软件工程上机报告

问题描述

> In a box bounded by $[-1, 1]$, given m balloons(they cannot overlap) with variable radio r and position mu, find the optimal value of r and mu which maximize sum r^2

问题分析

可以在方框的空白部分寻找最大的空白，然后用最大的圆进行填充，如图所示：

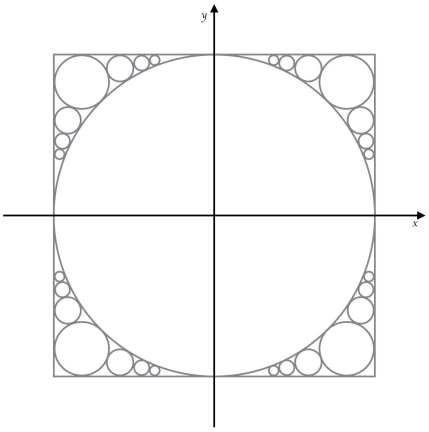


图1

我们按上述方案从大到小依次填满空隙，但是可能会出现下面的情况：

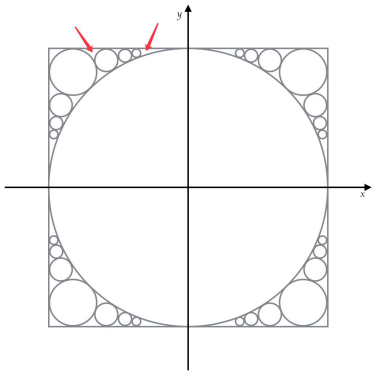


图2

箭头表明的部分右边的可能会出现比左箭头小的情况，这种情况由于计算过于复杂，所以我们寻求次优解，将圆按图1的方式填满

图1由大到小的顺序顺序依次填，可以分为多种圆，各种圆的个数分别为$1, 4, 8, 8, 8, \ldots$

第一个圆的半径为$R_1 = r_1 = 1$**，第二总圆的半径为**$R_2 = r_2 = r_3 = r_4 = r_5 = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

当圆的类别大于三时，可以用一下公式计算半径

$$\begin{cases} R_n = \left(\frac{1-C_{n-1}}{2(1+R_{n-1})}\right)^2, n = 3, 4, 5, \ldots \\ C_n = \sum_{i=2}^{n-1} 2\sqrt{R_i R_{i-1}} + R_2, n = 3, 4, 5, \ldots \\ C_2 = R_2 \\ r_{5+8(n-3)+i} = R_n, i = 1, 2, \ldots, 8, n = 3, 4, 5, \ldots \end{cases}$$

测试用例：

$m = 1, R_1 = 1$
$m = 2\ 6, R_1 = 1, R_2 = 3 - 2\sqrt{2}$
$m = 7\ 15, R_1 = 1, R_2 = 3 - 2\sqrt{2}, R_3 = 0.0857864376 \ldots$
$m = 16\ 24, R_1 = 1, R_2 = 3 - 2\sqrt{2}, R_3 = 0.0857864376 \ldots, R_4 = 0.0513207883 \ldots$

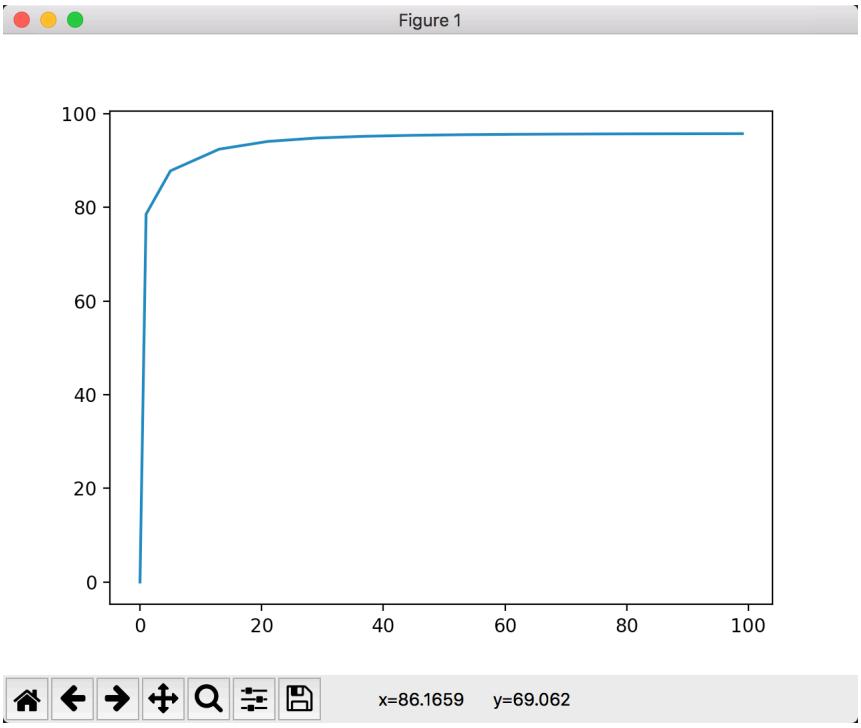
运行结果：

```
Center: ( 0.000000,  0.000000)  Radius:1.0000000000
Center: ( 0.828427,  0.828427)  Radius:0.1715728753
Center: ( 0.828427, -0.828427)  Radius:0.1715728753
Center: (-0.828427,  0.828427)  Radius:0.1715728753
Center: (-0.828427, -0.828427)  Radius:0.1715728753
Center: ( 0.914214,  0.414214)  Radius:0.0857864376
Center: ( 0.414214,  0.914214)  Radius:0.0857864376
Center: ( 0.914214, -0.414214)  Radius:0.0857864376
Center: (-0.414214,  0.914214)  Radius:0.0857864376
```

```
Center: (-0.914214,  0.414214)  Radius:0.0857864376
Center: ( 0.414214, -0.914214)  Radius:0.0857864376
Center: (-0.914214, -0.414214)  Radius:0.0857864376
Center: (-0.414214, -0.914214)  Radius:0.0857864376
Center: ( 0.948679,  0.281509)  Radius:0.0513207883
Center: ( 0.281509,  0.948679)  Radius:0.0513207883
Center: ( 0.948679, -0.281509)  Radius:0.0513207883
Center: (-0.281509,  0.948679)  Radius:0.0513207883
Center: (-0.948679,  0.281509)  Radius:0.0513207883
Center: ( 0.281509, -0.948679)  Radius:0.0513207883
Center: (-0.948679, -0.281509)  Radius:0.0513207883
Center: (-0.281509, -0.948679)  Radius:0.0513207883
Center: ( 0.965886,  0.197825)  Radius:0.0341137321
Center: ( 0.197825,  0.965886)  Radius:0.0341137321
Center: ( 0.965886, -0.197825)  Radius:0.0341137321
Center: (-0.197825,  0.965886)  Radius:0.0341137321
Center: (-0.965886,  0.197825)  Radius:0.0341137321
Center: ( 0.197825, -0.965886)  Radius:0.0341137321
Center: (-0.965886, -0.197825)  Radius:0.0341137321
Center: (-0.197825, -0.965886)  Radius:0.0341137321
Center: ( 0.975694,  0.140235)  Radius:0.0243059818
...
```

结论

当m从0增长到99时，计算每个m值下的覆盖率，得出一下函数图像：



从图中可以看出，用此种方案填放并不能达到最优，当m达到一定大小时，对覆盖率的提供并不大，空缺的部分就是图2中类似左箭头的部分已经成为主要的空缺部分

附录

Gitlog

```Git
commit 9e9c441469f16ee13c8797c1fb0817bc846a0eb2
Author: BluesJiang <763400095@qq.com>
Date:   Thu Jun 1 11:43:50 2017 +0800

    fix when m < 3 ,result goes wrong

commit 834494816cb0729923c505ecef89a2ee0131e7bd
Author: BluesJiang <763400095@qq.com>
Date:   Thu Jun 1 10:56:47 2017 +0800

    bug fixed

commit 017a074fc3b1b28dc07fd1314ae2fb13aa485ecd
Author: BluesJiang <763400095@qq.com>
Date:   Thu Jun 1 10:30:55 2017 +0800

    clear the formula

commit c851ac95780a02f307bbac35886900cb99cc4af2
Author: BluesJiang <763400095@qq.com>
Date:   Fri May 5 20:45:27 2017 +0800

    sub_solution
```