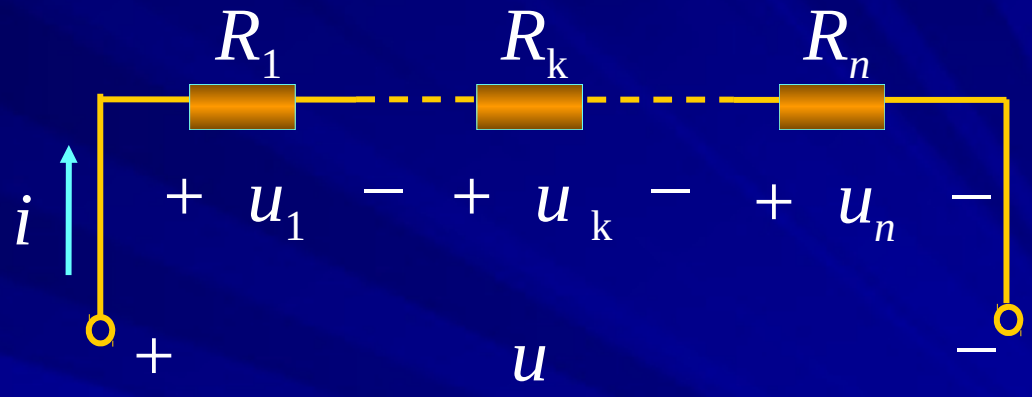
2.2 电阻的串联和并联

1. 电阻串联

① 电路特点

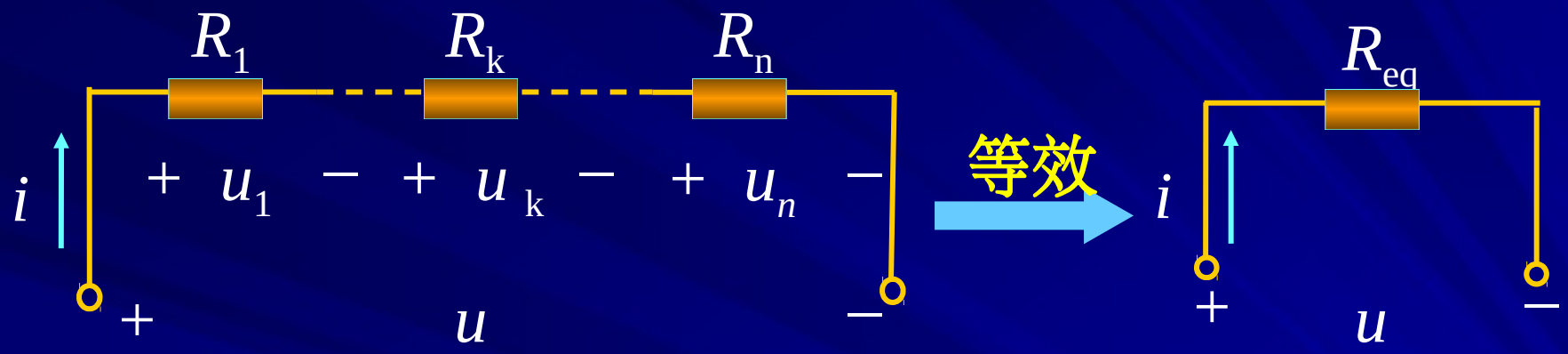


(a) 各电阻顺序连接，流过同一电流 (KCL)；

(b) 总电压等于各串联电阻的电压之和 (KVL)。

$$u = u_1 + \cdots + u_k + \cdots + u_n$$

② 等效电阻



由欧姆定律

$$u = R_1 i + \cdots + R_K i + \cdots + R_n i = (R_1 + \cdots + R_n) i = R_{eq} i$$

$$R_{eq} = R_1 + \cdots + R_k + \cdots + R_n = \sum_{k=1}^{n} R_k > R_k$$

结论 串联电路的总电阻等于各分电阻之和。

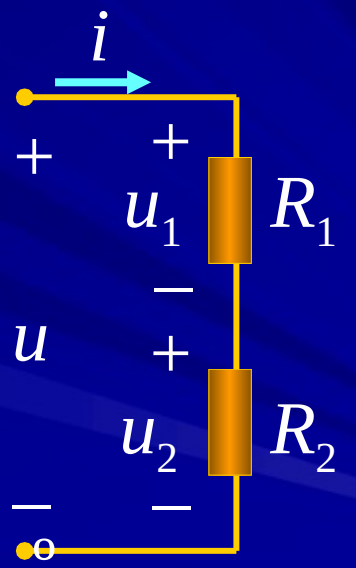
③ 串联电阻的分压

$$u_k = R_k i = R_k \frac{u}{R_{eq}} = \frac{R_k}{R_{eq}} u < u$$

表明 电压与电阻成正比，因此串联电阻电路可作分压电路。

例 两个电阻的分压：

$$u_1 = \frac{R_1}{R_1 + R_2} u \qquad u_2 = \frac{R_2}{R_1 + R_2} u$$



④ 功率

$$p_1=R_1i^2\ ,\quad p_2=R_2i^2\ ,\quad \cdots,\quad p_n=R_ni^2$$

$$p_1:p_2:\ldots:p_n=R_1:R_2:\ldots:R_n$$

总功率 $$p=R_{eq}i^2=(R_1+R_2+\ldots+R_n)\,i^2$$

$$=R_1i^2+R_2i^2+\ldots+R_ni^2$$

$$=p_1+p_2+\ldots+p_n$$

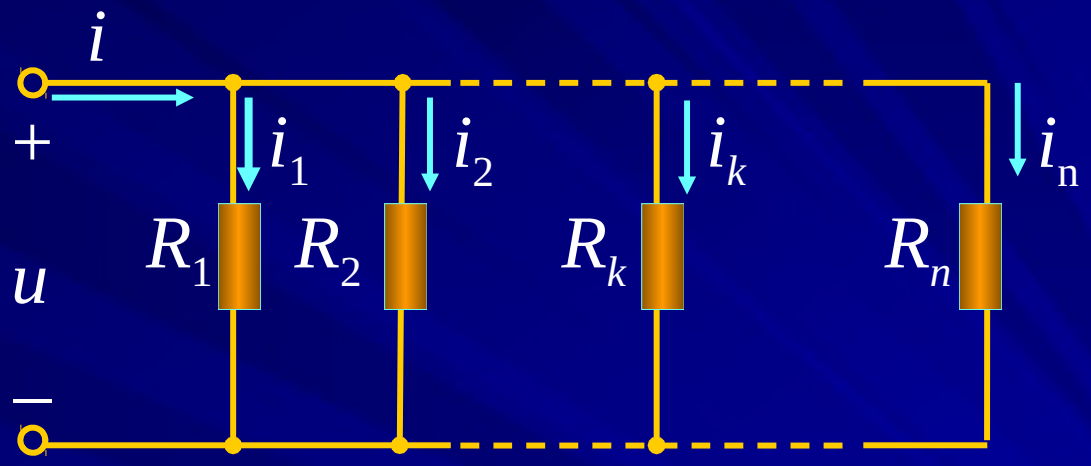
表明

① 电阻串联时，各电阻消耗的功率与电阻大小成正比；

② 等效电阻消耗的功率等于各串联电阻消耗功率的总和。

2. 电阻并联

① 电路特点

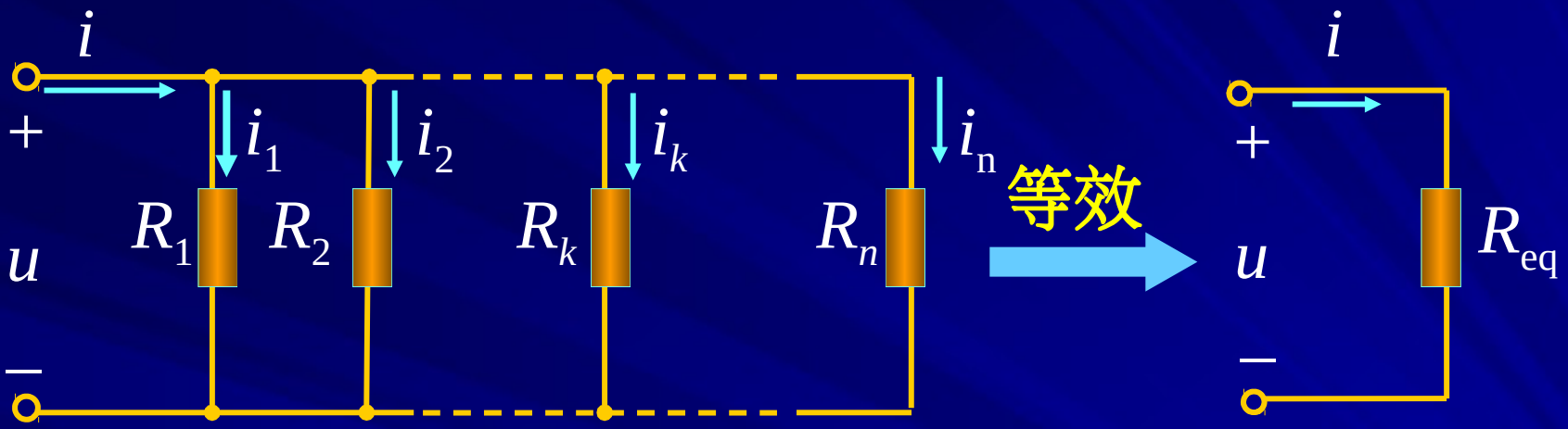


(a) 各电阻两端为同一电压（KVL)；

(b) 总电流等于流过各并联电阻的电流之和 (KCL)。

$$i = i_1 + i_2 + \ldots + i_k + \ldots + i_n$$

② 等效电阻



由 KCL：

$$i = i_1 + i_2 + \ldots + i_k + \ldots + i_n$$

$$= u/R_1 + u/R_2 + \ldots + u/R_n$$

$$= u(1/R_1 + 1/R_2 + \ldots + 1/R_n) = uG_{eq}$$

$$G_{eq} = G_1 + G_2 + \cdots + G_n = \sum_{k=1}^{n} G_k > G_k$$

结论 等效电导等于并联的各电导之和。

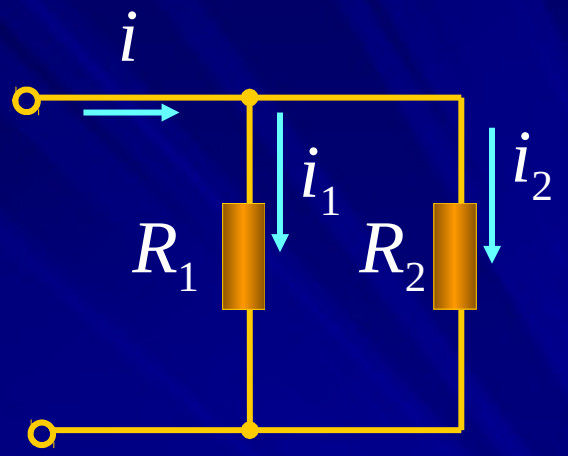
$$\frac{1}{R_{eq}}=G_{eq}=\frac{1}{R_1}+\frac{1}{R_2}+\cdots+\frac{1}{R_n} \quad \text{即} \quad R_{eq}<R_k$$

③ 并联电阻的分流

电流分配与电导成正比

$$\frac{i_k}{i}=\frac{u/R_k}{u/R_{eq}}=\frac{G_k}{G_{eq}} \longrightarrow i_k=\frac{G_k}{G_{eq}}i$$

例　两电阻的分流：



$$R_{eq}=\frac{1}{1/R_1+1/R_2}=\frac{R_1R_2}{R_1+R_2}$$

$$i_1=\frac{1/R_1}{1/R_1+1/R_2}i=\frac{R_2i}{R_1+R_2}$$

$$i_2=\frac{1/R_2}{1/R_1+1/R_2}i=\frac{R_1i}{R_1+R_2}=(i-i_1)$$

④ 功率

$$p_1=G_1u^2\ ,\quad p_2=G_2u^2\ ,\quad \cdots,\quad p_n=G_nu^2$$

$$p_1:p_2:\ldots:p_n=G_1:G_2:\ldots:G_n$$

总功率

$$p=G_{eq}u^2=(G_1+G_2+\ldots+G_n)\,u^2$$

$$=G_1u^2+G_2u^2+\ldots+G_nu^2$$

$$=p_1+p_2+\ldots+p_n$$

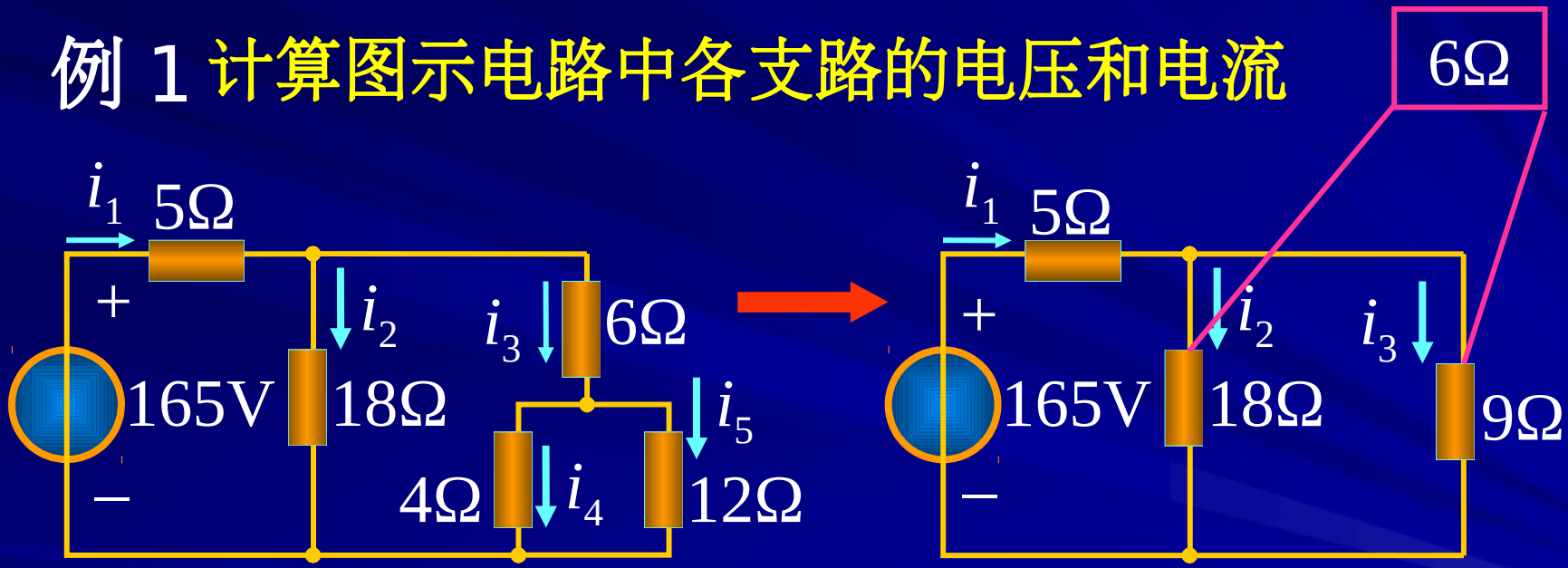
表明 ① 电阻并联时，各电阻消耗的功率与电阻大小成反比；

② 等效电阻消耗的功率等于各并联电阻消耗功率的总和

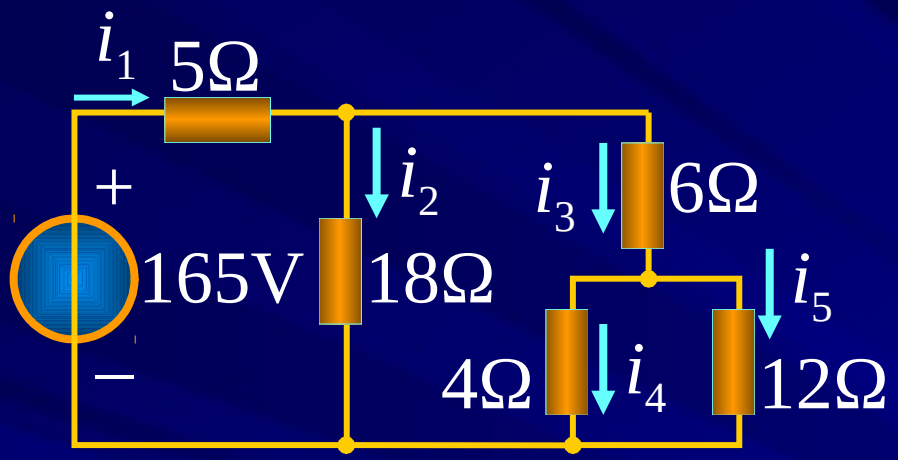
3. 电阻的串并联

电路中有电阻的串联，又有电阻的并联，这种连接方式称电阻的串并联。

例 1 计算图示电路中各支路的电压和电流



$i_1 = 165/11 = 15\text{A}$　　$u_2 = 6i_1 = 6 \times 15 = 90\text{V}$

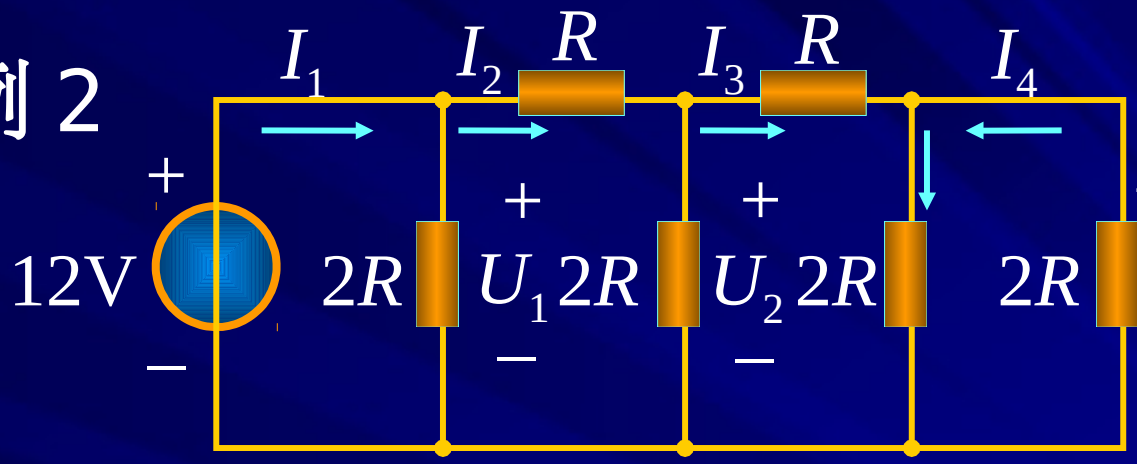


$i_2 = 90/18 = 5\text{A}$　　$u_3 = 6i_3 = 6 \times 10 = 60\text{V}$

$i_3 = 15 - 5 = 10\text{A}$　　$u_4 = 3i_3 = 30\text{V}$

$i_4 = 30/4 = 7.5\text{A}$　　$i_5 = 10 - 7.5 = 2.5\text{A}$

例 2



求：I_1，I_4，

解

① 用分流方法做

$$I_4 = -\frac{1}{2}I_3 = -\frac{1}{4}I_2 = -\frac{1}{8}I_1 = -\frac{1}{8}\frac{12}{R} = -\frac{3}{2R}$$

$$U_4 = -I_4 \times 2R = 3\text{V} \qquad I_1 = \frac{12}{R}$$

② 用分压方法做

$$U_4 = \frac{U_2}{2} = \frac{1}{4}U_1 = 3\text{V} \qquad I_4 = -\frac{3}{2R}$$

从以上例题可得求解串、并联电路的一般步骤:

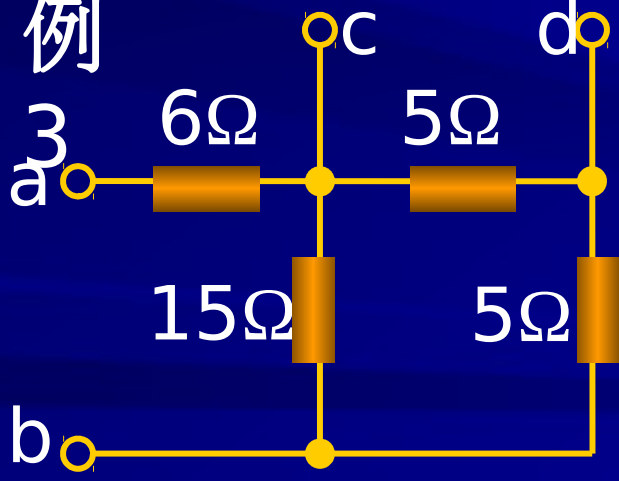
① 求出等效电阻或等效电导;

② 应用欧姆定律求出总电压或总电流;

③ 应用欧姆定律或分压、分流公式求各电阻上的电流和电压

以上的关键在于识别各电阻的串联、并联关系!

例3



求: R_{ab}, R_{cd}

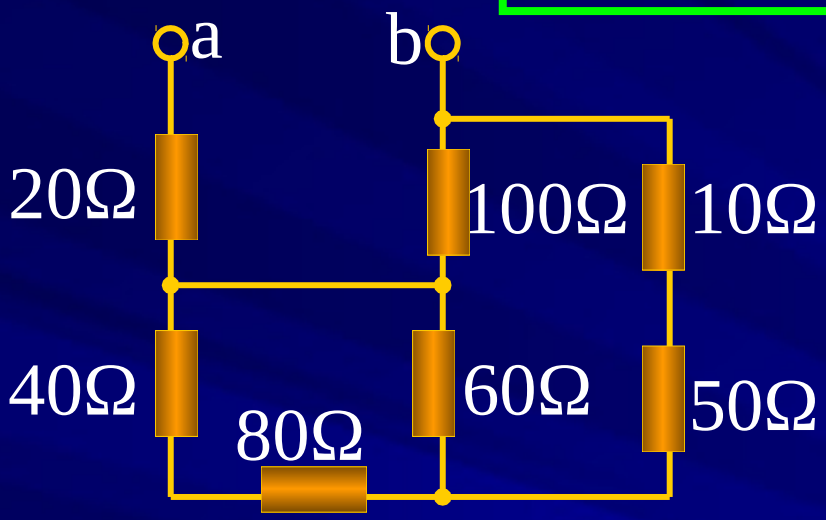
$$R_{ab} = (5+5)//15+6 = 12\Omega$$

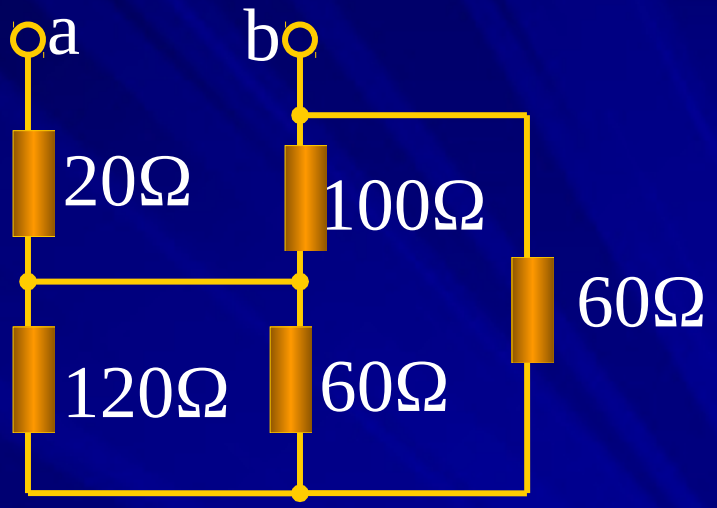
$$R_{cd} = (15+5)//5 = 4\Omega$$

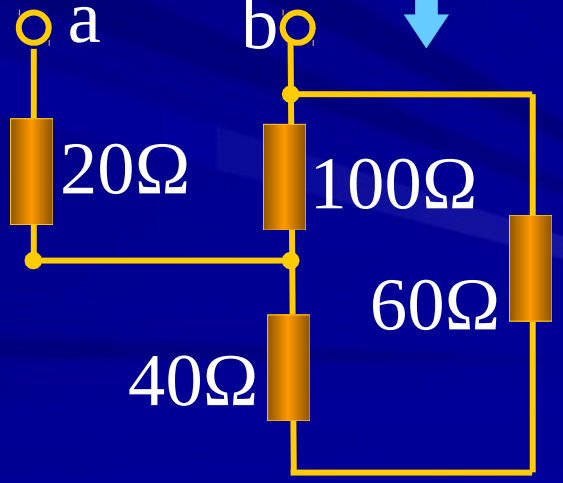
注意 等效电阻针对端口而言

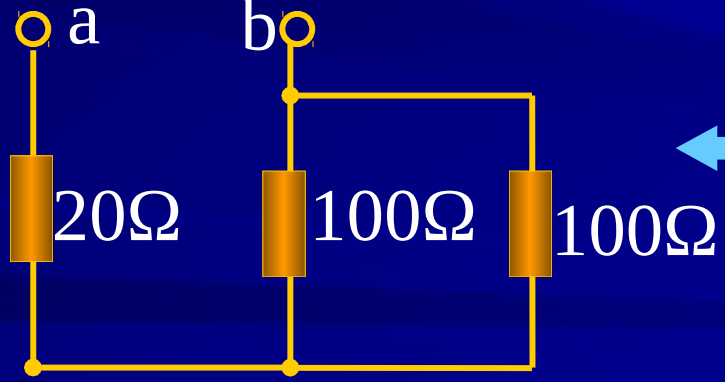
例 4 求：R_{ab}

$R_{ab} = 70\Omega$

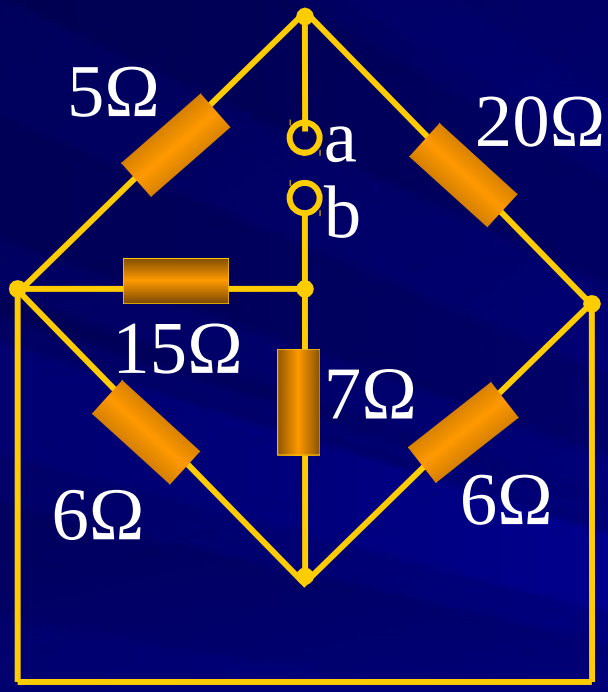




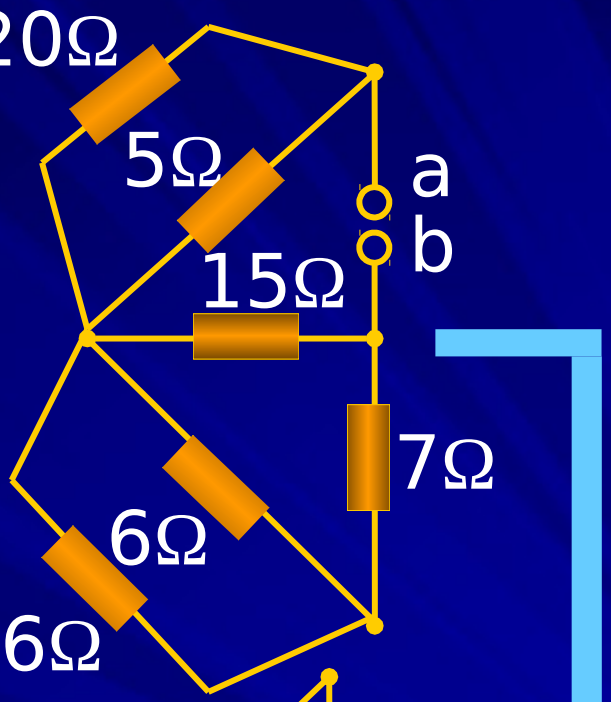


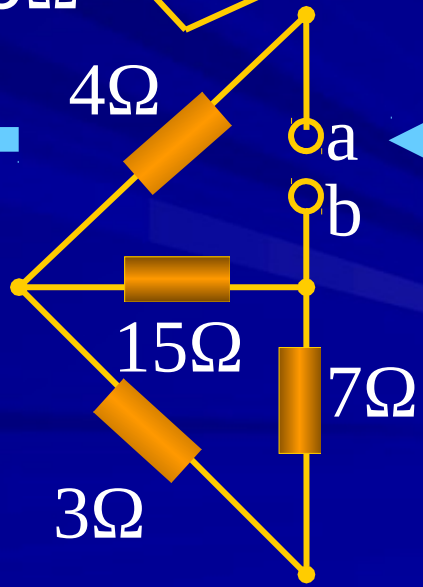


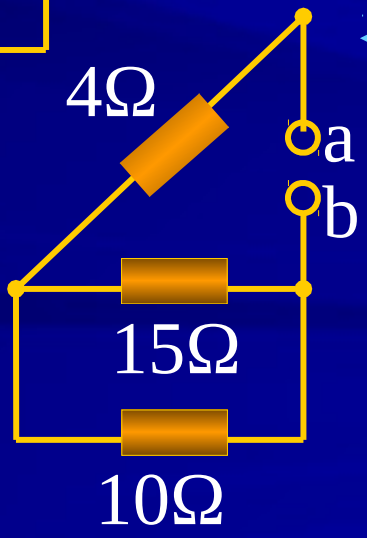
例 5 求：R_{ab}



缩短无电阻支路



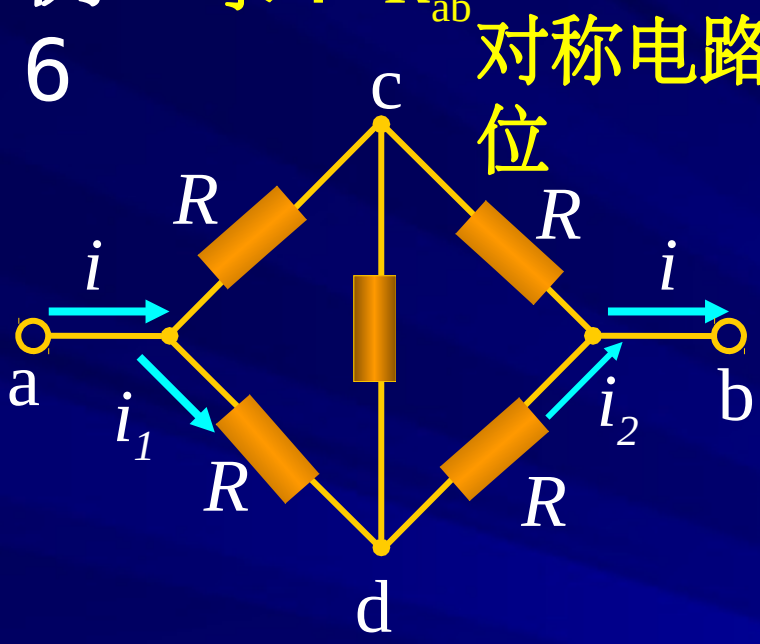




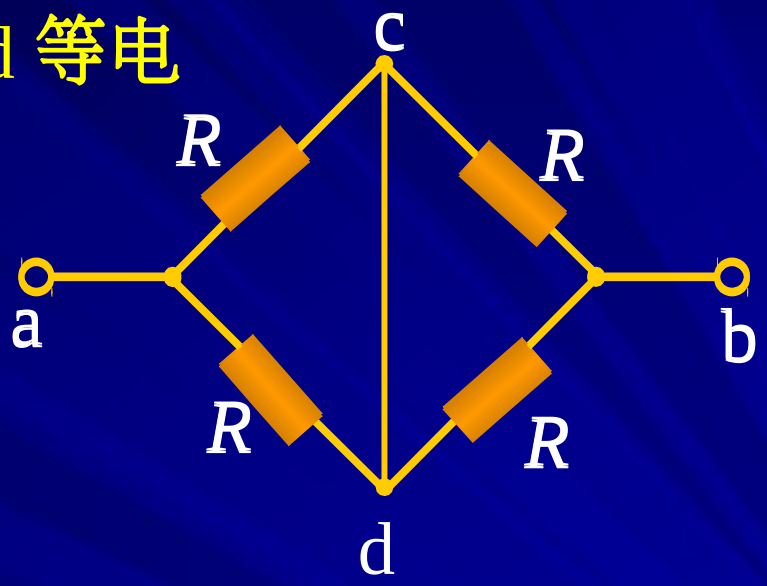
$R_{ab} = 10\Omega$

例6 求：R_{ab}

对称电路 c、d 等电位



短路



根据电流分配 $\longrightarrow$ $i_1 = \frac{1}{2} i = i_2$ $\qquad R_{ab} = R$

$$u_{ab} = i_1 R + i_2 R = (\frac{1}{2} i + \frac{1}{2} i) R = iR$$

$$R_{ab} = \frac{u_{ab}}{i} = R$$