Task 5

To proof:

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(s) e^{-\frac{s^2}{2}} ds = \int_0^1 f(\Phi^{-1}(t)) dt$$

Proof:

$$\begin{aligned}
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(s) e^{-\frac{s^2}{2}} ds &= \lim_{a \to \infty} \int_{-a}^{a} f(s) \phi'(s) ds \\
&= \lim_{a \to \infty} \int_{\Phi(-a)}^{\Phi(a)} f(\Phi^{-1}(t)) dt \\
&= \int_0^1 f(\Phi^{-1}(t)) dt
\end{aligned}$$

With substitution $s \to \Phi^{-1}(t)$.

□