Week 5 Questions

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Question 1

(a)

$$\text{Chance of picking both same} = \frac{1}{2} * \frac{4}{9} * 2 = \frac{4}{9}$$

$$\text{Chance of picking both different} = \frac{1}{2} * \frac{5}{9} * 2 = \frac{5}{9}$$

$$\text{Expected Value} = \frac{4}{9} * 1.1 + \frac{5}{9} * (-1) = -0.0667$$

(b)

$$(1.1 - (-0.0667))^2 * \frac{4}{9} = 0.605$$

$$(-1 - (-0.0667))^2 * \frac{5}{9} = 0.484$$

Variance =

$$0.605 + 0.484 = 1.089$$

Question 2

(a) n people selected
X is bernoulli random variable
for any given i, $P(Xi = 1) = 0.6$

$$E(Xi) = P(Xi = 1) = 0.6$$

$$Var(Xi) = 0.6 * (1 - 0.6) = 0.24$$

(c) E[Y] is the sum of all the E[Xi] for i=0 to i=n hence it is different.

(d) due to linearity of expected value

$$E\left[\frac{1}{n} * Y\right] = \frac{1}{n} * E[Y] =$$

$$E[Xi] = 0.6$$

for any given i since

$$E[Y] = n * E[Xi]$$

(e)

$$Var(Xi) = 0.24$$

$$Var(Y) = n * Var(X)$$

as Y is a binomial random variable

$$Var\left(\frac{1}{n} * Y\right) = \left(\frac{1}{n}\right)^2 * Var(Y) =$$

$$\frac{1}{n} * Var(X)$$

Question 3

(a) 13 balls 5 white 8 red

$$P(X1 = 1) = \frac{5}{13}$$

$$P(X1 = 0) = \frac{8}{13}$$

$$P(X2 = 1|X1 = 1) = \frac{4}{12}$$

$$P(X2 = 1|X1 = 0) = \frac{5}{12}$$

$$P(X2 = 1) = \frac{5}{13} * \frac{4}{12} + \frac{8}{13} * \frac{5}{12} = 0.385 = \frac{5}{13}$$

$$P(X2 = 0|X1 = 1) = \frac{8}{12}$$
$$P(X2 = 0|X1 = 0) = \frac{7}{12}$$

$$P(X1 = 1 \cap X2 = 1) = \frac{5}{13} * \frac{4}{12} = 0.128$$
$$P(X1 = 1 \cap X2 = 0) = \frac{5}{13} * \frac{8}{12} = 0.256$$
$$P(X1 = 0 \cap X2 = 1) = \frac{8}{13} * \frac{5}{12} = 0.256$$
$$P(X1 = 0 \cap X2 = 0) = \frac{8}{13} * \frac{7}{12} = 0.359$$

(b) No they are not independent

$$P(X1 = 1 \cap X2 = 1) = 0.128$$

But $P(X1 = 1) = \frac{5}{13}$

$$P(X2 = 1) = \frac{5}{13}$$
$$\frac{5}{13}^{\,2} = 0.148$$

Since P(X1=1)*P(X2=1) does not equal$P(X1 = 1 \cap X2 = 1)$ they are not independent

(c)

$$P(X2 = 1) = \frac{5}{13}$$
$$P(X2 = 0) = \frac{8}{13}$$

Bernoulli random variable hence

$$E(X2) = P(X2 = 1) = \frac{5}{13} = .385$$

(d) Similar logic as q before.

$$P(X2 = 1|X1 = 1) = \frac{4}{12} = 0.333$$