

# EE 220 Notes

Silas Kinnear

# Contents

<b>1</b>	<b>Basic Concepts</b>	<b>4</b>
1.1	SI Units . . . . .	4
1.2	CGS Units . . . . .	4
1.3	Electric Charge . . . . .	4
1.4	Current . . . . .	5
1.5	Voltage . . . . .	5
1.6	Power and Energy . . . . .	5
1.6.1	Power . . . . .	5
1.6.2	Energy . . . . .	5
1.7	Circuit Elements . . . . .	5
1.7.1	Resistor . . . . .	6
1.7.2	Capacitor . . . . .	6
1.7.3	Inductor . . . . .	6
<b>2</b>	<b>Basic Laws</b>	<b>7</b>
2.1	Ohm's Law . . . . .	7
2.2	Nodes, Branches, and Loops . . . . .	7
2.3	Kirchhoff's Laws . . . . .	7
2.3.1	Kirchhoff's Current Law (KCL) . . . . .	7
2.3.2	Kirchhoff's Voltage Law (KVL) . . . . .	7
2.4	Series Resistors and Voltage Division . . . . .	7
2.5	Parallel Resistors and Current Division . . . . .	8
2.6	Wye-Delta Transformations . . . . .	8
<b>3</b>	<b>Methods of Analysis</b>	<b>9</b>
3.1	Nodal Analysis . . . . .	9
3.1.1	Nodal Analysis with Voltage Sources . . . . .	9
3.2	Mesh Analysis . . . . .	9
3.2.1	Mesh Analysis with Current Sources . . . . .	10
3.3	Nodal and Mesh Analyses by Inspection . . . . .	10
3.4	Nodal versus Mesh Analysis . . . . .	10
<b>4</b>	<b>Circuit Theorems</b>	<b>11</b>
4.1	Linearity Property . . . . .	11
4.1.1	Implications of Linearity . . . . .	11
4.2	Superposition . . . . .	11
4.2.1	Procedure for Superposition . . . . .	11
4.3	Source Transformation . . . . .	12
4.3.1	Transformation Equations . . . . .	12
4.4	Thevenin's Theorem . . . . .	12
4.4.1	Determining Thevenin's Equivalent . . . . .	12
4.5	Norton's Theorem . . . . .	12
4.5.1	Determining Norton's Equivalent . . . . .	12

4.6	Derivations of Thevenin's and Norton's Theorems . . . . .	13
4.7	Maximum Power Transfer Theorem . . . . .	13
4.7.1	Derivation of Maximum Power Transfer . . . . .	13
<b>5</b>	<b>Operational Amplifiers</b>	<b>14</b>
5.1	Ideal Op-Amp . . . . .	14
5.2	Inverting Amplifier . . . . .	14
5.3	Noninverting Amplifier . . . . .	14
5.4	Summing Amplifier . . . . .	15
5.5	Difference Amplifier . . . . .	15
5.6	Cascaded Op-Amp Circuits . . . . .	15
<b>6</b>	<b>Capacitors and Inductors</b>	<b>16</b>
6.1	Capacitors . . . . .	16
6.1.1	Capacitance . . . . .	16
6.1.2	Types of Capacitors . . . . .	16
6.2	Series and Parallel Capacitors . . . . .	16
6.2.1	Series Capacitors . . . . .	16
6.2.2	Parallel Capacitors . . . . .	17
6.3	Inductors . . . . .	17
6.3.1	Inductance . . . . .	17
6.3.2	Types of Inductors . . . . .	17
6.4	Series and Parallel Inductors . . . . .	17
6.4.1	Series Inductors . . . . .	17
6.4.2	Parallel Inductors . . . . .	17
6.5	Conclusion . . . . .	18

# Chapter 1

## Basic Concepts

In electrical engineering, several systems of units are commonly used, including the SI (International System of Units) and the CGS (Centimeter-Gram-Second) system.

### 1.1 SI Units

The SI units relevant to electrical engineering are:

- Length: meter (m)
- Mass: kilogram (kg)
- Time: second (s)
- Electric current: ampere (A)
- Voltage: volt (V)
- Power: watt (W)
- Energy: joule (J)

### 1.2 CGS Units

In the CGS system:

- Electric charge: statcoulomb (esu)
- Voltage: statvolt
- Current: abampere

### 1.3 Electric Charge

Electric charge ( $Q$ ) is a fundamental property of matter that causes it to experience a force when placed in an electromagnetic field. The SI unit of charge is the coulomb (C).

## 1.4 Current

Electric current ( $I$ ) is the flow of electric charge and is defined as the rate of flow of charge. It is measured in amperes (A).

$$I = \frac{Q}{t}$$

where  $I$  is the current in amperes,  $Q$  is the charge in coulombs, and  $t$  is the time in seconds.

**Example:** If 5 coulombs of charge flow through a wire in 10 seconds, the current is:

$$I = \frac{5 \text{ C}}{10 \text{ s}} = 0.5 \text{ A}$$

## 1.5 Voltage

Voltage ( $V$ ), or electric potential difference, is the work done per unit charge to move a charge between two points in an electric field. The SI unit of voltage is the volt (V).

$$V = \frac{W}{Q}$$

where  $V$  is the voltage in volts,  $W$  is the work done in joules, and  $Q$  is the charge in coulombs.

**Example:** If 20 joules of work is done to move 4 coulombs of charge, the voltage is:

$$V = \frac{20 \text{ J}}{4 \text{ C}} = 5 \text{ V}$$

## 1.6 Power and Energy

### 1.6.1 Power

Power ( $P$ ) is the rate at which energy is transferred or converted. The SI unit of power is the watt (W), which is equivalent to one joule per second (J/s).

$$P = IV$$

where  $P$  is the power in watts,  $I$  is the current in amperes, and  $V$  is the voltage in volts.

**Example:** If a circuit has a current of 3 A flowing through a voltage of 12 V, the power is:

$$P = 3 \text{ A} \times 12 \text{ V} = 36 \text{ W}$$

### 1.6.2 Energy

Energy ( $E$ ) is the capacity to do work and is measured in joules (J). The relationship between power and energy over time is given by:

$$E = Pt$$

where  $E$  is energy in joules,  $P$  is power in watts, and  $t$  is time in seconds.

**Example:** If a device operates at 50 W for 2 hours (7200 seconds), the energy consumed is:

$$E = 50 \text{ W} \times 7200 \text{ s} = 360000 \text{ J} \quad (\text{or } 360 \text{ kJ})$$

## 1.7 Circuit Elements

Common circuit elements include resistors, capacitors, and inductors.

### 1.7.1 Resistor

A resistor opposes the flow of current and is characterized by its resistance ( $R$ ) measured in ohms ( $\Omega$ ).

$$V = IR$$

where  $V$  is the voltage across the resistor,  $I$  is the current through it, and  $R$  is resistance.

**Example:** For a resistor with a resistance of  $4\ \Omega$  and a current of  $2\ \text{A}$  flowing through it, the voltage is:

$$V = 2\ \text{A} \times 4\ \Omega = 8\ \text{V}$$

### 1.7.2 Capacitor

A capacitor stores electric charge and energy in an electric field. The capacitance ( $C$ ) is measured in farads (F).

$$Q = CV$$

where  $Q$  is the charge in coulombs,  $C$  is capacitance in farads, and  $V$  is voltage across the capacitor.

**Example:** A capacitor with a capacitance of  $10\ \mu\text{F}$  charged to a voltage of  $5\ \text{V}$  will store:

$$Q = 10 \times 10^{-6}\ \text{F} \times 5\ \text{V} = 50 \times 10^{-6}\ \text{C} \quad (\text{or } 50\ \mu\text{C})$$

### 1.7.3 Inductor

An inductor stores energy in a magnetic field when electric current flows through it. The inductance ( $L$ ) is measured in henries (H).

$$V = L \frac{dI}{dt}$$

where  $V$  is the voltage across the inductor,  $L$  is inductance, and  $\frac{dI}{dt}$  is the rate of change of current.

**Example:** If an inductor has an inductance of  $2\ \text{H}$  and the current changes at a rate of  $3\ \text{A/s}$ , the voltage across the inductor is:

$$V = 2\ \text{H} \times 3\ \text{A/s} = 6\ \text{V}$$

# Chapter 2

## Basic Laws

### 2.1 Ohm's Law

Ohm's Law states that the current through a conductor between two points is directly proportional to the voltage across the two points. Mathematically, it can be expressed as:

$$V = IR$$

where  $V$  is voltage,  $I$  is current, and  $R$  is resistance.

### 2.2 Nodes, Branches, and Loops

In a circuit:

- A **node** is a point where two or more components are connected.
- A **branch** is a path connecting two nodes that contains a component.
- A **loop** is any closed path within a circuit.

### 2.3 Kirchhoff's Laws

#### 2.3.1 Kirchhoff's Current Law (KCL)

The total current entering a junction equals the total current leaving the junction.

#### 2.3.2 Kirchhoff's Voltage Law (KVL)

The sum of the electrical potential differences (voltage) around any closed network is zero.

### 2.4 Series Resistors and Voltage Division

In a series circuit, resistors are connected end-to-end. The total resistance is the sum of the individual resistances:

$$R_{\text{total}} = R_1 + R_2 + \dots + R_n$$

The voltage across each resistor can be calculated using voltage division:

$$V_i = \frac{R_i}{R_{\text{total}}} V_{\text{total}}$$

## 2.5 Parallel Resistors and Current Division

In a parallel circuit, resistors are connected across the same two nodes. The total resistance  $R_{\text{total}}$  can be calculated as:

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

The current through each resistor can be found using current division:

$$I_i = \frac{V}{R_i}$$

## 2.6 Wye-Delta Transformations

Wye-Delta transformations are techniques used to simplify the analysis of circuits. In a Wye (Y) configuration, three resistors connect to a common node, while in a Delta ( $\Delta$ ) configuration, the resistors are arranged in a triangle.

**Wye to Delta Conversion:**

$$R_{AB} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$R_{AC} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

$$R_{BC} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

**Delta to Wye Conversion:**

$$R_a = \frac{R_{AB} R_{AC}}{R_{AB} + R_{BC} + R_{AC}}$$

$$R_b = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC} + R_{AC}}$$

$$R_c = \frac{R_{AC} R_{BC}}{R_{AB} + R_{BC} + R_{AC}}$$



## Chapter 3

# Methods of Analysis

### 3.1 Nodal Analysis

Nodal analysis is a systematic method used to determine the voltage at each node relative to a common reference point (ground) in an electrical circuit. The primary principle behind nodal analysis is Kirchhoff's Current Law (KCL), which states that the total current entering a node must equal the total current leaving the node. This method is particularly useful for analyzing circuits with multiple nodes and branches.

To perform nodal analysis, follow these steps:

1. Choose a reference node (ground) and label the remaining nodes with voltages  $V_1, V_2, \dots, V_n$ .
2. Apply KCL at each node, expressing the currents in terms of the node voltages and the resistances of the branches connected to those nodes.
3. Solve the resulting system of equations to find the node voltages.

#### 3.1.1 Nodal Analysis with Voltage Sources

When a voltage source is connected to a node, it complicates the analysis because the current through the voltage source is not directly known. In this case, we can treat the voltage source and its connected node as a supernode. A supernode encompasses both the voltage source and the nodes it connects.

**Example:** Consider a circuit with a 10 V voltage source connected between nodes  $A$  and  $B$  and resistors  $R_1 = 2\Omega$  connected to node  $A$  and  $R_2 = 4\Omega$  connected to node  $B$ .

1. Set node  $B$  to ground (0 V).
2. The voltage at node  $A$  is 10 V because of the voltage source.
3. Applying KCL at node  $A$ :

$$\frac{V_A - V_B}{R_1} + \frac{V_A}{R_2} = 0$$

Substituting  $V_B = 0$  and  $V_A = 10$ :

$$\frac{10 - 0}{2} + \frac{10}{4} = 0$$

4. This simplifies to:

$$5 + 2.5 = 0 \quad (\text{not balanced, confirms analysis})$$

5. Thus, you can determine that the current through  $R_1$  and  $R_2$  can be calculated once the voltages are determined.

### 3.2 Mesh Analysis

Mesh analysis is another fundamental technique used to analyze electrical circuits, focusing on the loop currents flowing in a circuit. It is based on Kirchhoff's Voltage Law (KVL), which states that the sum of all voltage drops around a closed loop must equal zero.

To perform mesh analysis:

1. Identify the meshes (independent loops) in the circuit.
2. Assign a mesh current to each loop.
3. Write KVL equations for each mesh, summing the voltage drops and sources.
4. Solve the system of equations to find the mesh currents.

### 3.2.1 Mesh Analysis with Current Sources

When a current source is present in a mesh, we can apply mesh analysis directly to the loops that include it. The current source imposes a specific current in that mesh, simplifying our equations.

**Example:** Consider a circuit with two meshes. Mesh 1 contains a current source of 2 A and resistors  $R_1 = 3\Omega$  and  $R_2 = 5\Omega$ . Mesh 2 contains  $R_2$  and another resistor  $R_3 = 2\Omega$ .

1. Assign mesh currents  $I_1$  for Mesh 1 and  $I_2$  for Mesh 2. 2. Write KVL for Mesh 1:

$$-I_1 R_1 + I_2 R_2 = 0$$

Substituting  $R_1$  and  $R_2$ :

$$-3I_1 + 5I_2 = 0$$

3. Write KVL for Mesh 2:

$$-I_2 R_2 + I_1 R_2 + I_2 R_3 = 0$$

This gives:

$$-5I_2 + 3I_1 + 2I_2 = 0 \Rightarrow 3I_1 - 3I_2 = 0$$

4. Now, using the current source  $I_1 = 2\text{ A}$ :

$$3(2) - 3I_2 = 0 \Rightarrow I_2 = 2\text{ A}$$

Thus, both mesh currents are determined to be 2 A.

## 3.3 Nodal and Mesh Analyses by Inspection

In some simple circuits, it is possible to perform nodal and mesh analyses by inspection. This approach allows for quick evaluations of circuits without needing to set up extensive equations.

**Example:** For a simple series circuit with a single voltage source  $V = 12\text{ V}$  and two resistors  $R_1 = 2\Omega$  and  $R_2 = 3\Omega$ :

1. The total resistance  $R_{\text{total}} = R_1 + R_2 = 2 + 3 = 5\Omega$ . 2. The total current can be found using Ohm's Law:

$$I = \frac{V}{R_{\text{total}}} = \frac{12}{5} = 2.4\text{ A}$$

3. The voltage drop across  $R_1$  is:

$$V_{R_1} = I \times R_1 = 2.4 \times 2 = 4.8\text{ V}$$

4. The voltage drop across  $R_2$  is:

$$V_{R_2} = I \times R_2 = 2.4 \times 3 = 7.2\text{ V}$$

This allows for a quick verification of voltages and currents without detailed equations.

## 3.4 Nodal versus Mesh Analysis

Nodal and mesh analyses are both valuable techniques for circuit analysis, each with its strengths and weaknesses.

- **Nodal Analysis:** More efficient for circuits with many nodes and fewer loops. It is particularly useful when there are many components connected to a single node.

- **Mesh Analysis:** Preferable for circuits with fewer nodes and more loops. It is often easier to apply when the circuit has several series components.

Understanding the strengths of each method allows engineers to choose the most effective analysis technique based on the characteristics of the circuit being analyzed.

## Chapter 4

# Circuit Theorems

### 4.1 Linearity Property

The linearity property in electrical circuits states that the output response (voltage or current) of a linear circuit is directly proportional to its input. This implies that if the input is scaled (multiplied) by a constant factor, the output will scale by the same factor. Mathematically, if  $f(x)$  is a linear function, then:

$$f(ax) = af(x)$$

for any constant  $a$ .

#### 4.1.1 Implications of Linearity

Linearity allows for simpler analysis of circuits because we can use superposition and other linear techniques. Linear components include resistors, inductors, capacitors, and ideal operational amplifiers. However, non-linear components such as diodes and transistors do not exhibit this property.

### 4.2 Superposition

The superposition theorem states that in a linear circuit with multiple independent sources, the total response (voltage or current) at any component is the sum of the responses caused by each independent source acting alone while all other independent sources are turned off. For voltage sources, this means replacing them with short circuits, and for current sources, replacing them with open circuits.

#### 4.2.1 Procedure for Superposition

1. Identify all independent sources in the circuit.
2. For each independent source:
  - Turn off all other sources.
  - Analyze the circuit to find the response (voltage or current).
3. Sum the individual responses to find the total response.

**Example:** Consider a circuit with two voltage sources  $V_1 = 10\text{ V}$  and  $V_2 = 5\text{ V}$  connected to a resistor  $R = 4\ \Omega$ : - For  $V_1$  alone,  $I_1 = \frac{V_1}{R} = \frac{10}{4} = 2.5\text{ A}$ . - For  $V_2$  alone,  $I_2 = \frac{V_2}{R} = \frac{5}{4} = 1.25\text{ A}$ . - Total current  $I = I_1 + I_2 = 2.5 + 1.25 = 3.75\text{ A}$ .

## 4.3 Source Transformation

Source transformation is a technique that allows you to simplify a circuit by converting a voltage source in series with a resistor into an equivalent current source in parallel with the same resistor, or vice versa.

### 4.3.1 Transformation Equations

Given a voltage source  $V_s$  with a series resistor  $R_s$ , the equivalent current source  $I_s$  with parallel resistor  $R_p$  is given by:

$$I_s = \frac{V_s}{R_s} \quad \text{and} \quad R_p = R_s$$

Conversely, given a current source  $I_s$  in parallel with  $R_s$ , the equivalent voltage source is:

$$V_s = I_s R_s \quad \text{and} \quad R_p = R_s$$

**Example:** For a voltage source of 12 V in series with a 6  $\Omega$  resistor, the equivalent current source is:

$$I_s = \frac{12}{6} = 2 \text{ A} \quad \text{and} \quad R_p = 6 \Omega$$

## 4.4 Thevenin's Theorem

Thevenin's theorem states that any linear circuit with voltage sources and resistors can be replaced by an equivalent circuit consisting of a single voltage source  $V_{th}$  in series with a single resistor  $R_{th}$ .

### 4.4.1 Determining Thevenin's Equivalent

To find the Thevenin equivalent:

1. Remove the load resistance from the circuit.
2. Calculate  $V_{th}$ , the open-circuit voltage across the terminals.
3. Calculate  $R_{th}$ , the equivalent resistance seen from the terminals with independent sources turned off:
  - Voltage sources are shorted.
  - Current sources are opened.

**Example:** In a circuit with a 10 V source and 5  $\Omega$  resistor, removing a load resistor gives: -  $V_{th} = 10 \text{ V}$   
 -  $R_{th} = 5 \Omega$ .

## 4.5 Norton's Theorem

Norton's theorem is similar to Thevenin's theorem, stating that any linear circuit can be replaced by an equivalent current source  $I_n$  in parallel with a resistor  $R_n$ .

### 4.5.1 Determining Norton's Equivalent

To find Norton's equivalent:

1. Remove the load resistance from the circuit.
2. Calculate  $I_n$ , the short-circuit current through the terminals.
3. Calculate  $R_n$ , the equivalent resistance seen from the terminals as in Thevenin's theorem.

**Example:** For the same circuit with a 10 V source and 5  $\Omega$  resistor: - If the short-circuit current  $I_n = 2 \text{ A}$ ,  
 - Then,  $R_n = 5 \Omega$ .

## 4.6 Derivations of Thevenin's and Norton's Theorems

Thevenin's and Norton's theorems can be derived from each other, as they are fundamentally based on the same linear circuit principles.

1. **From Thevenin to Norton:** The equivalent current source  $I_n$  can be derived from Thevenin's voltage:

$$I_n = \frac{V_{th}}{R_{th}}$$

with the same  $R_n = R_{th}$ .

2. **From Norton to Thevenin:** The equivalent voltage source  $V_{th}$  can be derived from Norton's current:

$$V_{th} = I_n R_n$$

with  $R_{th} = R_n$ .

## 4.7 Maximum Power Transfer Theorem

The maximum power transfer theorem states that maximum power is delivered to a load when the load resistance  $R_L$  is equal to the Thevenin resistance  $R_{th}$  of the source network.

### 4.7.1 Derivation of Maximum Power Transfer

1. The power delivered to the load is given by:

$$P = \frac{V_{th}^2 R_L}{(R_{th} + R_L)^2}$$

2. To find the maximum power, take the derivative of  $P$  with respect to  $R_L$ , set it to zero, and solve for  $R_L$ :

$$\frac{dP}{dR_L} = 0 \quad \Rightarrow \quad R_L = R_{th}$$

**Example:** If  $V_{th} = 10\text{ V}$  and  $R_{th} = 4\ \Omega$ , the load resistance for maximum power transfer is:

$$R_L = 4\ \Omega.$$

Substituting into the power equation gives:

$$P = \frac{10^2 \cdot 4}{(4 + 4)^2} = \frac{100 \cdot 4}{64} = 6.25\text{ W}.$$

Thus, the maximum power delivered to the load is 6.25 W.

## Chapter 5

# Operational Amplifiers

Operational amplifiers (op-amps) are versatile components widely used in electronic circuits for signal amplification, filtering, and mathematical operations. An ideal op-amp has infinite open-loop gain, infinite input impedance, and zero output impedance.

### 5.1 Ideal Op-Amp

An ideal op-amp has the following characteristics: - **Infinite Open-Loop Gain** ( $A_{OL} \rightarrow \infty$ ): Any non-zero difference between the inverting ( $V_-$ ) and non-inverting ( $V_+$ ) inputs results in a large output. - **Infinite Input Impedance** ( $Z_{in} \rightarrow \infty$ ): No current flows into the input terminals, allowing the op-amp to not affect the circuit it is connected to. - **Zero Output Impedance** ( $Z_{out} = 0$ ): The output can drive any load without affecting its output voltage. - **Infinite Bandwidth**: The op-amp can amplify signals of any frequency without attenuation.

### 5.2 Inverting Amplifier

The inverting amplifier configuration produces an output that is inversely proportional to the input. The relationship is defined as:

$$V_{out} = -\frac{R_f}{R_{in}} V_{in}$$

where:

$V_{out}$  is the output voltage,  
 $R_f$  is the feedback resistor,  
 $R_{in}$  is the input resistor,  
 $V_{in}$  is the input voltage.

**Example:** If  $R_f = 10\Omega$  and  $R_{in} = 2\Omega$  with an input voltage of  $5V$ :

$$V_{out} = -\frac{10}{2} \times 5 = -25V$$

### 5.3 Noninverting Amplifier

‘ In the noninverting amplifier configuration, the output is in phase with the input and is given by:

$$V_{out} = \left(1 + \frac{R_f}{R_{in}}\right) V_{in}$$

where  $R_f$  and  $R_{in}$  are the same as described in the inverting amplifier.

**Example:** If  $R_f = 10\Omega$  and  $R_{in} = 5\Omega$  with an input voltage of  $2V$ :

$$V_{out} = \left(1 + \frac{10}{5}\right) \times 2 = 5 \times 2 = 10V$$

## 5.4 Summing Amplifier

A summing amplifier is used to produce an output that is the weighted sum of multiple input signals. The output voltage is given by:

$$V_{out} = - \left( \frac{R_f}{R_{in1}} V_{in1} + \frac{R_f}{R_{in2}} V_{in2} + \dots + \frac{R_f}{R_{inN}} V_{inN} \right)$$

For equal input resistors:

$$V_{out} = - \frac{R_f}{R_{in}} (V_{in1} + V_{in2} + \dots + V_{inN})$$

## 5.5 Difference Amplifier

A difference amplifier outputs the difference between two input voltages. The output voltage is given by:

$$V_{out} = \left( \frac{R_f}{R_{in}} \right) (V_2 - V_1)$$

where  $V_1$  and  $V_2$  are the input voltages.

**Example:** For  $R_f = 10\Omega$  and  $R_{in} = 5\Omega$ , if  $V_1 = 2V$  and  $V_2 = 5V$ :

$$V_{out} = \frac{10}{5} (5 - 2) = 2 \times 3 = 6V$$

## 5.6 Cascaded Op-Amp Circuits

Cascading op-amps involves connecting multiple op-amp configurations to achieve a desired overall gain or functionality. The overall voltage gain of cascaded amplifiers can be calculated by multiplying the gains of individual stages:

$$V_{out} = V_{in} \times A_1 \times A_2 \times \dots \times A_n$$

where  $A_1, A_2, \dots, A_n$  are the voltage gains of each stage.

**Example:** If you have two stages with gains of  $A_1 = 2$  and  $A_2 = 3$  and an input voltage  $V_{in} = 1V$ :

$$V_{out} = 1 \times 2 \times 3 = 6V$$

This allows for complex signal processing tasks, including filtering and analog computation.

## Chapter 6

# Capacitors and Inductors

### 6.1 Capacitors

Capacitors are passive electrical components that store electrical energy in an electric field. They consist of two conductive plates separated by an insulating material known as a dielectric. The ability of a capacitor to store charge is quantified by its capacitance, denoted by the symbol  $C$ , and is measured in farads (F).

#### 6.1.1 Capacitance

The capacitance of a capacitor is defined as the ratio of the charge  $Q$  stored on one plate to the voltage  $V$  across the plates:

$$C = \frac{Q}{V}$$

Where:  $C$  = Capacitance in farads (F)  $Q$  = Charge in coulombs (C)  $V$  = Voltage in volts (V)

#### 6.1.2 Types of Capacitors

There are several types of capacitors, including:

- **Ceramic Capacitors:** Small, non-polarized capacitors used for high-frequency applications.
- **Electrolytic Capacitors:** Polarized capacitors that offer high capacitance values, often used in power supply applications.
- **Tantalum Capacitors:** Another type of polarized capacitor with stable capacitance and smaller size.

### 6.2 Series and Parallel Capacitors

Capacitors can be connected in two primary configurations: series and parallel. The overall capacitance of these configurations is different.

#### 6.2.1 Series Capacitors

When capacitors are connected in series, the total capacitance  $C_{total}$  can be calculated using the formula:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

Where:  $C_1, C_2, C_3, \dots, C_n$  are the capacitances of the individual capacitors.

In series, the total capacitance is always less than the smallest capacitor in the series.



### 6.2.2 Parallel Capacitors

When capacitors are connected in parallel, the total capacitance  $C_{total}$  is simply the sum of the individual capacitances:

$$C_{total} = C_1 + C_2 + C_3 + \dots + C_n$$

This configuration allows for a larger overall capacitance, making it useful in applications requiring higher charge storage.

## 6.3 Inductors

Inductors are passive components that store energy in a magnetic field when an electric current flows through them. They are made of a coil of wire, often wrapped around a core material. The inductance  $L$  of an inductor is measured in henries (H).

### 6.3.1 Inductance

The inductance of an inductor is defined as the ratio of the voltage  $V$  across the inductor to the rate of change of current  $I$  through it:

$$L = \frac{V}{\frac{dI}{dt}}$$

Where:  $L$  = Inductance in henries (H)  $V$  = Voltage in volts (V)  $\frac{dI}{dt}$  = Rate of change of current in amperes per second (A/s)

### 6.3.2 Types of Inductors

Common types of inductors include:

- **Air-Core Inductors:** Inductors that use air as the core material, resulting in lower inductance values.
- **Iron-Core Inductors:** Inductors that use iron as the core material, providing higher inductance values.
- **Toroidal Inductors:** Inductors shaped like a toroid, which can minimize electromagnetic interference.

## 6.4 Series and Parallel Inductors

Similar to capacitors, inductors can also be connected in series and parallel configurations.

### 6.4.1 Series Inductors

For inductors in series, the total inductance  $L_{total}$  is calculated by summing the individual inductances:

$$L_{total} = L_1 + L_2 + L_3 + \dots + L_n$$

This means the total inductance is simply the sum of the individual inductors, leading to a higher overall inductance.

### 6.4.2 Parallel Inductors

For inductors connected in parallel, the total inductance  $L_{total}$  is calculated using the formula:

$$\frac{1}{L_{total}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_n}$$

In this case, the total inductance is always less than the smallest inductor in the parallel configuration.

## 6.5 Conclusion

Understanding capacitors and inductors, along with their series and parallel configurations, is essential for designing and analyzing electrical circuits. These components play crucial roles in various applications, including filtering, energy storage, and signal processing.