SM_Assignment 3, 202135592 한웅재

1)

a) 11 and 19 are prime. $15 = 3\times5$. so their gcd is 1. so 11, 15, 19 are coprime.

b) 14, 15, 21 are coprime. $14 = 2\times7$, $21 = 3\times7$, $15 = 3\times5$. so their gcd is 1.

c) 12, 17, 31, 37 are coprime. 17, 31, 37 are prime. $12 = 2\times6$. so their gcd is 1.

d) 7, 8, 9, 11 are coprime. 7, 11 are prime. $8 = 2^3$, $9 = 3^2$. so their gcd is 1.

2)

- $1000 = 2^3 \times 5^3 \xrightarrow{\times 5} 2^3\times5^4 = 5000$
- $625 = 2^0 \times 5^4 \xrightarrow{\times 2^3} 2^3\times5^4 = 5000$

$gcd(1000,625) = 2^0\times5^3 = 125$

$lcm(1000,625) = 2^3\times5^4 = 5000$

$1000\times625 = 2^3\times5^3\times2^0\times5^4 = 2^3\times5^7$

$gcd(1000,625)\times lcm(1000,625) = 2^0\times5^3\times2^3\times5^4 = 2^3\times5^7$

so $gcd(1000,625)\times lcm(1000,625) = 1000\times625$

#3)

a) $18 = 12\times1+6$

$12 = 6\times2+0$

$\gcd(12,18) = 6$

b) $201 = 111\times1+90$

$111 = 90\times1+21$

$90 = 21\times4+6$

$21 = 6\times3+3$

$6 = 3\times2+0$

$\gcd(111,201) = 3$

c) $1331 = 1001\times1+330$

$1001 = 330\times3+11$

$330 = 11\times30+0$

$\gcd(1001,1331) = 11$

d) $54321 = 12345\times4+4941$

$12345 = 4941\times2+2463$

$4941 = 2463\times2+15$

$2463 = 15\times164+3$

$15 = 3\times5+0$

$\gcd(12345,54321) = 3$

e) $5040 = 1000\times5+40$

$1000 = 40\times25+0$

$\gcd(1000,5040) = 40$

f) $9888 = 6060\times1+3828$

$6060 = 3828\times1+2232$

$3828 = 2232\times1+1596$

$2232 = 1596\times1+636$

$1596 = 636\times2+324$

$636 = 324\times1+312$

$324 = 312\times1+12$

$312 = 12\times26+0$

$\gcd(9888,6060) = 12$

#4)

- Basis step

$P(n) = n(n+1)(n+2)(n+3)/4$

$P(1) = 1\cdot(2)(3)(4)/4 = 1\times2\times3$, so $P(1)$ is true

- Inductive step

Assume true for $P(n)$.

$$1\cdot2\cdot3 + 2\cdot3\cdot4 + \cdots + n(n+1)(n+2) + \{(n+1)(n+2)(n+3)\} = P(n) + \{(n+1)(n+2)(n+3)\}$$

$$= \frac{n(n+1)(n+2)(n+3)}{4} + \{(n+1)(n+2)(n+3)\}$$

$$= \frac{n(n+1)(n+2)(n+3) + 4(n+1)(n+2)(n+3)}{4}$$

$$= \frac{(n+1)(n+2)(n+3)(n+4)}{4}$$

$$= P(n+1) = (n+1)(n+2)(n+3)(n+4)/4$$

$P(n+1)$ is also true. so it is true for every positive integer.

#5

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procedure oddsum(n: positive integer, sum = 0)
if n > 0 then
    if n % 2 = 0 then
        return oddsum(n-1)
    else
        sum := sum + n
        return oddsum(n-1)
else
    return sum
```