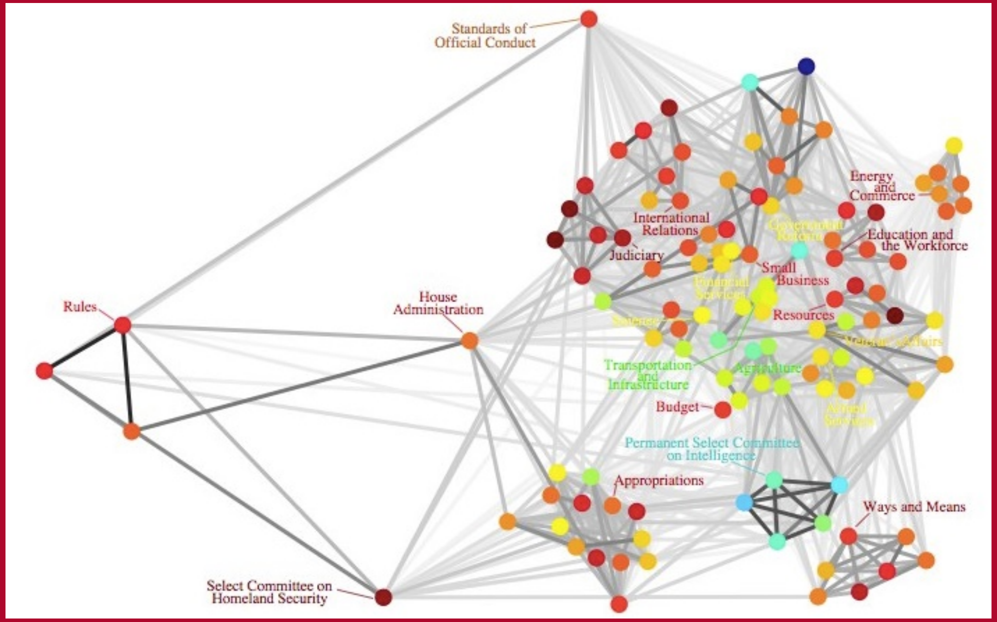
Automatic Control Theory

Chapter 3



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Main contents

1.Typical test signals for the time response of control systems.

2.The unit-step response and time-domain specifications.

3.Time response of first-order and second-order systems.

4.Improvement performance of second systems.

5. Condition for a feedback system to be stable

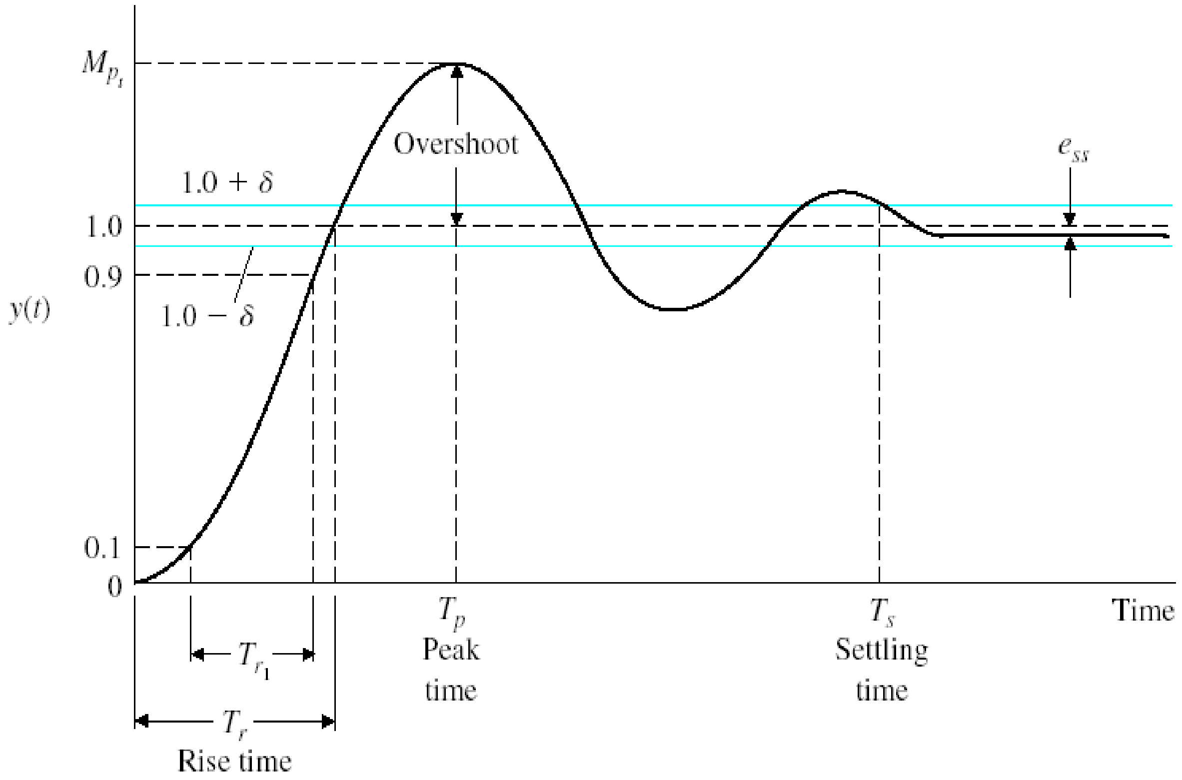
6. Routh-Hurwitz criterion

7. The steady-state error of feedback control system.

The unit-step response and time-domain specifications

Review

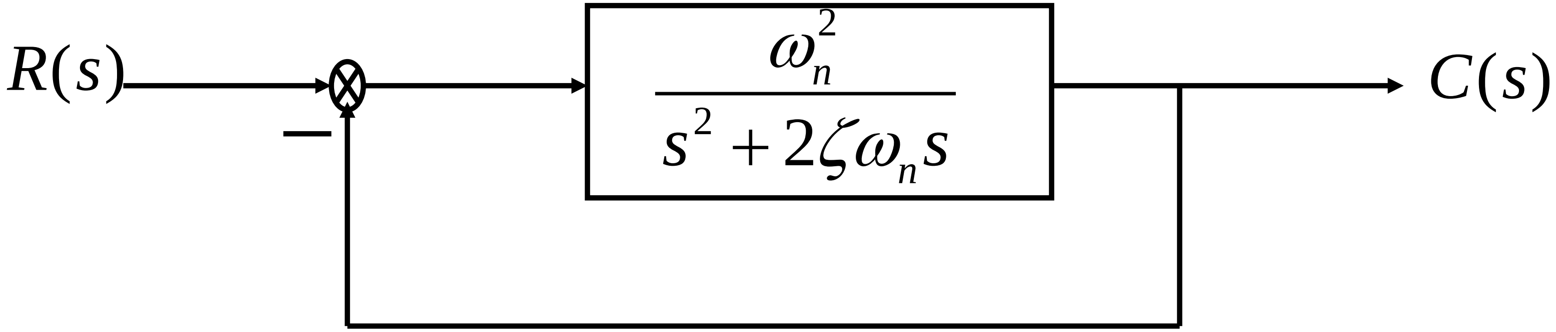
- **Time-domain specifications**
- **Time response of a first-order system**



what is next

Time response of a second-order system

Time response of a second-order system



$$T(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

The unit step response for this system

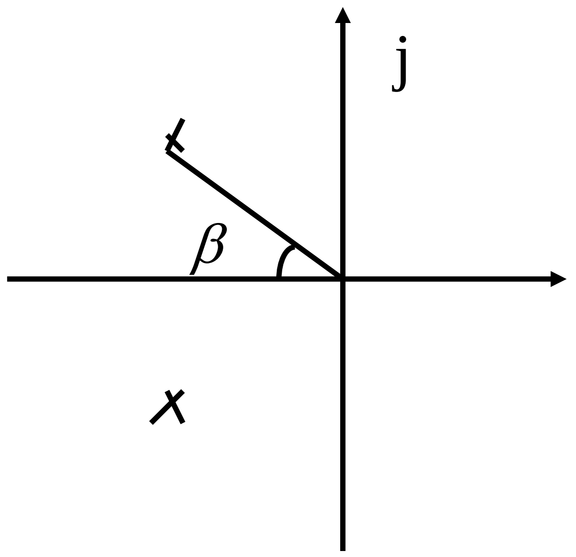
$$c(t)=L^{-1}[\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{1}{s}]$$

Time response of a second-order system

Underdamped 欠阻尼

when $0 < \zeta < 1$ $s_{1,2} = -\zeta\omega_n \pm j\sqrt{1-\zeta^2}\ \omega_n$

$$C(s) = \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$



$$c(t) = 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right) \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_d t + \beta) \qquad \beta = \cos^{-1}\zeta$$

Time response of a second-order system

ω_n Natural undamped frequency

ζ Damping ratio

ω_d Damped frequency

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d}$$

$$\sigma\% = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$$

$$t_r = \frac{\pi - \beta}{\omega_d}$$

工程化因取值范围
不同书不同公式

$$t_s = \frac{3.5}{\zeta\omega_n} \qquad t_s = \frac{4}{\zeta\omega_n}$$

for 5% steady-state error

$$t_s = \frac{4.5}{\zeta\omega_n}$$

for 2% steady-state error

Time response of a second-order system

$\zeta = 1$ $\quad s_{1,2} = -\omega_n$ $\quad$ critically damped 临界阻尼

$$C(s) = \frac{1}{s} - \frac{1}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2} \qquad c(t) = 1-(1+\omega_n t)e^{-\omega_n t}$$

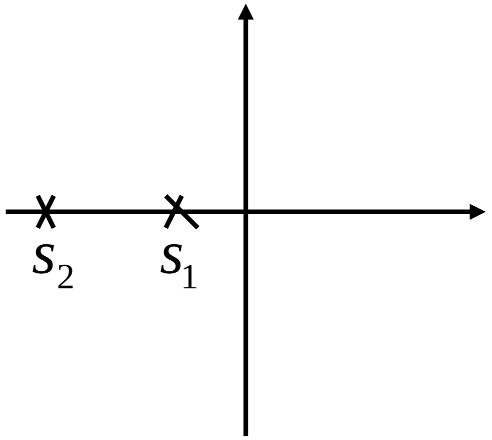
$$t_s = \frac{4.75}{\omega_n} \qquad \sigma\% = 0$$

$\zeta = 0$ $\quad s_{1,2} = \pm j\omega_n$ $\quad$ undamped 无阻尼

$$c(t) = 1-\cos(\omega_n t)$$

Time response of a second-order system

$\zeta > 1$ $\qquad s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$ $\qquad$ overdamped 过阻尼



$$c(t) = 1 + \frac{1}{\frac{s_1}{s_2} - 1} e^{s_1 t} + \frac{1}{\frac{s_2}{s_1} - 1} e^{s_2 t} \qquad t_s = \frac{3}{|s_1|}$$

S_1 dominates !

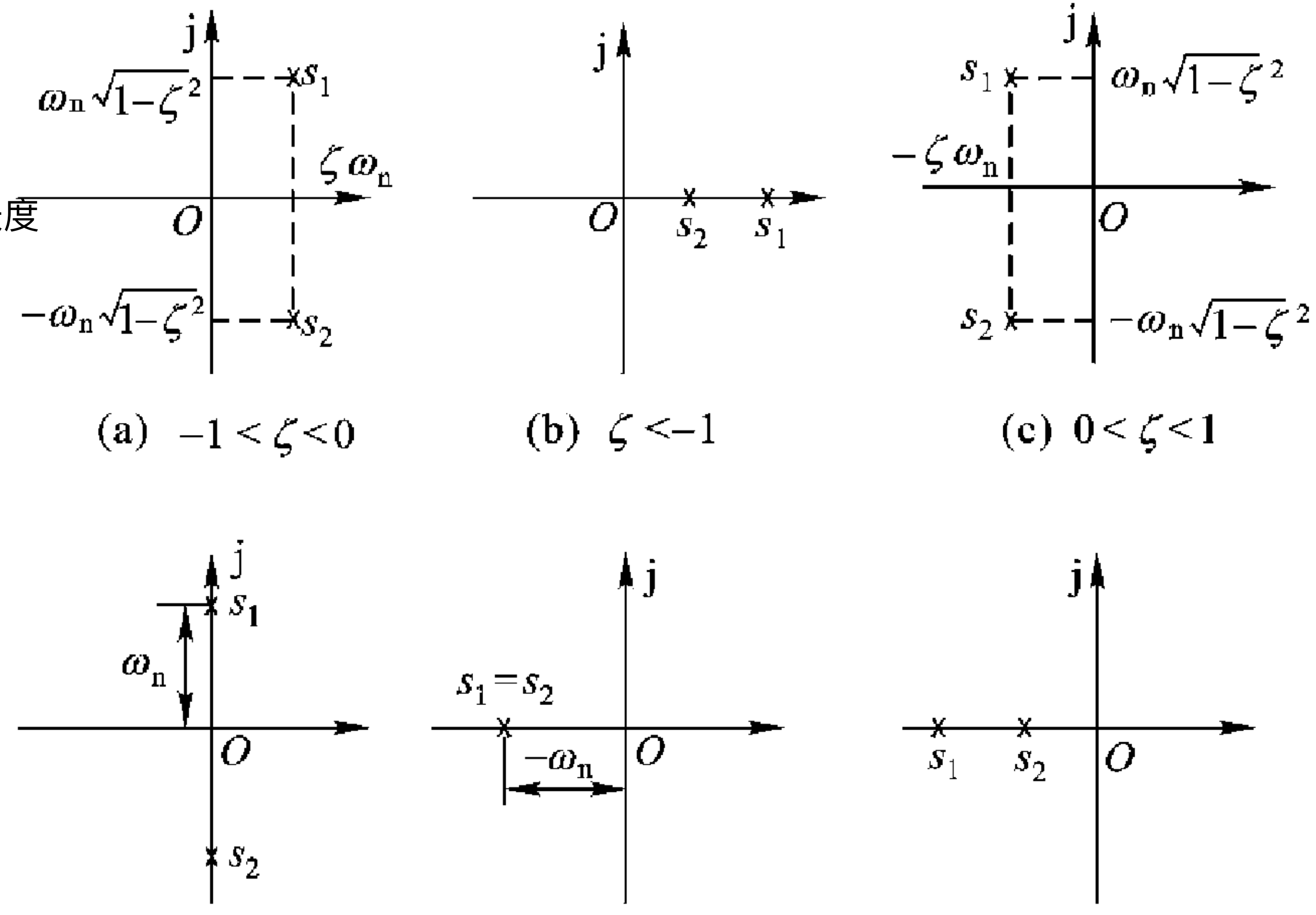
$\zeta < 0$ $\qquad s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}$ $\qquad$ negatively damped 负阻尼

The system is unstable.

Time response of a second-order system

Poles distribution for different Damping ratio

wn对映图中的长度

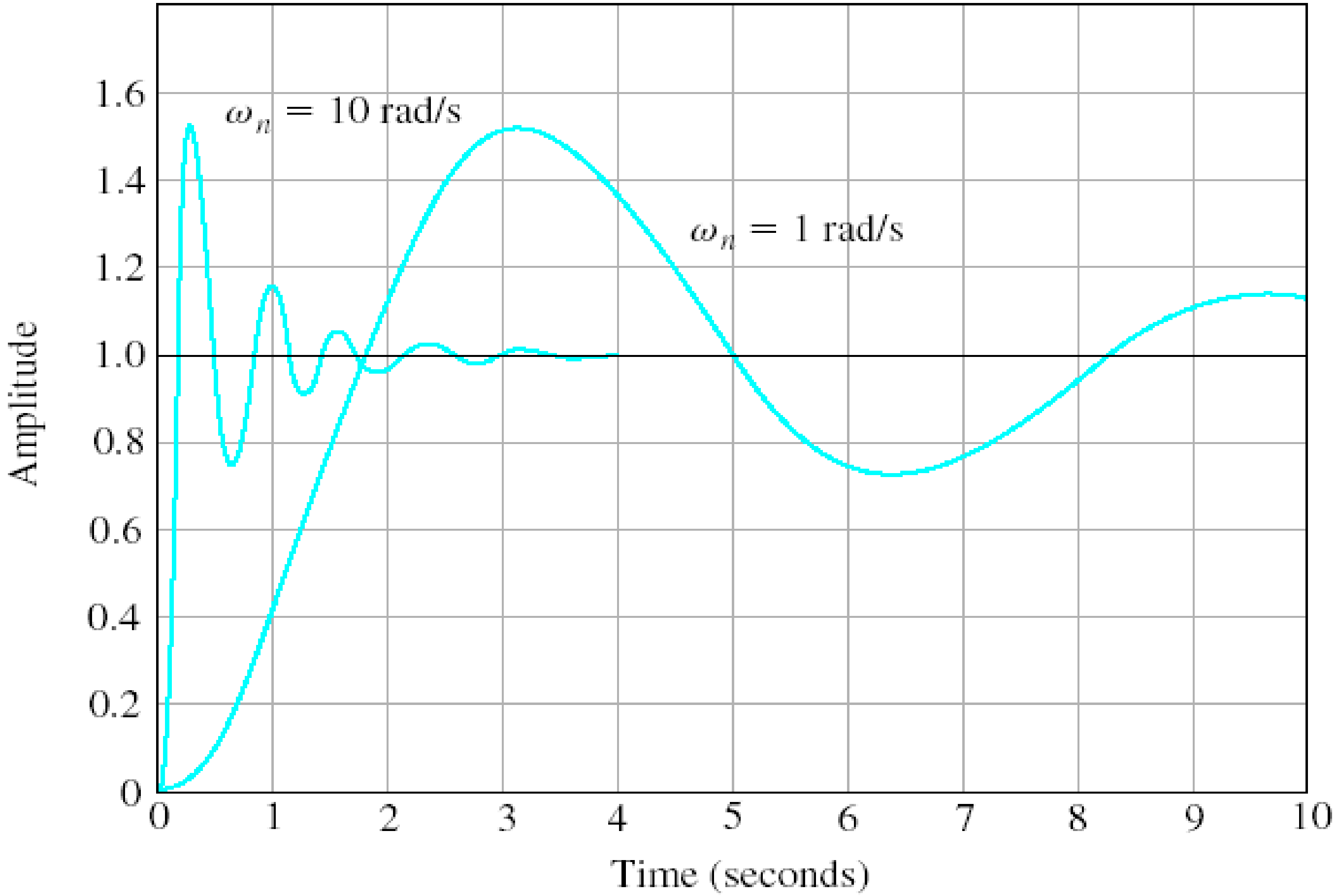


(a) $-1<\zeta<0$ (b) $\zeta<-1$ (c) $0<\zeta<1$

(d) $\zeta=0$ (e) $\zeta=1$ (f) $\zeta>1$

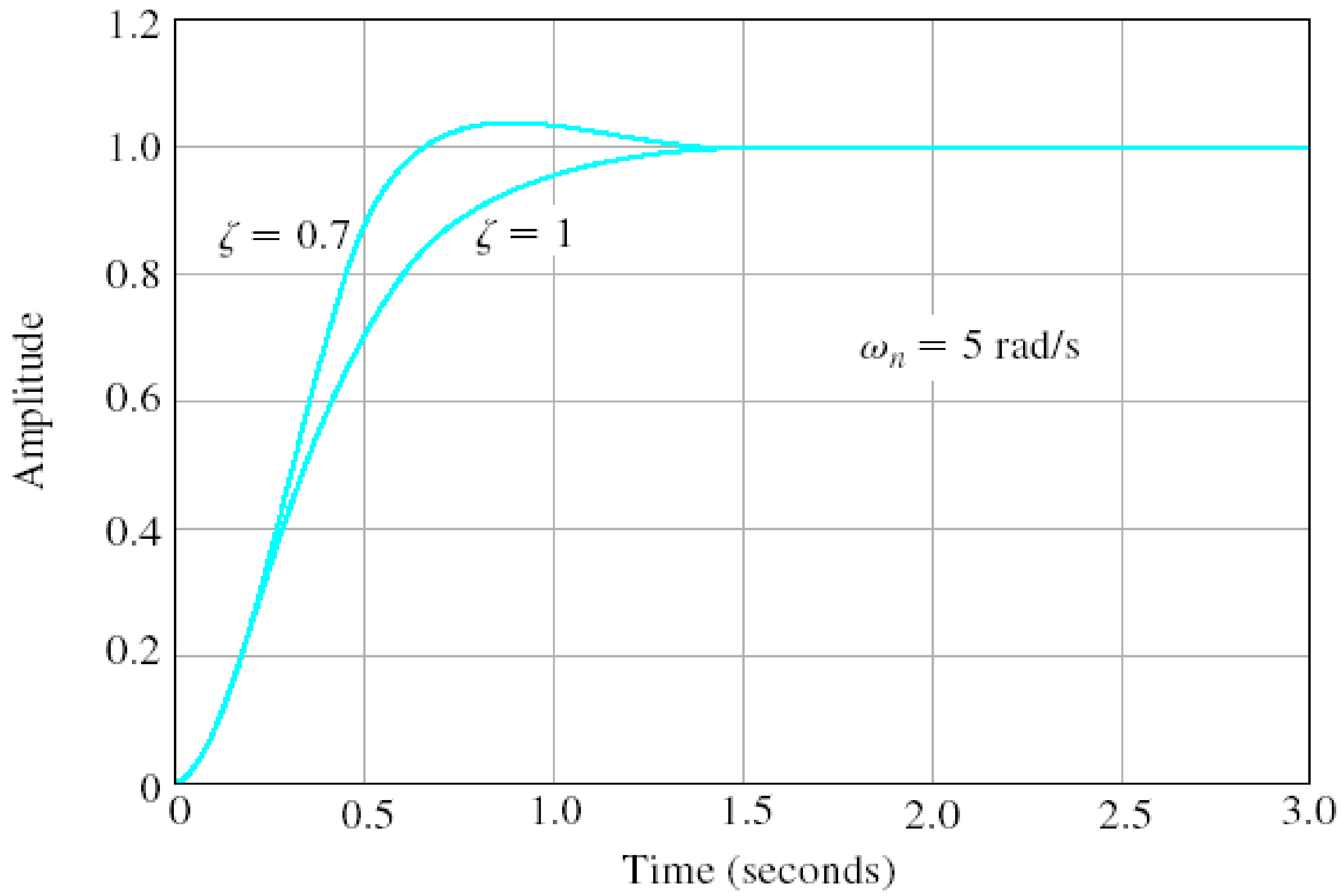
Time response of a second-order system

The step response for $\zeta = 0.2$ for $\omega_n = 1$ and $\omega_n = 10$.



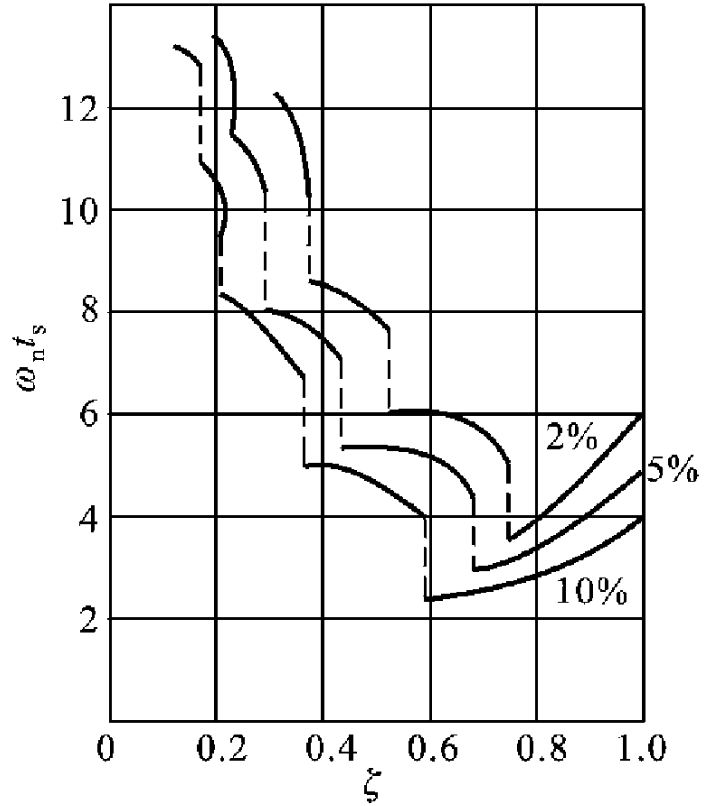
Time response of a second-order system

The step response for $\omega_n = 5$ with $\zeta = 0.7$ and $\zeta = 1$.

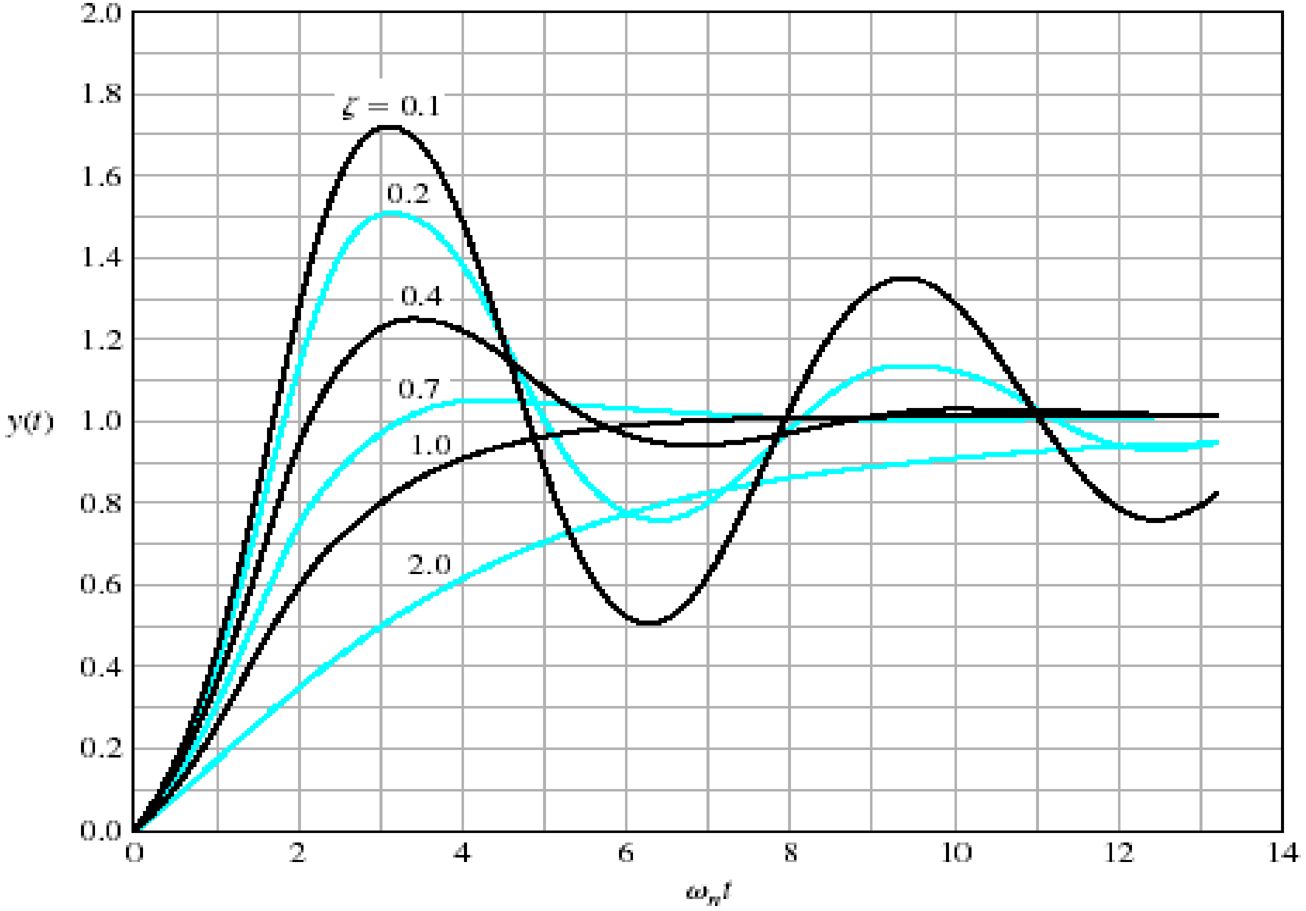


Time response of a second-order system

$\zeta = 0.707$ *Best !*



t_s: **discontinuous**



Time response of a second-order system

Example

$$G(s)=\frac{5K_A}{s(s+34.5)}$$

For the unit-step input

Try to get:

- Time-domain specifications for K_A=200: t_p, t_s, $\sigma\%$
- Time response changes if K_A=1500 or 13.5

Time response of a second-order system

Solution

Transfer function

$$\Phi(s)=\frac{5K_A}{s^2+34.5s+5K_A}$$

K_A=200

$$\Phi(s)=\frac{1\ 000}{s^2+34.5s+1\ 000}$$

$$\omega_n=\sqrt{1\ 000}=31.6 \quad \mathrm{rad\cdot s^{-1}}$$

$$\zeta=\frac{34.5}{2\omega_n}=0.545$$

$$t_p=\frac{\pi}{\omega_d}=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}=0.12\ \mathrm{s}$$

$$t_s=\frac{3.5}{\zeta\omega_n}=0.2\ \mathrm{s}$$

$$\sigma\%=e^{-\pi\zeta/\sqrt{1-\zeta^2}}\times 100\%=13\%$$

Time response of a second-order system

Solution

K_A=1500

$\omega_n = 86.2\ \mathrm{rad \cdot s^{-1}}$

$\zeta = 0.2$

$t_p = 0.037\ \mathrm{s}$

$t_s = 0.2\ \mathrm{s}$

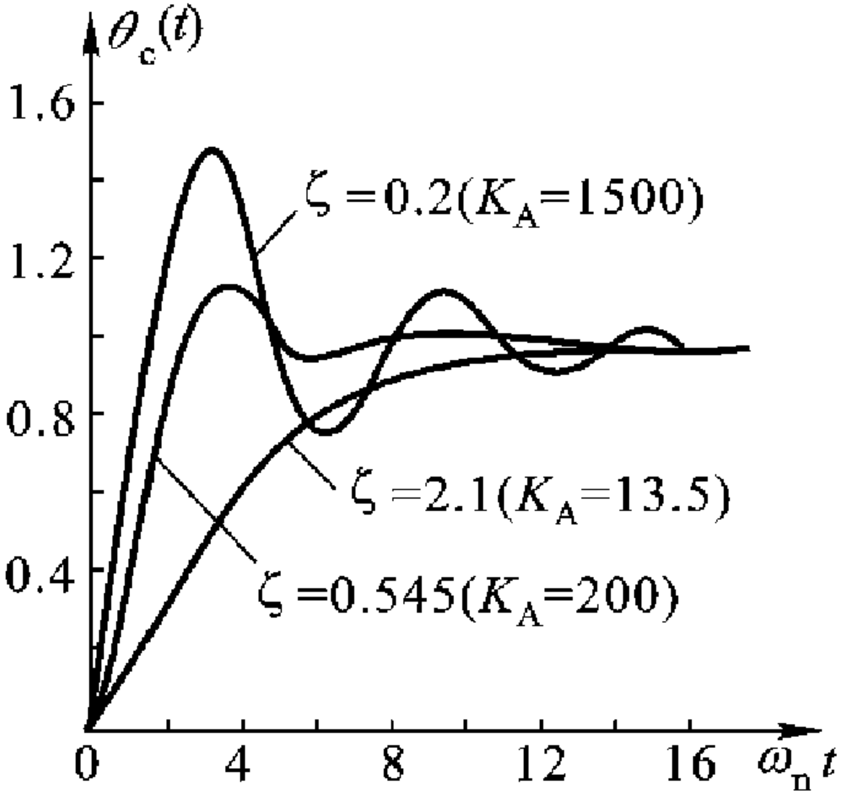
$\sigma\% = 52.7\%$

K_A=13.5

$\omega_n = 8.22\ \mathrm{rad \cdot s^{-1}}$

$\zeta = 2.1$ **Overdamped !**

$t_s = 3T_1 = 1.46\ \mathrm{s}$



Time response of a second-order system

核心

- **Poles distribution for different Damping ratio**
- **Time response of a second-order system**

续

- **Improvement performance of second systems**