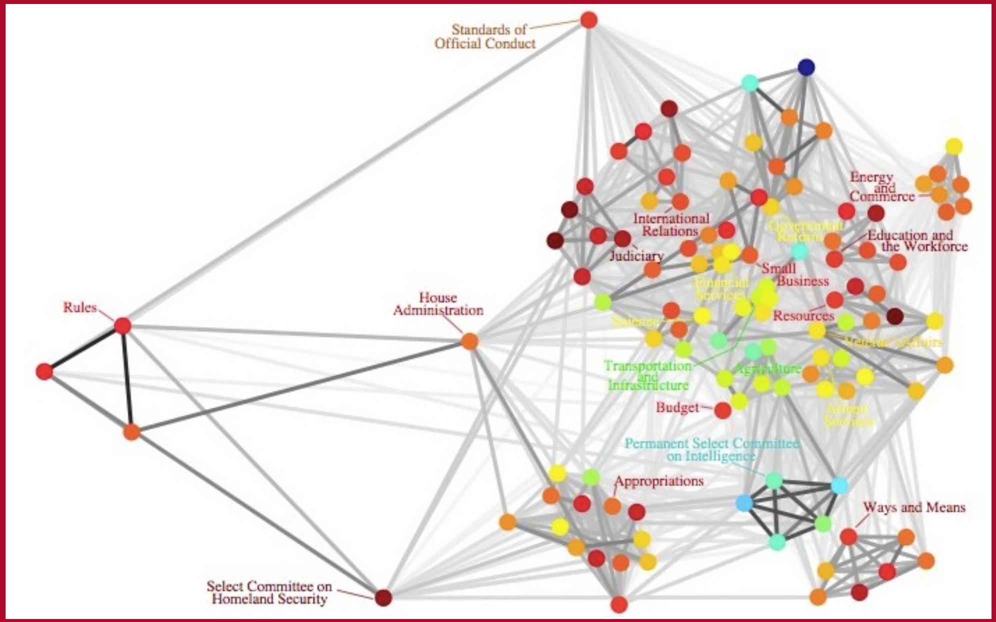
Automatic Control Theory

Chapter 4



Fan zichuan
School of Computer and Information Science
Southwest University

The root locus method

Main contents

1、The root locus concept and root locus equation

2、The root locus procedure

3、General root loci (Zero degree root loci)

The root locus method

Review

- **Open loop transfer function**
- **Phase equation**
- **The root locus procedure**

D(s)=1+G(s)H(s)=0

$$G(s)H(s) = \frac{K\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)} = -1$$

what is next

General root loci (Zero degree root loci)

The root locus method

Tips

- The number of separate loci is **equal to** the number of **poles**

- The root loci is continuous, and must be **symmetrical** with respect to the horizontal real axis

- The locus begins at the poles of P(s) and end at the zeros of P(s) as **K** increases form 0 to infinity

- The root locus on the **real** axis always lies in a section of the real axis to the left of an **odd** number of poles and zeros

- Asymptotes of the root loci

$$\sigma_A = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m} \qquad \varphi_A = \frac{(2k+1)\pi}{n-m}$$

The root locus method

- Angle of departure of the locus from a pole and the angle of arrival of the locus at a zero

$$\theta_{p_i} = (2k+1)\pi + \sum_{j=1}^{m} \angle(p_i - z_j) - \sum_{\substack{j=1 \\ j\neq i}}^{n} \angle(p_i - p_j)$$

$$\theta_{z_i} = (2k+1)\pi + \sum_{j=1}^{n} \angle(z_i - p_j) - \sum_{\substack{j=1 \\ j\neq i}}^{m} \angle(z_i - z_j)$$

- Breakaway point

$$\sum_{i=1}^{m} \frac{1}{d - z_i} = \sum_{i=1}^{n} \frac{1}{d - p_i}$$

- Breakaway angle and arrival angle

$$\boldsymbol{\theta_d = \frac{1}{l}[(2k+1)\pi + \sum_{j=1}^{m} \angle(d - z_j) - \sum_{i=l+1}^{n} \angle(d - s_i)]}$$

$$\boldsymbol{\Psi_d = \frac{1}{l}[(2k+1)\pi + \sum_{i=1}^{n} \angle(d - p_i) - \sum_{i=l+1}^{n} \angle(d - s_i)]}$$

- Locus crosses the imaginary axis

$$1+KP(jw)=0$$

- Sum and Product of all roots of closed-loop characteristic equation

$$-\sum_{i=1}^{n} s_i = a_1 \qquad (-1)^n \prod_{i=1}^{n} s_i = a_n$$

Zero degree root loci

The characteristic equation of **positive** feedback system is

$$D(s)=1-G(s)H(s)$$

And its root locus equation is

$$G(s)H(s)=1$$

The module equation is

$$K^*\frac{\prod_{i=1}^{m}|s-z_i|}{\prod_{i=1}^{n}|s-p_i|}=1$$

The phase equation is

$$\sum_{i=1}^{m}\angle(s-z_i)-\sum_{i=1}^{n}\angle(s-p_i)=2k\pi$$

Zero degree root loci

So some rules must be modified :

- The root locus on the real axis always lies in a section of the real axis to the left of an **even** number of poles and zeros.

- Asymptotes of the root loci $\varphi_A = \dfrac{2k\pi}{n-m}$ $\quad k = 0,1,2\cdots n-m-1$

- the angle of departure of the locus from a pole and the angle of arrival of the locus at a zero.

$$\theta_{p_i} = 2k\pi + \sum_{j=1}^{m} \varphi_{z_j p_i} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \theta_{p_j p_i}$$

$$\varphi_{z_i} = 2k\pi - \sum_{\substack{j=1 \\ j\neq i}}^{m} \varphi_{z_j z_i} + \sum_{j=1}^{m} \theta_{p_j z_i}$$

- Breakaway angle and arrival angle

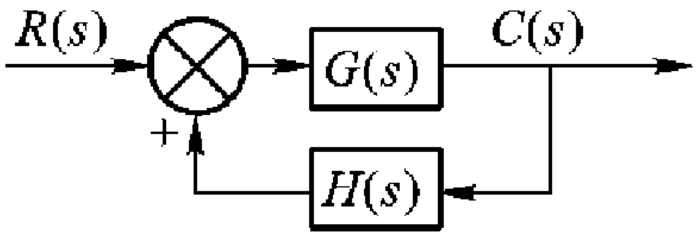
$$\theta_d = \frac{1}{l}[2k\pi + \sum_{j=1}^{m} \angle(d-z_j) - \sum_{i=l+1}^{n} \angle(d-s_i)]$$

$$\Psi_d = \frac{1}{l}[(2k+1)\pi + \sum_{i=1}^{n} \angle(d-p_i) - \sum_{i=l+1}^{n} \angle(d-s_i)]$$

会合角证明过程中，注意符号改变导致新系统为180°根轨迹

Zero degree root loci

Example 1



$$G(s)=\frac{K^*(s+2)}{(s+3)(s^2+2s+2)},H(s)=1$$

Get the root locus

Solution

$n=3, m=1$ $\quad p_1=-3, p_2=-1+j1, p_3=-1-j1 \quad z_1=-2$

root locus on the **real** axis $(-3,-\infty)$ $\quad (-2,+\infty)$

Zero degree root loci

Asymptotes equation

$$\sigma_a = \frac{\sum_{i=1}^{3} p_i - \sum_{i=1}^{1} z_i}{n - m} = \frac{-3-1+\mathrm{j}1-1-\mathrm{j}1+2}{2} = -\frac{3}{2} \qquad \varphi_a = \frac{2k\pi}{n-m} = \frac{2k\pi}{2}$$

$$k=0, \quad \varphi_a = 0^\circ$$
$$k=1, \quad \varphi_a = 180^\circ$$

Strat point angle

$$\theta_{p_2} = \varphi_{z_1 p_2} - \theta_{p_1 p_2} - \theta_{p_3 p_2} = 45^\circ - 26.6^\circ - 90^\circ = -71.6^\circ \qquad \theta_{p_3} = +71.6^\circ$$

Breakaway point

$$\sum_{i=1}^{3} \frac{1}{d - p_i} = \sum_{i=1}^{1} \frac{1}{d - z_i}$$

$$\frac{1}{d+3} + \frac{1}{d+1-\mathrm{j}1} + \frac{1}{d+1+\mathrm{j}1} = \frac{1}{d+2}$$

$$d_1 = -0.8$$

$$d_2 = -2.35 + \mathrm{j}0.85, \; d_3 = -2.35 - \mathrm{j}0.85$$

abandon

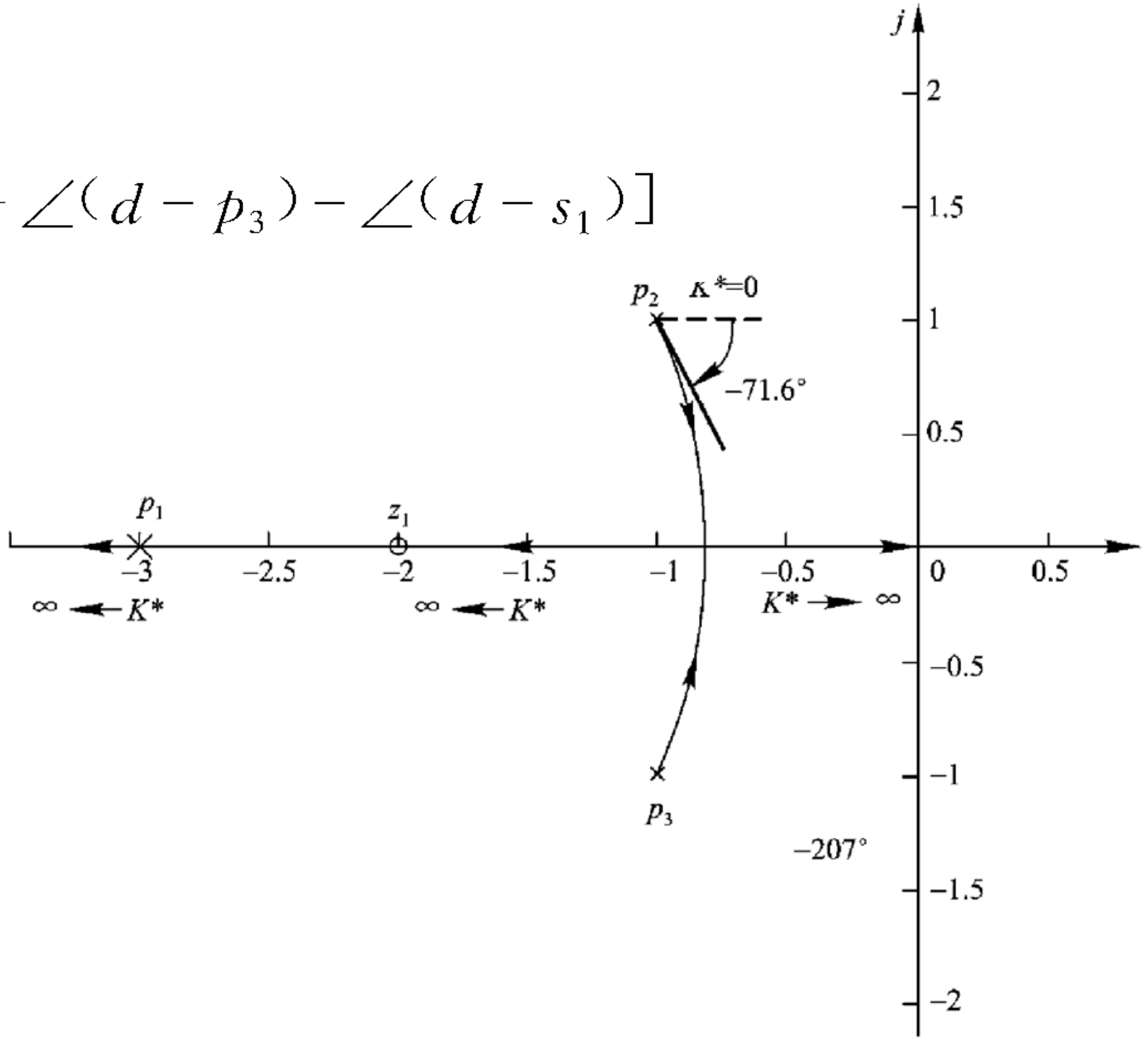
Zero degree root loci

Arrival angle

$$\begin{aligned}\varphi_{\mathrm{d}} &= \frac{1}{2}[(2k+1)\pi + \angle(d-p_1) + \angle(d-p_2) + \angle(d-p_3) - \angle(d-s_1)] \\ &= \frac{1}{2}[(2k+1)\pi + 0 + 0 + 0]\end{aligned}$$

$$k=0,\quad \varphi_{\mathrm{d}} = \frac{\pi}{2}$$

$$k=1,\quad \varphi_{\mathrm{d}} = -\frac{\pi}{2}$$



Root loci varies with arbitrary parameters

$$G(s)H(s)+1=0 \qquad A\frac{P(s)}{Q(s)}+1=0$$

Equivalent open loop transfer function $G_1(s)H_1(s)=A\frac{P(s)}{Q(s)}$

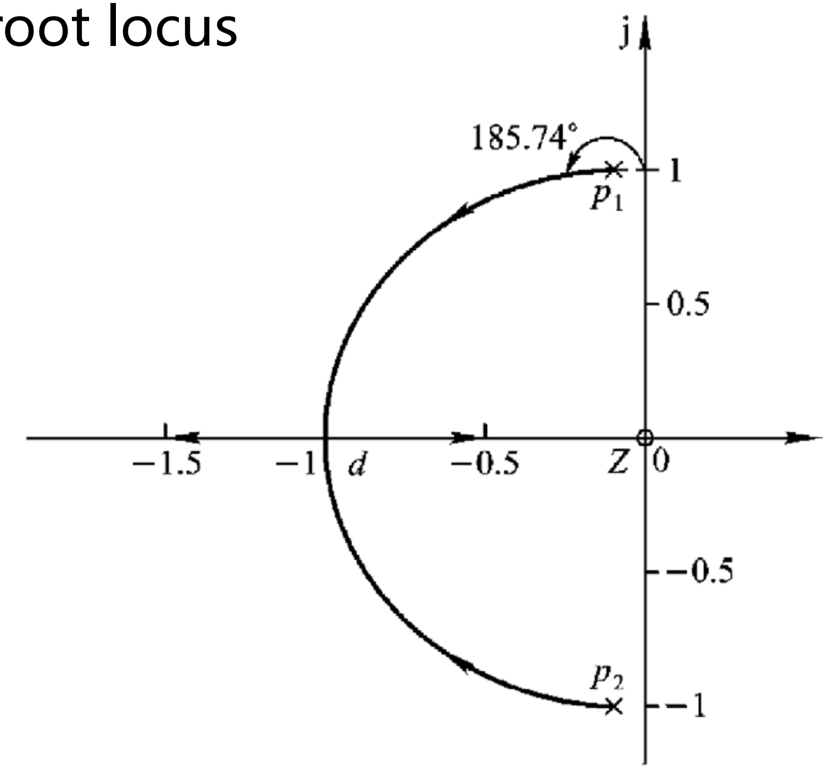
Example 2

Root loci varies with zeros of the open loop transfer function

$$G(s)H(s)=\frac{5(1+T_d s)}{s(1+5s)}$$

$T_d=0 \to -\infty$

Get the root locus



Solution

$$D(s)=5s^2+s+5T_d s+5=0$$

$$G_1(s)H_1(s)=A\frac{P(s)}{Q(s)}=\frac{T_d s}{s^2+0.2s+1}$$

Example 3

Root loci varies with poles of the open loop transfer function

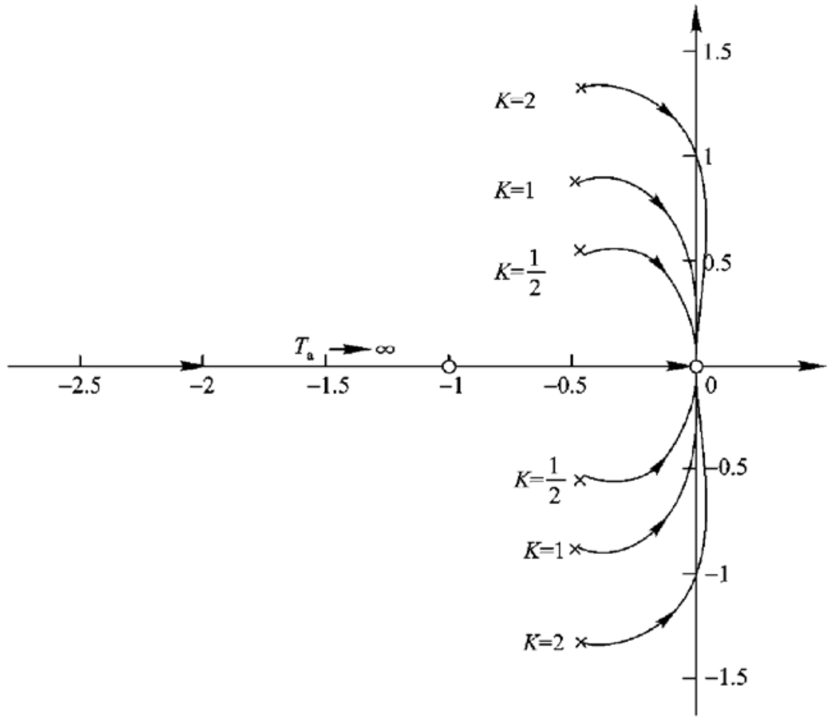
$$G(s)H(s)=\frac{K}{s(s+1)(T_a s+1)}$$

$T_a = 0 \rightarrow -\infty$ Get the root locus (K=0.5,1,2)

Solution

$$D(s)=s(s+1)(T_a s+1)+K=0$$

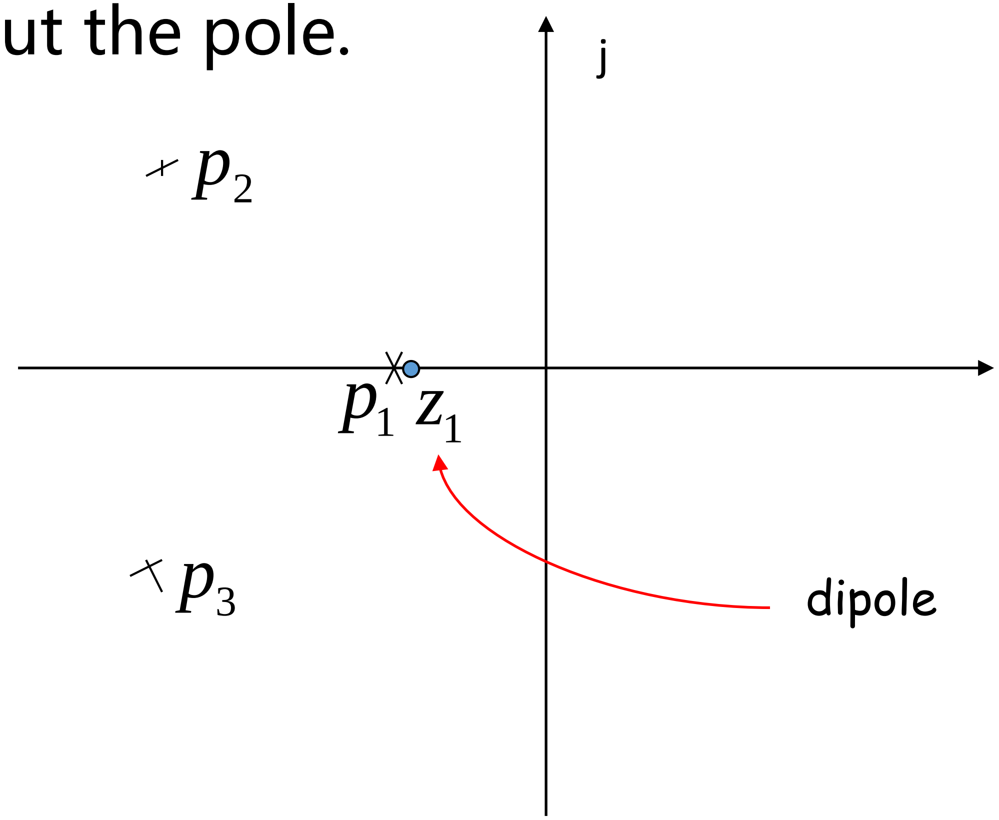
$$G_1(s)H_1(s)=\frac{T_a \bullet s^2(s+1)}{s^2+s+K}$$



The dominant pole and dipole

The dominant pole:

The dominant pole of a system are the poles that are closest to the *jw* axis in the left-half s-plane, and no has the zero about the pole.



The root locus method

核心

- **Definition of root locus**
- **Open loop transfer function**
- **Phase equation**

续

- **Frequency Response methods**