

Interpolierende Subdivision nach Dubuc Alk

$$\vec{f}_0^{(0)} := (x_0, y_0)^T := (-4, 0)^T \quad \vec{f}_1^{(0)} := (x_1, y_1)^T := (-1, 3)^T$$

$$\vec{f}_2^{(0)} := (x_2, y_2)^T := (0, 0)^T \quad \vec{f}_3^{(0)} := (x_3, y_3)^T := (3, -2)^T$$

Erste Stufe der Dubuc-Subdivision ($k=0$)

$$\vec{f}_i^{(k+1)} := \vec{f}_i^{(k)}$$

$$\vec{f}_{2i+1}^{(k+1)} := -\frac{1}{16} \vec{f}_{i-1}^{(k)} + \frac{9}{16} \vec{f}_i^{(k)} + \frac{9}{16} \vec{f}_{i+1}^{(k)} - \frac{1}{16} \vec{f}_{i+2}^{(k)}$$

$$\text{Rand: } \vec{f}_{-1}^{(k)} := \vec{f}_0^{(0)} \quad \vec{f}_{n+1}^{(k)} := \vec{f}_n^{(0)}$$

$$\vec{f}_0^{(1)} := \vec{f}_0^{(0)} = (-4, 0)^T$$

$$\vec{f}_1^{(1)} := \frac{1}{2} \vec{f}_0^{(0)} + \frac{9}{16} \vec{f}_1^{(0)} - \frac{1}{16} \vec{f}_2^{(0)} = \frac{1}{2} \begin{pmatrix} -4 \\ 0 \end{pmatrix} + \frac{9}{16} \begin{pmatrix} -1 \\ 3 \end{pmatrix} - \frac{1}{16} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -41/16 \\ 27/16 \end{pmatrix}$$

$$\vec{f}_2^{(1)} := \vec{f}_1^{(0)} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\frac{4}{16} - \frac{9}{16} - \frac{3}{16} = -\frac{8}{16}$$

$$\frac{27}{16} + \frac{21}{16} = \frac{48}{16} = 3$$

$$\vec{f}_3^{(1)} := -\frac{1}{16} \vec{f}_0^{(0)} + \frac{9}{16} \vec{f}_1^{(0)} + \frac{9}{16} \vec{f}_2^{(0)} - \frac{1}{16} \vec{f}_3^{(0)}$$

$$= -\frac{1}{16} \cdot \begin{pmatrix} -4 \\ 0 \end{pmatrix} + \frac{9}{16} \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \frac{9}{16} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \frac{1}{16} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 3 \end{pmatrix}$$

$$\vec{f}_4^{(1)} := \vec{f}_2^{(0)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\vec{f}_5^{(1)} := -\frac{1}{16} \vec{f}_1^{(0)} + \frac{9}{16} \vec{f}_2^{(0)} + \frac{1}{2} \vec{f}_3^{(0)}$$

$$= -\frac{1}{16} \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \frac{9}{16} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 25/16 \\ -17/16 \end{pmatrix}$$

$$\vec{f}_6^{(1)} := \vec{f}_3^{(0)} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$