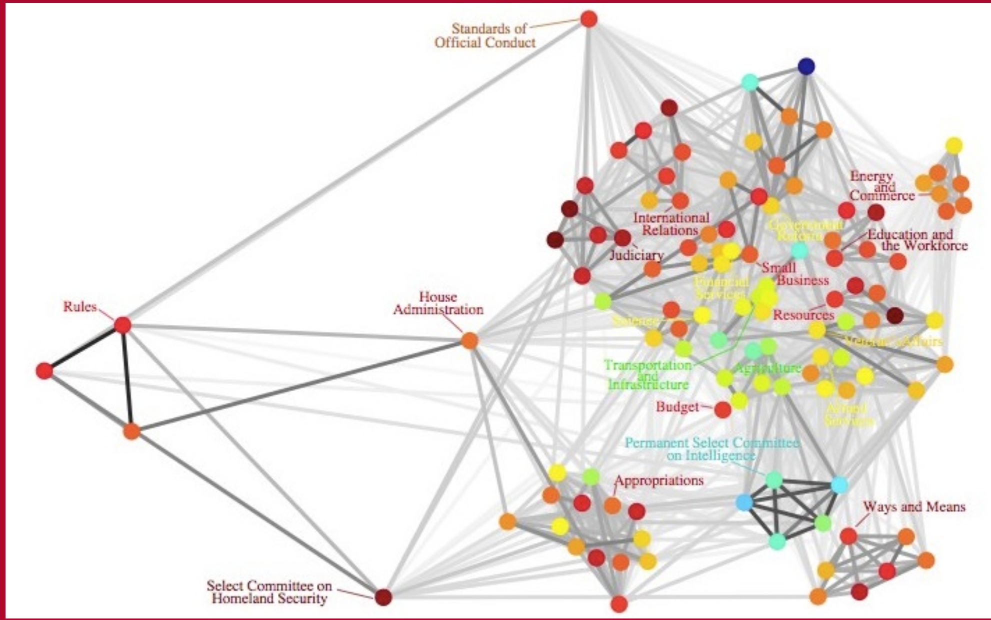
Automatic Control Theory

Chapter 2



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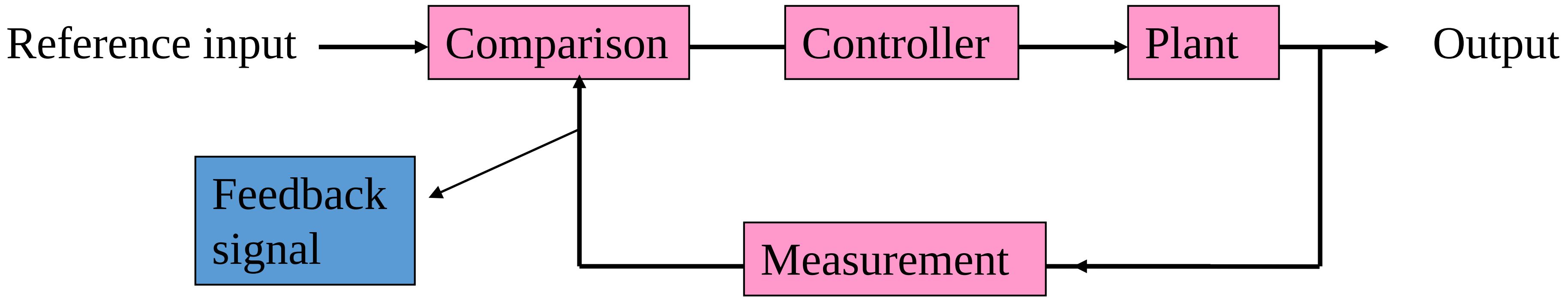
review

Definition of control system



what is next

Mathematical Models of Control Systems

CH2: Mathematical Models of Systems

Main contents

和差分方程区分 物理世间的所有系统

- Differential Equations of Physical Systems.

微分方程核心：动态特征 变化率

- The Transfer function of Linear Systems.

传递函数：描述转换关系

(The Laplace Transform and Inverse Transform)

- Block Diagram.
- Block Diagram Reduction (Mason’s gain formula)

Definition of Mathematical model of system

Mathematical model:

Descriptions of the behavior of a system using mathematics.

描述系统的输入、输出变量以及系统内部各个变量之间的数学表达式。

Types of mathematical models

1、*Differential Equation*

2、*Transfer Function*

3、*Frequency Response*

4、*State Equation*

5、*Difference Equation*

Differential Equations of Physical Systems

How to get the differential equations of physical systems?

The differential equations describing the dynamic performance of a physical system are obtained by utilizing the physical laws of the process.

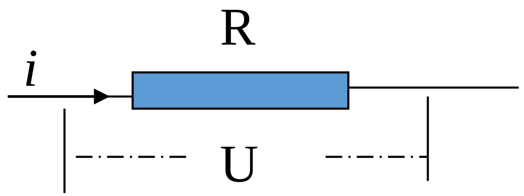
Step1: **确定系统中各元件的输入、输出变量。**

Step2: 按信号传递顺序列写微分方程。

Step3: 化简（线性化、消去中间变量），写出输入、输出变量间的数学表达式。

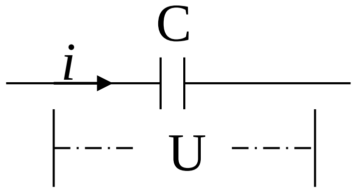
Differential Equations for Ideal Elements

(1) Electrical Resistance



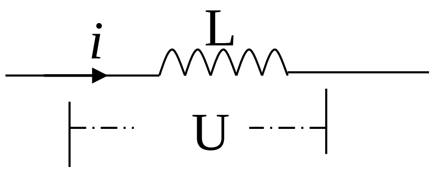
$$i = \frac{U}{R}$$

(2) Electrical Capacitance



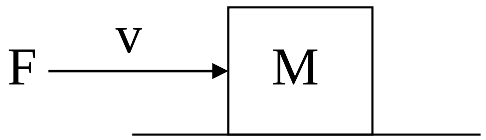
$$i = C\frac{dU}{dt}$$

(3)Electrical Inductance



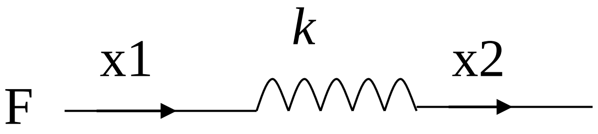
$$U = L\frac{di}{dt}$$

(4) Mass block



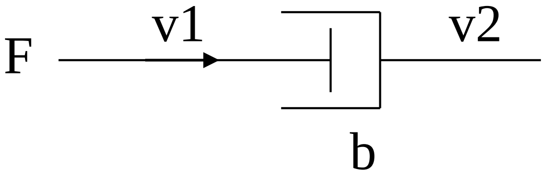
$$F = M\frac{dv}{dt}$$

(5) Spring



$$F = k(x_1 - x_2)$$

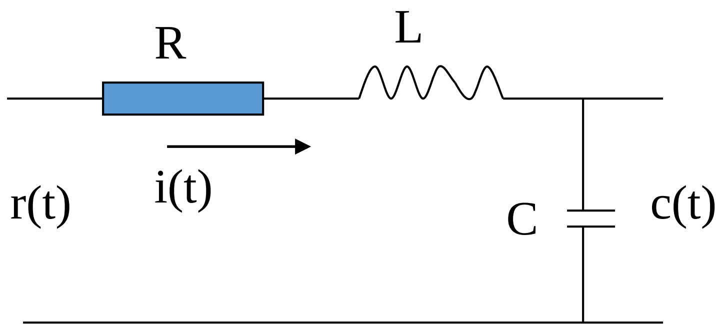
(6) Damper



$$F = b(v_1 - v_2)$$

Examples

Example 1 : RLC circuit

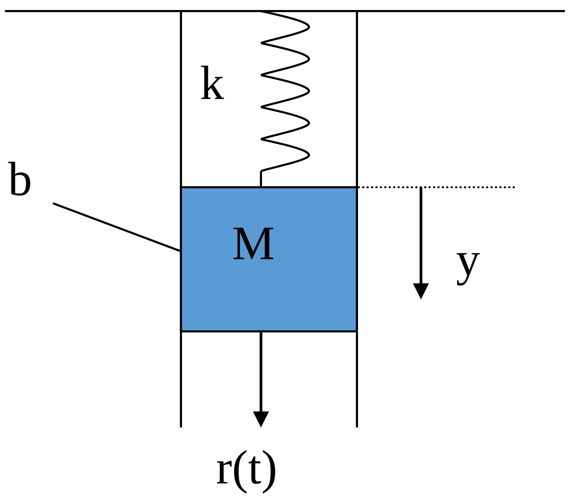


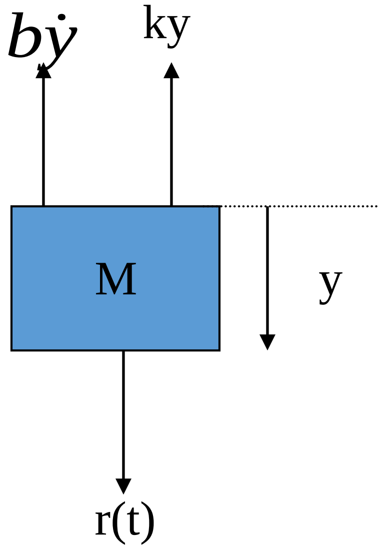
$$\begin{cases} r(t) = Ri(t) + L\dfrac{di(t)}{dt} + c(t) \\ i(t) = C\dfrac{dc(t)}{dt} \end{cases}$$

$$LC\frac{d^2c(t)}{dt^2} + RC\frac{dc(t)}{dt} + c(t) = r(t)$$

Examples

Example 2: Mass-spring-damper





重力可以在选取新的y的初始位置消去
初始位置弹簧伸缩还是拉长？假设初始位置

$$M\frac{d^2y(t)}{dt^2}+b\frac{dy}{dt}+ky(t)=r(t)$$

Linear Approximations of Physical Systems

What is the linear system?

A linear system satisfies the properties of superposition and Homogeneity: (Principle of Superposition).

满足叠加原理的系统称为线性系统。叠加原理又可分为可加性和齐次性。

Principle of superposition

Superposition Property

$$r_1 \longrightarrow \boxed{\text{system}} \longrightarrow y_1$$

$$r_2 \longrightarrow \boxed{\text{system}} \longrightarrow y_2$$

$$r_1 + r_2 \longrightarrow \boxed{\text{system}} \longrightarrow y_1 + y_2$$

Homogeneity Property

$$r \longrightarrow \boxed{\text{system}} \longrightarrow y \qquad ar \longrightarrow \boxed{\text{system}} \longrightarrow ay$$

Example

ps：分配律、交换律是有使用条件的（参考矩阵）

(1) $y = kx$

(2) $y = kx + b$ $\longrightarrow$ Does not satisfy the homogeneity property

ps！齐次和可加都不满足

(3) $y = x^2$ $\longrightarrow$ Does not satisfy the superposition property

When $x = x_0 + \Delta x$ and $y = y_0 + \Delta y$ Equation (2) can be rewritten as

$$y_0 + \Delta y = kx_0 + k\Delta x + b$$

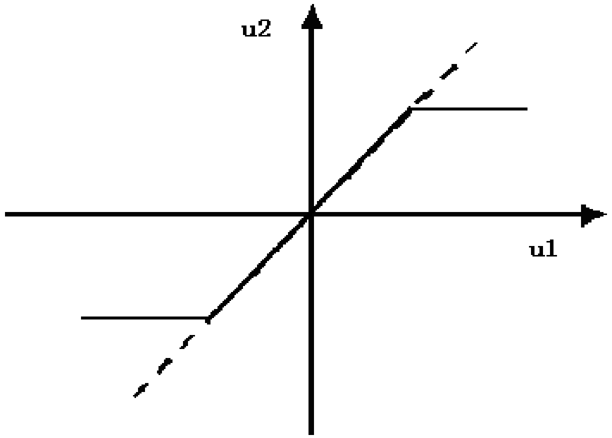
We have

$$\Delta y = k\Delta x$$

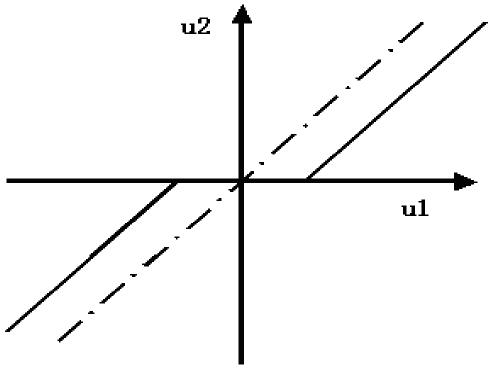
or

$$y = kx$$

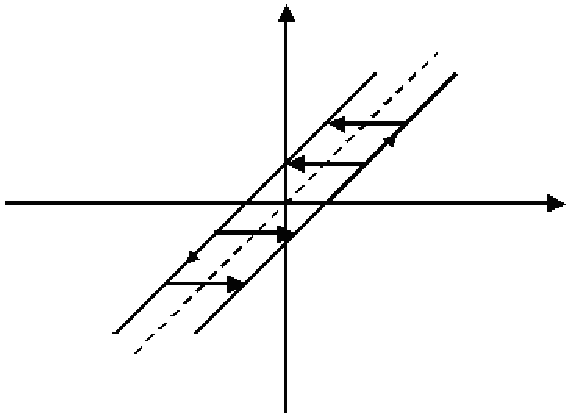
Linearization of Weak Nonlinear Characteristic



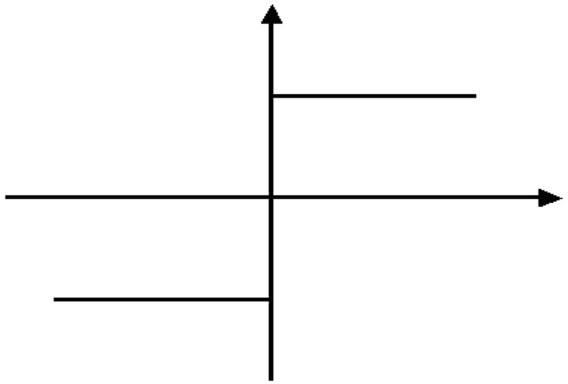
Saturation 饱和



Deadband



Backlash

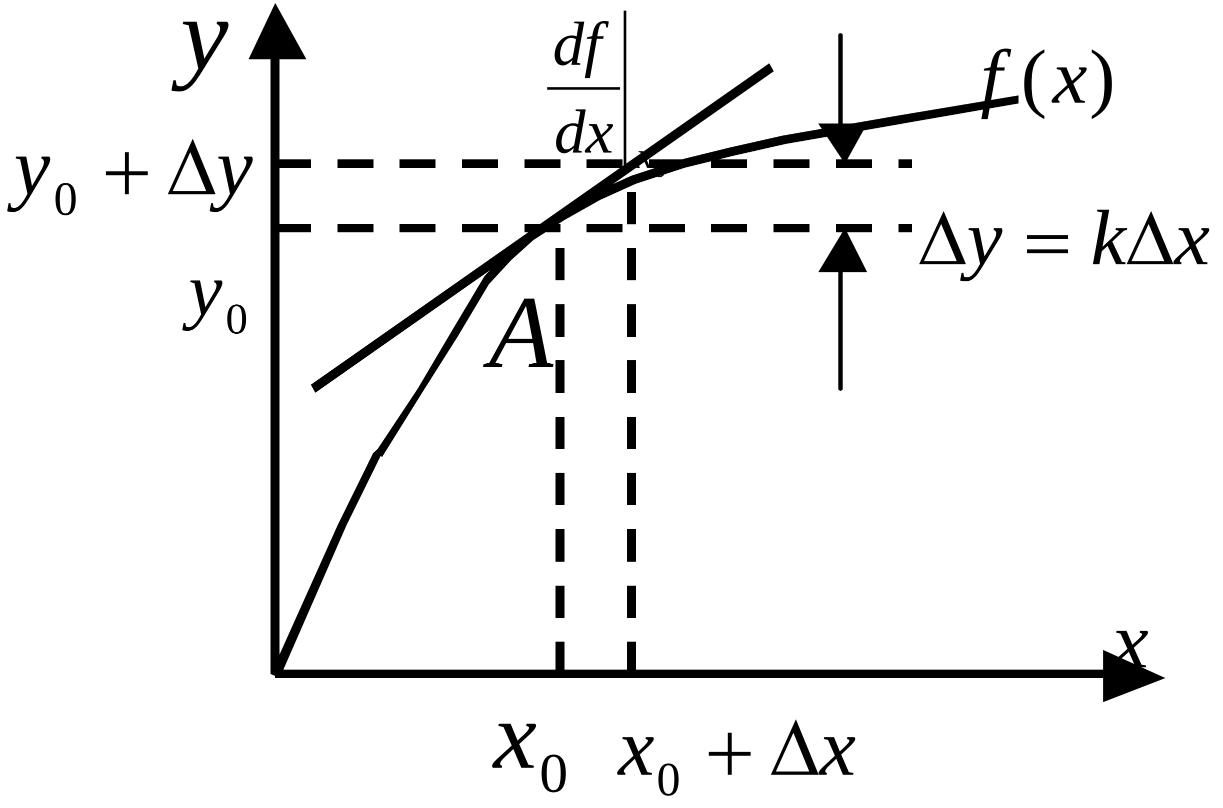


Ideal relay

Linearization using Taylor series expansion about the operating point (Equilibrium Position)

The output-input nonlinear characteristic of y= f (x) is illustrated in the following figure:

非线性的线性化 取很小的一段斜率表达
变成很多的多项式
泰勒多项展开



Linearization using Taylor series expansion about the operating point (Equilibrium Position)

So we get: $$y = f(x) = f(x_0) + \left.\frac{df}{dt}\right|_{x_0} \Delta x$$

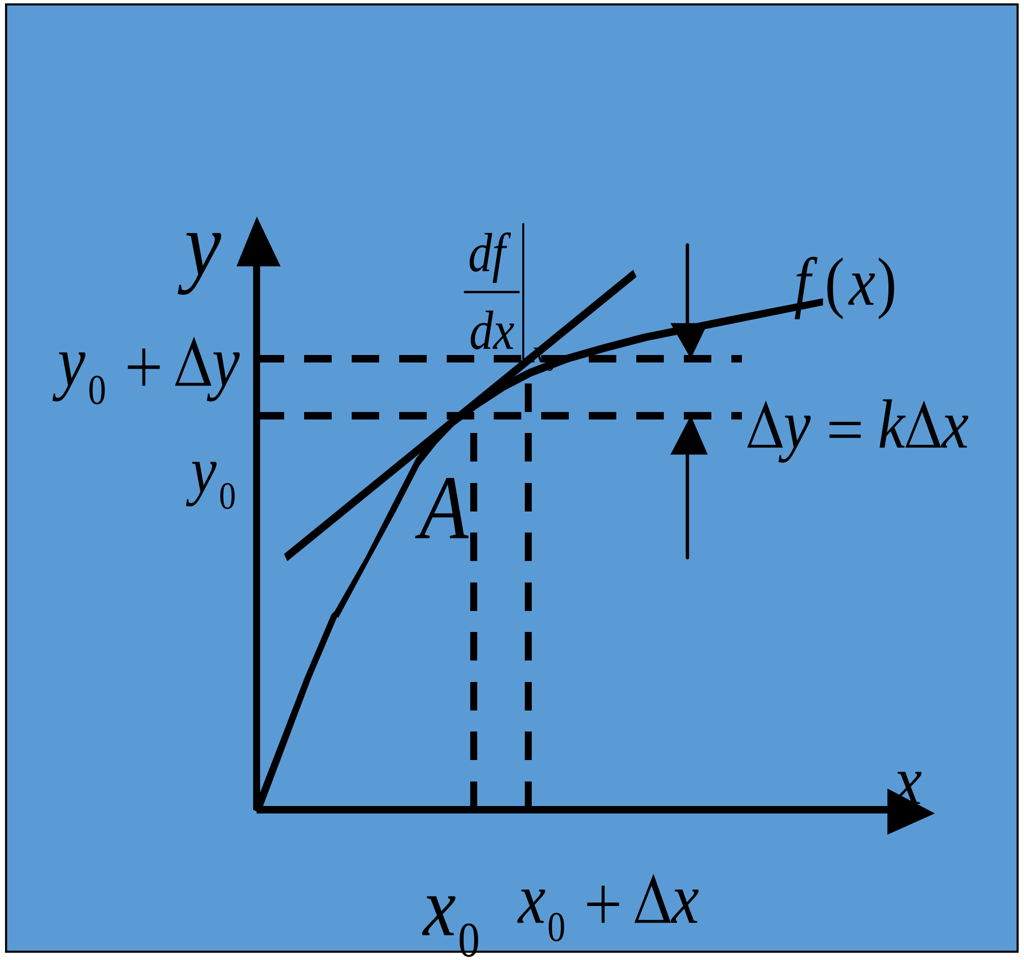
Set $\Delta y = f(x) - f(x_0)$, so we have

$$\Delta y = \left.\frac{df}{dx}\right|_{x_0} \Delta x$$

Set $$\left.\frac{df}{dx}\right|_{x_0} = k$$

We get $\Delta y = k\Delta x$

or $y = kx$



CH2: Mathematical Models of Systems

核心

- Differential Equations of Physical Systems
- Physical laws of the process
- Linear system

续

The Transfer function of Linear Systems.

(The Laplace Transform and Inverse Transform)