Stage 7 Mathematics: Intensive Revision

Topic 1: Factors | **Topic 2:** 2D & 3D Shapes

Topic 1: Factors, Primes, HCF, and LCM (20 Questions)

First Principles Focus: The Building Blocks of Numbers: Factors divide completely into a number; Multiples are the extended tables of a number. Prime factorization breaks a number down to its fundamental "atoms."

- **HCF (Intersection):** Find the shared primes, take the *lowest* powers. Used for cutting, splitting, and grouping equally.
- **LCM (Union):** Take all unique primes, use the *highest* powers. Used for synchronicity, repeating events, and future meetings.

Q1. List all the distinct factors of 36.

Q2. From the following list of numbers: $\{15, 19, 21, 27, 29, 33\}$, identify and list all the prime numbers. Justify why the remaining numbers are composite.

Q3. Express the number 120 as a product of its prime factors using index (power) notation.

Q4. Express the number 315 as a product of its prime factors using index notation.

Q5. A number is expressed as $2^3 \times 3^2 \times 5$. What is the numerical value of this number?

Q6. Find the Highest Common Factor (HCF) of 24 and 36.

Q7. Find the Lowest Common Multiple (LCM) of 15 and 20.

Q8. Find the HCF of three numbers: 45, 60, and 75.

Q9. Find the LCM of three numbers: 8, 12, and 18.

Q10. Two numbers are given in prime factorization form: $A = 2^3 \times 3^2 \times 7$ and $B = 2^2 \times 3^3 \times 5$. Determine the HCF of A and B in index form.

Q11. Using the same numbers A and B from Q10, determine the LCM of A and B in index form.

Q12. **(Application)** A florist has 48 roses and 72 tulips. She wants to arrange them into identical bouquets such that no flowers are left over. What is the maximum number of bouquets she can create?

Q13. **(Application)** Two wooden planks of length 64 cm and 96 cm need to be cut into equal pieces of the maximum possible length. What should be the length of each piece?

Q14. **(Application)** A teacher wants to distribute 90 pencils, 120 pens, and 150 erasers equally among her students with none left over. What is the greatest number of students she can distribute them to?

Q15. **(Application)** Two lighthouse beams flash at different intervals. Lighthouse A flashes every 12 seconds, and Lighthouse B flashes every 18 seconds. If they flash together at 8:00:00 PM, at what exact time will they next flash together?

Q16. **(Application)** Three planets orbit a star. Planet X takes 8 Earth-years, Planet Y takes 12 Earth-years, and Planet Z takes 16 Earth-years. If they align today, how many Earth-years will it take for them to align again?

Q17. **(Advanced Core Identity)** The HCF of two distinct numbers is 6, and their LCM is 72. If one of the numbers is 18, use the formula HCF $\times$ LCM $= a \times b$ to find the other number.

Q18. **(First Principles)** Let p and q be two distinct prime numbers. What is their Highest Common Factor (HCF) and Lowest Common Multiple (LCM) in terms of p and q?

Q19. (Advanced Remainders) Find the greatest number that will divide 50 and 70, but leaves a remainder of 2 in each case.

Q20. (Synthesis) Find the smallest number which when divided by 10, 14, and 18 leaves a remainder of 3 in each case.

Topic 2: 2D and 3D Shapes (20 Questions)

First Principles Focus: Spatial Reasoning & Dimensions:

- **1D (Length):** Perimeter. Unit conversion is linear (e.g., $\times 10$).
- **2D (Area/Surface Area):** Flat space. Unit conversion is squared (e.g., $\times 10^2$). Area is Additive/Subtractive.
- **3D (Volume):** Spatial capacity. Unit conversion is cubed (e.g., $\times 10^3$).
- **Surface Area Alert:** When joining 3D objects, the total surface area *decreases* because connecting faces are hidden inside!

Q1. Convert an area of 5 m^2 into cm^2.

Q2. Convert an area of $3,000,000$ m^2 into square kilometers (km^2).

Q3. Calculate the perimeter and area of a rectangle with a length of 1.5 m and a width of 80 cm. Provide your final area in cm^2.

Q4. A square has an area of 144 cm^2. Calculate its perimeter.

Q5. Calculate the area of a right-angled triangle with a base of 12 cm and a height of 5 cm.

Q6. The perimeter of a regular hexagon is 42 cm. Find the length of one side.

Q7. **(Additive Area)** An L-shaped room is formed by two rectangles. Rectangle A is 8 m by 3 m. Rectangle B, attached to the bottom, is 5 m by 4 m. Find the total area.

Q8. **(Subtractive Area)** A rectangular wall measures 6 m by 4 m. There is a square window of side 1.5 m in the middle of the wall. What is the area of the wall that can be painted?

Q9. A rectangle has an area of 108 cm^2 and a width of 9 cm. What is its length?

Q10. **(Nets)** A student draws a cross-shaped net consisting of exactly 5 equal squares. Can this net fold to form a completely closed 3D cube? Explain why or why not.

Q11. Name the 3D solid that has exactly 2 identical triangular faces and 3 rectangular faces.

Q12. Calculate the volume of a cube with a side length of 6 cm.

Q13. Calculate the volume of a rectangular cuboid measuring 10 cm in length, 5 cm in width, and 4 cm in height.

Q14. A cuboid has a total internal volume of 240 cm^3 and a base area of 48 cm^2. What is the height of the cuboid?

Q15. **(Capacity Conversion)** A water tank has a volume of $2,000$ cm^3. Convert this capacity into Liters (L).

Q16. Calculate the total surface area of a cube with a side length of 4 cm.

Q17. Calculate the total surface area of a closed rectangular box measuring 6 cm by 4 cm by 3 cm.

Q18. **(Real-world SA)** You need to paint the outside of a wooden toy box measuring 1.2 m long, 0.5 m wide, and 0.4 m high. You will *not* paint the bottom face. Calculate the total area to be painted.

Q19. **(Compound Surface Area)** Two identical wooden cubes, each with a side length of 3 cm, are glued together face-to-face to form a single cuboid. Calculate the total surface area of this new cuboid. Explain why it is not 108 cm^2.

Q20. (First Principles Dimensional Scaling) Imagine a cube of side length x. If you strictly double every side length to $2x$, by what exact factor does the total volume increase? Show your algebraic proof.

Appendix: Detailed Step-by-Step Solutions

Topic 1: Factors Solutions

A1. Factors of 36: Think in pairs: 1×36, 2×18, 3×12, 4×9, 6×6. List in order: 1, 2, 3, 4, 6, 9, 12, 18, 36.

A2. Primes: 19 and 29. *Justification:* They have exactly two distinct factors (1 and themselves). The others are composite: 15 (divisible by 3,5), 21 (3,7), 27 (3,9), 33 (3,11).

A3. Prime Factorization of 120: $120 = 12 \times 10 = (4 \times 3) \times (2 \times 5) = (2^2 \times 3) \times (2 \times 5) = 2^3 \times 3 \times 5$.

A4. Prime Factorization of 315: Ends in 5, so divide by 5: $315 = 5 \times 63$. We know $63 = 9 \times 7 = 3^2 \times 7$. $3^2 \times 5 \times 7$.

A5. Value: $2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 72 \times 5 = 360$.

A6. HCF of 24 and 36: $24 = 2^3 \times 3$; $36 = 2^2 \times 3^2$. HCF takes the lowest powers of common primes: $2^2 \times 3^1 = 4 \times 3 = 12$.

A7. LCM of 15 and 20: $15 = 3 \times 5$; $20 = 2^2 \times 5$. LCM takes the highest powers of all primes: $2^2 \times 3^1 \times 5^1 = 4 \times 3 \times 5 = 60$.

A8. HCF of 45, 60, 75: $45 = 3^2 \times 5$; $60 = 2^2 \times 3 \times 5$; $75 = 3 \times 5^2$. Lowest powers of common primes (3 and 5): $3^1 \times 5^1 = 15$.

A9. LCM of 8, 12, 18: $8 = 2^3$; $12 = 2^2 \times 3$; $18 = 2 \times 3^2$. Highest powers: $2^3 \times 3^2 = 8 \times 9 = 72$.

A10. HCF of A and B: Lowest powers of shared primes (2 and 3). HCF $= 2^2 \times 3^2$.

A11. LCM of A and B: Highest powers of all primes (2, 3, 5, 7). LCM $= 2^3 \times 3^3 \times 5 \times 7$.

A12. Grouping (HCF): Find HCF of 48 and 72. $48 = 16 \times 3 = 2^4 \times 3$. $72 = 8 \times 9 = 2^3 \times 3^2$. HCF $= 2^3 \times 3 = 24$. Maximum of 24 bouquets.

A13. Cutting (HCF): HCF of 64 and 96. $64 = 2^6$; $96 = 2^5 \times 3$. HCF $= 2^5 = 32$. Each piece should be 32 cm long.

A14. Distribution (HCF): Find HCF of 90, 120, and 150. All end in 0 (factor of 10). Divide by 10: 9, 12, 15. HCF of 9, 12, 15 is 3. Total HCF $= 10 \times 3 = 30$. Greatest number of students is 30.

A15. Synchronicity (LCM): LCM of 12 and 18 is 36. They will flash together 36 seconds later, exactly at 8:00:36 PM.

A16. Orbit Alignment (LCM): Find LCM of 8, 12, and 16. LCM of 8 and 16 is 16. LCM of 16 and 12: $16 = 2^4$, $12 = 2^2 \times 3$. LCM $= 2^4 \times 3 = 16 \times 3 = 48$. They will align in 48 Earth-years.

A17. Core Identity: HCF $\times$ LCM $= a \times b \implies 6 \times 72 = 18 \times b \implies 432 = 18b \implies b = 432/18 = 24$.

A18. Prime Variables: Since p and q are prime, they share no factors other than 1. HCF = 1. To find the LCM, take the highest powers of all primes present: LCM $= p \times q$.

A19. Remainders (HCF): If remainder is 2, the number exactly divides $(50 - 2 = 48)$ and $(70 - 2 = 68)$. We need the HCF of 48 and 68. $48 = 16 \times 3 = 2^4 \times 3$. $68 = 4 \times 17 = 2^2 \times 17$. HCF $= 2^2 = 4$.

A20. Synthesis (LCM): Find the number exactly divisible first (LCM of 10, 14, 18). $10 = 2 \times 5$; $14 = 2 \times 7$; $18 = 2 \times 3^2$. LCM $= 2 \times 3^2 \times 5 \times 7 = 630$. Since we need a remainder of 3, add 3. 633.

Topic 2: 2D & 3D Shapes Solutions

A1. Area Conv.: 1 m $= 100$ cm. Therefore 1 m^2 $= (100$ cm$)^2$ $= 10,000$ cm^2. $5 \times 10,000 = 50,000$ cm^2.

A2. Area Conv.: 1 km $= 1000$ m. Therefore 1 km^2 $= (1000$ m$)^2 = 1,000,000$ m^2. $3,000,000/1,000,000 = 3$ km^2.

A3. Perimeter/Area: Convert units first! Length $= 150$ cm. Width $= 80$ cm. Perimeter $= 2(150 + 80) = 460$ cm. Area $= 150 \times 80 = 12,000$ cm^2.

A4. Square Area: Area $= s^2 = 144 \implies$ side $(s) = \sqrt{144} = 12$ cm. Perimeter $= 4 \times 12 = 48$ cm.

A5. Triangle Area: Area $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 12 \times 5 = 6 \times 5 = 30$ cm^2.

A6. Hexagon Perimeter: A regular hexagon has 6 equal sides. Side length $= 42/6 = 7$ cm.

A7. Additive Area: Area of Rect A $= 8 \times 3 = 24$ m^2. Area of Rect B $= 5 \times 4 = 20$ m^2. Total Area $= 24 + 20 = 44$ m^2.

A8. Subtractive Area: Total wall area $= 6 \times 4 = 24$ m^2. Window void $= 1.5 \times 1.5 = 2.25$ m^2. Paintable Area $= 24 - 2.25 = 21.75$ m^2.

A9. Missing Length: Area $= L \times W \implies 108 = L \times 9 \implies L = 108/9 = 12$ cm.

A10. Nets: No. A completely closed cube requires exactly 6 square faces (Top, Bottom, Front, Back, Left, Right). A net with 5 squares will leave one face entirely open.

A11. 3D ID: A triangular prism (like a standard Toblerone box).

A12. Cube Vol.: Volume $= s^3 = 6 \times 6 \times 6 = 216$ cm^3.

A13. Cuboid Vol.: Volume $= L \times W \times H = 10 \times 5 \times 4 = 200$ cm^3.

A14. Missing Height: Volume $=$ Base Area $\times$ Height $\implies 240 = 48 \times H \implies H = 240/48 = 5$ cm.

A15. Capacity: 1 Liter $= 1000$ cm^3 (or 1000 mL). 2000 cm$^3/1000 = 2$ Liters.

A16. Cube SA: A cube has 6 identical square faces. Area of one face $= 4 \times 4 = 16$ cm^2. Total SA $= 6 \times 16 = 96$ cm^2.

A17. Cuboid SA: SA $= 2(LW + LH + WH) = 2(6 \times 4 + 6 \times 3 + 4 \times 3) = 2(24 + 18 + 12) = 2(54) = 108$ cm^2.

A18. Real-World SA: Top face $= 1.2 \times 0.5 = 0.6$ m^2 (Bottom excluded). Two Side faces $= 2 \times (1.2 \times 0.4) = 0.96$ m^2. Front/Back faces $= 2 \times (0.5 \times 0.4) = 0.4$ m^2. Total Paint $= 0.6 + 0.96 + 0.4 = 1.96$ m^2.

A19. Compound SA: Single cube SA $= 6 \times (3 \times 3) = 54$ cm^2. Two cubes independent $= 108$ cm^2. However, when glued, two faces (one from each cube) are hidden inside. Hidden area $= 2 \times 9 = 18$ cm^2. True SA $= 108 - 18 = 90$ cm^2.

A20. Dimensional Scaling (Proof): Initial Volume $V_1 = x \times x \times x = x^3$. Scaled Volume $V_2 = (2x) \times (2x) \times (2x) = 8x^3$. Since $V_2 = 8(V_1)$, the volume increases by an exact factor of 8 (which is 2^3).