Trapez- und Simpson-Regel AK

Berechnen Sie die Integrale

(a) $\int_1^3 \frac{1}{x}\,dx$ und (b) $\int_0^{\pi} \sin(x)\,dx$

sowohl exakt, als auch näherungsweise mit der Trapez- und der Simpson-Regel

(a) $\int_1^3 \frac{1}{x}\,dx$

Exakt

$$\int_1^3 \frac{1}{x}\,dx = [\ln(|x|)]_1^3 = \ln(3) - \underbrace{\ln(1)}_{0}$$

$$= \ln(3) \approx 1{,}099$$

Trapez-Regel $\int_a^b f(x)\,dx \approx (b-a)\left(\frac{1}{2}f(a) + \frac{1}{2}f(b)\right)$

$$\int_1^3 \frac{1}{x}\,dx \approx (3-1)\left(\frac{1}{2}f(1) + \frac{1}{2}f(3)\right) = 2\cdot\left(\underbrace{\frac{1}{2}\cdot 1}_{\frac{3}{6}} + \underbrace{\frac{1}{2}\cdot\frac{1}{3}}_{\frac{1}{6}}\right)$$

$$= 2\cdot\frac{4}{6} = 2\cdot\frac{2}{3} = \frac{4}{3}$$

Simpson-Regel $\int_a^b f(x)\,dx \approx (b-a)\left(\frac{1}{6}f(a) + \frac{2}{3}f\left(\frac{a+b}{2}\right) + \frac{1}{6}f(b)\right)$

$$\int_1^3 \frac{1}{x}\,dx \approx (3-1)\left(\underbrace{\frac{1}{6}\cdot\frac{1}{1}}_{\frac{1}{6}} + \frac{2}{3}\cdot f\left(\frac{1+3}{2}\right) + \underbrace{\frac{1}{6}\cdot\frac{1}{3}}_{\frac{1}{18}}\right)$$

$$= 2\cdot\left(\underbrace{\frac{1}{6}}_{\frac{3}{18}} + \underbrace{\frac{2}{3}\cdot\frac{1}{2}}_{\frac{6}{18}} + \frac{1}{18}\right) = 2\cdot\frac{10}{18} = \frac{10}{9} \approx 1{,}111$$

(b) $\int_0^{\pi} \sin(x)\,dx$

Exakt

$$\int_0^{\pi} \sin(x)\,dx = [-\cos(x)]_0^{\pi} = \underbrace{-\cos(\pi)}_{1} - \left(\underbrace{-\cos(0)}_{1}\right) = 2$$

Trapez-Regel

$$\int_0^{\pi} \sin(x)\,dx \approx \pi\left(\underbrace{\frac{1}{2}\cdot\sin(0)}_{0} + \underbrace{\frac{1}{2}\sin(\pi)}_{0}\right) = 0$$

Simpson-Regel

$$\int_0^{\pi} \sin(x)\,dx \approx \pi\left(\underbrace{\frac{1}{6}\sin(0)}_{0} + \frac{2}{3}\underbrace{f\left(\frac{\pi}{2}\right)}_{\sin(\pi/2)=1} + \underbrace{\frac{1}{6}\sin(\pi)}_{0}\right)$$

$$= \pi\left(\frac{2}{3}\right) = \frac{2}{3}\pi \approx 2{,}094$$