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Lattice hydrodynamic model based delay feedback control of vehicular traffic flow considering the effects of density change rate difference

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ABSTRACT

This study proposes a control strategy of vehicular traffic based on the lattice hydrodynamic model by taking the difference of density change rate as a feedback signal. In particular, a delay feedback control scheme is designed based on Nagatani's lattice model considering the effects of density change rate difference under the open boundary condition. Stability analysis of the lattice hydrodynamic model with the control signal is performed using the small gains theorem to obtain the stability condition. Results from numerical experiments including the single perturbation and multiple perturbations illustrate that the proposed control strategy can suppress traffic jam effectively and make the vehicular traffic flow to move into a comparatively homogeneous state.

1. Introduction

Traffic problems have been widely concerned and investigated in recent years. To understand the complex mechanism behind the traffic phenomenon, various traffic flow models including the microscopic and macroscopic models have been proposed. At the microscopic level, car-following models are used to describe the interactions between each pair of leading and following vehicles, and many related achievements have been obtained [1–14]; while at the macroscopic level, lattice hydrodynamic models have drawn considerable attention recently [15–24]. An important issue of traffic study is to avoid traffic jam and make the traffic flow move homogeneously. However, many previous studies focus on the traffic jam transitions and the mechanism of the traffic jam phenomena [1–24]. Therefore, it is valuable to study how to control the vehicular traffic system from jam to homogeneous state.

Car-following models generally use the microscopic variables including velocity, position and acceleration to capture the characteristics of traffic flow. In 1999, Konishi et al. [25] investigate the traffic jam phenomena under periodic boundary condition and derive a simple stability condition of the optimal velocity (OV) model. And then, Konishi et al. [26] study the traffic jam suppression using a decentralized delayed-feedback control method. Zhao et al. [27] propose a feedback control of traffic jam based on the OV model, where the velocity difference between the leading and following vehicle is designed as the feedback signal. Li et al. [28] further study the traffic jam suppression based on the full velocity difference (FVD) model using the

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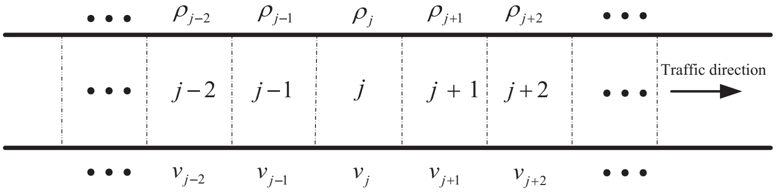


Fig. 1. Sketch of the lattice traffic flow.

acceleration feedback control method, which is motivated that the acceleration of vehicle is the underlying factor that causes a change in vehicular traffic system state. Zhou et al. [29] present a feedback control scheme for traffic jam considering the effect of the safe headway based on the coupled map car-following model. The literature review heretofore illustrates the importance of traffic jam suppression and the effectiveness of feedback control method. However, the aforementioned studies mainly rely on the OV-based traffic models, which belong to a microscopic control method.

Unlike the car-following models, lattice hydrodynamic models use the collective variables such as the local density and local average speed to describe the traffic flow. In 1998, Nagatani [15] proposes the first lattice hydrodynamic model based on the continuum model. Later, Nagatani [16] performs the nonlinear analysis of the lattice model to obtain the TDGL and MKdV equations. Li et al. [17] carry out the stability analysis of the lattice model considering the relative current of traffic flow. Zhu et al. [18] propose a generalized optimal current lattice model with a consideration of multi-interaction of the front lattice sites. Peng et al. [20,21] study the performance of lattice model considering the effects of the honk and driver's memory, respectively. Recently, Wang et al. [24] propose a new lattice model considering the effects of multiple density difference. All the foregoing lattice models study the vehicular traffic flow from the macroscopic viewpoint. As we know, the difference of density change rate between adjacent lattice sites has potential impacts on the state evolution of traffic flow. However, to the best of our knowledge, there are seldom studies on lattice models considering the effects of density change rate difference.

The primary objective in this paper is to develop a feedback control strategy of vehicular traffic from the macroscopic viewpoint. In particular, the lattice hydrodynamic model is used to study the traffic flow. A delay feedback control scheme is designed based on the lattice model. The difference of density change rate between adjacent lattice sites is designed as the feedback signal. The linear stability of the lattice model with the feedback signal is analyzed using the small gains theorem to obtain the stability condition. Finally, to verify the effectiveness of the proposed control strategy, numerical experiments including the single perturbation and multiple perturbations are performed for comparison using simulation under the open boundary condition.

The rest of paper is organized as follows: Section 2 reviews the lattice hydrodynamic model. Section 3 proposes the delay feedback control strategy of vehicular traffic flow and designs the corresponding control law. Section 4 performs the stability analysis of the lattice model with the control signal. Section 5 conducts the numerical experiments and comparisons. The final section concludes this study.

2. Lattice hydrodynamic model

To capture the characteristics of vehicular traffic flow, Nagatani [15] proposes the continuum models as follows:

$$\partial_t \rho + \rho_0 \partial_x(\rho v) = 0 \tag{1}$$

$$\partial_t(\rho v) = a\rho_0 V(\rho(x+\delta)) - a\rho v \tag{2}$$

where ρ and $\rho(x+\delta)$ are the local density at position x and $x+\delta$ at time t, respectively. ρ_0 is the local average density, and δ represents the average space headway, i.e., $\delta = 1/\rho_0$. $a \in \mathbb{R}$ and $a > 0$ is the sensitivity of a driver. v is the local average speed and $V(\rho)$ represents the optimal speed of the traffic flow at the density of ρ.

Considering the following scenario in the vehicular traffic system as shown in Fig. 1, the road is divided into N lattice sites. Nagatani proposed the lattice hydrodynamic model based on the continuum models as follows [15]:

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0 \tag{3}$$

$$\partial_t(\rho_j v_j) = a\rho_0 V(\rho_{j+1}) - a\rho_j v_j \tag{4}$$

where j denotes the site of the road on the one-dimensional lattice, ρ_j denotes the local density on site j at time t. v_j denotes the local average speed on site j at time t. And the optimal speed is used as in [6] except for density variable:

$$V(\rho) = \frac{v_{\max}}{2}\left[\tanh\left(\frac{1}{\rho} - h_c\right) + \tanh(h_c)\right]$$

where $v_{\max}$ is the maximal speed and h_c is the safety distance. $\tanh(\cdot)$ is the hyperbolic tangent function.

Based on Eqs. (3) and (4) and eliminating the variable v_j, the lattice hydrodynamic model is given by [15]

$$\partial_t^2 \rho_j + a\partial_t \rho_j + a\rho_0^2(V(\rho_{j+1}) - V(\rho_j)) = 0. \tag{5}$$

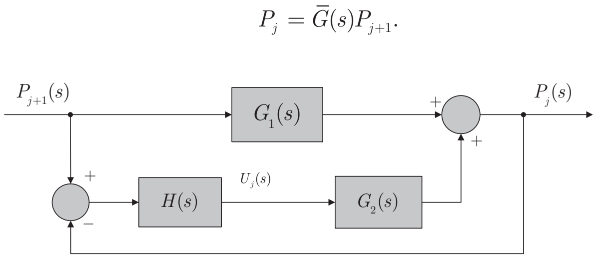


Fig. 2. Block diagram of the control system.

3. Delay feedback control

To begin with, we define $\delta\rho_j = \rho_j - \rho_0$ and substitute it into Eq. (5). Then the resulting equation can be linearized using the Taylor expansion as follows:

$$\partial_t^2 \delta\rho_j = -a[\partial_t \delta\rho_j + \Lambda {\rho_0}^2 (\delta\rho_{j+1} - \delta\rho_j)] \tag{6}$$

where $\Lambda = \frac{\mathrm{d}V(\rho)}{\mathrm{d}\rho}|_{\rho=\rho_0}$.

Adding a feedback control term $u_j(t)$ to Eq. (6), and we have

$$\partial_t^2 \delta\rho_j = -a[\partial_t \delta\rho_j + \Lambda {\rho_0}^2 (\delta\rho_{j+1} - \delta\rho_j)] + u_j(t). \tag{7}$$

Considering the effects of density change rate difference on the vehicular traffic flow, the feedback control term $u_j(t)$ is designed as follows:

$$u_j(t) = k[\partial_t \rho_{j+1}(t-1) - \partial_t \rho_j(t-1)]. \tag{8}$$

That is

$$u_j(t) = k[\partial_t \delta\rho_{j+1}(t-1) - \partial_t \delta\rho_j(t-1)] \tag{9}$$

where k is the adjustable feedback gain. In Eqs. (8) and (9), the control signal $u_j(t)$ at time t is proportional to the difference of density change rate between the adjacent lattice site j and $j+1$ at time $t-1$. Therefore, the asynchronization results in the delay feedback control of the vehicular traffic flow. Note that the control signal acts in the vehicular traffic system only if the state is unstable, and it will vanish under the stable state [27,28].

4. Stability analysis

To facilitate stability analysis of the lattice model with the feedback signal, we perform the Laplace transform with respect to time and the resulting system will be obtained based on Eqs. (7) and (9):

$$\begin{cases} P_j = G_1(s)P_{j+1} + G_2(s)U_j \\ U_j = H(s)(P_{j+1} - P_j) \end{cases} \tag{10}$$

where $P_j = L(\delta\rho_j),\ P_{j+1} = L(\delta\rho_{j+1}),\ G_1(s) = \frac{-a\rho_0^2\Lambda}{s^2+as-a\rho_0^2\Lambda},\ G_2(s) = \frac{1}{s^2+as-a\rho_0^2\Lambda},\ H(s) = kse^{-s},\ U_j = L(u_j)$.

Fig. 2 illustrates the block diagram of the control system and the relation between P_{j+1} and P_j is described as

$$P_j = \bar{G}(s)P_{j+1}. \tag{11}$$

The transfer function $\bar{G}(s)$ in Eq. (11) is given by

$$\bar{G}(s) = \frac{P_j}{P_{j+1}} = \frac{kse^{-s} - a\Lambda\rho_0^2}{\bar{d}(s)} \tag{12}$$

The characteristic polynomial $\bar{d}(s)$in Eq. (12) is

$$\bar{d}(s) = s^2 + (a + ke^{-s})s - a\Lambda\rho_0^2 \tag{13}$$

Definition 1. [26] No traffic jam occurs in the vehicular traffic flow if the following conditions are satisfied:

(i) The characteristic polynomial $\bar{d}(s)$ is stable;
(ii) The transfer function $\|\bar{G}(s)\|_\infty = \sup_{\omega\in[0,\infty)} |G(\mathrm{i}\omega)| \le 1$.

Theorem 1. *There is no traffic jam in the controlled system (10) if the parameters are selects to satisfy*

$$\begin{cases} k > 0, \\ a = nk, \quad \forall n \in \mathbb{R}, \quad n > 1, \\ \Lambda \geq -\dfrac{[(n-1)^2 - 1]k^2}{2a\rho_0^2}. \end{cases} \tag{14}$$

Proof. Based on the designed feedback control signal as in Eq. (9), it is important to adjust the value of the parameter k in the controlled system to suppress traffic jam. In the following, we will prove that the parameter selection in Eq. (14) validate the two conditions in Definition 1; thus the traffic jam is prevented.

First, we will show that the condition 1 of Definition 1 holds. Follow the approach proposed by Konishi et al. [26] and note that $G_2(s)$ and $H(s)$ are in the same path, so $\bar{d}(s)$ is stable using the small gain theorem as follows:

$$\|G_2(s)H(s)\|_\infty \|1\|_\infty < 1. \tag{15}$$

To guarantee that the condition (15) holds, we obtain the magnitude of $G_2(jw)H(jw)$ as follows:

$$|G_2(jw)H(jw)| = \sqrt{\frac{k^2}{g(w)}}, \tag{16}$$

where $g(w) := w^2 + \frac{a^2\rho_0^4\Lambda^2}{w^2} + a(a + 2\rho_0^2\Lambda)$. Note that $\Lambda = \frac{dV(\rho)}{d\rho}|_{\rho=\rho_0} = -\frac{v_{\max}}{2\rho_0^2}$ with $v_{\max} > 0$, we know that $\Lambda < 0$. In addition, considering the fact that $a > 0$, it is straightforward that $a\rho_0^2\Lambda < 0$. The lower bound of $g(w)$ is obtained when $w^2 = \frac{a^2\rho_0^4\Lambda^2}{w^2}$, namely, $w^2 = a\rho_0^2|\Lambda| = -a\rho_0^2\Lambda$. Thus, we have

$$\inf_{w\in[0,\infty)} g(w) = a^2. \tag{17}$$

Hence, H_∞-norm of $G_2(s)H(s)$ is

$$\|G_2(s)H(s)\|_\infty = \sqrt{\frac{k^2}{a^2}}. \tag{18}$$

Based on the parameters chosen in (14), note that $a = nk$, n is an arbitrary constant and $n > 1$. Thus we know that $\|G_2(s)H(s)\|_\infty = \sqrt{\frac{1}{n^2}} < 1$. Hence, the condition (15) holds and thus $\bar{d}(s)$ is stable based on the small gain theorem.

Second, we will show that the condition 2 of Definition 1 holds. It follows directly from (14) that

$$(n-1)^2k^2 + 2a\Lambda\rho_0^2 - k^2 \geq 0, \ \forall \quad \omega \in [0, \infty). \tag{20}$$

From (14), it is known that $a = nk$, n is an arbitrary constant and $n > 1$. Thus we have $|a + k\cos w| = k|n + \cos w| = k(n + \cos w) > k(n-1)$, which further implies from (20) that

$$(a + k\cos\omega)^2 + 2a\Lambda\rho_0^2 - k^2 \geq 0, \quad \forall\, \omega \in [0, \infty). \tag{21}$$

If follows from (21) that

$$(\omega - k\sin\omega)^2 + (a + k\cos\omega)^2 + 2a\Lambda\rho_0^2 - k^2 \geq 0, \quad \forall\, \omega \in [0, \infty), \tag{22}$$

Eq. (22) can be re-written as

$$\omega^2 + 2a\Lambda\rho_0^2 + a^2 + 2ak\cos\omega - 2k\omega\sin\omega \geq 0, \quad \forall\, \omega \in [0, \infty). \tag{23}$$

Based on (23), it is straightforward to obtain

$$(\omega^2 + 2a\Lambda\rho_0^2 + a^2 + 2ak\cos\omega - 2k\omega\sin\omega)\omega^2 \geq 0, \quad \forall\, \omega \in [0, \infty), \tag{24}$$

Eq. (23) can be further converted into the fractional form as

$$\frac{(k\omega\cos\omega)^2 + (k\omega\sin\omega - a\Lambda\rho_0^2)^2}{(k\omega\sin\omega - \omega^2 - a\Lambda\rho_0^2)^2 + (a\omega + k\omega\cos\omega)^2} \leq 1, \quad \forall\, \omega \in [0, \infty) \tag{25}$$

Based on the definition of infinity norm, we obtain from (25) that

$$\left\|\bar{G}(s)\right\|_\infty = \sup_{\omega\in[0,\infty)} |\bar{G}(i\omega)| = \sup_{\omega\in[0,\infty)} \sqrt{\frac{(k\omega\cos\omega)^2 + (k\omega\sin\omega - a\Lambda\rho_0^2)^2}{(k\omega\sin\omega - \omega^2 - a\Lambda\rho_0^2)^2 + (a\omega + k\omega\cos\omega)^2}} \leq 1. \tag{26}$$

Namely, the second condition of Definition 1 is also satisfied, thus it is concluded from Definition 1 that there is no traffic jam. □

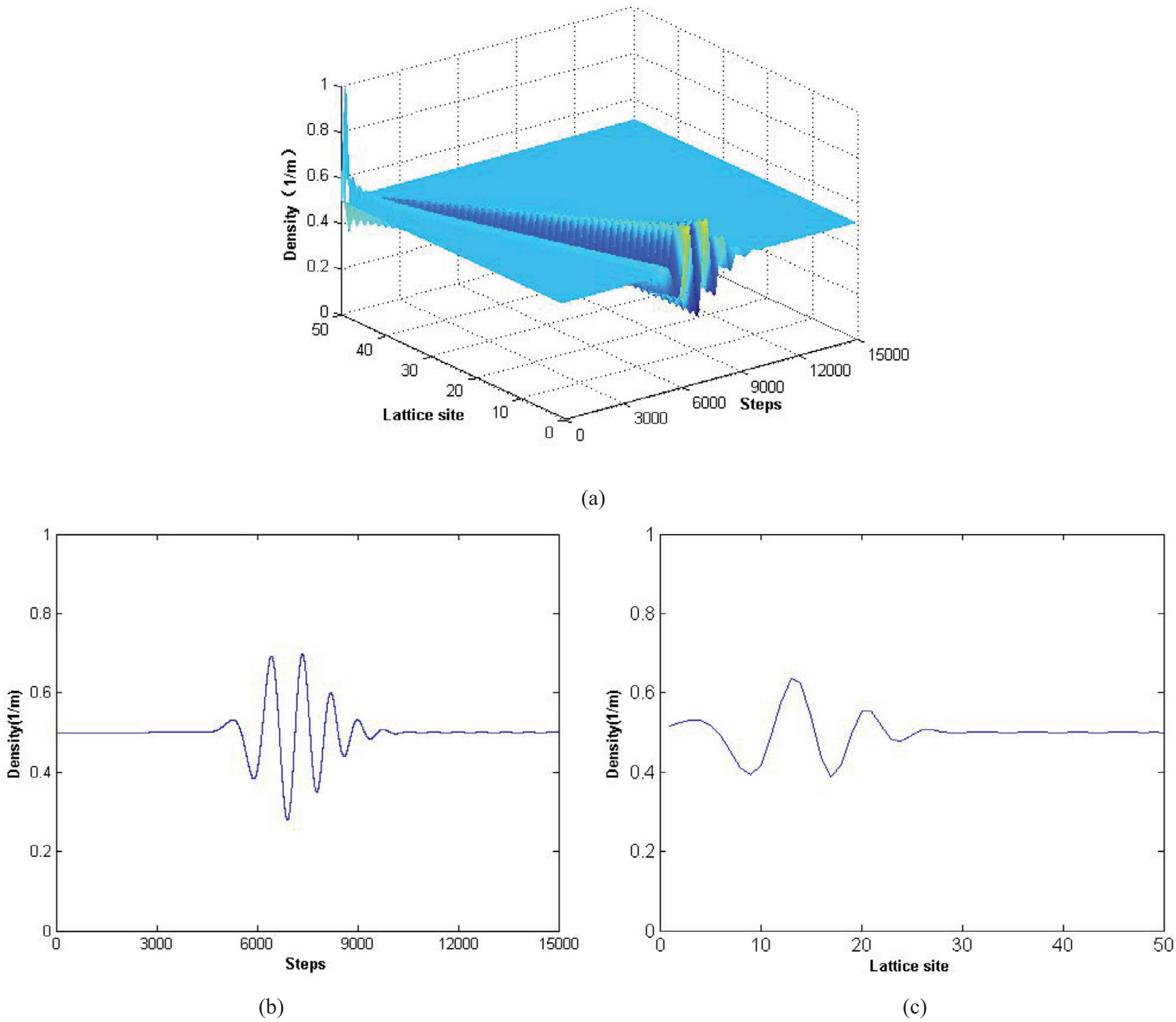


Fig. 3. The local density distribution under the single perturbation without control signal: (a) space-time evolutions of local density; (b) the local density distribution of the last lattice site; and (c) the local density distribution at $t = 50$ s.

5. Numerical experiments

Based on the foregoing theoretical analyses, we perform the numerical experiments using the simulation with and without control signal to verify the effectiveness of the proposed control strategy. Suppose that there are N lattice sites on a road under an open boundary condition. The related parameters are set as

$$a = 1.4 \text{ m}^{-1}, \quad \rho_0 = 0.5 \text{ m}^{-1}, \quad h_c = 2 \text{ m}, \quad v_{\max} = 2 \text{ m/s}, \quad N = 50.$$

For all results, we use the Runge–Kutta algorithm for numerical integration with time-step $\Delta t = 0.01$ s. To illustrate the comparative results, two cases are studied in terms of the perturbations in the vehicular traffic flow under the assumption that the initial vehicular traffic flow is homogenous.

Case 1: Single perturbation

Considering the scenario that a single sudden deceleration (perturbation) of the Nth lattice site occurs, which results in the decrease of the following space headway. Note that the local density is associated with the space headway. Consequently, the local density fluctuation will appear in the following lattice site. Therefore, the initial condition in this case can be described as follows

$$\rho_{N-1} = \rho_0 + 0.5. \tag{27}$$

Fig. 3(a) shows the space–time evolutions of the local density in the vehicular traffic flow under the single perturbation without the control signal. From Fig. 3(a), the traffic jam appears in the homogenous traffic flow when the perturbation occurs in

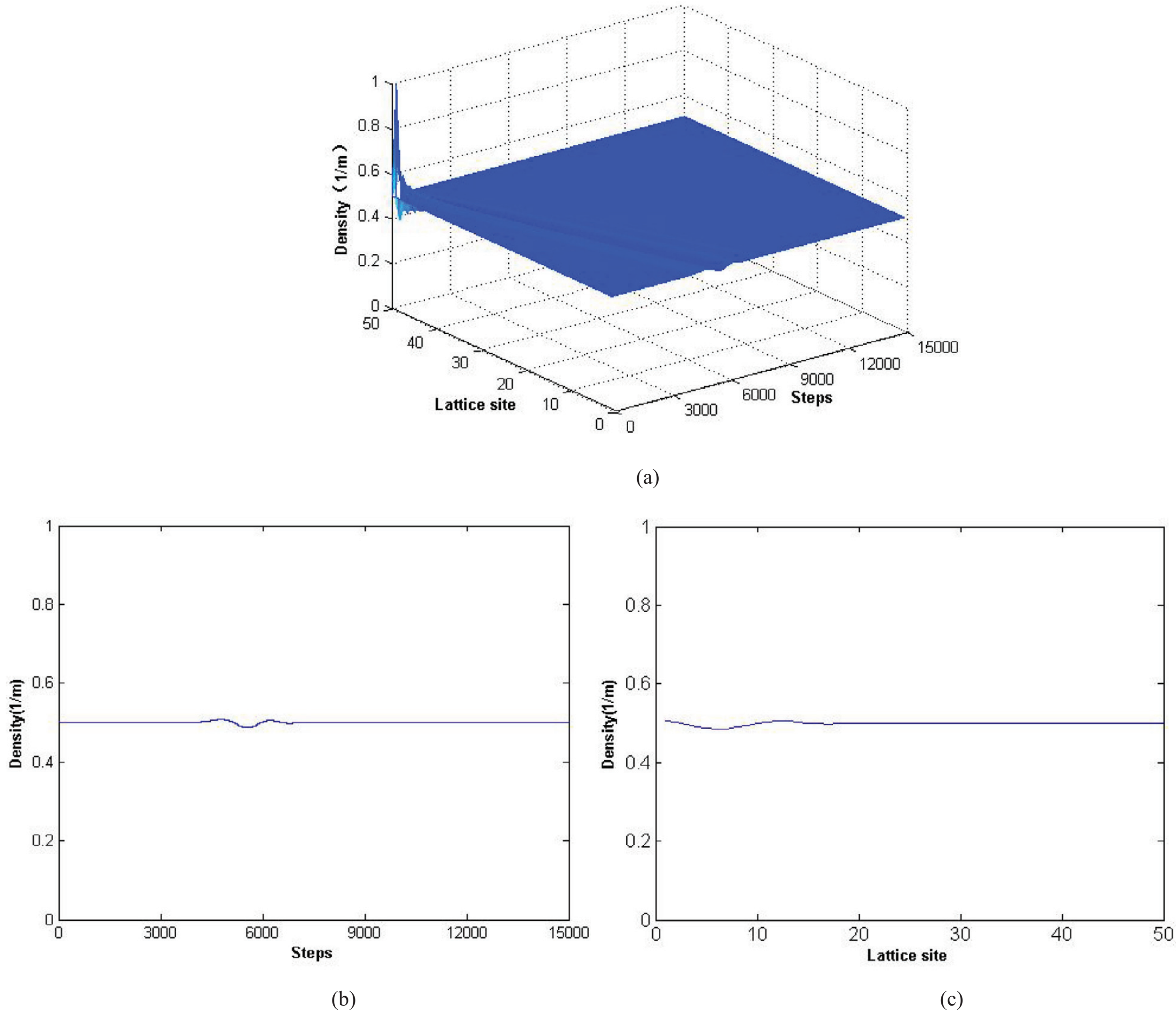


Fig. 4. The local density distribution under the single perturbation with control signal: (a) space-time evolutions of local density; (b) the local density distribution of the last lattice site; (c) the local density distribution at $t = 50$ s.

the first lattice site. Fig. 3(b) shows the local density distribution of last lattice site along with the simulation time, while Fig. 3(c) shows the local density distribution of each lattice site in the vehicle traffic flow at $t = 50$ s. From Fig. 3(b) and (c), the oscillations appear in the local density distribution. Hence, the traffic jam exists in the vehicular traffic flow.

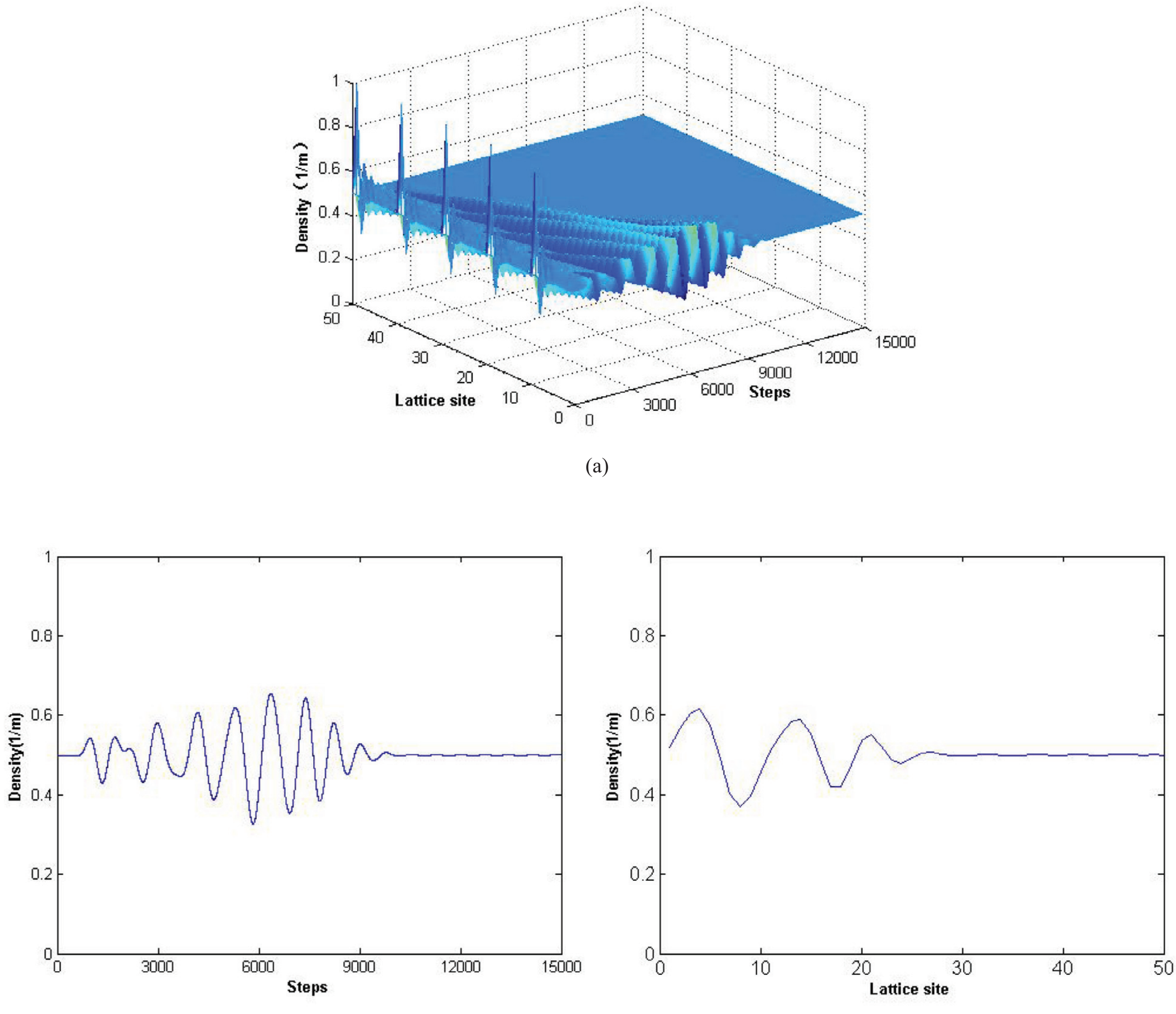
Next, we perform the experiments with the feedback control signal in the lattice model according to Eq. (10) and the feedback gain is chosen as $k = 0.3$. Fig. 4(a) shows the space-time evolutions of the local density in the vehicular traffic flow under the single perturbation with the designed control signal. In Fig. 4, the state of the local density in the vehicular traffic flow is approximately uniform, which illustrates the effectiveness of the proposed control strategy. In order to compare with Fig. 3(b) and (c), we also provide the local density distribution of last lattice site and all lattice sites in the vehicular traffic flow as shown in Fig. 4(b) and (c). Therefore, the traffic jam can be effectively suppressed by the designed control signal.

Case 2: Multiple perturbations

To further verify the effectiveness of the proposed control strategy under the complex environment, we study the scenario that the multiple perturbations happen in some lattice sites from the upstream to downstream in the vehicular traffic flow. In particular, considering the deceleration or acceleration (perturbations) of some lattice sites, it accordingly results in the increase or decrease of the corresponding local densities of adjacent lattice sites. To account for this phenomenon, we thereby describe the initial conditions as follows:

$$\rho_{N-1}=\rho_{N-11}=\rho_{N-21}=\rho_{N-31}=\rho_{N-41}=\rho_0+0.5 \tag{28}$$

$$\rho_{N-2}=\rho_{N-12}=\rho_{N-22}=\rho_{N-32}=\rho_{N-42}=\rho_0-0.167 \tag{29}$$



(a)

(b)

(c)

Fig. 5. The local density distribution under multiple perturbations without control signal: (a) space-time evolutions of local density; (b) the local density distribution of the last lattice site; (c) the local density distribution at $t = 50$ s.

Fig. 5(a) shows the space-time evolutions of the local density in the vehicular traffic flow under the multiple perturbations without the control signal. In Fig. 5(a), the spike phenomenon represents the corresponding sudden variations of the local density as shown in Eqs. (28) and (29). And the traffic jam appears in the homogenous traffic flow when the perturbations occur. Figs. 5(b) and 5(c) show the local density distribution of last lattice site and all lattice sites in the vehicle traffic flow. In Fig. 5(b) and (c), the oscillations appear in the local density distribution and the amplitude of vibration is significant.

To compare, the experiments with the feedback control signal in the lattice model are carried out according to Eq. (10) and the feedback gain is also set as $k = 0.3$. Fig. 6(a) shows the space-time evolutions of the local density in the vehicular traffic flow under multiple perturbations with the designed control signal. In Fig. 6(a), although the spike phenomenon exists, the state of the local density in the vehicular traffic flow is smoother than that of Fig. 5(a). In addition, Fig. 6(b) and (c) shows the local density distribution of last lattice site and all lattice sites in the vehicle traffic flow. According to Figs. 5(b) and 6(b) as well as Figs. 5(c) and 6(c), the amplitude of vibration of the local density in the vehicular flow is comparatively suppressed.

In summary, based on the discussion under the case 1 and case 2, we can obtain that: (i) the vehicular traffic flow is generally stable, although there is a oscillation of the local density. That is because the values of the parameters satisfy the stable condition as shown in Eq. (14); (ii) the local density waves generating from the perturbation always propagate backwards; (iii) the perturbation occurring somewhere in the vehicular traffic flow will result in the traffic jam, which can be suppressed effectively by the designed control strategy.

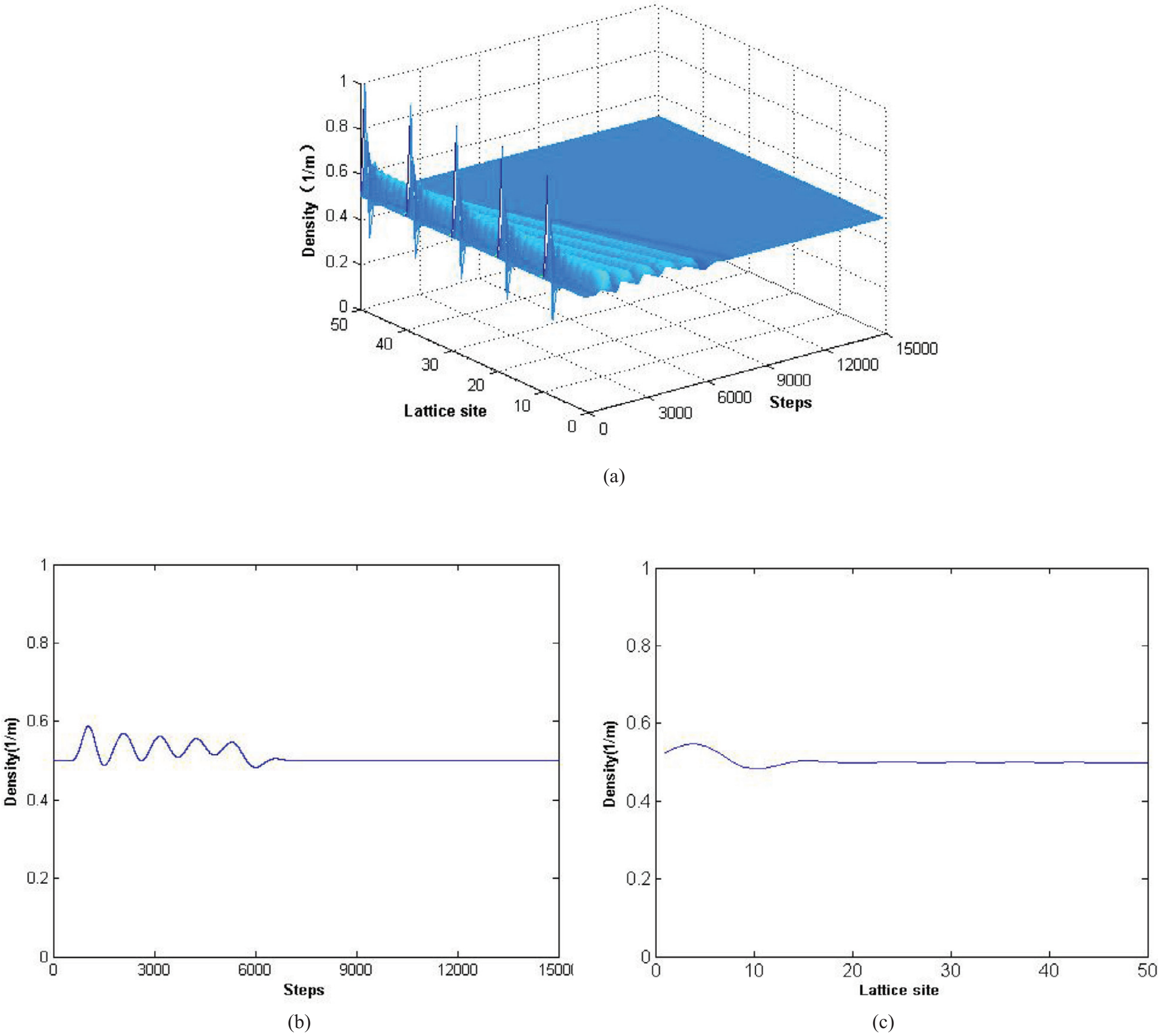


Fig. 6. The local density distribution under multiple perturbations with control signal: (a) space-time evolutions of local density; (b) the local density distribution of the last lattice site; and (c) the local density distribution at $t = 50$ s.

6. Conclusions

To suppress the traffic jam effectively from the macroscopic viewpoint, a delay feedback control scheme is proposed based on the lattice hydrodynamic model considering the effects of the local density change rate difference between adjacent lattice sites. Theoretical analysis illustrates the lattice model with the control signal is stable if the feedback gain k is designed reasonably. To verify the effectiveness of the proposed control strategy, numerical experiments of two scenarios including the single perturbation and multiple perturbations are performed. Comparative results from simulation-based numerical experiments demonstrate that the traffic jam in the vehicular traffic flow can be suppressed by the designed control signal.

Acknowledgments

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