MATH210 – Applied Complex Variables

– Review Problem – Part I –

1. Let $f(z) = z^3 + 3z^2 - 2z + 1$. Show that if $|z| < 2$, then $|f(z)| < 25$.

2. Find all $z \in \mathbb{C}$ to satisfy $|z - i| \leq |z - 2|$.

3. Evaluate

(1) $(-1)^{1/4}$
(2) $(-8i)^{1/3}$

4. Solve

$$z^2 - z + 1 + i = 0$$

5. Let $a, b, c \in \mathbb{C}$ with $a \neq 0$. Show that

$$z = -\frac{b + (b^2 - 4ac)^{1/2}}{2a}$$

6. Sketch the region onto which the sector $r \leq 1,\ 0 \leq \theta \leq \frac{\pi}{4}$ is mapped by

(1) $w = z^2$
(2) $w = z^3$

7. Are the following functions analytic?

(1) $f(z) = i\bar{z}z$
(2) $f(z) = e^x(cos\, y - i\, sin\, y)$
(3) $\mathrm{f(z)} = \mathrm{Arg\, z}$ on $\mathbb{C} \setminus \{z : \mathrm{Re}\, z \leq 0,\ \mathrm{Im}\, z = 0\}$

8. Find a and b such that $u(x, y) = ax^3 + bxy$ and $u(x, y)$ is a real part of an analytic function.

9. Suppose that $f(z)$ is analytic and $\mathrm{Re}\, f(z)$ is constant. Then what can we say about $f(z)$?

10. Determin whether the following functions are entire or not.

(1) $f(z) = \exp z^2$
(2) $f(z) = \exp \bar{z}$
(3) $f(z) = \sin(\bar{z})$

11. Find in the form $x + iy$

(1) $\log(-5)$
(2) $(1+i)^i$
(3) $\cos(5-2i)$

12. Find the image of the set $\{x + iy : 1 \leq x^2 + y^2 \leq 2,\ x \geq 0,\ y \geq 0\}$ under the map

$$f(z) = \operatorname{Log} z$$