Week 3 Questions

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Question 1

(a) $\frac{1}{6^6} = 0.00002$ Every sequence has equal chance of occuring

(b) $\binom{6}{4}$ ways to arrange the results and $\frac{1}{6}^4$ chance of four 3s $+\frac{5}{6}^2$ chance of other numbers that arent three

$$\binom{6}{4} * \left(\frac{1}{6}\right)^4 * \left(\frac{5}{6}\right)^2 = 0.0080$$

(c) $\binom{6}{1}$ ways positions for the 1 =

$$\binom{6}{1} * \frac{1}{6} * \left(\frac{5}{6}\right)^5$$

$$= 0.4019$$

(d) enumeration of all possible ways to get a 1

$$\binom{6}{1} * \frac{1}{6}^1 * \frac{5}{6}^5 = 0.4019 + //\text{one 1}$$

$$\binom{6}{2} * \frac{1}{6}^2 * \frac{5}{6}^4 = 0.2009 + //\text{two ones}$$

$$\binom{6}{3} * \frac{1}{6}^3 * \frac{5}{6}^3 = 0.0536 + //\text{three ones}$$

$$\binom{6}{4} * \frac{1}{6}^4 * \frac{5}{6}^2 = 0.0080 + //\text{four ones}$$
$$\binom{6}{5} * \frac{1}{6}^5 * \frac{5}{6}^1 = 0.0006 + //5 \text{ ones}$$
$$\binom{6}{6} * \frac{1}{6}^6 * \frac{5}{6}^0 = 0.00002//\text{all ones}$$
$$= 0.6651$$

Question 2

No because $P(B|A)$ does not equal $P(A \cup B)$
Without any knowledge of A the probability of B is {1,1} over the entire sample space

$$= \frac{1}{6 * 20}$$

The probability of A is $\frac{1}{6}$
The probability of $P(B|A)$ is $\frac{1}{20}$
As we can see $\frac{1}{6} * \frac{1}{120}$ does not equal $\frac{1}{20}$

Question 3

(a) $\frac{1}{n}$ correct answers and $\frac{n-1}{n}$ wrong answers first time
$\frac{1}{n-1}$ correct answers and $\frac{n-2}{n-1}$ wrong answers second time

$\frac{1}{n}$ chance of first try when k = 1
$\frac{1}{n-1} * \frac{n-1}{n}$ chance on second try when k = 2

$$\frac{n-1}{n} * \frac{n-2}{n-1} * \frac{1}{n-2}$$

... and so on

$$\frac{(n-1) * (n-2) * ...(n-(k-1))}{(n-1+1) * (n-2+1) * ...(n-(k-1)+1)} * \frac{1}{n-k+1}$$

*note: we have k-1 wrong answers so $n-(k-1) = n-k+1$ all except last term of top sequence and first term of bottom sequence cancel

$\frac{n-k+1}{n} * \frac{1}{n-k+1}$ which cancels to
$\frac{1}{n}$

(b)

$$\frac{1}{n} = \frac{1}{6} = 0.1667$$

$$\frac{5}{6} * \frac{4}{5} * \frac{1}{4} = \frac{1}{6} = 0.1667$$

(c) $\frac{1}{n}$ chance of correct password each time
$\frac{n-1}{n}$ chance of wrong each time
$\left(\frac{n-1}{n}\right)^{k-1} * \frac{1}{n}$

(d)

$$\left(\frac{5}{6}\right)^2 * \frac{1}{6} = 0.1157$$

Question 4

(a) .3 chance of failing one
.7 chance of success

$$\binom{3}{1} * .7^1 * .3^2 +$$

$$\binom{3}{2} * .7^2 * .3^1 +$$

$$\binom{3}{3} * .7^3 * .3^0 = 0.973$$

(b) Same logic as before but with new numbers

$$\binom{3}{1} * .05^1 * .95^2 +$$

$$\binom{3}{2} * .05^2 * .95^1 +$$

$$\binom{3}{3} * .05^3 * .95^0 = 0.143$$

(c)

$$\text{P(F) is a robot} = \frac{1}{10} = 0.10$$

$$\text{P(E) is flagged} = \frac{1}{10} * 0.973 + \frac{9}{10} * 0.143 = 0.226$$

$$P(E|F) = 0.973$$

$$P(F|E) = ?$$

$$P(F|E) = \frac{P(F) * P(E|F)}{P(E)}$$

$$= \frac{.1 * .973}{.226}$$

$$= 0.4305$$