MATH 3110 - INFINITE SERIES

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1. Definition

An infinite series is an expression of the form

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots$$

It is better defined as the limit of a sequence $\{s_k\}$ whose terms are given by

$$s_k = \sum_{n=1}^{k} a_n$$

That is, $\sum_{n=1}^{\infty} a_n = \lim_{k\to\infty} s_k = \lim_{k\to\infty} \sum_{n=1}^{k} a_n$.

1.1. **Repeating decimals.** A repeating decimal is in fact an infinite series. For example, $0.\overline{9} = 0.999... = 0.9 + 0.09 + 0.009 + \cdots = \sum_{n=1}^{\infty} \frac{9}{10^n}$. To evaluate this series, express it as the infinite limit of a finite sum:

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{9}{10^n} &= \lim_{k\to\infty} \left(\sum_{n=1}^{k} \frac{9}{10^n} \right) \\ &= \lim_{k\to\infty} \left(1 - \frac{1}{10^k} \right) \\ &= 1 - 0 = 1\end{aligned}$$