Multiple Signal Classification (MUSIC) Algorithm for Time Delay Estimation

Multiple Signal Classification (MUSIC) is a classic algorithm for both angular and time delay estimations in wireless channels. In a previous article, I have presented the MUSIC algorithm for angular information estimation. In this article, I will describe the MUSIC algorithm for the time delay estimation in a multi-carrier communication systems. The simulation codes will be attached for check.

I. Multi-Carrier Channel Model

We consider a single-input-single-output (SISO) multi-carrier system where a transmitter sends pilot signal x to the receiver at N subcarrier. The central frequency at subcarrier $n \in \{1, 2, \dots, N\}$ is denoted by $f_c + (n-1)\Delta f$, where Δf is the interval between two neighbouring subcarriers. For ease of notations, we focus on the baseband signal and replace the central frequency $f_c + (n-1)\Delta f$ by $(n-1)\Delta f$.

The time-domain of a multi-path channel model is expressed as

$$h(t) = \sum_{l=1}^{L} a_l \delta(t - \tau_l), \tag{1}$$

where L is the number of paths in the channel, a_l and $\tau_l)$ are the attenuation and time delay of the path l, respectively. Along with wireless channel in (1), the received signal in time domain is given by

$$y(t) = \sum_{l=1}^{L} a_l \delta(t - \tau_l) * x(t), \tag{2}$$

where $*$ is the operator for convolution. Conducting the Fourier transformation, we can transform the time domain signal in (2) into frequency domain expression given by

$$Y(f) = X(f) \sum_{l=1}^{L} a_l e^{-j2\pi f \tau_l}. \tag{3}$$

According to (3), the received frequency domain signals at all subcarriers with central frequency $f = (n-1)\Delta f, n = 1, 2, \dots, N$ is given by

$$\mathbf{y} = \begin{bmatrix} Y(f_1) \\ Y(f_2) \\ \vdots \\ Y(f_N) \end{bmatrix} = \underbrace{\begin{bmatrix} a_1 & a_2 & \dots & a_L \\ a_1 e^{-j2\pi\Delta f \tau_1} & a_2 e^{-j2\pi\Delta f \tau_2} & \dots & a_L e^{-j2\pi\Delta f \tau_L} \\ \vdots & & \dots & \\ a_1 e^{-j2\pi(N-1)\Delta f \tau_1} & a_2 e^{-j2\pi(N-1)\Delta f \tau_2} & \dots & a_L e^{-j2\pi(N-1)\Delta f \tau_L} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} X(f_1) \\ X(f_2) \\ \vdots \\ X(f_N) \end{bmatrix}}_{\mathbf{x}} + \mathbf{n}, \tag{4}$$

where $\mathbf{n} \in \mathcal{C}^{N \times 1}$ is the noise with norm distribution $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{N \times N})$.

II. MUSIC Algorithm

According to (4), the received signal can be expressed as

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}, \tag{5}$$

which has the same structure with the (5) in [1]. Hence, one directly reads the analysis in [1] to understand the MUSIC algorithm form time delay estimation. Details are omitted and once can check the [1].

REFERENCES

[1] GitHub, “MUSIC Algorithm for AoA Estimation,” https://github.com/Bluegyfrys/Wireless-Communication-Techniques/tree/main/MUSIC%20Algorithm%20for%20AoA%20Estimation.