

习题 5.4

1. 求下列不定积分:

- (1) $\int (\sin x + 3e^x) dx$;
- (2) $\int (x^a + a^x) dx \quad (a > 0 \text{ 且 } a \neq 1)$;
- (3) $\int (3 + \cot^2 x) dx$;
- (4) $\int \sec x (\sec x - \tan x) dx$;
- (5) $\int (\sqrt{x} + 1)(x - \sqrt{x} + 1) dx$;
- (6) $\int \left(x + \frac{1}{x}\right)^2 dx$;
- (7) $\int (1 - x^2) \sqrt{x} \sqrt{x} dx$;
- (8) $\int \left(\frac{3}{1+x^2} - \frac{2}{\sqrt{1-x^2}}\right) dx$;
- (9) $\int \frac{x^4}{1+x^2} dx$;
- (10) $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} dx$;
- (11) $\int \cos^2 \frac{x}{2} dx$;
- (12) $\int \frac{\cos 2x}{\cos x + \sin x} dx$;
- (13) $\int \frac{dx}{\cos^2 x \sin^2 x}$;
- (14) $\int \frac{1 + \cos^2 x}{1 + \cos 2x} dx$;

2. 曲线 $y = f(x)$ 经过点 $(e, -1)$, 且在任一点处的切线斜率为该点横坐标的倒数, 求该曲线的方程.

3. 求下列不定积分:

- (1) $\int (5 - 3x)^2 dx$;
- (2) $\int \frac{dx}{\sqrt[3]{3-2x}}$;
- (3) $\int x e^{x^2} dx$;
- (4) $\int \cos^2 5x dx$;
- (5) $\int e^x \sin(e^x) dx$;
- (6) $\int \frac{dx}{e^x - e^{-x}}$;
- (7) $\int \frac{dx}{x \ln x}$;

$$(8) \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx;$$

$$(9) \int \frac{dx}{2+5x^2};$$

$$(10) \int \frac{x^2}{4+x^6} dx;$$

$$(11) \int \frac{2x-5}{(x^2-5x+8)^2} dx;$$

$$(12) \int \frac{dx}{x^2-2x+2};$$

$$(13) \int \frac{x^2}{\sqrt[4]{1-2x^3}} dx;$$

$$(14) \int \frac{x dx}{\sqrt{5+x-x^2}};$$

$$(15) \int \frac{1-x}{\sqrt{9-4x^2}} dx;$$

$$(16) \int \frac{\sin x + \cos x}{\sqrt[3]{\sin x - \cos x}} dx;$$

$$(17) \int \frac{1+\ln x}{(x \ln x)^2} dx;$$

$$(18) \int \frac{dx}{1+e^x};$$

$$(19) \int \frac{\arctan \sqrt{x}}{\sqrt{x}(1+x)} dx;$$

$$(20) \int \frac{x \tan \sqrt{1+x^2}}{\sqrt{1+x^2}} dx;$$

4. 求下列不定积分:

$$(1) \int \frac{dx}{1+\sqrt{2x}};$$

$$(2) \int \frac{dx}{\sqrt{(1-x^2)^3}};$$

$$(3) \int \frac{dx}{x\sqrt{x^2-1}};$$

$$(4) \int \frac{dx}{x\sqrt{a^2-x^2}} \quad (a>0);$$

$$(5) \int \frac{dx}{x^2\sqrt{1+x^2}};$$

$$(6) \int \frac{\sqrt{x^2-9}}{x} dx;$$

$$(7) \int x^2 \cdot \sqrt[3]{1-x} dx;$$

- (8) $\int \frac{dx}{1+\sqrt{1-x^2}};$
- (9) $\int \frac{x^2}{\sqrt{a^2-x^2}} dx \quad (a>0);$
- (10) $\int \frac{dx}{\sqrt{1+e^x}};$
- (11) $\int e^x \sqrt{1-e^{2x}} dx;$
- (12) $\int \frac{dx}{\sqrt{(x-a)(b-x)}} \quad (a<x<b);$

5. 求下列不定积分:

- (1) $\int x e^{2x} dx;$
- (2) $\int x \ln(x-1) dx;$
- (3) $\int x \cos^2 x dx;$
- (4) $\int \arctan x dx;$
- (5) $\int x^2 \arctan x dx;$
- (6) $\int x^2 \ln x dx;$
- (7) $\int e^{-2x} \sin \frac{x}{2} dx;$
- (8) $\int \frac{\arcsin x}{\sqrt{1-x}} dx;$
- (9) $\int (\arcsin x)^2 dx;$
- (10) $\int \ln(x+\sqrt{1+x^2}) dx;$
- (11) $\int x \ln \frac{1+x}{1-x} dx;$
- (12) $\int \frac{\ln \cos x}{\cos^2 x} dx;$
- (13) $\int \cos(\ln x) dx;$
- (14) $\int \sin x \ln(\tan x) dx;$
- (15) $\int \cos \sqrt{x} dx;$
- (16) $\int \sqrt{x} e^{\sqrt{x}} dx;$
- (17) $\int \frac{\arctan \sqrt{x}}{\sqrt{1+x}} dx;$
- (18) $\int \frac{\arcsin x}{x^2} dx;$
- (19) $\int \frac{x+\sin x}{1+\cos x} dx;$
- (20) $\int \ln(\sqrt{1+x}+\sqrt{1-x}) dx;$

6. 求分别满足下列条件的函数 $f(x)$ 的表达式:

(1) $f'(x^2) = 1 + x \quad (x > 0)$;

(2) $f'(\sin^2 x) = \cos x + \tan^2 x$;

7. 已知 $f(x)$ 的一个原函数为 $\frac{\sin x}{1+x \sin x}$, 求 $\int f(x) f'(x) dx$.

8. 设 $f(\ln x) = \frac{\ln(1+x)}{x}$, 求 $\int f(x) dx$.

9. 求不定积分 $I_1 = \int \frac{\cos x}{\sin x + \cos x} dx$ 与 $I_2 = \int \frac{\sin x}{\sin x + \cos x} dx$.

10. 求下列不定积分的递推表达式 ($n \in \mathbb{N}_+$):

(1) $I_n = \int \sin^n x dx$;

(2) $I_n = \int \tan^n x dx$;

(3) $I_n = \int x^\alpha \ln^n x dx \quad (\alpha \neq -1)$;

(4) $I_n = \int \frac{x^n}{\sqrt{1-x^2}} dx$;

11. 求下列不定积分:

(1) $\int \frac{x+1}{x^2-3x+2} dx$;

(2) $\int \frac{dx}{(x-1)(x+1)^2}$;

(3) $\int \frac{2x+3}{(x^2-1)(x^2+1)} dx$;

(4) $\int \frac{x^5+x^4-8}{x^3+x} dx$;

(5) $\int \frac{x^4+1}{(x-1)(x^2+1)} dx$;

(6) $\int \frac{x^3}{(1+x^8)^2} dx$;

(7) $\int \frac{x^2}{(x+1)^{100}} dx$;

(8) $\int \frac{x^2+1}{x^4+1} dx$.

12. 求下列不定积分:

(1) $\int \frac{dx}{1-\sin x}$;

(2) $\int \frac{dx}{4+5\cos x}$;

(3) $\int \frac{\tan x}{4\sin^2 x + 9\cos^2 x} dx$;

(4) $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x} \quad (ab \neq 0)$;

$$(5) \int \sin 5x \cos 3x dx ;$$

$$(6) \int \frac{\sin x \cos x}{1 + \sin^4 x} dx ;$$

$$(7) \int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx ;$$

$$(8) \int \frac{\sin^2 x}{1 + \sin^2 x} dx ;$$

$$(9) \int \frac{dx}{\sin x \cos^4 x} ;$$

$$(10) \int \frac{\sin^2 x}{\cos^3 x} dx ;$$

13. 求下列不定积分:

$$(1) \int \frac{\sqrt{x}}{1 + \sqrt[4]{x^3}} dx ;$$

$$(2) \int \frac{\sqrt{3+2x}}{x} dx ;$$

$$(3) \int \sqrt{\frac{x+1}{x-1}} dx ;$$

$$(4) \int \frac{dx}{\sqrt{x(1+x)}} ;$$

$$(5) \int \frac{x^2}{\sqrt{1+x-x^2}} dx ;$$

$$(6) \int \frac{dx}{x\sqrt{x^2+3x-4}} ;$$

$$(7) \int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx ;$$

$$(8) \int \frac{dx}{\sqrt[3]{(x-2)(x+1)^2}} .$$