

# **Image Analysis and Understanding: Exercises and Practicals 1**

**Feb 21, 2025 8:00 - 10:25**

**KU LEUVEN**

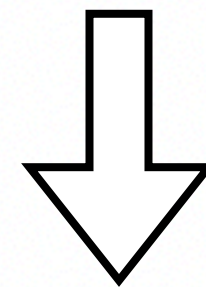


processing  
speech &  
images

- Python basics
- Pinhole Camera Model
- Distortion (radial and tangential)

# Pinhole Camera Model

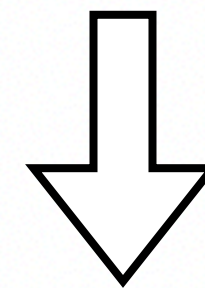
$$sp = K[R|t]P'$$



$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \left. \vphantom{\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}} \right\} \begin{array}{l} \text{3D point in world} \\ \text{coordinate} \end{array}$$

# Pinhole Camera Model

$$sp = K[R|t]P'$$



$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix}}_{\text{Extrinsic parameters (camera pose)}} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Extrinsic parameters  
(camera pose)

$R$  is rotation matrix

$t$  is translation vector



# Pinhole Camera Model

$$sp = K[R|t]P'$$

↓

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Camera Intrinsic}} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

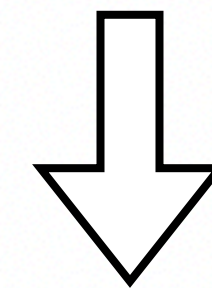
Camera Intrinsic

$(c_x, c_y)$  is principal point (in pixel)

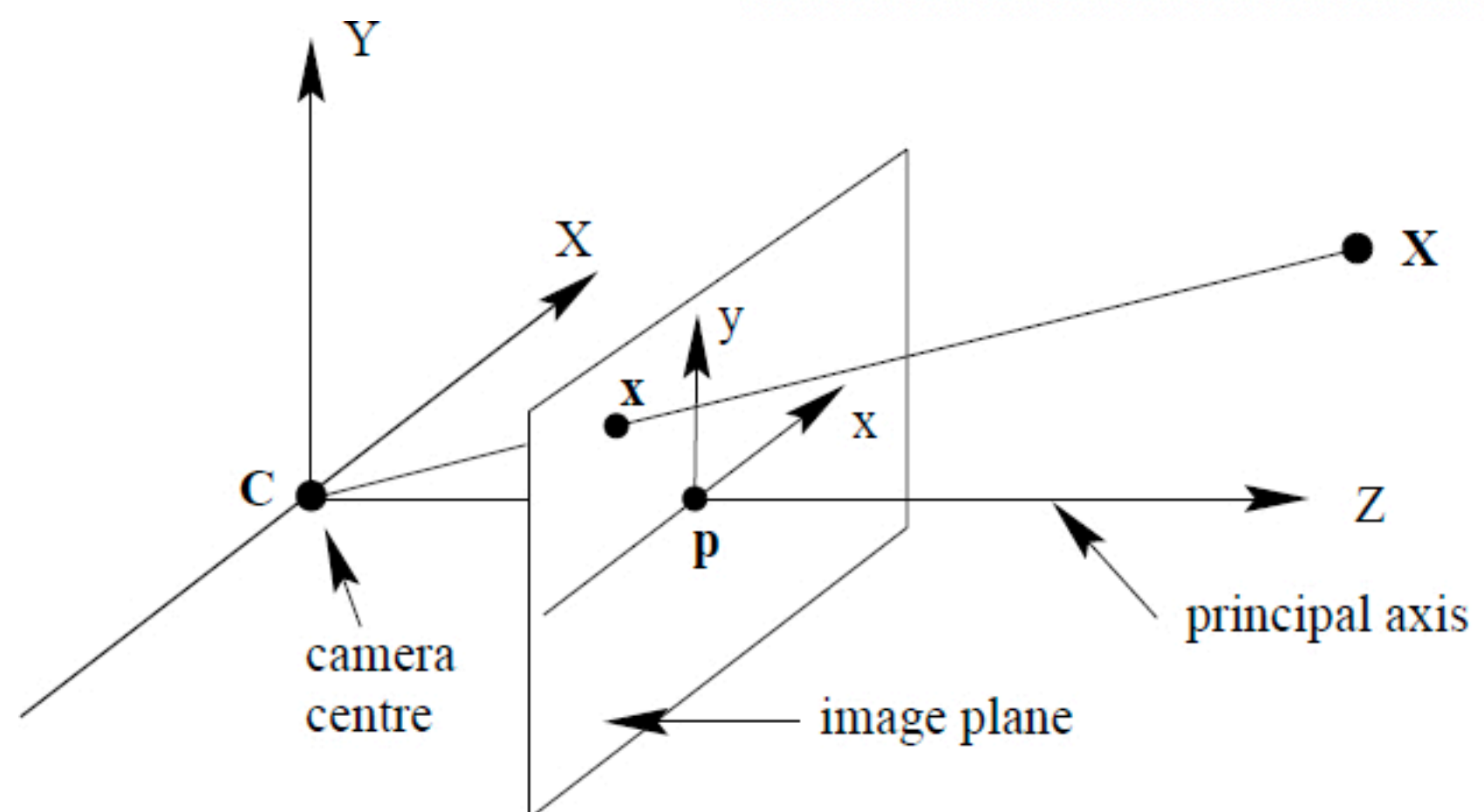
$(f_x, f_y)$  is focal length (in pixel)

# Pinhole Camera Model

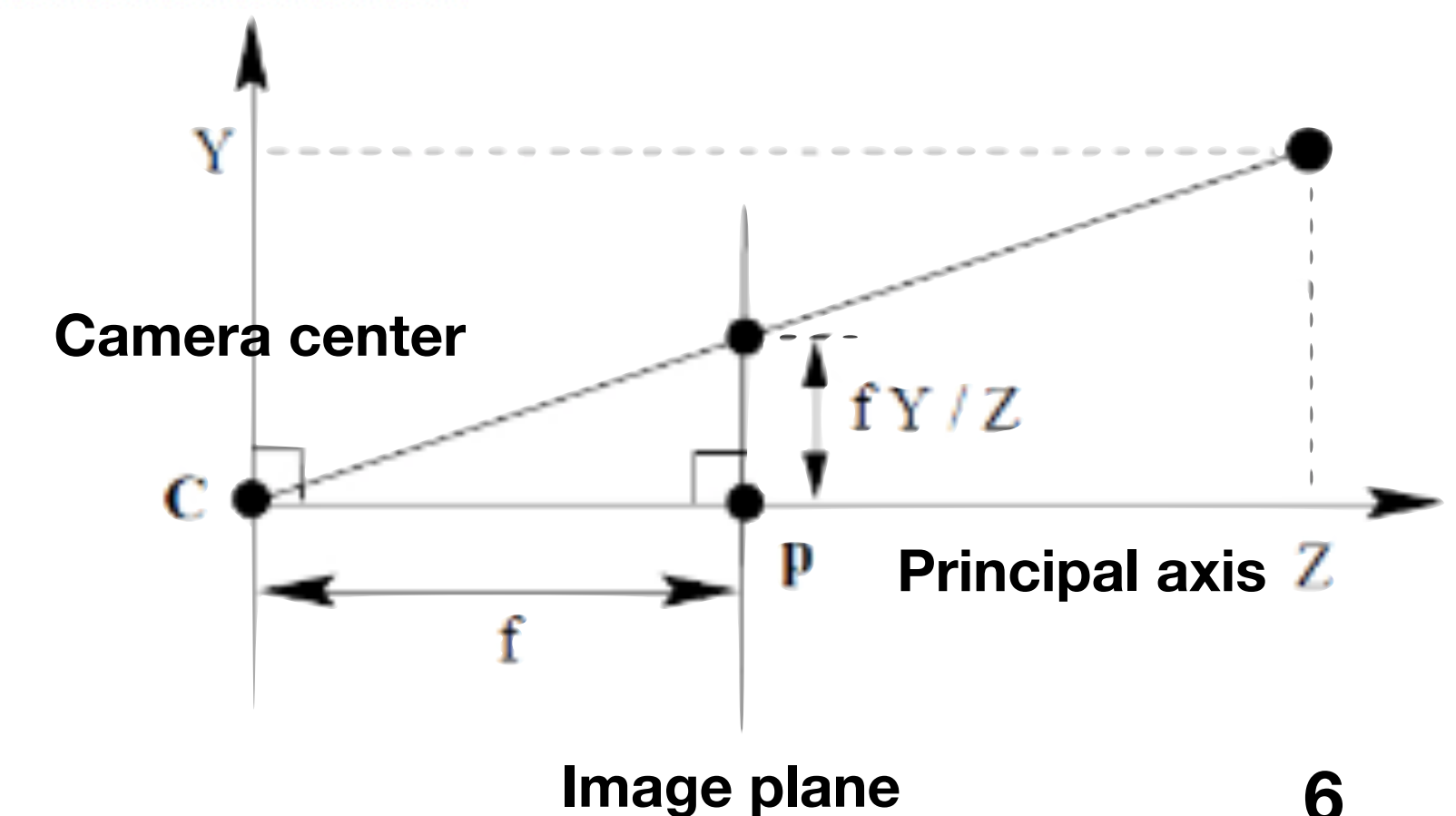
$$sp = K[R|t]P'$$



$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



**Example:** only look at y component,  
in camera coordinate





# Pinhole Camera Model

$$sp = K[R|t]P'$$

↓

pixel coordinate

depth value

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Pinhole Camera Model

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t$$

$$x' = x/z$$

$$y' = y/z$$

$$u = f_x * x' + c_x$$

$$v = f_y * y' + c_y$$

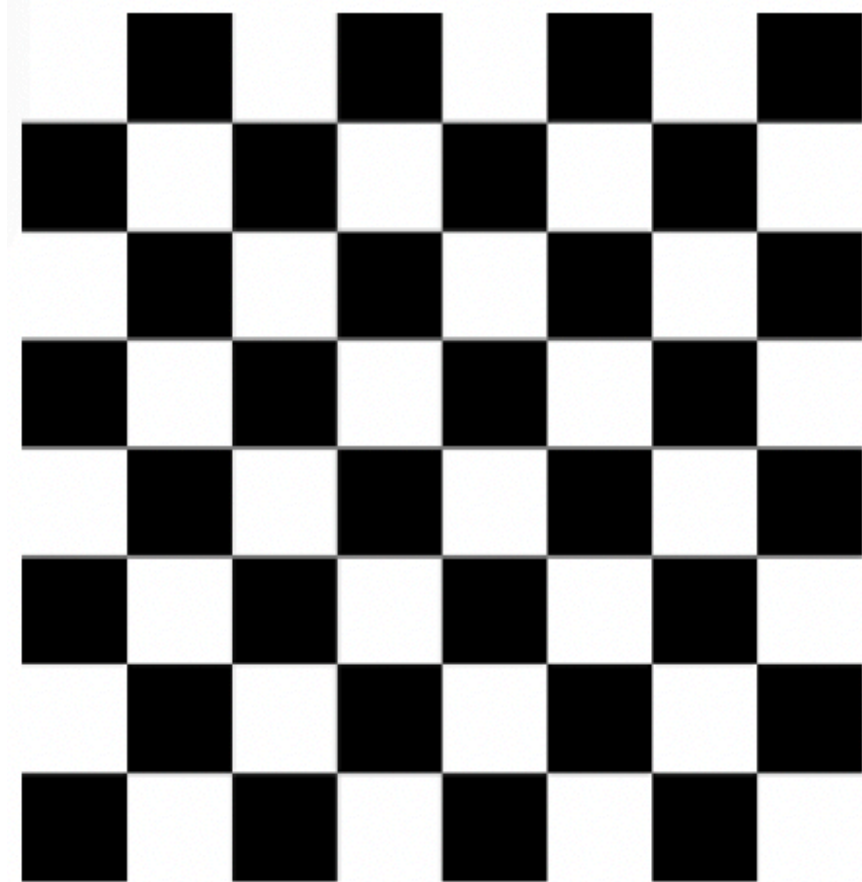
The formula in the assignment materials can be interpreted as:

1. 3D world coordinate => 3D camera coordinate
2. 3D camera coordinate => **unit** 3D camera coordinate
3. Unit 3D camera coordinate => 2D pixel coordinate

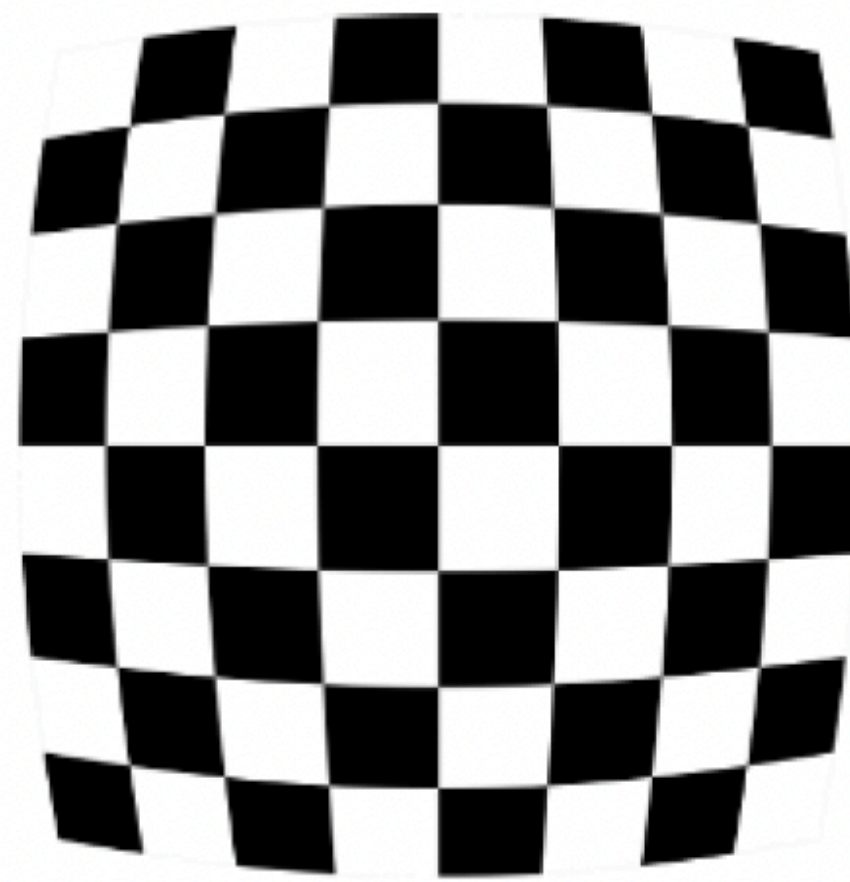


# Radial Distortion

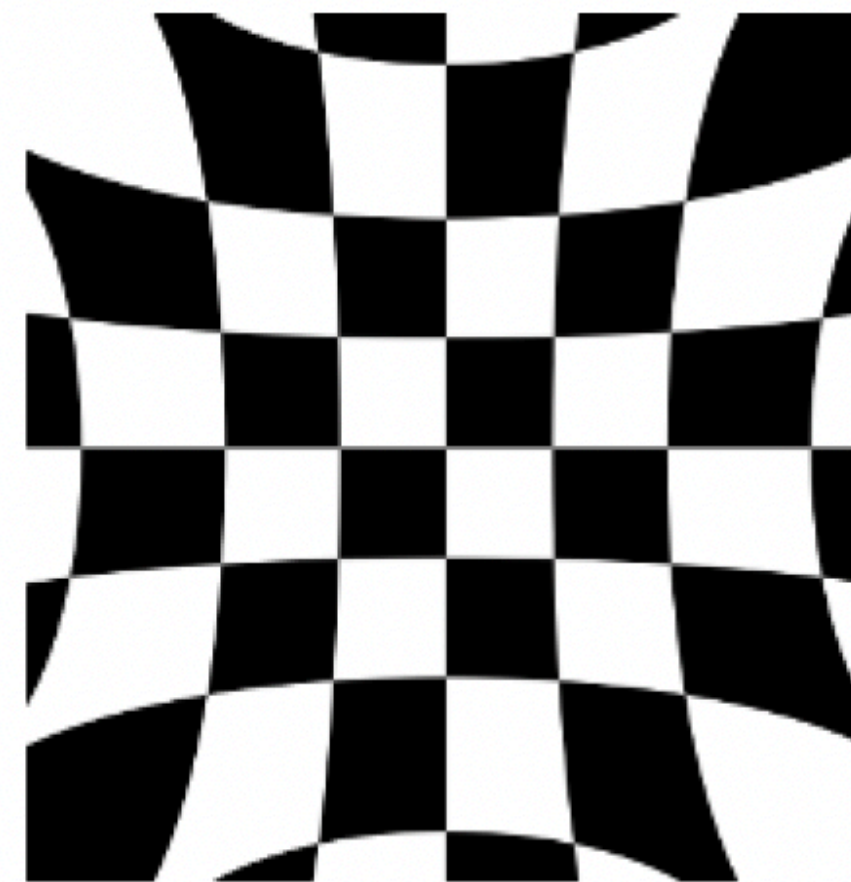
Radial distortion causes straight lines to appear curved. This effect is more pronounced the further away a point is from the center of the image.



No distortion



Positive radial distortion  
(Barrel distortion)



Negative radial distortion  
(Pincushion distortion)

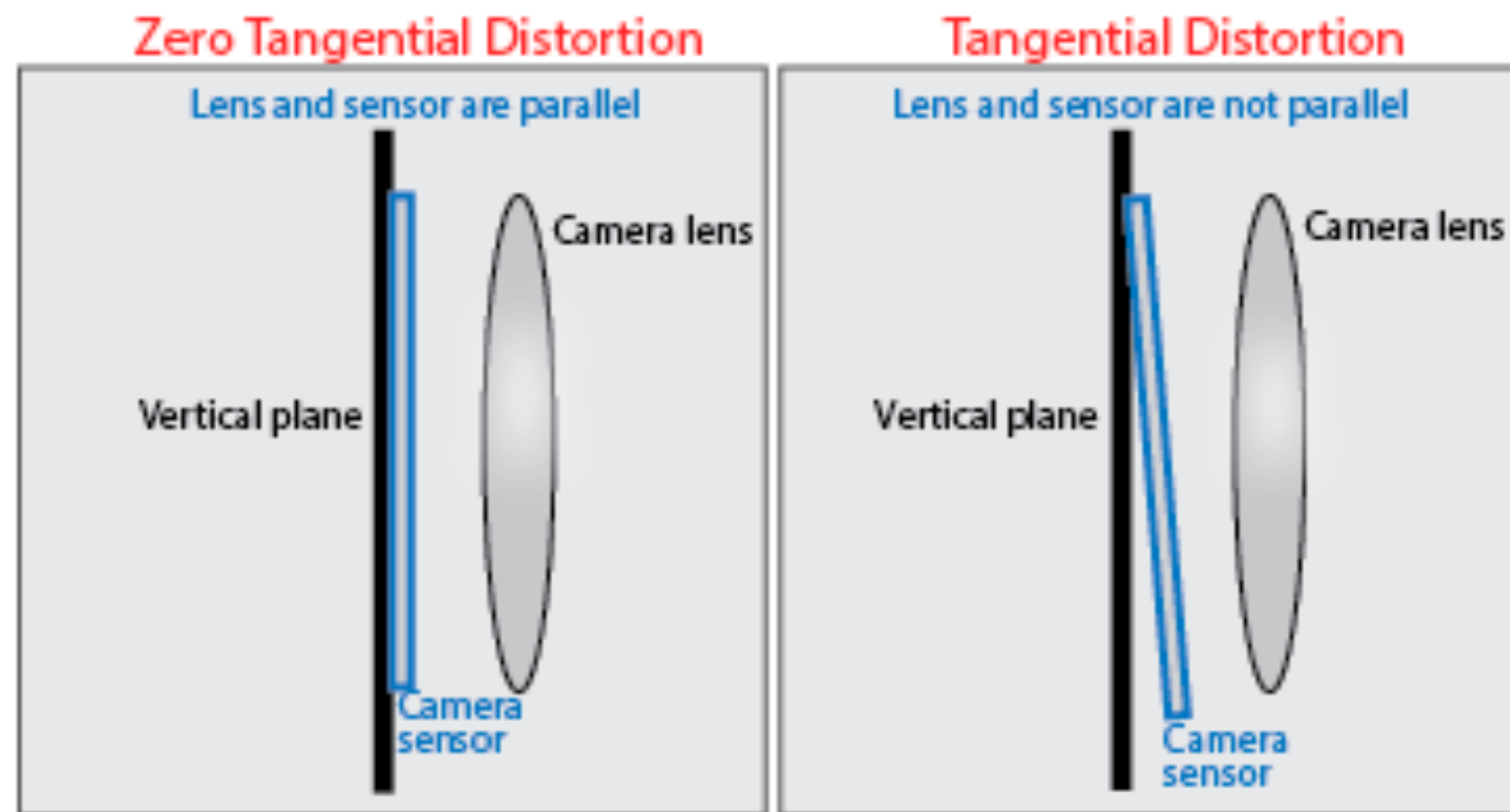
$$x'' = x'(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

$$y'' = y'(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

Note:  $(x', y')$   $(x'', y'')$  are the normalized coordinates of a point

# Tangential Distortion

Tangential distortion occurs when the lens is not aligned perfectly parallel to the imaging plane



$$x'' = 2p_1x'y' + p_2(r^2 + 2x'^2)$$
$$y'' = p_1(r^2 + 2y'^2) + 2p_2x'y'$$

# Undistortion procedure

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t$$

1. World to camera coordinate

$$x' = x/z$$

2. normalize to unit camera coordinate

$$y' = y/z$$

$$x'' = x'(1 + k_1 r^2 + k_2 r^4 + k_3 r^6) + 2p_1 x' y' + p_2 (r^2 + 2x'^2)$$

3. Undistortion

$$y'' = y'(1 + k_1 r^2 + k_2 r^4 + k_3 r^6) + p_1 (r^2 + 2y'^2) + 2p_2 x' y'$$

$$\text{Where } r^2 = x'^2 + y'^2$$

$$u = f_x * x'' + c_x$$

4. Camera coordinate to pixel space

$$v = f_y * y'' + c_y$$