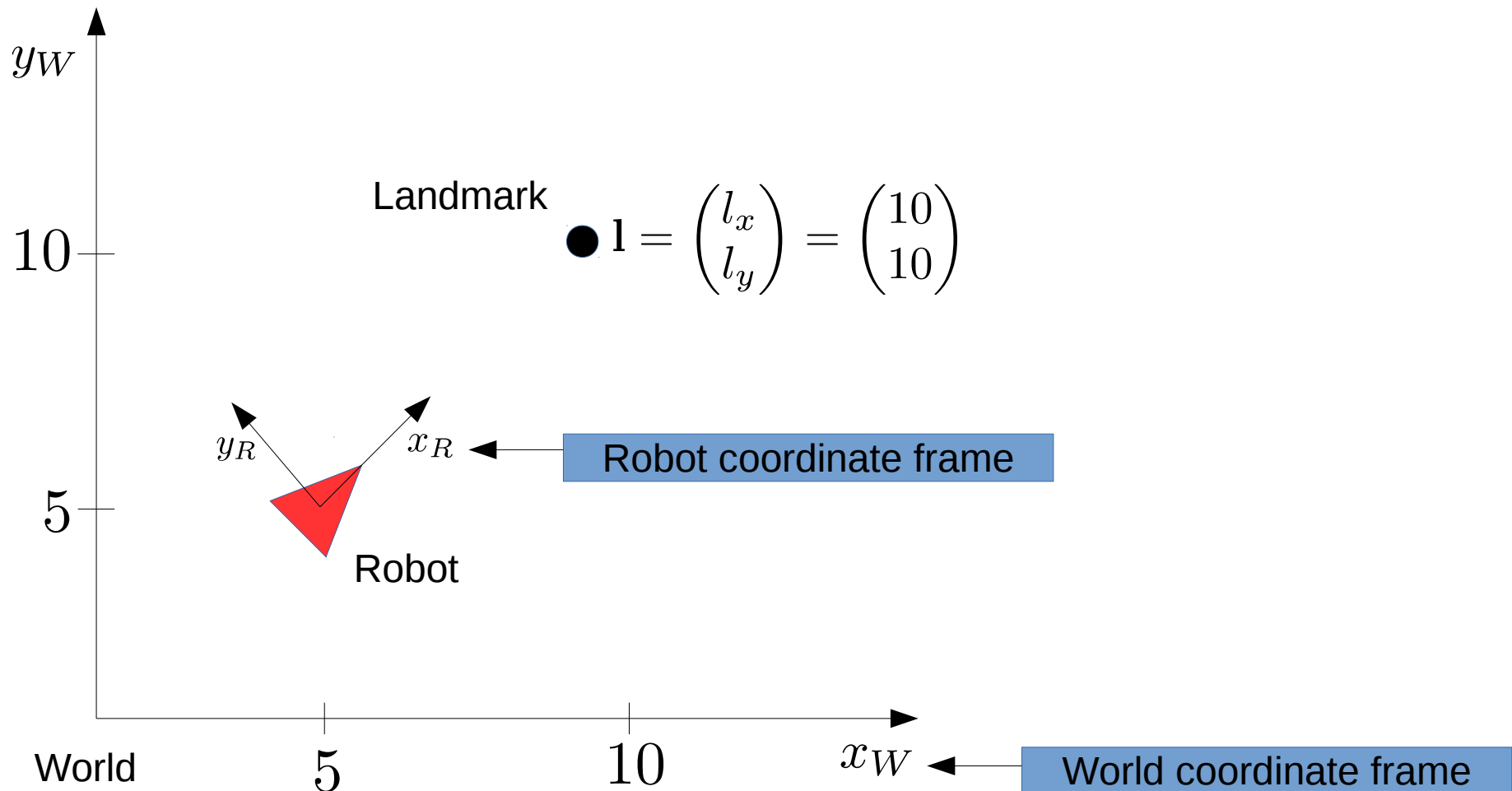# Probabilistic Robotics Course

EKF Scenarios

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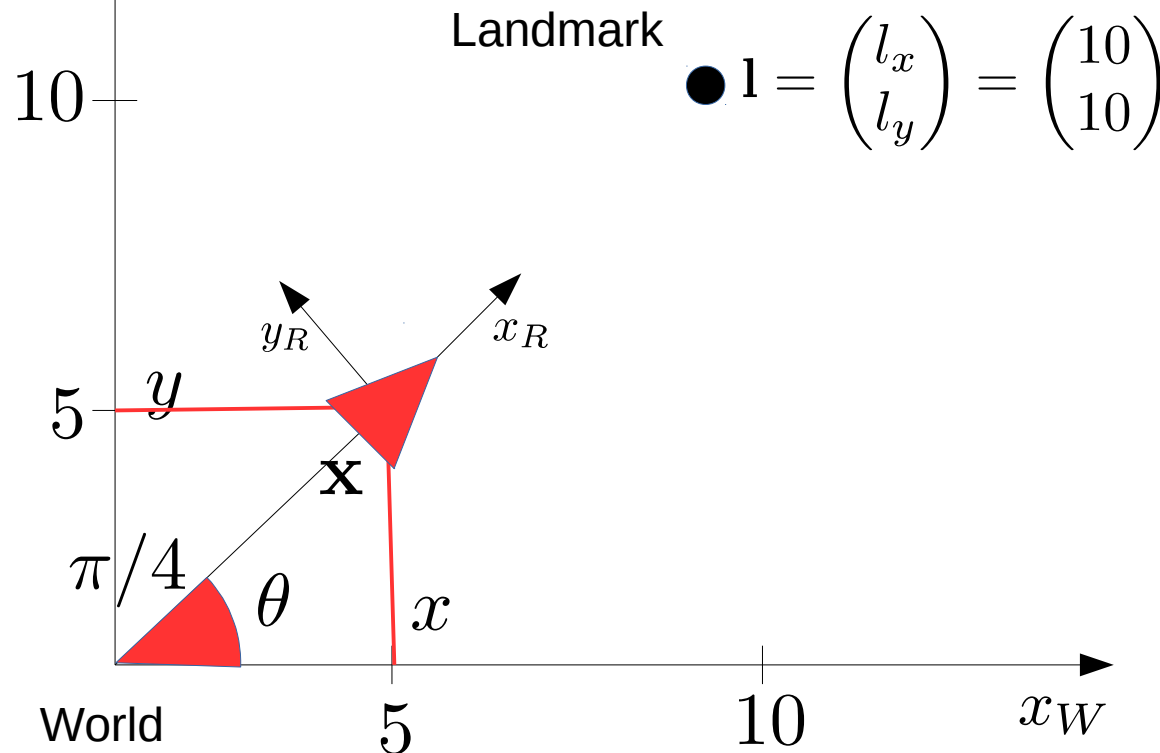Sapienza University of Rome

EKF Localization

- The <u>map</u> (i.e. the landmarks) is known to the robot!

Landmark

$$\bullet \, \mathbf{l} = \begin{pmatrix} l_x \\ l_y \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$$

y_R x_R ◄── Robot coordinate frame

Robot

y_W

10

5

World   5   10 x_W ◄── World coordinate frame

EKF Localization

- The <u>robot</u> doesn't know if its exact state **x**

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ \pi/4 \end{pmatrix} : \left\{ \mathbf{R} = \begin{pmatrix} c(\theta) & -s(\theta) \\ s(\theta) & c(\theta) \end{pmatrix}, \mathbf{t} = \begin{pmatrix} x \\ y \end{pmatrix} \right\}$$ State

y_W

Landmark

$\bullet \, \mathbf{l} = \begin{pmatrix} l_x \\ l_y \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$

10 —

y_R x_R

y

5 —

x

$\pi/4$ θ x

World   5   10 x_W

EKF Localization
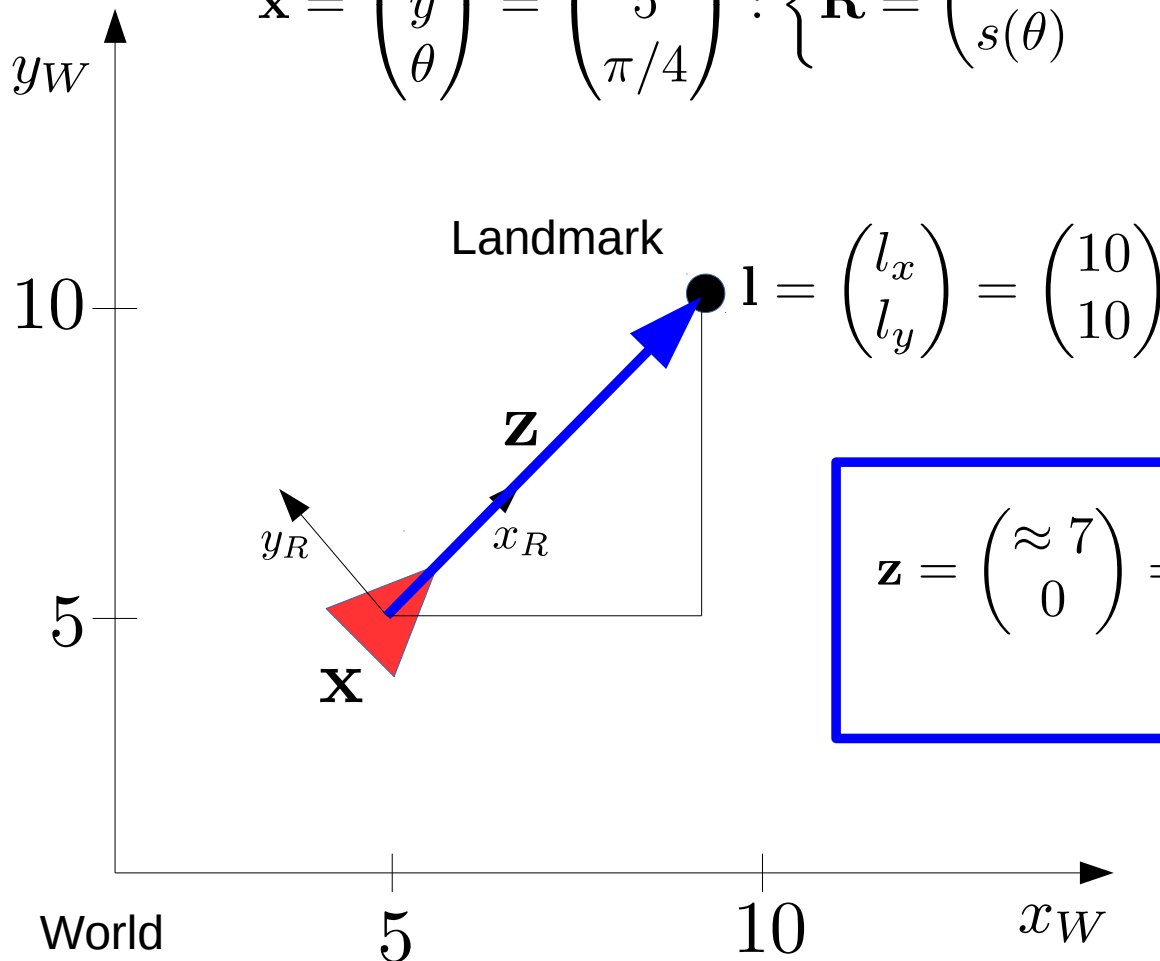
- The robot obtains <u>relative</u> measurements!

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ \pi/4 \end{pmatrix} : \left\{ \mathbf{R} = \begin{pmatrix} c(\theta) & -s(\theta) \\ s(\theta) & c(\theta) \end{pmatrix}, \mathbf{t} = \begin{pmatrix} x \\ y \end{pmatrix} \right\}$$

Landmark

$$\mathbf{l} = \begin{pmatrix} l_x \\ l_y \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$$
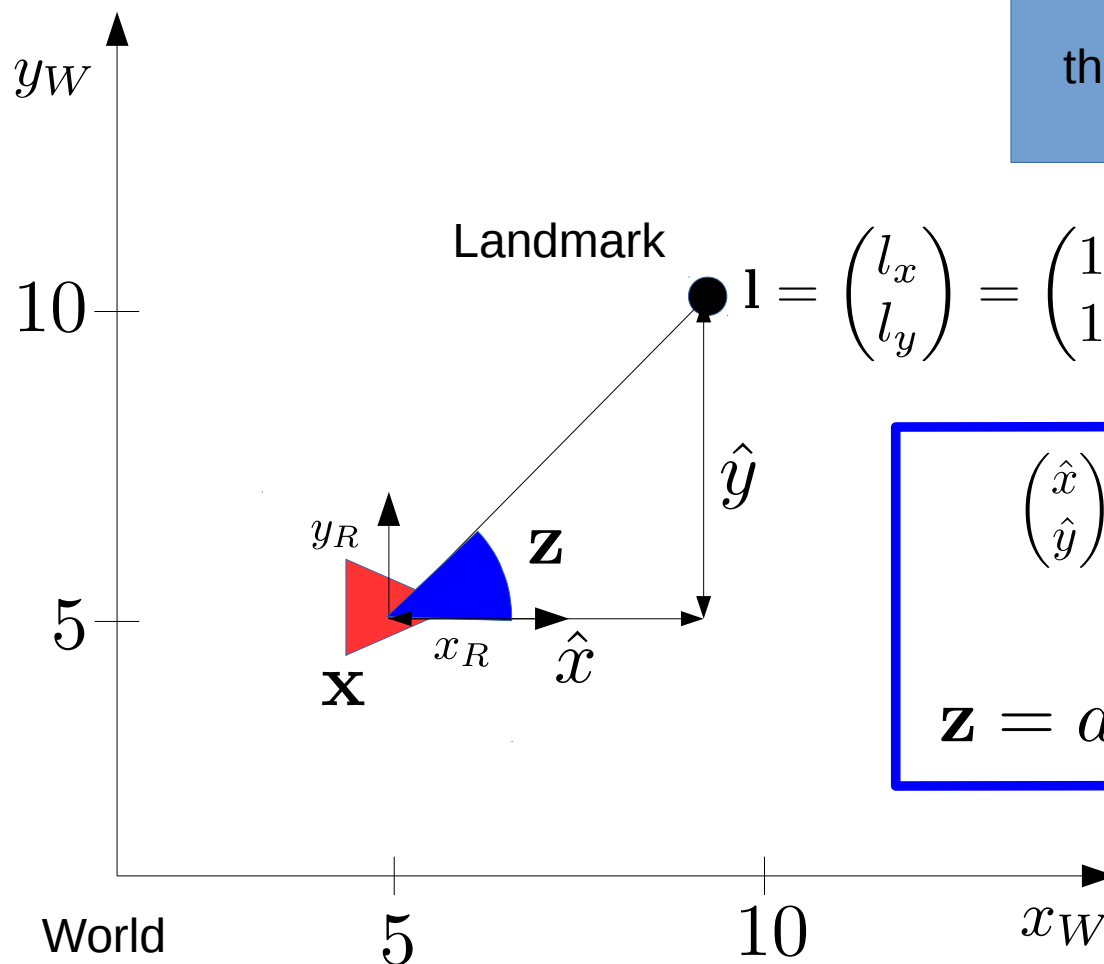
$$\approx 0.7$$

$$\mathbf{z} = \begin{pmatrix} \approx 7 \\ 0 \end{pmatrix} = \underbrace{\begin{pmatrix} c(\pi/4) & s(\pi/4) \\ -s(\pi/4) & c(\pi/4) \end{pmatrix}}_{\mathbf{R}^{\top}(\theta)} \underbrace{\begin{pmatrix} 10 - 5 \\ 10 - 5 \end{pmatrix}}_{(\mathbf{l} - \mathbf{t})}$$

Measurement: a vector

World

EKF Localization Bearing-only

- The map is still known, but we only measure bearings (angles to landmarks)

Relative coordinates to the landmark, the robot can perceive them only through the bearing angle!

Landmark

$$\mathbf{l} = \begin{pmatrix} l_x \\ l_y \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$$
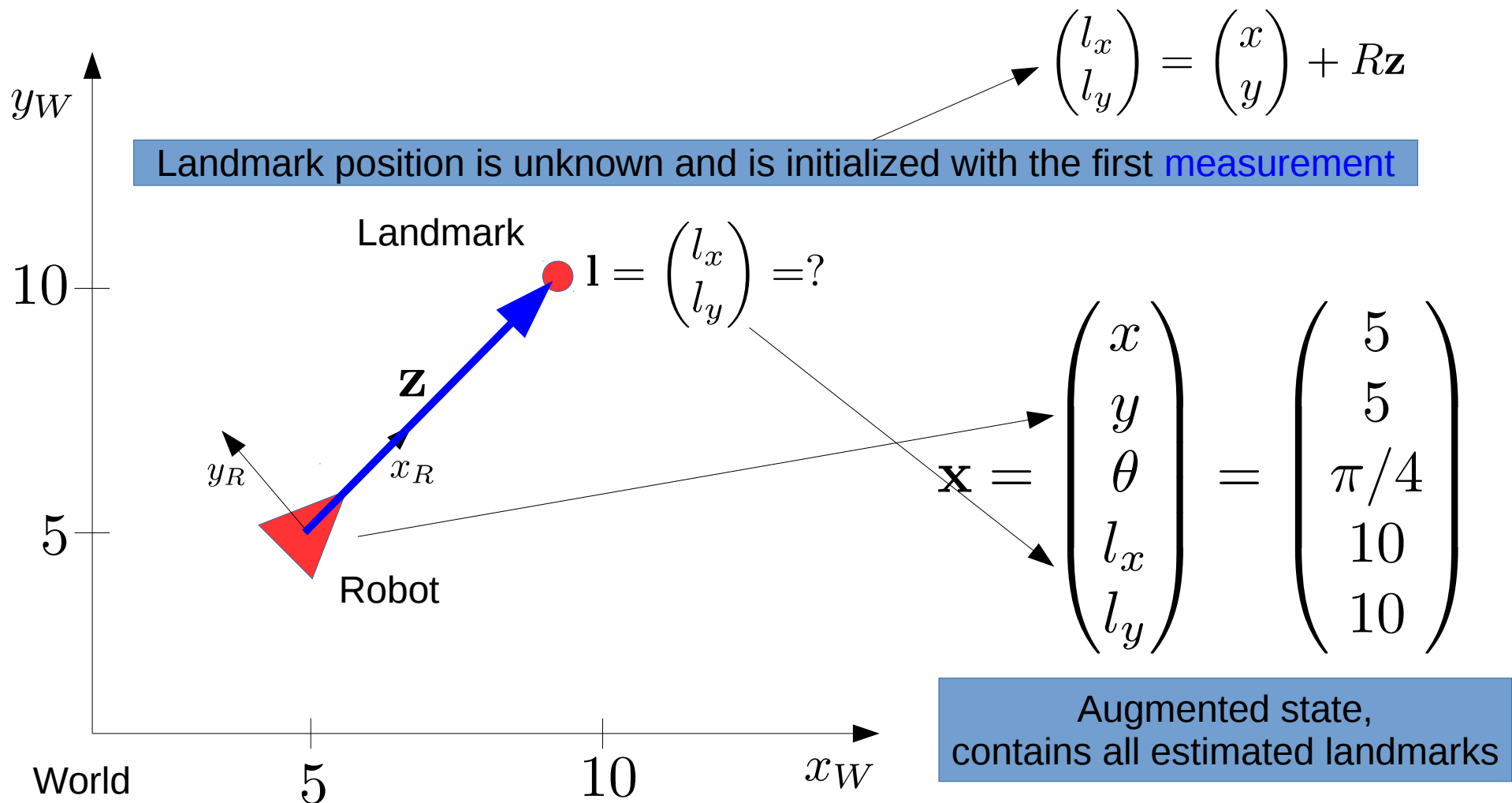
$$\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} = \underbrace{\begin{pmatrix} c(0) & s(0) \\ -s(0) & c(0) \end{pmatrix}}_{\mathbf{R}^\top(\theta)} \underbrace{\begin{pmatrix} 10-5 \\ 10-5 \end{pmatrix}}_{(\mathbf{l}-\mathbf{t})}$$

$$\mathbf{z} = atan(\hat{y}/\hat{x}) = atan(1) = \pi/4$$

Measurement: a scalar

World

EKF SLAM

- The <u>map is unknown</u>, so the robot <u>also</u> has to estimate the positions of the landmarks

$$\begin{pmatrix} l_x \\ l_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + R\mathbf{z}$$

Landmark position is unknown and is initialized with the first measurement

Landmark

$$\mathbf{l} = \begin{pmatrix} l_x \\ l_y \end{pmatrix} = ?$$

$\mathbf{z}$

y_R x_R

Robot

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ \theta \\ l_x \\ l_y \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ \pi/4 \\ 10 \\ 10 \end{pmatrix}$$

y_W

10

5

World   5   10 x_W

Augmented state, contains all estimated landmarks

EKF SLAM Bearing-only

- How to compute the landmark position $\mathbf{l}$ based on two measurements $\mathbf{z}_0, \mathbf{z}_1$?

y_W

Landmark

$\mathbf{l} = \begin{pmatrix} l_x \\ l_y \end{pmatrix}$

Relative coordinates to the landmark, computed at the current robot position

10

$\hat{y} = \dfrac{\sin(\mathbf{z}_0)\sin(\mathbf{z}_1)}{\sin(\mathbf{z}_1 - \mathbf{z}_0)} u_0^x, \; \hat{x} = ?$

$\mathbf{z}_0$ $\mathbf{z}_1$

5

$\mathbf{x}_0$ u_0^x $\mathbf{x}_1$

Control input in x-axis

World   5   10 x_W
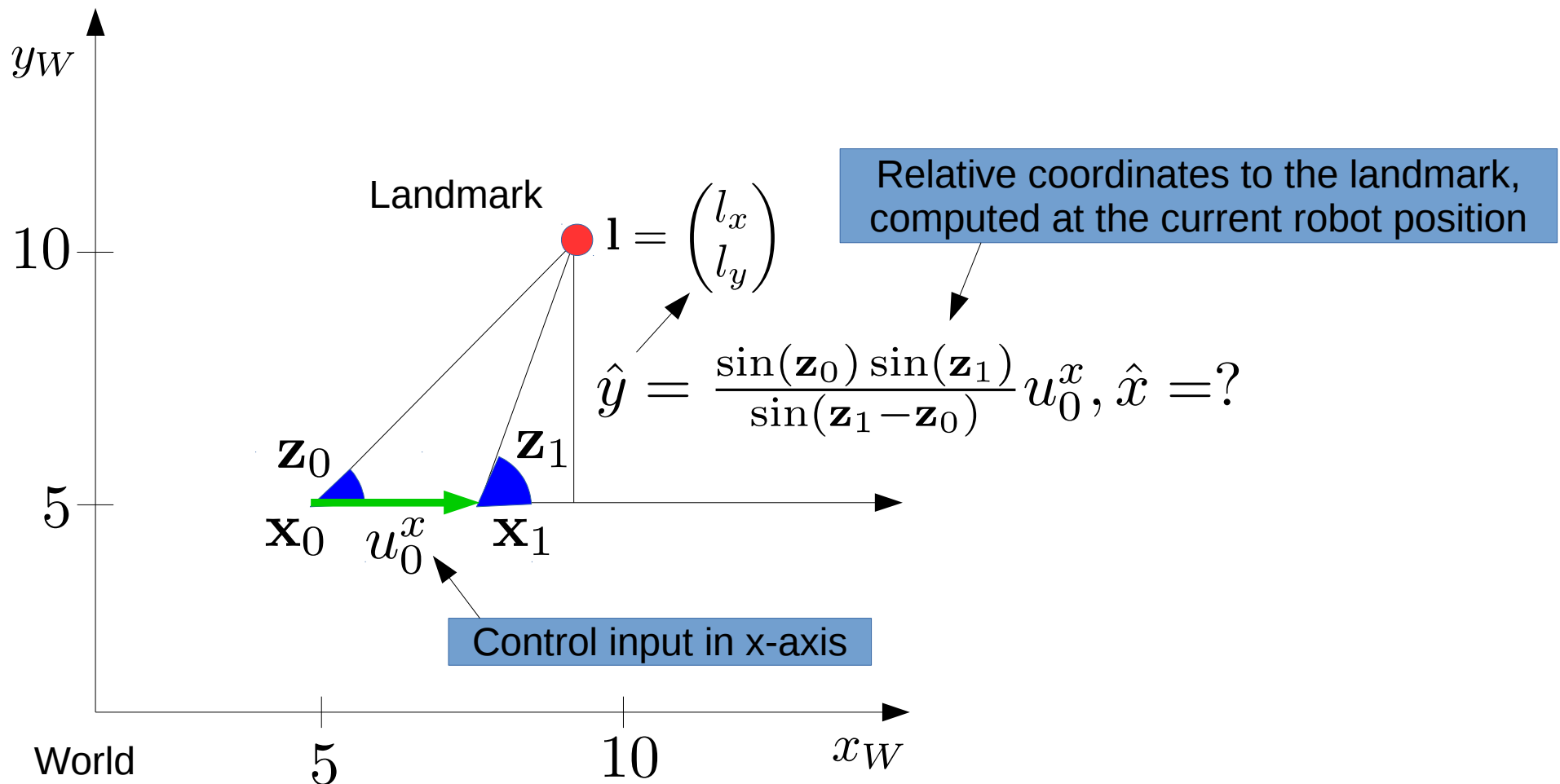
EKF Mapping

- What's different with respect to SLAM?

- What's different with respect to classical Localization?