

Week 4 Questions

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Question 1

(a) $\{(1,1)\}$

(b) $\{(2,1),(1,2)\}$

(c) $\{(1,3),(3,1),(2,2)\}$

(d)

$$\frac{3}{6^2} = 0.08333$$

since 3 events and a total sample space of 6^2 different events

Question 2

(a) -3,-1,1,3

If all tails then $X = -1 -1 -1 = -3$ If one heads then $X = -1 -1 +1 = -1$

Similarly we can get X for all possible outcomes after three flips

(b)

$$\{(T, T, T)\}$$

Are the events corresponding to $X = -3$ hence given 2^3 different possible events

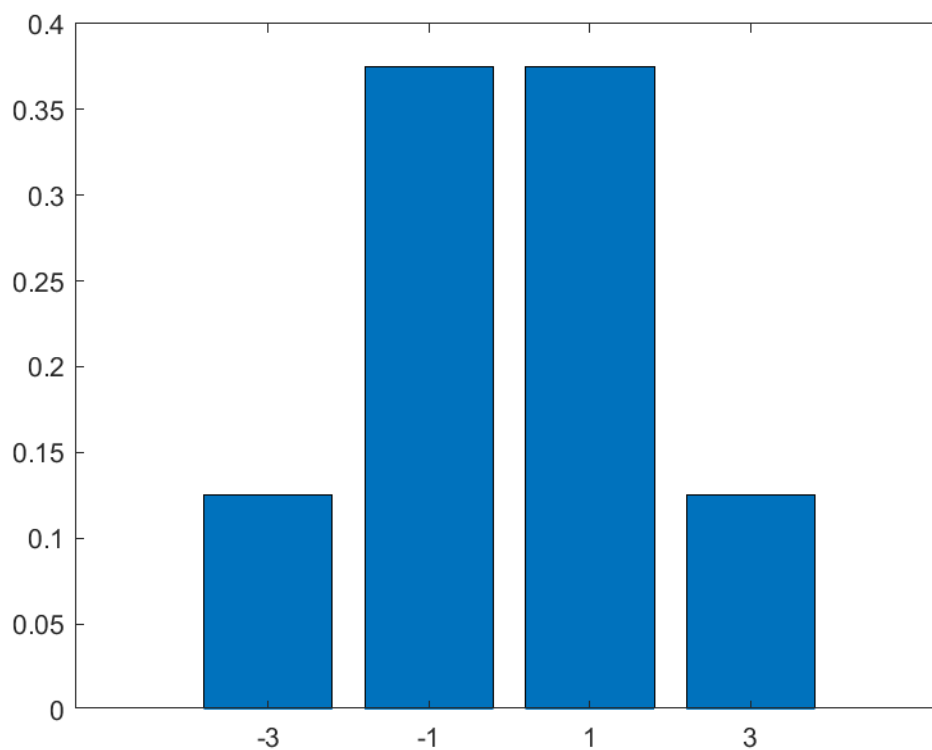
$$P(X = -3) = \frac{1}{2^3} = 0.125$$

(c) Using a similar logic as (b) above

$$\{(H, T, T), (T, H, T), (T, T, H)\} = 0.375$$

(d) Probability of $P(X=1) = P(X=-1)$ and $P(X=3) = P(X=-3)$

PMF = plotting out these values =



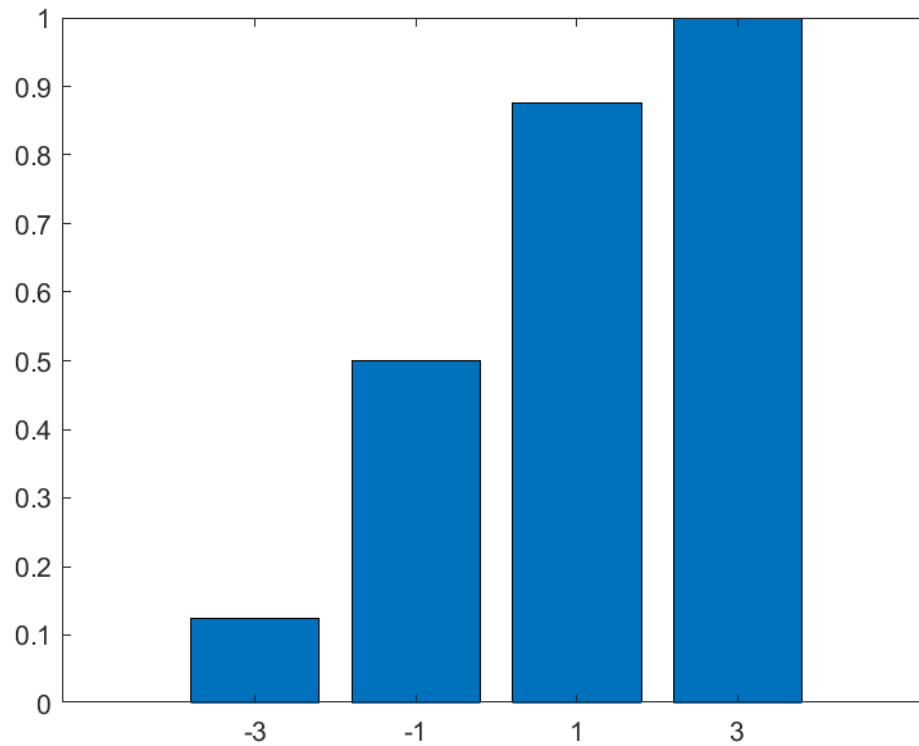
CDF =

$$P(X = -3) = .125$$

$$P(X = -1) = .375 + .125 = .5$$

$$P(X = 1) = .875$$

$$P(X = 3) = 1$$



Question 3

(a) 1 since all numbers on the dice are ≥ 1

(b) Exactly 1 possibility in 6 can't be landed on all 4 times, since if the die rolls a 1 then $X = 1$.

$$\left(\frac{5}{6}\right)^4 = 0.4823$$

(c) $P(X \leq k)$ for all k .

$$P(X \leq 1)$$

can be derived through counting and summing up the following:

$$\binom{4}{1} * \left(\frac{5}{6}\right)^3 * \left(\frac{1}{6}\right)^1 = \frac{125}{324}$$

$$\binom{4}{2} * \left(\frac{5}{6}\right)^2 * \left(\frac{1}{6}\right)^2 = \frac{25}{216}$$

$$\binom{4}{3} * \left(\frac{5}{6}\right)^1 * \left(\frac{1}{6}\right)^3 = \frac{5}{324}$$

$$\binom{4}{4} * \left(\frac{5}{6}\right)^0 * \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$$

$$P(X \leq 1) = \frac{671}{1296}$$

$$P(X \leq 2) = P(X \leq 1) + P(X = 2)$$

$$P(X \leq 1) = \frac{671}{1296}$$

$$\binom{4}{1} * \left(\frac{4}{6}\right)^3 * \left(\frac{1}{6}\right)^1 = \frac{16}{81}$$

$$\binom{4}{2} * \left(\frac{4}{6}\right)^2 * \left(\frac{1}{6}\right)^2 = \frac{2}{27}$$

$$\binom{4}{3} * \left(\frac{4}{6}\right)^1 * \left(\frac{1}{6}\right)^3 = \frac{1}{81}$$

$$\binom{4}{4} * \left(\frac{4}{6}\right)^0 * \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$$

$$P(X \leq 2) = \frac{65}{81}$$

...All these answers so far are correct but I have been enlightened as to just get 1 - (chance of not getting any roll $\leq X$)

$$P(X \leq 3) = \frac{15}{16}$$

$$P(X \leq 4) = \frac{80}{81}$$

$$P(X \leq 5) = \frac{1295}{1296}$$

$$P(X \leq 6) = 1$$