HW 1

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These problems are based on chapter 1 of Harris's book, *Algebraic Geoemtry*. Please reference it for definitions and theorems. You may choose at least 2 of these problems to do by March 1, 2024. Every problem you do gives you two points. On that note, particularly ingenious solutions will be rewarded three points.

1. Show that the space of homogeneous polynomials of degree d on $\mathbb{P}V$ is naturally identified with the vector space $\mathrm{Sym}^d(V^*)$. If it appears trivial, give examples for $V = \mathbb{R}^2$, $V = \mathbb{R}^n$, $V = \mathbb{C}^2$, and any other vector spaces you like. - Homework for Kyle: Investigate the case when $V = (\mathbb{Z}/7\mathbb{Z})^3$.

2. A subset $X \subseteq \mathbb{P}^n$ is a projective variety if and only if its intersections $X_i = X \cap U_i$ are all affine varieties.

3. Show in general that for $k \geq 2$ any collection Γ of $d \leq kn$ points in general position may be described by polynomials of degree k and less (as we will see in Exercise 1.15, this is sharp).

4. Show that any finite set of points on a twisted cubic curve are in general position, i.e., any four of them span $\mathbb{P}^3$.

5. The locus of points $p(\lambda)$ as λ varies in $\mathbb{P}^1$ is a rational normal curve (see Page 14 of Harris).

6. Show that $C_{\alpha,\beta}$ is indeed an algebraic variety, and that it may by described as the zero locus of one quadratic and two cubic polynomials.