PLEASE ATTACH THIS SINGLE SHEET FACING OUT ON THE BACK OF YOUR HW. We are attempting to reduce bias in grading by keeping the names off of the front of the HW. Studies show that we can easily be unintentionally biased by names, even when we are trying to be fair (search names and bias if you want to learn more). Thanks!

Reminder: Please familiarize yourself with the HW expectations policy on Sakai as well as the information about collaboration in the class syllabus. Graders are aware of these policies and may take points off. The purpose of the guidelines is for you to practice good communication and to make things do-able for your dear graders.

Name: ______________________________

Section (Circle one) 1: MW 1:15 2: MW 2:45 3: TTH 1:15

Due date: Friday Feb 3 at noon in the Engineering lounge.

Outside sources and how I used them: ______________________________

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Collaborators: ______________________________.

I spent about ______________________________ **hours on this assignment.**

I still need to better understand: ______________________________

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POINTS (graders can fill for # problems on this HW)

1		6	
2		7	
3		8	
4		9	
5		Σ	

(please turn over)

Engineering 72
Homework 2
Due 3 February 2017

1, 2, 3, 4

Reminder: Explain your solution process in complete sentences. You may lose points due to poor presentation. Review policy on collaboration in the class syllabus. If you use any outside resources be sure to cite your source.

1 As shown in Figure.1, the signal $x(t)$ is approximated by the two building blocks, i.e. $x(t) \cong \hat{x}(t) = a_1\phi_1(t) + a_2\phi_2(t)$.

(a) Determine the coefficients a_1 and a_2 such that integrated square error (ISE) is minimized. Sketch the approximated signal $\hat{x}(t)$. Label the relevant amplitudes and times.

(b) Consider a circuit system that is linear and time-invariant. If the response to the input waveform $\phi_1(t)$ is $y_1(t)$, and the response to the input waveform $\phi_2(t)$ is $y_2(t)$, what is the response of the system to $\hat{x}(t)$? Please explain your results.

(c) Sketch another non-trivial basis function $\phi_3(t)$, defined in the region $0 \le t \le 1$, that is orthogonal to both $\phi_1(t)$ and $\phi_2(t)$. Label the relevant amplitudes and times. Let $x(t) \cong \hat{x}(t) = a_1\phi_1(t) + a_2\phi_2(t) + a_3\phi_3(t)$, what are the new coefficients? Describe the advantage of using orthogonal basis functions.

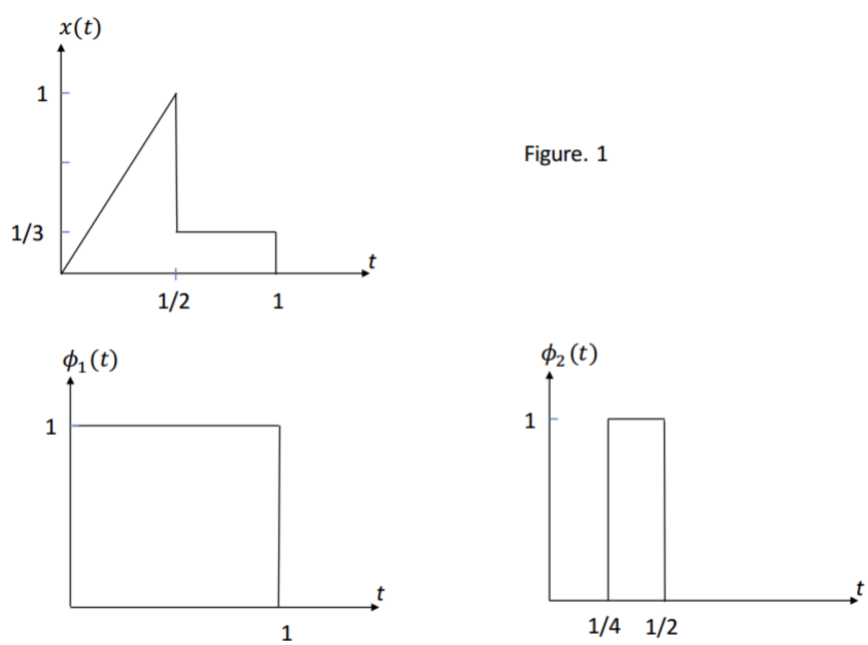

Figure. 1

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2 A Hadamard matrix consists of elements ± 1 and has the property that its rows are orthogonal to each other (recall that if two vectors are orthogonal their dot product is zero). Starting with a 2x2 matrix $[H_2]$, a matrix of $[H_{2n}]$ of size 2nx2n can be constructed recursively using the equation given below.

$$[H_2] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad [H_{2n}] = \begin{bmatrix} [H_n] & [H_n] \\ [H_n] & -[H_n] \end{bmatrix}$$

(a) Construct $[H_4]$ and $[H_8]$.

(b) If c_{ik} denotes the elements in the ith row and kth column of $[H_4]$, sketch the following waveforms:

$$\phi_i(t) = \sum_{k=1}^{4} c_{ik} p(t-(k-1)T), \quad i = 1,2,3,4$$

where $p(t)$ is a pulse of unit amplitude on the interval $0 \leq t < T$, and T is an arbitrary time interval. The waveforms $\phi_i(t)$ are known as **Walsh functions** and among other things are used to separate channels in Code Division Multiple Access (CDMA) cell phones. When the codes used in CDMA system are orthogonal to each other, interference from other users (codes) can be minimized at the detection system when the user channels are properly synchronized.

(c) Are the $\phi_i(t)$ of part (b) orthogonal to each other? Why or why not? Discuss the relationship of result to the orthogonal nature of the pair of the row vectors.

(d) Determine the coefficients c_k that minimize the ISE in the series approximation of the signal shown in Figure 2, i.e.

$$x(t) \approx \widehat{x}(t) = \Sigma_{k=1}^{4} c_k \phi_k(t)$$

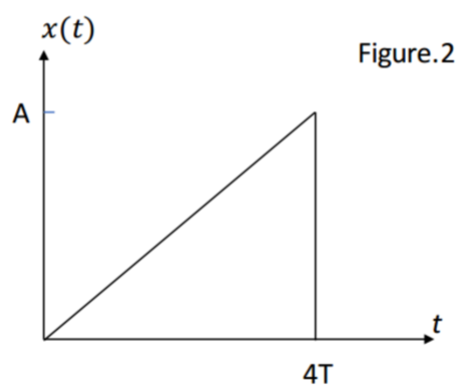

Figure.2

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3 As discussed in class, Fourier Series can be used to represent a periodic signal because the complex exponential building blocks are also periodic with the same T_0. In order to apply the Fourier Series method to a non-periodic signal $x(t)$ over a given time interval, say T, we can instead construct the periodic extension of the signal as $x_p(t)$, by choosing a T_0 that is something larger than T (i.e. including all the interested time).
Mathematically,

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(t - nT_0)$$

(a) For the signal $x(t)$ shown in Figure 3, determine the expression for the Fourier coefficients of $x_p(t)$ if we choose $T_0 = T$. We assume the Fourier Series form is given as $x(t) \cong \widehat{x}(t) = \Sigma_{k=-K}^{K} c_k \phi_k(t)$, where $\phi_k(t) = e^{jk\omega_0 t}$, and $\omega_0 = \frac{2\pi}{T_0}$.

(b) Create a Matlab function to generate the graphs (include a couple periods at least) of $\widehat{x}(t)$ for A=2, T=1, T_0=1 and K=10, 20 and 50. Estimate the ISE for each choice of K and display the result on the graph.

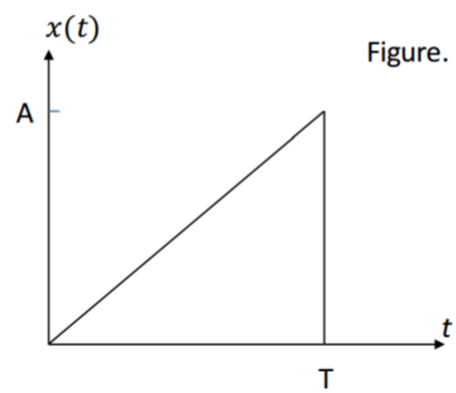

Figure. 3

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4 Let the pulse train $x(t)$ used in the class example pass through a RC circuit system below. The voltage across the capacitor C be the output $y(t)$. Assume A=1 volt, τ = 0.25 msec, T_0 = 1 msec. R=0.5 kΩ, C=$\frac{1}{2\pi}$ μF. **From E79**, you have learned that each complex exponential component $c_k e^{jk\omega_0 t}$ of the pulse train after the LTI system is scaled by $H(jk\omega_0)$ or its frequency response function at the corresponding exciting frequency of $k\omega_0$.

(a) Use your Matlab code from the class to sketch the input $x(t)$ and output waveform $y(t)$ for comparison. You should decide on a proper number of terms K to use.

(b) Explain your results and what the RC circuit does to the input signal.