SM_Assignment4, 2021 35592 한웅재

#1 $u+v=\begin{bmatrix}-4\\1\end{bmatrix}$

$u-2v=\begin{bmatrix}-1\\2\end{bmatrix}-2\begin{bmatrix}-3\\-1\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}+\begin{bmatrix}6\\2\end{bmatrix}=\begin{bmatrix}5\\4\end{bmatrix}$

#2 4 vectors are in 2-dimensional space.

A necessary condition to be independence is at most n-vectors in n-dimensional vector.

this problem is linearly dependent.

#3

1.) $[-1\ \ 4\ \ -3]\begin{bmatrix}5\\2\\1\end{bmatrix}=-5+8-3=0$

$[-1\ \ 4\ \ -3]\begin{bmatrix}3\\-4\\-7\end{bmatrix}=-3-16+21=2$

$[5\ \ 2\ \ 1]\begin{bmatrix}3\\-4\\-7\end{bmatrix}=15-8-7=0$

$\begin{bmatrix}-1\\4\\-3\end{bmatrix}$ is orthogonal with $\begin{bmatrix}5\\2\\1\end{bmatrix}$ and $\begin{bmatrix}3\\-4\\-7\end{bmatrix}$

3.) $[2\ \ -7\ \ -1]\begin{bmatrix}-6\\-3\\9\end{bmatrix}=-12+21-9=0$

$[2\ \ -7\ \ -1]\begin{bmatrix}3\\1\\-1\end{bmatrix}=6-7+1=0$

$[-6\ \ -3\ \ 9]\begin{bmatrix}3\\1\\-1\end{bmatrix}=-18-3-9=-30$

$\begin{bmatrix}2\\-7\\-1\end{bmatrix}$ is orthogonal with $\begin{bmatrix}-6\\-3\\9\end{bmatrix}$ and $\begin{bmatrix}3\\1\\-1\end{bmatrix}$

#4

17.) $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} = -\frac{1}{6} + 0 + \frac{1}{6} = 0$

$\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = u$, $\begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} = v$

$\|u\| = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{1}{9}} = \sqrt{\frac{1}{3}}$

$\|v\| = \sqrt{\frac{1}{4} + 0 + \frac{1}{4}} = \sqrt{\frac{1}{2}}$

So u and v are orthogonal but not orthonormal.

$\frac{1}{\|u\|} = \sqrt{3}$, $\frac{1}{\|v\|} = \sqrt{2}$

$\frac{1}{\|u\|} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \frac{u}{\|u\|}$, $\left\| \frac{u}{\|u\|} \right\| = \frac{1}{\|u\|} \times \|u\| = \sqrt{3} \times \sqrt{\frac{1}{3}} = 1$

$\frac{1}{\|v\|} \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} = \frac{v}{\|v\|}$, $\left\| \frac{v}{\|v\|} \right\| = \frac{1}{\|v\|} \times \|v\| = \sqrt{2} \times \sqrt{\frac{1}{2}} = 1$

normalizing u and v. $\rightarrow$ $\sqrt{3} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$, $\sqrt{2} \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}$

18.) $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = 0 - 1 + 0 = -1$

It isn't orthogonal. so It isn't also orthonormal