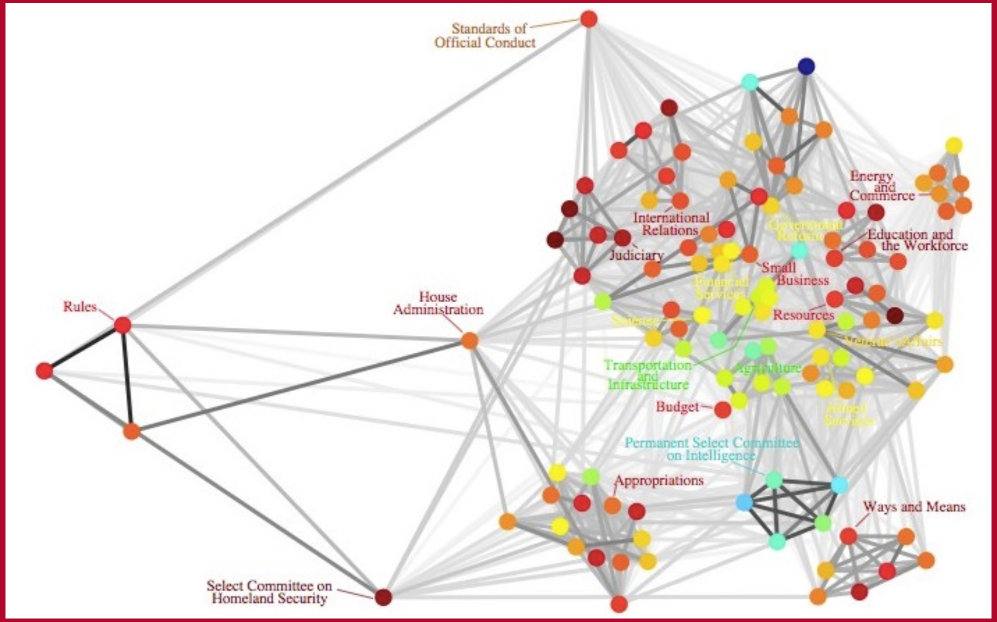
Automatic Control Theory

Chapter 4



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The root locus method

Main contents

1、The root locus concept and root locus equation

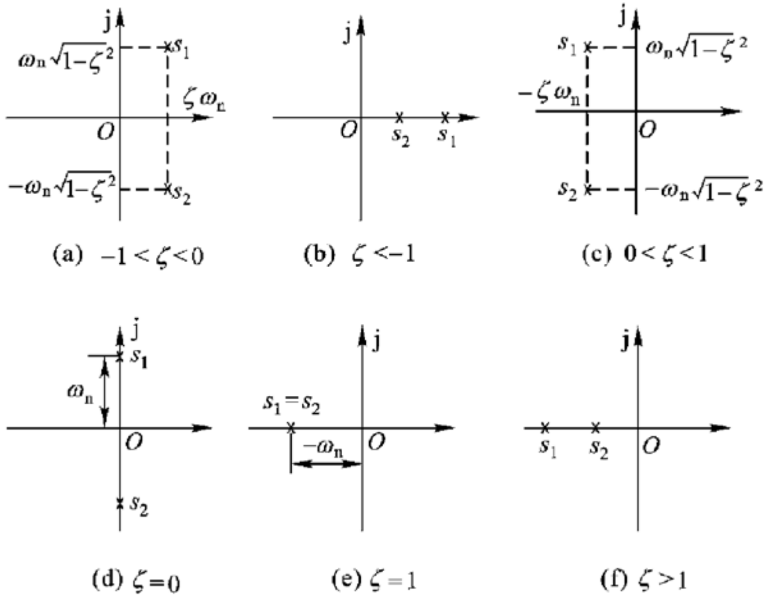
2、The root locus procedure

3、General root loci (Zero degree root loci)

The root locus concept and root locus equation.

Review

- **Differential Equations of Physical Systems**
- **The Transfer function of Linear Systems**
- **The performance of feedback control systems**
- **Condition for a feedback system to be stable**

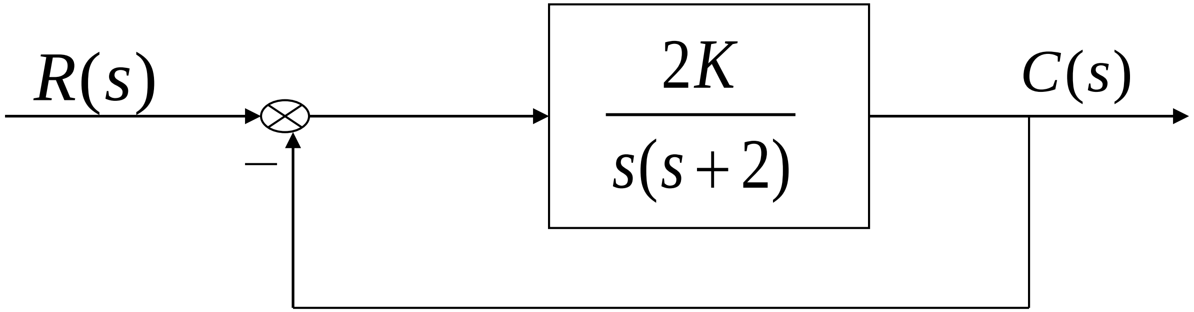


$$\lim_{t\to\infty}\sum_{i=1}^{n} A_i e^{s_i t} = 0$$

what is next

The root locus concept and root locus equation

The root locus concept and root locus equation.



$$T(s) = \frac{2K}{s^2 + 2s + 2K}$$

$$D(s) = s^2 + 2s + 2K = 0$$

The roots of the closed-loop system

$$s_1 = -1 + \sqrt{1 - 2K}$$

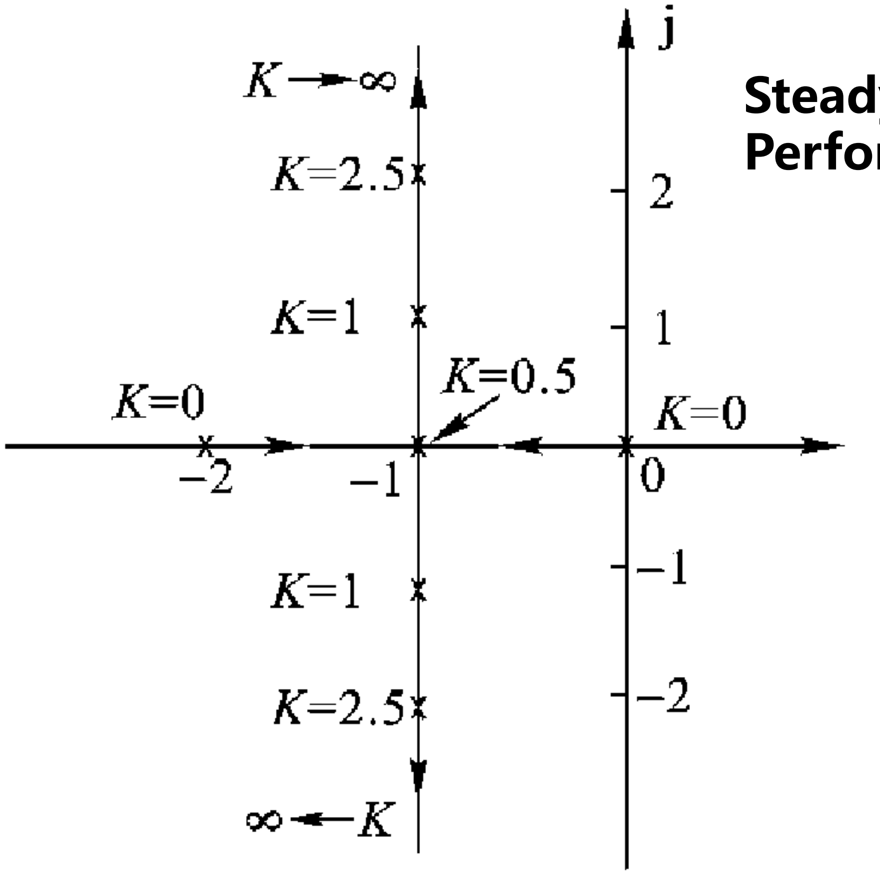
$$s_2 = -1 - \sqrt{1 - 2K}$$

The root locus concept and root locus equation.

$$s_1 = -1 + \sqrt{1-2K}$$

$$s_2 = -1 - \sqrt{1-2K}$$

$K=0$	$s_1=0,$	$s_2=-2$
$K=0.5$	$s_1=-1,$	$s_2=-1$
$K=1$	$s_1=-1+\mathrm{j},$	$s_2=-1-\mathrm{j}$
$K=2.5$	$s_1=-1+2\mathrm{j},$	$s_2=-1-2\mathrm{j}$
$K=+\infty$	$s_1=-1+\mathrm{j}\infty,$	$s_2=-1-\mathrm{j}\infty$



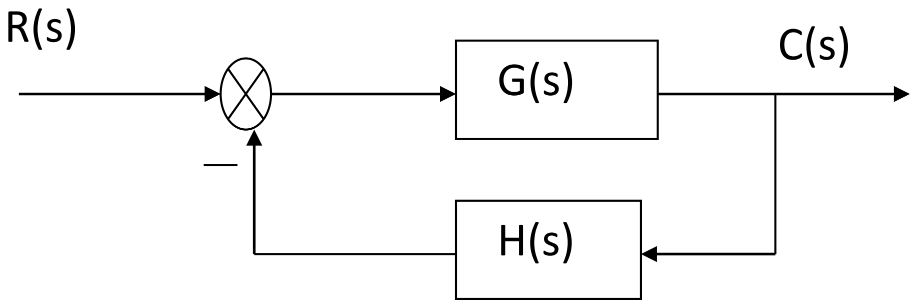
Steady ?
Performance ?

Definition

The roots locus is the path of the roots of the characteristic equation traced out in the s-plane as a system parameter is changed.

根轨迹是指系统中**某个**参数由0→∞变动时, **闭环特征根**在s平面上移动的轨迹。

The root locus concept and root locus equation.



$$\Phi(s)=\frac{G(s)}{1+G(s)H(s)}$$

The characteristic equation is

D(s)=1+G(s)H(s)=0

$$G(s)H(s)=\frac{K\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)}$$

Zeros of open loop transfer function.

Poles of open loop transfer function.

Open loop transfer function

Where

K : the root locus gain

The root locus concept and root locus equation.

$$G(s)=\frac{K_G(\tau_1 s+1)(\tau_2^2 s^2+2\tau_1\tau_2 s+1)\cdots}{s^{\nu}(T_1 s+1)(T_2^2 s^2+2\tau_2 T_2 s+1)\cdots} = K_G^* \frac{\prod_{i=1}^{f}(s-z_i)}{\prod_{i=1}^{q}(s-p_i)} \qquad H(s) = K_H^* \frac{\prod_{j=1}^{l}(s-z_j)}{\prod_{j=1}^{h}(s-p_j)}$$

Where $K_G^* = K_G \dfrac{\tau_1 \tau_2^2 \cdots}{T_1 T_2^2 \cdots}$ K_G: the gain of forward path

K_G^*: the root locus gain of forward path K_H^*: the root locus gain of feedback

$$G(s)H(s) = K_G^* K_H^* \frac{\prod_{i=1}^{f}(s-z_i)\prod_{j=1}^{l}(s-z_j)}{\prod_{i=1}^{q}(s-p_i)\prod_{i=1}^{h}(s-p_j)} = K^* \frac{\prod_{i=1}^{f}(s-z_i)\prod_{j=1}^{l}(s-z_j)}{\prod_{i=1}^{q}(s-p_i)\prod_{j=1}^{h}(s-p_j)}$$

$$f+l=m$$
$$q+h=n$$

The root locus concept and root locus equation.

$$\Phi(s)=\frac{G(s)}{1+G(s)H(s)}$$

Open loop transfer function

$$G(s)H(s)=K_G^* K_H^* \frac{\prod_{i=1}^{f}(s-z_i)\prod_{j=1}^{l}(s-z_j)}{\prod_{i=1}^{q}(s-p_i)\prod_{i=1}^{h}(s-p_j)}=K^*\frac{\prod_{i=1}^{f}(s-z_i)\prod_{j=1}^{l}(s-z_j)}{\prod_{i=1}^{q}(s-p_i)\prod_{j=1}^{h}(s-p_j)}$$

$$f+l=m$$
$$q+h=n$$

Close loop transfer function

$$\Phi(s)=\frac{K_G^*\prod_{i=1}^{f}(s-z_i)\prod_{j=1}^{h}(s-p_j)}{\prod_{i=1}^{q}(s-p_i)\prod_{j=1}^{h}(s-p_j)+K_G^* K_H^*\prod_{i=1}^{f}(s-z_i)\prod_{j=1}^{l}(s-z_j)}=K_G^*\frac{\prod_{k=1}^{f+h}(s-z_k)}{\prod_{k=1}^{n}(s-p_k)}$$

K_G^* is also the root locus gain of close loop transfer function

The root locus concept and root locus equation.

$$G(s)H(s)=\frac{K\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)}=-1$$

Root locus equation

The equation may be rewritten in polar form as

$$\frac{K\prod_{i=1}^{m}|s-z_i|}{\prod_{i=1}^{n}|s-p_i|}\angle G(s)H(s)=-1$$

Then

$$\frac{K\prod_{i=1}^{m}|s-z_i|}{\prod_{i=1}^{n}|s-p_i|}=1$$

$\longrightarrow$ Magnitude equation

$$\angle G(s)H(s)=\sum_{i=1}^{m}\angle(s-z_i)-\angle\sum_{i=1}^{n}(s-p_i)=(2k+1)\pi$$

$\longrightarrow$ **Phase equation**

Dominates !

The root locus concept and root locus equation.

Example 1

$$G(s)H(s)=\frac{2K}{s(s+2)}$$

$$\frac{2K}{|s||s+2|}=1$$ Magnitude equation S =-8 , When K =24

Phase equation ? $s_{1,2}=-1\pm j\sqrt{47}$

Example 2

$$G(s)H(s)=2K/(s+2)^2$$

$$s_1=-2+j4, s_2=-2-j4$$

Poles of close loop transfer function ?

Solution

$$\sum_{i=1}^{m}\angle(s-z_i)-\sum_{i=1}^{n}\angle(s-p_i)=(2k+1)\pi$$

Phase equation

$$-\sum_{i=1}^{2}\angle(s-p_i)=-\angle(s-p_1)-\angle(s-p_2)=(2k+1)\pi$$

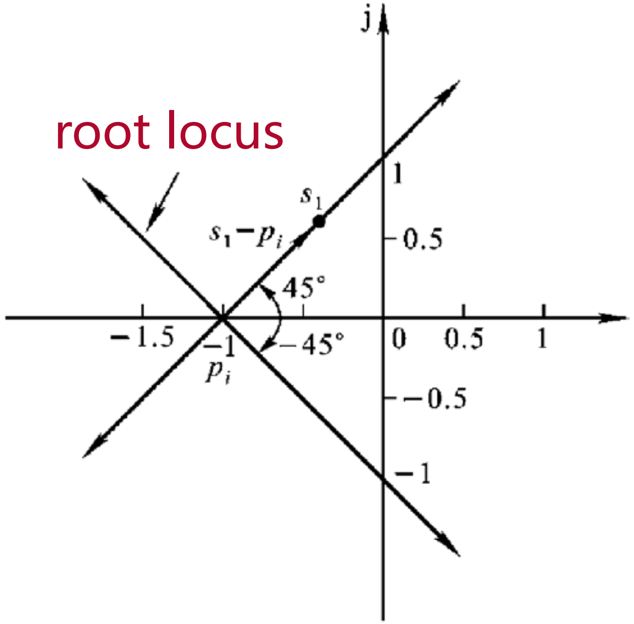
$$p_1=-2, p_2=-2$$

$$-\angle(s_1-p_1)-\angle(s_1-p_2)=-90^\circ-90^\circ$$

$$-\angle(s_1-p_1)-\angle(s_1-p_2)=90^\circ+90^\circ$$

Example 3

$$G(s)H(s) = K/(s+1)^4 \qquad s_1 = -0.5 + j0.5 \qquad K\ ?$$



Solution

$$\frac{K^* \prod_{i=1}^{m} |s - z_i|}{\prod_{i=1}^{n} |s - p_i|} = 1$$

Magnitude equation

$$K = \frac{1}{4}$$

The root locus concept and root locus equation.

核心

- **Definition of root locus**
- **Open loop transfer function**
- **Magnitude and Phase equation**

$$G(s)H(s)=\frac{K\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)}=-1$$

续

- **The root locus procedure**