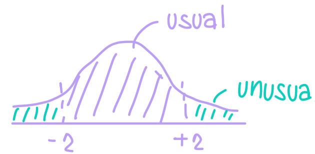
Hypothesis Testing → population

(C.I.) Testing claims → true / false



The average payload of trucks on the highway is 18,000 lbs.

Rare event rule

guilty not guilty

มีความผิด

gender selection :

claim : There is at least 50% chance of having a girl.

Random sample of 100 couples are studied.

Assumption : The drug does not work. → 50% chance of boy & girl.

1. 52 had girls.
2. 17 had girls.

Parts of a hypothesis test

null hypothesis vs. **alternative hypothesis**

null hypothesis — H_0

states that the population parameter (μ, p, σ) is equal to a certain value.

p: proportion

e.g.: $H_0: \mu = 5$; $p = 0.5$

alternative hypothesis — H_1 / H_a

states that the population parameter has a value different from H_a ($<$, $>$, $\neq$)

e.g: $p < 0.54$, $\mu > 82$, $p \neq 0.5$

How to test hypothesis ?

$H_0 \leftrightarrow H_a$

* begin by assuming that H_0 is true.
* then you have to use the evidence (given by the sample) to reach a conclusion.
  - reject H_0 (enough evidence to prove that H_0 is wrong)
  - fail to reject H_0 $\approx$ accepting H_0 ?

* if you want to support a claim, always state it as H_1 (not H_0)

  e.g. suppose I want to prove that the drug works

  $H_0: p = 0.50$ (×)

  $H_1: p > 0.50$ (✓)

  inconclusive rejection region

How to identify H_0 and H_1

1. state the original claim symbolically.
2. state the opposite of the claim also.

e.g.1. The mean fluid volume in a can is at least 12 oz.

claim : $\mu \geq 12$ $\rightarrow$ $H_0 : \mu = 12$ (claim)

opposite : $\mu < 12$ $H_a : \mu < 12$ (opposite)

e.g.2. The proportion of male CEO's is greater than 0.5

claim : $p > 0.5$ $H_a : p > 0.5$ (claim)

opposite : $p \leq 0.5$ $\rightarrow$ $H_0 : p = 0.5$ (opposite)

e.g.3. The mean weight of babies is at most 8.9 lbs.

claim : $\mu \leq 8.9$ $\rightarrow$ $H_0 : \mu = 8.9$ (claim)

opposite : $\mu > 8.9$ $H_a : \mu > 8.9$ (opposite)

e.g.4. The mean IQ score is 100.

claim : $\mu = 100$ $\rightarrow$ $H_0 : \mu = 100$ (claim)

opposite : $\mu \neq 100$ $H_a : \mu \neq 100$ (opposite)

Test statistics

proportion : $p = z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}}$

mean (μ) = $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ σ unknown

$T = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$ σ unknown

e.g. A sample of 406 companies found that 61% of the CEO's are male. Claim that most CEO's are male.

claim: $p > 0.5$; $H_a: p > 0.5$

opposite : $p \leq 0.5$; $H_0: p = 0.5$

$\hat{p} = 0.61$

$p = 0.5$

$q = 0.5$

$n = 406$

$$z = \frac{0.61 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{406}}}$$

$z = 5.84$

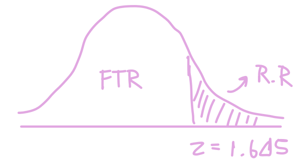
How to make the decision ?

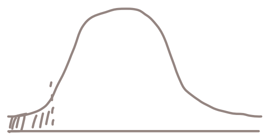
significance level (α)

common α values : 0.10, 0.05, 0.01

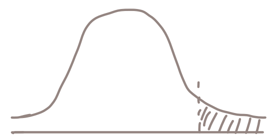
critical value : seperates the rejection region from fail to reject region.

$\alpha = 0.05 \rightarrow z = 1.645$

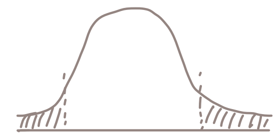




right tail



left tail

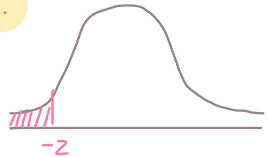


two-tail test

2-ways to make the decision

traditional method :

1.



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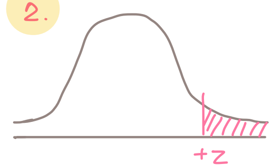

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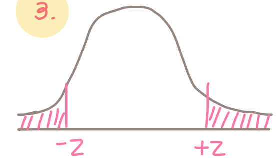
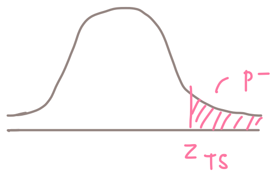


p-value method :

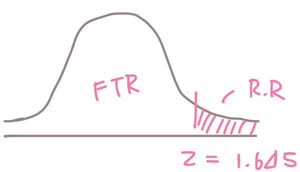


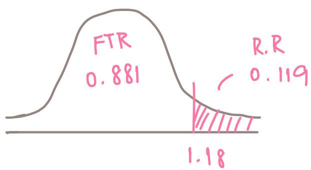
p-value (look up this area)

p-value $\leq \alpha$ → reject

p-value $> \alpha$ → fail to reject H_0

e.g. Assume $\alpha = 0.05$, $H_1: p > 0.25$, $z_{TS} = 1.18$





$0.119 > 0.05$