant 0.1$$

$$K_t\geqslant 0.3$$

# The performance of feedback control systems

## 核心

- **Time-domain specifications**
- **Time response of a first-order system**

## 续

- **Time response of a second-order system**