MTH 2401 Midterm Test II - Practice, Spring 2024

Problem 1: A particular brand of dishwasher soap is sold in three sizes: 25 oz, 40 oz, and 65 oz. Twenty percent of all purchasers select a 25-oz box, 50% select a 40-oz box, and the remaining 30% choose a 65-oz box. Let X1 and X2 denote the package sizes selected by two independently selected purchasers.
(a) Determine the sampling distribution of $\bar{X}$, calculate $E(\bar{X})$, and compare to μ.
(b) Determine the sampling distributions of the sample variance S^2, calculate $E(S^2)$, and compare to σ^2.

a). 25 40 65 / 20% 50% 30% =>

X_2 \ X_1	25	40	65	
25	.04	.10	.06	.2
40	.1	.25	.15	.5
60	.06	.15	.09	.3
	.2	.5	.3	

$E(\bar{x}) = (25 \cdot .04) + (32.5 \cdot .2) + (40 \cdot .25) + (45 \cdot .12) + (52.5 \cdot .3) + (65 \cdot .09) \Rightarrow \boxed{44.5}$

$\mu = 25(.2) + (40(.5) + (65 \cdot .3) = \boxed{44.5}$

$E(\bar{x}) = \mu$

b). $s^2 = \frac{1}{2-1} \sum_{i=1}^{2} (x_i - \bar{x})^2 =$

s^2	0	112.5	312.5	800
$p(s)^2$	.38	.20	.3	.12

$E(s)^2 = (0 \cdot .38) + (112.5 \cdot .2) + (312.5 \cdot .3) + (800 \cdot .12) = \boxed{212.25}$

$\sigma^2 = (25 - 44.5)^2(.2) + (40 - 44.5)^2(.5) + (65 - 44.5)^2(.3) = \boxed{212.25}$

$\checkmark$ $E(s)^2 = \sigma^2$

Problem 2: There are 40 students in an elementary statistics class. On the basis of years of experience, the instructor knows that the time needed to grade a randomly chosen first examination paper is a random variable with an expected value of 6 min and a standard deviation of 6 min.
(a). If grading times are independent and the instructor begins grading at 6:50 p.m. and grades continuously, what is the (approximate) probability that he is through grading before the 11:00 p.m. TV news begins?
(b). If the sports report begins at 11:10, what is the probability that he misses part of the report if he waits until grading is done before turning on the TV?

a). 11: - 6:50 = 4h10m = 250min => 40·6 = 240 min $\sigma T_0 = \underline{37.95}$

$P\left(z \le \frac{250-240}{37.95}\right) = P(z \le .26) = \boxed{0.6026}$

b). $P\left(z \le \frac{260-240}{37.95}\right) = P(z > .53) = 0.2981$

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Problem 3: The lifetime of a certain type of battery is normally distributed with mean value 10 hours and standard deviation 1 hour. There are four batteries in a package. What lifetime value is such that the total lifetime of all batteries in a package exceeds that value for only 5% of all packages?

4 bat $\cdot$ 1h $\cdot$ 10 h = 40 -> T_0 40 + (1.645(2)) = 43.29 h

Problem 4: Consider the following sample of observations on coating thickness for low viscosity paint ("Achieving a Target Value for a Manufacturing Process: A Case Study," J. of Quality Technology, 1992: 22–26):

.83	.88	.88	1.04	1.09	1.12	1.29	1.31
1.48	1.49	1.59	1.62	1.65	1.71	1.76	1.83

Assume that the distribution of coating thickness is normal (a normal probability plot strongly supports this assumption).
(a) Calculate a point estimate of the mean value of coating thickness, and state which estimator you used.
(b) Calculate a point estimate of the median of the coating thickness distribution, and state which estimator you used.

(c) Estimate $P(X < 1.5)$, i.e., the proportion of all thickness values less than 1.5. [Hint: If you knew the values of m and s, you could calculate this probability. These values are not available, but they can be estimated.]
(d) What is the estimated standard error of the estimator that you used in part (b)?

a). mean $= (x_1 + x_2 \ldots x_{16})/16 = \bar{x} = 1.3481$

$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = 0.3385$

b). $x = 1.3481$

c). $P\left(z < \frac{1.5 - 1.3481}{0.3385}\right) = P(z < 0.45) = 0.6736$

d). $\bar{x} = \frac{0.3385}{4} = 0.0846$

Problem 5: Each of 150 newly manufactured items is examined and the number of scratches per item is recorded (the items are supposed to be free of scratches), yielding the following data:

Number of scratches per item	0	1	2	3	4	5	6	7
Observed frequency	18	37	42	30	13	7	2	1

Let $X = 5$ the number of scratches on a randomly chosen item, and assume that X has a Poisson distribution with parameter m.
(a) Find an unbiased estimator of m and compute the estimate for the data.
(b) What is the standard deviation (standard error) of your estimator? Compute the estimated standard error.

a). $(0 \cdot 18) + (1 \cdot 37) + (2 \cdot 42) + (3 \cdot 30) +$
$(4 \cdot 13) + (5 \cdot 7) + (6 \cdot 2) + (7 \cdot 1)$
$= \frac{317}{150} = 2.11$

b). $\sigma_x = \sqrt{\frac{\mu}{\sqrt{n}}} = \frac{\sqrt{2.11}}{\sqrt{150}} = 0.119$

Problem 6: Consider a random sample $X_1, \ldots, X_n$ from the pdf

$$f(x;\theta) = 0.5(1+\theta x), \qquad -1 \le x \le 1$$

where $-1 \le \theta \le 1$. Is $\hat{\theta} = 3\bar{X}$ is an unbiased estimtor of θ? Explain why?

$$\mu = E(x) = \hat{\theta} = 3\bar{x} \Rightarrow E(3\bar{x}) = 3\mu = 3\left(\frac{1}{3}\right)\theta = \theta$$

$$\mu = E(x) = \int_{-1}^{1} x \cdot \frac{1}{2}(1+\theta x)\,dx = \frac{x^2}{4} + \frac{\theta x^3}{6}\Big|_{-1}^{1} = \frac{1}{3}\theta = 3\mu$$