Probabilistic Robotics Course

Modeling Dependencies Bayesian Networks

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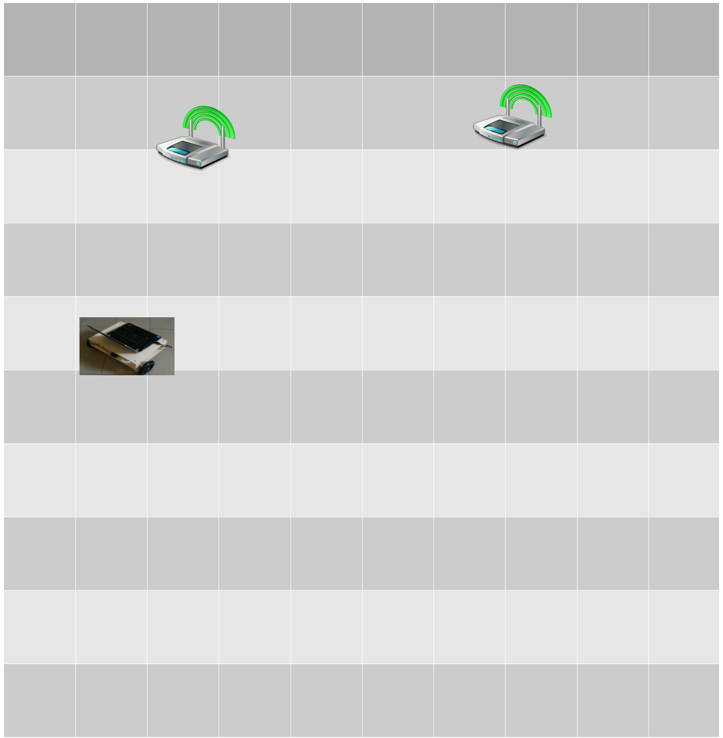
Outline

- Considerations about dimensions
- Role of Conditional Independence
- Modeling Phenomena: Bayesian Networls
- Inference on Bayesian Networks

Orazio!

Let us go back to our robot. The scene was

- 2 access points (AP)
- orazio moving
- one elevator



Quantities

- signal strengths from the 2 APs
- floor of the elevator
- location of Orazio

Queries

To perform a query of the type

$$p(S^A|S^B_2)$$

we can start from a joint probability distribution

$$P(S^A,S^B_2,I^{00},..I^{10,10},L,E)$$

eliminate all variables that are non relevant through *marginalization*

$$P(S^A,S^B_2,\cancel{I^{00},..I^{10,10},L,E})$$

and use the *Bayes*' *Rule* to get the answer

Dimensionality

Having

- 4 possible levels of strength per access point
- 2 access points
- 6 floors
- 100 locations

How many possible independent outcomes of events do I have?

Dimensionality

Having

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- 2 access points
- 6 floors
- 100 locations

How many possible disjoint outcomes of events do I have?

4*4*6*100 = **9600**

Dimensionality

What if we gift Orazio a monochromatic camera with resolution "poorVGA" (10x10), and 16 levels of gray

- number of disjoint events becomes:

  4*4*6*100*(10x10)^16 =**9600***10^32**

Dimensionality

What if we gift Orazio a monochromatic camera with resolution “poorVGA” (10x10), and 16 levels of gray

- number of disjoint events becomes:

  4*4*6*100*(10x10)^16 =**9600***10^32**

Enough to challenge some good computer

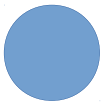
Conditional Independence

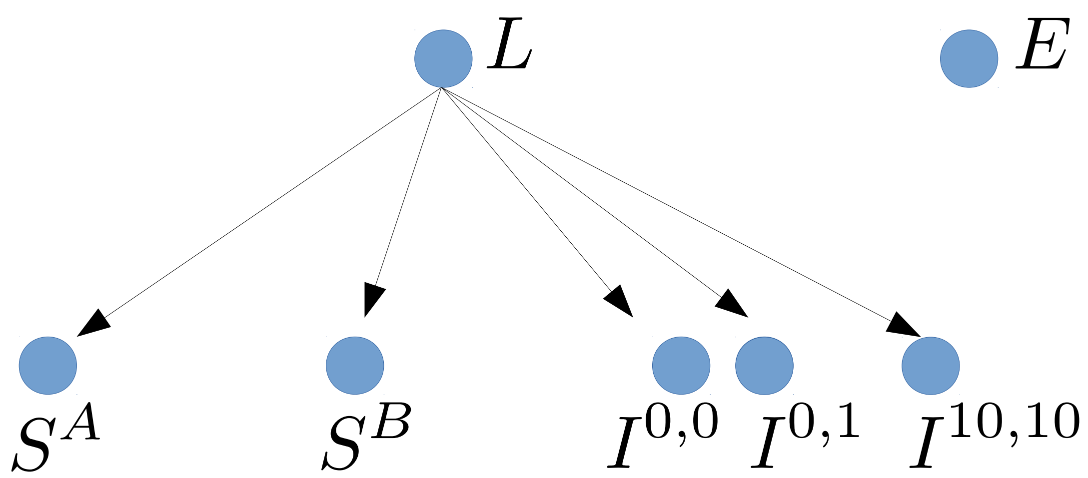
Using our wisdom we can notice some astonishing facts:

- The signal strength of an access point depends only on the location
- The pixels in the image depend only on the location
  - if nothing but Orazio move
  - if the light conditions do not change
- The elevator lives in his own world (as long as Orazio does not take it)

Making Order

We can represent this phenomena as a *Bayesian Network*

- One  per variable
- One  if the source influences the destination
- No loops, it’s a directed acyclic graph!



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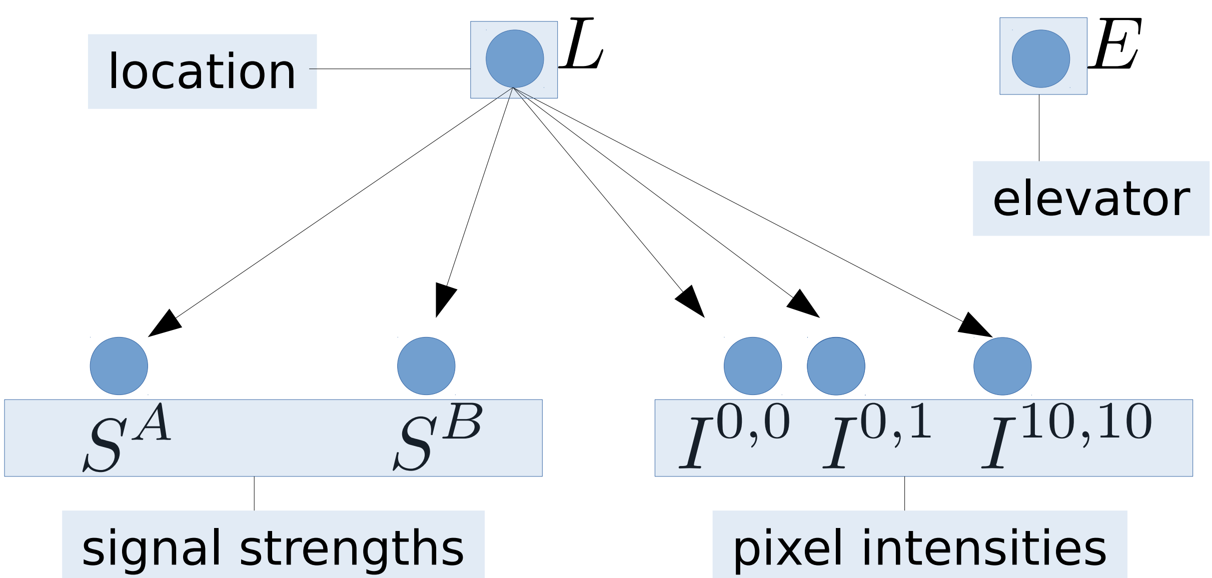


Making Order

For storing the conditional probabilities we need:

- 100 numbers for robot locations
- 4*100 numbers for signal strength A
- 4*100 numbers for signal strength B
- 16*100 numbers for each pixel, as they are independent GIVEN the location
- 6 numbers for the elevator

$100+2*4*100+100*16*100+6=160906$

Much less than before

Using the Order

By storing the conditional probability we need much less numbers

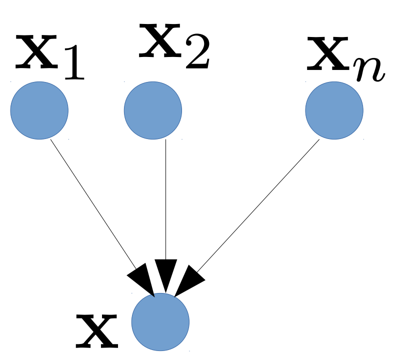
We can anyway recover the joint distribution and perform *inference* on that, by using the chain rule

Bayesian Networks are a formalism to highlight independence between variables, without loss of information

Each node

- represents a variable $\mathbf{x}$
- stores a conditional probability:

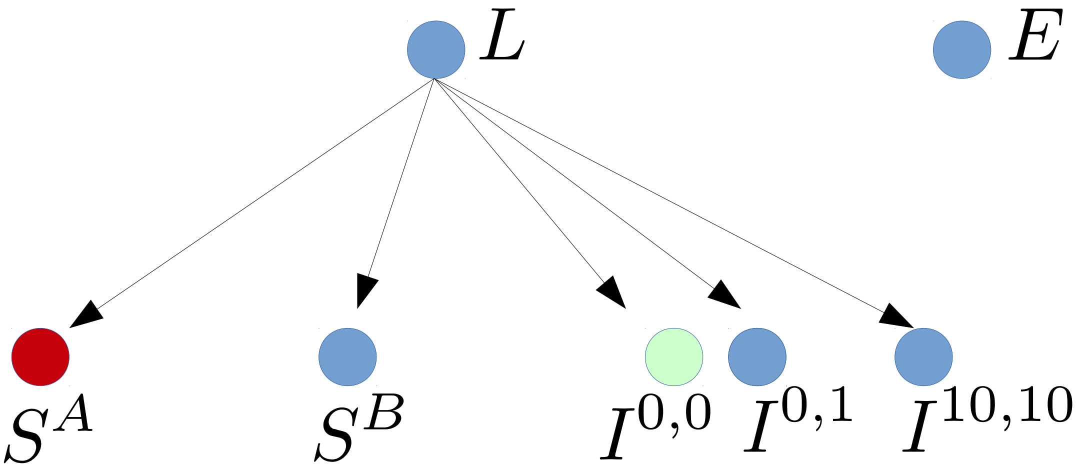
$$p(\mathbf{x} \mid \mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_n)$$



Example of Inference

What is the probability distribution of S^A,

given the pixel 0,0 has intensity 7 [$I^{0,0} = 7$] ?



Example of Inference

We want $\mathrm{P}(\mathrm{S}^A | I_7^{0,0}) = ?$

- compute the joint

it is the product of all distributions

$$P(S^A, S^B, I_7^{0,0}, \ldots I^{10,10}, L, E) =$$
$$P(S^A|L)P(S^B|L)P(I_{10}^{0,0}|L)(I^{10,10}|L)P(L)P(E)$$

- marginalize out the variables we don't need

$$P(S^A, S^B, I_7^{0,0}, \ldots I^{10,10}, L, E)$$

to get a joint distribution over $\mathrm{S}^A, I_7^{0,0}$

$$P(S^A, I_7^{0,0}) = \sum_{S^B} \sum_{I^{0,1}} \cdots \sum_{I^{10,10}} \sum_L \sum_E P(S^A, S^B, I_7^{0,0}, \ldots I^{10,10}, L, E)$$

sum over all values of variables to suppress

- use conditioning to get the answer

$$P(S^A \mid I_7^{0,0}) = \frac{P(S^A, I_7^{0,0})}{\sum_{S^A} P(S^A, I_7^{0,0})}$$

Considerations

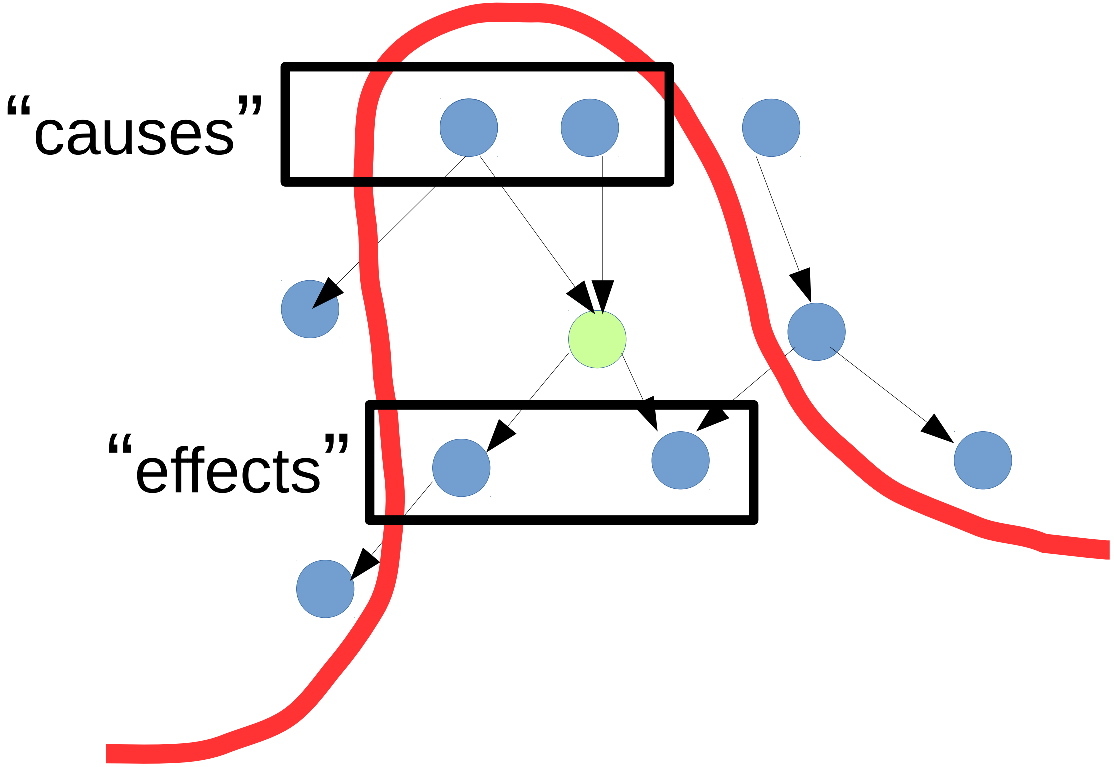
- Would we get the same answer if the elevator was not in the domain

- Knowing the intensity of another pixel would provide us with more information?

- Does the same hold if we know the location?

Local Semantics

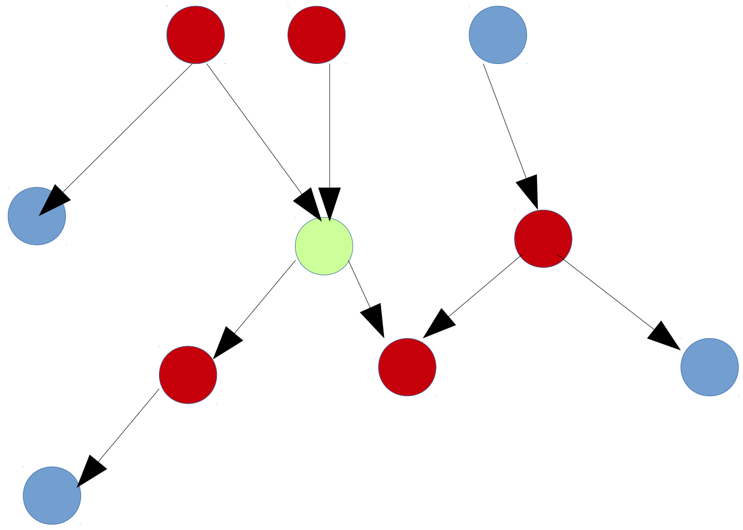
Each node is conditionally independent of its non descendants, given its parents



Global Semantics

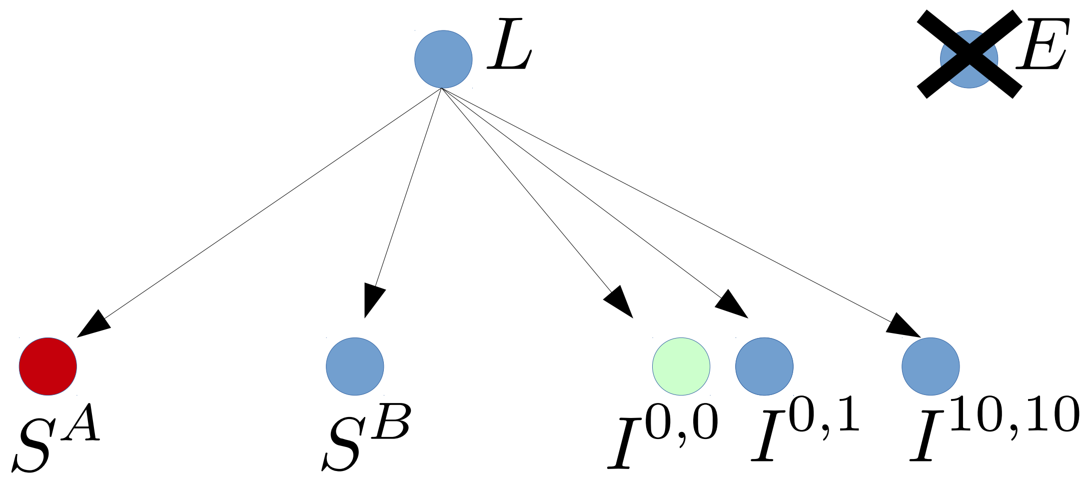
Each node is independent from the rest, given

- its parents
- its children
- the parents of its children



Using Semantics

Elevator is disconnected, thus independent

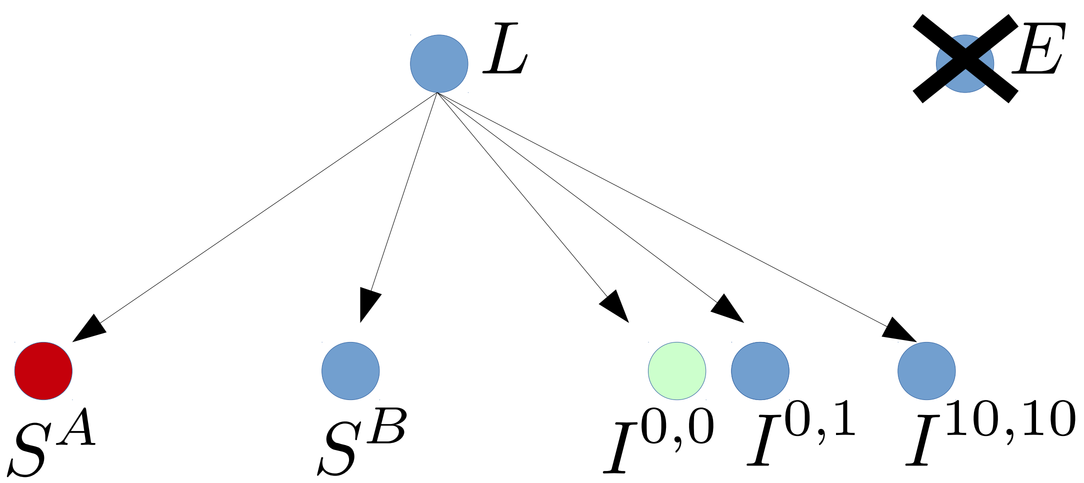


Using Semantics

Elevator is disconnected, thus independent

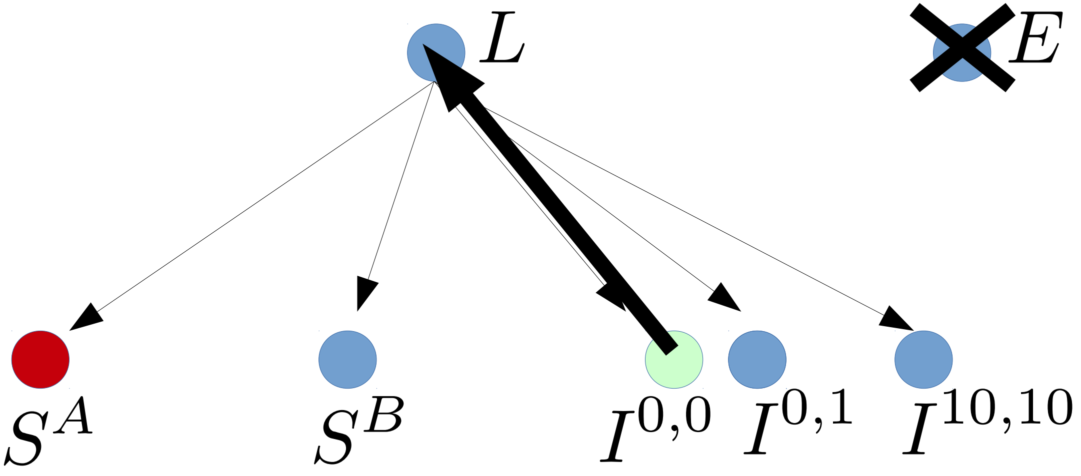
If we would know the location, the task would be easy

The only consequence of the location is $I^{0,0}$



Using Semantics

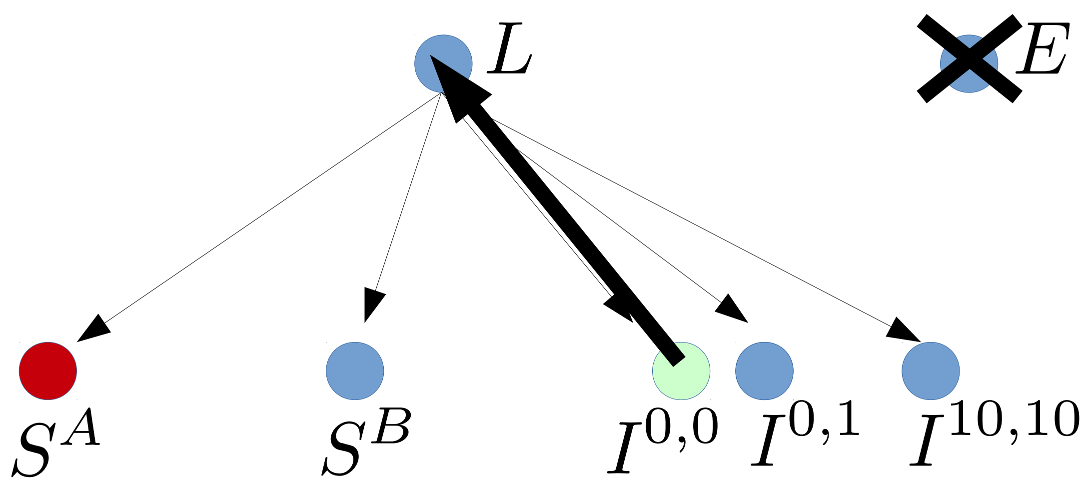
We can compute $P(L \mid I_7^{0,0})$ ignoring all the rest



Using Semantics

We can compute $P(L \mid I_7^{0,0})$ ignoring all the rest

This will alter our prior about the location, incorporating the knowledge of the intensity

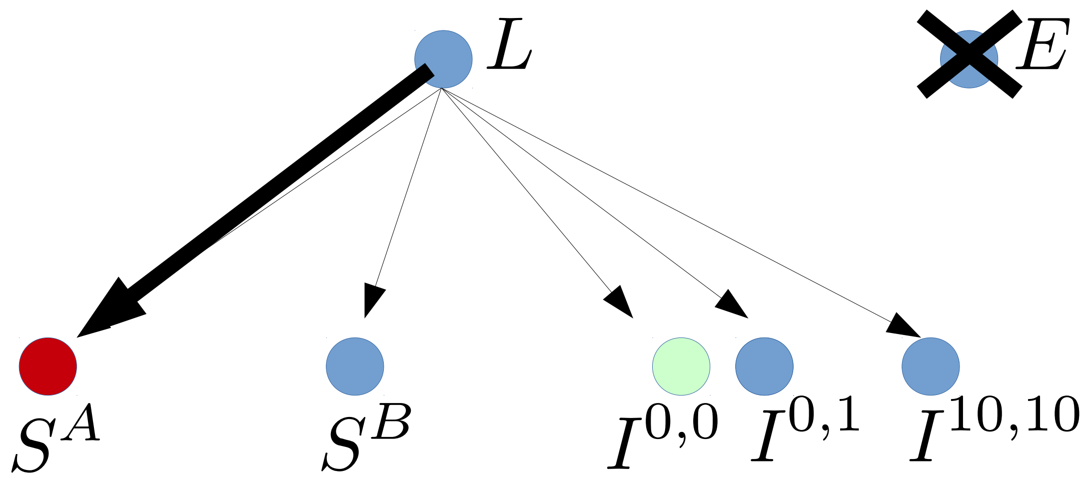


HOW:
- chain rule on P($I^{0,0}$|L)and P(L) to get P(L,I)
- conditioning on P(L,I), to get P(L|I)

Using Semantics

We can compute $P(L \mid I_7^{0,0})$ ignoring all the rest

Finally, we determine the signal strength, from the improved location estimate



HOW:
- chain rule on $P(S^A|L)$and $P(L|I)$

References

An excellent tutorial on Bayes networks and graphical models:

http://www.cs.ubc.ca/~murphyk/Bayes/bnintro.html