习题 11.6 答案与提示

1.

(1) $\dfrac{1}{2}-\dfrac{\pi}{4}+\dfrac{2}{\pi}\sum\limits_{n=1}^{\infty}\dfrac{\cos(2n-1)x}{(2n-1)^2}-\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n(1+\pi)-1}{n\pi}\sin nx=\begin{cases}1, & -\pi<x<0,\\ -x, & 0\le x<\pi,\\ \dfrac{1}{2}, & x=0,\\ \dfrac{1-\pi}{2}, & x=\pm\pi.\end{cases}$

(2) $\dfrac{2}{3}\pi^2+4\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{n^2}\cos nx=\pi^2-x^2\quad(-\pi\le x\le\pi)$;

(3) $\dfrac{2}{\pi}-\dfrac{4}{\pi}\sum\limits_{n=1}^{\infty}\dfrac{\cos 2nx}{4n^2-1}=|\sin x|\quad(-\pi\le x\le\pi)$.

2.

(1) $\sum\limits_{n=1}^{\infty}\dfrac{\sin 2nx}{2n}=\begin{cases}\dfrac{\pi}{4}-\dfrac{x}{2}, & 0<x<\pi,\\ 0, & x=0,\pi.\end{cases}$

(2) $\dfrac{h}{\pi}+\dfrac{2}{\pi}\sum\limits_{n=1}^{\infty}\dfrac{\sin nh}{n}\cos nx=\begin{cases}1, & 0\le x<h,\\ 0, & h\le x\le\pi,\\ \dfrac{1}{2}, & x=h.\end{cases}$

3. $\dfrac{8}{\pi}\sum\limits_{k=1}^{\infty}\dfrac{\sin(2k-1)x}{(2k-1)^3}=x(\pi-x)$.

4.

(1) $\dfrac{l}{2}-\dfrac{4l}{n^2}\sum\limits_{n=1}^{\infty}\dfrac{1}{(2n-1)^2}\cos\dfrac{(2n-1)\pi}{l}x=|x|\quad(-l\le x\le l)$.

(2) $\dfrac{h}{2}+\dfrac{2h}{\pi}\sum\limits_{n=1}^{\infty}\dfrac{1}{2n-1}\sin\dfrac{(2n-1)\pi}{2}x=\begin{cases}0, & -2<x\le 0,\\ h, & 0<x\le 2,\\ \dfrac{h}{2}, & x=0,\pm 2.\end{cases}$