Week 1 Questions

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Question 1.

(a) 10! since pick 10 choices with no replacement, each unique and no other limitations

$$= 3628800$$

(b) E and F must appear together so treat as one set taking two slots so now 9 slots. 9! but also order of E and F can swap so $9! * 2!$ since 2 elements in set $\{E, F\}$

$$= 725760$$

(c) Permutation formula. 6 slots so 6! but set of $\{N, N\}$ and $\{A, A, A\}$ has 2! and 3! ordering within the sets that don't matter. Hence $\frac{6!}{(2! * 3!)}$

$$= 60$$

(d) 3 slots. 5 unique choices. Assume order matters since letter arrangements. Since 3 slots we don't care about the order of the 2 we don't pick. All elements are unique Hence $\frac{5!}{2!}$

$$= 60$$

Correction:

Without any indication apparently order doesnt matter for this question so instead its $\binom{5}{3}$ if order doesn't matter

$$= 10$$

Question 2.

(a) 6^4 Since 4 repetitions and 10 choices each time (with replacement) and order matters.

$$= 1296$$

(b) $\binom{4}{2}$ ways to pick exactly 2 slots being 3 and $5^{(4-2)}$ choices for each non-three slot hence $\binom{4}{2} * (5^2)$

$$= 150$$

(c)$\binom{4}{2} * (5^2)$ where 2 3s + $\binom{4}{3} * (5)$ where 3 3s and $\binom{4}{4}$ where all 3s = $150 + 20 + 1$

$$= 171$$

Question 3.

(a) 8! ways to order cards but 4 sets with 2 duplicate elems each $\frac{8!}{(2!)^4}$

$$= 2520$$

(b) Both same set = $\binom{4}{1} = 4$ ways. Different sets = $\binom{4}{2} = 6$ ways Order doesnt matter Hence 6+4 = 10 unique combinations of two aces from the 8 cards

$$= 10$$

Correction: Both have to be different apparently so ignore the 4 extra choices and we get just

$$= 6$$

(c) $\binom{2}{1} + \binom{2}{2} = 2 + 1 = 3$from same logic as before

$$= 3$$

Alternative way to same answer:
$\frac{4!}{2! * 2! * 2!}$ because $\frac{4!}{2! * 2!}$ unique permutations of the 4 good cards. But also we don't care about order of the final two cards we selected hence divide by 2! again.

$$= 3$$