Landau's clock synchronization

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Let us write the interval, separating the space and time coordinates:

$$\mathrm{d}s^2 = g_{00}\,\mathrm{d}t^2 + 2g_{0i}\,\mathrm{d}t\,\mathrm{d}x^i + g_{ij}\underset{\cdot}{\mathrm{x}}^i\underset{\cdot}{\mathrm{x}}^j \tag{1}$$

For null signals, $ds^2 = 0$:

$$0 = g_{00}\,\mathrm{d}t^2 + 2g_{0i}\,\mathrm{d}t\,\mathrm{d}x^i + g_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j \tag{2}$$

Solving the quadratic equation for $\mathrm{d}t$:

$$\mathrm{d}t = \frac{-g_{0i}\,\mathrm{d}x^i \pm \sqrt{(g_{0i}g_{0j} - g_{00}g_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j}}{g_{00}} \tag{3}$$

The two roots are:

$$\begin{aligned} \mathrm{d}t_1 &= \frac{-g_{0i}\,\mathrm{d}x^i + \sqrt{(g_{0i}g_{0j} - g_{00}g_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j}}{g_{00}}, \\ \mathrm{d}t_2 &= \frac{-g_{0i}\,\mathrm{d}x^i - \sqrt{(g_{0i}g_{0j} - g_{00}g_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j}}{g_{00}} \end{aligned} \tag{4}$$

We should regard as simultaneous with the moment t at the point A that thereading of the clock at B which is halfway between the moments of departure and return of the signal to that point, i.e. the moment

$$t + \mathrm{d}t = t + \frac{1}{2}(\mathrm{d}t_1 + \mathrm{d}t_2). \tag{5}$$

Substituting (**??**), we thus find that the difference in the values of the "time" t for two simultaneous events occurring at infinitely near points is given by

$$\mathrm{d}t = \frac{-g_{0i}\,\mathrm{d}x^i}{g_{00}} \tag{6}$$

And this is equivalent of saying that "the 'covariant differential' $\mathrm{d}x_0$ between two infinitely near simultaneous events must be zero":

$$g_{00}\,\mathrm{d}x^0 + g_{0i}\,\mathrm{d}x^i \equiv \mathrm{d}x_0 = 0 \tag{7}$$

This relation enables us to synchronize clocks in any infinitesimal region of space.

However, sychronization of clocks along a closed contour turns out to be impossible unless the following relation can be satisfied:

$$\oint \mathrm{d}t = \oint \left(-\frac{g_{0i}}{g_{00}} \right) \mathrm{d}x^i = 0 \tag{8}$$

or its sufficient condition is found:

$$g_{0i} = 0 \tag{9}$$

This is Landau's theory of clock synchronization.