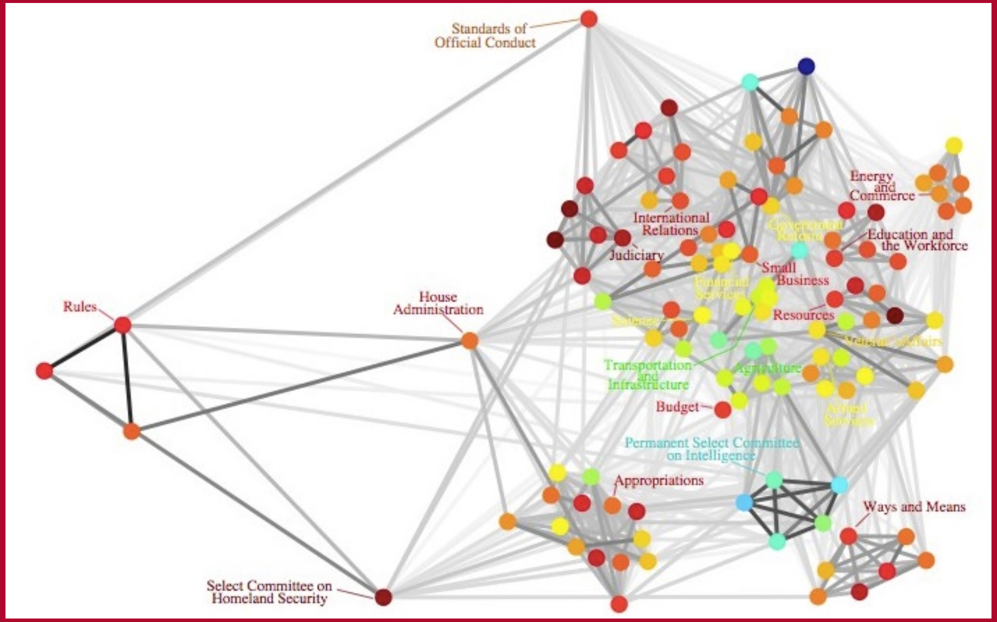
Automatic Control Theory

Chapter 2



Fan zichuan
School of Computer and Information Science
Southwest University

CH2: Mathematical Models of Systems

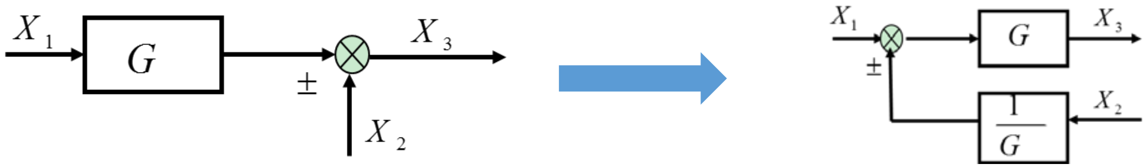
- Differential Equations of Physical Systems.
- The Transfer function of Linear Systems.

  (The Laplace Transform and Inverse Transform)
- Block Diagram.
- Block Diagram Reduction

Review

Block Diagram reduction by Transformations

Moving a summing point ahead a block



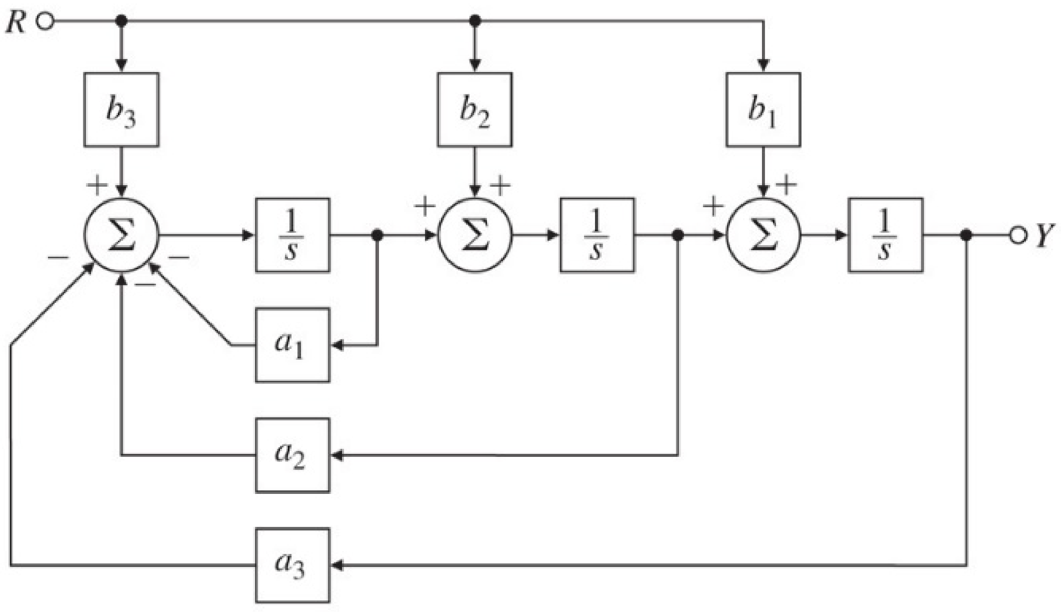
Moving a pickoff point behind a block

what is next

Block Diagram Reduction (Mason’s gain formula) !

Mason’ s signal-flow gain formula

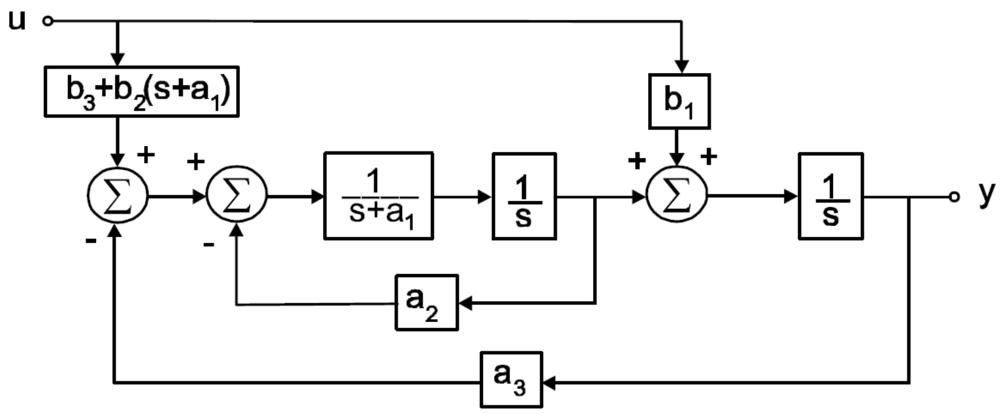
Homework

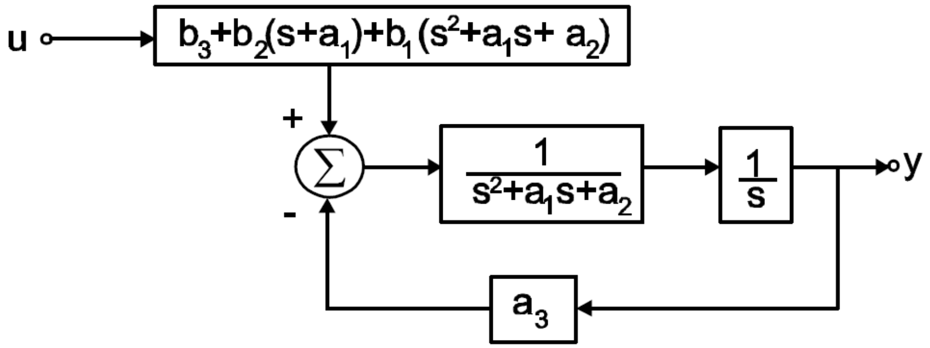


(c)

Complicated !

Solution





$$\frac{Y(s)}{R(s)} = \frac{b_1 s^2 + (a_1 b_1 + b_2)s + a_1 b_2 + a_2 b_1 + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

Mason’ s signal-flow gain formula

$$T = \frac{\sum_k P_k \Delta_k}{\Delta}$$

P_k : k[th] forward path gain

$\triangle$: determinant of the path

Δ_k : **cofactor** of the path P_k

$$\Delta = 1 - \sum L_n + \sum L_m L_q - \sum L_r L_s L_t + \cdots$$

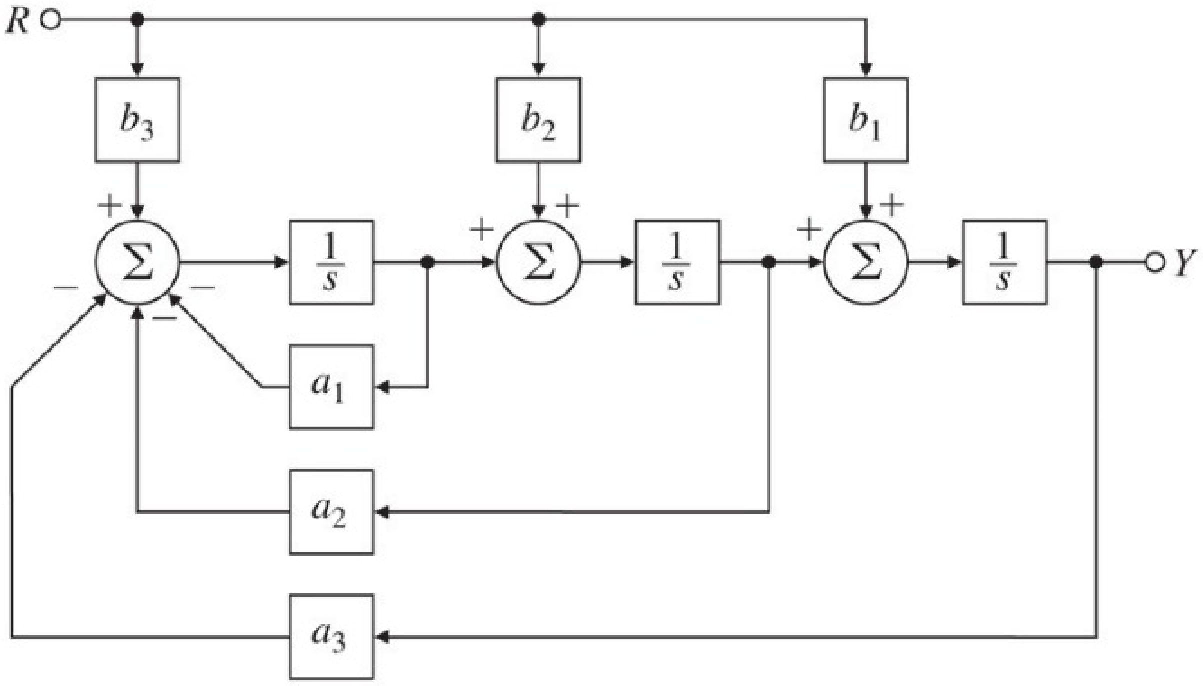
$\sum L_n$ Sum of all different **loop** gains

$\sum L_m L_q$ Sum of the gain products of all combinations of **two** non-touching **loops**

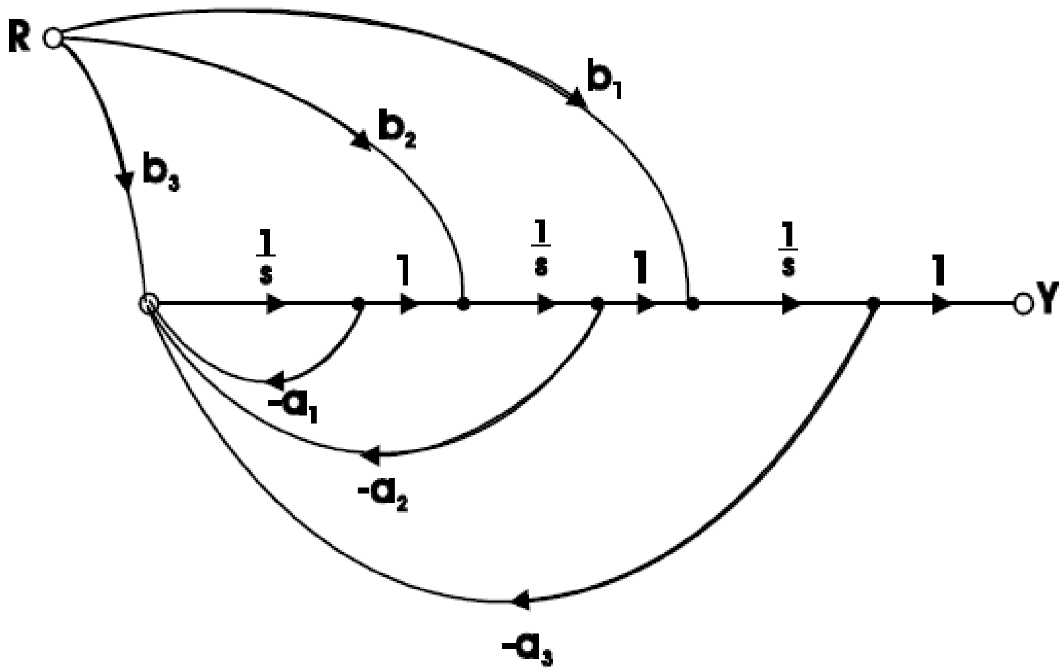
$\sum L_r L_s L_t$ Sum of the gain products of all combinations of **Three** non-touching **loops**

Mason's signal-flow gain formula

Solution (by Mason's signal-flow gain formula)

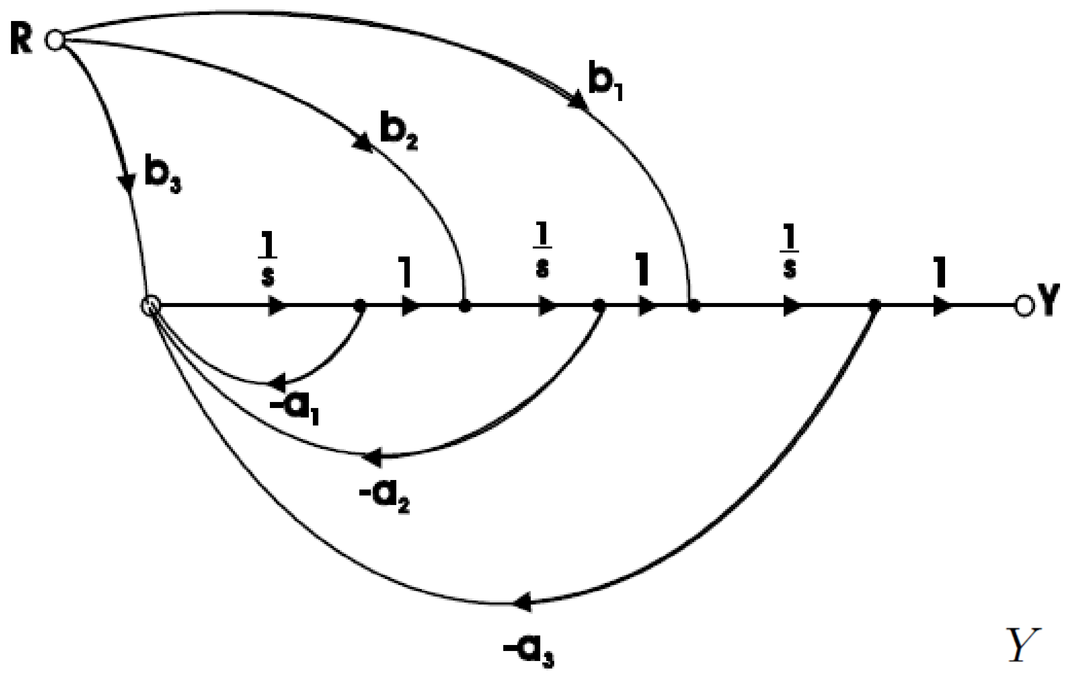


(c)



Mason's signal-flow gain formula

Solution



Forward path gains and cofactor:

$$p_1 = \frac{b_3}{s^3}, \qquad p_2 = \frac{b_2}{s^2}[1 + \frac{a_1}{s}], \qquad p_3 = \frac{b_1}{s}[1 + \frac{a_1}{s} + \frac{a_2}{s^2}]$$

Loop path gains:

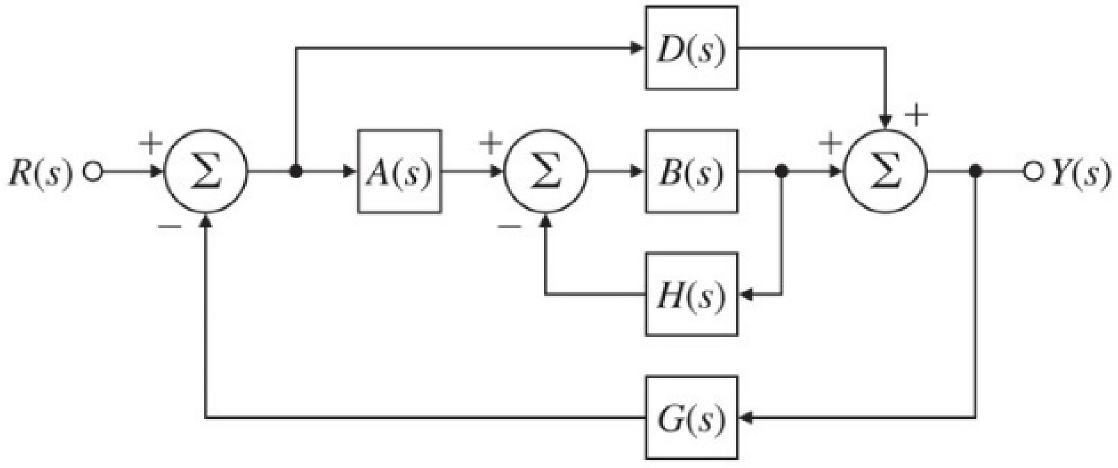
$$\ell_1 = -\frac{a_3}{s^3}, \qquad \ell_2 = -\frac{a_2}{s^2}, \qquad \ell_3 = -\frac{a_1}{s}$$

$$\frac{Y}{R} = \frac{p_1 + p_2 + p_3}{1 - \ell_1 - \ell_2 - \ell_3} = \frac{b_1 s^2 + (a_1 b_1 + b_2)s + a_1 b_2 + a_2 b_1 + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

Mason’ s signal-flow gain formula

Homework

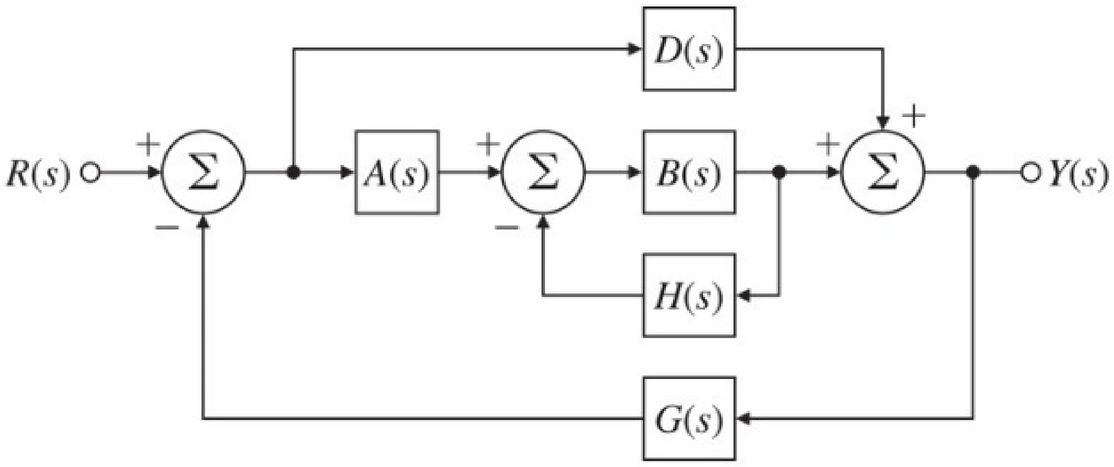
Find the transfer functions for the block diagrams as shown below, using Mason’s rule



(d)

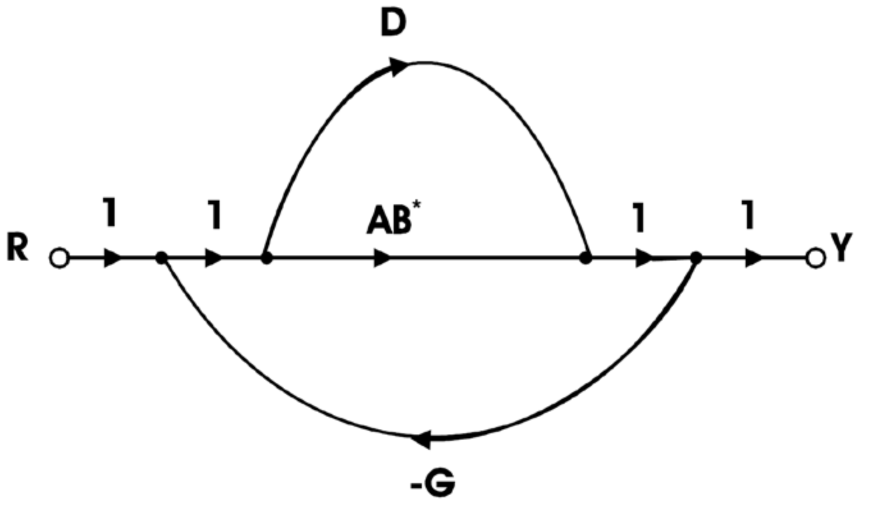
Mason's signal-flow gain formula

Solution



(d)

$$B^* = \frac{B}{1+BH}$$



Forward path gains: $p_1 = D, \quad p_2 = AB^*$

Loop path gains: $\ell_1 = -GD, \quad \ell_2 = -AB^*G$

$$\frac{Y}{R} = \frac{p_1 + p_2}{1 - \ell_1 - \ell_2} = \frac{D + AB^*}{1 + G(D + AB^*)}$$

Mason’ s signal-flow gain formula

核心

Block Diagram Reduction by:

- **Transformations**
- **Mason’ s gain formula**

续

- **Unit impulse response function**
- **Different types of Transfer function**

Some functions of MATLAB

(1) conv(p,q) multiply polynomial p and polynomial q

(2) plot(t,y) plots variable y versus variable t.

(3) roots(p) calculate roots of p(s)=0

(4) pole(sys) calculate poles of sys

(5) zero(sys) calculate zeros of sys

(6) series(sys1,sys2) series interconnection of sys1 and sys2.

(7) feedback(sys1,sys2,sign) feedback interconnection of sys1 and sys2,sign=1or -1.