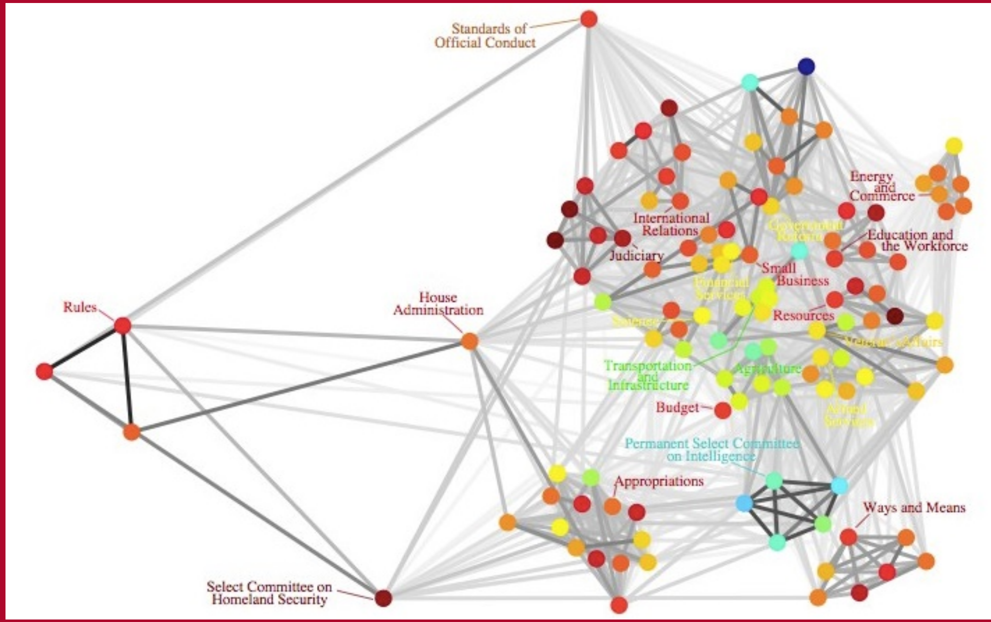
Automatic Control Theory

Chapter 3



Fan zichuan
School of Computer and Information Science
Southwest University

Main contents

1.Typical test signals for the time response of control systems.

2.The unit-step response and time-domain specifications.

3.Time response of first-order and second-order systems.

4.Improvement performance of second systems.

5. Condition for a feedback system to be stable

6. Routh-Hurwitz criterion

7. The steady-state error of feedback control system.

The performance of feedback control systems

Review

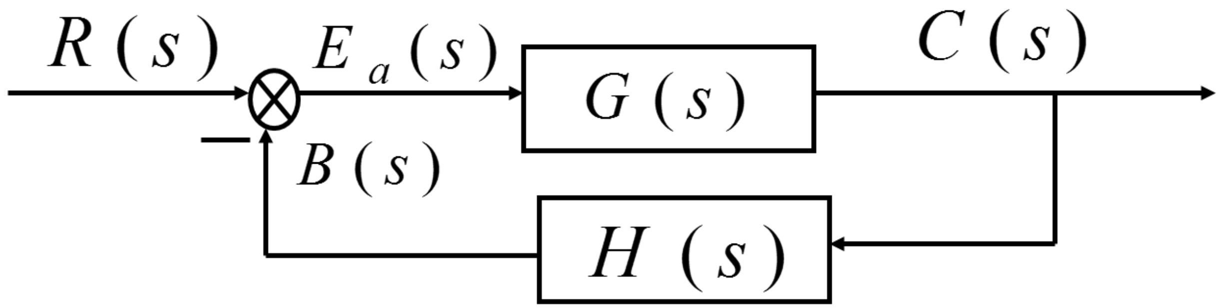
- **Condition for a feedback system to be stable**
- **Routh-Hurwitz criterion**

$$c(t) = \sum_{j=1}^{l} B_j e^{s_{rj} t} + \sum_{i=1}^{n} A_i e^{s_i t}$$

what is next

The steady-state error of feedback control system

The steady-state error of feedback control system



Definition of error

$$E(s) = R(s) - C(s) \qquad E_a(s) = R(s) - B(s)$$

The steady-state error is defined as

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

The steady-state error of feedback control system

set $E_a(s) = \dfrac{R(s)}{1+G(s)H(s)}$ $\quad G(s)H(s) = \dfrac{K\Pi(\tau_j s+1)\Pi(\tau_l^2 s^2 + 2\zeta_k \tau_l s+1)}{s^\nu \Pi(T_i s+1)\Pi(T_k^2 s^2 + \zeta_k T_k s+1)} = \dfrac{K \cdot N_0(s)}{s^\nu \cdot D_0(s)}$

where $\quad K \longrightarrow \text{Open loop gain}$

The number of integrations is called type number of system

$\nu = 0$ Type 0 system $\quad \nu = 1$ Type 1 system $\quad \nu = 2$ Type 2 system

$$e_{ss} = \lim_{s\to 0} sE_a(s) = \lim_{s\to 0} \frac{s^{\nu+1} D_0(s)}{s^\nu D_0(s) + KN_0(s)} R(s) \quad = \lim_{s\to 0} \frac{s^{\nu+1}}{s^\nu + K} R(s)$$

Steady-state error due to input signal

(1).Steady State Error in the Step input $R(s)=\dfrac{r_0}{s}$

$$e_{ss}=\lim_{s\to 0}\frac{s^{v+1}}{s^{v}+K}\cdot\frac{r_0}{s}=\lim_{s\to 0}\frac{r_0 s^{v}}{s^{v}+K}$$

$$e_{ss}=\begin{cases}\dfrac{r_0}{1+K} & , \quad v=0\\ 0 & , \quad v\geq 1\end{cases}$$

(2).Steady State Error in the slop input

$$R(s)=\frac{V_0}{s^2}$$

$$e_{ss}=\lim_{s\to 0}\frac{s^{v+1}}{s^v+K}\cdot\frac{V_0}{s^2}=\lim_{s\to 0}\frac{V_0 s^{v-1}}{s^v+K}$$

$$e_{ss}=\begin{cases}\infty, & v=0\\ \dfrac{V_0}{K}, & v=1\\ 0, & v\geq 2\end{cases}$$

(3). Steady State Error in the accelerator input $R(s)=\frac{a_0}{s^3}$

$$e_{ss}=\lim_{s\to 0}\frac{s^{v+1}}{s^v+K}\cdot\frac{a_0}{s^3}=\lim_{s\to 0}\frac{a_0 s^{v-2}}{s^v+K}$$

$$e_{ss}=\begin{cases}\infty & , \quad v\le 1\\ \frac{a_0}{K} & , \quad v=2\\ 0 & , \quad v\ge 3\end{cases}$$

The steady-state error of feedback control system

Error constant

$$E(s) = \frac{R(s)}{1+G(s)H(s)} \qquad e_{ss} = \lim_{s\to 0} s \cdot \frac{R(s)}{1+G(s)H(s)}$$

when $$R(s) = \frac{1}{s} \qquad e_{ss} = \frac{1}{1+\lim\limits_{s\to 0} G(s)H(s)}$$

Defined position error constant K_p as $$K_P = \lim_{s\to 0} G(s)H(s)$$

The steady-state error of feedback control system

when $R(s) = \dfrac{1}{s^2}$ $e_{ss} = \dfrac{1}{\lim\limits_{s \to 0} sG(s)H(s)}$

Defined velocity error constant K_v as $K_v = \lim\limits_{s \to 0} sG(s)H(s)$

---

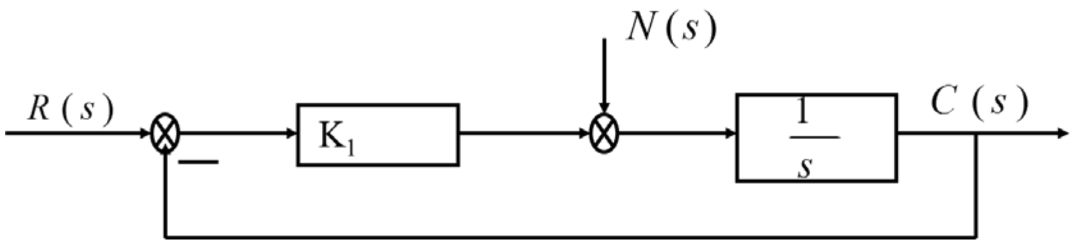
when $R(s) = \dfrac{1}{s^3}$ $e_{ss} = \dfrac{1}{\lim\limits_{s \to 0} s^2 G(s)H(s)}$

Defined acceleration error constant K_a as $K_a = \lim\limits_{s \to 0} s^2 G(s)H(s)$

The steady-state error of feedback control system

系统型别	静态误差系数			阶跃输入 $r(t)=r_0$	斜坡输入 $r(t)=v_0 t$	加速度输入 $r(t)=\frac{a_0 t^2}{2}$
v	K_p	K_v	K_a	$e_{ss}=\frac{r_0}{1+K_p}$	$e_{ss}=\frac{v_0}{K_v}$	$e_{ss}=\frac{a_0}{K_a}$
0	K	0	0	$\frac{r_0}{1+K}$	∞	∞
1	∞	K	0	0	$\frac{v_0}{K}$	∞
2	∞	∞	K	0	0	$\frac{a_0}{K}$
3	∞	∞	∞	0	0	0

Steady-state error due to disturbance signal input



$$E(s) = \frac{-1}{s+K_1} N(s)$$

When $N(s) = \frac{1}{s}$, then $e_{ss} = \frac{-1}{K_1}$

If K_1 change to $G_1(s)$ $e_{ss} = \lim_{s\to 0} \frac{-1}{s+G_1(s)} = \frac{-1}{\lim_{s\to 0} G_1(s)}$

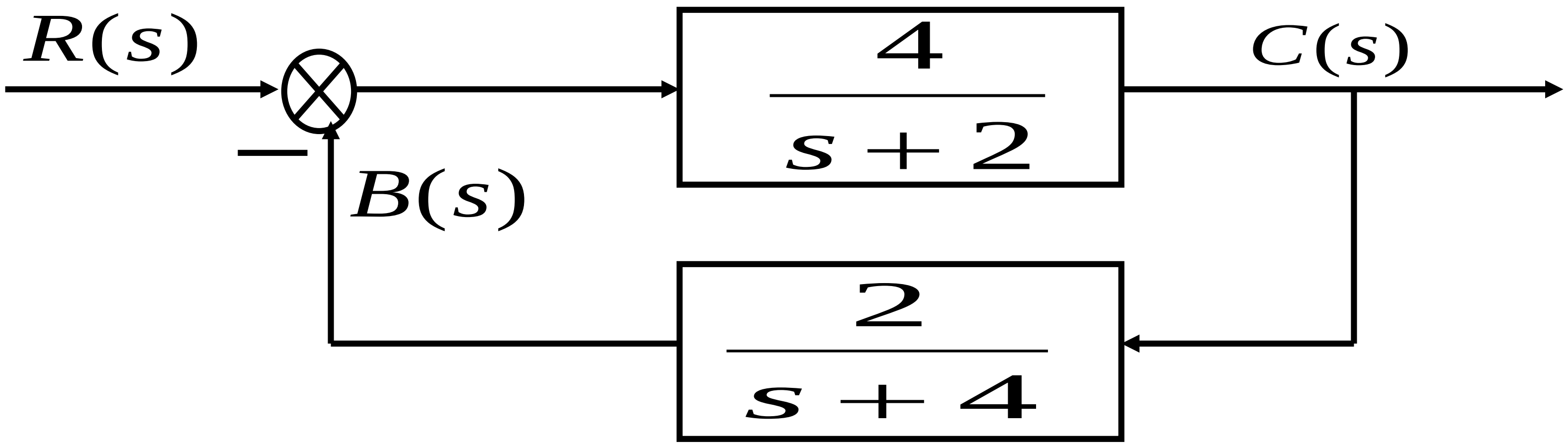
If $e_{ss} = 0,$ $G_1(s)$ at least has one integration.

Conclusion

The steady-state error due to **disturbance signal** input is decided by the **front block of disturbance signal input point**.

Steady-state error due to disturbance signal input

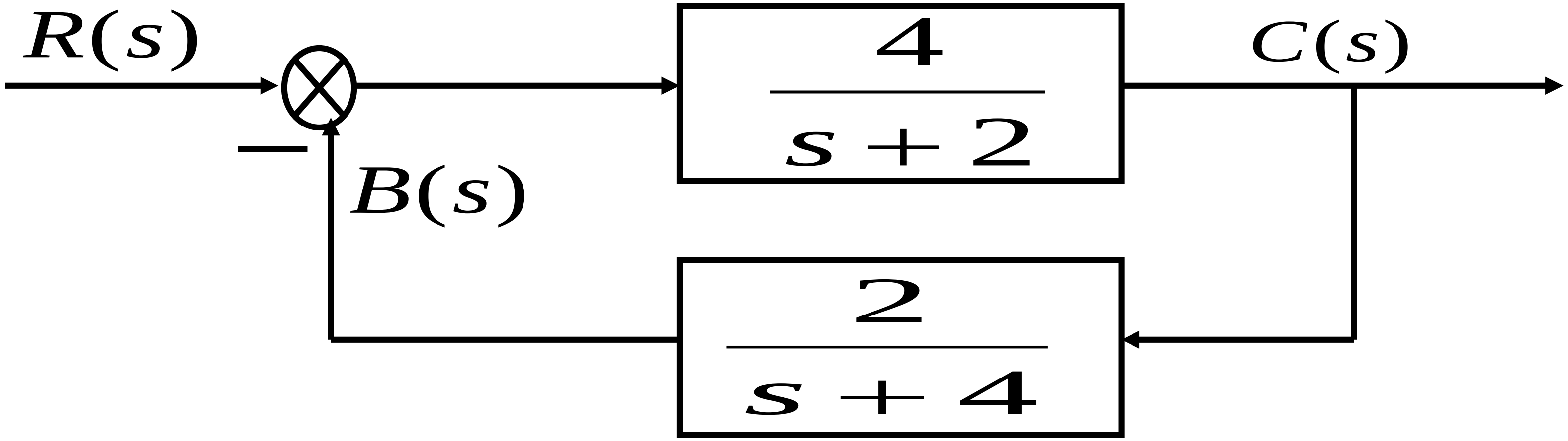
Example 1



Find the steady-state due to a unit step input. The error is defined as $e(t)=r(t)-c(t)$ and $e(t)=r(t)-b(t)$.

Steady-state error due to disturbance signal input

Solution



$$T(s)=\frac{4s+16}{s^2+6s+16}$$

s^2	1	16
s^1	6	
s^0	16	

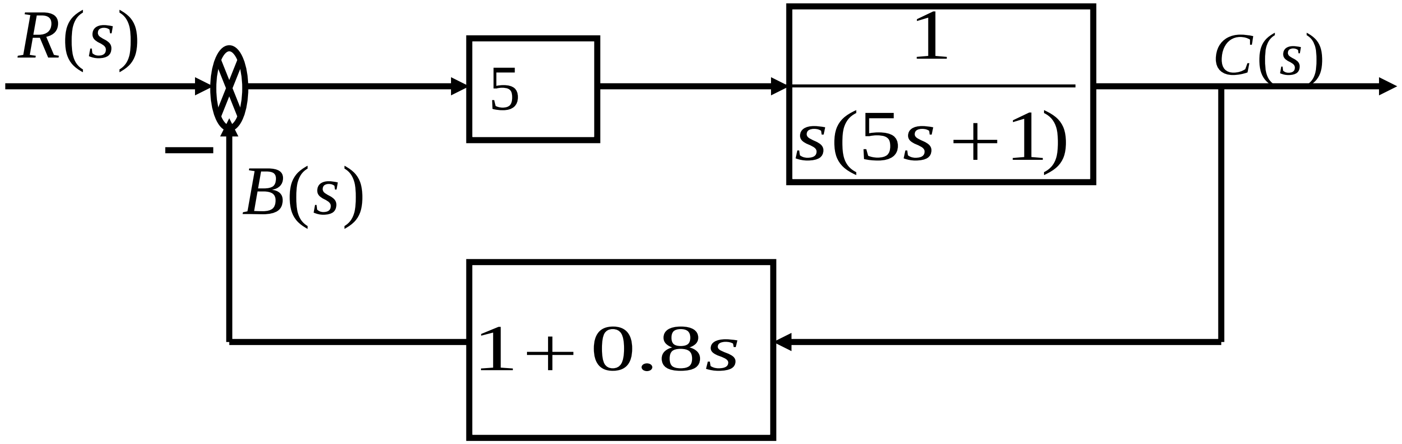
steady

$$e_{ss}=\lim_{t\to\infty} e(t)=\lim_{s\to 0} sE(s)=0$$

$$e_{ss}=\lim_{t\to\infty} e(t)=\lim_{s\to 0} sE_a(s)=1/2$$

Steady-state error due to disturbance signal input

Example 2

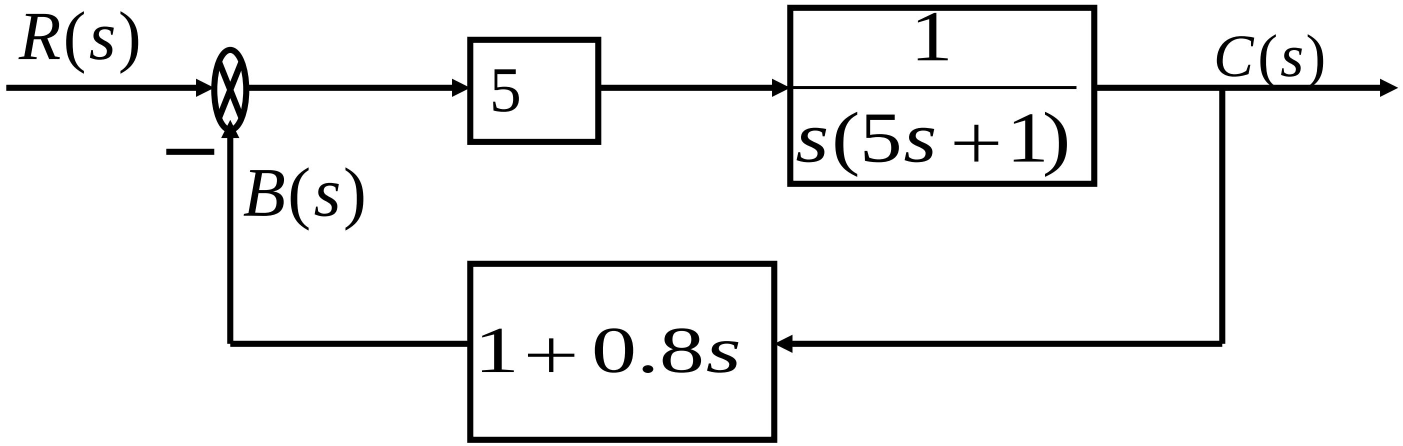


Find the steady-state due to the input $r(t) = 1 + t + \frac{t^2}{2}$

The error is defined as $e(t) = r(t) - c(t)$.

Steady-state error due to disturbance signal input

Solution



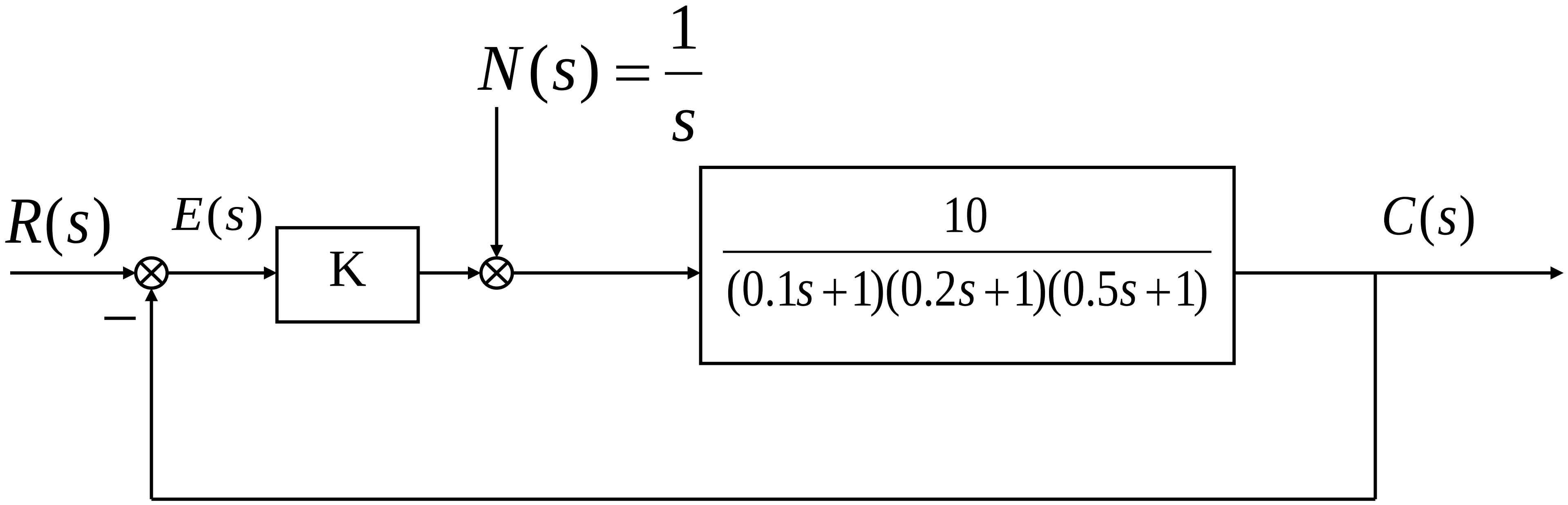
$$T(s)=\frac{1}{s^2+s+1}$$

steady

$$e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to 0}sE(s)=\infty$$

Example 3:

For the control system shown in following Fig, find the value *K*, so that the steady-state error due to disturbance input is –0.099.



Steady-state error due to disturbance signal input

核心

- **Condition for a feedback system to be stable**
- **Steady-state error**

续

- *Chapter 4 The root locus method*