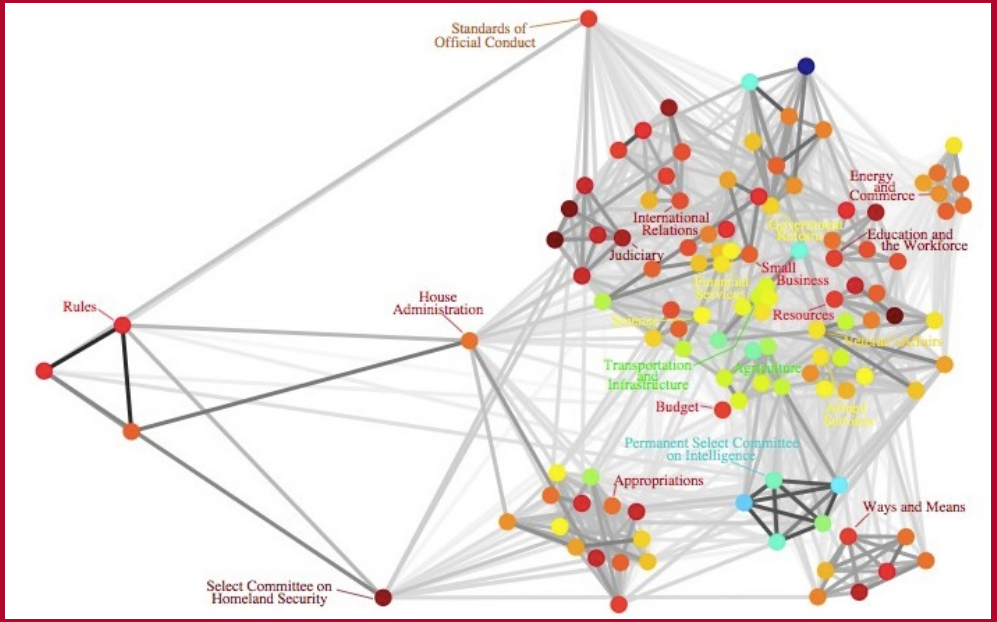
Automatic Control Theory

Chapter 3



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Main contents

1.Typical test signals for the time response of control systems.

2.The unit-step response and time-domain specifications.

3.Time response of first-order and second-order systems.

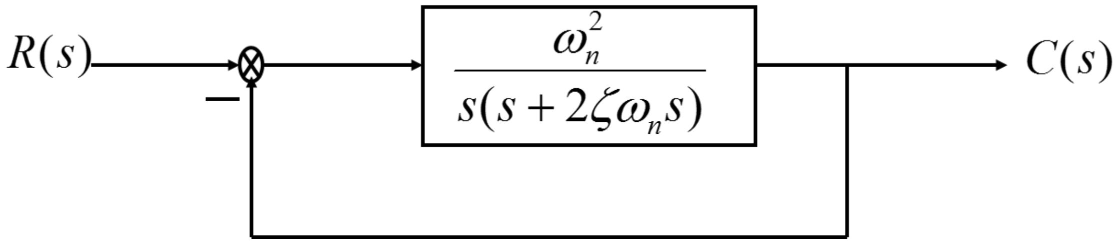
4.Improvement performance of second systems.

5. Condition for a feedback system to be stable

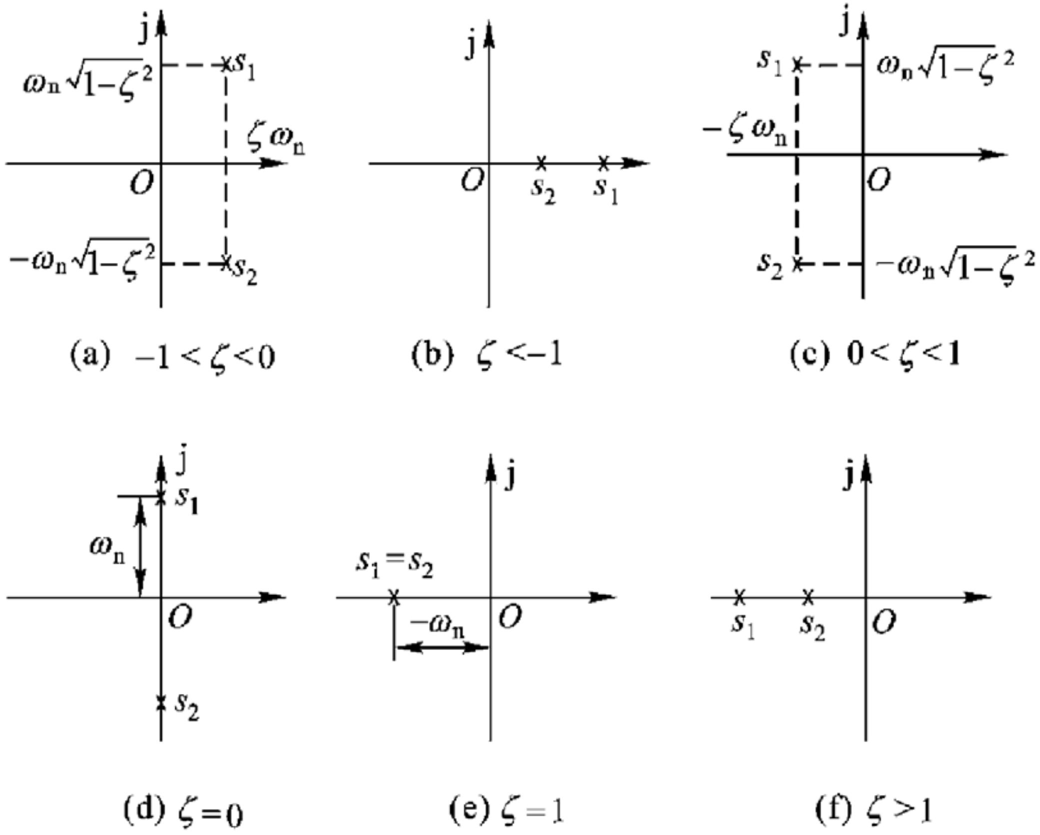
6. Routh-Hurwitz criterion

7. The steady-state error of feedback control system.

Review



$$T(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$



(a) $-1<\zeta<0$ (b) $\zeta<-1$ (c) $0<\zeta<1$

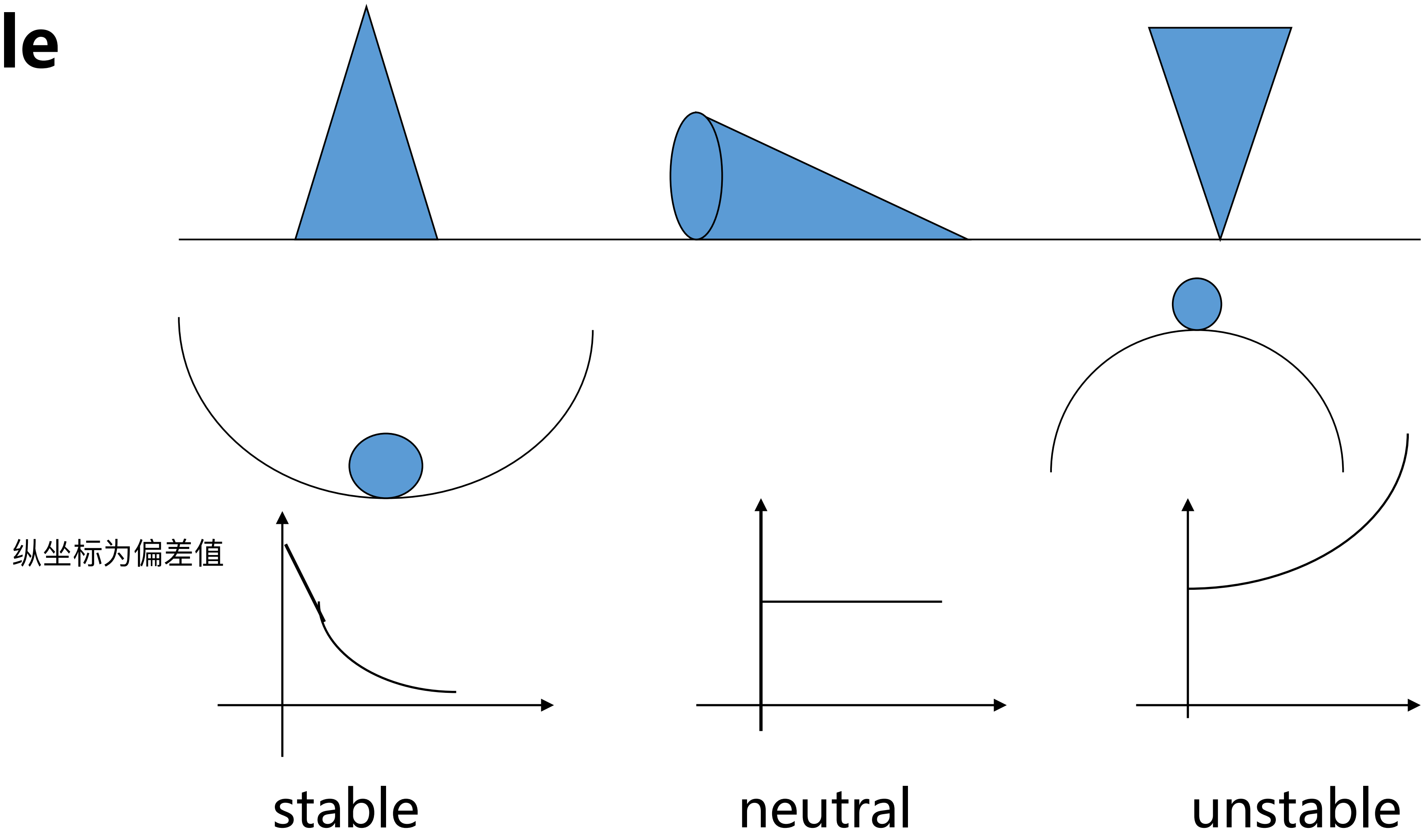
(d) $\zeta=0$ (e) $\zeta=1$ (f) $\zeta>1$

what is next

Condition for a feedback system to be stable (Routh-Hurwitz criterion)

Condition for a feedback system to be stable

Example



Condition for a feedback system to be stable

Consider the transfer function of a closed-loop system as

$$T(s)=\frac{N(s)}{D(s)}=\frac{N(s)}{\prod_{i=1}^{n}(s-s_i)}$$

⚠️写法
不管分子
分母一定是因式相乘

稳定性和分子无关 和极点分别有关

Where $s_i, i=1,2\cdots n$, are roots of the characteristic equation D(s) . Assume that all roots are simple, then we have

$$C(s)=T(s)R(s)=\frac{N(s)}{\prod_{i=1}^{n}(s-s_i)}R(s)=\sum_{j=1}^{l}\frac{B_j}{s-s_{rj}}+\sum_{i=1}^{n}\frac{A_i}{s-s_i}$$

由输入r决定

由传递函数决定
分析稳定性时主要分析

Condition for a feedback system to be stable

$$c(t)=\sum_{j=1}^{l}B_j e^{s_{rj}t}+\sum_{i=1}^{n}A_i e^{s_i t}$$

Steady-state response

Transient response

$$\lim_{t\to\infty}\sum_{i=1}^{n}A_i e^{s_i t}=0$$

When s_i is real number, letting $s_i=\sigma_i$ we have

$\sigma_i<0 \qquad \lim_{t\to\infty}A_i e^{s_i t}=0$

$\sigma_i=0 \qquad \lim_{t\to\infty}A_i e^{s_i t}=A_i$

$\sigma_i>0 \qquad \lim_{t\to\infty}A_i e^{s_i t}=\infty$

Condition for a feedback system to be stable

When s_i is complex number,letting $s_i = \sigma_i \pm j\omega_i$,we have

$$A_i e^{(\sigma_i + j\omega_i)t} + A_{i+1} e^{(\sigma_i - j\omega_i)t} = Ae^{\sigma_i t}\cos(\omega_i t + \psi)$$

$$\sigma_i < 0 \qquad \lim_{t\to\infty} Ae^{\sigma_i t}\cos(\omega_i t + \psi) = 0$$

$$\sigma_i = 0 \qquad \lim_{t\to\infty} Ae^{\sigma_i t}\cos(\omega_i t + \psi) = A\cos(\omega_i t + \psi)$$

$$\sigma_i > 0 \qquad \lim_{t\to\infty} Ae^{\sigma_i t}\cos(\omega_i t + \psi) = \infty$$

A necessary and sufficient condition for a feedback system to be stable is that all the **poles** of the system transfer function have **negative real parts**.

Routh-hurwitz criterion

The characteristic equation of a feedback system

$$a_n s^n + a_{n-1} s^{n-1} + \cdots a_1 s + a_0 = 0$$

Assume: $a_n > 0$, when n=5, Routh table is

s^5	a_5	a_3	a_1
s^4	a_4	a_2	a_0
s^3	$\frac{a_3 a_4 - a_2 a_5}{a_4} = b_1$	$\frac{a_1 a_4 - a_0 a_5}{a_4} = b_2$	0
s^2	$\frac{b_1 a_2 - b_2 a_4}{b_1} = c_1$	a_0	
s^1	$\frac{c_1 b_2 - b_1 a_0}{c_1} = d_1$		
s^0	a_0		

Routh-hurwitz criterion

Example 1 :

Determine the stability of the closed-loop system that has the following characteristic equations.

(1) $$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$$

(2) $$s^3 - 3s + 4 = 0$$

(3) $$s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10 = 0$$

Routh-hurwitz criterion

Answer

(1) $$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$$

$$\begin{array}{llll} s^4 & 1 & 3 & 5 \\ s^3 & 2 & 4 & \\ s^2 & 1 & 5 & \\ s^1 & -6 & & \\ s^0 & 5 & & \end{array}$$

0.2878 + 1.4161i

0.2878 - 1.4161i

-1.2878 + 0.8579i

-1.2878 - 0.8579i

Routh-hurwitz criterion

Answer

(2) $s^3 - 3s + 4 = 0$

$(s^3 - 3s + 4)(s+1) = s^4 + s^3 - 3s^2 + s + 4 = 0$

$$\begin{array}{lll} s^3 & 1 & -3 \\ s^2 & \varepsilon & 4 \\ s^1 & \dfrac{-3\varepsilon - 4}{\varepsilon} & \\ s^0 & 4 & \end{array}$$

$$\begin{array}{llll} s^4 & 1 & -3 & 4 \\ s^3 & 1 & 1 & \\ s^2 & -4 & 4 & \\ s^1 & 2 & & \\ s^0 & 4 & & \end{array}$$

Routh-hurwitz criterion

(3) $$s^5+2s^4+2s^3+4s^2+11s+10=0$$

$$\begin{array}{llll}
s^5 & 1 & 2 & 11 \\
s^4 & 2 & 4 & 10 \\
s^3 & 0 & 6 \to \varepsilon & 6 \\
s^2 & \dfrac{4\varepsilon-12}{\varepsilon} & 10 & \\
s^1 & \dfrac{24}{\varepsilon}-\dfrac{72}{\varepsilon^2}-10 & & \\
s^0 & 10 & &
\end{array}$$

```
a=[1 2 2 4 11 10];
>> p=roots(a)
p =
   0.8950 + 1.4561i
   0.8950 - 1.4561i
  -1.2407 + 1.0375i
  -1.2407 - 1.0375i
  -1.3087
```

Routh-hurwitz criterion

应用ε 法要注意，否则可能得出错误的结论

例 $s^6+s^5+3s^4+3s^3+3s^2+2s+1=0$

$$
\begin{array}{lllll}
s^6 & 1 & 3 & 3 & 1 \\
s^5 & 1 & 3 & 2 & \\
s^4 & \varepsilon & 1 & 1 & \\
s^3 & 3-\dfrac{1}{\varepsilon} & 2-\dfrac{1}{\varepsilon} & & \\
s^2 & v & 1 & & \\
s^1 & u & & & \\
s^0 & 1 & & &
\end{array}
$$

$$v=1-\frac{2\varepsilon-1}{3-1/\varepsilon}$$

$$u=2-\frac{1}{\varepsilon}-\frac{3-1/\varepsilon}{1-\dfrac{2\varepsilon-1}{3-1/\varepsilon}}$$

原因是该方程有虚轴上的根。

Routh-hurwitz criterion

(4) $s^6 + s^5 - 2s^4 - 3s^3 - 7s^2 - 4s - 4 = 0$

s^6	1	−2	−7	−4
s^5	1	−3	−4	
s^4	1	−3	−4	
s^3	0	0	0	

$$\boxed{F(s) = s^4 - 3s^2 - 4 = 0}$$

$$\frac{dF(s)}{ds} = 4s^3 - 6s = 0$$

辅助方程
求导
得到并替代

s^6	1	−2	−7	−4
s^5	1	−3	−4	
s^4	1	−3	−4	
s^3	4	−6	0	
s^2	−1.5	−4		
s^1	−16.7	0		
s^0	−4			

但不能直接反应稳定性的几个极点
只能反应实根
虚根被解决掉了 要从辅助方程里求

$\pm 2, \pm j, (-1 \pm j\sqrt{3})/2$

The performance of feedback control systems

核心

- **Condition for a feedback system to be stable** $$c(t)=\sum_{j=1}^{l}B_j e^{s_{rj}t}+\sum_{i=1}^{n}A_i e^{s_i t}$$
- **Routh-Hurwitz criterion**

续

- **The steady-state error of feedback control system**

Routh-hurwitz criterion

Example 2 : Consider that the characteristic equation of a closed-loop control system is

$$s^3 + 3Ks^2 + (K+2)s + 4 = 0$$

Find the rang of K so that the system is stable. Answer: $K > 0.528$

Example 3: Determine the following characteristic equations, how many roots are to the right of the line s=-1 in the s-plane.

(1) $$s^3 + 3s^2 + 3s + 1 = 0$$

(2) $$s^3 + 4s^2 + 3s + 10 = 0$$