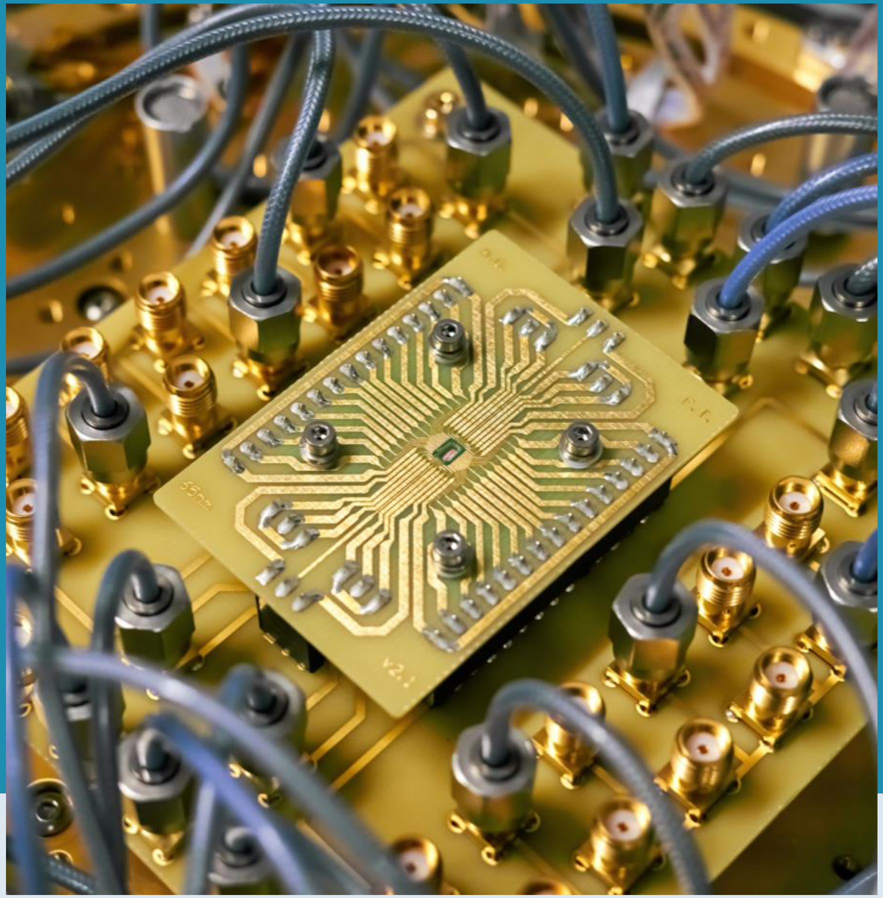
Design of Analog and Mixed-Signal Integrated Circuits B-KUL-H05E3A

Feedback Theory and Exercises



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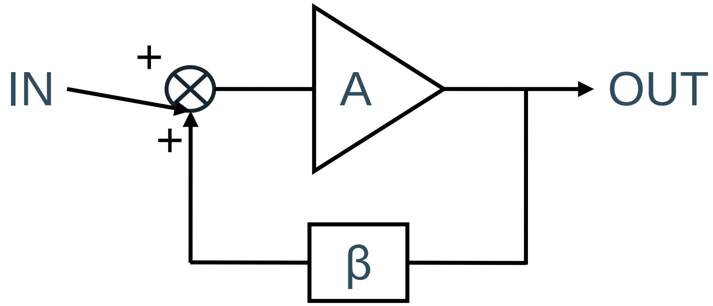
Feedback Theory

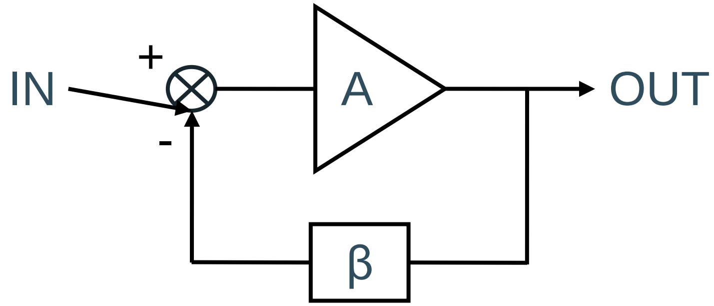
Feedback

- Positive



$$\frac{OUT}{IN} = A_{CL} = \frac{A}{1 - A\beta}$$

- Negative



$$\frac{OUT}{IN} = A_{CL} = \frac{A}{1 + A\beta}$$

A – Feedforward Gain
β – Feedback factor
Aβ – Loop gain

Loop gain cases

Considering negative feedback topology,

$$\frac{OUT}{IN} = A_{CL} = \frac{A}{1 + A\beta}$$

- **$A\beta$ >> 1**

$$A_{CL} \approx \frac{1}{\beta} \cdot \frac{1}{1+1/A_{OL}\beta} \approx \frac{1}{\beta}$$

Good!

- **$A\beta$ = -1**

$$A_{CL} = \infty$$

Bad!

- **$A\beta$ < -1**

$$A_{CL} = ?$$

Positive feedback stable?

$$\frac{OUT}{IN} = A_{CL} = \frac{A}{1 + A\beta}$$

Take A = 1000 and β = -0.1,

$$A_{CL} = \frac{1000}{-99} \approx -10.1$$

Finite!

- The simulator with ideal elements also shows finite output.
- How can the gain be finite for positive feedback?

Positive feedback stable?

$$\frac{OUT}{IN} = A_{CL} = \frac{A}{1 + A\beta}$$

Take A = 1000 and β = -0.1,

$$A_{CL} = \frac{1000}{-99} \approx -10.1$$

Finite!

- The simulator with ideal elements also shows finite output.
- How can the gain be finite for positive feedback?

No practical system has infinite bandwidth.
There’s always a delay or phase lag within the loop.

Phase is important!

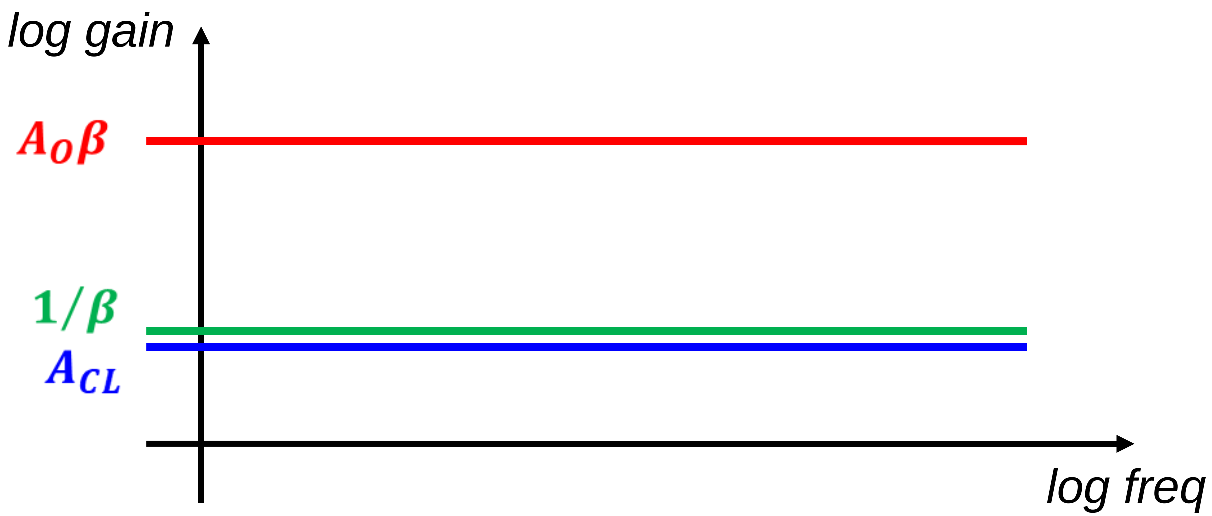
Ideal amplifier

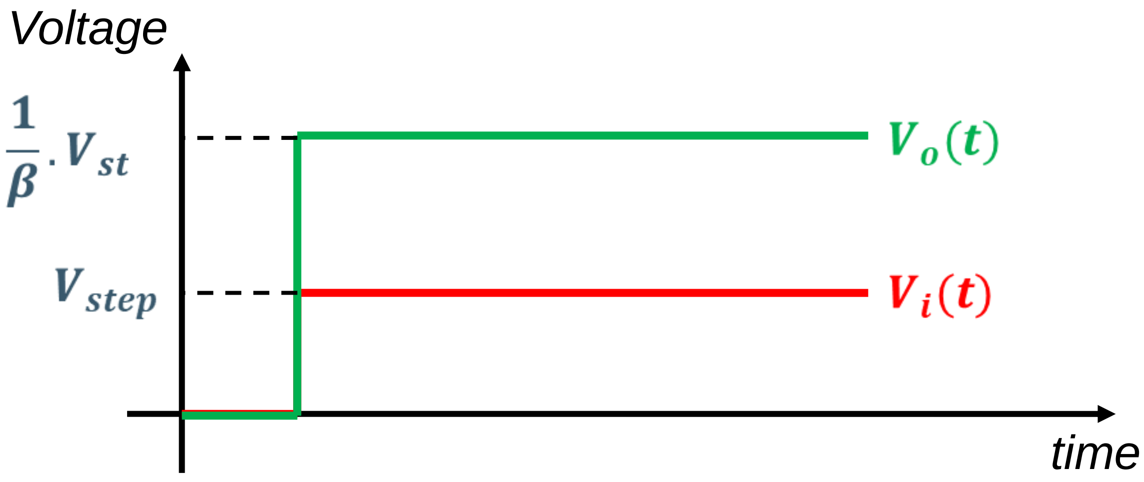
- $A = A_O$ (constant)

$$\boxed{A_{CL}(s) = \frac{1}{\beta} \cdot \frac{A_O\beta}{1 + A_O\beta}}$$

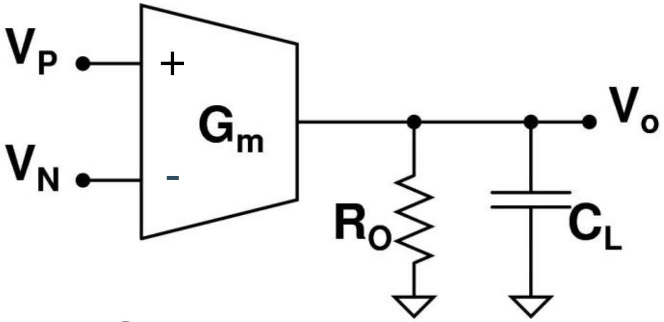
Step response:

$$\boxed{V_o(s) \approx \frac{1}{\beta} \cdot \frac{V_{step}}{s}}$$





Real amplifier

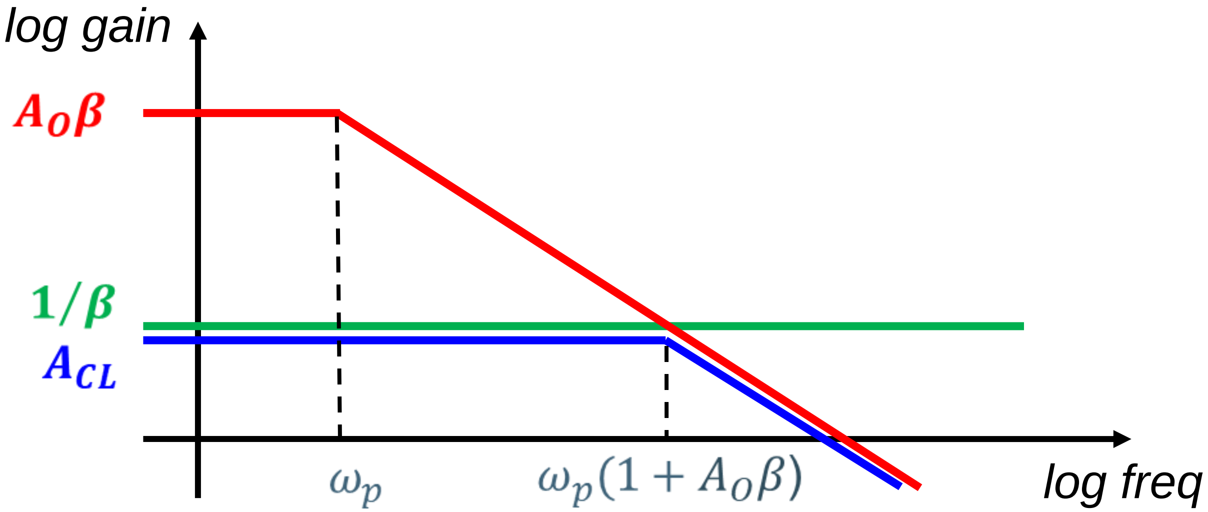


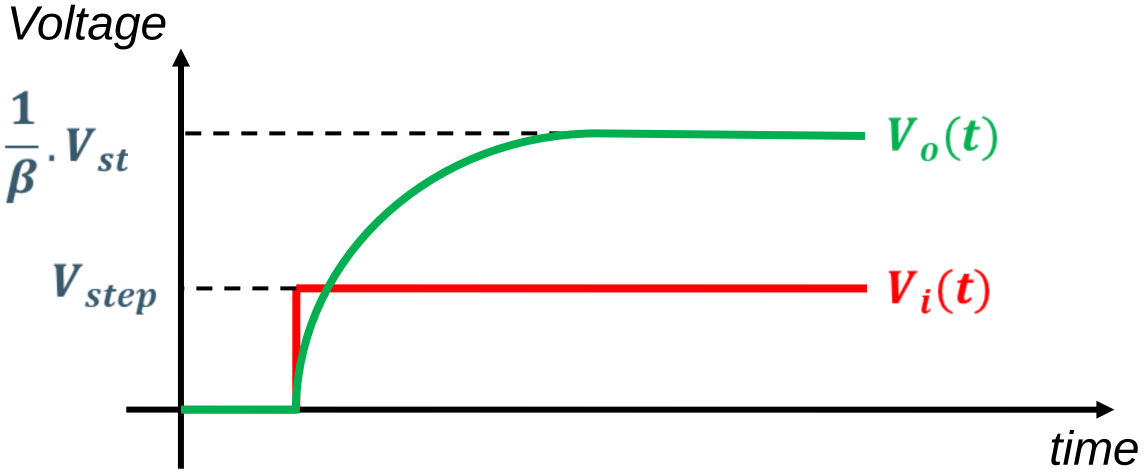
$$A = \frac{A_O}{1+s/\omega_p}$$

$$A_{CL}(s) = \frac{1}{\beta} \cdot \frac{A_O\beta}{1+A_O\beta} \cdot \frac{1}{1+s/\omega_p(1+A_O\beta)}$$

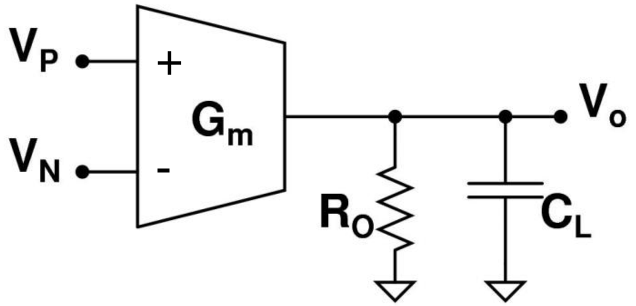
Step response:

$$V_o(s) \approx \frac{1}{\beta} \cdot \frac{1}{1+s/\omega_p(1+A_O\beta)} \cdot \frac{V_{step}}{s}$$





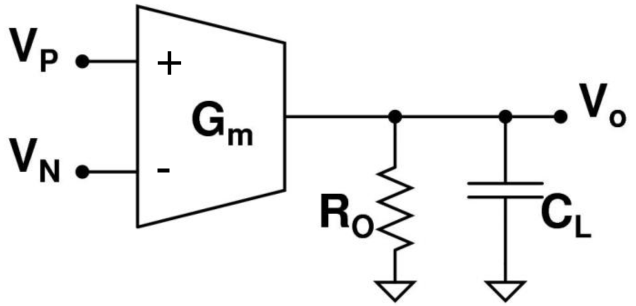
Problem with single stage



$$\text{Relative error} \approx {}^{1}/_{(G_m R_O)\beta}$$

We need higher gain!

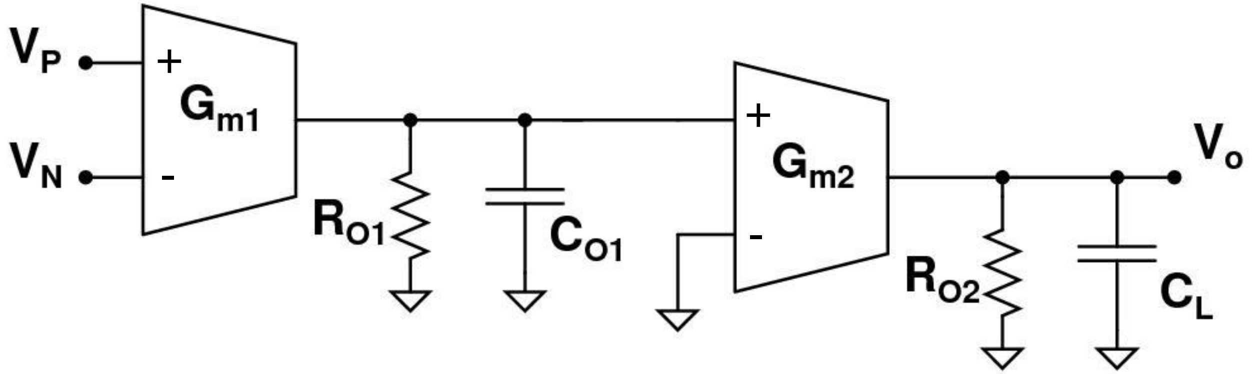
Problem with single stage



Relative error $\approx {}^{1}/_{(G_m R_O)\beta}$

We need higher gain!

- **Let's cascade two amplifiers**



Relative error $\approx {}^{1}/_{(G_{m1} R_{O1} G_{m1} R_{O1})\beta}$

This brings an additional pole!

Two pole system

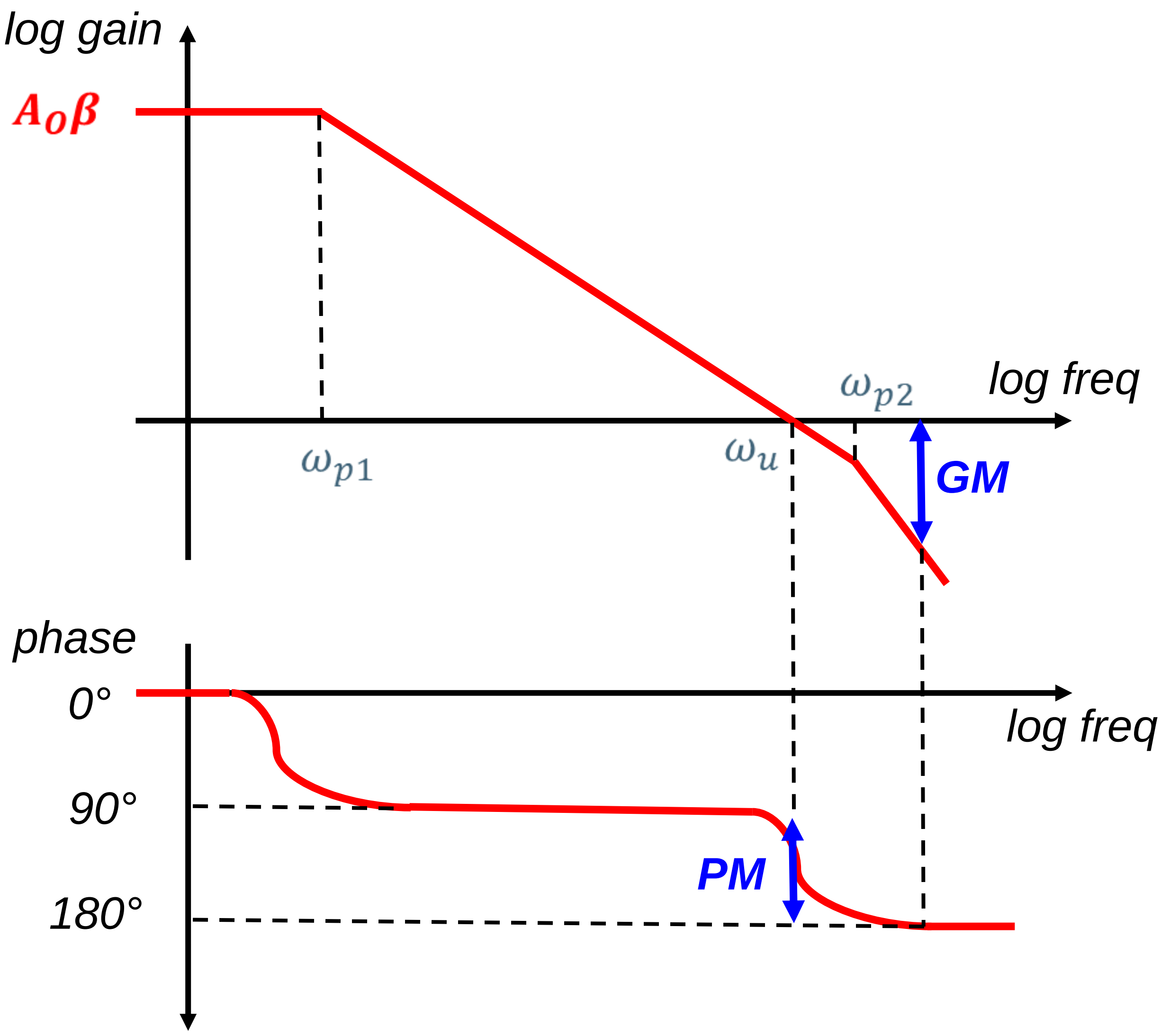
- $A = \dfrac{A_O}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$

Considering worst case ($\beta = 1$),

$$\boxed{A_{CL}(s) = \frac{A_O}{\left(\frac{s^2}{\omega_{p1}\omega_{p2}} + s.\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + (1 + A_O)\right)}}$$

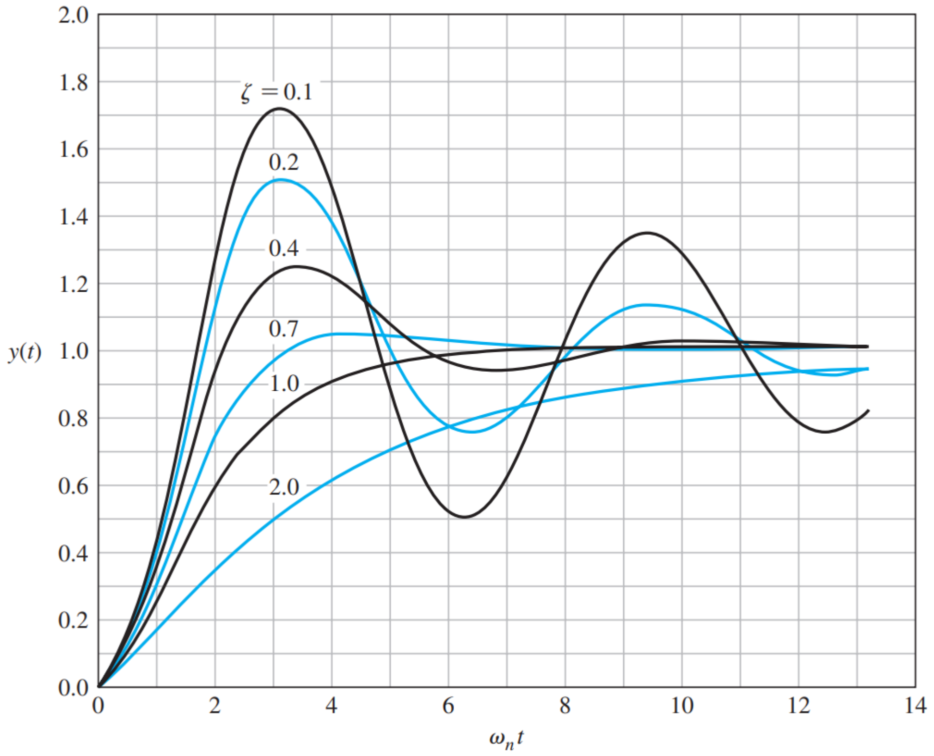
Equation is of the form:

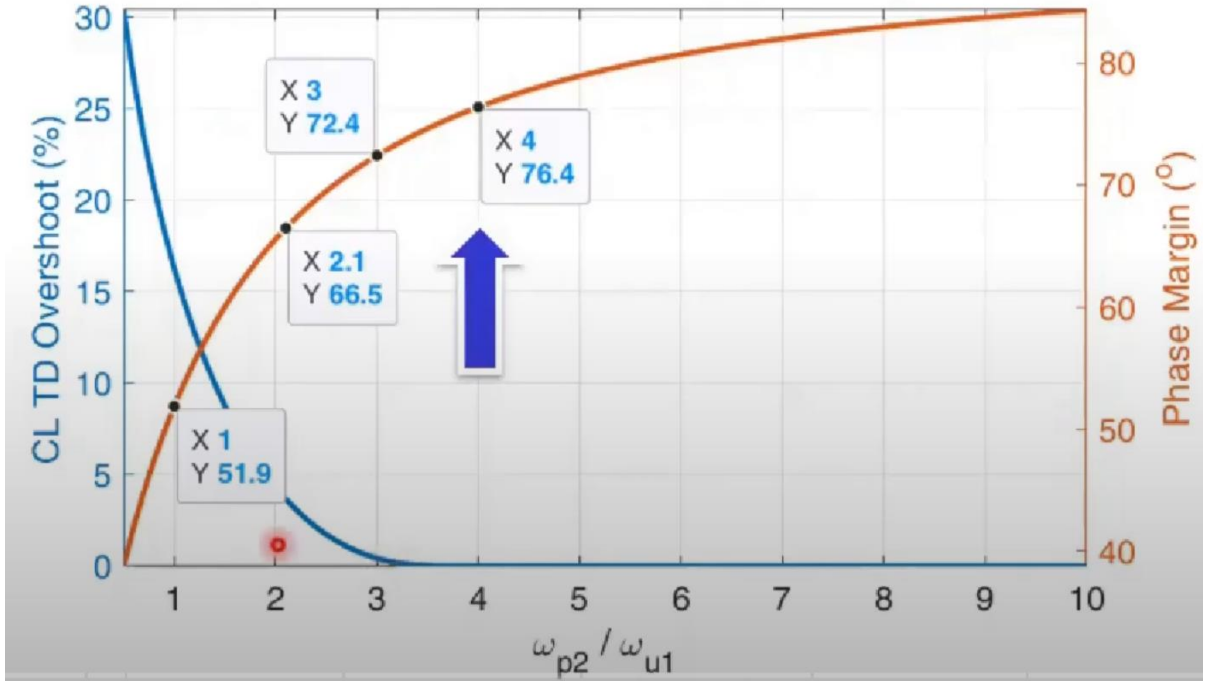
$$\boxed{A_{CL}(s) = \frac{{\omega_n}^2}{({\omega_n}^2 + 2\zeta\omega_n s + s^2)}}$$



Designing for certain PM

$$\zeta = \frac{1}{2}\sqrt{\frac{\omega_{p2}}{\boxed{A_O\omega_{p1}}}}$$ $\longleftarrow \omega_u$



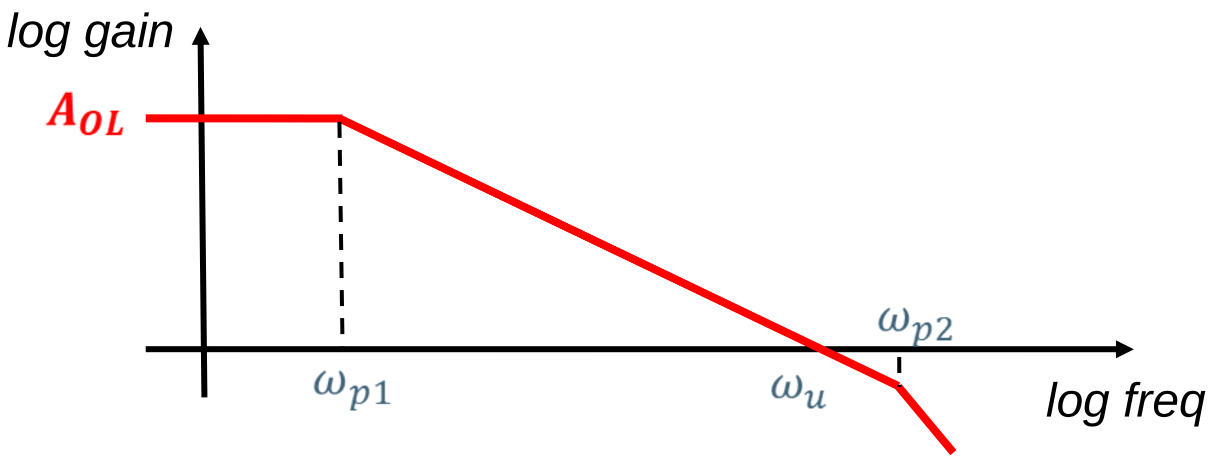


For critically damped system, **ζ = 1**

Based on required PM, we can decide $\left({}^{\omega_{p2}}/_{\omega_u}\right)$

Source: https://youtu.be/PT31xAEd_v4?feature=shared

Need for compensation



$\boxed{\omega_u \approx \omega_{p1}.A_{OL}}$

$\boxed{\omega_{p1} = 1/R_{O1}C_{O1}}$

$\boxed{\omega_{p2} = 1/R_{O2}C_L}$

Let's take an example,

A_{OL} = *1000*, $C_L = 10\,pF$, *PM* = *72°*

From last slide for PM = 72°, $\frac{\omega_{p2}}{\omega_u} = 3$

$\frac{R_{O1}C_{O1}}{A_{OL}R_{O2}C_L} = 3$

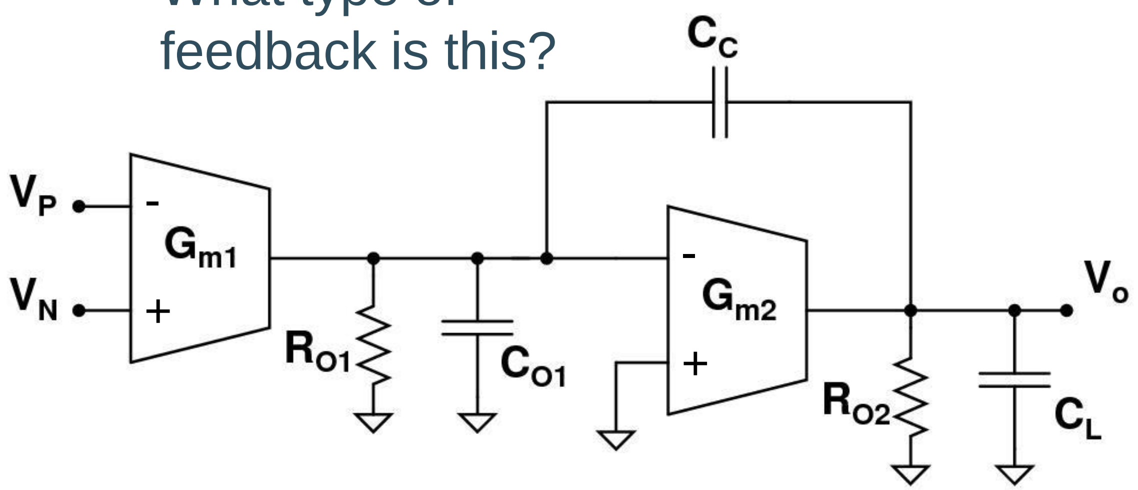
Assuming, $R_{O1} = R_{O2}$

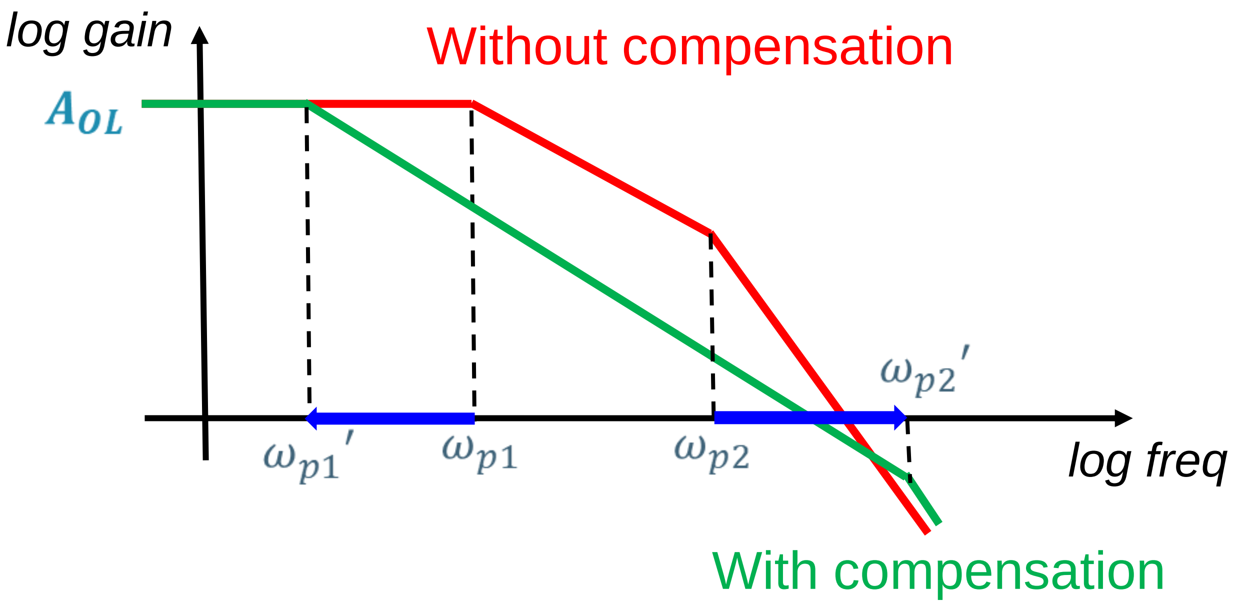
$C_{O1} =$ **30 *nF*** **Very high!**

We need a smart way to separate the poles i.e. do compensation.

Miller compensation

What type of feedback is this?





$$\omega_{p1}{}' = \frac{1}{R_{O1}[C_C(1+G_{m2}R_{O2})+C_1]+R_{O2}(C_C+C_L)} \approx \frac{1}{R_{O1}\boxed{(G_{m2}R_{O2})C_C}}$$

$$\omega_{p2}{}' = \frac{G_{m2}}{(C_{O1}+C_L)+{}^{C_{O1}C_L}/_{C_C}} \approx \frac{G_{m2}}{(C_{O1}+C_L)}$$

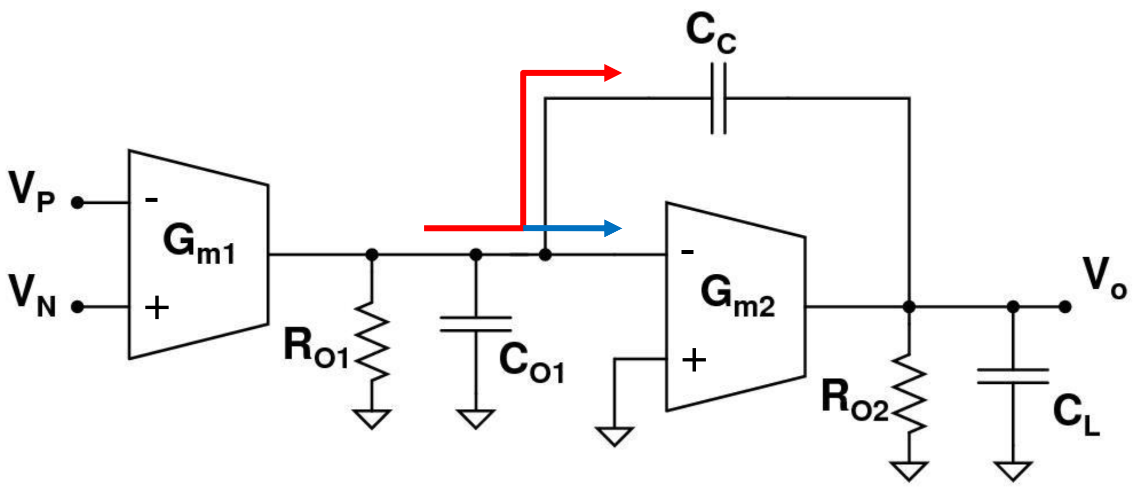
$$\omega_z{}' = \frac{G_{m2}}{C_C}$$

Right Half plane Zero!

A small cap appears as big due to miller effect.

We don't need a very high C_{O1} to get required PM!

2 Stage Miller OTA



The 2 parallel paths introduce an RHP zero and reduces PM!

We need a way to block the forward path though C_C.

$$A_O = G_{m1}R_{O1}G_{m2}R_{O2}$$

$$\omega_u \approx \frac{G_{m1}}{C_C}$$

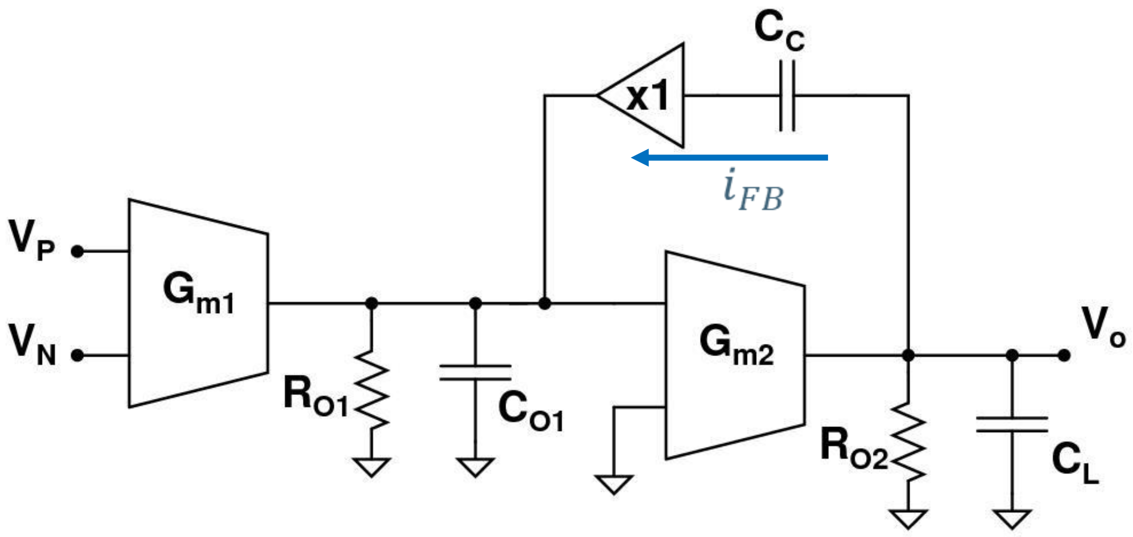
$$SR \approx \frac{I_{BIAS,1}}{C_L}$$

$$\omega_{p1} \approx \frac{1}{R_{O1}(G_{m2}R_{O2})C_C}$$

$$\omega_{p2} \approx \frac{G_{m2}}{(C_{O1}+C_L)}$$

$$\omega_z = \frac{G_{m2}}{C_C}$$

2 Stage Indirect Compensated Miller OTA



$$A_O = G_{m1}R_{O1}G_{m2}R_{O2}$$

$$\omega_u \approx \frac{G_{m1}}{C_C}$$

$$SR \approx \frac{I_{BIAS,1}}{C_L}$$

$$\omega_{p1} \approx \frac{1}{R_{O1}(G_{m2}R_{O2})C_C}$$

$$\omega_{p2} \approx \frac{G_{m2}C_C}{C_L C_{O1}}$$

$$\omega_{p3} \approx \left(\frac{1}{R_{O1}C_{O1}} + \frac{1}{R_C C_C}\right)$$

$$\omega_z = \frac{1}{R_C C_C}$$

The RHP zero goes but this adds an extra pole!

R_C needs to be small to ensure ω_{p2} and ω_{p3} are separated.

R_C is looking in impedance of current buffer

Exercise (see syllabus for details)

- Try the positive feedback simulation with and without ‘*tdelay*’.

- Design and simulate ideal 2-stage OTA for *PM* of 45°, 60°, 72°
- $A_O = 50\,dB,\ C_L = 10\,pF,\ \omega_u = 3\,\frac{Mrad}{sec},\ SR = 1\,\frac{V}{usec}$

- Add a current buffer in series with feedback capacitor and check the results.

References

- https://cmosedu.com/jbaker/students/theses/Indirect%20Feedback%20Compensation%20Techniques%20for%20Multi-Stage%20Operational%20Amplifiers.pdf

- https://in.ncu.edu.tw/ncume_ee/harvard-es154/lect_18_feedback.pdf

- https://youtu.be/PT31xAEd_v4?feature=shared

Appendix

2 Stage Miller OTA design steps

- Derive the OTA parameters from given specifications.
  (Ensure that your calculated parameters meets all required specifications)

- Start the design with 5T OTA at first. (only DC op. simulation at first!)

  - For current sources take L more than at least 10*Lmin
  - Decide $V_{ov} = (V_{gs} - V_{th})$ based on $\frac{G_m}{I_d}$ ratio for all transistors
  - Small $\frac{G_m}{I_d}$ ratio → small width → high power consumption → less $C_{parasitic}$
  - High $\frac{G_m}{I_d}$ ratio → large width → low power consumption → high $C_{parasitic}$

  - Always observe i_d, v_{dsat}, v_{ds}, g_m, $g_{moverid}$ and $region$ of transistors.
    (You will require to choose a few of them from annotation setup at a time)

2 Stage Miller OTA design steps

- Once you meet desired G_m, start to increase L for diff. pair and load MOSFETs to meet the gain requirement, ensuring that you are not changing W/L i.e. G_m.

- Once done with 1st stage, use the same dimension of PMOS load for 2nd stage CS amplifier and of NMOS IBIAS transistor for 2nd stage NMOS load but with **(multiplier factor based on your hand calculated parameters)**.

- Connect the feedback capacitor and simulate for STB analysis.

- Check which parameters are affecting the PM you want to reach and how to optimize OTA parameters to reach there.

- Be mindful while doing that ;)