

#3.

P	q	r	$P \rightarrow r$	$q \rightarrow r$	$P \wedge q$	$(P \rightarrow r) \vee (q \rightarrow r)$	$(P \wedge q) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

#4.

Let $P(x)$ is " x is a positive number", $Q(x)$ is " x is a negative number", the domain U are nonzero integers. In $\forall x P(x) \vee \forall x Q(x)$, what $\forall x P(x)$ means to determine if x is a positive number about nonzero integers. If $\forall x P(x)$ is true, all values are positive numbers. But nonzero integers include negative numbers. So $\forall x P(x)$ has false value. $\forall x Q(x)$ also has false value because nonzero integers include positive numbers. In conclusion, Truth value of $\forall x P(x) \vee \forall x Q(x)$ is false.

Next, Let's look at the truth value of $\forall x (P(x) \vee Q(x))$ under the same condition. $\forall x (P(x) \vee Q(x))$ means that x is positive or negative number about nonzero integers. At this point, whether x is positive or negative number, $P(x) \vee Q(x)$ is true by disjunction. In conclusion, $P(x) \vee Q(x)$ is true about nonzero integers. So $\forall x (P(x) \vee Q(x))$ is true.

If $\forall x P(x) \vee \forall x Q(x)$ and $\forall x (P(x) \vee Q(x))$ are logically equivalent, these have to be same under the same condition. But these are not equal. So two expressions are not logically equivalent.