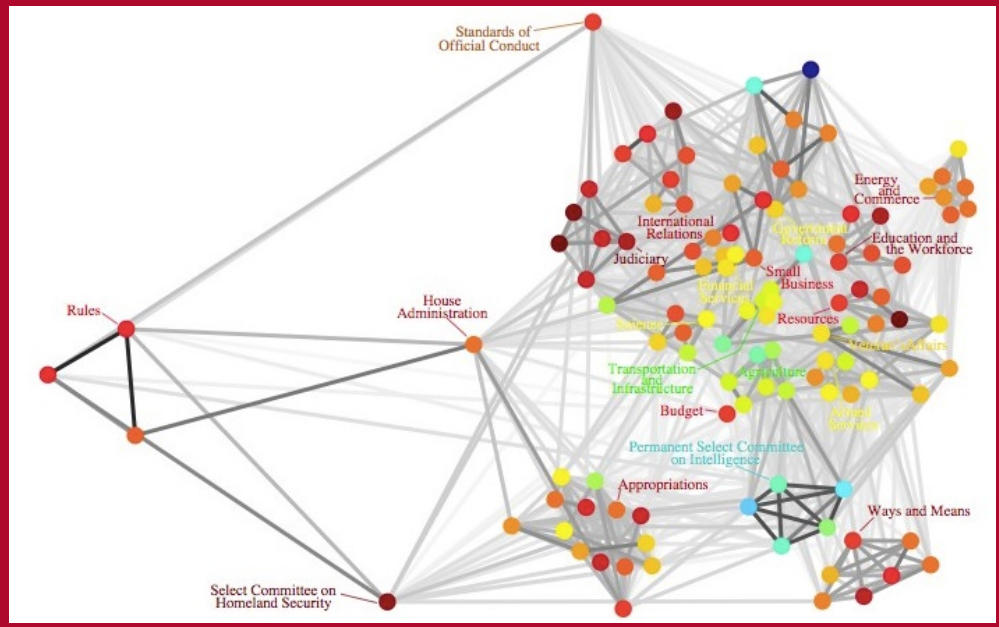
Automatic Control Theory

Chapter 2



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CH2: Mathematical Models of Systems

Main contents

- Differential Equations of Physical Systems.
- The Transfer function of Linear Systems.

  (The Laplace Transform and Inverse Transform)
- Block Diagram.
- Block Diagram Reduction (Mason’ s gain formula)

CH2: Mathematical Models of Systems

Review

1. The concept of transfer function only applies to the LTI system.

2. Transfer function is only determined by the structure and parameter of system.

3. The method of the transfer function has some limitation.

(1). It only applies to the SISO system.

(2). It only can reflect the relationship of input and output.

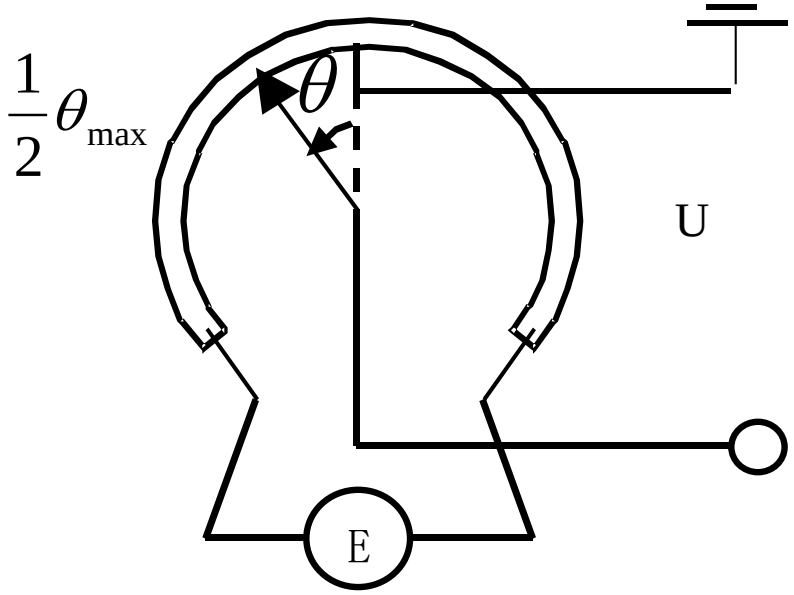
(3). It only can analyze the motion characteristic of zero initial conditions.

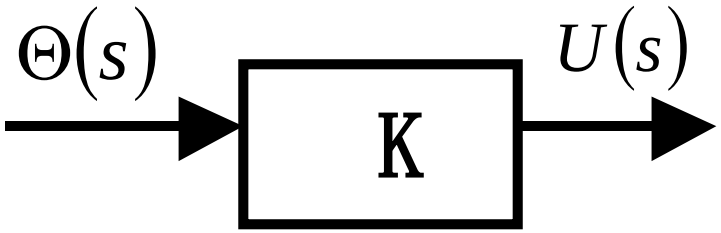
what is next

- **Block Diagram represented by using Transfer function**

The transfer functions of some components

1. Potentiometer.

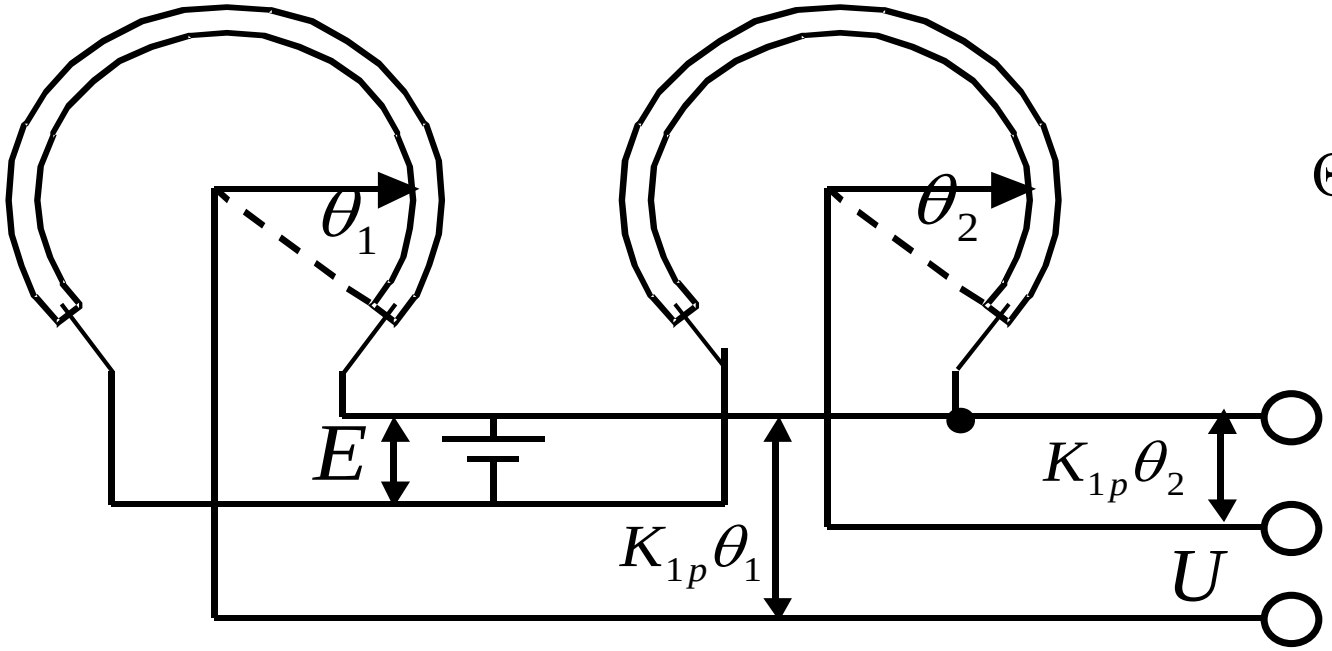


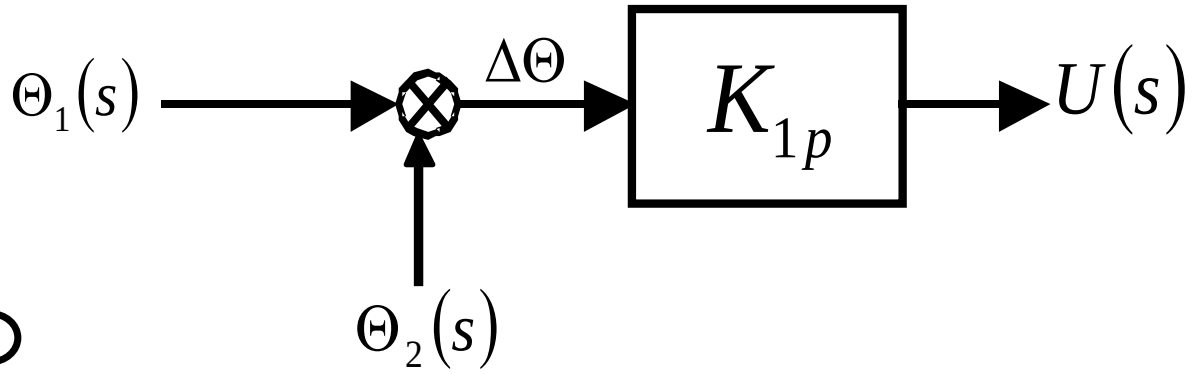


$$K = \frac{E}{\theta_{\max}}$$

The transfer functions of some components

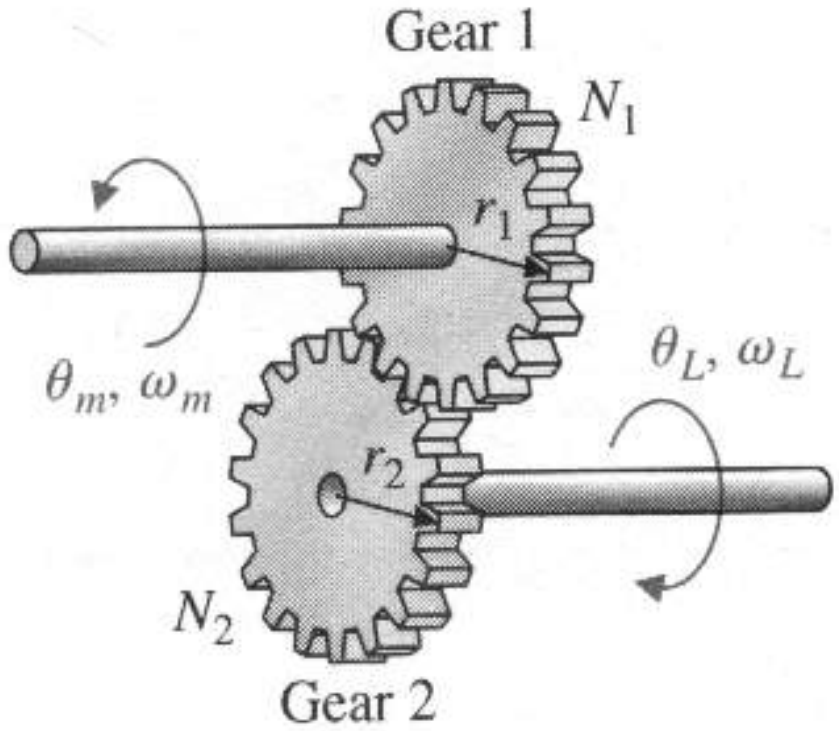
2. Potentiometer Bridge.





The transfer functions of some components

3. Gear train



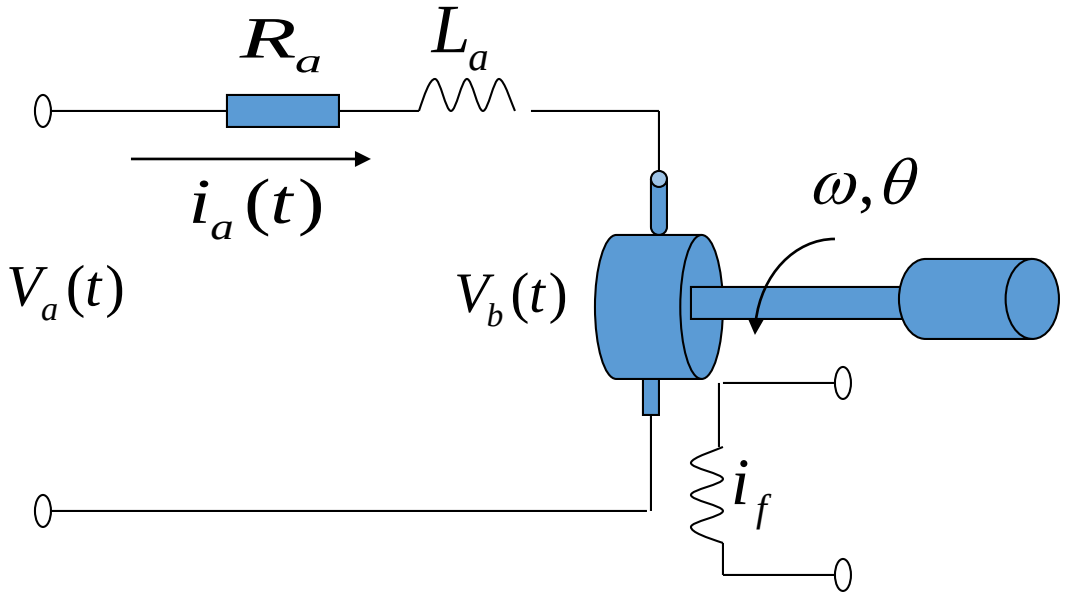
$$Gear\ ratio = n = \frac{N_1}{N_2}$$

$$N_2\theta_L = N_1\theta_m, \theta_L = n\theta_m$$

$$\omega_L = n\omega_m$$

The transfer functions of some components

4. Armature-controlled DC motor



Inertial = J

Friction = b

$$V_a(s) = (R_a + L_a s) I_a(s) + V_b(s)$$

$$V_b(s) = K_b \omega(s) = K_b s \theta(s)$$

Useful tips !

$$s = \frac{d}{dt} \qquad \frac{1}{s} = \int_0^t dt$$

$$I_a(s) = \frac{V_a(s) - K_b \omega(s)}{(R_a + L_a s)}$$

The transfer functions of some components

Motor torque

$$T_m(s) = K_m I_a(s)$$

$$T_m(s) = T_L(s) + T_d(s)$$

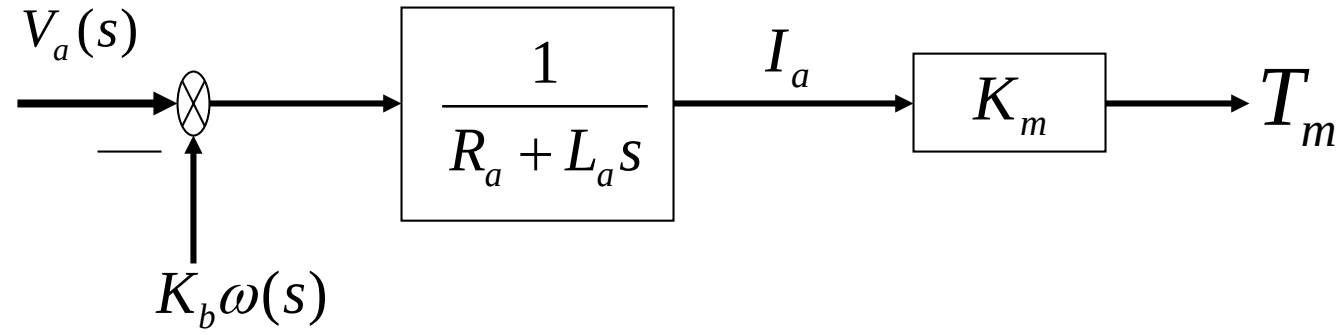
where $T_L(s)$**: load torque** $T_d(s)$ **: disturbance torque**

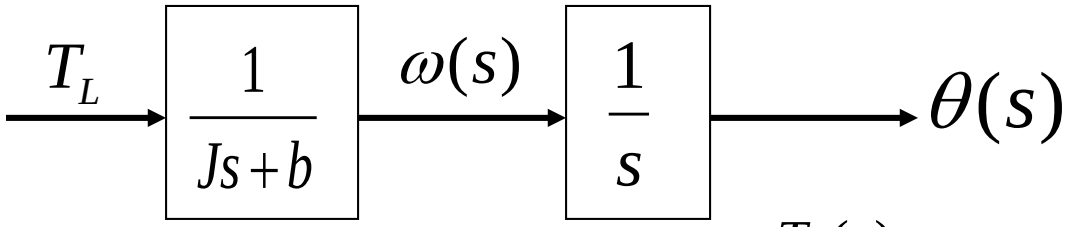
$$T_L(s) = Js^2\theta(s) + bs\theta(s)$$

We can obtain the transfer function (with $T_d(s) = 0$ **)**

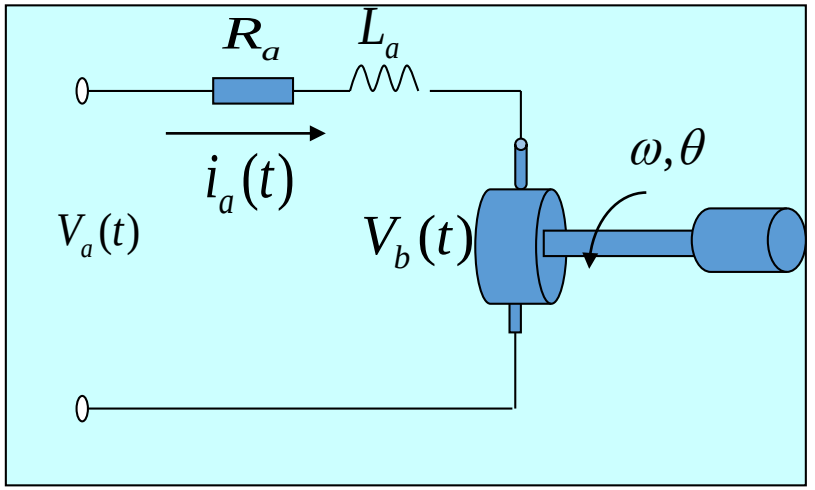
$$G(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_m}{s[(R_a + L_a s)(Js + b) + K_b K_m]}$$

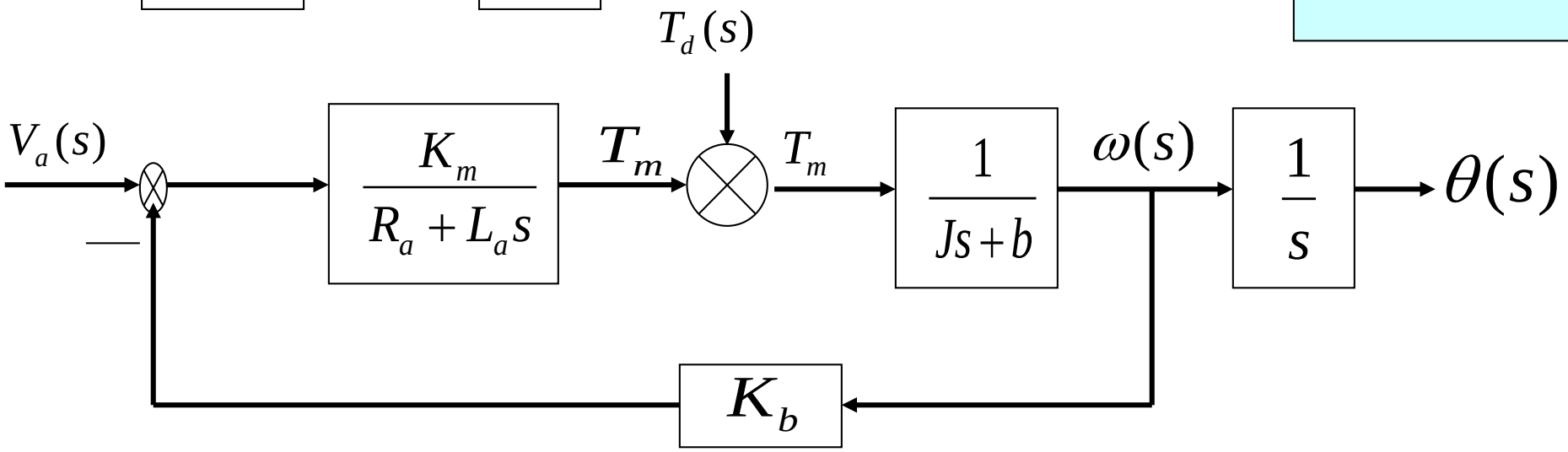
The transfer functions of some components





The block diagram of DC-motor

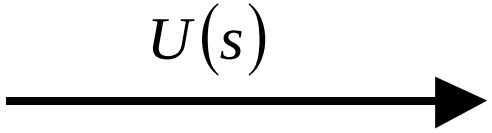




Components of block diagram

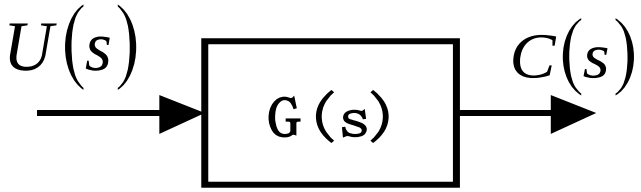
1. Signal line:

A line with arrow that indicate the direction of signal transform.



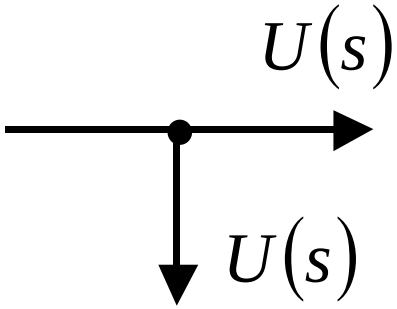
2. Block:

It expresses the transfer function.



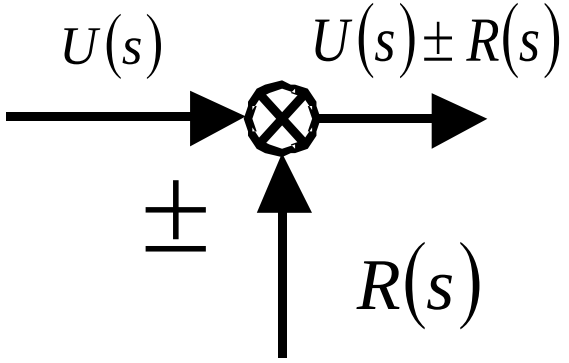
3. Derivation point

(measuring point).



4. Synthesis Point

(Comparing point).



Components of block diagram

核心

Laplace Transform

Differential Equations → **Transfer function**

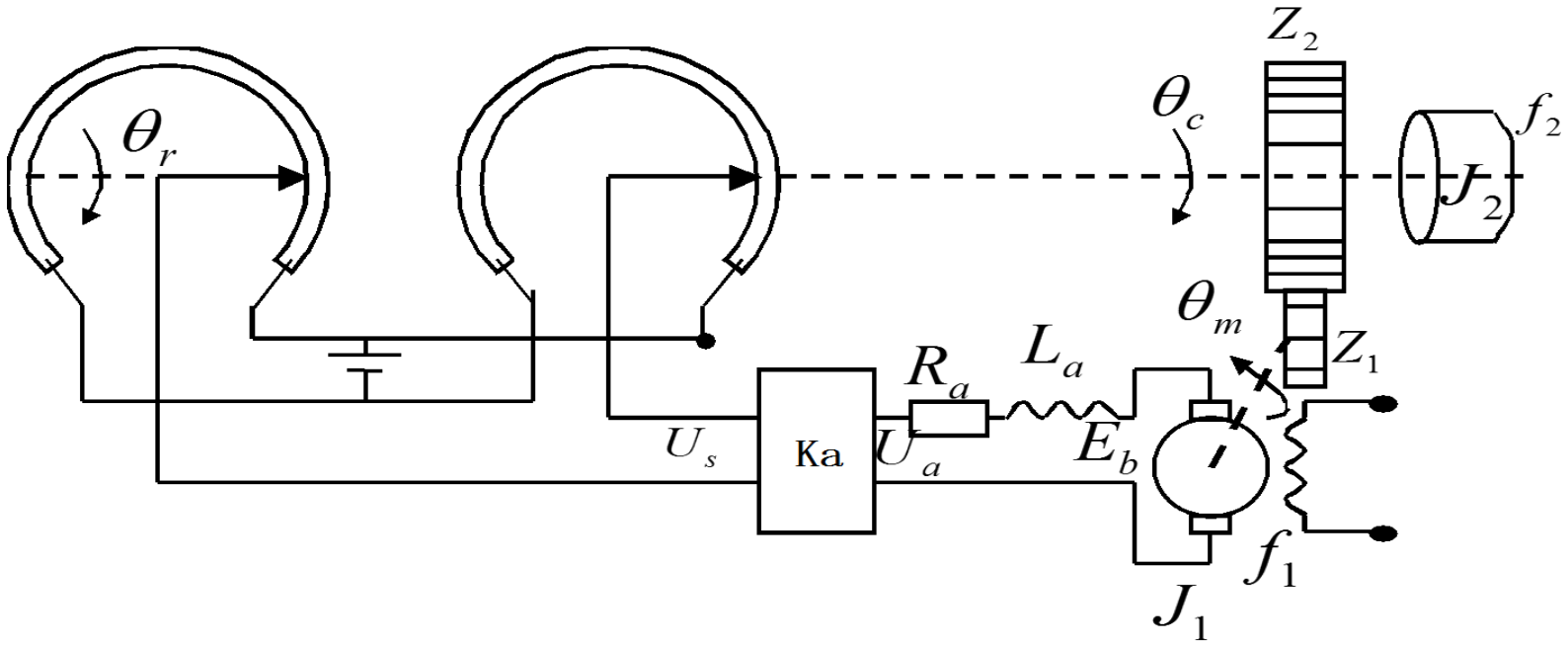
Block Diagram

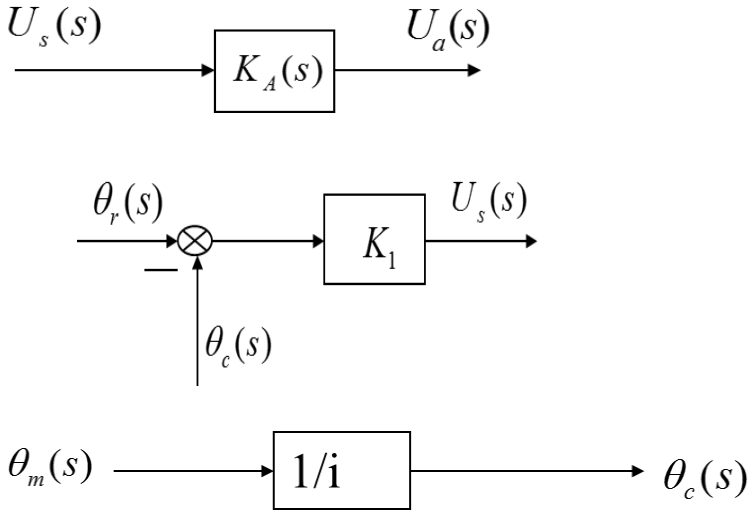
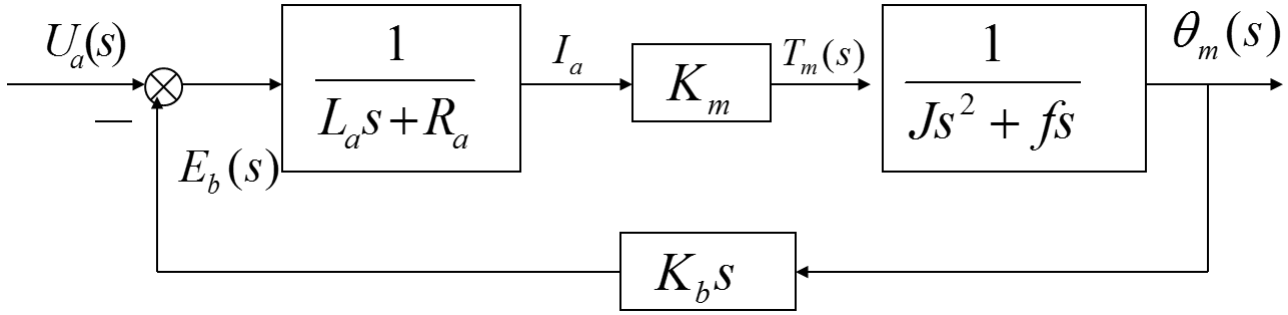
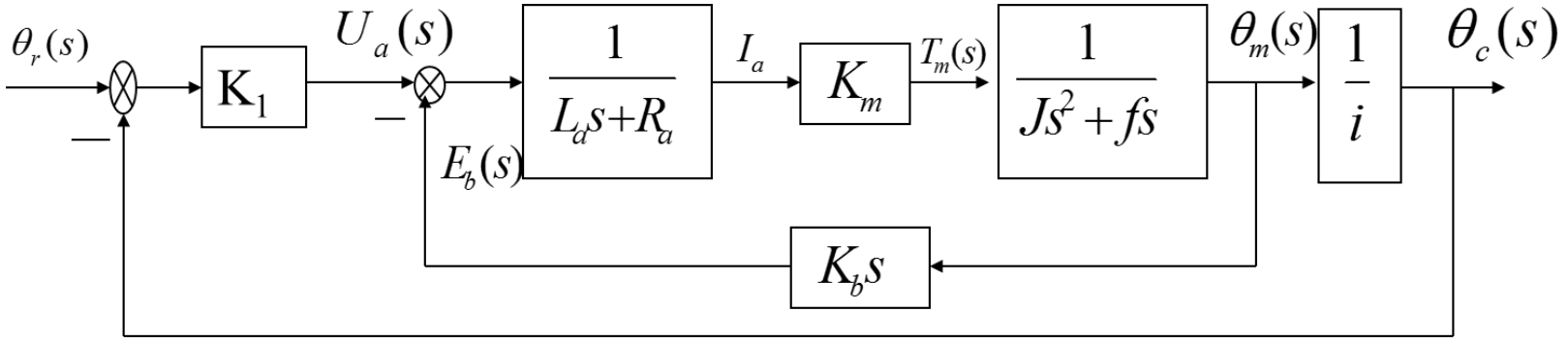
续

Block Diagram Reduction !

Components of block diagram

Homework: Position slave system



Components of block diagram

Solution: