

习题 2.2

1. 计算下列数列的极限:

$$(1) \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n}}{1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^n}};$$

$$(2) \lim_{n \rightarrow \infty} \frac{5^n + (-2)^n}{5^{n+1} + (-2)^{n+1}};$$

$$(3) \lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + \cdots + n^2}{n^3};$$

$$(4) \lim_{n \rightarrow \infty} \left(\frac{1}{n^2} + \frac{3}{n^2} + \cdots + \frac{2n-1}{n^2} \right);$$

$$(5) \lim_{n \rightarrow \infty} \left[\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{n(n+2)} \right]; \quad (6) \lim_{n \rightarrow \infty} \left[\frac{1^2}{n^3} + \frac{3^2}{n^3} + \cdots + \frac{(2n-1)^2}{n^3} \right];$$

$$(7) \lim_{n \rightarrow \infty} \left(1 - \frac{1}{2^2} \right) \left(1 - \frac{1}{3^2} \right) \cdots \left(1 - \frac{1}{n^2} \right); \quad (8) \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} \right) \left(1 + \frac{1}{2^2} \right) \left(1 + \frac{1}{2^4} \right) \cdots \left(1 - \frac{1}{2^{2^n}} \right).$$