

#1.

a)
$$\begin{array}{r} 101110 \\ 0100001 \\ \hline \text{OR } 111111 \\ \text{AND } 000000 \\ \text{XOR } 111111 \end{array}$$

b)
$$\begin{array}{r} 11110000 \\ 10101010 \\ \hline \text{OR } 11111010 \\ \text{AND } 10100000 \\ \text{XOR } 01011010 \end{array}$$

c)
$$\begin{array}{r} 0001110001 \\ 1001001000 \\ \hline \text{OR } 1001111001 \\ \text{AND } 0001000000 \\ \text{XOR } 1000111001 \end{array}$$

d)
$$\begin{array}{r} 11111111 \\ 00000000 \\ \hline \text{OR } 11111111 \\ \text{AND } 00000000 \\ \text{XOR } 11111111 \end{array}$$

#2.

P	q	$\neg P$	$\neg q$	$P \wedge q$	$P \vee q$	$P \rightarrow q$	$\neg(P \rightarrow q)$
T	T	F	F	T	T	T	F
T	F	F	T	F	T	F	T
F	T	T	F	F	T	T	F
F	F	T	T	F	F	T	F

Application

d) $(P \wedge q) \rightarrow P$

T	\rightarrow	T
F	\rightarrow	T
F	\rightarrow	F
F	\rightarrow	F

b) $P \rightarrow (P \vee q)$

T	\rightarrow	T
T	\rightarrow	T
F	\rightarrow	T
F	\rightarrow	F

c) $\neg P \rightarrow (P \rightarrow q)$

F	\rightarrow	T
F	\rightarrow	F
T	\rightarrow	T
T	\rightarrow	T

d) $(P \wedge q) \rightarrow (P \rightarrow q)$

T	\rightarrow	T
F	\rightarrow	F
F	\rightarrow	T
F	\rightarrow	T

e) $\neg(P \rightarrow q) \rightarrow P$

F	\rightarrow	T
T	\rightarrow	T
F	\rightarrow	F
F	\rightarrow	F

f) $\neg(P \rightarrow q) \rightarrow \neg q$

F	\rightarrow	F
T	\rightarrow	T
F	\rightarrow	F
F	\rightarrow	T

#3.

P	q	r	$P \rightarrow r$	$q \rightarrow r$	$P \wedge q$	$(P \rightarrow r) \vee (q \rightarrow r)$	$(P \wedge q) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

#4.

Let $P(x)$ is " x is a positive number", $Q(x)$ is " x is a negative number", the domain U are nonzero integers. In $\forall x P(x) \vee \forall x Q(x)$, what $\forall x P(x)$ means to determine if x is a positive number about nonzero integers. If $\forall x P(x)$ is true, all values are positive numbers. But nonzero integers include negative numbers. So $\forall x P(x)$ has false value. $\forall x Q(x)$ also has false value because nonzero integers include positive numbers. In conclusion, Truth value of $\forall x P(x) \vee \forall x Q(x)$ is false.

Next, Let's look at the truth value of $\forall x (P(x) \vee Q(x))$ under the same condition. $\forall x (P(x) \vee Q(x))$ means that x is positive or negative number about nonzero integers. At this point, whether x is positive or negative number, $P(x) \vee Q(x)$ is true by disjunction. In conclusion, $P(x) \vee Q(x)$ is true about nonzero integers. So $\forall x (P(x) \vee Q(x))$ is true.

If $\forall x P(x) \vee \forall x Q(x)$ and $\forall x (P(x) \vee Q(x))$ are logically equivalent, these have to be same under the same condition. But these are not equal. So two expressions are not logically equivalent.

#5.

- a) some student has sent an e-mail message to some student.
- b) some student has sent an email message to Every student.
- c) Every student has sent an e-mail message to some student.
- d) An e-mail message has been sent to some student by Every student.
- e) An e-mail message has been sent to every student by some student.
- f) Every student has sent an e-mail message to every student.

6.

- a) π, y are positive real numbers. Some y belong to $0 < y < \pi$ for some number π , for every number y . when square of some y in $0 < y < \pi$ are smaller than π in $y < \pi$, Many y exist under the same conditions. For example, when $\pi = 4, y = 2$, it satisfies $\pi \leq y^2$. but if the range of y is $0 < y < 2$, these don't satisfy $\pi \leq y^2$. so $\exists x \forall y (\pi \leq y^2)$ is false.
- b) If $\pi = 0$, y^2 is always same or greater than 0. $\exists x \forall y (\pi \leq y^2)$ is true.
- c) If $\pi < 0$, y^2 is always greater than 0. $\exists x \forall y (\pi \leq y^2)$ is true.