Finite sample space

To define a probability function $\mathbb{P}$ on a set of events $S = \{s_1, s_2 \cdots s_n\}$ is to define a set of numbers $\{p_1, p_2 \cdots p_n\}$ such that $p_i = \mathbb{P}(\{s_i\})$, and:

1. $0 \leq p_i \leq 1 \; \forall \; 1 \leq i \leq n$
2. $\sum_{i=1}^{n} p_i = 1$

If $A \subset S$, then $\mathbb{P}(A)$ evaluates to the **sum** of all the p_i values associated with the outcomes s_i that make up A.

For example, take the following experiment. We have 5 fibers of lengths 1, 2, 3, 4 and 5. We want to see what fiber takes the least time to break at a certain tensile stress. Then, our set of outcomes S has 5 members: one for each fiber length. An example of A, the subset event, is "the fiber that breaks first is no longer than two". What is the value of $\mathbb{P}(A)$?

We can model our experiment by making the assumption of proportionality: that $p_i = \alpha i$, for some $\alpha > 0$. Then we can calculate our constant:

$$1 = \sum_{i=1}^{5} p_i = \sum_{i=1}^{5} \alpha i = \alpha(1 + 2 + 3 + 4 + 5) = 15\alpha$$

Thus we know that $\alpha = \frac{1}{15}$. We also know, by definition, that $\mathbb{P}(A) = \mathbb{P}(\{s_1, s_2\}) = \mathbb{P}(\{s_1\} \cup \{s_2\}) = \mathbb{P}(\{s_1\}) + \mathbb{P}(\{s_2\}) = \frac{1}{15} + \frac{2}{15} = \frac{3}{15}$.

If $A \subset S$, S consists of *equally likely outcomes* and $|A| = m, |S| = n$, then $\mathbb{P}(A) = \frac{m}{n}$.