

软件工程上机报告

问题描述

In a box bounded by $[-1, 1]$, given m balloons(they cannot overlap) with variable radio r and position mu , find the optimal value of r and mu which maximize $\sum r^2$

问题分析

可以在方框的空白部分寻找最大的空白，然后用最大的圆进行填充，如图所示：

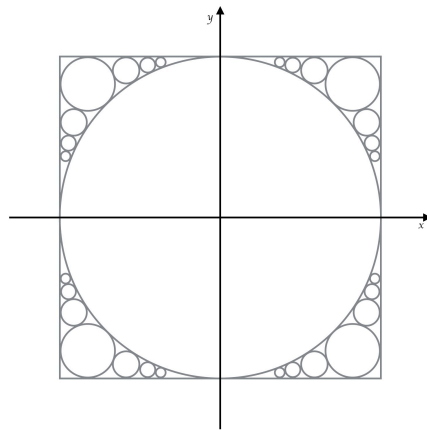


图1

我们按上述方案从大到小依次填满空隙，但是可能会出现下面的情况：

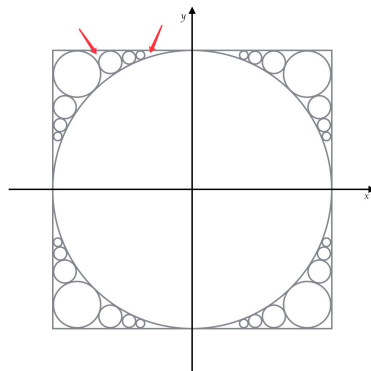


图2

箭头表明的部分右边的可能会出现比左箭头小的情况，这种情况由于计算过于复杂，所以我们寻求次优解，将圆按图1的方式填满

图1由大到小的顺序顺序依次填，可以分为多种圆，各种圆的个数分别为1, 4, 8, 8, 8, ...

第一个圆的半径为 $R_1 = r_1 = 1$ ，第二总圆的半径为 $R_2 = r_2 = r_3 = r_4 = r_5 = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

当圆的类别大于三时，可以用一下公式计算半径

$$\begin{cases} R_n = \left(\frac{1-C_{n-1}}{2(1+R_{n-1})} \right)^2, n = 3, 4, 5, \dots \\ C_n = \sum_{i=2}^{n-1} 2\sqrt{R_i R_{i-1}} + R_2, n = 3, 4, 5, \dots \\ C_2 = R_2 \\ r_{5+8(n-3)+i} = R_n, i = 1, 2, \dots, 8, n = 3, 4, 5, \dots \end{cases}$$

测试用例：

$m = 1, R_1 = 1$

$m = 2 \ 6, R_1 = 1, R_2 = 3 - 2\sqrt{2}$

$m = 7 \ 15, R_1 = 1, R_2 = 3 - 2\sqrt{2}, R_3 = 0.0857864376 \dots$

$m = 16 \ 24, R_1 = 1, R_2 = 3 - 2\sqrt{2}, R_3 = 0.0857864376 \dots, R_4 = 0.0513207883 \dots$

运行结果：

1	Center: (0.000000, 0.000000)	Radius:1.0000000000
2	Center: (0.828427, 0.828427)	Radius:0.1715728753
3	Center: (0.828427, -0.828427)	Radius:0.1715728753
4	Center: (-0.828427, 0.828427)	Radius:0.1715728753
5	Center: (-0.828427, -0.828427)	Radius:0.1715728753
6	Center: (0.914214, 0.414214)	Radius:0.0857864376
7	Center: (0.414214, 0.914214)	Radius:0.0857864376
8	Center: (0.914214, -0.414214)	Radius:0.0857864376
9	Center: (-0.414214, 0.914214)	Radius:0.0857864376

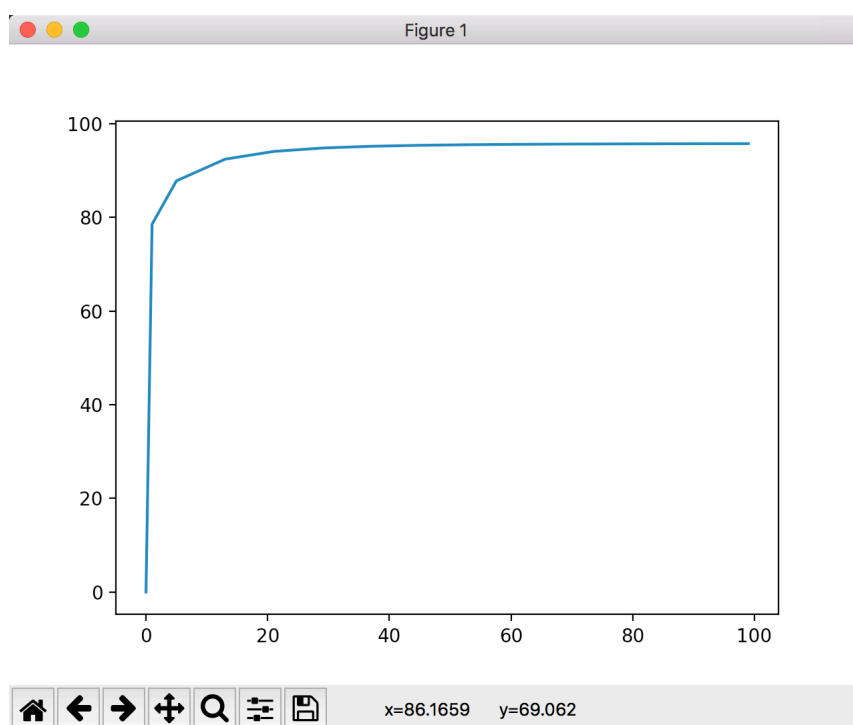
```

1 Center: (-0.914214, 0.414214) Radius:0.0857864376
2 Center: ( 0.414214, -0.914214) Radius:0.0857864376
3 Center: (-0.914214, -0.414214) Radius:0.0857864376
4 Center: (-0.414214, -0.914214) Radius:0.0857864376
5 Center: ( 0.948679, 0.281509) Radius:0.0513207883
6 Center: ( 0.281509, 0.948679) Radius:0.0513207883
7 Center: ( 0.948679, -0.281509) Radius:0.0513207883
8 Center: (-0.281509, 0.948679) Radius:0.0513207883
9 Center: (-0.948679, 0.281509) Radius:0.0513207883
10 Center: ( 0.281509, -0.948679) Radius:0.0513207883
11 Center: (-0.948679, -0.281509) Radius:0.0513207883
12 Center: (-0.281509, -0.948679) Radius:0.0513207883
13 Center: ( 0.965886, 0.197825) Radius:0.0341137321
14 Center: ( 0.197825, 0.965886) Radius:0.0341137321
15 Center: ( 0.965886, -0.197825) Radius:0.0341137321
16 Center: (-0.197825, 0.965886) Radius:0.0341137321
17 Center: (-0.965886, 0.197825) Radius:0.0341137321
18 Center: ( 0.197825, -0.965886) Radius:0.0341137321
19 Center: (-0.965886, -0.197825) Radius:0.0341137321
20 Center: (-0.197825, -0.965886) Radius:0.0341137321
21 Center: ( 0.975694, 0.140235) Radius:0.0243059818
22 ...

```

结论

当m从0增长到99时，计算每个m值下的覆盖率，得出一下函数图像：



从图中可以看出，用此种方案填放并不能达到最优，当 m 达到一定大小时，对覆盖率的提供并不大，空缺的部分就是图2中类似左箭头的部分已经成为主要的空缺部分

附录

Gitlog

```
1 | commit 9e9c441469f16ee13c8797c1fb0817bc846a0eb2
2 | Author: BluesJiang <763400095@qq.com>
3 | Date: Thu Jun 1 11:43:50 2017 +0800
4 |
5 |     fix when  $m < 3$  ,result goes wrong
6 |
7 | commit 834494816cb0729923c505ecef89a2ee0131e7bd
8 | Author: BluesJiang <763400095@qq.com>
9 | Date: Thu Jun 1 10:56:47 2017 +0800
10 |
11 |     bug fixed
12 |
13 | commit 017a074fc3b1b28dc07fd1314ae2fb13aa485ecd
14 | Author: BluesJiang <763400095@qq.com>
15 | Date: Thu Jun 1 10:30:55 2017 +0800
16 |
17 |     clear the formula
18 |
19 | commit c851ac95780a02f307bbac35886900cb99cc4af2
20 | Author: BluesJiang <763400095@qq.com>
21 | Date: Fri May 5 20:45:27 2017 +0800
22 |
23 |     sub_solution
```

Git