

$$I f'(0) = 0$$

$$V(r) = \frac{4}{3} \pi r^3$$

$$II f(s) = 0$$

$$III f'(s) = 0$$

$C \in R = \text{constant}$

$K(s)$

$$IV \int_0^R f_s(r) dr = C = \int_0^R f_s(r) dr$$

$$C = \int_0^R \int_0^{2\pi} \int_0^\pi f_s(r) r^2 dr d\varphi d\vartheta$$

$$(\Rightarrow) C = \int_0^R f_s(r) r^2 dr \int_0^{2\pi} d\varphi \int_0^\pi \sin \vartheta d\vartheta$$

$$(\Rightarrow) C = \int_0^R f_s(r) r^2 dr \cdot (2\pi \cdot 2)$$

$$(\Rightarrow) \frac{C}{4\pi} = \int_0^R f_s(r) r^2 dr$$

$$A \vec{a} = \vec{b} \quad (\Rightarrow) \vec{a} = A^{-1} \vec{b}$$

$$I \begin{pmatrix} 1 & 0 & 25 & 35^2 \\ 0 & 1 & s^2 & s^3 \\ 0 & 0 & 2s & \frac{1}{6}s^5 \\ 0 & 0 & \frac{1}{3}s^3 & \frac{1}{6}s^5 \end{pmatrix} \begin{pmatrix} a_0 \\ a_2 \\ a_3 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{C}{4\pi} \\ -\frac{C}{4\pi} \end{pmatrix}$$

$$II \begin{pmatrix} 1 & 0 & 25 & 35^2 \\ 0 & 1 & s^2 & s^3 \\ 0 & 0 & 2s & \frac{1}{6}s^5 \\ 0 & 0 & \frac{1}{3}s^3 & \frac{1}{6}s^5 \end{pmatrix} \begin{pmatrix} a_0 \\ a_2 \\ a_3 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{C}{4\pi} \\ -\frac{C}{4\pi} \end{pmatrix}$$

$$= \int_0^R \int_0^{2\pi} \int_0^\pi f_s(r) r^2 \sin \vartheta dr d\varphi d\vartheta$$

$$\int_0^{2\pi} \int_0^\pi r^2 \sin \vartheta d\varphi d\vartheta = 2\pi \cdot \frac{2}{2} R^2 = \pi R^2$$

$$f_s(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$f_s'(x) = 3a_3 x^2 + 2a_2 x + a_1$$

$$I f_s'(0) = 0 = a_1$$

$$II f_s(s) = 0 = a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

$$III f_s'(s) = 0 = 3a_3 s^2 + 2a_2 s + a_1$$

$$IV \int_0^R (a_3 r^3 + a_2 r^2 + a_1 r + a_0) r^2 dr = \frac{C}{4\pi}$$

$$(\Rightarrow) \int_0^R (a_3 r^5 + a_2 r^4 + a_1 r^3 + a_0 r^2) dr = \frac{C}{4\pi}$$

$$(\Rightarrow) \left[\frac{1}{6} a_3 r^6 + \frac{1}{5} a_2 r^5 + \frac{1}{3} a_1 r^3 + \frac{1}{2} a_0 r^2 \right]_0^R = \frac{C}{4\pi}$$

$$(\Rightarrow) \frac{1}{6} a_3 R^6 + \frac{1}{5} a_2 R^5 + \frac{1}{3} a_1 R^3 + \frac{1}{2} a_0 R^2 = \frac{C}{4\pi}$$

$$(\Rightarrow) 4\pi \left(\frac{1}{6} a_3 R^6 + \frac{1}{5} a_2 R^5 + \frac{1}{3} a_1 R^3 + \frac{1}{2} a_0 R^2 \right) = C$$

$$\begin{pmatrix} 1 & s^2 & s^3 \\ 0 & 2s & 3s^2 \\ \frac{1}{3}s^3 & \frac{1}{5}s^5 & \frac{1}{6}s^6 \end{pmatrix} = \begin{pmatrix} a_0 & a_1 & a_2 \\ 0 & 0 & 0 \\ \frac{1}{4\pi} & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & s^2 & s^3 \\ 0 & 2s & 3s^2 \\ 0 & (\frac{1}{5}-\frac{1}{3})s^5 & (\frac{1}{6}-\frac{1}{3})s^6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{3}s^3 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & s^2 & s^3 \\ 0 & 1 & \frac{3}{2}s \\ 0 & -\frac{2}{15}s^5 & -\frac{1}{6}s^6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{7}{2}s & 0 \\ -\frac{1}{3}s^3 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & s^2 & s^3 \\ 0 & 1 & \frac{3}{2}s \\ 0 & 0 & (\frac{1}{5}-\frac{1}{6})s^6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{7}{2}s & 0 \\ -\frac{1}{3}s^3 & \frac{1}{15}s^4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & s^2 & s^3 \\ 0 & 1 & \frac{3}{2}s \\ 0 & 0 & \frac{1}{30}s^6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{7}{2}s & 0 \\ -\frac{10}{s^3} & \frac{2}{s^2} & \frac{30}{s^6} \end{pmatrix}$$

$$\begin{pmatrix} 1 & s^2 & s^3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 15/s^2 & (\frac{1}{25}-\frac{3}{25}) & -45/s^5 \\ -\frac{10}{s^3} & \frac{2}{s^2} & \frac{30}{s^6} \end{pmatrix}$$

$$\begin{pmatrix} 1 & s^2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -2s & -20/s^3 \\ 15/s^2 & -\frac{1}{25} & -45/s^5 \\ -10/s^3 & 2/s^2 & (45-30)/s^3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -4 & -\frac{15}{s^2} & -\frac{70}{s^3} \\ -\frac{15}{s^2} & -\frac{25}{s^5} & \frac{2}{s^2} \\ -\frac{10}{s^3} & \frac{2}{s^2} & \frac{30}{s^6} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -4 & \frac{15}{s^2} & -\frac{70}{s^3} \\ -2/s^5 & -45/s^5 & -7 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{4\pi} \end{pmatrix}$$

(=)

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{4\pi} & 0 & 0 \end{pmatrix} = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

$$\frac{C}{4\pi} \begin{pmatrix} \frac{15}{s^3} & \frac{6}{4\pi} \\ -45 & 1/s^5 \\ 30 & 1/s^6 \end{pmatrix} = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

Check of function properties

polynomial sampling
weighting function

$$\frac{C}{4\pi} \left(\frac{30}{s^6} x^3 - \frac{45}{s^5} x^2 + \frac{15}{s^3} \right) = f'_s(x)$$

$$\frac{C}{4\pi} \left(\frac{90}{s^6} x^2 - \frac{90}{s^5} x \right) = f_s'(x)$$

I $f'_s(0) = \frac{C}{4\pi} \left(\frac{90}{s^6} \cdot 0^2 - \frac{90}{s^5} \cdot 0 \right) = 0$ ✓

III $f'_s(s) = \frac{C}{4\pi} \left(\frac{90}{s^6} s^2 - \frac{90}{s^5} s \right) = 0$ ✓

II $f_s(s) = \frac{C}{4\pi} \left(\frac{30}{s^6} (s^3)^2 - \frac{45}{s^5} s^2 + \frac{15}{s^3} \right) = 0$ ✓

IV $\int_0^s f_s(r) r^2 dr = \frac{C}{4\pi}$

$$\Rightarrow \int_0^s \left(\frac{30}{s^6} r^3 - \frac{45}{s^5} r^2 + \frac{15}{s^3} \right) r^2 dr = \frac{C}{4\pi}$$

$$\Rightarrow \int_0^s \frac{C}{4\pi} \left[\frac{30}{s^6} r^5 - \frac{45}{s^5} r^4 + \frac{15}{s^3} r^2 \right] dr = \frac{C}{4\pi}$$

$$\Rightarrow \frac{C}{4\pi} \left[\frac{30}{s^6} \cdot \frac{1}{6} r^6 - \frac{45}{s^5} \cdot \frac{1}{5} r^5 + \frac{15}{s^3} \cdot \frac{1}{3} r^3 \right]_0^s = \frac{C}{4\pi}$$

$$\Rightarrow \frac{30}{s^6} \cdot \frac{1}{6} s^6 - \frac{45}{s^5} \cdot \frac{1}{5} s^5 + \frac{15}{s^3} \cdot \frac{1}{3} s^3 = 1$$

$$\Rightarrow 5 - 9 + 5 = 1$$

$$\Rightarrow 1 = 1 \quad \checkmark$$