PLEASE ATTACH THIS SINGLE SHEET FACING OUT ON THE BACK OF YOUR HW. We are attempting to reduce bias in grading by keeping the names off of the front of the HW. Studies show that we can easily be unintentionally biased by names, even when we are trying to be fair (search names and bias if you want to learn more). Thanks!

Reminder: Please familiarize yourself with the HW expectations policy on Sakai as well as the information about collaboration in the class syllabus. Graders are aware of these policies and may take points off. The purpose of the guidelines is for you to practice good communication and to make things do-able for your dear graders.

Name: ______________________________

Section (Circle one) 1: MW 1:15 2: MW 2:45 3: TTH 1:15

Due date: Friday Feb 17 at noon in the Engineering lounge.

Outside sources and how I used them: ______________________________

____

Collaborators: ______________________________.

I spent about ______________________________ **hours on this assignment.**

I still need to better understand: ______________________________

____

POINTS (graders can fill for # problems on this HW)

1		6	
2		7	
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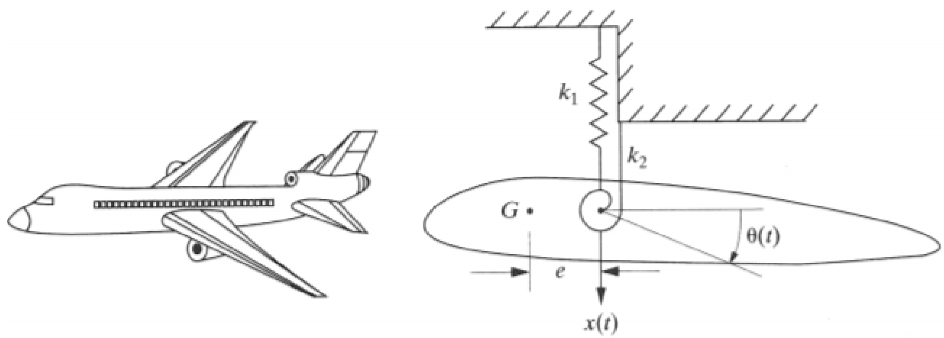
4		9	
5		Σ	

(please turn over)

Engineering 72
Homework 4
Due 17 February 2017

1,2,3

1 A simple model of the vibration modes of a wing on a flying airplane is given below. It accounts for bending and twisting motion by modeling the wing as a mass attached to the aircraft body by a linear spring with constant k_1 and a torsional spring with constant k_2. Linear damping with constant c_1 and torsional damping with constant c_2 are not indicated in the figure but can also be considered.



The vertical motion $x(t)$ and rotational motion $\theta(t)$ of this system are governed by these linearized differential equations:

$$m\ddot{x} - me\ddot{\theta} + c_1\dot{x} + k_1 x = 0$$
$$(J + me^2)\ddot{\theta} - me\ddot{x} + c_2\dot{\theta} + k_2\theta = 0$$

Here, m is the mass of the wing, J is the moment of inertia of the wing about its center of mass, and e is the horizontal distance between the center of mass and the effective connection point of the spring and damping elements. Use the following parameter values: $m = 10000$ kg, $J = 5000$ kg $\cdot$ m^2, $e = 1$ m, $k_1 = 106$ N/m and $k_2 = 105$ Nm/rad.

(a) Develop a set of four first-order differential equations for this system and write them in the matrix form $\dot{\mathbf{Y}} = A\mathbf{Y} + \mathbf{F}(t)$ where

$$\mathbf{Y} = \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

1 (cont.)

(b) Suppose that there is no damping in the system ($c_1 = c_2 = 0$). What kind of motion characterizes the transient response of the system, and what are its vibrational frequencies?

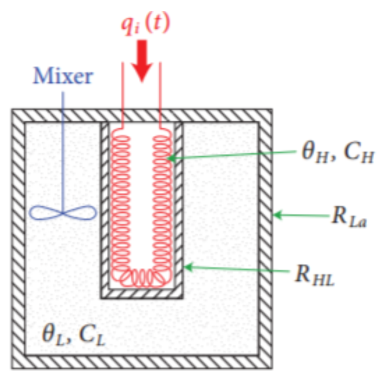
(c) Now suppose $c_1 = 10^5$ Ns/m and $c_2 = 10^5$ Nm/(rad/s). What kind of motion characterizes the transient response of the system?

Hint: Having a hard time computing eigenvalues of a 4 4 matrix? Use Matlab.

2 Producing chemicals in a batch process involves filling a vessel with liquid, sealing it, then heating it to a prescribed temperature. In the design of such a process it is important to calculate the time required for the liquid to reach the desired temperature. The vessel shown is a small model designed for quick processing. It has an electrical heating element contained within an inner metal jacket that has thermal resistance R_{HL} (heater-to-liquid resistance). The thermal resistance of the vessel and its outer layer of insulation is R_{La} (liquid-to-ambient resistance). The heater and liquid have thermal capacitances C_H and C_L, respectively.

All temperatures are to be measured relative to the ambient temperature. Initially both the heater and the liquid are at this ambient temperature with the heater turned off, so the heater (relative) temperature $\theta_H(t)$ and the liquid (relative) temperature $\theta_L(t)$ (which is assumed to be spatially uniform because there is a mixer in the vessel) have initial values of zero.

The rate at which energy is supplied to the heating element is $q_i(t)$.



At time $t = 0$ the heater is connected to an electrical source that supplies energy at a constant rate. The goal is to determine the response of the liquid temperature θ_L and to calculate the time required for the liquid to reach a desired temperature.

The system of equations that governs this system is

$$\dot{\theta}_H = \frac{1}{C_H}\left[q_i - q_{HL}\right]$$
$$\dot{\theta}_L = \frac{1}{C_L}\left[q_{HL} - q_{La}\right]$$

where $q_{HL} = (\theta_H - \theta_L)/R_H L$ and $q_{La} = \theta_L/R_{La}$. Assuming the values $C_H = 1 \times 10^3$ J/K, $C_L = (40/19) \times 10^4$ J/K, $R_{HL} = 1 \times 10^{-4}$ s $\cdot$ K/J, $R_{La} = (19/21)10^{-4}$ s $\cdot$ K/J, and $q_i = 500$ kW, find the liquid temperature as a function of time (suggestion: stick with the fractions, we're not being unnecessarily mean). What is the steady state temperature of the liquid (relative to ambient)? What is the time required for the liquid temperature to increase by 40 degrees?

2 (cont.) **Note #1:** If you would like to understand the nature of each of these quantities further, the analogous electrical circuit might be helpful. See `hw4-ElectricalThermalAnalogy.pdf` in the Homework folder on Sakai for an outline of the analogy, that circuit, and derivation of the equations.

Note #2: Most of the work on this problem should be done by hand. Be strategic about when you introduce numerical values into your work and when you use decimal numbers like 0.9047619 instead of exact expressions like 19/21. There are ways to avoid lots of fractions when diagonalizing your matrix.

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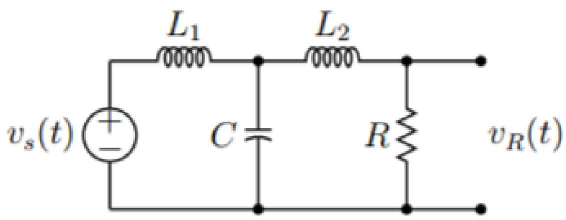
3 To reduce the possibility of damage caused by impulsive voltage spikes (due to surges from equipment or lightning), the following electrical filter is installed in the power line for some sensitive equipment.



The values of the filter components are $L_1 = L_2 = 10$ mH, $C = 1$ μF. The load resistance is 50 Ω.

(a) Derive a set of three first-order differential equations for this system in terms of the variables v_R, v_C and i_C and their derivatives, where v_C is the voltage across the capacitor and i_C is the current through the capacitor. The voltage spike is described by the function $v_s(t)$. Explain the governing principle(s) behind each differential equation. Write the equations in the matrix form $\dot{\mathbf{Y}} = A\mathbf{Y} + \mathbf{F}(t)$ where

$$\mathbf{Y} = \begin{bmatrix} v_R \\ v_C \\ i_C \end{bmatrix}$$

(b) Compute $v_R(t)$, the output system voltage, in the case where $v_s(t) = 10^{-3}\delta(t)$. This function models a voltage spike with amplitude 100 V and duration 10 μs. Plot $v_R(t)$ and determine the maximum system voltage that is reached.

Hint: You do not need to compute the entire matrix exponential. Think about what parts of the matrix exponential you really need to get the information you want.

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