Homework 1 solutions

Academic integrity

I used the mathematics stackexchange (math.stackexchange.com) to help with some questions. If I asked a question on the site, I only did so after spending some time on the problem and making some progress, I also made it clear that it was for a homework question, and asked that I not be given the full solution but only a hint in the right direction.

(1.4.6)

Half of the cards are even, and half are blue, so $P(A) = P(B) = \frac{1}{2} = \frac{5}{10}$. 40% of the cards have a number less than 5, so $P(C) = \frac{2}{5} = \frac{4}{10}$.

1. All the cards that are both blue and whose number is even and less than 5: $A \cap B \cap C = \{2b, 4b\}$.
2. All the cards that are blue and whose number is greater than or equal to 5: $B \cap C^{\complement} = \{5b, 6b, 7b, 8b, 9b, 10b\}$.
3. All the cards that are either blue or whose number is either even or less than 5, non-exclusively: $\{1b, 2b, 3b, 4b, 5b, 6b, 7b, 8b, 9b, 10b, 1r, 2r, 3r, 4r, 6r, 8r, 10r\}$.
4. All the cards whose number is even and whose colour is either blue or whose number is less than 5: $\{2b, 4b, 6b, 8b, 10b, 2r, 4r\}$.
5. All the cards that are both not blue and whose number is odd and greater than or equal to 5: $\{5r, 7r, 9r\}$.

(1.4.7)

1. $\{x|x < 1, x > 5\}$.
2. $\{x|1 \leq x \leq 7\}$.
3. $\{x|3 < x \leq 7\}$.
4. $\{x|0 < x < 1, x > 7\}$.
5. $\varnothing$.

(1.5.4)

Let A be the event that student A fails the exam and B be the event that student B fails it. Then, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.2 - 0.1 = 0.6$.

(1.5.5)

$P((A \cup B)^{\complement}) = 1 - P(A \cup B) = 1 - 0.6 = 0.4$.

(1.5.8)

Let M be event that a family subscribes to the morning paper, and A be the event that a familiy subscribes to the afternoon paper. Then, $P(M \cap A) = P(M) + P(A) - P(M \cup A) = 0.5 + 0.65 - 0.85 = 0.3$.

(1.6.6)

Let $H_i = 0.5$ be the event that coin i lands heads-up, and $T_i = H_i^{\complement} = 0.5$ be the event that coin i lands tails-up. The three coins are independent of each other, and the two cases of all the coins landing heads-up and all the coins landing tails up are mutually exclusive, therefore $P((H_1 \cap H_2 \cap H_3) \cup (T_1 \cap T_2 \cap T_3)) = P(H_1 \cap H_2 \cap H_3) + P(T_1 \cap T_2 \cap T_3) = P(H_1) \cdot P(H_2) \cdot P(H_3) + P(T_1) \cdot P(T_2) \cdot P(T_3) = 0.5^3 + 0.5^3 = 0.25$.

(1.7.6)

The total number of possible ways 6 dice can roll is 6^6. The total number of ways 6 numbers can be arranged on 6 dice is 6!. Since all the outcomes have equal probability, the answer is $\frac{6!}{6^6} = 0.0154$.

(1.7.8)

The total number of ways 5 passengers can get off at 7 different floors is 7^5. The total number of permutations of 5 passengers getting off at 5 different floors out of 7 is $P_5^7 = P_2^7 = 2520$. Since the passengers are equally likely to get off at any floor and do so independently of each other, the probability of the latter event occurring is $\frac{2520}{7^5} = 0.150$.

(1.8.6)

The total amount of ways n people can be arranged in n seats is $n!$. The total amount of ways this can happen with A and B next to each other can be calculated by thinking of A and B as one person AB, taking up one seat, and then multiplying by two, since they can also be next to each other the other way around (BA) in equally many ways: $2 \cdot (n-1)!$. Since all events are equally likely to occur, we have the probability of A and B being next to each other as $\frac{2 \cdot (n-1)!}{n!} = \frac{2}{n}$.

(1.8.10)

The amount of combinations of 10 lightbulbs that can be selected from a box of 24 is $C_{10}^{24} = 1961256$. The amount of such combinations that include the two defective lightbulbs can be calculated as follows: assume both of the defective light bulbs has already been selected, and calculate the amount of combinations of the remaining 8 being selected, i.e. $C_8^{22} = 319770$. Since the lightbulbs are selected at random, the probability of the two defective lightbulbs being selected is $\frac{319770}{1961256} = 0.163$.

(1.8.18)

~~The total number of ways the awards can be distributed is $C_{10}^{100} = \frac{100!}{10! \cdot 90!}$. The total number of ways two awards can be distributed among one class of 20 is $a = C_2^{20} = 190$. There are 10 awards, and the total number of ways 10 awards can be split up into groups of two can be calculated by modeling this as first taking two awards, then another two, and then another until there are none left. Therefore, this number is $b = \frac{C_2^{10} \cdot C_2^8 \cdot C_2^6 \cdot C_2^4 \cdot C_2^2}{5!} = \frac{45 \cdot 28 \cdot 15 \cdot 6 \cdot 1}{120} = 945$ (divide by 5! since the ordering of the 5 groups doesn't matter). Since there are 5 classes, the total number of ways the awards can be distributed with 2 awards per class is $a^5 \cdot b = 28078962660000000$. Since all of the distributions occur with equal probability, the probability of the former happening is~~

Unfortunately I couldn't get this question...

(1.9.2)

$\frac{50!}{18! \cdot 12! \cdot 8! \cdot 12!} = 51349876166812729593087000 0$.

(1.9.4)

The total number of ways of arranging three *s*s, three *t*s, two *i*s, one *a* and one *c* is $\frac{10!}{3!\cdot 3!\cdot 2!}$. The number of ways the letters can form *statistics* is $3!\cdot 3!\cdot 2!$, since the identical letters can be arranged in any order. Therefore, the probability is $\frac{1}{10!} = \frac{1}{3628800}$.

(1.10.6)

There are C_{10}^{90} ways to select ten balls out of the 90 that are there. There are $3 \cdot C_{10}^{60}$ ways to select ten balls of two colours; the multiplicative factor is to account for the fact that there are three combinations of two colours out of three. There are also $3 \cdot C_{10}^{30}$ ways to select ten balls of one colour; again, the multiplicative factor is to account for the fact that there are three colours. Therefore, since all combinations have an equal probability of happening, the probability of one colour missing is $\frac{C_{10}^{90}-3\cdot C_{10}^{60}-3\cdot C_{10}^{30}}{C_{10}^{90}} = \frac{5720645481903-226182082698-90135045}{5720645481903} = 0.960$.