Përmbajtja.

Permbledhje

Kristian Blido

03, 06 2021

1 Kinematika

1.1 Shpejtesia Mesatare

$$v = \frac{\Delta l}{\Delta t}$$

1.2 Levizja me nxitim

$$\begin{aligned} a &= \frac{\Delta v}{\Delta t} \\ v &= v_0 + a \cdot t \\ v^2 &= v_0^2 + 2 \cdot a \cdot l \\ l &= v_{avg} \cdot t_{total} = \frac{v + v_0}{2} \cdot t \\ l &= l_0 + v_0 \cdot t + \frac{a \cdot t^2}{2} \end{aligned}$$

1.3 Levizja 2 permasore

$$\begin{aligned} v_x &= v \cdot \cos\theta \\ v_y &= v \cdot \sin\theta \\ t_{ajer} &= \frac{2 \cdot v \cdot \sin\theta}{g} \\ l_{max} &= v_x \cdot \Delta t = \frac{v^2 \cdot \sin 2\theta}{g} \\ h_{max} &= \frac{v^2 \sin^2\theta}{2 \cdot g} \end{aligned}$$

1.4 Levizja rrethore

$$\begin{aligned} \theta &= \theta_0 + \omega \cdot \Delta t \\ \omega &= \frac{\Delta\theta}{\Delta t} = 2 \cdot \pi \cdot f \\ v &= r \cdot \omega \\ a &= r \cdot \omega^2 = \frac{v^2}{r} \\ \vec{F} &= m \cdot a = m \cdot \omega^2 \cdot r = \frac{m \cdot v^2}{r} \end{aligned}$$

2 Dinamika

$$\sum | \vec{F} | = m \cdot a$$

2.1 Terheqja Gravitacionale

$$\begin{aligned} g &= \frac{\vec{G}}{m} \\ \vec{G} &= \gamma \cdot \frac{m_1 \cdot m_2}{r^2} \end{aligned}$$

2.2 Pesha

Forca qe vepron nga trupi tek mbeshtetsja ku eshte vendosur ose mbi fijen e varur.

$$P = m \cdot (g - a)$$

Nxitimi me kah lart $\rightarrow$ poshte e ben nxitimin rezultant me te vogel se ai fillestar (g) dhe anasjelltas.

2.3 Momenti

$$M = \vec{F} \cdot d$$

2.4 Impulsi

$$\vec{p} = m \cdot \vec{v}$$
$$\Delta\vec{p} = \vec{F} \cdot \Delta t$$

2.4.1 Ruajtja e Impulsit

$$m_1 \cdot v_1 + m_2 \cdot v_2 = m_1 \cdot v_1' + m_2 \cdot v_2'$$

2.5 Susta

k : koeficienti i sustes $\frac{N}{m}$

$$\vec{F} = -k \cdot x$$

$$\omega = \sqrt{\frac{k}{m}}$$
$$f = \frac{\omega}{2 \cdot \pi} = \frac{\sqrt{\frac{k}{m}}}{2 \cdot \pi}$$
$$T = 2 \cdot \pi \cdot \sqrt{\frac{m}{k}}$$

2.5.1 Lidhja e sustave

Lidhja ne seri

$$\frac{1}{k_r} = \frac{1}{k_1} + \frac{1}{k_2}$$

Lidhja ne paralel

$$k_r = k_1 + k_2$$

2.6 Lavjerresi

$$\omega = \sqrt{\frac{g}{l}}$$
$$f = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{g}{l}}$$

3 Puna, Energjia, Fuqia

3.1 Puna

$$A = \Delta E$$
$$A = \vec{F} \cdot \vec{x} = F \cdot x \cdot \cos\theta$$

3.2 Energjia

3.2.1 Mekanike

$$E_m = E_k + E_p$$

3.2.2 Kinetike

$$E_k = \frac{m \cdot v^2}{2}$$

3.2.3 Potenciale e lartesise

$$E_p = m \cdot g \cdot h$$

3.2.4 Potenciale elastike

$$E_{p_{elastike}} = \frac{k \cdot \Delta x^2}{2}$$

3.3 Fuqia

$$P = \frac{A}{\Delta t} = \vec{F} \cdot \vec{v} = F \cdot v \cdot \cos\theta$$

4 Lenda dhe Materialet

4.1 Dendeisa

$$\rho = \frac{m}{v} = d$$

4.2 Shtypja

$$P = \frac{F}{S}$$

4.2.1 Shtypja brenda lengut

$$P = \rho \cdot g \cdot h$$

4.3 Sforcimi

$$\sigma = \frac{F}{S}$$

4.4 Shformimi

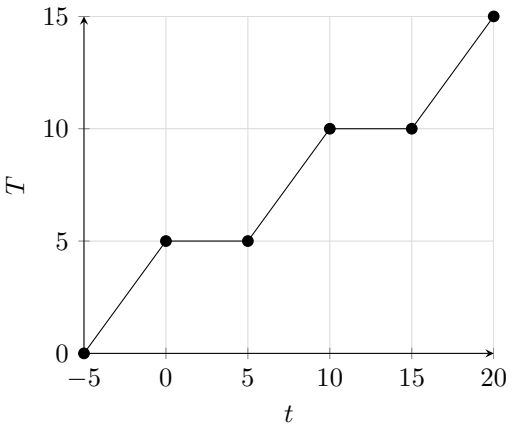
$$\epsilon = \frac{x}{L}$$

4.5 Moduli i Young-ut

$$E = \frac{\sigma}{\epsilon}$$

5 Fizika Termike

5.1 Ndryshimi i Gjendjes dhe Energjise



$$]-5,0[\quad \rightarrow \quad \text{Ngrohje} \quad Q = c \cdot m \cdot \Delta T$$

$$]0,5[\quad \rightarrow \quad \text{Shkrirje} \quad Q = \lambda \cdot m$$

$$]5,10[\quad \rightarrow \quad \text{Ngrohje} \quad Q = c \cdot m \cdot \Delta T$$

$$]10,15[\quad \rightarrow \quad \text{Avullim} \quad Q = q \cdot m$$

$$]15,20[\quad \rightarrow \quad \text{Ngrohje} \quad Q = c \cdot m \cdot \Delta T$$

6 Gazet Ideale

$$n = \frac{m}{M} = \frac{N}{N_A}$$

$$T(K) = T(^\circ C) + 273.15$$

6.1 Ligji i gazeve

$$P \cdot V = N \cdot k_b \cdot T$$
$$P \cdot V = n \cdot (N_A \cdot k_B) \cdot T$$
$$P \cdot V = n \cdot R \cdot T$$
$$P \cdot M = d \cdot R \cdot T$$

6.2 Energjia e Brendshme

$$U = \begin{cases} \frac{3}{2} \cdot R \cdot T \cdot n, & 1 \quad \text{atom} \\ \frac{5}{2} \cdot R \cdot T \cdot n, & 2 \quad \text{atome} \\ 3 \cdot R \cdot T \cdot n, & 3+ \quad \text{atome} \end{cases}$$

$$\begin{aligned} R &= N_A \cdot k_B \\ &= 6.02 \cdot 10^{23} \frac{}{mol} \cdot 1.38 \cdot \frac{m^2 kg}{10^{23} \cdot s^2 \cdot K^1} \\ &= 8.31 \frac{m^2 \cdot kg}{s^2 \cdot K \cdot mol} \\ &= 8.31 \frac{J}{mol \cdot K} \end{aligned}$$

6.3 Energjia Kinetike

$$P = \frac{1}{3} \cdot \frac{N}{V} \cdot m \cdot < v^2 >$$

$$P = \frac{2}{3} \cdot \frac{N}{V} \cdot < \epsilon_k >$$

$$< \epsilon_k > = \frac{3}{2} \cdot k_B \cdot T$$

6.4 Puna e gazeve

$$A = P \cdot \Delta V$$

6.5 Parimi i pare i Termodinamikes

$$Q = \Delta U + A$$

"Sasia e nxehtesise qe merr nje sistem shkon pjeserisht per ndryshimin e energjise se brendshme dhe pjeserisht per kryerjen e punes"

6.6 Izoproceset

6.6.1 Ciklik

- 2 rruge Termodinamike

- Sisteme *Quasi-Statike*

$$\left\{\begin{array}{rcl} T_1 & = & T_2 \\ \Delta U & = & 0 \\ Q & = & A \end{array}\right\}$$

6.6.2 Izotermik

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

$$\left\{\begin{array}{rcl} T_1 & = & T_2 \\ \Delta U & = & 0 \\ Q & = & A \end{array}\right\}$$

6.6.3 Izobarik

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$\left\{\begin{array}{rcl} P_1 & = & P_2 \\ Q & = & \Delta U + A \end{array}\right\}$$

6.6.4 Izohorik

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\left\{\begin{array}{rcl} V_1 & = & V_2 \\ A & = & 0 \\ Q & = & \Delta U \end{array}\right\}$$

6.6.5 Adiabatik

$$\left\{\begin{array}{rcl} Q & = & 0 \\ A & = & -\Delta U \end{array}\right\}$$

6.7 Parimi i dyte i Termodinamikes

"Nuk mund te ekzistoje motorri i perjetshem"

$$A = Q_i - Q_f$$

Rendimenti Rendimenti $\rightarrow \eta$

$$\left\{\begin{array}{rcl} \eta & = & \frac{A}{Q_i} \\ \eta & < & 1 \end{array}\right\}$$

7 Fusha Elektrike

7.1 Intensiteti i Fushes Elektrike

$$E = \frac{F}{q}\left(\frac{N}{C}\right)$$

7.2 Ligji i Kulonit

$$\begin{array}{rcl} |\vec{F}| & = & k \cdot \frac{Q_1 \cdot Q_2}{\epsilon \cdot r^2} \\ & = & \frac{1}{4 \cdot \pi \cdot \epsilon_0} \cdot \frac{Q_1 \cdot Q_2}{\epsilon \cdot r^2} \\ & = & \frac{Q_1 \cdot Q_2}{4 \cdot \pi \cdot \epsilon_0 \cdot \epsilon \cdot r^2} \end{array}$$

Ku $\epsilon_0 = 8.85 \cdot 10^{-12} \frac{F}{m}$ dhe $k = 9 \cdot 10^9 \frac{N\,m^2}{C^{-2}}$

7.3 Intensiteti i Fushes Elektrike Qendrore

$$\begin{array}{rcl} E & = & \frac{F}{q} \\ & = & \frac{\frac{Q_1 \cdot q}{4 \cdot \pi \cdot \epsilon_0 \cdot \epsilon \cdot r^2}}{q} \\ & = & \frac{Q}{4 \cdot \pi \cdot \epsilon_0 \cdot \epsilon \cdot r^2} \end{array}$$

7.4 Puna e fushes elektrike

$$A = q \cdot E \cdot d$$
$$A = q \cdot \Delta V$$

7.5 Potenciali Elektrik

$$V = \frac{A_P}{q}$$

7.6 Intensiteti i Fushes se Njetrajtshme

$$\begin{aligned} A &= A_P \\ F \cdot \Delta d &= \Delta V \cdot q \\ \frac{F}{q} &= \frac{\Delta V}{\Delta d} \\ E &= -\frac{\Delta V}{\Delta d} \end{aligned}$$

7.7 Potenciali i Fushes Qendrore

$$V = \frac{q}{4 \cdot \pi \cdot \epsilon_0 \cdot \epsilon \cdot r}$$

8 Kondensatoret

8.1 Kapaciteti

$$C = \frac{q}{V}(F)$$

8.1.1 Kapaciteti i Percjellesit

$$C = \frac{q}{-\Delta V} = \frac{q}{U}$$

8.2 Energjia e Kondesatorit

$$\begin{aligned} W &= \frac{q \cdot -\Delta V}{2} \\ &= \frac{(C \cdot -\Delta V) \cdot -\Delta V}{2} \\ &= \frac{C \cdot \Delta V^2}{2} \\ &= \frac{q^2}{2 \cdot C} \end{aligned}$$

8.3 Kapaciteti i Kondensatorit

$$E = \frac{q}{S \cdot \epsilon \cdot \epsilon_0}$$
$$E = \frac{V}{d}$$
$$\frac{Q}{S \cdot \epsilon \cdot \epsilon_0} = \frac{V}{d}$$
$$\frac{q}{V} = C = \frac{\epsilon \cdot \epsilon_0 \cdot S}{d}$$

8.3.1 Depertueshmeria Elektrike

$$\epsilon = \frac{C}{C_0}$$
$$\epsilon_0 = \frac{1}{\mu_0 \cdot c}$$

ϵ_0 $\rightarrow$ Pershkueshmeria elektrike ne vakum

μ_0 $\rightarrow$ Pershkueshmeria magnetike vakum

c $\rightarrow$ Shpejtesia e drietes ne vakum

8.4 Lidhja e Kondensatoreve

8.4.1 Ne Paralel

$$\begin{aligned} C &= \sum C_i \\ \Delta V &= V_1 = V_2 = V_3 = \ldots = V_i \\ q &= \sum q_i \end{aligned}$$

8.4.2 Ne Seri

$$\begin{aligned} \frac{1}{C} &= \sum \frac{1}{C_i} \\ \Delta V &= \sum V_i \\ q &= q_1 = q_2 = q_3 = \ldots = q_i \end{aligned}$$

9 Rryma Elektrike

9.1 Rryma

$$I = \frac{\Delta Q}{\Delta t} \quad (A)$$

9.2 Dendesia e Ngarkesave

$$\begin{aligned} \text{lineare} &\rightarrow \lambda, \quad \lambda = \tfrac{q}{l} \\ \text{siperfaqje} &\rightarrow \sigma, \quad \sigma = \tfrac{q}{s} \\ \text{vellim} &\rightarrow \rho, \quad \rho = \tfrac{q}{v} \end{aligned}$$

9.3 Dendesia e Rrymes

$$J = \frac{I}{S}$$

9.4 Forca Elektro Motorre

$$\epsilon = \frac{A}{q} = \frac{q \cdot V}{q} = \Delta V$$

9.5 Rezistenca Elektrike

$$R = \rho \cdot \frac{l}{S}$$

9.6 Ligji i Ohmit

$$I = \frac{\epsilon}{R + r}$$

9.7 Fuqia Elektrike

$$\begin{aligned} P &= \frac{W}{\Delta t} \\ &= \frac{V \cdot \Delta Q}{\Delta T} \\ &= V \cdot I \\ &= I^2 \cdot R \\ &= \frac{V^2}{R} \end{aligned}$$

9.8 Ligji i Joul-Lencit

$$Q = I^2 \cdot R \cdot \Delta t$$

10 Qarqet elektrike

10.1 Ligji i pare i Kirkofit

"Shuma algjebrike e intensiteteve te rrymave qe hyjne ne nje pike cfaredo te qarkut jane te barabarta me shumen e intesiteteve qe dalin nga ajo pike"

$$\sum I_{in} = \sum I_{out}$$

10.2 Ligji i dyte i Krikofit

"Shuma e drejtuar e diferencave te potencialit rreth nje laku te mbyllur eshte 0"

$$\sum_{k=1}^{n} V_k = 0$$

10.3 Lidhja e Rezistencave

10.3.1 Ne Seri

$$\begin{aligned} \Delta V &= \sum V_i \\ I &= I_1 = I_2 = I_3 = \ldots = I_i \\ R &= \sum R_i \end{aligned}$$

10.3.2 Ne Paralel

$$\begin{aligned} \Delta V &= V_1 = V_2 = V_3 = \ldots = V_i \\ I &= \sum I_i \\ \tfrac{1}{R} &= \sum \tfrac{1}{R_i} \end{aligned}$$

10.4 ΔV ne skajet e burimit

$$V = \epsilon - I \cdot r$$

11 Fusha Magnetike

11.1 Induksioni

$\mu_0 = 4\pi \cdot 10^{-7} \frac{H}{m}$

$$\vec{B} = \frac{F_A}{I \cdot L \cdot \sin\theta}$$

Fusha magnetike e percjellesit drejtvizor ne largesine d

$$\vec{B} = \frac{\mu_0 \cdot I}{2 \cdot \pi \cdot d}$$

Fusha magnetike ne qender te spires me rreze r

$$\vec{B} = \frac{\mu_0 \cdot I}{2 \cdot r}$$

Fusha magnetike e bobines me gjatesi l dhe N spira

$$\vec{B} = \frac{\mu_0 \cdot I}{l} \cdot N$$

11.2 Forca e Amperit

Vepron mbi rrymen.

$$F_A = B \cdot I \cdot L \cdot \sin\theta$$

Forca e percjellesit 1 mbi percjellesin 2

$$\begin{cases} \vec{B_1} = \dfrac{\mu_0 \cdot I_1}{2 \cdot \pi \cdot d} \\ \vec{F_{1\to 2}} = \vec{B_1} \cdot I_2 \cdot l_2 \cdot \sin\theta \end{cases}$$

$$\implies \vec{F_{1\to 2}} = \frac{\mu_0 \cdot I_1}{2 \cdot \pi \cdot d} \cdot I_2 \cdot l_2 \cdot \sin\theta$$

$$\vec{F_{1\to 2}} = \frac{\mu_0 \cdot I_1 \cdot I_2}{2 \cdot \pi \cdot d} \cdot l_2 \cdot \sin\theta$$

11.3 Momenti magnetik dhe efekti rrotullues

$$\begin{aligned} M &= F \cdot d \\ &= B \cdot \sin\theta \cdot [I \cdot (L \cdot d)] \\ &= B \cdot \sin\theta \cdot [I \cdot S] \\ &= B \cdot \sin\theta \cdot P \end{aligned}$$

ku $P \to$ Momenti magnetik i spires.

11.4 Forca e Lorencit

Vepron mbi ngarkesen.

$$\begin{aligned} F_L &= F_A \\ &= B \cdot I \cdot L \cdot \sin\theta \\ &= B \cdot \frac{Q}{\Delta t} \cdot L \cdot \sin\theta \\ &= B \cdot Q \cdot \frac{L}{\Delta t} \cdot \sin\theta \\ &= B \cdot Q \cdot v \cdot \sin\theta \end{aligned}$$

11.5 Orbita e ngrkesave

$$F_q = F_L$$

$$\frac{m \cdot v^2}{r} = B \cdot Q \cdot v \cdot \sin\theta$$

$$r = \frac{m \cdot v}{Q \cdot B \cdot \sin\theta}$$

11.6 Raporti $\frac{q}{m}$

$$\frac{m \cdot v^2}{r} = B \cdot Q \cdot v \cdot \sin\theta$$

$$\frac{q}{m} = \frac{v}{r \cdot B \cdot \sin\theta}$$

$$\frac{m \cdot v^2}{2} = V \cdot q$$

$$\frac{q}{m} = \frac{v^2}{2 \cdot V}$$

$$\frac{v}{r \cdot B \cdot \sin\theta} = \frac{v^2}{2 \cdot V}$$

$$v = \frac{2 \cdot V}{B \cdot r \cdot \sin\theta}$$

$$\frac{q}{m} = \frac{2 \cdot V}{B^2 \cdot r^2 \cdot \sin^2\theta}$$

12 Induksioni Elektromagnetik

12.1 Fluksi Magnetik

$$\Phi = B_N \cdot S = B \cdot S \cdot \cos(\theta) \quad (Wb = T \cdot m^2)$$

ku $B_N \rightarrow$ Perbersja e Induksionit sipas normales se siperfaqjes.

12.2 Ligji i Faradei-Lencit

$$\epsilon = -\frac{\Delta\phi}{\Delta t}$$

12.3 Induktiviteti

$$L = \frac{d\phi}{dI} \cdot N$$

13 Rryma Alternative

$$\begin{cases} I = & I_0 \cdot \sin(\omega \cdot t) \\ U = & U_0 \cdot \sin(\omega \cdot t) \end{cases}$$

13.1 Rryma & Tensioni Efektiv

$$\begin{cases} I_{ef} = & \frac{I_0}{\sqrt{2}} \\ U_{ef} = & \frac{U_0}{\sqrt{2}} \end{cases}$$

13.2 Fuqia

$$P = V \cdot I = I^2 \cdot R$$

13.3 Transformatori

$$\frac{N_d}{N_p} = \frac{V_d}{V_p} = \frac{I_d}{I_p}$$

14 Lekundjet

14.1 Lekundjet Harmonike

$$\omega = 2 \cdot \pi \cdot f = \frac{2 \cdot \pi}{T}$$

14.1.1 Zhvendosja $x(t)$

$$x(t) = A \cdot \sin(\omega \cdot t)$$

14.1.2 Shpejtesia $v(t)$

$$v(t) = \frac{dx}{dt} = A \cdot \omega \cdot \cos(\omega \cdot t)$$

Shpejtesia Maksimale

$$v_{max} = \omega \cdot A$$

14.1.3 Nxitimi $a(t)$

$$a(t) = \frac{dv}{dt} = -A \cdot \omega^2 \cdot \sin(\omega \cdot t) = -\omega^2 \cdot x(t)$$

14.1.4 Energjia e lekundjeve

$$E = \frac{k \cdot A^2}{2}$$

14.1.5 Shpejtesia e lekundjeve ne korde

$$\mid \vec{v} \mid = \sqrt{\frac{F_t}{\frac{m}{l}}} = \sqrt{\frac{F_t \cdot l}{m}}$$

14.1.6 Frekuenca e lekundjeve te sustes

$$f = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{k}{m}}$$

15 Valët

15.1 Shpejtësia

$$v = f \cdot \lambda = \frac{\lambda}{T}$$

Shpejtesia e drites

$$c_m = \frac{c}{\sqrt{\epsilon \cdot \mu}}$$

ϵ : Pershkueshmeria dielektrike e materialit.
μ : Pershkueshmeria magnetike e materialit.

15.2 Intensiteti

$$I = \frac{P}{S}, \; I = c^{t\ddot{e}} \cdot A^2$$

15.3 Efekti Doppler

$$f_v = \frac{v + v_v}{v + v_b} \cdot f_b$$

f_v → frekuenca e vezhgusesit

v → shpejtesia e vales

v_v → shpejtesia e vezhguesit

v_b → shpejtesia e burimit

f_b → frekuenca normale e burimit

16 Mbivendosja e valeve

d : distanca midis te carave

θ : kendi midis valeve

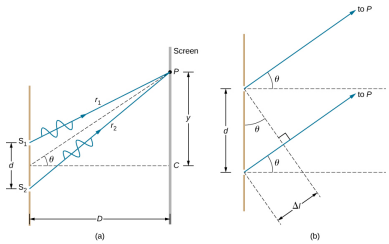
m : rendi, $m \in \mathbb{Z}$ gjendet edhe si k ose n

λ : gjatesia e vales

Δl : diferenca e rrugeve te valeve

D : distanca nga te carat tek ekrani

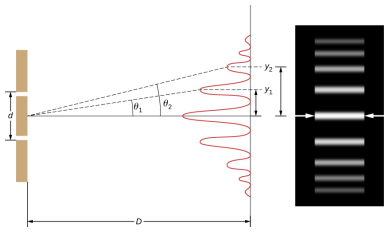
16.1 Interferenca



$$\Delta l = m \cdot \lambda$$

16.1.1 Interferenca konstruktive

$$d \cdot \sin\ \theta = m \cdot \lambda$$



$$y_m = \frac{m \cdot \lambda \cdot D}{d}$$

16.1.2 Interferenca destruktive

$$d \cdot \sin\ \theta = \left(m + \frac{1}{2}\right) \cdot \lambda$$

$$y_m = \frac{(m + \frac{1}{2}) \cdot \lambda \cdot D}{d}$$

16.2 Difraksioni

Difraksioni eshte perkulja e valeve rreth pengesave.

$$MAX \quad : \quad \begin{cases} d \cdot \sin\theta = \left(m + \dfrac{1}{2}\right) \cdot \lambda \\ y_m = \dfrac{(m + \dfrac{1}{2}) \cdot \lambda \cdot D}{d} \end{cases}$$

$$min \quad : \quad \begin{cases} d \cdot \sin\theta = m \cdot \lambda \\ y_m = \dfrac{m \cdot \lambda \cdot D}{d} \end{cases}$$

17 Optika

17.1 Perthyerja e drites

$$\frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

17.2 Thjerrat Pembledhese

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_1}$$

17.3 Thjerrat Shpehapese

$$\frac{1}{f} = \frac{1}{d_1} - \frac{1}{d_0}$$

18 Fizika Kuantike

18.1 Shpejtesia e drites

$$c = f \cdot \lambda \approx 3 \cdot 10^8 \frac{m}{s}$$

18.2 Energjia e rrezatimit elektromagnetik

$$E = h \cdot f = \frac{h \cdot c}{\lambda}$$

$h = 6.62 \cdot 10^{-34} J \cdot s$

18.3 Fotoefekti

"Emetimi i fotoneve nga siperfaqja e nje metali kur mbi te dergohet drite."

18.3.1 Ekuacioni i Ajnshtajnit

$$h \cdot f = \phi + Ek_{max} = \phi + e \cdot U$$

$\phi \rightarrow$ puna minimale e daljes e metalit

Frekuenca e pragut

$$f_{prag} = \frac{\phi}{h}$$

Gjatesia e vales e pragut

$$\lambda_{prag} = \frac{h \cdot c}{\phi}$$

Tensioni Frenues

$$E = \phi + Ek_{max}$$
$$h \cdot f = \phi + q \cdot U$$
$$U = \frac{h \cdot f - \phi}{q}$$

18.4 Gjatesia e vales se De Brojl

$$\lambda = \frac{h}{p} = \frac{h}{m \cdot v}$$

19 Relativiteti

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$