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Evaluating the performance of vehicular platoon control under different network topologies of initial states

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HIGHLIGHTS

- A feedback based protocol for platoon control is proposed considering the longitudinal and lateral gaps simultaneously.
- The stability and consensus of the vehicular platoon is analyzed using the Lyapunov technique.
- Effects of different network topologies of initial states on convergence time and robustness of platoon control are investigated.
- Results demonstrate the effectiveness of the proposed protocol with respect to the position and velocity consensus.
- The findings illustrate the convergence time is associated with the initial states, while the robustness is not significantly.

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ABSTRACT

This study proposes a feedback-based platoon control protocol for connected autonomous vehicles (CAVs) under different network topologies of initial states. In particularly, algebraic graph theory is used to describe the network topology. Then, the leader–follower approach is used to model the interactions between CAVs. In addition, feedback-based protocol is designed to control the platoon considering the longitudinal and lateral gaps simultaneously as well as different network topologies. The stability and consensus of the vehicular platoon is analyzed using the Lyapunov technique. Effects of different network topologies of initial states on convergence time and robustness of platoon control are investigated. Results from numerical experiments demonstrate the effectiveness of the proposed protocol with respect to the position and velocity consensus in terms of the convergence time and robustness. Also, the findings of this study illustrate the convergence time of the control protocol is associated with the initial states, while the robustness is not affected by the initial states significantly.

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1. Introduction

Over the past few decades, traffic problems such as traffic congestion, traffic management, transportation planning have received much attention. To address these problems, there has been a lot of focus on capturing the complex mechanisms behind the phenomena of vehicular traffic flow from the microscopic and macroscopic viewpoints. Consequently, various traffic flow models such as car-following (CF) models [1–14] and lattice hydrodynamic models [15–18] have been proposed.

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However, these models focus on understanding the characteristics of traffic flow rather than the vehicular formation forming. Especially regarding to the stability analysis of the above traffic flow models, the assumption that the traffic flow is a steady-state platoon with identical space headway and velocity is made first [2–13,15–17]. As we know that platoon-based driving pattern (a special formation) referred to a group of vehicles traveling together with short inter-vehicle headway is regarded as a promising driving manner. Benefits of platoon driving pattern include the road capacity increase, traffic congestion mitigation, and energy consumption and exhaust emissions reduction [19,20].

Recently, autonomous vehicles (AVs), i.e., Google driverless car, have received much attention. Compared to human-driven vehicles, AVs can be operated without direct driver input to control the steering, acceleration, and braking so that the vehicles can travel with higher mobility, less faults, and smarter choices. In this context, AVs equipped with the communication technology (i.e., vehicle-to-vehicle (V2V) or vehicle-to-infrastructure (V2I) communications), connected autonomous vehicles (CAVs), can generate and capture environmentally relevant real-time transportation data and use this data to create actionable information to support and facilitate green transportation choices [21], such as improving the efficiency of intersections [22] and applying to the real-time adaptive signal control [23]. In the context of CAVs, an AV can cooperate with other AVs based on the information of position, speed, and acceleration via the V2V communications. Consequently, it leads to an efficient cooperative task. On the other hand, in many developing countries where multiple vehicles can travel in parallel in a road without lane discipline. And past studies illustrate that the vehicles travel with lateral gaps consume more energy under non-lane-discipline environment [14]. Hence, there is a research need to study the control protocol to facilitate the CAVs to form a platoon considering the longitudinal and lateral gaps simultaneously under the non-lane-discipline environment.

Existing approaches to vehicular formation control can be classified as the leader–follower approach and the virtual leader approach. The leader–follower approach is of significance for the multi-agent control problem, where the leader moves along a predefined trajectory while the others, the followers, maintain a desired distance and orientation to the leader [24]. By applying the local leader–follower control to small fraction of agents, Guan et al. [25] proved that the states of the pinning-controlled multi-agent system can approach the states of the leader in finite time. Ma et al. [26] and Djaidja et al. [27] studied the leader-following consensus control for multi-agent systems under measurement noise. However, the reference leader to track is considered constant. To address this issue, Su et al. [28] and Peng et al. [29] considered the situation that the leader moves with a varying velocity, and then designed the protocol which enables followers to asymptotically track the leader. Although the leader–follower approach depends heavily on the leader for achieving the goal. However, this approach is widely used in the cooperative systems for its simplicity and scalability.

Unlike the leader–follower approach, the virtual leader approach assumes each agent behaves like particles embedded in a rigid virtual structure. Desired trajectories are not assigned to each single agent but to the entire formation as a whole. Lu et al. [30] investigated the tracking control problems with a virtual leader under the conditions of fixed and switching topologies in finite time, respectively. Ren et al. [31] proposed a feedback protocol in order to improve the performance of the system in terms of the robustness. Shi et al. [32] introduced a set of coordination control laws that enables the group to generate the desired stable state, and then studied the coordinated control for multiple agents using the virtual leader approach. Zhang et al. [33] proposed a distributed feedback law related to the virtual leader to address the problem of coordinated path tracking. This approach avoids the problems with disturbance rejection inherent in the leader–follower approach, but at the cost of high communication and computation capabilities needed to synthesize the virtual leader and communicate its position in time to support real-time control of other vehicles. In addition, it may cause unexpected instability for algorithms without considering the communication delay and feedback [34,35].

The literature review heretofore illustrates that the formation control is an important issue in the cooperative systems. However, the above mentioned studies do not consider the effects of different network topologies of initial states. Also, they do not consider the space constraints as well. Hence, motivated by the CAVs context, this study restricts its focus to protocol design to facilitate the CAVs to form a platoon under the non-lane-discipline road. The emphasis is on the effects of different network topologies of initial states on the platoon control performance. Specifically, we firstly transform a general problem into an analytical formation to describe the network topology using the algebraic graph theory. Then, the interactions between CAVs are modeled using the leader–follower approach. In addition, feedback-based protocol is designed to control the platoon considering the longitudinal and lateral gaps simultaneously. The stability and consensus of the platoon is analyzed using the Lyapunov technique. Effects of different network topologies of initial states on convergence time and robustness are investigated. Results from numerical experiments demonstrate the effectiveness of the proposed protocols with respect to the position and velocity consensus in terms of the convergence time and robustness.

The rest of the paper is organized as follows: Section 2 presents the problem formulation and some preliminaries. Section 3 proposes control protocol for CAVs, and analyzes the stability and consensus problems. Section 4 performs numerical experiments under different initial states. Section 5 concludes this study.

2. Problem formulation and preliminaries

As shown in Fig. 1, the vehicular systems include a leading vehicle (noted as the leader), and other following vehicles (noted as followers, vehicle i, $i+1$, $i+2$). Considering the characteristics of AVs, we treat each CAV as an agent using the multi-agent systems theory. The goal of this study is to design control protocol to facilitate CAVs in Fig. 1 to form a platoon as shown in Fig. 2 in the context of V2V communications, which the initial longitudinal and lateral gaps between any vehicles

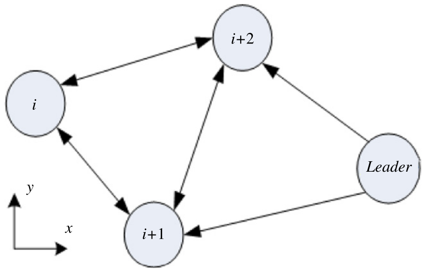

Fig. 1. Network topology.

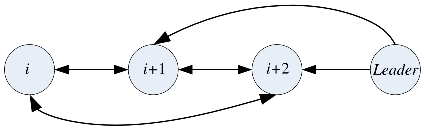

Fig. 2. Vehicular platoon.

can satisfy the desired platoon of vehicle formation. Each vehicle can adjust its speed to track platoon alignment according to leader vehicle speed. And the vehicular platoon runs in a road and employs the information flow topologies, where CAVs can sensor its own velocity and relative distance from the vehicle ahead. Moreover, from Fig. 2, the ith vehicle not only can receive the information from the $i+1$th vehicle, but also can communicate with the $i+2$th vehicle.

To describe the relationship between CAVs, we give the algebraic graph theory as follows. In this theory, each vertex represents an individual CAV and each edge represents an active communication link. A digraph (directed graph) consists of a pair (N, ε), where N is a finite nonempty set of nodes and $\varepsilon \in N^2$ is a set of ordered pairs of nodes, called edges. As a comparison, the pairs of nodes in an undirected graph are unordered. If there is a directed edge from node v_i to node v_j, then v_i is defined as the parent node and v_j is defined as the child node [36]. The adjacency matrix $A = [a_{ij}]$ of a weighted graph is defined as $a_{ii} = 0$ and $a_{ii} > 0$ if $(j, i) \in \varepsilon$where $i \neq j$. The Laplacian matrix of the weighted graph is defined as $L = [l_{ij}]$, where $l_{ii} = \sum_{j \neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, $\forall i \neq j$. And let $\mathbf{1}$ denote the $n \times 1$ column vector of all ones. We consider another graph $\bar{G}$ associated with the system consisting of n agents (followers) and one leader denoted by v_L [37]. Let the linked matrix be $K = \mathrm{diag}(k_1, k_2, \dots, k_n)$, if agent i connects with the leader, then $k_i > 0$, otherwise $k_i = 0$.

If a graph $\bar{G}$ with $\forall (i, j) \in E \Rightarrow (j, i) \in E$, we say that graph $\bar{G}$ is an undirected graph. Then the node v_L is said to be globally reachable in $\bar{G}$, if there is a path from every other node to v_L in $\bar{G}$. The edges of undirected graph denote that the communications between vehicles are mutual [38]. In this study, we consider the followers network topologies under the undirected graph.

Following the leader–follower approach, we consider the followers of the system as [39]:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases} \tag{1}$$

where $x_i(t) \in R$ and $v_i(t) \in R$ are the position and velocity of vehicle i. $u_i(t) \in R$ is the control input.

The leader of the system is described as [39]:

$$\begin{cases} \dot{x}_L(t) = v_L(t) \\ \dot{v}_L(t) = u_L(t) \end{cases} \tag{2}$$

where $x_L(t) \in R$ and $v_L(t) \in R$ are the position and velocity of the leader. $u_L(t) \in R$ is the control input of the leader.

3. Control protocol design

To address the scenario as shown in Fig. 1, we propose the feedback-based control protocol as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = \dot{v}_L(t) - \sum_{j=1}^{n} a_{ij}[(x_i(t) - x_j(t) - r_{ij}) + \beta(v_i(t) - v_j(t))] - k_i[(x_i(t) - x_L(t) - r_i) + \gamma(v_i(t) - v_L(t))] \end{cases} \tag{3}$$

where a_{ij} is adjacency weight between vehicle i and vehicle j; $r_{ij} = -r_{ji}$ is the vector of position, which represent longitudinal and lateral gaps between vehicle i and vehicle j; $r_i = (r_{i_x}, r_{i_y})$,r_{i_x}, r_{i_y} is the desired longitudinal gap and lateral gap; the linked weight $k_i > 0$ if vehicle i receives information from leader and 0 otherwise. β and γ are the gain parameters of system.

Following the approach proposed by Ren in Ref. [36], we define the position error and velocity error as $\tilde{x}_i(t) = x_i(t) - \boldsymbol{1}\int_{\boldsymbol{0}}^{\mathbf{t}} v_L(\tau)\mathrm{d}\tau - r_i$, and $\tilde{v}_i(t) = v_i(t) - \boldsymbol{1}v_L(t)$, respectively, where $\tilde{x}_i(t)$, $\tilde{v}_i(t)$ both possess two vectors in horizontal and vertical coordinates, such as $\tilde{x}_i(t) = [\tilde{x}_{i_x}(t), \tilde{x}_{i_y}(t)]^{\mathrm{T}}$, $\tilde{v}_i(t) = [\tilde{v}_{i_x}(t), \tilde{v}_{i_y}(t)]^{\mathrm{T}}$. And the $\int_{\boldsymbol{0}}^{\mathbf{t}} v_L(\tau)\mathrm{d}\tau$ is an auxiliary notation for obtaining the error position corresponding to the error velocity. Hence, the error equation can be obtained as follows:

$$\begin{cases} \dot{\tilde{x}}_i(t) = \dot{\tilde{v}}_i(t) \\ \dot{\tilde{v}}_i(t) = -\sum_{j=1}^{n} a_{ij}[(\tilde{x}_i(t) - \tilde{x}_j(t)) + \beta(\tilde{v}_i(t) - \tilde{v}_j(t))] - k_i(\tilde{x}_i(t) + \gamma\tilde{v}_i(t)) \end{cases} \tag{4}$$

where $i = 1, 2, \ldots, n$.

The following lemma is used to analyze the stability and consensus of the vehicular platoon.

Lemma 1 (*[40]*). *Let $x(t)$ be a solution of $\dot{x}(t) = f(x)$, $x(0) = x_0 \in R^n$, where $f : U \to R^n$ is continuous with an open subset U of R^n, and $V : U \to R$ be a locally Lipschitz function such that $D^+V(x(t)) \le 0$, where D^+ denotes the upper Dini derivative. Then $\Theta^+(x_0) \cap U$ is contained in the union of all solutions that remain in $S = \{x \in U : D^+V(x) = 0\}$, where $\Theta^+(x_0)$ denotes the positive limit set.*

Theorem 1. *Suppose that the initial position and velocity of each vehicle are given. Assume that under protocol (3) with the feedback gain $K = diag(k_1, k_2, \ldots k_n)$ satisfying $k_i > 0$ if vehicle i connects with the leader, otherwise $k_i = 0$, the states of vehicles can be directly or indirectly affected by the state of the leader vehicle.*

Then we have:

(i) $\lim_{t\to\infty} \|\tilde{x}_{i_x}\| = \|x_{i_x} - x_{L_x} - r_{i_x}\| = 0$, $\lim_{t\to\infty} \|\tilde{x}_{i_y}\| = \|x_{i_y} - x_{L_y} - r_{i_y}\| = 0$;
(ii) $\lim_{t\to\infty} \|\tilde{v}_{i_x}\| = \|v_{i_x} - v_{L_x}\| = 0$, $\lim_{t\to\infty} \|\tilde{v}_{i_y}\| = \|v_{i_y} - v_{L_y}\| = 0$.

Namely, the CAVs in a road can form a platoon formation. It means that the longitudinal gaps $x_{i_x}(t) - x_{L_x}(t)$ and the lateral gaps $x_{i_y}(t) - x_{L_y}(t)$ can converge to desired distance r_{i_x} and r_{i_y}, respectively; while the velocities of x-coordinate and y-coordinate also can converge to the desired speed $v_{L_x}$and v_{L_y}, respectively.

Proof. For the error dynamic Eq. (4), inspired by the Lyapunov technique [41], the candidate Lyapunov function is defined as [25]:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{5}$$

where $V_1 = \frac{1}{2}\sum_{i=1}^{n}\tilde{v}_i(t)^2$, $V_2 = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^{\tilde{x}_i(t)-\tilde{x}_j(t)} a_{ij}s\ \mathrm{d}s$, and $V_3 = \sum_{i=1}^{n}\int_0^{\tilde{x}_i(t)} k_i s\ \mathrm{d}s$.

Considering the time derivative of $V_k(t)$, $(k = 1, 2, 3)$ along the solution of Eq. (4), we can obtain

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^{n}\tilde{v}_i(t)\dot{\tilde{v}}_i(t) = \sum_{i=1}^{n}\tilde{v}_i(t)\left[-\sum_{j=1}^{n}a_{ij}((\tilde{x}_i(t) - \tilde{x}_j(t)) + \beta(\tilde{v}_i(t) - \tilde{v}_j(t))) - k_i(\tilde{x}_i(t) + \gamma\tilde{v}_i(t))\right] \\ \dot{V}_2 &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\tilde{v}_i(t) - \tilde{v}_j(t))(\tilde{x}_i(t) - \tilde{x}_j(t)) \\ \dot{V}_3 &= \sum_{i=1}^{n}k_i\tilde{v}_i(t)\tilde{x}_i(t). \end{aligned} \tag{6}$$

Consequently, we have

$$\begin{aligned} \dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \\ &= \sum_{i=1}^{n}\tilde{v}_i(t)\left[-\sum_{j=1}^{n}a_{ij}((\tilde{x}_i(t) - \tilde{x}_j(t)) + \beta(\tilde{v}_i(t) - \tilde{v}_j(t))) - k_i(\tilde{x}_i(t) + \gamma\tilde{v}_i(t))\right] \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\tilde{v}_i(t) - \tilde{v}_j(t))(\tilde{x}_i(t) - \tilde{x}_j(t)) + \sum_{i=1}^{n}k_i\tilde{v}_i(t)(\tilde{x}_i(t)). \end{aligned} \tag{7}$$

Note that the network topology of the system is undirected. Consequently, it follows from Eq. (7) that

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n}\tilde{v}_i(t)\left[-\sum_{j=1}^{n}a_{ij}\beta(\tilde{v}_i(t) - \tilde{v}_j(t)) - k_i\gamma\tilde{v}_i(t)\right] \\ &= -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\beta(\tilde{v}_i(t) - \tilde{v}_j(t))^2 - \sum_{i=1}^{n}k_i\gamma\tilde{v}_i^2(t) \\ &\le 0. \end{aligned} \tag{8}$$

If $\dot{V} = 0$, it follows from Eq. (8) that

$$-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\beta(\tilde{v}_i(t) - \tilde{v}_j(t))^2 - \sum_{i=1}^{n} k_i\gamma\tilde{v}_i^2(t) = 0. \tag{9}$$

From condition of Theorem 1, we know that parameters $a_{ij}, k_i \geqslant 0$. Then, according to Eq. (9), we can obtain $\tilde{v}_i = \tilde{v}_j = 0$, which implies $\dot{\tilde{v}}_i = \dot{\tilde{v}}_j = 0$. Hence, we have

$$\begin{aligned}\dot{\tilde{v}}_i(t) &= -\sum_{j=1}^{n} a_{ij}[(\tilde{x}_i(t) - \tilde{x}_j(t)) + \beta(\tilde{v}_i(t) - \tilde{v}_j(t))] - k_i(\tilde{x}_i(t) + \gamma\tilde{v}_i(t))\\ &= -\sum_{j=1}^{n} a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t)) - k_i\tilde{x}_i(t) = 0.\end{aligned} \tag{10}$$

Based on Eq. (10), we can obtain that

$$\begin{aligned}\sum_{i=1}^{n}\tilde{x}_i(t)\dot{\tilde{v}}_i(t) &= -\sum_{i=1}^{n}\tilde{x}_i(t)\left[\sum_{j=1}^{n} a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t)) - k_i\tilde{x}_i(t)\right]\\ &= -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t))^2 - \sum_{i=1}^{n} k_i\tilde{x}_i^2(t)\\ &= 0.\end{aligned} \tag{11}$$

From Eq. (11), we know $\tilde{x}_i = \tilde{x}_j = 0$. According to Lemma 1, we have $x_i(t) - x_L(t) - r_i \to 0$, $v_i(t) - v_L(t) \to 0$ for all $i \in I$, as $t \to \infty$. That means $\lim_{t\to\infty}\|\tilde{x}_i\| = \|x_i(t) - x_L(t) - r_i\| \to 0$ and $\lim_{t\to\infty}\|\tilde{v}_i\| = \|v_i(t) - v_L(t)\| \to 0$ in horizontal and vertical coordinates. In other words, we have $\lim_{t\to\infty}\|\tilde{x}_{i_x}\| = \|x_{i_x} - x_{L_x} - r_{i_x}\| = 0$, $\lim_{t\to\infty}\|\tilde{x}_{i_y}\| = \|x_{i_y} - x_{L_y} - r_{i_y}\| = 0$ and $\lim_{t\to\infty}\|\tilde{v}_{i_x}\| = \|v_{i_x} - v_{L_x}\| = 0$, $\lim_{t\to\infty}\|\tilde{v}_{i_y}\| = \|v_{i_y} - v_{L_y}\| = 0$. The proof is completed. □

Remark 1. According to Theorem 1, we know that CAVs in Eq. (2) with control protocol (3) can achieve the platoon formation with the desired distance and speed. Particularly, the longitudinal gap between CAVs can be kept in the safe range r_{ij}, and the velocity of each CAV can track the desired speed v_L. It means that the consensus of the control protocol is guaranteed. In addition, the consensus problem is analyzed using the Lyapunov technique, hence, the stability of the control protocol is also guaranteed.

4. Numerical experiments

In this section, we perform the numerical experiments to verify the effectiveness of the proposed control protocol for the platoon of CAVs. To investigate the effects of network topologies of initial states on the performance associated with the convergence time and robustness, we discuss three cases with different initial states. For comparison, we give the same simulation conditions as follows:

(i) Gain parameters: we set $\beta = \gamma = 1$;
(ii) Desired distance: the desired longitudinal and lateral gaps between followers and leader are set as $r_{i,L} = (-15, 0)^{\mathrm{T}}$, $r_{i+1,L} = (-10, 0)^{\mathrm{T}}$, and $r_{i+2,L} = (-5, 0)^{\mathrm{T}}$, respectively.

Case I: Initial states: we suppose the position of CAVs as below:

$$\begin{aligned}&x_i(0) = (6, 60)^{\mathrm{T}}, \quad x_{i+1}(0) = (10, 40)^{\mathrm{T}}, \quad x_{i+2}(0) = (16, 70)^{\mathrm{T}}, \quad x_L(0) = (20, 50)^{\mathrm{T}}\\ &v_i(0) = (10, 5)^{\mathrm{T}}, \quad v_{i+1}(0) = (8, 4)^{\mathrm{T}}, \quad v_{i+2}(0) = (9, 3)^{\mathrm{T}}, \quad v_L(0) = (6, 0)^{\mathrm{T}}.\end{aligned}$$

To address this scenario, we adopt the proposed control protocol as shown in Eq. (3). Fig. 3 shows the position trajectories in two-dimensional plane. From Fig. 3, the leader keep the moving direction and the followers gradually track the leader. Specifically, the longitudinal gaps between the followers and leader converge to the desired distance as shown in Fig. 4(a), while the corresponding lateral gaps converge to zero as shown in Fig. 4(b). In addition, from Fig. 4, we know that the convergence time of longitudinal and later gaps is 7.9 s and 9.2 s, respectively. Fig. 5 shows the velocity trajectories. Fig. 5(a) shows that the velocities of followers in x-coordinate can converge to the desired velocity (the leader's velocity) and keep moving constantly. Fig. 5(b) shows that the velocities of followers in y-coordinate can converge to zero. Also, the convergence time of x-coordinate and y-coordinate is 7.9 s and 9.2 s, respectively. Hence, Fig. 5 shows that the velocity of CAVs can reach a consensus state. Therefore, according to Figs. 3–5, the CAVs under the non-lane discipline can form a platoon with the designed control protocol.

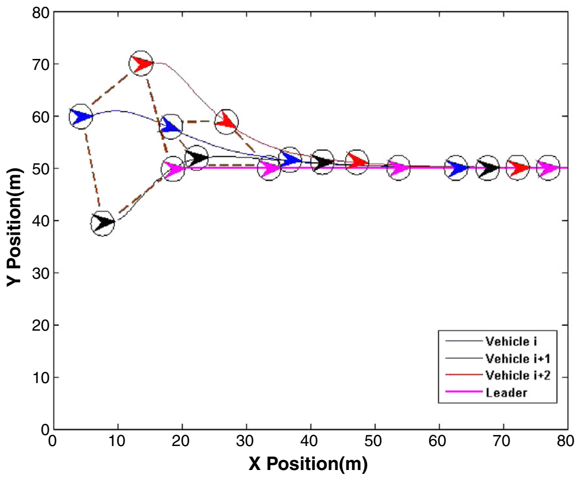

Fig. 3. The position trajectories of CAVs.

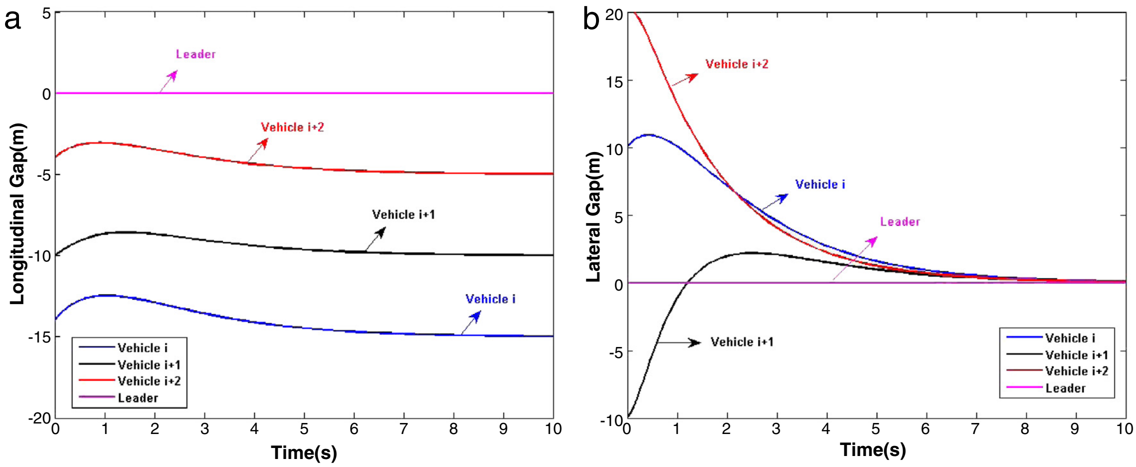

Fig. 4. The gap trajectories of CAVs: (a) The longitudinal gap; (b) The lateral gap.

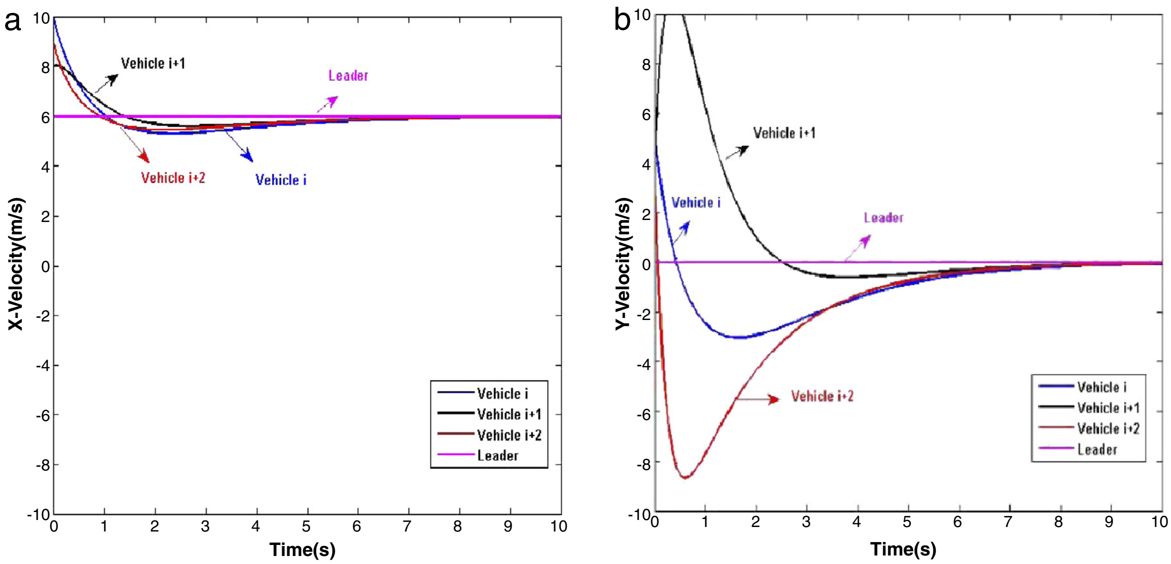

Fig. 5. The velocity trajectories of CAVs: (a) x-velocity; (b) y-velocity.

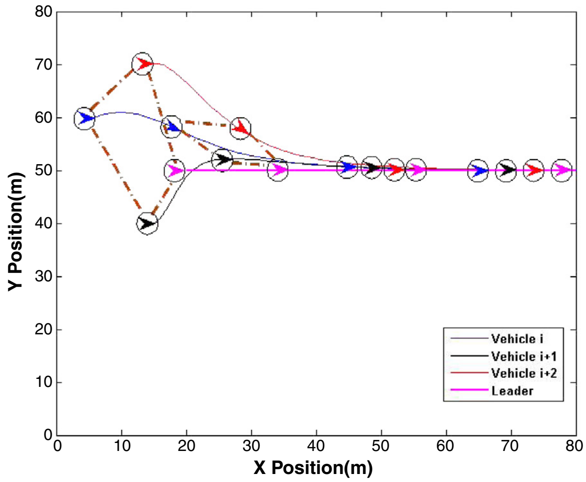

Fig. 6. The position trajectories of CAVs.

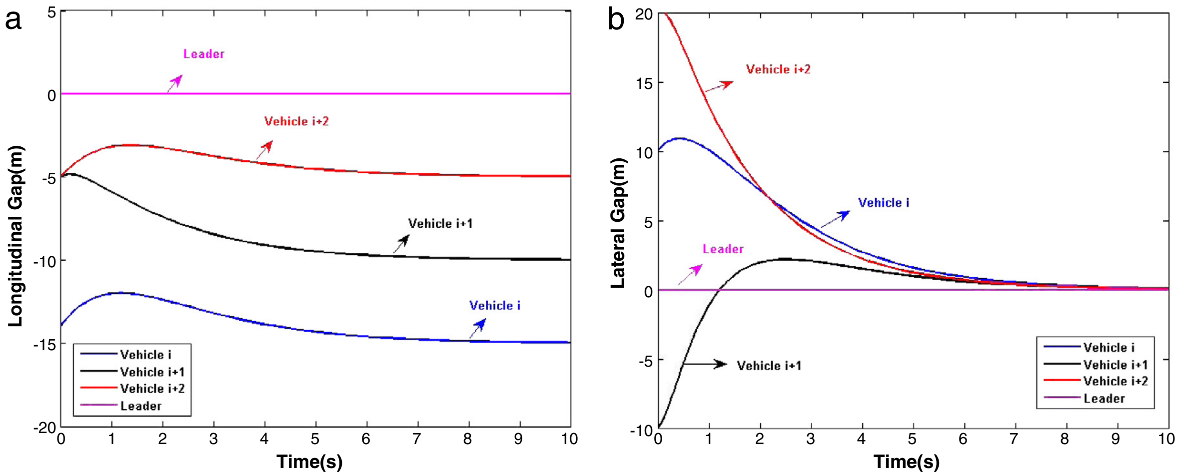

Fig. 7. The gap trajectories of CAVs: (a) The longitudinal gap; (b) The lateral gap.

Case II: Initial states: we suppose the position of CAVs as below:

$$x_i(0) = (6, 60)^{\mathrm{T}}, \quad x_{i+1}(0) = (15, 40)^{\mathrm{T}}, \quad x_{i+2}(0) = (15, 70)^{\mathrm{T}}, \quad x_L(0) = (20, 50)^{\mathrm{T}}$$
$$v_i(0) = (10, 5)^{\mathrm{T}}, \quad v_{i+1}(0) = (8, 4)^{\mathrm{T}}, \quad v_{i+2}(0) = (9, 3)^{\mathrm{T}}, \quad v_L(0) = (6, 0)^{\mathrm{T}}.$$

In practice, the platoon runs on a flat road, and can have different network topologies. To address this scenario, we adopt the proposed control protocol as shown in Eq. (3) with different network topologies of initial states. From Fig. 6, we know that the leader keep the moving direction and the followers also can gradually track the leader. Meanwhile the longitudinal gaps between the followers and leader from Fig. 7 can achieve the same effect of formation as case I. In addition, from Fig. 7, we know that the convergence time of longitudinal and later gaps is 8.1 s and 9.4 s, respectively. Fig. 8 shows the velocity trajectories. Also, the convergence time of x-coordinate and y-coordinate is 8.1 s and 9.4 s, respectively.

Hence, From Figs. 6–8, we can see that the position and velocity of CAVs can reach the consensus state with the proposed control protocol considering the different network topologies. In addition, compare Figs. 4, 5 and Figs. 7, 8, we can see that the convergence time of case II is longer than that of case I, while the amplitude of oscillation including the gap and velocity of case II is similar to that of case I. According to Figs. 6–8, the proposed control protocol is also readily applied in the case of the network topology of case II.

Case III: Initial states: we suppose the position of CAVs as below:

$$x_i(0) = (6, 60)^{\mathrm{T}}, \quad x_{i+1}(0) = (16, 40)^{\mathrm{T}}, \quad x_{i+2}(0) = (10, 70)^{\mathrm{T}}, \quad x_L(0) = (20, 50)^{\mathrm{T}}$$
$$v_i(0) = (10, 5)^{\mathrm{T}}, \quad v_{i+1}(0) = (8, 4)^{\mathrm{T}}, \quad v_{i+2}(0) = (9, 3)^{\mathrm{T}}, \quad v_L(0) = (6, 0)^{\mathrm{T}}.$$

For comparison, in this case we also use the same control protocol shown in Eq. (3) with another different network topology of initial states. Fig. 9 shows the position trajectories in two-dimensional plane. From Fig. 9, the leader keep the moving

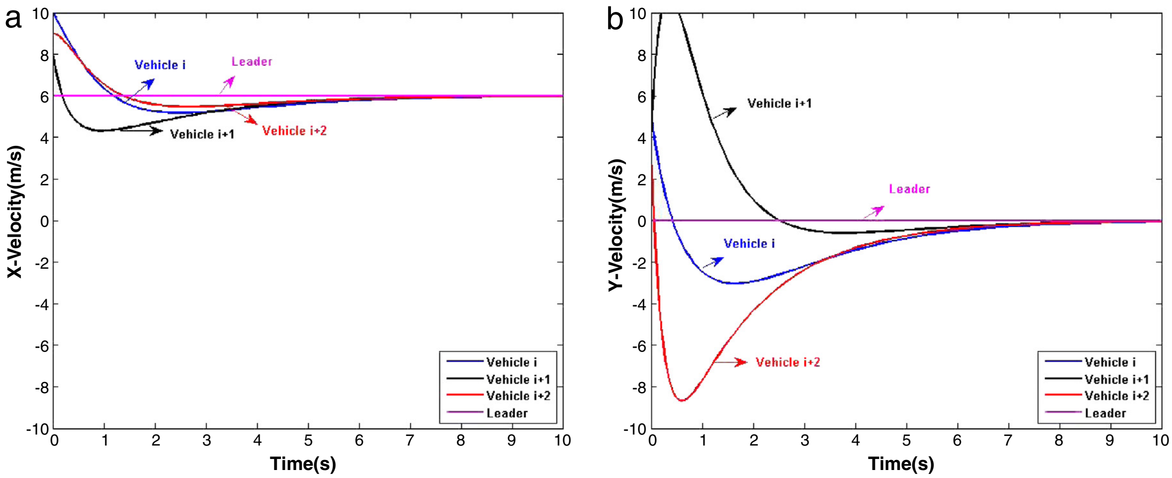

Fig. 8. The velocity trajectories of CAVs: (a) x-velocity; (b) y-velocity.

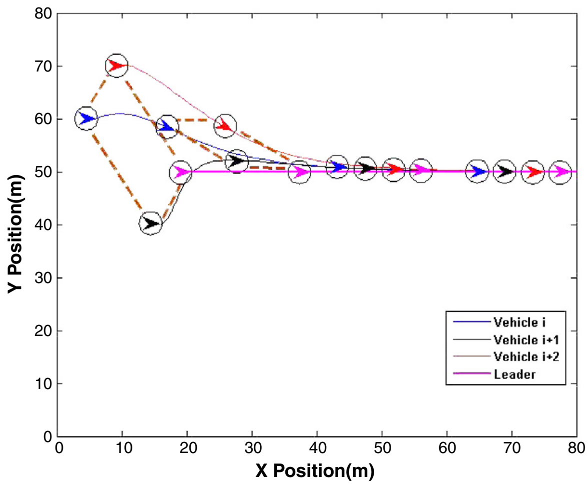

Fig. 9. The position trajectories of CAVs.

direction and the followers gradually track the leader. Specifically, the longitudinal gaps between the followers and leader converge to the desired distance as shown in Fig. 10(a), while the lateral gaps converge to zero as shown in Fig. 10(b). In addition, from Fig. 10, we know that the convergence time of longitudinal and later gaps is 8.4 s and 9.6 s, respectively. From Figs. 9 and 10, we also can see that the position of CAVs can reach the consensus state with the proposed control protocol. Fig. 11 shows the velocity trajectories. Fig. 11(a) shows that the velocities of followers in x-coordinate can converge to the desired velocity (the leader's velocity) and keep moving constantly. Fig. 11(b) shows that the velocities of followers in y-coordinate can converge to zero. Also, the convergence time of x-coordinate and y-coordinate is 8.4 s and 9.6 s, respectively. Hence, Fig. 11 shows that the velocity of CAVs can reach a consensus state.

Table 1 summarizes the performance of the proposed control protocols with different network topologies of initial states. Based on the above discussion, we conclude that: (i) The proposed control protocols can facilitate CAVs in the non-lane-discipline road to form a platoon; (ii) The position and velocity trajectories of CAVs can achieve the consensus state. Particularly, the longitudinal gap can converge to the desired distance, and the lateral gap can converge to zero. Moreover, the velocity can converge to the desired speed; (iii) The robustness of the system can be guaranteed under the different network topologies of initial states.

5. Conclusion

A feedback-based protocol is designed to control CAV vehicular platoon under the non-lane-discipline environment. Stability and consensus of the proposed protocol is analyzed using the Lyapunov technique. Theoretical analysis proves that the longitudinal and lateral gaps as well as the velocity can simultaneously reach the consensus state. In addition, the

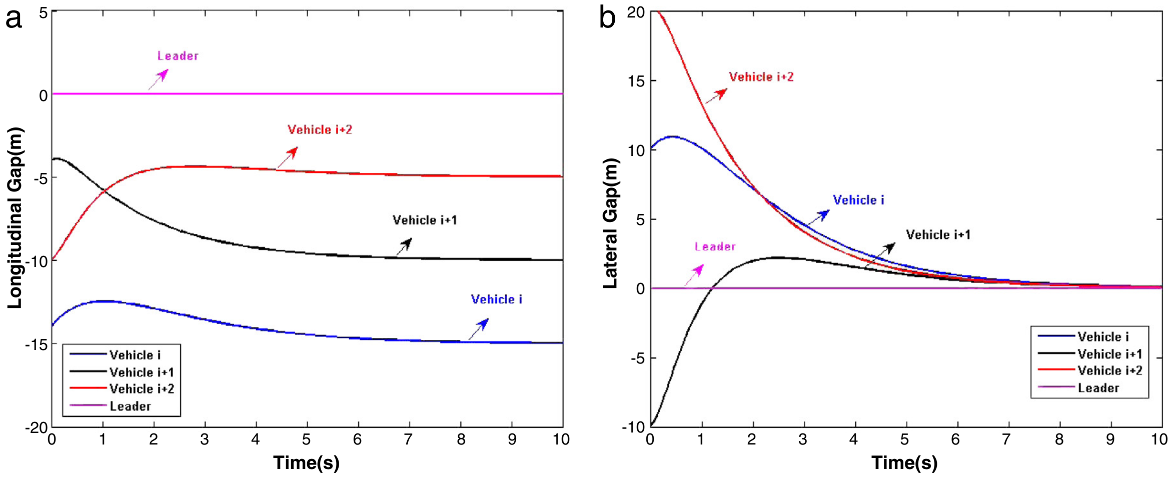

Fig. 10. The gap trajectories of CAVs: (a) The longitudinal gap; (b) The lateral gap.

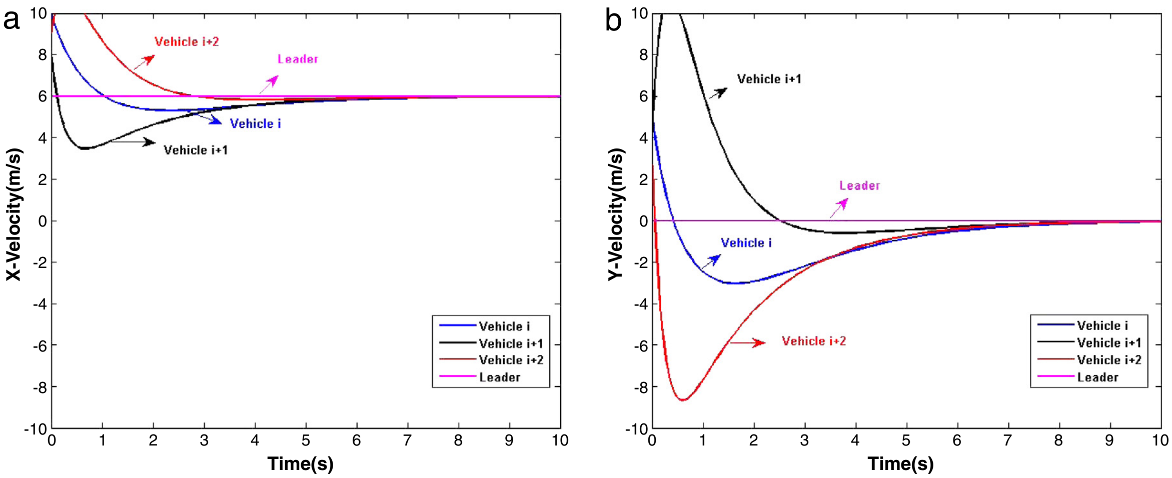

Fig. 11. The velocity trajectories of CAVs: (a) x-velocity; (b) y-velocity.

Table 1
Performance comparisons.

Index		Case		
		Case I	Case II	Case III
Convergence time	Longitudinal gap x-velocity	7.9 s	8.1 s	8.4 s
	Lateral gap y-velocity	9.2 s	9.4 s	9.6 s
Oscillation amplitude		Medium	Medium	Medium

effects of different network topologies of initial states are also analyzed. Results from numerical experiments verify the effectiveness of the proposed control protocol in terms of the convergence time and robustness.

The findings of this study illustrate that the convergence time of the control protocol is associated with the initial states, while the robustness is not affected by the initial states significantly. Also, this study provides insights on the efficient control protocol design for the vehicular platoon with respect to the convergence time and robustness under the V2V communication environment.

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