Multiple Signal Classification (MUSIC) Algorithm for Time Delay Estimation

In a previous article, I presented the Multiple Signal Classification (MUSIC) for angular estimation in wireless channels. In this article, the MUSIC algorithm is extended for the time delay estimation in a multi-carrier communication system. The simulation codes is same to the Matlab codes for angular estimation.

I. Multi-Carrier Channel Model

We consider a single-input-single-output (SISO) multi-carrier system where a transmitter sends pilot signal x to the receiver at N subcarriers. The system bandwidth is B, and the central frequency at subcarrier $n \in \{1, 2, \ldots, N\}$ is denoted by $f_c + (n-1)\Delta f$, where $\Delta f = B/N$. For ease of notations, we focus on the baseband signal and replace the central frequency $f_c + (n-1)\Delta f$ by $(n-1)\Delta f$.

The time-domain of a multi-path channel model is expressed as

$$h(t) = \sum_{l=1}^{L} a_l \delta(t - \tau_l), \tag{1}$$

where L is the number of paths, a_l and τ_l) are the attenuation and time delay of the path l, respectively. Along with the wireless channel in (1), the time domain received signal is given by

$$y(t) = \sum_{l=1}^{L} a_l \delta(t - \tau_l) * x(t), \tag{2}$$

where $*$ is the convolution operator. Conducting Fourier transformation, we can transform the time domain signal in (2) into its frequency domain expression, given by

$$Y(f) = X(f) \sum_{l=1}^{L} a_l e^{-j2\pi f \tau_l}. \tag{3}$$

According to (3), the frequency domain received signals at all subcarriers with central frequency $f_n = (n-1)\Delta f, n = 1, 2, \ldots, N$ is given by

$$\mathbf{y}=\begin{bmatrix} Y(f_1) \\ Y(f_2) \\ \vdots \\ Y(f_N) \end{bmatrix} = \underbrace{\begin{bmatrix} a_1 & a_2 & \ldots & a_L \\ a_1 e^{-j2\pi\Delta f \tau_1} & a_2 e^{-j2\pi\Delta f \tau_2} & \ldots & a_L e^{-j2\pi\Delta f \tau_L} \\ \vdots & & \ldots & \\ a_1 e^{-j2\pi(N-1)\Delta f \tau_1} & a_2 e^{-j2\pi(N-1)\Delta f \tau_2} & \ldots & a_L e^{-j2\pi(N-1)\Delta f \tau_L} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} X(f_1) \\ X(f_2) \\ \vdots \\ X(f_N) \end{bmatrix}}_{\mathbf{x}} + \mathbf{n}, \tag{4}$$

where $X(f_n)$ is the symbol on subcarrier n, $\mathbf{n} \in \mathcal{C}^{N\times 1}$ is the noise with norm distribution $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{N\times N})$.

II. MUSIC Algorithm

According to (4), the frequency domain received signal is given by

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}, \tag{5}$$

which has the same structure with the (5) in [1]. Hence, one directly reads the analysis in [1] to understand the MUSIC algorithm form time delay estimation. Details are omitted and once can check the [1].

REFERENCES

[1] GitHub, "MUSIC Algorithm for AoA Estimation," https://github.com/Bluegyfrys/Wireless-Communication-Techniques/tree/main/MUSIC%20Algorithm%20for%20AoA%20Estimation.