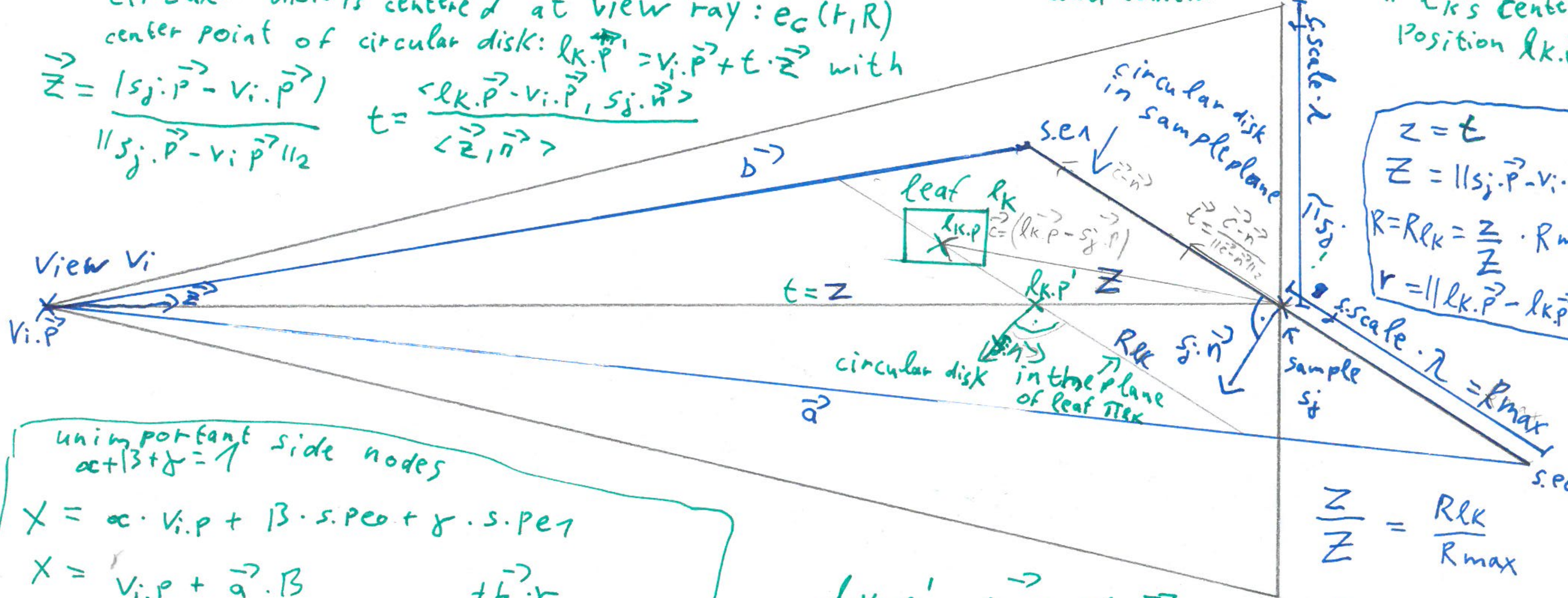
oblique circular view cone

1. emptiness function along view ray: $e_r(z, Z)$
2. emptiness function ~~along~~ over circular disk in plane $\Pi_{\ell k}$ (parallel to sample s_j's plane Π_{s_j}) and coincident with leaf ℓ_k's center position $\ell k.\vec{p}$,
   circular disk is centered at view ray: $e_c(r, R)$
   center point of circular disk: $\ell k.\vec{p}' = v_i.\vec{p} + t \cdot \vec{z}$ with

$$\vec{z} = \frac{(s_j.\vec{p} - v_i.\vec{p})}{\|s_j.\vec{p} - v_i.\vec{p}\|_2} \qquad t = \frac{\langle \ell k.\vec{p} - v_i.\vec{p}, s_j.\vec{n} \rangle}{\langle \vec{z}, \vec{n} \rangle}$$



$z = t$

$Z = \|s_j.\vec{p} - v_i.\vec{p}\|_2$

$R = R_{\ell k} = \frac{z}{Z} \cdot R_{max}$

$r = \|\ell k.\vec{p} - \ell k.\vec{p}'\|_2$

unimportant side nodes
$\alpha + \beta + \gamma = 1$

$X = \alpha \cdot v_i.p + \beta \cdot s.pe_0 + \gamma \cdot s.pe_1$

$X = v_i.p + \vec{a} \cdot \beta + \vec{b} \cdot \gamma$

$X = v_i.p + (s.pe_0 - v_i.p) \cdot \beta + (s.pe_1 - v_i.p) \cdot \gamma$

$\iiint e(z, Z, r, R) \, dV$ viewcone $= 1$?

given: $\mathrm{I} \int_0^{2\pi} \int_0^R r \cdot e_c(r, R) \, dr \, d\varphi \overset{\text{compact support}}{=} \int_0^{2\pi} \int_{-\infty}^{\infty} r \, e_c(r, R) \, dr \, d\varphi = 1$

$\mathrm{II} \int_{-\infty}^{\infty} e_r(z, Z) \, dz \overset{\text{compact support } Z}{=} \int_0^Z e_r(z, Z) \, dz = 1$

$\Rightarrow \int_0^Z \int_0^R \int_0^{2\pi} \overbrace{e_r(z, Z) \cdot e_c(r, R)}^{e(z, Z, r, R)} \, d\varphi \, dr \, dz$

$= \int_0^Z e_r(z, Z) \int_0^R \int_0^{2\pi} e_c(r, R) \, d\varphi \, dr \, dz \overset{\mathrm{I}}{=} \int_0^Z e_r(z, Z) \, dz \overset{\mathrm{II}}{=} 1$ ▫

$\frac{z}{Z} = \frac{R_{\ell k}}{R_{max}}$

$\vec{z} = \frac{s_j.\vec{p} - v_i.\vec{p}}{\|s_j.\vec{p} - v_i.\vec{p}\|_2}$

~~$\ell k.p' = \ell k.\vec{p} + t \cdot \vec{t}$~~

~~$\ell k.p' = v_i.\vec{p} + s \cdot \vec{s}$~~

$\frac{z}{Z} = \frac{R_{\ell k}}{R_{max}}$

$\Pi_{\ell k}$: $\langle \vec{X} - \ell k.\vec{p}, s_j.\vec{n} \rangle = 0$

Π_{s_j}: $\langle \vec{X} - s_j.\vec{p}, s_j.\vec{n} \rangle = 0$

$\ell k.p' = \text{intersection}\left(\Pi_{\ell k}, \vec{z} = \frac{s_j.\vec{p} - v_i.\vec{p}}{\|s_j.\vec{p} - v_i.\vec{p}\|_2}\right) = v_i.\vec{p} + t \cdot \vec{z}$ with $\langle v_i.\vec{p} + t \cdot \vec{z} - \ell k.\vec{p}, s_j.\vec{n} \rangle = 0$