PLEASE ATTACH THIS SINGLE SHEET FACING OUT ON THE BACK OF YOUR HW. We are attempting to reduce bias in grading by keeping the names off of the front of the HW. Studies show that we can easily be unintentionally biased by names, even when we are trying to be fair (search names and bias if you want to learn more). Thanks!

Reminder: Please familiarize yourself with the HW expectations policy on Sakai as well as the information about collaboration in the class syllabus. Graders are aware of these policies and may take points off. The purpose of the guidelines is for you to practice good communication and to make things do-able for your dear graders.

Name: ____________________

Section (Circle one) 1: MW 1:15 2: MW 2:45 3: TTH 1:15

Due date: Friday, February 10 at noon in the Engineering lounge.

Outside sources and how I used them: ____________________

____________________.

Collaborators: ____________________.

I spent about ____________________ **hours on this assignment.**

I still need to better understand: ____________________

____________________.

POINTS (graders can fill for # problems on this HW)

1		6	
2		7	
3		8	
4		9	
5		Σ	

(please turn over)

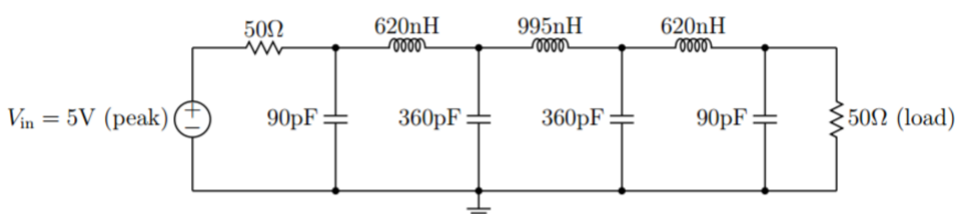
Engineering 72
Homework 3
Due 10 February 2017

1,2,3

1 Kirchhoffs current law (KCL) can be used to write *algebraic* equations for RLC circuits using the concept of impedance, which you learned in E59. If we think of impedance as a complex-valued resistance, then

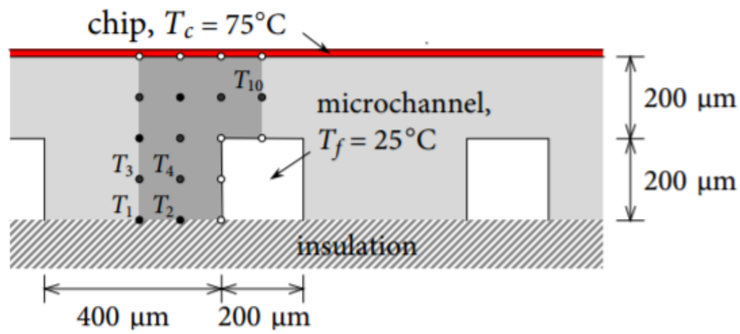
- for a resistor, impedance $Z_R = R$
- for an inductor, impedance $Z_L = j\omega L$
- for a capacitor, impedance $Z_C = 1/(j\omega C)$.

Use KCL to write a set of equations in the form $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x}$ is the vector of (complex) node voltages. Write a function to solve your system of equations using Matlab for an AC input with amplitude $V_{in} = 5$ V and frequency f (note that in your system of equations this can be represented by the amplitude only). This circuit is a low-pass Butterworth filter with a cutoff frequency of 12 MHz. To see the effect of frequency on the output voltage, make a plot of log magnitude (dB) $= 20\log_{10}(\frac{V_{out}}{V_{in}})$ (with V_{out} as the magnitude of the complex voltage across the load resistor) versus frequency from 1 to 100 MHz.



■

2 A heat sink for cooling computer chips is fabricated from copper with machined microchannels that carry a cooling fluid with temperature $T_f = 25°C$. Assume that this is also the temperature of the walls of the channel (that is, convective heat transfer from the wall to the fluid is very effective). The lower edge of the heat sink is in contact with a thermal insulator that does not remove any heat. The figure shows a cross section of the heat sink, with the channels running into the page. Our goal is to compute the temperature distribution for the maximum allowable chip temperature of $T_c = 75°C$ and the flux of waste heat that can be removed.



(a) Instead of trying to compute the temperature distribution everywhere, we overlay a grid on the cross-section and determine the temperatures at a discrete set of points. Because of the symmetry in the problem, it is enough to compute the temperatures in the region covered by the points shown. The temperatures at the surface points are known (shown by open dots). The temperatures at the closed dots are unknown. Since this is a steady-state problem, the temperature distribution in any cross-section of the heat sink (into the page) is the same and the temperature at any grid point is approximately equal to the average of the temperatures at the four grid points connected to it. Use this approximation to set up a system of equations for the unknown temperatures. This is called a "finite difference" formulation of the problem. The grid points on the insulated surface require special treatment to enforce a no-flux condition. For example,

$$T_2 = \frac{T_1 + T_4 + T_4 + T_f}{4} = \frac{T_1 + 2T_4 + T_f}{4}$$

(Effectively, we mirror T_4 on the bottom side of T_2. Come talk to us if you want to know why this is the correct method.) Make use of symmetry in a similar way when you think of the four grid points surrounding points on the left and right internal edges of the domain you are analyzing. Use Matlab to solve your system of equations for the unknown point temperatures.

(b) Use Matlab's `contourf` command to produce a graph of the steady-state temperature distribution. Include a printout of your graph with your homework. Use your graph to check if your temperature distribution seems reasonable.

2 (cont.)

(c) Fourier's law of heat conduction says that the flux of heat energy across a surface is proportional to the outward normal temperature gradient, and the constant of proportionality is the thermal conductivity, k, which for copper is 400 W/(m K). In this situation, the heat transferred from the chip to the heat sink per unit length (in W per meter into the page) at the top surface (whose normal is aligned with the y direction) of the shaded domain we are analyzing is

$$\begin{aligned} Q_{\text{top}} &= k \int_{\text{left end of shaded domain}}^{\text{right end of shaded domain}} \frac{\partial T}{\partial y} dx \\ &\approx k \cdot \frac{\Delta}{2} \left[\frac{T_7 - T_c}{\Delta} + 2\frac{T_8 - T_c}{\Delta} + 2\frac{T_9 - T_c}{\Delta} + \frac{T_{10} - T_c}{\Delta} \right] \\ &= \frac{k}{2} \left[(T_7 - T_c) + 2(T_8 - T_c) + 2(T_9 - T_c) + (T_{10} - T_c) \right] \end{aligned}$$

In the equation above, Δ is the horizontal or vertical distance between any two adjacent grid points, the integral over the top edge of the domain is approximated using the trapezoidal rule, and the temperature gradient $\partial T/\partial y$ is approximated by the temperature difference between adjacent grid points, divided by Δ. What is the rate of heat transfer (in W) from a 10 mm x 10 the chip to the heat sink? [Note that this calculation is an overestimate of the heat that can be removed for several reasons. The coarse grid limits the precision of the temperature distribution (so several steps of refining the grid until the further refinement does not significantly change the answer is warranted), we have idealized the heat transfer between the chip and the copper and between the copper and the fluid, and we assumed the fluid temperature stays constant as it flows into the page, but it would heat up. Extending the model to remove these approximations would be within your abilities if you needed a more reliable answer].

■

3 In this problem, we will reconsider the truss problem that we studied in class.

(a) Recalculate the optimal location of joint B that minimizes the sum of the bar lengths of the truss, this time using the `fminsearch` function in Matlab. What is that minimum total bar length? (By minimizing the total bar length, we are minimizing the weight and cost of the truss.) For this part of the problem, you will need to modify your `e72truss.m` code so that it outputs the total length of the truss, given a 2×1 vector representing the coordinates of the joint B as input.

(b) Suppose there is an obstacle that requires $x \geq 3$, where x is the x-coordinate of joint B. Calculate the optimal location of joint B that minimizes the total bar lengths of the truss in this situation. What is that optimal length? You might be interested in Matlabs `fmincon` function.

(c) Calculate the optimal location of joint B that maximizes ϕ, the max load per weight of the truss. What is the resulting optimal value of ϕ and the maximum load of the truss?

(d) Calculate the optimal location of joint B that maximizes ϕ, subject to the constraint $x \geq 3$. What is the resulting optimal value of ϕ and the maximum load of the truss?

Note: If you want to know more about whats happening with Matlabs `fmincon`, you might look into using the `optimset` function to set values for the options `TolX` and `TolFun`. Youll find more information in the Matlab documentation for `fmincon` function.