Notes on Hydro&CME&HIC

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1 Hydrodyanmics

We start by Minkowski metric,

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + \,\mathrm{d}z^2 + \,\mathrm{d}x_\perp^2, \tag{1}$$

where z is the direction of the moving particle, and $x_\perp$ denotes perpendicular coordinates i.e, $\mathrm{d}x_\perp^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2$. It is convenient to introduce proper time (τ) and rapidity (η) coordinates in the longitudinal position plane:

$$\begin{cases} t = \tau \cosh \eta \\ z = \tau \sinh \eta, \end{cases} \tag{2}$$

in this coordinates,

$$\mathrm{d}s^2 = -\,\mathrm{d}\tau^2 + \tau^2\,\mathrm{d}\eta^2 + \,\mathrm{d}x_\perp^2, \tag{3}$$

the covariant and contra-variant metric components are given by:

$$\begin{cases} g_{\mu\nu} = \mathrm{diag}\{-1, \tau^2, 1, 1\} \\ g^{\mu\nu} = \mathrm{diag}\{-1, \tau^{-2}, 1, 1\}, \end{cases} \tag{4}$$

then we could find that the only non-vanishing component containing the derivative is

$$g_{\eta\eta,\tau} = 2\tau, \tag{5}$$

thus the Christoffel symbol has only three terms left, which are

$$\Gamma^{\eta}{}_{\eta\tau} = \Gamma^{\eta}{}_{\tau\eta} = \frac{1}{2} g^{\eta\eta} g_{\eta\eta,\tau} = \frac{1}{\tau}, \qquad \Gamma^{\tau}{}_{\eta\eta} = \frac{1}{2} g^{\tau\tau}(-g_{\eta\eta,\tau}) = \tau. \tag{6}$$

In this coordinates we expand the covariant form of the conservation law of the energy-momentum tensor as

$$0 = D_\nu T^{\mu\nu} = \partial_\nu T^{\mu\nu} + \Gamma^{\mu}{}_{\nu\alpha} T^{\alpha\nu} + \Gamma^{\nu}{}_{\nu\alpha} T^{\mu\alpha}. \tag{7}$$

Here and after we assume that $T^{\mu\nu}$ has only the following terms: $T^{\tau\tau}$, $T^{\eta\eta}$, and $T^{x_1x_1} = T^{x_2x_2} = T^{xx}$ (, which holds well in the case of perfect fluid).
We take $\mu = \tau$ and get

$$0 = \partial_\tau T^{\tau\tau} + \Gamma^{\tau}{}_{\eta\eta} T^{\eta\eta} + \Gamma^{\eta}{}_{\eta\tau} T^{\tau\tau} = \frac{\partial}{\partial\tau} T^{\tau\tau} + \tau T^{\eta\eta} + \frac{1}{\tau} T^{\tau\tau}, \tag{8}$$

or equivalently, we get the first constraint as

$$\tau \frac{\partial}{\partial\tau} T^{\tau\tau} + \tau^2 T^{\eta\eta} + T^{\tau\tau} = 0, \tag{9}$$

and then take $\mu = \eta$ to get

$$0 = \partial_\eta T^{\eta\eta} + \Gamma^{\eta}{}_{\eta\tau} T^{\tau\eta} + \Gamma^{\nu}{}_{\nu\eta} T^{\eta\eta} = \partial_\eta T^{\eta\eta}, \tag{10}$$

which means $T^{\eta\eta}$ does not depend on η.
Moreover, the traceless condition is given by

$$g_{\mu\nu} T^{\mu\nu} = 0, \tag{11}$$

from which we get the second constraint as

$$-T^{\tau\tau} + \tau^2 T^{\eta\eta} + 2T^{xx} = 0. \tag{12}$$

From the above two constraints we can express $T^{\eta\eta}$ and T^{xx} in terms of $T^{\tau\tau}$,

2 Chiral Magnetic Effect

2.1 Micellaneous

Chirality depends on the reference frame.

3 Heavy Ion Collisions

The Fourier series expansion is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos nx + b_n \sin nx\right], \tag{13}$$

where the basis $\cos nx$ s and $\sin nx$ s are orthogonal under the relation,

$$\langle f|g\rangle = \int_0^{2\pi} f^*(x) g(x)\,\mathrm{d}x, \tag{14}$$

and the normalization integrals are given by

$$\begin{aligned} \langle \sin nx| \sin nx\rangle &= \pi \\ \langle \cos nx| \cos nx\rangle &= (1+\delta_{0n})\pi. \end{aligned} \tag{15}$$

From the fundamental relations

$$|f\rangle = \sum c_m |\phi_m\rangle \implies c_i = \frac{\langle \phi_i|f\rangle}{\langle \phi_i|\phi_i\rangle}, \tag{16}$$

we can extract the Fourier coefficients as

$$\begin{cases} a_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} f(x)\cos nx\,\mathrm{d}x, \quad n = 0, 1, 2, ..., \\ b_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} f(x)\sin nx\,\mathrm{d}x, \quad n = 1, 2, ..., \end{cases} \tag{17}$$

the factor 1/2 attached to a_0 allows the formula for an to apply without change for $n = 0$.

Now we apply it to the background particle azimuthal distributions $\frac{\mathrm{d}\langle N_\pm\rangle}{\mathrm{d}\phi}$, but before that, we can drop the sin terms due to the reflection symmetry of the reaction plane, and then we make a little modification to the original equation as,

$$\frac{\mathrm{d}\langle N_\pm\rangle}{\mathrm{d}\phi} = \frac{\langle N_\pm\rangle}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos n(\phi - \Psi_{RP})\right], \tag{18}$$

where $v_n = a_n/a_0$ and a_0 is easy to be verified as

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} \frac{\mathrm{d}\langle N_\pm\rangle(\phi)}{\mathrm{d}\phi}\,\mathrm{d}\phi = \frac{1}{\pi}(\langle N_\pm\rangle(2\pi) - \langle N_\pm\rangle(0)) = \frac{\langle N_\pm\rangle}{\pi}. \tag{19}$$

Now to construct the total particle azimuthal distribution, we add a CME term, which should be symmetric about the $\vec{B}$ field, orthogonal toe the reaction plane, taking a simple ansatz as a sin term.

$$\frac{\mathrm{d}N_\pm}{\mathrm{d}\phi} = \frac{\langle N_\pm\rangle}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos n(\phi - \Psi_{RP})\right] + \frac{1}{4}\Delta_\pm \sin(\phi - \Psi_{RP}) \tag{20}$$