PLEASE ATTACH THIS SINGLE SHEET FACING OUT ON THE BACK OF YOUR HW. We are attempting to reduce bias in grading by keeping the names off of the front of the HW. Studies show that we can easily be unintentionally biased by names, even when we are trying to be fair (search names and bias if you want to learn more). Thanks!

Reminder: Please familiarize yourself with the HW expectations policy on Sakai as well as the information about collaboration in the class syllabus. Graders are aware of these policies and may take points off. The purpose of the guidelines is for you to practice good communication and to make things do-able for your dear graders.

Name: ______________________________

Section (Circle one) 1: MW 1:15 2: MW 2:45 3: TTH 1:15

Due date: Friday Feb 24 at noon in the Engineering lounge.

Outside sources and how I used them: ______________________________

______________________________.

Collaborators: ______________________________.

I spent about ______________________________ **hours on this assignment.**

I still need to better understand: ______________________________

______________________________.

POINTS (graders can fill for # problems on this HW)

1		6	
2		7	
3		8	
4		9	
5		Σ	

(please turn over)

Engineering 72
Homework 5
Due 24 February 2017

1,2,3

1 In some periodic vibratory systems, external energy is supplied to the system over part of a period and dissipated within the system in another part of the period. This involves nonlinear damping and is known as a relaxation oscillation or a van der Pol oscillation. Van der Pol was a telecommunications engineer who discovered these stable oscillations while building electronic circuit models of the human heart. The model has since been used to describe many other physical phenomena, which you can search on the internet to see.
The van der Pol equation is

$$\ddot{x} + \mu(x^2 - 1)\dot{x} + x = 0$$

where μ is a parameter to be varied and x is the pacemaker signal in the cardiac model case.

(a) Write the equation as a system of first order equations, determine the equilibrium point(s) of the system, linearize about each equilibrium point and characterize the stability of the system for $-3 < \mu < 3$.

(b) The interesting van der Pol results are for $\mu > 0$. Use Matlab to create separate phase portraits for $\mu = 0, 1$ and 3. Comment on the relation between these plots and the expectations from part (a).

Some hints if you are using Matlab:

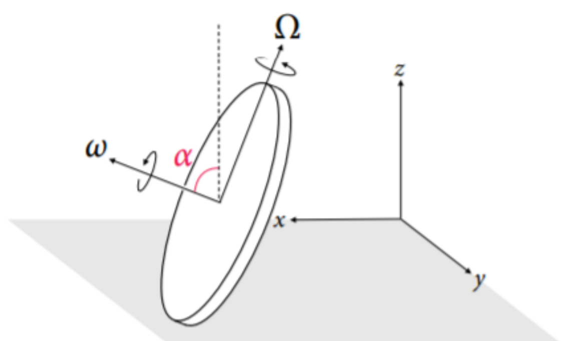
- The `quiver` command shows both the magnitude and the direction at each point in the vector field. Sometimes the scaling is set by large magnitude vectors in a way that makes small ones impossible to see. To have it show direction only, normalize all of the vectors. Instead of using a command like `quiver(x,y,u,v)`, use `quiver(x,y,u./L,v./L)` where `L=sqrt(u.^2+v.2)`
- You can add streamlines to the quiver plots to help you visualize the trajectories. For example try `startx = 0.5; starty = 0; streamline(x,y,u,v,startx,starty)`

■

2 A first step in evaluating the dynamics of wheeled vehicles such as bicycles and motorcycles is understanding the stability of a rolling disk. This disk rolls without slipping on a horizontal plane. Its angular velocity is characterized by $\dot{\alpha}$, the rate of change of the lean angle α, Ω the rate of spin about the vertical $\mathbf{z}$ direction, and ω the rate of spin about its axial direction. The equations governing the orientation are

$$\begin{aligned}
(2k+1)\dot{\omega} + \dot{\alpha}\Omega\sin\alpha &= 0 \\
k\Omega^2 \sin\alpha\cos\alpha + (2k+1)\omega\Omega\sin\alpha - (k+1)\ddot{\alpha} &= \frac{g}{r}\cos\alpha \\
\dot{\Omega}\sin\alpha + 2\dot{\alpha}\Omega\cos\alpha + 2\omega\dot{\alpha} &= 0
\end{aligned}$$

where r is the radius of the disk and g is the gravitational constant. For a solid disk as we are considering $k = \frac{1}{4}$.
Arbitrary motion may be simulated with these equations, but in this problem we are interested in equilibrium states of the system and their stability. Equilibrium states of this system correspond to steady motion of the disk. You may want to check for yourself that all equilibrium states of the system must have $\alpha = \alpha_0$, $\dot{\alpha} = 0$, $\omega = \omega_0$, and $\Omega = \Omega_0$, where α_0, ω_0, and Ω_0 are constants that satisfy a nonlinear equation. In particular, we're interested in two particular equilibrium states with $\alpha_0 = \pi/2$, which corresponds to the disc being vertical.



(a) If $\Omega_0 = 0$ the disk rolls along a straight line and ω_0 is the rolling angular velocity. For what angular velocities ω_0 does local stability analysis predict unstable motion? (You answer will be in terms of r and g.)

(b) If $\omega_0 = 0$ the disk spins in place about the vertical axis with angular velocity Ω_0. For what angular velocities Ω_0 does local stability analysis predict unstable motion?

There are plenty of other interesting cases (rolling in a circle, various precessions). If you are interested come discuss some time, or take E175 (Dynamics of Rigid Bodies).

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3 Three weeks after teaching at Harvey Mudd College began in the fall of 1957, the Soviet Union launched the first-ever artificial Earth satellite: the spherically shaped Sputnik I. They launched Sputnik II the following month. There was a lot of pressure for the US to keep up, and in January 1958 the US launched Explorer I. Explorer I was very different looking—long and narrow—and it was supposed to rotate about its own long axis centerline. A radio astronomer named Ronald Bracewell had tracked and analyzed the flight of Sputnik I and understood its stable behavior. He also knew that Explorer I would not be stable and would tumble end over end. He tried to reach engineers at JPL to warn them but security concerns prevented him from getting his information to the right people and he was only able to share it by publishing it in Nature later in 1958. Too late–Explorer I made just one stably-spinning orbit of the Earth then began to tumble. It was not unreasonable for the JPL engineers to plan for spin about the long axis. This would be spinning about the axis with the minimum moment of inertia, which should be stable.

However once in flight, its flexible antennas were deployed, and their vibration dissipated energy and the axis changed. We're not going to analyze the transition, but we can study the effect of rotation about different axes.

The equations of motion governing the orientation for a spinning rigid body with at least two axes of symmetry are

$$\begin{aligned} 0 &= I_x\dot{\omega}_x + (I_z - I_y)\omega_z\omega_y \\ 0 &= I_y\dot{\omega}_y + (I_x - I_z)\omega_x\omega_z \\ 0 &= I_z\dot{\omega}_z + (I_y - I_x)\omega_y\omega_x \end{aligned}$$

where ω_x, ω_y and ω_z are the angular velocities of the body about the x-, y- and z-axes, respectively, and I_x, I_y and I_z are the corresponding central moments of inertia of the body about each axis.

(a) Show that constant rotation of the body along each of the axes is an equilibrium state of the differential equations above. In other words, show that $(\omega_x, \omega_y, \omega_z) = (c, 0, 0)$ or $(0, c, 0)$ or $(0, 0, c)$, where c is a constant, is an equilibrium state. What information does the local stability analysis provide about these equilibrium states? Explain why local stability analysis does not fully explain the stability of the three equilibrium states.

(b) Use Matlab to investigate the stability of each of three types of equilibrium states. The moment of inertia about the long axis is $I_x = 0.06\ \mathrm{kg}\cdot\mathrm{m}^2$, and the other two moments of inertia, while nominally the same, will be shifted by just 1% to model a redistribution due to antenna flexibility: $I_y = 0.099\ \mathrm{kg}\cdot\mathrm{m}^2$ and $I_z = 0.101\ \mathrm{kg}\cdot\mathrm{m}^2$. Write a few sentences about how you carried out your investigation and whether your work demonstrates the expected instability. Include figures that best explain your claims.

3 (cont.) **More history**: In addition to not having Matlab or OpenRocket in 1958, here is another detail to give context to the work being done at that time. "During the 1940s and 1950s, JPL used the word 'computer' to refer to a person rather than a machine. The all-female computer team, many of the members recruited right out of high school, were responsible for doing all the math by hand required to plot satellite trajectories and more." (from `http://www.nasa.gov/mission_pages/explorer/computers.html`)