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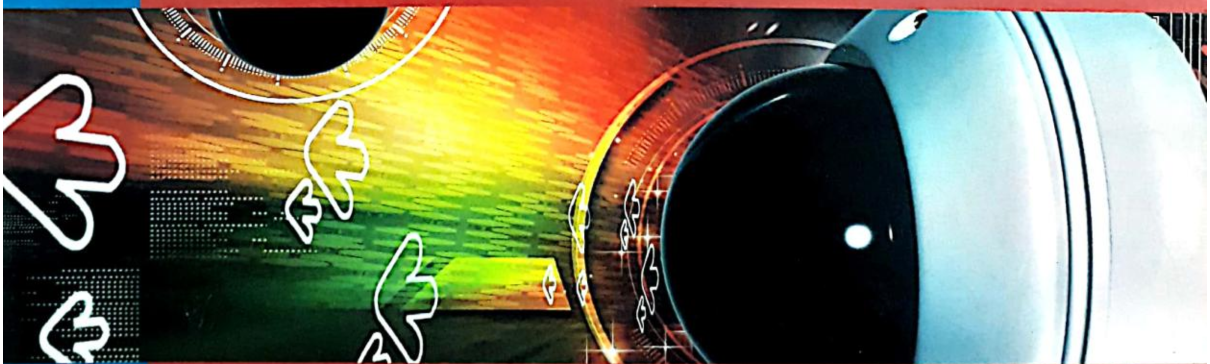
Discussions on IE Irodov's

Problems in General Physics

Discussion 2 (Electrodynamics, Oscillations & Sound, Optics & Modern Physics)



DB Singh

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DB Singh
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Inspiring Minds.Inspiring Lives

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Preface

I heartily thank the students and the teachers for their response given to Volume 2 of my book **'Discussions on IE IRODOV 'S Problems in General Physics'**. This book has achieved a great success within a very short span of time.

Many authors might have tried to give shape to such a book, but this book is one of the best of its kind which incisively clears the concepts on **Problems in General Physics**. The contents of the book are designed in a particular way having surprising methods, and made so interesting that to pitchfork students to learn more and more. I have gone in very much depth to find out minute clues that could be useful for students.

In the heading ***Concepts,*** clues are provided to clear your concept. In ***Discussions and Solutions,*** the problems are discussed in details and solutions provided step by step. In ***Your Step,*** we have provided self-made problems for the students to shake their brain to have more practice in solving any sort of problems, and that should surely help them getting more confidence. The students are advised to read the problems again and again till the concept is cleared, and then attempt to solve it.

We shall be highly thankful if the mistakes are brought to our notice that might have crept in. The same shall be rectified in the subsequent editions.

DB Singh

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3.1.

CONSTANT ELECTRIC FIELD IN VACUUM

§ 3.1

➢ CONCEPT

Gravitational force of interaction is given by

$$F_g = \frac{Gm_1m_2}{r^2} \quad \text{(In magnitude)}$$

The force of electrostatic interaction between two point charges is given by

$$F_e = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} \quad \text{(In magnitude)}$$

SOLUTION : For proton, $q_1 = q_2 = +e = 1.6 \times 10^{-19}$C,

$$m_1 = m_2 = m = 1.67 \times 10^{-27} \text{ kg}$$

$$\therefore \quad \text{Ratio} = \frac{F_e}{F_g} = \frac{\dfrac{e^2}{4\pi\varepsilon_0 r^2}}{\dfrac{Gm^2}{r^2}} = \frac{e^2}{4\pi\varepsilon_0 m^2 G}$$

Here, $G = 6.67 \times 10^{-11}$ Nm^2/kg^2, $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2/C^2

Putting the values, ratio for proton is

$$\frac{F_e}{F_g} = 4 \times 10^{42}$$

For electron, $m_1 = m_2 = m = 9.1 \times 10^{-31}$ kg, $q_1 = q_2 = -e = 1.6 \times 10^{-19}$ C

$$\therefore \quad \text{Ratio} = 10^{36}$$

For being both forces equal in magnitude,

$$F_e = F_g$$

or
$$\frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{Gm^2}{r^2}$$

or
$$\frac{q^2}{m^2} = 4\pi\varepsilon_0 G$$

or
$$\frac{q}{m} = \sqrt{4\pi\varepsilon_0 G} \quad \text{...(i)}$$

Here, $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2

Putting the values, in equation (i) $\dfrac{q}{m} = 0.86 \times 10^{-10}$ C/kg

Remarks : Coulomb's law of electrostatic interaction and Newton's law of gravitation are only applicable for point body.

YOUR STEP

1. Two positive point charge q_1 and q_2 are placed at a distance r apart in air. If a dielectric slab of thickness t ($< r$) and relative permittivity K is put between the charges. Find the force of electrostatic interaction between the charges.
2. Two point charges q_1 and q_2 are separated by a distance l apart and the space between them is filled with a medium of variable dielectric constant. Near the charge q_1 dielectric constant is K_1 and increases linearly with the distance from q_1 and becomes K_2 near the charge q_2. Find the force of interaction between the charges.

$$\left\{ 1.\ F = \frac{q_1 q_2}{4\pi\varepsilon_0 \{r + t(\sqrt{K} - 1)\}^2} \qquad 2.\ F = \frac{q_1 q_2}{4\pi\varepsilon_0 l^2} \frac{\left(ln \frac{K_2}{K_1}\right)}{(K_2 - K_1)} \right\}$$

§ 3.2

➢ **CONCEPT**

For solving the problem, spheres are assumed as point charges.

SOLUTION : The atomic weight of copper is 63.55 g

The number of atoms in 63.55 g $= 6.022 \times 10^{23}$ atoms

∴ The number of atoms in 1 g of copper is

$$N_1 = \frac{6.022 \times 10^{23}}{63.55}$$

The number of protons in 1 g of copper is

$$n_1 = N_1 \times \text{atomic number of copper} = 29N_1$$

∴ Total charge on the nuclei of 1 g of copper atoms is

$$q_1 = n_1 e = 29N_1 e$$

According to problem, charge on each sphere is

$$q = q_1 \times 1\% = \frac{q_1}{100}$$

$$= \frac{29N_1 e}{100} = \frac{29e}{100} \times \frac{6.022 \times 10^{23}}{63.55}$$

∴
$$F = \frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{9 \times 10^9 \left(\frac{29e \times 6.022 \times 10^{23}}{63.55}\right)^2}{(1)^2}$$

Here, $e = 1.6 \times 10^{-19}$ C

Putting the value of e, $F = 1.74 \times 10^{15}$ N

YOUR STEP

A particle of mass m having negative charge q moves along an ellipse around a fixed positive charge Q so that its maximum and minimum distances from fixed charge are equal to r_1 and r_2 respectively. Calculate angular momentum L of this particle.

$$\left\{ \sqrt{\frac{m r_1 r_2 Q q}{2\pi\varepsilon_0 (r_1 + r_2)}} \right\}$$

§ 3.3

➢ **CONCEPT**

All arrangements of problem are shown in the Fig. 3.3 (A)

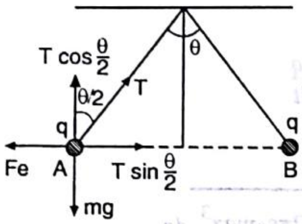


Fig. 3.3A

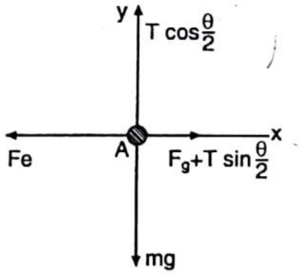


Fig. 3.3B

For equilibrium condition of sphere A, net force on the sphere is zero. Forces acting on sphere A are.

(a) Electrostatic force, $F_e = \dfrac{q^2}{4\pi\varepsilon_0 x^2}$ along BA

(b) Gravitational force of interaction between spheres,

$$F_g = \frac{Gm^2}{r^2} \text{ along } AB$$

(c) Weight of the sphere, $A = w = mg$ in downward direction.

(d) The force of tension $= T$

For equilibrium of spheres A, $\Sigma F_x = 0$, $\Sigma F_y = 0$

$\therefore$ $$F_e = F_g + T\sin\frac{\theta}{2}$$

or $$F_e - F_g = T\sin\frac{\theta}{2}$$

But $$F_g <<< F_e$$

So, F_g can be neglected with respect to F_e.

$\therefore$ $$T\sin\frac{\theta}{2} = F_e \quad \text{...(i)}$$

Also, $$T\cos\frac{\theta}{2} = mg \quad \text{...(ii)}$$

$\therefore$ $$\frac{T\sin\frac{\theta}{2}}{T\cos\frac{\theta}{2}} = \frac{F_e}{mg} \quad \text{or} \quad \tan\frac{\theta}{2} = \frac{q^2}{4\pi\varepsilon_0 mgx^2}$$

According to geometry of figure, $$\sin\frac{\theta}{2} = \frac{x}{2l}$$

Since, $$x << l,$$

$\therefore$ $$\sin\frac{\theta}{2} = \frac{\theta}{2} = \tan\frac{\theta}{2} = \frac{x}{2l}$$

$\therefore$ $$\frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 mgx^2}$$

or $$q^2 = \frac{2\pi\varepsilon_0 mgx^3}{l} \quad \text{...(iii)}$$

or $$x^3 = \frac{q^2 l}{2\pi\varepsilon_0 mg}$$

or $$\frac{d(x^3)}{dt} = \frac{l}{2\pi\varepsilon_0 mg}\frac{d(q^2)}{dt}$$

or $$3x^2\frac{dx}{dt} = \frac{l}{2\pi\varepsilon_0 mg}(2q)\frac{dq}{dt}$$

$$3x^2 v = \frac{l}{2\pi\varepsilon_0 mg}2q\frac{dq}{dt}$$

or $$\frac{3}{2}x^2 v = \left(\frac{l}{2\pi\varepsilon_0 mg}\right)\sqrt{\frac{2\pi\varepsilon_0 mgx^3}{l}}\frac{dq}{dt} \quad \text{(from Eq. iii)}$$

or $$\frac{3}{2}x^2\frac{a}{\sqrt{x}} = \sqrt{\frac{l}{2\pi\varepsilon_0 mg}}\,x^{3/2}\frac{dq}{dt} \quad \left(\because v = \frac{a}{\sqrt{x}}\right)$$

$\therefore$ $$\frac{dq}{dt} = \frac{3}{2}a\sqrt{\frac{2\pi\varepsilon_0 mg}{l}}$$

YOUR STEP

Three identically charged, small spheres each of mass m are suspended from a common point by insulated light strings each of length l. The spheres are always on vertices of an equilateral triangle of length of the sides x ($<< l$). Calculate the rate dq/dt with which charge on each sphere increases, if length of the sides of the equilateral triangle increases slowly according to law $\frac{dx}{dt} = \frac{a}{\sqrt{x}}$

$\left\{\sqrt{\frac{3\pi\varepsilon_0 mga^2}{l}}\right\}$

§ 3.4

➢ **CONCEPT**

The electric force on q_1 due to q_2 is

$$\vec{F}_{12} = \frac{q_1 q_2(\vec{r}_1 - \vec{r}_2)}{4\pi\varepsilon_0|\vec{r}_1 - \vec{r}_2|^3}, \quad \vec{F}_{31} = \frac{q_3 q_1(\vec{r}_3 - \vec{r}_1)}{4\pi\varepsilon_0|\vec{r}_3 - \vec{r}_1|^3}, \quad \vec{F}_{32} = \frac{q_3 q_2(\vec{r}_3 - \vec{r}_2)}{4\pi\varepsilon_0|\vec{r}_3 - \vec{r}_2|^3}$$

Similarly, $$\vec{F}_{13} = \frac{q_1 q_3(\vec{r}_1 - \vec{r}_3)}{4\pi\varepsilon_0|\vec{r}_1 - \vec{r}_3|^3}$$

and $$\vec{F}_{23} = \frac{q_2 q_3(\vec{r}_2 - \vec{r}_3)}{4\pi\varepsilon_0|\vec{r}_2 - \vec{r}_3|^3}$$

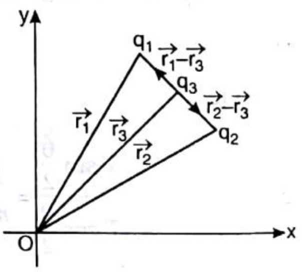


Fig. 3.4A

SOLUTION :

For equilibrium of q_3, $$\vec{F}_{31} + \vec{F}_{32} = 0$$

or $$\frac{q_3 q_1(\vec{r}_3 - \vec{r}_1)}{4\pi\varepsilon_0|\vec{r}_3 - \vec{r}_1|^3} + \frac{q_3 q_2(\vec{r}_3 - \vec{r}_2)}{4\pi\varepsilon_0|\vec{r}_3 - \vec{r}_2|^3} = 0 \quad \text{...(i)}$$

For equilibrium of q_3, the unit vector along. $\vec{F}_{31}$ should be in opposite direction of unit vector along $\vec{F}_{32}$

i.e., $$\frac{\vec{F}_{31}}{|\vec{F}_{31}|} = \frac{-\vec{F}_{32}}{|\vec{F}_{32}|}$$

But $$\vec{F}_{31} \parallel (\vec{r}_3 - \vec{r}_1)$$

So, unit vector along $\vec{F}_{31}$ will be same as unit vector along $(\vec{r}_3 - \vec{r}_1)$

$$\therefore \qquad \frac{\vec{F}_{31}}{|\vec{F}_{31}|} = \frac{(\vec{r}_3 - \vec{r}_1)}{|\vec{r}_3 - \vec{r}_1|}$$

Also,
$$\vec{F}_{32} \parallel (\vec{r}_3 - \vec{r}_2)$$

$$\therefore \qquad \frac{\vec{F}_{32}}{|\vec{F}_{32}|} = \frac{\vec{r}_3 - \vec{r}_2}{|\vec{r}_3 - \vec{r}_2|}$$

But
$$\frac{\vec{F}_{32}}{|\vec{F}_{32}|} = -\frac{\vec{F}_{31}}{|\vec{F}_{31}|}$$

or
$$\frac{\vec{r}_3 - \vec{r}_2}{|\vec{r}_3 - \vec{r}_2|} = -\frac{(\vec{r}_3 - \vec{r}_1)}{|\vec{r}_3 - \vec{r}_1|} \qquad \text{...(ii)}$$

From Eq. (i) and (ii), we get

$$\therefore \qquad \frac{q_3 q_1 (\vec{r}_3 - \vec{r}_1)}{4\pi\varepsilon_0 |\vec{r}_3 - \vec{r}_1|^3} + \frac{q_2 q_3 (\vec{r}_3 - \vec{r}_2)}{4\pi\varepsilon_0 |\vec{r}_3 - \vec{r}_2|^3} = 0$$

or
$$\frac{q_1 q_3}{4\pi\varepsilon_0 |\vec{r}_3 - \vec{r}_1|^2} \frac{\vec{r}_3 - \vec{r}_1}{|\vec{r}_3 - \vec{r}_1|} = \frac{-q_2 q_3}{4\pi\varepsilon_0 |\vec{r}_3 - \vec{r}_2|^2} \frac{\vec{r}_3 - \vec{r}_2}{|\vec{r}_3 - \vec{r}_2|}$$

or
$$\frac{q_1}{|\vec{r}_3 - \vec{r}_1|^2} = \frac{q_2}{|\vec{r}_3 - \vec{r}_2|^2}$$

or
$$\frac{\sqrt{q_1}}{|\vec{r}_3 - \vec{r}_2|} = \frac{\sqrt{q_2}}{|\vec{r}_3 - \vec{r}_2|}$$

or
$$\sqrt{q_1}\, |\vec{r}_3 - \vec{r}_2| = \sqrt{q_2}\, |\vec{r}_3 - \vec{r}_1| \qquad \left(\because \frac{\vec{r}_3 - \vec{r}_1}{|\vec{r}_3 - \vec{r}_1|} = -\frac{(\vec{r}_3 - \vec{r}_2)}{|\vec{r}_3 - \vec{r}_2|} \right)$$

or
$$\sqrt{q_1}\, (\vec{r}_3 - \vec{r}_2) = \sqrt{q_2}\, [-(\vec{r}_3 - \vec{r}_2)]$$

or
$$\vec{r}_3 (\sqrt{q_1} + \sqrt{q_2}) = \sqrt{q_1}\, \vec{r}_2 + \sqrt{q_2}\, \vec{r}_1$$

$$\therefore \qquad \vec{r}_3 = \frac{\sqrt{q_1}\, \vec{r}_2 + \sqrt{q_2}\, \vec{r}_1}{\sqrt{q_1} + \sqrt{q_2}} \qquad \text{...(iii)}$$

Similarly for equilibrium for q_1,

$$\vec{r}_1 = \frac{\sqrt{q_3}\, \vec{r}_2 + \sqrt{q_2}\, \vec{r}_3}{\sqrt{q_2} + \sqrt{q_3}} \qquad \text{...(iv)}$$

On putting the value of $\vec{r}_3$ from Eq. (iii) in Eq. (iv), we get

$$\therefore \qquad \vec{r}_1 = \frac{\sqrt{q_3}\, \vec{r}_2 + \sqrt{q_2}\, \vec{r}_3}{\sqrt{q_2} + \sqrt{q_3}}$$

or
$$\vec{r}_1 (\sqrt{q_2} + \sqrt{q_3}) = \sqrt{q_3}\, \vec{r}_2 + \sqrt{q_2}\, \vec{r}_3$$

or
$$\vec{r}_1 (\sqrt{q_2} + \sqrt{q_3}) = \sqrt{q_3}\, \vec{r}_2 + \sqrt{q_2} \left\{ \frac{\sqrt{q_1}\, \vec{r}_2 + \sqrt{q_2}\, \vec{r}_1}{(\sqrt{q_1} + \sqrt{q_2})} \right\}$$

or
$$\vec{r}_1 (\sqrt{q_2} + \sqrt{q_3}) = \sqrt{q_3}\, \vec{r}_2 + \sqrt{q_2} \left\{ \frac{\sqrt{q_1}\, \vec{r}_2 + \sqrt{q_2}\, \vec{r}_1}{(\sqrt{q_1} + \sqrt{q_2})} \right\}$$

or
$$\vec{r}_1 (\sqrt{q_2} + \sqrt{q_3}) = \frac{\sqrt{q_3}\, (\sqrt{q_1} + \sqrt{q_2})\, \vec{r}_2 + \sqrt{q_2}\, \{\sqrt{q_1}\, \vec{r}_2 + \sqrt{q_2}\, \vec{r}_1\}}{(\sqrt{q_1} + \sqrt{q_2})}$$

or $$\vec{r_1}(\sqrt{q_2}+\sqrt{q_3})(\sqrt{q_1}+\sqrt{q_2})=\sqrt{q_3}(\sqrt{q_1}+\sqrt{q_2})\vec{r_2}+\sqrt{q_2}\{\sqrt{q_1}\vec{r_2}+\sqrt{q_2}\vec{r_1}\}$$

On solving, we get $$q_3=\frac{-q_1q_2}{(\sqrt{q_1}+\sqrt{q_2})^2}$$

and from Eq. (iii), we get $$\vec{r_3}=\frac{\vec{r_1}\sqrt{q_2}+\vec{r_2}\sqrt{q_1}}{\sqrt{q_1}+\sqrt{q_2}}$$

YOUR STEP

Two equal positive point charges q are held a fixed distance $2a$ apart. A point test charge is located in a plane that is normal to the line joining these charges and midway between them. Find the radius R of the circle in this plane for which the force on the test particle has a maximum value see Fig. 3.4(B). $\left\{\frac{a}{\sqrt{2}}\right\}$

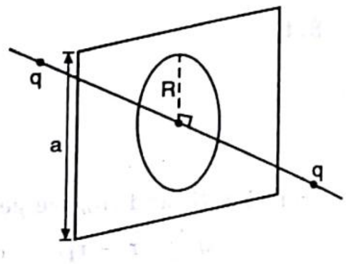


Fig. 3.4B

§ 3.5

➢ **CONCEPT**

Due to force of electrical interaction, ring has a tendency to expand. So, a force of tension comes into play which balances the electrical forces acted on different element of ring.

SOLUTION : For calculation of force of tension, we consider an element $AB=\Delta l$ on the circumference of ring.

The charge on considered element is $\Delta q=\lambda\Delta l$ where, $\lambda=\frac{q}{2\pi r}$

The force of interaction between considered element on ring and charge q_0 is.

$$F_e=\frac{q_0\,\Delta q}{4\pi\varepsilon_0 r^2}=\frac{q_0\,\lambda\Delta l}{4\pi\varepsilon_0 r^2}$$

Here, $$\Delta l=r\theta$$

$\therefore$ $$F_e=\frac{q_0\lambda\theta}{4\pi\varepsilon_0 r}$$

From Fig. 3.5(a), $$2T\sin\frac{\theta}{2}=F_e \qquad \left\{\because \sin\frac{\theta}{2}=\frac{\theta}{2}\right\}$$

or $$T\theta=\frac{q_0\lambda\theta}{4\pi\varepsilon_0 r}$$

$\therefore$ $$T=\frac{q_0\lambda}{4\pi\varepsilon_0 r}$$

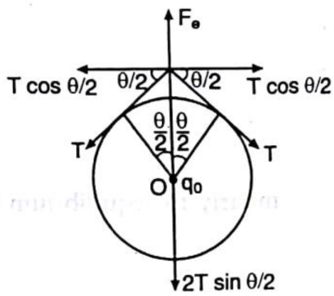


Fig. 3.5A

Putting the value of $\lambda=\frac{q}{2\pi r}$, $$T=\frac{q_0q}{8\pi^2\varepsilon_0 r^2}$$

YOUR STEP

Two point charges Q_1 and Q_2 are situated as shown in Fig. 3.5(B). Q_1 is a positive charge and Q_2 is a negative charge; the magnitude of Q_1 is greater than Q_2. A third point charge, which is positive, is now placed in such a position, x, that it experiences no resultant electrostatic force due to Q_1 and Q_2. Explain carefully why x must lie somewhere on the line AB which passes through Q_1 and Q_2.

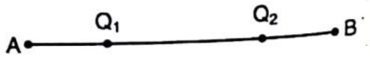


Fig. 3.5B

§ 3.6

➢ **CONCEPT**

The ~~electric~~ field at a point P due to a point charge is given by

$$\vec{E} = \frac{q\,(\vec{r}_p - \vec{r}_0)}{4\pi\varepsilon_0\,|\vec{r}_p - \vec{r}_0|^3}$$

where, $\vec{r}_0$ = position vector of point charge q, $\vec{r}_p$ = position vector of the point P.

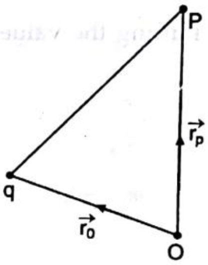


Fig. 3.6A

SOLUTION : According to problem, $q = 50 \times 10^{-6}$ C

$$\vec{r}_p = \vec{r} = 8\,\hat{i} - 5\hat{j}$$
$$\vec{r}_0 = 2\hat{i} + 3\hat{j}$$

$\therefore$ $$\vec{E} = \frac{9 \times 10^9 \times 50 \times 10^{-6}\,(8\hat{i} - 5\hat{j} - 2\hat{i} - 3\hat{j})}{|8\hat{i} - 5\hat{j} - 2\hat{i} - 3\hat{j}|^3}$$

$\therefore$ $$\vec{E} = (2.7 \times 10^3\,\hat{i} - 3.6 \times 10^3\,\hat{j}) \text{ (In V/m)}$$

$\therefore$ $$\vec{E} = (2.7\,\hat{i} - 3.6\,\hat{j}) \text{ (In kV/m)}$$

$\therefore$ $$E = \sqrt{(2.7)^2 + (-3.6)^2} = 4.5 \text{ kV/m}$$

Magnitude of electric field $\vec{E}$ is $|\vec{E}| = E = \sqrt{(2.7)^2 + (-3.6)^2} = 4.5$ kV/m

YOUR STEP

A point charge of 2 μC lies at point $A\,(3^m, -4^m, 7^m)$. Find the electric field at a point $P\,(10^m, 2^m, 6^m)$.

$$\left\{\vec{E}_p = \frac{2 \times 10^{-6}}{4\pi\varepsilon_0}\,\frac{(7\hat{i} + 6\hat{j} - \hat{k})}{86\sqrt{86}} \text{ N/C}\right\}$$

§ 3.7

➢ **CONCEPT**

For solving this problem, superposition principle of electric field is applicable.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

$$= \frac{q_1\,(\vec{r}_p - \vec{r}_1)}{4\pi\varepsilon_0\,|\vec{r}_p - \vec{r}_1|^3} + \frac{q_2\,(\vec{r}_p - \vec{r}_2)}{4\pi\varepsilon_0\,|\vec{r}_p - \vec{r}_2|^3} + \frac{q_3\,(\vec{r}_p - \vec{r}_3)}{4\pi\varepsilon_0\,|\vec{r}_p - \vec{r}_3|^3} + \frac{q_4\,(\vec{r}_p - \vec{r}_4)}{4\pi\varepsilon_0\,|\vec{r}_p - \vec{r}_4|^3}$$

SOLUTION : The arrangement of charges is shown in Fig. 3.7A.

According to problem, $q_1 = +q$, $\vec{r}_1 = l\hat{j}$, $q_2 = +q$, $\vec{r}_2 = l\,\hat{i}$

$$q_3 = -q, \quad \vec{r}_3 = -l\,\hat{i}\;;\; q_4 = -q, \quad \vec{r}_4 = -l\,\hat{j}$$
$$\vec{r}_p = x\,\hat{k}$$

$\therefore$ $\vec{r}_p - \vec{r}_1 = x\,\hat{k} - l\,\hat{i}$, $|\vec{r}_p - \vec{r}_1| = \sqrt{x^2 + l^2} = (x^2 + l^2)^{1/2}$

$\vec{r}_p - \vec{r}_2 = x\,\hat{k} - l\hat{j}$, $|\vec{r}_p - \vec{r}_2| = (x^2 + l^2)^{1/2}$

$\vec{r}_p - \vec{r}_3 = x\,\hat{k} + l\,\hat{i}$, $|\vec{r}_p - \vec{r}_3| = (x^2 + l^2)^{1/2}$

and $\vec{r}_p - \vec{r}_4 = x\,\hat{k} + l\hat{j}$, $|\vec{r}_p - \vec{r}_4| = (x^2 + l^2)^{1/2}$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

Fig. 3.7A

or $$\vec{E}=\frac{q_1(\vec{r_p}-\vec{r_1})}{4\pi\varepsilon_0\,|\vec{r_p}-\vec{r_1}|^3}+\frac{q_2(\vec{r_p}-\vec{r_2})}{4\pi\varepsilon_0\,|\vec{r_p}-\vec{r_2}|^3}+\frac{q_3(\vec{r_p}-\vec{r_3})}{4\pi\varepsilon_0\,|\vec{r_p}-\vec{r_3}|^3}+\frac{q_4(\vec{r_p}-\vec{r_4})}{|\vec{r_p}-\vec{r_4}|^3}$$

Putting the values, we get $$\vec{E}=\frac{q}{4\pi\varepsilon_0(x^2+l^2)^{3/2}}\{(x\hat{k}-l\hat{i})+(x\hat{k}-l\hat{j})-(x\hat{k}+l\hat{i})-(x\hat{k}+l\hat{j})\}$$

$$=\frac{q}{4\pi\varepsilon_0(x^2+l^2)^{3/2}}(-2l\hat{j}-2l\hat{i})$$

$\therefore$ $$E=\frac{q}{4\pi\varepsilon_0(x^2+l^2)^{3/2}}\sqrt{(-2l)^2+(-2l)^2}$$

$\therefore$ $$E=\frac{2\sqrt{2}\,lq}{4\pi\varepsilon_0(x^2+l^2)^{3/2}}=\frac{ql}{\sqrt{2}\pi\varepsilon_0(x^2+l^2)^{3/2}}$$

YOUR STEP

A cube of side 'a' has a charge q at seven of its vertices. Determine the electric field and potential at the vertex without charge.

$$\left\{E=\frac{q}{4\pi\varepsilon_0 a^2}\left(\frac{6\sqrt{3}+3\sqrt{6}+2}{6}\right);\ V=\frac{q}{24\pi\varepsilon_0 a}(18+9\sqrt{2}+2\sqrt{3})\right\}$$

§ 3.8

➢ CONCEPT

Let a circular arc of radius R making an angle α at the centre of curvature of the arc. The arc is uniformly charged with charge q.

The arc is shown in Fig. 3.8A.

An element of charge dq is considered on the arc (shown in Fig. 3.8B).

Axis yy' is bisector of angle α.

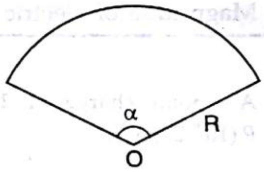


Fig. 3.8A

The electric field at point O due to considered element is

$$dE=\frac{dq}{4\pi\varepsilon_0 R^2}$$

According to Fig. 3.8B, $$dq=\frac{q}{\alpha}d\theta$$

Also, $$d\vec{E}=-dE\cos\theta\,\hat{j}-dE\sin\theta\,\hat{i}$$

$$=\frac{-dq}{4\pi\varepsilon_0 R^2}\cos\theta\,\hat{j}-\frac{dq}{4\pi\varepsilon_0 R^2}\sin\theta\,\hat{i}$$

$$=\frac{-q}{4\pi\varepsilon_0 R^2\alpha}\cos\theta\,d\theta\,\hat{j}-\frac{q}{4\pi\varepsilon_0 R^2\alpha}\sin\theta\,d\theta\,\hat{i}$$

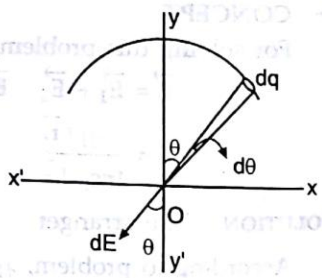


Fig. 3.8B

$\therefore$ The component of electric field along y-axis is

$$E_y=\frac{-q}{4\pi\varepsilon_0 R^2\alpha}\int_{-\alpha/2}^{+\alpha/2}\cos\theta\,d\theta=\frac{-q}{4\pi\varepsilon_0 R^2\alpha}[\sin\theta]_{-\alpha/2}^{\alpha/2}$$

or $$E_y=\frac{-q}{4\pi\varepsilon_0 R^2\alpha}\left[\sin\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right]=\frac{-q}{2\pi\varepsilon_0 R^2\alpha}\sin\frac{\alpha}{2}$$

Also, $$E_x=\int_{-\alpha/2}^{+\alpha/2}-dE\sin\theta=-\int_{-\alpha/2}^{+\alpha/2}\frac{dq}{4\pi\varepsilon_0 R^2}\sin\theta$$

$$=\frac{-q}{4\pi\varepsilon_0\alpha R^2}\int_{-\alpha/2}^{+\alpha/2}\sin\theta\,d\theta=\frac{-q}{4\pi\varepsilon_0\alpha R^2}[-\cos\theta]_{-\alpha/2}^{+\alpha/2}$$

$\therefore$ $E_x = 0$

$\therefore$ Net electric field is $E = \sqrt{E_x^2 + E_y^2}$

$\therefore$ $E = \dfrac{q}{2\pi\alpha\varepsilon_0 R^2} \sin\dfrac{\alpha}{2}$ (along bisector of angle α)

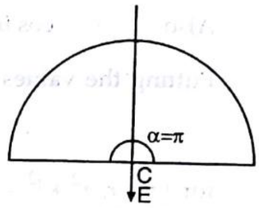


Fig. 3.8C

SOLUTION : For semi-circular ring, $\alpha = \pi$

$\because$ $E = \dfrac{q}{2\pi\alpha\varepsilon_0 R^2} \sin\dfrac{\alpha}{2}$

$\therefore$ $E = \dfrac{q}{2\pi^2\varepsilon_0 R^2} \sin\dfrac{\pi}{2} = \dfrac{q}{2\pi^2\varepsilon_0 R^2}$ (shown in Fig. 3.8 C).

Putting the values, we get $E = 0.1$ kV/m

YOUR STEP

The strength of the electric field produced by charges uniformly distributed over the surface of a hemisphere at its centre O is E_0. A part of the surface is isolated from this hemisphere by two planes passing through the same diameter and forming an angle α with each other as shown in Fig. 3.8(D).

Determine the electric field strength E produced at the same point O by the charges located on the isolated surface [on the "mericarp"].

$\left\{E = E_0 \sin\left(\dfrac{\alpha}{2}\right)\right\}$

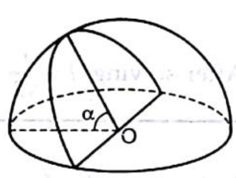


Fig. 3.8D

§ 3.9

➢ **CONCEPT**

The electric field at a point due to a continuous distribution of charge is obtained by integration.

Mathematically, $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int \dfrac{dq}{r^3}\vec{r}$

where dq is the charge on small element of the charge distribution.

SOLUTION : We consider an element dq on the circular ring (shown in Fig. 3.9A).

$\therefore$ $dE = \dfrac{dq}{4\pi\varepsilon_0 r^2}$

From Fig. 3.9A, $d\vec{E} = dE\cos\theta\,\hat{i} - dE\sin\theta\,\hat{j}$

$\therefore$ $dE_x = dE\cos\theta$

and $dE_y = -\,dE\sin\theta$

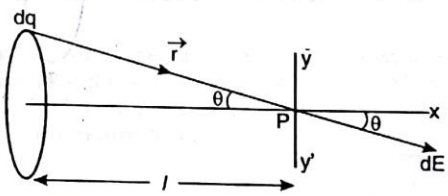


Fig. 3.9A

But according to symmetry of figure (shown in Fig. 3.9B).

$$E_y = \int dE_y = 0$$

$\therefore$ Net electric field at point P is

$$E = E_x = \int_{q=0}^{q=q} dE_x = \int_{q=0}^{q=q} dE\cos\theta$$

$\therefore$ $E = \displaystyle\int_0^q \dfrac{dq}{4\pi\varepsilon_0 R^2}\cos\theta = \dfrac{\cos\theta}{4\pi\varepsilon_0 R^2}\int_0^q dq$

$\therefore$ $E = \dfrac{q\cos\theta}{4\pi\varepsilon_0 R^2}$

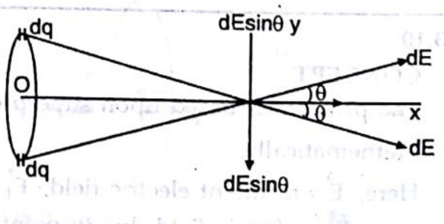


Fig. 3.9B

From Fig. 3.9A, $R^2 = OP^2 + r^2$, $R^2 = l^2 + r^2$ $(\because OP = l)$

Also, $\cos\theta = \frac{l}{\sqrt{l^2 + r^2}}$

Putting the values of r and $\cos\theta$, we get

$$E = \frac{ql}{4\pi\varepsilon_0 (r^2 + l^2)^{3/2}} \text{ along the axis of ring.}$$

for $l >> r$, $r^2 + l^2 \approx l^2$

$\therefore$ $$E \approx \frac{ql}{4\pi\varepsilon_0 l^3} = \frac{q}{4\pi\varepsilon_0 l^2}$$

For E_{max}, $\frac{dE}{dL} = 0$

or $$\frac{d}{dl}\left\{\frac{ql}{4\pi\varepsilon_0 (r^2 + l^2)^{3/2}}\right\} = 0$$

After solving, $l = \frac{r}{\sqrt{2}}$ $\therefore$ $E_{max} = \frac{q}{6\sqrt{3}\pi\varepsilon_0 r^2}$

YOUR STEP

1. A ring made of wire with a radius of $R = 10$ cm is charged negatively and carries a charge of $Q = -5 \times 10^{-9}$ C.
   (a) Find the intensity of an electric field on the axis of the ring at points lying at a distance of l equal to 0, 5, 8, 10 and 15 cm from the ring centre. Plot a diagram $E = f(l)$.
   (b) At what distance l from the ring centre will the intensity of the electric field be maximum?
2. An electron is constrained to move along the axis of the ring of charge q as shown in Fig. 3.9(C). Show that the electron can perform small oscillations, through the centre of the ring, with a frequency given by $\omega = \sqrt{\frac{eq}{4\pi\varepsilon_0 mR^3}}$.

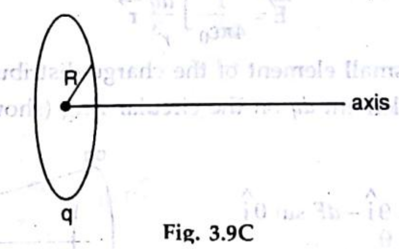


Fig. 3.9C

1. (a) $E = \frac{Q}{4\pi\varepsilon_0\varepsilon_r l^2}$,
Putting $l = 0, 5, 8, 10, 15$ cm we get,
$E = 0, 1600, 1710, 1600, 1150$ V/m
(b) $l = 7.1 \times 10^{-2}$ m

§ 3.10

➢ **CONCEPT**

The problem is based upon superposition principle of electric field.

Mathematically, $\vec{E} = \vec{E_1} + \vec{E_2}$

Here, $\vec{E}$ = resultant electric field, $\vec{E_1}$ = electric field due to ring

$\vec{E_2}$ = electric field due to point charge on the centre of ring.

According to superposition principle $\vec{E}_1$ and $\vec{E}_2$ are physically independent to each other.

SOLUTION : The electric field at point P due to point charge q is

$$\vec{E}_2 = \frac{q}{4\pi\varepsilon_0 x^2}\,\hat{i} \quad \text{(along axis of ring)}$$

Electric field at point P due to ring is

$$\vec{E}_1 = \frac{-qx}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}}\,\hat{i}$$

$\cdot$(from the solution of problem 3.10A)

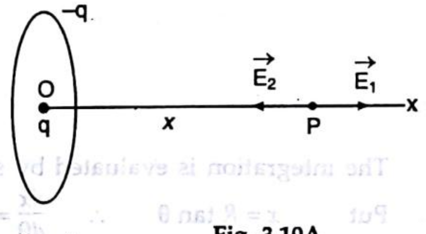


Fig. 3.10A

$\therefore$
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x^2} - \frac{x}{(R^2 + x^2)^{3/2}}\right]\hat{i}$$

or
$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{(R^2 + x^2)^{3/2} - x^3}{x^2 (R^2 + x^2)^{3/2}}\right]\hat{i} = \frac{q}{4\pi\varepsilon_0}\left[\frac{x^3\left(1 + \frac{R^2}{x^2}\right)^{3/2} - x^3}{x^2 (R^2 + x^2)^{3/2}}\right]$$

or
$$\vec{E} = \frac{\frac{q}{4\pi\varepsilon_0}\left[x^3\left(1 + \frac{3R^2}{2x^2} + \ldots\right) - x^3\right]}{x^2 (R^2 + x^2)^{3/2}}$$

Since, $x >> R$, so in binomial expansion higher power may be neglected.

$\therefore$
$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\,\frac{\left[\frac{3R^2 x}{2}\right]\hat{i}}{x^2 (R^2 + x^2)^{3/2}}$$

But R^2 may be neglected with respect to x^2.

$\therefore$
$$\vec{E} = \frac{3qR^2 x}{8\pi\varepsilon_0 x^5} = \frac{3qR^2}{8\pi\varepsilon_0 x^4}$$

Remarks : The answer of this problem in answer sheet of the book 'I.E. Irodov' is wrong.

YOUR STEP

A circular ring of radius R with uniform positive charge density λ per unit length is located in the $y-z$ plane with its centre at the origin. A particle of mass m and positive charge q is projected from the point P $(\sqrt{3}R, 0, 0)$ on the positive x-axis directly towards O, with initial speed v. Find the smallest (non zero) value of the speed such that the particle does not return to P.

$$\left\{v_{\min} = \sqrt{\left(\frac{\lambda q}{2\varepsilon_0 m}\right)}\right\}$$

§ 3.11

➢ **CONCEPT**

The problem is based upon interaction between two definite shape bodies. Coulomb's law is not directly applicable for such types of problems. In this case, applicable formula is

$$F = \int E\,dq$$

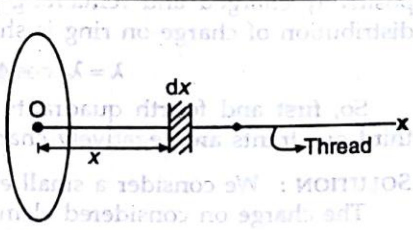


Fig. 3.11A

SOLUTION : We have to find force of interaction between the ring and the thread. The situation of problem is shown in Fig. 3.11A. From symmetry of figure, electric field due to ring at every point of thread is directed along the x-axis. We consider an element dx on thread which carries the charge $dq = \lambda dx$.

The electrical force experienced by considered element is $dF = Edq$ along x-axis.

But electric field due to ring at the site of considered element is

$$E = \frac{qx}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}} \quad \text{(from the solution of 3.9)}$$

$$\therefore \quad F = \int_{x=0}^{x=\infty} Edq = \int_0^\infty \frac{\lambda qx dx}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}}$$

The integration is evaluated by substitution method.

Put $x = R \tan\theta \quad \therefore \quad \frac{dx}{d\theta} = R\frac{d(\tan\theta)}{d\theta}$ or $\frac{dx}{d\theta} = R\sec^2\theta$

$$\therefore \quad dx = R\sec^2\theta \, d\theta$$

Conversion of limits : For lower limit, $x = 0 = R\tan\theta \Rightarrow \theta = 0$

For upper limit, $x = \infty = R\tan\theta$ or $\tan\theta = \infty \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore \quad F = \int_0^{\pi/2} \frac{q\lambda (R\tan\theta)(R\sec^2\theta\, d\theta)}{4\pi\varepsilon_0 (R^2 + R^2\tan^2\theta)^{3/2}} \quad \left\{\begin{aligned} x &= R\tan\theta \\ dx &= R\sec^2\theta\, d\theta \end{aligned}\right\}$$

$$= \frac{q\lambda}{4\pi\varepsilon_0}\int_0^{\pi/2} \frac{R^2 \tan\theta \sec^2\theta\, d\theta}{R^3 \sec^3\theta}$$

$$= \frac{q\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2} \sin\theta\, d\theta = \frac{q\lambda}{4\pi\varepsilon_0 R}[-\cos\theta]_0^{\pi/2}$$

$$\therefore \quad F = \frac{q\lambda}{4\pi\varepsilon_0 R}$$

Remarks : Coulomb's law of electrostatic interaction is only applicable in the case of point charges.

YOUR STEP

Two thin straight uniform rods AB and CD of oppositely charged are pivoted together at their middle points. Show that the attraction between them reduces to a couple of moment.

$$\frac{\lambda\lambda' (AC - BC) \operatorname{cosec}\alpha}{2\pi\varepsilon_0}$$

where, λ and λ' are their linear charge densities and α is the angle between them.

§ 3.12

➢ CONCEPT

The meaning of azimuthal angle is the angle with radius. Since, λ is cosine function of ϕ. So, ring is not uniformly charged. A part of ring is positively charged and remaining part of ring is negatively charged. The distribution of charge on ring is shown in Fig. 3.12A.

$$\because \quad \lambda = \lambda_0 \cos\phi$$

So, first and fourth quadrants are positively charged but second and third quadrants are negatively charged.

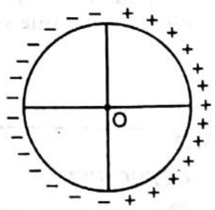


Fig. 3.12A

SOLUTION : We consider a small element $dx = Rd\phi$ on the ring shown in Fig. 3.12B.

The charge on considered element is

$$dq = \lambda dx = \lambda_0 \cos\phi\, Rd\phi = \lambda_0 R\cos\phi\, d\phi$$

$\therefore$ Net electric field at point P is

$$E = E_1 + E_2 = \frac{\lambda_0 R^2}{4\varepsilon_0\,(R^2 + x^2)^{3/2}}$$

when $x >> R$, R^2 may be neglected with respect to x^2.

$$\therefore \quad E = \frac{\lambda_0 R^2}{4\varepsilon_0 x^3} \quad \text{when } x >> R \quad = \frac{\lambda_0 \pi R^2}{4\pi\varepsilon_0 x^3} = \frac{p}{4\pi\varepsilon_0 x^3} \quad \text{where, } p = \lambda_0 \pi R^2$$

(a) For electric field at the centre of ring, $x = 0$,

$$\because \quad E = \frac{\lambda_0 R^2}{4\varepsilon_0\,(R^2 + x^2)^{3/2}}$$

At centre, $x = 0$ $\therefore$ $E = \dfrac{\lambda_0 R^2}{4\varepsilon_0 R^3} = \dfrac{\lambda_0}{4\varepsilon_0 R}$

(b) Let the ring plane coincides with y-z plane (shown in fig. 3.12B). We consider a small element AB (of length dl) on ring.

Here $dl = Rd\theta$ where R is the radius of ring.

Also, from fig. 3.12B, $y = R\sin\theta$ and $z = R\cos\theta$

The electric charge on the considered element is $dq = \lambda dl$

$= \lambda_0 \cos\theta\,(R\,d\theta) = \lambda_0 R\cos\theta d\theta$

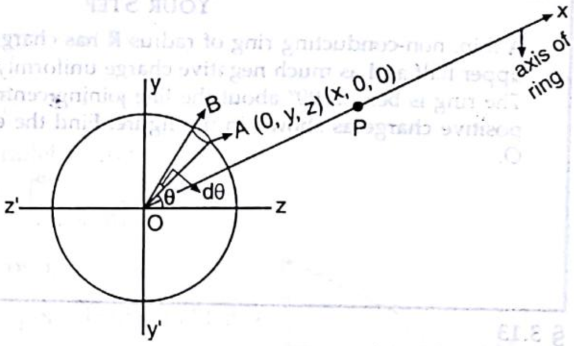


Fig. 3.12B

The axis of the ring is X-axis.

The electric field at point P due to considered element is

$$\overrightarrow{d\mathbf{E}} = \frac{dq\,(\overrightarrow{\mathbf{r}_P} - \overrightarrow{\mathbf{r}_A})}{4\pi\varepsilon_0\,|\overrightarrow{\mathbf{r}_P} - \overrightarrow{\mathbf{r}_A}|^3} \quad \text{or } d\overrightarrow{\mathbf{E}} = \frac{(\lambda_0 R\cos\theta\,d\theta)\,(x\hat{\mathbf{i}} - y\hat{\mathbf{j}} - z\hat{\mathbf{k}})}{4\pi\varepsilon_0\,|x\hat{\mathbf{i}} - y\hat{\mathbf{j}} - z\hat{\mathbf{k}}|^3}$$

$$\text{or} \quad d\overrightarrow{\mathbf{E}} = \frac{\lambda_0 R\cos\theta d\theta}{4\pi\varepsilon_0}\,\frac{(x\hat{\mathbf{i}} - R\sin\theta\,\hat{\mathbf{j}} - R\cos\theta\,\hat{\mathbf{k}})}{(x^2 + y^2 + z^2)^{3/2}}$$

$$= \frac{\lambda_0 R\cos\theta\,d\theta}{4\pi\varepsilon_0}\,\frac{(x\hat{\mathbf{i}} - R\sin\theta\hat{\mathbf{j}} - R\cos\theta\,\hat{\mathbf{k}})}{(x^2 + R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}}$$

$$= \frac{\lambda_0 R\cos\theta\,d\theta}{4\pi\varepsilon_0}\,\frac{(x\hat{\mathbf{i}} - R\sin\theta\hat{\mathbf{j}} - R\cos\theta\,\hat{\mathbf{k}})}{(x^2 + R^2)^{3/2}}$$

$$\therefore \quad d\overrightarrow{\mathbf{E}} = \frac{\lambda_0 R}{4\pi\varepsilon_0\,(R^2 + x^2)^{3/2}}\,(x\cos\theta\,d\theta\,\hat{\mathbf{i}} - R\sin\theta\cos\theta\,d\theta\,\hat{\mathbf{j}} - R\cos^2\theta\,d\theta\,\hat{\mathbf{k}})$$

$$\therefore \quad dE_x = \frac{\lambda_0 R\,x\cos\theta\,d\theta}{4\pi\varepsilon_0\,(R^2 + x^2)^{3/2}} \qquad dE_y = \frac{-\lambda_0 R^2\sin\theta\cos\theta\,d\theta}{4\pi\varepsilon_0\,(R^2 + x^2)^{3/2}}$$

$$\text{and} \quad dE_2 = \frac{-\lambda_0 R^2\cos^2\theta d\theta}{4\pi\varepsilon_0\,(R^2 + x^2)^{3/2}} \qquad \therefore \quad E_x = \int dE_x = \frac{\lambda_0 R\,x}{4\pi\varepsilon_0\,(R^2 + x^2)^{3/2}}\int_0^{2\pi}\cos\theta\,d\theta$$

After integrating, $E_x = 0$ and $\quad E_y = \int dE_y = \dfrac{-\lambda_0 R^2}{4\pi\varepsilon_0\,(R^2 + x^2)^{3/2}}\displaystyle\int_0^{2\pi}\frac{\sin 2\theta}{2}\,d\theta$

$$= \frac{-\lambda_0 R^2}{8\pi\varepsilon_0\,(R^2 + x^2)^{3/2}}\left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi} = 0$$

$$\therefore \qquad E_y = 0$$

Similarly, $$E_z = \int dE_z = \frac{-\lambda_0 R^2}{4\pi\varepsilon_0 (R^2+x^2)^{3/2}} \int_0^{2\pi} \cos^2\theta \, d\theta = \frac{-\lambda_0 R^2}{4\varepsilon_0 (R^2+x^2)^{3/2}}$$

$$\therefore \qquad \vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k} \qquad \therefore \quad |\vec{E}| = E = |E_z \hat{k}| \qquad (\because E_x = 0, E_y = 0)$$

$$= \frac{\lambda_0 R^2}{4\varepsilon_0 (R^2+x^2)^{3/2}}$$

For $x >> R$, $R^2 + x^2 \approx x^2$ $\quad \therefore \quad E = \dfrac{\lambda_0 R^2}{4\varepsilon_0 x^3} = \dfrac{P}{4\pi\varepsilon_0 x^3}$ where $P = \lambda_0 \pi R^2$

YOUR STEP

A thin, non-conducting ring of radius R has charge q uniformly spread on its upper half and as much negative charge uniformly spread over its lower half. The ring is bent at 90° about the line joining centre of negative and centre of positive charge as shown in the figure. Find the electric field E at the centre O.

$$\left\{\frac{\lambda}{\pi\varepsilon_0 R}\right\}$$

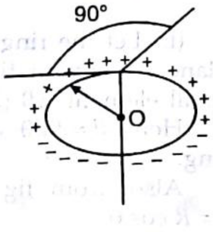


Fig. 3.12C

§ 3.13

➢ **CONCEPT**

Let us consider a thin rod of length L having uniform linear charge density λ. We have to find electric field at point P on a line perpendicular to the rod (shown in figure 3.13A).

$$x = r\tan\theta$$

$$\therefore \qquad \frac{dx}{d\theta} = r\sec^2\theta$$

$$\therefore \qquad dx = r\sec^2\theta \, d\theta$$

Also, $$\sec\theta = \frac{Z}{r}$$

$$\therefore \qquad Z = r\sec\theta$$

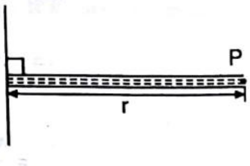


Fig. 3.13A

The electric field at point P due to considered element is

$$dE = \frac{dq}{4\pi\varepsilon_0 Z^2} = \frac{\lambda\, dx}{4\pi\varepsilon_0 Z^2}$$

Putting the values of Z and dx,

We get $$dE = \frac{\lambda r \sec^2\theta \, d\theta}{4\pi\varepsilon_0 (r\sec\theta)^2} = \frac{\lambda\, d\theta}{4\pi\varepsilon_0 r}$$

The component of electric field perpendicular to the length of rod is

$$E_\perp = \int_{\theta=-\theta_1}^{\theta=\theta_2} dE\cos\theta$$

Step I. Find the angle made by both ends at the point P.

The net electric field at point P is $\quad E = \sqrt{E_\perp^2 + E_{||}^2}$

and $$\tan\alpha = \frac{E_{||}}{E_\perp}$$

Step II. Consider a small element dx of the rod at distance x from the point O (shown in Fig. 3.13 C).

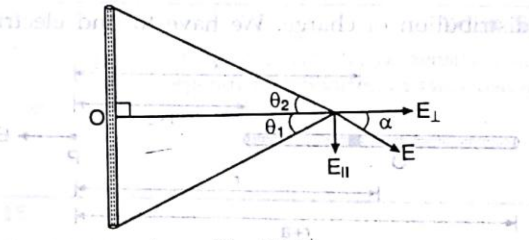


Fig. 3.13B

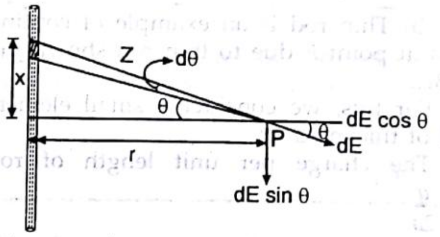


Fig. 3.13C

The electric charge on the considered element is $dq = \lambda dx$.

From Fig. 3.13C, $\tan\theta = \dfrac{x}{r} \quad \therefore \quad x = r\tan\theta$

or $$E_\perp = \int_{-\theta_1}^{\theta_2} \frac{\lambda}{4\pi\varepsilon_0 r}\cos\theta d\theta$$

or $$E_\perp = \frac{\lambda}{4\pi\varepsilon_0 r}[\sin\theta_1 + \sin\theta_2]$$

Similarly, the component of electric field parallel to rod is

$$E_{||} = \int_{\theta=-\theta_1}^{\theta=\theta_2} dE\sin\theta = \int_{-\theta_1}^{\theta_2} \frac{\lambda}{4\pi\varepsilon_0 r}\sin\theta d\theta$$

$\therefore$ $$E_{||} = \frac{\lambda}{4\pi\varepsilon_0 r}[\cos\theta_1 - \cos\theta_2]$$

Remarks : For solving the problem based upon electric field due to uniformly charged rod, we can use formula,

$$E_{||} = \frac{\lambda}{4\pi\varepsilon_0 r}[\cos\theta_1 - \cos\theta_2] \quad \text{and} \quad E_\perp = \frac{\lambda}{4\pi\varepsilon_0 r}[\sin\theta_1 + \sin\theta_2]$$

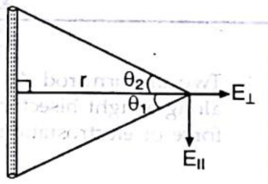


Fig. 3.13D

SOLUTION : According to statement of problem,

$$\theta_1 = \theta_2 = \alpha$$

The arrangement of problem is shown in Fig. 3.13E.

From Fig. 3.13E,

$OA = OB = a$, $OP = r$ and $AP = BP = \sqrt{r^2 + a^2}$

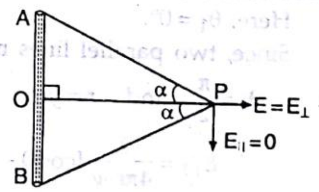
$\therefore$ $\sin\alpha = \dfrac{a}{\sqrt{a^2 + r^2}}$ and $\cos\alpha = \dfrac{r}{\sqrt{a^2 + r^2}}$

Also, $\lambda = \dfrac{q}{2a}$

$\therefore$ $E_{||} = \dfrac{\lambda}{4\pi\varepsilon_0 r}[\cos\theta_1 - \cos\theta_2] = \dfrac{\lambda}{4\pi\varepsilon_0 r}[\cos\alpha - \cos\alpha]$

$\therefore$ $E_{||} = 0$ But $E_\perp = \dfrac{\lambda}{4\pi\varepsilon_0 r}[\sin\theta_1 + \sin\theta_2]$

or $E_\perp = \dfrac{\lambda}{4\pi\varepsilon_0 r}[2\sin\alpha]$ or $E_\perp = \dfrac{\lambda\sin\alpha}{2\pi\varepsilon_0 r} = \dfrac{\lambda a}{2\pi\varepsilon_0 r\sqrt{r^2 + a^2}}$



$\because$ Net electric field at point P is $E = \sqrt{E_{||}^2 + E_\perp^2} = E_\perp$

$\therefore$ $$E = \frac{\lambda a}{2\pi\varepsilon_0 r\sqrt{r^2 + a^2}} = \frac{q}{4\pi\varepsilon_0 r\sqrt{r^2 + a^2}}$$

Fig. 3.13E

when $r >> a$, a^2 may be neglected with respect to r^2

$\therefore$ $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ just like a field due to point charge.

(b) Thin rod is an example of continuous distribution of charge. We have to find electric field at point P due to thin rod shown in Fig. 3.13F.

For this, we consider a small element of rod of thickness dx.

The charge per unit length of rod is $\lambda = \frac{q}{2a}$

The electric charge on the considered element is $dq = \lambda dx$

The electric field at point P due to considered element is

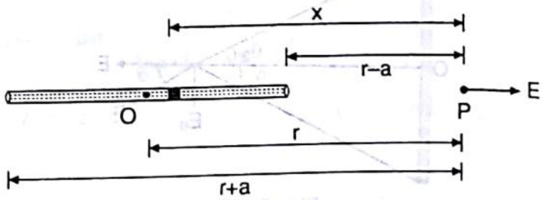


Fig. 3.13F

$$dE = \frac{dq}{4\pi\varepsilon_0 x^2} = \frac{\lambda dx}{4\pi\varepsilon_0 x^2}$$

Since, electric field at the point P due to all elements of rod are in the same direction.

$\therefore$ Net electric field at point P due to rod is $E = \int_{r-a}^{r+a} \frac{\lambda\, dx}{4\pi\varepsilon_0 x^2} = \frac{q}{4\pi\varepsilon_0 (r^2 - a^2)}$

when $r >> a$, a^2 may be neglected with respect to r^2

Hence, $E = \frac{q}{4\pi\varepsilon_0 r^2}$

YOUR STEP

Two uniform rod AB and CD of length 'a' and charge λ per unit length are placed so that CD lies along a right bisector of AB and C and D are distance a and $2a$ respectively from rod AB. Find the force of electrostatic interaction between the rods.

$$\left\{F = \frac{\lambda^2}{8\pi\varepsilon_0}\left[ln\, 2\left(\frac{\sqrt{5}+1}{\sqrt{17}+1}\right)\right]\right\}$$

§ 3.14

➢ **CONCEPT**

The concept is similar to previous problem.

According to concept of problem 3.13.

$$E_{||} = \frac{\lambda}{4\pi\varepsilon_0 R}[\cos\theta_1 - \cos\theta_2] \qquad E_{\perp} = \frac{\lambda}{4\pi\varepsilon_0 r}[\sin\theta_1 + \sin\theta_2]$$

The net electric field is $E = \sqrt{E_{\perp}^2 + E_{||}}$

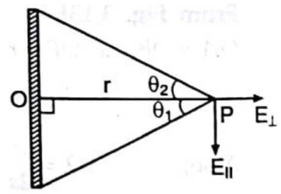


Fig. 3.14A

SOLUTION : The situation of problem is shown in Fig. 3.14B.

Here, $\theta_1 = 0°$.

Since, two parallel lines meets at infinity.

$\therefore$ $\theta_2 = \frac{\pi}{2}$ and $r = y$ $\quad E_{\perp} = \frac{\lambda}{4\pi\varepsilon_0 y}\left[\sin 0 + \sin\frac{\pi}{2}\right]$

$\therefore$ $E_{||} = \frac{\lambda}{4\pi\varepsilon_0 y}[\cos 0 - \cos \pi/2] = \frac{\lambda}{4\pi\varepsilon_0 y}$

The net electric field is $E = \sqrt{E_{\perp}^2 + E_{||}^2}$

or $E = \frac{\sqrt{2}\,\lambda}{4\pi\varepsilon_0 y}$ $\qquad$ Also, $\tan\alpha = \frac{E_{\perp}}{E_{||}} = 1$

$\Rightarrow$ $\alpha = 45°$

It means electric field $\vec{E}$ is directed at the angle 45° to the thread.

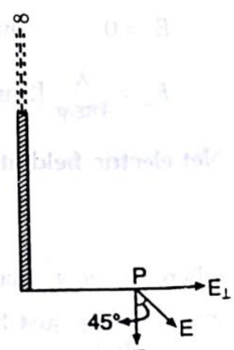


Fig. 3.14B

YOUR STEP

Two long wires have uniform charge density λ-per unit length each. The wires are non-coplanar and mutually perpendicular. Shortest distance between them is d. Calculate interaction force between them.

$$\left\{\frac{\lambda^2}{2\varepsilon_0}\right\}$$

§ 3.15

➢ **CONCEPT**

The problem is based upon superposition principle of electric field for continuous distribution of charge.

Mathematically, $\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3}$

SOLUTION : (a) The thread of the problem may be divided into three parts.

(i) Semi-infinite vertical thin thread (shown in Fig. 3.15B).

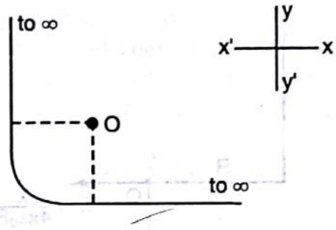


Fig. 3.15A

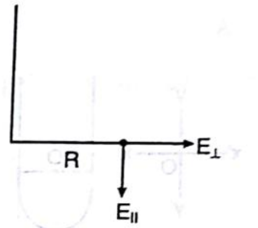


Fig. 3.15B

According to solution of problem 3.14. $E_\perp = \dfrac{\lambda}{4\pi\varepsilon_0 R}$

$$E_{||} = \frac{\lambda}{4\pi\varepsilon_0 R}$$

So, the net electric field due to this part of thread at point O is

$$\vec{E_1} = \vec{E_\perp}\,\hat{i} - E_{||}\,\hat{j} \quad \text{(See Fig. 3.15B)}$$

Putting the values of $E_\perp$ and $E_{||}$, we get

We get $$\vec{E_1} = \frac{\lambda}{4\pi\varepsilon_0 R}\,\hat{i} - \frac{\lambda}{4\pi\varepsilon_0 R}\,\hat{j}$$

(ii) **Second part :** A circular arc which is making angle $\frac{\pi}{2}$ at the centre of curvature O (Shown in Fig. 3.15C).

According to concept of problem 3.8,
the electric field due to this part is

$$E_2 = \frac{q}{2\pi\varepsilon_0\alpha R^2}\sin\frac{\alpha}{2} = \frac{(\lambda R\alpha)}{2\pi\alpha\varepsilon_0 R^2}\sin\frac{\alpha}{2} = \frac{\lambda}{2\pi\varepsilon_0 R}\sin\frac{\alpha}{2}$$

From Fig. 3.15C, $\alpha = \frac{\pi}{2} = 90°$ $\therefore$ $E_2 = \dfrac{\lambda\sin\frac{\pi}{4}}{2\pi\varepsilon_0 R} = \dfrac{\lambda}{2\sqrt{2}\,\pi\varepsilon_0 R}$

In vector form,

$\because$ $$\vec{E_2} = E_2\cos 45\,\hat{i} + E_2\sin 45\,\hat{j}$$

or $$\vec{E_2} = \frac{\lambda}{2\sqrt{2}\,\pi\varepsilon_0 R}\times\frac{1}{\sqrt{2}}\,\hat{i} + \frac{\lambda}{2\sqrt{2}\,\pi\varepsilon_0 R}\left(\frac{1}{\sqrt{2}}\right)\hat{j}$$

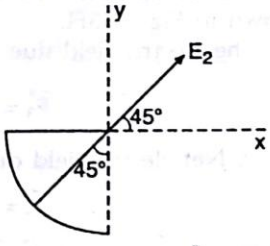


Fig. 3.15C

or
$$\vec{E}_2 = \frac{\lambda}{4\pi\varepsilon_0 R}\hat{i} + \frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

(iii) Third part : This consists of a semi-infinite thin horizontal thread (shown in Fig. 3.15D).

$$E_{\perp} = \frac{\lambda}{4\pi\varepsilon_0 R} \qquad E_{\parallel} = \frac{\lambda}{4\pi\varepsilon_0 R}$$

The electric field at point O due to this part is

$$\vec{E}_3 = -E_{\parallel}\hat{i} + E_{\perp}\hat{j} = \frac{-\lambda}{4\pi\varepsilon_0 R}\hat{i} + \frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

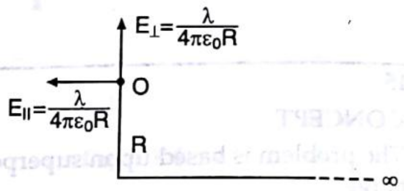


Fig. 3.15D

The net electric field at point O is $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$.

Putting the values of $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$, we get

$$\vec{E} = \vec{E}_2 = \frac{\lambda}{4\pi\varepsilon_0 R}\hat{i} + \frac{\lambda}{4\pi\varepsilon_0 R}\hat{j} \quad \therefore E = \frac{\sqrt{2}\,\lambda}{4\pi\varepsilon_0 R}$$

(b) The thread of problem may be divided into three parts :

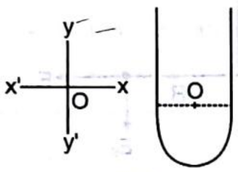


Fig. 3.15E

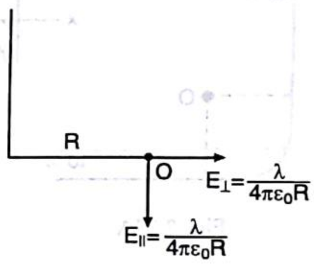


Fig. 3.15F

(i) First part : Semi-infinite vertical thread which is shown in Fig. 3.15F.
The electric field at point O due to this part is

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 R}\hat{i} - \frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

(ii) Second part : This part is semicircular thread of radius R (shown in Fig. 3.15G).
From solution of 3.15 (a), the electric field due to circular arc is

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0 R}\sin\frac{\alpha}{2}$$

In the case of semicircular arc, $\alpha = \pi$ $\quad \therefore E_2 = \dfrac{\lambda}{2\pi\varepsilon_0 R}$

In vector form, $\quad \vec{E}_2 = \dfrac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$

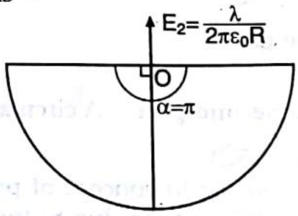


Fig. 3.15G

(iii) Third part : Semi-infinite vertical thin thread which is shown in Fig. 3.15H.

The electric field due to this part is

$$\vec{E}_3 = \frac{-\lambda}{4\pi\varepsilon_0 R}\hat{i} - \frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

$\therefore$ Net electric field due to thread at point O is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

Putting the values of $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$, we get, $\vec{E} = \vec{0}$.

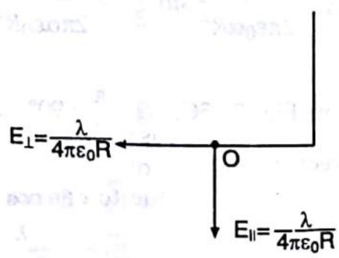


Fig. 3.15H

YOUR STEP

Two line charges are placed along x-axis and y-axis as such their middle points are at the origin (0, 0). The linear charge density of each line charge is λ. Find the electric intensity at point $P(x, y)$.

$$\left\{E = \frac{\lambda}{2\pi\varepsilon_0}\sqrt{\frac{1}{x^2}+\frac{1}{y^2}} \text{ with } \theta = \tan^{-1}\left(\frac{y}{x}\right)\right\}$$

§ 3.16

➢ **CONCEPT**

Here surface charge density is given. So, charge resides on outer surface. Hence, sphere is either conducting sphere or hollow sphere (spherical shell).

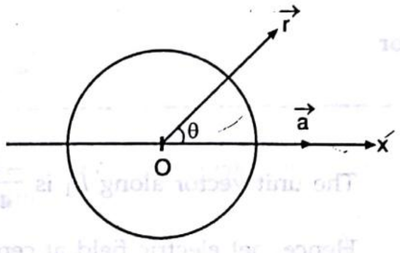


Fig. 3.16A

➢ **DISCUSSION**

Here, $\sigma = \vec{a} \cdot \vec{r}$

We take vector $\vec{a}$ along positive x-direction.

From Fig. 3.16A, $\sigma = ar\cos\theta$

where, θ is angle between $\vec{a}$ and $\vec{r}$. The value of $\cos\theta$ is positive for half right part of sphere and the value of $\cos\theta$ is negative for half left part of sphere. The distribution of charge is shown in Fig. 3.16B.

Fig. 3.16B

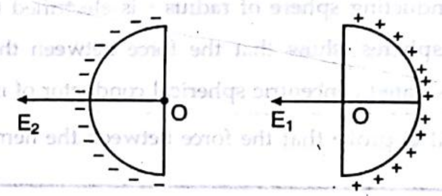


Fig. 3.16C

The sphere may be divided into two hemisphere, one of positive charge and other of negative charge (shown in Fig. 3.16 C).

From symmetry of charge distribution the directions and magnitudes of electric field due to both parts are same.

So, net electric field at the centre of sphere is $\vec{E} = \vec{E_1} + \vec{E_2} = 2\vec{E_1}$

SOLUTION : Calculation of electric field at centre of sphere due to right part hemisphere.

The hemisphere can be divided into a number of ring elements. The electric field at the point O (centre of sphere) is vector sum of the field due to each such rings. Let us consider on elemental ring located at an angle θ with the x-axis and subtending an angle $d\theta$ at the centre O (shown in Fig. 3.16D).

The area of considered element is

$$dA = 2\pi R dS = 2\pi R\,(r d\theta)$$

From Fig. 3.16D, $\sin\theta = \frac{R}{r}$ $\therefore$ $R = r\sin\theta$

$$\therefore \quad dA = 2\pi r^2 \sin\theta\, d\theta$$

The charge on the considered element is

$$dq = \sigma dA = (ar\cos\theta)\, 2\pi r^2 \sin\theta\, d\theta = 2\pi a r^3 \sin\theta \cos\theta\, d\theta$$

From the solution of problem 3.9,

the electric field due to considered ring at the centre O is

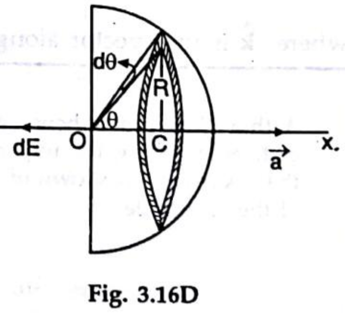


Fig. 3.16D

$$dE = \frac{dq\,(CO)}{4\pi\varepsilon_0\,[R^2 + (CO)^2]^{3/2}}$$ directed along negative x-direction

From Fig. 3.16D, $CO = r\cos\theta$

$$\therefore \qquad dE = \frac{(2\pi a r^3 \sin\theta\cos\theta\, d\theta)\,(r\cos\theta)}{4\pi\varepsilon_0\left\{(r\sin\theta)^2 + (r\cos\theta)^2\right\}^{3/2}} = \frac{ar\sin\theta\cos^2\theta\, d\theta}{2\varepsilon_0}$$

$\therefore$ The electric field due to hemisphere at centre O is

$$E_1 = \int_{\theta=0}^{\theta=\pi/2} dE$$

or

$$E_1 = \frac{ar}{2\varepsilon_0}\int_0^{\pi/2} \sin\theta\cos^2\theta\, d\theta$$

$$= \frac{ar}{2\varepsilon_0}\times\left(\frac{1}{3}\right) = \frac{ar}{6\varepsilon_0} \text{ (directed along negative } x\text{-direction)}$$

The unit vector along E_1 is $\frac{-\vec{a}}{a}$ $\qquad \therefore \ \vec{E}_1 = E_1\left(\frac{-\vec{a}}{a}\right) = \frac{-r\vec{a}}{6\varepsilon_0}$

Hence, net electric field at centre of sphere is $\vec{E} = 2\vec{E}_1$

Putting the value of $\vec{E}_1$, we get, $\vec{E} = \frac{-\vec{ar}}{3\varepsilon_0} = \frac{-\vec{\sigma}}{3\varepsilon_0}$

YOUR STEP

A conducting sphere of radius r is electrified to potential ϕ. If the sphere consists of two separate hemispheres, show that the force between them is $\frac{1}{8}\phi^2$ and if the whole be surrounded by an uninsulated concentric spherical conductor of internal radius R and the potential of the solid sphere is still ϕ, prove that the force between the hemisphere is $\frac{\phi^2 R^2}{8(R-r)^2}$.

---

§ 3.17

➢ **CONCEPT**

Here, $\sigma_0 = \rho a$

where ρ is volume charge density.

The electric field due to uniformly charged sphere (non-conducting solid sphere) is

$$\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0}$$

SOLUTION : In the given situation, $\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}\left(\vec{r} - \frac{\vec{a}}{2}\right) + \frac{(-\rho)}{3\varepsilon_0}\left(\vec{r} + \frac{\vec{a}}{2}\right)$

$$\therefore \qquad \vec{E} = \frac{-\rho\vec{a}}{3\varepsilon_0} = \frac{-\sigma_0}{3\varepsilon_0}\hat{k}$$

where, $\hat{k}$ is unit vector along the reference line from which θ is measured.

YOUR STEP

A thin glass rod is bent into a semicircle of radius r. A charge $+Q$ is uniformly distributed along the upper half and a charge $-Q$ is uniformly distributed along the lower half, as shown in Fig. 3.17(A). Calculate the electric field E at P, the centre of the semicircle.

$$\left\{E = \frac{Q}{\pi^2\varepsilon_0 r^2}\right\}$$

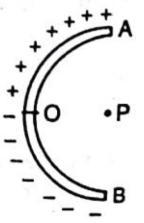


Fig. 3.17A

§ 3.18

➢ **CONCEPT**

If we consider a spherical shell of thickness dr and radius r, then,

$$\sigma = \rho dr = (\vec{a} \cdot \vec{r})\, dr$$

From the result of 3.16,

The electric field due to considered element is $d\vec{E} = \dfrac{-\vec{a} r dr}{3\varepsilon_0}$

SOLUTION : The resultant electric field is $\vec{E} = \dfrac{-\vec{a}}{3\varepsilon_0}\displaystyle\int_0^R r dr = \dfrac{R^2}{6\varepsilon_0}\vec{a}$

YOUR STEP

The volume charge density of a solid sphere of radius a is proportional to the depth below the surface. Find the depth at which the electric intensity is maximum.

$\left\{\dfrac{a}{3}\right\}$

§ 3.19

➢ **CONCEPT**

The solid angle subtended by a right circular cone on its vertex is $\Omega = 2\pi(1 - \cos\alpha)$, where α is semivertex angle (shown in Fig. 3.19A).

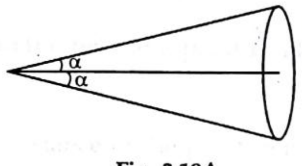


Fig. 3.19A

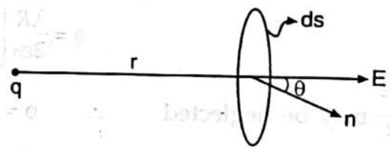


Fig. 3.19B

Electric flux through an area ds due to a point charge q.

Electric flux ϕ is defined as $\phi = \vec{E} \cdot d\vec{s} = \dfrac{q}{4\pi\varepsilon_0 r^2} ds \cos\theta = \dfrac{q}{4\pi\varepsilon_0} \dfrac{ds\cos\theta}{r^2}$

$$\phi = \vec{E} \cdot d\vec{s} = \frac{q}{4\pi\varepsilon_0 r^2} ds\cos\theta \qquad \therefore \qquad \phi = \frac{q\,\Omega}{4\pi\varepsilon_0} \qquad \left\{\because \frac{ds\cos\theta}{r^2} = \Omega\right\}$$

where, Ω is solid angle subtended by area ds on the point charge.

In similar fashion, electric flux passing through a given area due to an elementry charge dq is $d\phi = \dfrac{\Omega\, dq}{4\pi\varepsilon_0}$

where Ω is solid angle subtended on the elementry charge dq.

We consider an element dx at distance x from centre of circle (shown in Fig. 3.19C).

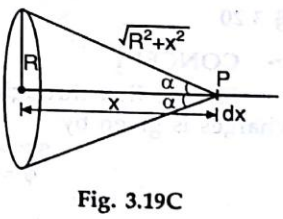


Fig. 3.19C

The electric charge on the considered element is $dq = \lambda dx$ (treated as point charge).

The solid angle subtended by circle on the considered element is $\Omega = 2\pi(1 - \cos\alpha)$,

Here, $\cos\alpha = \dfrac{x}{\sqrt{R^2 + x^2}}$ $\quad\therefore\quad$ $\Omega = 2\pi\left(1 - \dfrac{x}{\sqrt{R^2 + x^2}}\right)$

The electric flux passing through the circle due to considered element is

$$d\phi = \frac{\Omega dq}{4\pi\varepsilon_0} = \frac{2\pi}{4\pi\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)\lambda dx$$

$\therefore$ Total electric flux crossing through the circle due to large thread is

$$\phi = \int d\phi = \frac{\lambda}{2\varepsilon_0}\int_0^l \left(1 - \frac{x}{\sqrt{R^2+x^2}}\right) dx$$

Here, put $x = R\tan\theta$, $\quad dx = R\sec^2\theta d\theta$

$\therefore$
$$\phi = \frac{\lambda}{2\varepsilon_0}\int_{x=0}^{x=l}\left(1 - \frac{R\tan\theta}{R\sec\theta}\right) R\sec^2\theta d\theta$$

$$= \frac{\lambda}{2\varepsilon_0}\int_0^l (1-\sin\theta) R\sec^2\theta d\theta$$

$$= \frac{\lambda R}{2\varepsilon_0}\left[\int_{x=0}^{x=l}\sec^2\theta d\theta - \int_{x=0}^{x=l}\tan\theta\sec\theta d\theta\right]$$

$$= \frac{\lambda R}{2\varepsilon_0}\left[[\tan\theta]_{x=0}^{x=l} - [\sec\theta]_{x=0}^{x=l}\right]$$

$\therefore$
$$\phi = \frac{\lambda R}{2\varepsilon_0}[\tan\theta - \sec\theta]_{x=0}^{x=l}$$

$$\phi = \frac{\lambda R}{2\varepsilon_0}\left[\frac{x}{R} - \frac{\sqrt{R^2+x^2}}{R}\right]_0^l = \frac{\lambda R}{2\varepsilon_0}\left[\frac{l}{R} - \frac{\sqrt{R^2+l^2}}{R} + 1\right]$$

But thread is very long. $\quad \therefore \quad R >> l$

$\therefore$
$$\phi = \frac{\lambda R}{2\varepsilon_0}\left[\frac{l}{R} - \frac{l}{R}\sqrt{\frac{R^2}{l^2}+1} + 1\right]$$

$\frac{R^2}{l^2}$ may be neglected $\quad \therefore \quad \phi = \frac{\lambda R}{2\varepsilon_0}\left[\frac{l}{R} - \frac{l}{R} + 1\right]$

$\therefore$
$$\phi = \frac{\lambda R}{2\varepsilon_0}$$

YOUR STEP

A point charge q is placed at one corner of a cube of edge a. What is the flux through each of the cube faces?

$$\left\{\frac{1}{24}\frac{q}{\varepsilon_0}\right\}$$

§ 3.20

➢ CONCEPT

Electric flux through an area due to a number of point charges is given by

$$\phi = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n} q_i \Omega_i$$

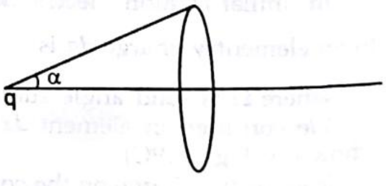


Fig. 3.20A

The solid angle due to circle on the point charge q is

$$\Omega = 2\pi(1-\cos\alpha)$$

SOLUTION : Since, charges are opposite sides of the circle. So, net flux passing through circle is $\phi = \phi_1 - \phi_2$

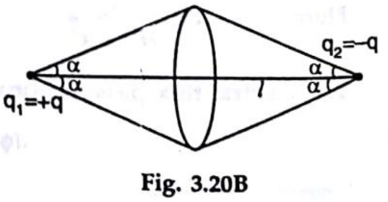


Fig. 3.20B

Here, $\quad \phi_1 = \frac{q_1\Omega}{4\pi\varepsilon_0} = \frac{q\, 2\pi(1-\cos\alpha)}{4\pi\varepsilon_0} = \frac{q(1-\cos\alpha)}{2\varepsilon_0}$

Similarly, $$\phi_2 = \frac{q_2\,\Omega}{4\pi\varepsilon_0} = \frac{-q\,2\pi\,(1-\cos\alpha)}{4\pi\varepsilon_0} = \frac{-q\,(1-\cos\alpha)}{2\varepsilon_0}$$

$\therefore$ $$\phi = \phi_1 - \phi_2 = \frac{q}{\varepsilon_0}(1-\cos\alpha)$$

From Fig. 3.20B, $\cos\alpha = \dfrac{l}{\sqrt{R^2+l^2}}$

$\therefore$ $$\phi = \frac{q}{\varepsilon_0}\left(1-\frac{l}{\sqrt{R^2+l^2}}\right) = \frac{q}{\varepsilon_0}\left(1-\frac{1}{\sqrt{1+\left(\frac{R}{l}\right)^2}}\right)$$

YOUR STEP

A fixed circle of radius a and centre O is drawn and a charge e is placed at a distance $3a/4$ from O on a line through O perpendicular to the plane of circle. Show that the flux of E through the circle is $4\pi e/5$. If a second charge e' is similarly placed at a distance $5a/12$ on the opposite side of the circle, and there is no net flux through the circle, prove that $e' = \dfrac{13e}{20}$.

§ 3.21

➢ **CONCEPT**

According to basic definition of flux, $$d\phi = \vec{E}\cdot d\vec{s}$$

The electric field at a point inside a non-conducting solid sphere is $$\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0}$$

where r is distance of the point from the centre of sphere.

SOLUTION : The section formed by the mentioned plane will be disc of radius $r_1 = \sqrt{R^2 - r_0^2}$ (shown in Fig 3.21A).

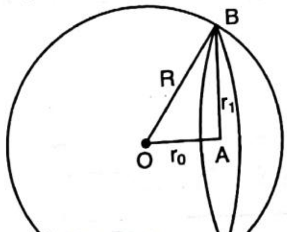


Fig. 3.21A

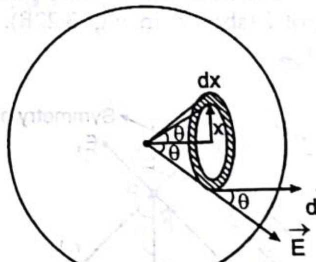


Fig. 3.21B

For solving the problem, we consider a ring element of radius x $\left[0 \le x \le \sqrt{R^2-x^2}\right]$ and radial thickness dx (shown in Fig. 3.21B). The area of considered ring is $ds = 2\pi x\,dx$. But electric field at each element of the ring is

$\vec{E} = \dfrac{\rho\vec{r}}{3\varepsilon_0}$. where r is distance of each element of ring from the centre of sphere.

The electric flux passing through the considered ring is $$d\phi = \vec{E}\cdot\vec{ds} = \frac{\rho\,\vec{r}\cdot\vec{ds}}{3\varepsilon_0} = \frac{\rho r ds\cos\theta}{3\varepsilon_0}$$

where θ is angle between $\vec{E}$ and $\vec{ds}$ (shown in Fig. 3.21B).

$\therefore$
$$d\phi = \frac{\rho r\,(2\pi x dx)}{3\varepsilon_0}\left(\frac{r_0}{r}\right) = \frac{2\pi\rho r_0\, x\, dx}{3\varepsilon_0}$$

$\therefore$ Total flux passing through the disc is

$$\phi = \int_0^{r_1=\sqrt{R^2-r_0^2}} \frac{2\pi\rho r_0}{3\varepsilon_0}\, x\, dx = \frac{2\pi\rho r_0}{3\varepsilon_0}\left[\frac{x^2}{2}\right]_0^{\sqrt{R^2-r_0^2}} = \frac{\pi\rho r_0\,(R^2-r_0^2)}{3\varepsilon_0}$$

YOUR STEP

1. A very small earthed conducting sphere is at a distance a from a point charge q_1 and at a distance b from a point charge q_2 ($a < b$). At a certain instant, the sphere starts expanding so that its radius grows according to the law $R = vt$.
Determine the time dependence $I(t)$ of the current in the earthing conductor, assuming that the point charges and the centre of the sphere are at rest, and in due time the initial point charges get into the expanding sphere without touching it (through small hole).
2. A charge Q is distributed over two concentric hollow spheres of radii r and R ($R > r$) such that the surface densities are equal. Find the potential at the common centre.
3. A charge Q is distributed over two concentric hollow spheres of radii r and R ($R > r$). Such that the surface densities are equal. Find the potential at the common centre.

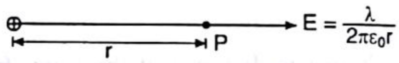
$\left\{ \textbf{1. } I(t) = -v\left(\frac{q_1}{a}+\frac{q_2}{b}\right); t < \frac{a}{v};\quad \frac{-vq_2}{b}, \frac{a}{v} \le t < \frac{b}{v}; 0, t \ge \frac{b}{v} \quad \textbf{2. } V = \frac{1}{4\pi\varepsilon_0}\frac{Q(R+r)}{(R^2+r^2)} \quad \textbf{3. } \frac{1}{4\pi\varepsilon_0}\frac{Q(R+r)}{(R^2+r^2)} \right\}$

§ 3.22

➢ **CONCEPT**

The electric field due to a long thread (thin rod) is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \text{(shown in Fig. 3.22A)}$$



The direction of electric field is along increasing r.

Fig. 3.22A

SOLUTION : Let both threads are placed perpendicular to the plane of paper (along z-axis) at a separation of l (shown in Fig. 3.22B).

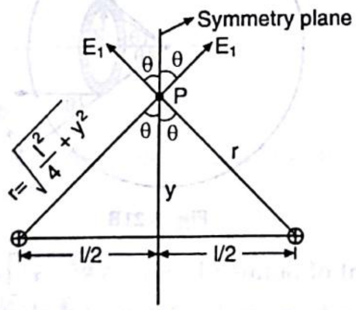


Fig. 3.22B

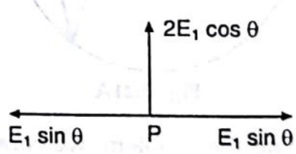


Fig. 3.22C

Horizontal components of electric field balance each other.

$\therefore$ Net electric field at point P is $E = 2E_1\cos\theta$

But electric field due to one of the thread is :

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \therefore \quad E = \frac{2\lambda}{2\pi\varepsilon_0 r}\cos\theta = \frac{\lambda}{\pi\varepsilon_0}\frac{\cos\theta}{r} = \frac{\lambda}{\pi\varepsilon_0}\frac{y}{r^2}$$

$$\therefore \qquad E = \frac{\lambda}{\pi\varepsilon_0} \frac{y}{\left(\frac{l^2}{4} + y^2\right)} \qquad \text{(from Fig. 3.22B)}$$

For maximum electric field, $\frac{dE}{dy} = 0$

After solving, we get $y = \frac{l}{2}$ $\quad \therefore \quad E_{max} = \frac{\lambda}{\pi\varepsilon_0} \frac{\left(\frac{l}{2}\right)}{\frac{l^2}{4} + \frac{l^2}{4}} = \frac{\lambda}{\pi\varepsilon_0 l}$

YOUR STEP

Two very long and thin insulating threads are placed parallel to one another, a distance $2r$ apart. A semicircular thread of radius r connects these two threads at one end as shown in Fig. 3.22D. All the threads are uniformly charged by a charge density of λ. Find the electric field at the centre of the semicircle.

{zero}

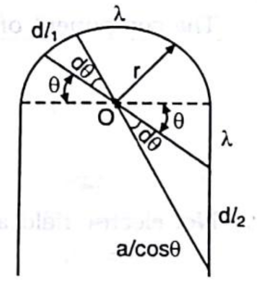


Fig. 3.22D

§ 3.23

➢ **CONCEPT**

Since, surface charge density is given. So, the cylinder may be treated as hollow charged cylinder.

SOLUTION : The given cylinder behaves as a long bomboo. So, the strip of the cylinder along length of the cylinder behaves as long thread.

The top view of cylinder is like a circular ring (shown in Fig. 3.23B).

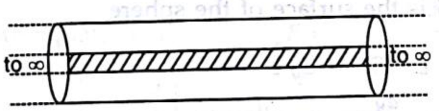


Fig. 3.23A

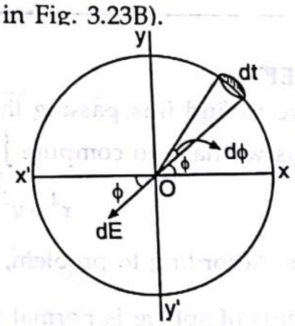


Fig. 3.23B

We consider a long strip of width $dt = Rd\phi$ (shown in Fig. 3.23B). If length of cylinder is l ($l \to \infty$).

The area of strip is $dA = ldt = lRd\phi$

The charge on the strip is $dq = \sigma dA = \sigma_0 \cos\phi \, lRd\phi = \sigma_0 lR \cos\phi d\phi$

The linear charge density of considered strip is, $\lambda = \frac{dq}{l} = \frac{\sigma_0 lR \cos\phi \, d\phi}{l} = \sigma_0 R \cos\phi \, d\phi$

The electric field at point O due to considered long strip is

$$dE = \frac{\lambda}{2\pi\varepsilon_0 R} \qquad \text{(direction is shown in Fig. 3.23B).}$$

$$= \frac{\sigma_0 R \cdot \cos\phi \, d\phi}{2\pi\varepsilon_0 R} = \frac{\sigma_0}{2\pi\varepsilon_0} \cos\phi \, d\phi$$

$\therefore$ $$d\vec{\mathbf{E}} = -dE \cos\phi \,\hat{\mathbf{i}} - dE \sin\phi \,\hat{\mathbf{j}}$$

The component of electric field along x-axis due to whole cylinder is

$$E_x = \int_{\phi=0}^{\phi=2\pi} -dE\cos\phi = \frac{-\sigma_0}{2\pi\varepsilon_0}\int_0^{2\pi} \cos^2\phi \, d\phi$$

$$= \frac{-\sigma_0}{2\pi\varepsilon_0}\int_0^{2\pi}\left(\frac{1+\cos 2\phi}{2}\right) d\phi$$

$$= \frac{-\sigma_0}{4\pi\varepsilon_0}\int_0^{2\pi}(1+\cos 2\phi)\, d\phi = \frac{-\sigma_0}{2\varepsilon_0}$$

The component of electric field due to whole cylinder along y-axis is

$$E_y = \int_{\phi=0}^{\phi=2\pi} -dE\sin\phi = -\frac{\sigma_0}{2\pi\varepsilon_0}\int_0^{2\pi}\cos\phi\sin\phi\, d\phi$$

$$= -\frac{\sigma_0}{4\pi\varepsilon_0}\int_0^{2\pi}\sin 2\phi\, d\phi = 0$$

$\therefore$ Net electric field at point O is $E = E_x\,\hat{\mathbf{i}} + E_y\,\hat{\mathbf{j}} = -\frac{\sigma_0}{2\varepsilon_0}\,\hat{\mathbf{i}}$

i.e., $E = \frac{\sigma_0}{2\varepsilon_0}$ along the direction $\phi = \pi$

YOUR STEP

A right circular cylinder of radius R and height L is oriented along the z-axis. It has a non-uniform volume density of charge given by $\rho(z) = \rho_0 + \beta z$ with reference to an origin at the center of the cylinder. Find the force on a point charge q placed at the centre of the cylinder.

$$\left\{E = \frac{\beta}{2\varepsilon_0}\left\{\frac{L}{2}\left(\frac{L}{2} - \sqrt{\frac{L^2}{4}+R^2}\right) + R^2 \log\left(\frac{L}{2R}\sqrt{1+\frac{L^2}{2R^2}}\right)\right\}\right\}$$

§ 3.24

➢ **CONCEPT**

We have to find flux passing through the surface.

It means we have to compute $\int_s \hat{\mathbf{E}} \cdot d\vec{\mathbf{S}}$ where S is the surface of the sphere

$$x^2 + y^2 + z^2 = R^2 \quad \text{and} \quad E = E_x\,\hat{\mathbf{i}} + E_y\,\hat{\mathbf{i}}$$

SOLUTION : According to problem, $E_x = \frac{ax}{x^2+y^2}$, $E_y = \frac{ay}{x^2+y^2}$

The radius of sphere is normal to surface of sphere (direction of area).

$\therefore$ The unit vector along the normal on the surface of sphere (direction of area vector).

$$\hat{\mathbf{n}} = \frac{x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}}{\sqrt{x^2+y^2+z^2}} = \frac{x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}}{R}$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = (\vec{\mathbf{E}} \cdot \vec{\mathbf{n}})\, dS$$

$$= (E_x\,\hat{\mathbf{i}} + E_y\,\hat{\mathbf{j}})\left(\frac{x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}}{R}\right) dS = (xE_x + yE_y)\frac{dS}{R}$$

$$= \left(\frac{ax^2}{x^2+y^2} + \frac{ay^2}{x^2+y^2}\right)\frac{dS}{R}$$ (On putting the values of E_x and E_y)

$$= a\frac{dS}{R}$$

$\therefore$ $$\phi = \int \vec{E}\cdot d\vec{S} = \int_s \frac{a\,dS}{R}$$

or $$\phi = \frac{a}{R}\int dS = \frac{a}{R}\times 4\pi R^2$$

$\therefore$ $$\phi = 4\pi aR$$

YOUR STEP

The components of electric intensity in a certain region are given by

$$E_x = \frac{-ky}{(x^2+y^2)^{1/2}} \text{ and } E_y = \frac{kx}{(x^2+y^2)^{1/2}}$$

Prove that the lines of force are the family of concentric circles.

§ 3.25

➢ **CONCEPT**

The problem is easily solved by using Gauss's law.

i.e. $$\int_c \vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0} \quad \text{or} \quad \int_c \vec{E}\cdot d\vec{S} = \frac{\int dq}{\varepsilon_0}$$

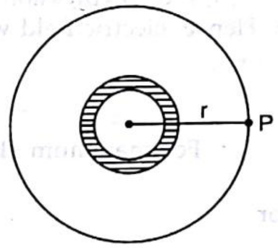


Fig. 3.25A

SOLUTION : Case I. When $r < R$

$$\int_c \vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0}$$

$$\int_c E\,dS\cos 0 = \frac{q_{in}}{\varepsilon_0}$$

$$E\,4\pi r^2 = \frac{q_{in}}{\varepsilon_0} \quad \therefore \quad E = \frac{q_{in}}{4\pi\varepsilon_0 r^2} \qquad \ldots(i)$$

where, q_{in} = Total electric charge enclosed in a sphere of radius r (shown in Fig. 3.25A).

Here, $$q_{in} = \int_0^r \rho 4\pi r^2 dr = \int_0^r \rho_0\left(1-\frac{r}{R}\right)4\pi r^2 dr$$

$$= 4\pi\rho_0\left[\int_0^r r^2 dr - \int_0^r \frac{r^3}{R}dr\right] = 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^4}{4R}\right] = \frac{4\pi\rho_0}{\varepsilon_0}\left[\frac{r^3}{3}-\frac{r^4}{4R}\right]$$

From equation (1), $$E = \frac{q_{in}}{4\pi\varepsilon_0 r^2} = \frac{4\pi\rho_0}{4\pi r^2}\left[\frac{r^3}{3}-\frac{r^4}{4R}\right]$$

$\therefore$ $$E = \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3}-\frac{r^2}{4R}\right] \text{ radially outward} \quad \therefore \quad E = \frac{\rho_0 r}{3\varepsilon_0}\left[1-\frac{3r}{4R}\right] \quad \text{where } r < R$$

Case II. When $r = R$ at the surface of sphere

$$E = \frac{\rho_0}{\varepsilon_0}\left(\frac{R}{3}-\frac{R^2}{4R}\right) = \rho_0 R\left[\frac{4-3}{12}\right]$$

$\therefore$ $$E = \frac{\rho_0 R}{12\varepsilon_0} \text{ radially outward}$$

Case III. When $r > R$

In this case, $$q_{in} = \int_0^R \rho_0\left(1-\frac{r}{R}\right)4\pi r^2 dr$$

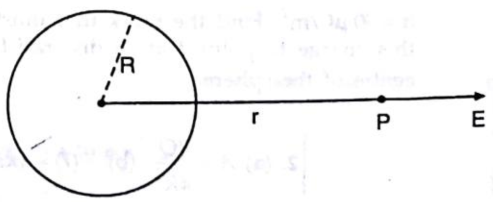


Fig. 3.25B

or $\quad q_{in} = 4\pi\rho_0\left[\frac{R^3}{3} - \frac{R^4}{4R}\right]$ $\quad$ or $\quad q_{in} = 4\pi\rho_0 R^3\left[\frac{1}{3} - \frac{1}{4}\right]$

or $\quad q_{in} = 4\pi\rho_0 R^3 \times \frac{1}{12} = \frac{\pi\rho_0 R^3}{3}$

From equation (i), we have $\quad E = \frac{q_{in}}{4\pi\varepsilon_0 r^2} = \frac{\pi\rho_0 R^3}{3\times 4\pi\varepsilon_0 r^2}$

$\therefore \quad E = \frac{\rho_0 R^3}{12\varepsilon_0 r^2}$ $\quad \therefore \quad E = \frac{\rho_0 r}{3\varepsilon_0}\left[1 - \frac{3r}{4R}\right]$ when $r \le R$

$E = \frac{\rho_0 R}{12\varepsilon_0}$ when $r = R$ and $E = \frac{\rho_0 R^3}{12\varepsilon_0 r^2}$ when $r \ge R$

(b) From expression of electric field, the electric field outside sphere decreases with increasing r. Hence, electric field will be maximum inside the sphere

$\because$
$$E = \frac{\rho_0 r}{3\varepsilon_0}\left[1 - \frac{3r}{4R}\right]$$

$\therefore$ For maximum electric field, $\quad \frac{dE}{dr} = 0$

or $\quad 1 - \frac{3r}{2R} = 0 \quad \therefore \quad r = \frac{2R}{3}$

At $r = \frac{2R}{3}$, electric field will be maximum.

$\therefore$
$$E_{max} = \frac{\rho_0\left(\frac{2R}{3}\right)}{3\varepsilon_0}\left[1 - \frac{3}{4R}\left(\frac{2R}{3}\right)\right] = \frac{\rho_0 R}{9\varepsilon_0}$$

YOUR STEP

1. A solid non-conducting sphere of radius R carries a non-uniform charge distribution, the charge density being $\rho = \rho_s r/R$, where ρ_s is a constant and r is the distance from the centre of the sphere. Show that (a) the total charge on the sphere is $Q = \pi\rho_s R^3$ and (b) the electric field inside the sphere is given by
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^4}r^2$$
2. A sphere of radius R contains space charge density $\rho(r) = A(R - r)$ for $0 < r < R$. The total electric charge inside the sphere is Q.
(a) Express A in terms of Q and R.
(b) Find the electric field inside and outside the sphere.
3. A point charge $q = 2\,\mu C$ (see Fig. 3.25(C)) is at point 1 at a distance $l_1 = 1.4$ m from the surface of a sphere having a radius $r = 20$ cm and a surface charge density $\sigma = 30\,\mu C/m^2$. Find the work that must be done to carry this charge to point 2 at a distance $l_2 = 40$ cm from the centre of the sphere.

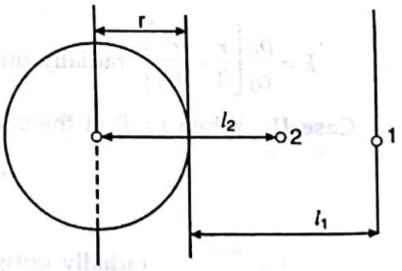


Fig. 3.25C

$\left\{2.\ (a)\ A = \frac{3Q}{\pi R^4}\ (b)\ E(r) = 4\pi A\left(\frac{rR}{3} - \frac{r^2}{4}\right) \text{ for } 0 < r < R,\ E(r) = \frac{Q}{r^2} \text{ for } r \ge R \quad 3.\ 0.51 \text{ Joule}\right\}$

§ 3.26

➢ **CONCEPT**

The electric field on the surface of a sphere is $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$.

This result is independent of distribution of charge inside the sphere.

➢ **DISCUSSION**

For discussion, we take origin at the centre of sphere. From $r = 0$ to $r = R$, space is uniformly charged (volume charge density is constant).

From $r = R$ to $r \to \infty$, volume charge density is $\rho = \dfrac{\alpha}{r}$. For convenience, the system may be assumed as combination of a uniformly charged sphere of radius R and a hollow sphere of inner radius $R_1 = R$ and outer radius $R_2 \to \infty$. The resultant electric field at a point is calculated by vector sum of fields due to both spheres.

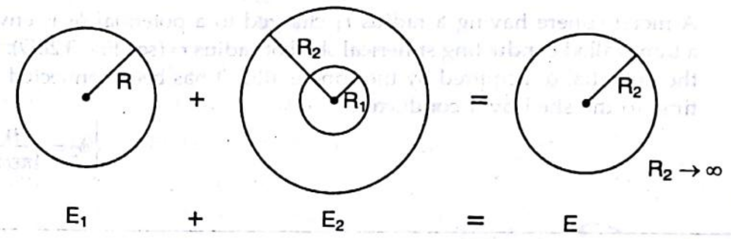


Fig. 3.26A

Mathematically,

$$\vec{E} = \vec{E_1} + \vec{E_2},\ E_1 = E_2 = E$$

Calculation of E_1 : $E_1 = \dfrac{q_0}{4\pi\varepsilon_0 x^2}$ at point p at a distance x from centre of sphere.

Here, q_0 = charge on the sphere.

Calculation for E_2 : $E_2 = \dfrac{q}{4\pi\varepsilon_0 x^2}$ where q is charge on the hollow sphere from $r = R$ to $r = x$.

Calculation for q.

We consider a hollow sphere of radius r $(R \le r \le x)$ and thickness dr.

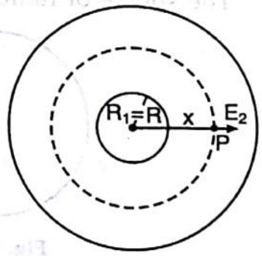


Fig. 3.26B

The volume of considered element is $dV = 4\pi r^2 dr$.

$\therefore$ Electric charge on the considered element is

$$dq = \rho dV = \frac{\alpha}{r} 4\pi r^2 dr = 4\pi\alpha r dr$$

The total charge enclosed from $r = R$ to $r = x$ is

$$q = \int dq = 4\pi\alpha \int_R^x r dr = 4\pi\alpha \left(\frac{x^2 - R^2}{2}\right) = 2\pi\alpha\,(x^2 - R^2)$$

$\therefore$ Net electric field is $E = E_1 + E_2$ (both are in the same direction).

$\therefore$
$$E = \frac{q_0}{4\pi\varepsilon_0 x^2} + \frac{q}{4\pi\varepsilon_0 x^2}$$

or
$$E = \frac{q_0}{4\pi\varepsilon_0 x^2} + \frac{2\pi\alpha\,(x^2 - R^2)}{4\pi\varepsilon_0 x^2} \quad \text{(on putting the value of } x\text{)}$$

or $E = \dfrac{q_0 + 2\pi\alpha\,(x^2 - R^2)}{4\pi\varepsilon_0 x^2}$ or $E = \dfrac{(q_0 - 2\pi\alpha R^2) + 2\pi\alpha x^2}{4\pi\varepsilon_0 x^2}$

According to problem, result should be independent of x (in the problem, r is taken at the place of x).

If $$q_0 - 2\pi\alpha R^2 = 0$$

Then $E = \dfrac{2\pi\alpha}{4\pi\varepsilon_0}$ which is independent of x.

Hence, $$q_0 - 2\pi\alpha R^2 = 0$$

$$\therefore \quad q_0 = 2\pi\alpha R^2 \quad \text{and} \quad E = \frac{\alpha}{2\varepsilon_0}$$

YOUR STEP

A metal sphere having a radius r_1 charged to a potential ϕ_1 is enveloped by a thin walled conducting spherical shell of radius r_2 (see Fig. 3.26D). Determine the potential ϕ_2 acquired by the sphere after it has been connected for a short time to the shell by a conductor.

$$\left\{\phi_2 = \frac{q_1}{4\pi\varepsilon_0 r_2} = \phi_1 \frac{r_1}{r_2}\right\}$$

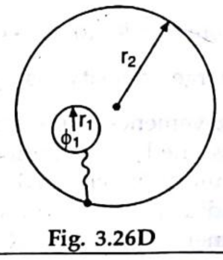


Fig. 3.26D

§ 3.27

➢ **CONCEPT**

The electric field on the surface of spherical distribution of charge, is

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

This result is applicable for uniform as well as non-uniform distribution of charge.
This result does not depend upon charge outside the surface.

SOLUTION : Step I. Draw a sphere of radius r

The sphere of radius r is shown in Fig. 3.27A.

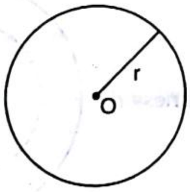


Fig. 3.27A

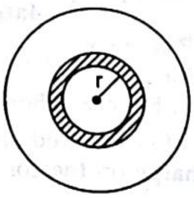


Fig. 3.27B

Step II. Calculate the net charge enclosed by the sphere, Fig. 3.27B.

For this, we consider a hollow concentric sphere of radius r and thickness dr. The volume of considered element is $dV = 4\pi r^2 dr$.

The electric charge in the considered element is.

$$dq = \rho dV = \rho_0 e^{-\alpha r^3} 4\pi r^2 dr = 4\pi\rho_0 r^2 e^{-\alpha r^3} dr$$

$\therefore$ Total charge enclosed by the sphere is $q = \int_0^r 4\pi\rho_0 r^2 e^{-\alpha r^3} dr$

$$\therefore \quad q = \frac{4\pi\rho_0}{3\varepsilon_0\alpha}(1 - e^{-\alpha r^3})$$

Step III. Apply the concept of the problem :

$$\because \quad E = \frac{q}{4\pi\varepsilon_0 r^2} \quad \text{radially outward}$$

$$= \frac{\rho_0}{3\varepsilon_0 \alpha r^2}[1 - e^{-\alpha r^3}]$$

Step IV. Discuss the condition of problem :

According to problem, $\alpha r^3 << 1$

$e^{-\alpha r^3} = 1 - \alpha r^3$ (see page 372 of the book I.E. Irodov).

$$\therefore \quad E = \frac{\rho_0 \alpha r^3}{3\varepsilon_0 \alpha r^2} = \frac{\rho_0 r}{3\varepsilon_0}$$

$e^{-\alpha r^3} \approx 0$ (when $\alpha r^3 >>> 1$,) $\therefore \quad E = \dfrac{\rho_0}{3\varepsilon_0 \alpha r^2}$

YOUR STEP

1. The region between two concentric spheres of radii $a < b$ contains volume charge density $\rho(r) = \dfrac{C}{r}$, where C is constant and r is the radial distance, as shown in Fig. 3.27(C). A point having charge q is placed at the origin, $r = 0$. Find the value of C for which the electric field in the region between the spheres is constant (*i.e.*, r is independent).
2. A distribution of electric charge is spherically symmetric about an origin O. If r is the distance measured from O and the total charge within a sphere of radius r, centre O, is $\dfrac{qr^2}{a^2}\left[e^{-r/a} - e^{-2r/a}\right]$, Show that potential ϕ is $\phi = \dfrac{q}{4\pi\varepsilon_0 a}\left[e^{-r/a} - \dfrac{1}{2}e^{-2r/a}\right]$ and that as $r \to 0$, the charge density (tends to) $\dfrac{3q}{\pi a^3}$

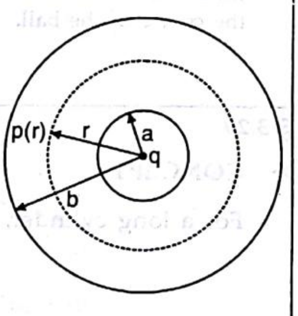


Fig. 3.27C

$\left\{ \textbf{1.}\ C = \dfrac{q}{a^2} \right\}$

§ 3.28

➢ **CONCEPT**

Electric field at a point = electric field due to sphere without cavity− electric field due to a sphere of radius equal to cavity.

Mathematically, $\vec{E} = \vec{E}_1 - \vec{E}_2$

SOLUTION : From Fig. 3.28A, $\vec{E} = \dfrac{\rho\vec{r}}{3\varepsilon_0} - \dfrac{\rho\vec{r}_1}{3\varepsilon_0} = \dfrac{\rho}{3\varepsilon_0}(\vec{r} - \vec{r}_1)$ ..(i)

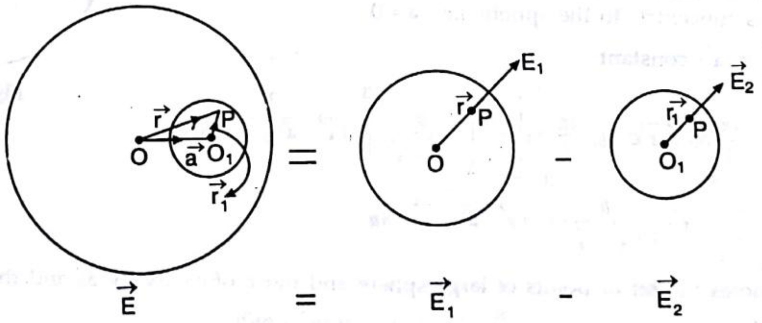


Fig. 3.28A

By addition law of vectors, $\overrightarrow{OO_1} + \overrightarrow{O_1P} = \overrightarrow{OP}$

or $\vec{a} + \vec{r_1} = \vec{r}$

$\therefore$ $\vec{a} = \vec{r} - \vec{r_1}$

From equation (i), $\vec{E} = \dfrac{\rho \vec{a}}{3\varepsilon_0}$

Remarks : Electric field inside the cavity remains constant with respect to magnitude and direction.

This electric field is directed along the line joining the centre of sphere and the centre of cavity.

> **YOUR STEP**
>
> The radius of a metal ball is 5 cm, the thickness of the spherical layer of a dielectric surrounding the ball is 5 cm, the permittivity of the layer is $\varepsilon = 3$ and the charge of the ball, $q = 10.8$ cgs electrostatic units. Calculate the intensity of the field at the points lying a distance $r_1 = 6$ cm and $r_2 = 12$ cm from the centre of the ball.
>
> $\{E_1 = \dfrac{q}{\varepsilon r_1^2} = 0.1$ cgs electrostatic unit; $E_2 = \dfrac{q}{r_2^2} = 0.075$ cgs electrostatic unit$\}$

§ 3.29

➢ **CONCEPT**

For a long cylinder, $E = \dfrac{\rho r}{2\varepsilon_0}$ when $r \le R$

$E = \dfrac{\rho R^2}{2\varepsilon_0 r}$ when $r \ge R$

SOLUTION : The top view of cylinder is shown in Fig. 3.29A.

From solution of previous problem, $\vec{E} = \vec{E_1} - \vec{E_2} = \dfrac{\rho \vec{r}}{2\varepsilon_0} - \dfrac{\rho \vec{r_1}}{2\varepsilon_0}$

$\therefore$ $\vec{E} = \dfrac{\rho}{2\varepsilon_0}(\vec{r} - \vec{r_1})$ $\therefore$ $\vec{E} = \dfrac{\rho}{2\varepsilon_0}\vec{a}$ (By addition law of vectors)

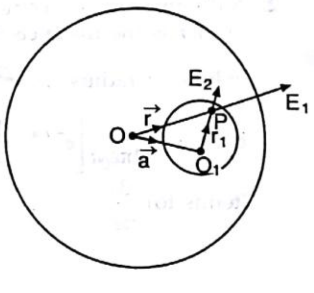


Fig. 3.29A

> **YOUR STEP**
>
> 1. A sphere of radius R has an uniform volume density ρ_0. A spherical cavity of radius b whose centre lies at $\vec{r} = \vec{a}$ is formed inside the sphere [see Fig. 3.29(B)].
> (a) Find the electric field at any point inside the spherical cavity.
> (b) Find the electric field outside the spherical cavity.
> (c) Find the electrostatic energy stored in the system in the case that the cavity is concentric to the sphere, *i.e.*, $\vec{a} = 0$.
>
> (a) $\vec{E} = \dfrac{4\pi}{3}\rho_0 \vec{a} = \text{constant}$
>
> (b) $\vec{E}(\vec{r}) = \dfrac{4\pi}{3}\rho_0 \vec{a}$; $\vec{r} \in S_b$, $\dfrac{4\pi}{3}\rho_0\left[\vec{r} - \left(\dfrac{b}{|\vec{r}-\vec{a}|}\right)^3 (\vec{r}-\vec{a})\right]$, $\vec{r} \in S_R/S_b$
>
> $\dfrac{4\pi}{3}\rho_0\left[\left(\dfrac{R}{r}\right)^3 \vec{r} - \left(\dfrac{b}{|\vec{r}-\vec{a}|}\right)^3 (\vec{r}-\vec{a})\right]$, $\vec{r} \in \overline{S_R}$
>
> where, S_R denotes the set of points of large sphere and those of cavity by S_b and those outside S_R by $\overline{S_R}$. Also $\dfrac{S_R}{S_b}$ is S_R less S_b. (c) $U = \dfrac{8}{15}\pi^2 \rho_0^2 [2R^5 - 5b^3R^2 + 3b^5]$

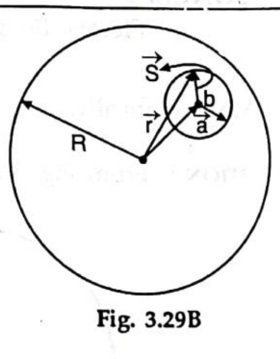


Fig. 3.29B

§ 3.30

➢ CONCEPT

Electric potential at point P (on the axis) due to ring is

$$\phi = \frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0\sqrt{R^2+a^2}}$$

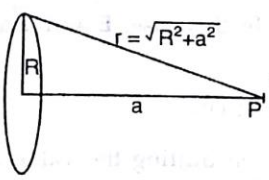


Fig. 3.30A

SOLUTION : Step I : Determine the net potential at the centre of first ring.

The electric potential on the centre of 1st ring is $\phi_1 = \phi'_1 + \phi''_1$

Here, ϕ'_1 = electric potential due to 1st ring on its centre $= \frac{q}{4\pi\varepsilon_0 R}$

ϕ''_1 = electric potential due to second ring at the centre of first ring $= \frac{-q}{4\pi\varepsilon_0\sqrt{R^2+a^2}}$

Step II. Determine the net potential at the centre of second ring.

$$\phi_2 = \phi'_2 + \phi''_2$$

where ϕ'_2 = Electric potential due to second ring at its centre

$$= \frac{-q}{4\pi\varepsilon_0 R}$$

and ϕ''_2 = Electric potential due to first ring on the centre of second ring $= \frac{q}{4\pi\varepsilon_0\sqrt{R^2+a^2}}$.

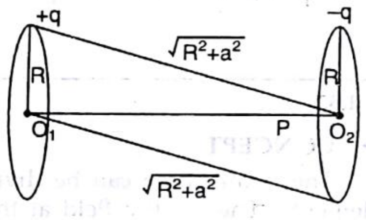


Fig. 3.30B

Step III. Determine potential difference. $\Delta\phi = \phi_1 - \phi_2$

On putting the values of ϕ_1 and ϕ_2, we get

$$\Delta\phi = \frac{q}{2\pi\varepsilon_0 R}\left(1 - \frac{1}{\sqrt{1+\left(\frac{a}{R}\right)^2}}\right)$$

YOUR STEP

Two circular rings A and B, each of radius $a = 30$ cm, are placed coaxially with their axes vertical as shown in Fig. 3.30 (C). Distance between centres of these ring is $h = 40$ cm. Lower ring A has a positive charge of 10 μC, while upper ring B has a negative charge of 20 μC. A particle of mass $m = 100$ g carrying a positive charge of $q = 10$ μC is released from rest at the centre of the ring A.

(i) Calculate initial acceleration of the particle.

(ii) Calculate velocity of particle when it reaches at the centre of upper ring B. [$g = 10$ m/sec^2]

{(i) 47.6 m/sec^2, (ii) 8 m/sec}

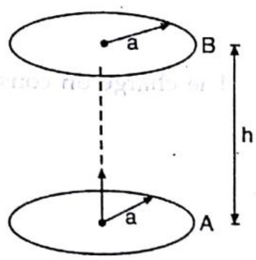


Fig. 3.30C

§ 3.31

➢ CONCEPT

The electric field due to long straight wire is $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ along increasing r.

Also, $d\phi = -\vec{E}\cdot d\vec{r}$

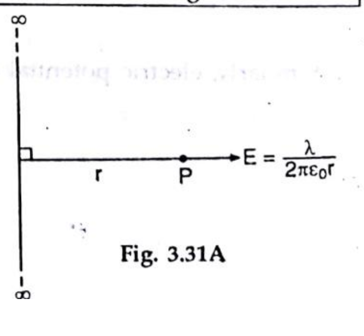


Fig. 3.31A

In this case, $\vec{E}$ and $\vec{r}$ are in the same direction.

$\therefore$ $$d\phi = -Edr\cos 0 = -Edr$$

SOLUTION : $\therefore$ $$d\phi = -Edr$$

On putting the value of E, $\int_{\phi_1}^{\phi_2} d\phi = -\int_{r_0}^{\eta r_0} \frac{\lambda}{2\pi\varepsilon_0 r} dr$ (see Fig. 3.31B)

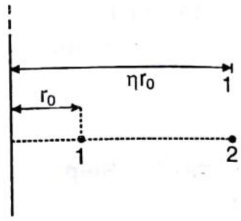


Fig. 3.31B

or $$\phi_1 - \phi_2 = \frac{\lambda}{2\pi\varepsilon_0} ln\,\eta$$

where, ϕ_1 and ϕ_2 are potentials of points (1) and (2)

$\therefore$ $$\Delta\phi_{12} = \phi_1 - \phi_2 = \frac{\lambda}{2\pi\varepsilon_0} ln\,\eta$$

> **YOUR STEP**
>
> Two long wires are placed on a smooth horizontal table. Wires have equal but opposite charges. Magnitude of linear charge density on each wire is λ. Calculate (for unit length of wires) work required to increase the separation between the wires from a to $2a$.
>
> $$\left\{\frac{\lambda^2}{2\pi\varepsilon_0}\log_e 2\right\}$$

§ 3.32

➢ CONCEPT

The hemisphere can be divided into a number of ring elements. The electric field at the point O is vector sum of the field due to each such rings.

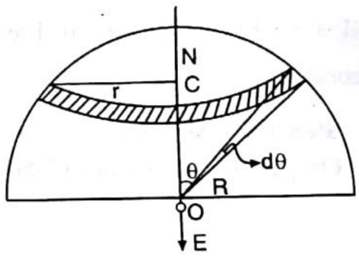


Fig. 3.32A

SOLUTION : Let us consider a ring element located at an angle θ with the reference line ON (shown in Fig. 3.32A), and subtending an angle $d\theta$ at the centre O. The area of considered element is $dA = 2\pi r ds$.

where,
$$r = R\sin\theta$$
$$dS = Rd\theta$$
$$OC = x = R\cos\theta$$

$\therefore$ $$dA = 2\pi R^2 \sin\theta\, d\theta$$

The charge on considered element is $dq = \sigma\, 2\pi R^2 \sin\theta d\theta$

$\therefore$ $$dE = \frac{xdq}{4\pi\varepsilon_0 (r^2 + x^2)^{3/2}}$$

or $$dE = \frac{2\pi\sigma\sin\theta\cos\theta\, d\theta}{4\pi\varepsilon_0} = \frac{\sigma\sin 2\theta\, d\theta}{4\pi\varepsilon_0}$$

$\therefore$ $$E = \int dE = \int_0^{\pi/2} \frac{\sigma\sin 2\theta\, d\theta}{4\pi\varepsilon_0}$$

$\therefore$ $$E = \frac{\sigma}{4\varepsilon_0} \text{ along } NO$$

Similarly, electric potential due to considered ring element at point O is
$$d\phi = \frac{dq}{4\pi\varepsilon_0 R} = \frac{\sigma R\sin\theta\, d\theta}{2\varepsilon_0}$$

$\therefore$ $$\phi = \int d\phi = \frac{\sigma R}{2\varepsilon_0}\int_0^{\pi/2} \sin\theta\, d\theta$$

$\therefore$ $$\phi = \frac{\sigma R}{2\varepsilon_0}$$

YOUR STEP

Determine the force F of interaction between two hemispheres of radius R touching each other along the equator if one hemisphere is uniformly charged with a surface density σ_1 and the other with a surface density σ_2.

$$\left\{F = \frac{\pi R^2}{2\varepsilon_0}\sigma_1\sigma_2\right\}$$

§ 3.33

➢ CONCEPT

Electric field due to a circular ring at a point on the axis of ring is

$$E = \frac{q\,x}{4\pi\varepsilon_0\,(R^2 + x^2)^{3/2}}$$

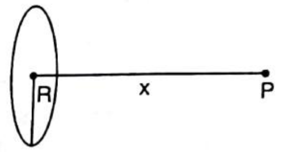


Fig. 3.33A

SOLUTION : We have to find electric field strength E on the axis of disc at distance l from the centre of disc.

From geometrical construction, circular disc charge distribution is a collection of concentric charged rings.

We consider a ring element of radius r and thickness dr.

The area of considered ring is

$$dA = \text{circumference} \times \text{thickenss} = 2\pi r dr$$

The charge on considered element is

$$dq = \sigma dA = 2\pi\sigma r dr$$

Electric field at point P due to considered ring is

$$dE = \frac{l dq}{4\pi\varepsilon_0\,(r^2 + l^2)^{3/2}}$$

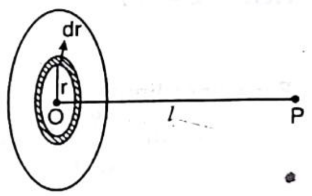


Fig. 3.33B

To find net electric field due to the disc, we must integrate from 0 to R (not from $-R$ to $+R$).

$$\therefore \quad E = \int_0^R \frac{2\pi\sigma r l dr}{4\pi\varepsilon_0\,(l^2 + r^2)^{3/2}} = \frac{\sigma l}{2\varepsilon_0}\int_0^R \frac{r dr}{(l^2 + r^2)^{3/2}}$$

This integration is evaluated by substitution method.

Put $\quad r = l\tan\theta$

For lower limit, $r = 0,\ \theta = 0$

When $l \to 0$, $\quad E = \dfrac{\sigma}{2\varepsilon_0}$

When $l >> R$, $\quad E = \dfrac{\sigma\pi R^2}{4\pi\varepsilon_0 l^2} = \dfrac{q}{4\pi\varepsilon_0\, l^2}$

Here, $\quad q = \sigma\pi R^2$

Expression for electric potential.

We have to find electric potential at point P on the axis of disc at distance l from the centre of disc. The corresponding situation is shown in Fig. 3.33C.

The electric field at point P is $\quad E = \dfrac{\sigma}{2\varepsilon_0}\left[1 - \dfrac{l}{\sqrt{R^2 + l^2}}\right]$

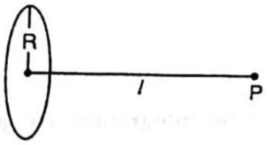


Fig. 3.33C

when $l \to o$, then $E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{0}{\sqrt{R^2 + 0^2}}\right] = \frac{\sigma}{2\varepsilon_0}$ for $l \to 0$

$\because$ $$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{l}{\sqrt{R^2 + l^2}}\right]$$

or $$E = \frac{\sigma}{2\varepsilon_0}\left[1 - l\,(R^2 + l^2)^{-\frac{1}{2}}\right]$$

or $$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{l}{l}\left(\frac{R^2}{l^2} + 1\right)^{-\frac{1}{2}}\right]$$

or $$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1 - \frac{1}{2}\frac{R^2}{l^2}\right)\right] \qquad \text{for } l >> R$$

or $E \approx \frac{\sigma}{2\varepsilon_0}\left[\frac{R^2}{2l^2}\right]$ or $E \approx \frac{\sigma\,\pi R^2}{4\pi\varepsilon_0\, l^2}$

or $E \approx \frac{q}{4\pi\varepsilon_0 l^2}$ Here $q = \sigma\pi R^2$ (for $l >> R$)

Here, $E = f(l)$

$\therefore$ $$\int_{\phi}^{0} d\phi = -\int_{l}^{\infty} E dL$$

For upper limit, $r = R$,

$\therefore$ $\theta = \tan^{-1}\left(\frac{R}{l}\right)$ $\therefore$ $r = l \tan\theta$

$$\frac{dr}{d\theta} = l \sec^2\theta$$

$\therefore$ $$dr = l \sec^2\theta\, d\theta$$

$\therefore$ $$E = \frac{\sigma}{2\varepsilon_0}\int_{0}^{\theta = \tan^{-1}\left(\frac{R}{l}\right)} \frac{l^3 \tan\theta \sec^3\theta\, d\theta}{l^3\,(1 + \tan^2\theta)^{3/2}}$$

or $$E = \frac{\sigma}{2\varepsilon_0}\int_{0}^{\theta} \sin\theta\, d\theta = \frac{\sigma}{2\varepsilon_0}[1 - \cos\theta] \qquad \text{...(i)}$$

$\because$ $$\theta = \tan^{-1}\frac{R}{l}$$

or $\tan\theta = \frac{R}{l}$ $\therefore$ $$\cos\theta = \frac{l}{\sqrt{R^2 + l^2}} \qquad \text{...(ii)}$$

Putting the values of $\cos\theta$ from eq. (ii) in equation (i), we get

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{l}{\sqrt{R^2 + l^2}}\right]$$

or $$\int_{\phi}^{0} d\phi = -\int_{l}^{\infty} \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{l}{\sqrt{R^2 + l^2}}\right]$$

After integrating, we get, $$\phi = \frac{\sigma l}{2\varepsilon_0}\left[\sqrt{1 + \left(\frac{R}{l}\right)^2} - 1\right]$$

For $l \to 0$, $\phi = \frac{\sigma R}{2\varepsilon_0}$ for, $l >> R$, $\phi \approx \frac{q}{4\pi\varepsilon_0 l}$ where, $q = \sigma\pi R^2$

YOUR STEP

1. A surface charge distribution of density σ is spread uniformly over a disc of radius a. Calculate the electric potential and the electric field at a point P a distance z above the disc surface, on an axis perpendicular to the disc surface, at its centre as shown in Fig. 3.33(D).
2. The diameter of a charged disc is 25 cm. At what maximum distance from the disc along a normal to its centre may an electric field be regarded as the field of an infinitely long plane ? The error should not exceed 5%.

Note : The error $\delta = \frac{E_2 - E_1}{E_2}$, where E_1 is the intensity of the field induced by the disc, and E_2 that of the field induced by an infinite plane.

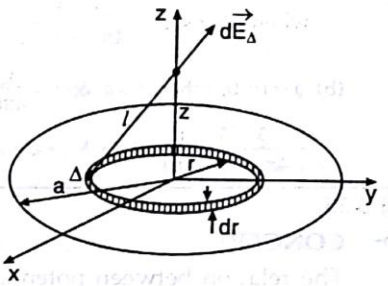


Fig. 3.22D

$$\left\{ 1.\ \phi(y) = \frac{Q}{4\pi\varepsilon_0 \, |z|} \text{ where, } Q = \pi a^2 \sigma,\ 2.\ \frac{a}{R} = \frac{\delta}{\sqrt{1-\delta^2}} = \delta \right.$$

$$\left. \text{when } \delta = 0.05 \text{ and } R = 0.25 \text{ m We get } a = 1.2 \times 10^{-2} \text{ m} \right\}$$

§ 3.34

➢ **CONCEPT**

Electric potential due to an element is $d\phi = \frac{dq}{4\pi\varepsilon_0 r}$.

SOLUTION : We consider an area element dA in the form of a part of ring of radius r and thickness dr (shown in Fig. 3.34A) Since, BDE is the part of a ring whose centre is at O,

So, $$OD = OB = OE = r$$

Also, $$\angle FBO = 90°,\ OF = 2R$$

In right angle triangle FBO, $$\cos\theta = \frac{OB}{OF} = \frac{r}{2R}$$

$\therefore$ $$r = 2R\cos\theta$$

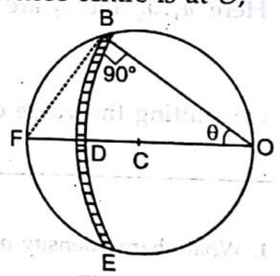


Fig. 3.34A

Differentiating both sides with respect to θ.

$\frac{dr}{d\theta} = -2R\sin\theta \qquad \therefore \qquad dr = -2R\sin\theta\, d\theta$

The area of the considered element is $dA = 2r\theta dr$

($\because$ arc ADE makes 2θ angle at the centre of ring)

$\therefore$ The charge on considered element $= dq$

$\therefore$ $$dq = \sigma dA \qquad \therefore \qquad dq = 2r\sigma\theta dr$$

$\therefore$ Electric potential at point O due to disc is

$$\phi = \int \frac{dq}{4\pi\varepsilon_0 r} = \int \frac{2r\theta\,\sigma dr}{4\pi\varepsilon_0 r}$$

$$\phi = \int_0^{\pi/2} \frac{2r\theta\,\sigma\,(-2R\sin\theta\, d\theta)}{4\pi\varepsilon_0 r}$$

On solving, we get $$\phi = \frac{\sigma R}{\pi\varepsilon_0}$$

YOUR STEP

A solid, insulating ball of radius a is surrounded by a conducting spherical shell with an inner radius of b and an outer radius of c [see Fig. 3.34B]. The inner ball has charge Q, which is uniformly distributed throughout its volume. The conducting spherical shell is charged with charge $-Q$.

(a) Compute the electric potential everywhere.

(b) The inner ball is grounded; *i.e.*, its potential is fixed to be zero, while its charge is no longer Q. Compute the electric potential everywhere. Calculate the total charge in the inner ball.

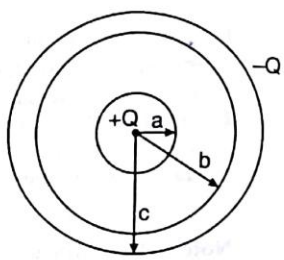


Fig. 3.34B

(a) $\phi(r) = \dfrac{Q}{8\pi\varepsilon_0 a}\left(\dfrac{r}{a}\right)^2 + \dfrac{\phi}{4\pi\varepsilon_0}\left(\dfrac{3}{2a} - \dfrac{1}{b}\right)$

when $0 \le r < a$ $= \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r} - \dfrac{1}{b}\right)$ when $a \le r < b$ $= 0$, when $b \le r$.

(b) $\phi(r) = 0$, when $r \le a$, $\phi(r) = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r} - \dfrac{1}{a}\right)$ when $a \le r \le b$

$= \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{b} - \dfrac{1}{a}\right)$ when $b < r < c$ and $\phi(r) = 0$ when $r \ge c$

§ 3.35

➢ **CONCEPT**

The relation between potential and electric field is defined as below :

$$\vec{E} = \frac{-\partial\phi}{\partial x}\hat{i} - \frac{\partial\phi}{\partial y}\hat{j} - \frac{\partial\phi}{\partial z}\hat{k}$$

This relation is applicable for solving the problem,

$$\phi = \vec{a}\cdot\vec{r}$$

Here, $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ (say)

Also, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore$ $\phi = xa_x + ya_y + za_z$

Here, a_x, a_y and a_z are constant.

$\therefore$ $\vec{E} = -\dfrac{\partial\phi}{\partial x}\hat{i} - \dfrac{\partial\phi}{\partial y}\hat{j} - \dfrac{\partial\phi}{\partial z}\hat{k}$

On putting the value of ϕ, we get, $\vec{E} = -a_x\hat{i} - a_y\hat{j} - a_z\hat{k}$

$\therefore$ $\vec{E} = -\vec{a}$

YOUR STEP

1. What charge density $\rho(r)$ would produce a field $E = \dfrac{q}{4\pi\varepsilon_0}\dfrac{r}{r^a}$? What is the potential of this field?

2. The screened coulomb potential $\phi = \dfrac{q}{4\pi\varepsilon_0}\dfrac{e^{-r/\lambda}}{r}$, commonly occurs in a conducting medium. Calculate the corresponding electric field and charge density.

$\left\{ \textbf{1. } \rho = \dfrac{(3-a)q}{4\pi r^a},\ \phi = \dfrac{q}{4\pi\varepsilon_0}(a-2)\,r^{a-2}\ (a \ne 2)\quad \textbf{2. } \rho = q\,[\delta(r) - 1/4\pi\lambda^2 r]\,e^{-r/\lambda},\ E = \dfrac{q}{4\pi\varepsilon_0}\left(\dfrac{1}{r} + \dfrac{1}{\lambda}\right)e^{-r/\lambda}\cdot\dfrac{r}{r^2} \right\}$

§ 3.36

➢ **CONCEPT**

The concept is similar to previous problem *i.e.*,

$$\vec{E} = -\frac{\partial\phi}{\partial x}\hat{i} - \frac{\partial\phi}{\partial y}\hat{j} - \frac{\partial\phi}{\partial z}\hat{k}$$

SOLUTION : (a) Here, $\phi = a(x^2 - y^2)$

$\therefore$ $\vec{E} = -\dfrac{\partial\phi}{\partial x}\hat{i} - \dfrac{\partial\phi}{\partial y}\hat{j} = -2a(x\hat{i} - y\hat{j})$

(b) $\because$ $\phi = axy$

$\therefore$ $$\vec{E} = \frac{-\partial\phi}{\partial x}\hat{i} - \frac{\partial\phi}{\partial y}\hat{j} = -ay\hat{i} - ax\hat{j}$$

YOUR STEP

1. (a) An insulating sphere of radius a has a uniform charge density ρ. The sphere is centred at $\vec{r} = \vec{b}$, not at the origin. Show that the electric field inside the sphere is given $\vec{E} = \frac{\rho(\vec{r} - \vec{b})}{3\varepsilon_0}$.

(b) An insulating sphere of radius R has a spherical hole of radius a located within its volume and centred a distance b from the centre of the sphere, where $a < b < R$ (a cross-section of the sphere is shown in Fig. 3.36 (A)). The solid part of the sphere has a uniform volume charge density ρ. Find the magnitude and direction of the electric field $\vec{E}$ inside the hole, and show that $\vec{E}$ is uniform over the entire hole.

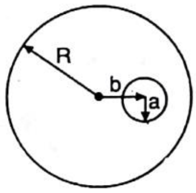


Fig. 3.36A

2. Find the shape of the lines of force if radial and transverse components of electric field intensity are given by $E_r = \frac{2P\cos\theta}{\varepsilon_0 r^3}$, $E_\theta = \frac{P\sin\theta}{\varepsilon_0 r^3}$ and find the charge contained by a sphere of radius b described about its centre.

{1. (b) $\vec{E} = \frac{\rho b}{3\varepsilon_0}$ 2. $q = 0$}

§ 3.37

➢ **CONCEPT**

The negative gradient of potential gives electrostatic field (conservation field).

Mathematically, $$\vec{E} = -\nabla\phi = -\frac{\partial\phi}{\partial x}\hat{i} - \frac{\partial\phi}{\partial y}\hat{j} - \frac{\partial\phi}{\partial z}\hat{k}$$

SOLUTION :

$$\phi = a(x^2 + y^2) + bz^2$$

$$\frac{\partial\phi}{\partial x} = \frac{\partial}{\partial x}\{a(x^2 + y^2) + bz^2\} = 2ax$$

$$\frac{\partial\phi}{\partial y} = \frac{\partial}{\partial y}\{a(x^2 + y^2) + bz^2\} = 2ay$$

Similarly, $$\frac{\partial\phi}{\partial z} = 2bz$$

$\therefore$ $$\vec{E} = -\frac{\partial\phi}{\partial x}\hat{i} - \frac{\partial\phi}{\partial y}\hat{j} - \frac{\partial\phi}{\partial z}\hat{k} = -2ax\,\hat{i} - 2ay\,\hat{j} - 2bz\,\hat{k}$$

$$= -2(ax\hat{i} + ay\hat{j} + bz\hat{k})$$

$$|\vec{E}| = \sqrt{(-2ax)^2 + (-2ay)^2 + (-2bz)^2}$$

$$= \sqrt{4a^2(x^2 + y^2) + 4b^2z^2} = 2\sqrt{a^2(x^2 + y^2) + b^2z^2}$$

YOUR STEP

A spherical charge distribution has a volume charge density that is a function only of r, the distance from the centre of the distribution. In other words, $\rho = \rho(r)$. If $\rho(r)$ is as given below, determine the electric field as a function of r. Integrate the result to obtain an expression for the electrostatic potential $\phi(r)$, subject to the restriction that $\phi(\infty) = 0$.

(a) $\rho = \frac{A}{r}$ with A a constant for $0 \le r \le R$, $\rho = 0$ for $r > R$ (b) $\rho = \rho_0$ (*i.e.*, constant) for $0 \le r \le R$; $\rho = 0$ for $r > R$

$$\left\{(a)\ \phi = \left(\frac{A}{\varepsilon_0}\right)\left(R - \frac{1}{2}r\right) \text{ for } r \le R;\ \phi = \frac{AR^2}{2\varepsilon_0 r} \text{ for } r \ge R\quad (b)\ \phi = \left(\frac{\rho_0}{2\varepsilon_0}\right)\left(R^2 - \frac{r^2}{3}\right) \text{ for } r \le R;\ \phi = \frac{R^3\rho_0}{3\varepsilon_0 r} \text{ for } r \ge R\right\}$$

§ 3.38

➢ **CONCEPT**

SOLUTION : We consider a uniformly charged non-conducting sphere of radius R having charge q. We have to calculate electric potential at a point P at distance r from centre of sphere $(r < R)$ and also at the centre of sphere.

(b) At an internal point $(r < R)$

As we know $E = \dfrac{\rho r}{3\varepsilon_0}$ when $r < R$ $\therefore$ $\partial\phi = -E \cdot dr$ or $-d\phi = Edr$

or $$-\int_{\phi}^{0} d\phi = \int_{r}^{R} Edr + \int_{R}^{\infty} Edr \quad \therefore \quad \phi = \int_{r}^{R} \frac{\rho r}{3\varepsilon_0} dr + \int_{R}^{\infty} \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

or $$\phi = \frac{\rho}{3\varepsilon_0}\left[\frac{R^2 - r^2}{2}\right] + \frac{Q}{4\pi\varepsilon_0 R}$$

or $$\phi = \frac{3Q}{3\varepsilon_0 4\pi R^3}\left[\frac{R^2 - r^2}{2}\right] + \frac{Q}{4\pi\varepsilon_0 R} \qquad \left\{\because \rho = \frac{3Q}{4\pi R^3}\right\}$$

$$\phi = \frac{Q}{4\pi\varepsilon_0 R^3}\left[\frac{R^2 - r^2}{2}\right] + \frac{Q}{4\pi\varepsilon_0 R}$$

$$\phi = \frac{Q(R^2 - r^2) + 2R^2\theta}{8\pi\varepsilon_0 R^3}$$

$$\phi = \frac{Q}{8\pi\varepsilon_0 R}\left[3 - \frac{r^2}{R^2}\right] = \frac{Q}{4\pi\varepsilon_0 R}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right]$$

(a) But at the centre of sphere $r = 0$. So the expression for the potential at the centre of sphere is $\phi_0 = \dfrac{3Q}{8\pi\varepsilon_0 R}$ $\quad\therefore$ Potential inside the sphere, $\phi = \phi_0\left[1 - \dfrac{r^2}{3R^2}\right]$ when $r < R$

YOUR STEP

1. A non-conducting hollow sphere having inner and outer radii a and b respectively is made of a material having dielectric constant K and has uniformly distributed charge over its entire solid volume. Volume density of charge is ρ. Calculate potential at a distance r from its centre when
(i) $r > b$ (ii) $r < a$ (iii) $a < r < b$

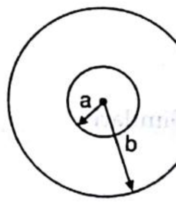


Fig. 3.38A

2. If the potential of point outside a conducting sphere of radius a is $-F\cos\theta\left(r - \dfrac{a^3}{r^2}\right)$, where r is the distance measured from the centre, θ is the angular distance measured from a fixed diameter and F is constant. Find the surface density at any point of the sphere and show that the total charge on the sphere is zero.
3. An insulated thin spherical shell of radius a, charge Q, potential ϕ_1, is surrounded by a concentric, thin, insulated, spherical shell of radius $2a$, charge $-2Q$, potential ϕ_2, which in turn is surrounded by a concentric, thin, insulated, spherical shell of radius $3a$, charge $3Q$, potential ϕ_3. Find ϕ_1, ϕ_2, ϕ_3 in terms of $\dfrac{Q}{a}$ and show that $3\phi_3 = \phi_1 + 2\phi_2$

$$\left\{ \begin{array}{l} 1.\ (i)\ V_1 = \dfrac{\rho(b^3 - a^3)}{3\varepsilon_0 r} \quad (ii)\ V_i = \dfrac{\rho(b^3 - a^3)}{3\varepsilon_0 b} + \dfrac{\rho}{3\varepsilon_0 K}\left[\left(\dfrac{b^2 - a^2}{2}\right) - \dfrac{a^3(b - a)}{ab}\right] \\ (iii)\ V = \dfrac{\rho(b^3 - a^3)}{3\varepsilon_0 b} + \dfrac{\rho}{3\varepsilon_0 K}\left[\left(\dfrac{b^2 - r^2}{2}\right) - \dfrac{a^3(b - r)}{rb}\right],\ 2.\ \sigma = \dfrac{3F\cos\theta}{4\pi} \quad 3.\ \phi_1 = \dfrac{Q}{a},\ \phi_2 = \dfrac{Q}{2a},\ \phi_3 = \dfrac{2Q}{3a} \end{array} \right\}$$

§ 3.39

SOLUTION : We have to find an expression for electric field intensity at point $S(r, \theta)$ due to a dipole of strength ($p = q \times 2a$), provided only that the point is not too close to the dipole. For this firstly we want to find the expression for potential.

Now, according to figure,

Electric potential at point S, $\phi = \phi_1 + \phi_2$.

$$\phi = \frac{-q}{4\pi\varepsilon_0 r_1} + \frac{+q}{4\pi\varepsilon_0 r_2}$$

$$\phi = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

or
$$\phi = \frac{q}{4\pi\varepsilon_0}\left[\frac{r_1 - r_2}{r_1 r_2}\right] \quad \text{...(i)}$$

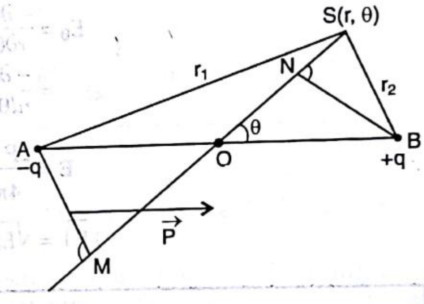


Fig. 3.39A

But according to basic assumption, $r >> 2a$

$\therefore$ $AS \approx MS$

or $r_1 \approx OS + OM$ (in Fig. 3.39(A))

But In ΔOAM, $\cos\theta = \frac{OM}{OA} = \frac{OM}{a}$

$\therefore$ $OM = a\cos\theta$

Now, In ΔONB, $\cos\theta = \frac{ON}{OB} = \frac{ON}{a}$

$\therefore$ $ON = a\cos\theta$ $\quad\therefore\quad r_1 \approx OS + OM = r + a\cos\theta$

Similarly, $BS \approx SO - ON = r - a\cos\theta$

or $r_2 \approx r - a\cos\theta$

But
$$\phi = \frac{q}{4\pi\varepsilon_0}\left[\frac{r_1 - r_2}{r_1 r_2}\right]$$

or
$$\phi = \frac{q}{4\pi\varepsilon_0}\left[\frac{r + a\cos\theta - (r - a\cos\theta)}{(r + a\cos\theta)(r - a\cos\theta)}\right] \quad \text{or} \quad \phi = \frac{q}{4\pi\varepsilon_0}\left[\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}\right]$$

or
$$\phi = \frac{1}{4\pi\varepsilon_0}\left[\frac{q \times 2a\cos\theta}{r^2 - a^2\cos^2\theta}\right] \quad \text{or} \quad \phi = \frac{1}{4\pi\varepsilon_0}\left[\frac{P\cos\theta}{r^2 - a^2\cos^2\theta}\right]$$

$a^2\cos^2\theta$ can be neglected with respect to r^2

$$\therefore \quad \phi = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{Pr\cos\theta}{4\pi\varepsilon_0 r^3} \qquad \therefore \quad \phi = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

It means $\phi = f(r, \theta)$

Let $\hat{e}_r$ = unit vector along increasing $\vec{r}\,(\vec{OS})$

$\hat{e}_\theta$ = unit vector along increasing angle θ.

Unit vectors $\vec{e}_r$ and $\vec{e}_\theta$ are perpendicular to each other.

The arc $SQ = \partial S$ (In fig. 3.39B).

$\therefore$ According to Fig. 3.39B,

$$\vec{E} = E_r\hat{e}_r + E_\theta\hat{e}_\theta$$

$$E_r = \frac{-\partial\phi}{\partial r} = \frac{-\partial}{\partial r}\left(\frac{P\cos\theta}{4\pi\varepsilon_0 r^2}\right)$$

$$E_r = \frac{-P\cos\theta}{4\pi\varepsilon_0}\frac{\partial(r^{-2})}{\partial r}$$

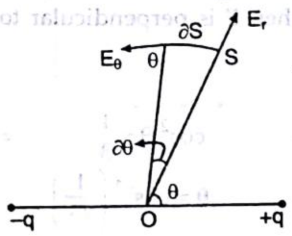


Fig. 3.39B

$$E_r = \frac{-P\cos\theta}{4\pi\varepsilon_0} \times (-2)\, r^{-2-1} = \frac{2P\cos\theta}{4\pi\varepsilon_0 r^3}$$

Now,
$$E_\theta = \frac{-\partial\phi}{\partial S}$$

$$E_\theta = \frac{-\partial\phi}{r\partial\theta} \quad \because \quad d\theta = \frac{\partial S}{dr} \qquad \therefore\ \partial S = r\partial\theta$$

$$= \frac{-\partial}{r d\theta}\left(\frac{P\cos\theta}{4\pi\varepsilon_0 r^2}\right) = \frac{-P}{4\pi\varepsilon_0 r^3}(-\sin\theta) = \frac{P\sin\theta}{4\pi\varepsilon_0 r^3}$$

$\therefore$
$$\vec{E} = \frac{2P\cos\theta}{4\pi\varepsilon_0 r^3}\hat{e}_r + \frac{P\sin\theta}{4\pi\varepsilon_0 r^3}\hat{e}_\theta$$

$$|\vec{E}| = \sqrt{E_r^2 + E_\theta^2} = \frac{P}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta} = \frac{P}{4\pi\varepsilon_0 r^3}\sqrt{1 + 3\cos^2\theta}$$

YOUR STEP

A thunder cloud which may be regarded as electric doublet with its axis vertical, is moving uniformly along a horizontal straight line and directly approaching an observer on the ground who is recording electric intensity close to the surface. Show that change of the rate of electric intensity vanishes when the elevation of the cloud is $\tan^{-1}\left(\frac{1}{2}\right)$.

§ 3.40

➢ **CONCEPT**

From the solution fo previous problem,

$$E_r = \frac{2P\cos\theta}{4\pi\varepsilon_0 r^3} \qquad \text{(shown in Fig. 3.40A)}$$

and
$$E_\theta = \frac{P\sin\theta}{4\pi\,\varepsilon_0\, r^3}$$

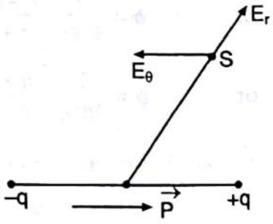


Fig. 3.40A

SOLUTION : From Fig. 3.40B,
$$E_\perp = E_x = E_r\sin\theta + E_\theta\cos\theta$$

Putting the values of E_r and E_θ,

$$E_\perp = \frac{2P\sin\theta\cos\theta}{4\pi\varepsilon_0 r^3} + \frac{P\sin\theta\cos\theta}{4\pi\varepsilon_0 r^3}$$

$\therefore$
$$E_\perp = \frac{3P\sin\theta\cos\theta}{4\pi\varepsilon_0 r^3} \qquad \text{...(i)}$$

Also,
$$E_z = E_r\cos\theta - E_\theta\sin\theta$$

Putting the values of E_r and E_θ, $E_z = \frac{P}{4\pi\varepsilon_0 r^3}(3\cos^2\theta - 1)$

when $\vec{E}$ is perpendicular to z-axis, then, $E_z = 0$.

$\therefore$
$$\frac{P}{4\pi\varepsilon_0 r^3}(3\cos^2\theta - 1) = 0$$

$\therefore \quad \cos^2\theta = \frac{1}{3} \quad \Rightarrow \quad \cos\theta = \frac{1}{\sqrt{3}}$

$\therefore \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad \Rightarrow \quad \theta_1 = 54.7°,\ \theta_2 = 123.5°.$

At these points, $E = E_\perp = \frac{P\sqrt{2}}{4\pi\varepsilon_0 r^3}$ (from equ. (i))

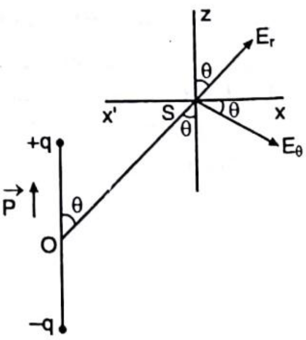


Fig. 3.40B

YOUR STEP

A dipole is placed in the field of a point charge, the distance between the dipole and the field source being much greater than the dipole separation. Find the force acting on the dipole and the torque, if the dipole is arranged;

(a) perpendicular to the line of force;

(b) in the direction of the line of force.

$$\left\{ \text{(a) } F_r = 0;\ M = \frac{QP_e}{4\pi\varepsilon_0 r^2};\ \text{(b) } F_r = \frac{-2QP_e}{4\pi\varepsilon_0 r^3};\ M = 0 \right\}$$

§ 3.41

➢ **CONCEPT**

Electric field is perpendicular to the equipotential surface. So, the component of electric field along equipotential surface is zero.

SOLUTION : Suppose that this equipotential surface is passing through point P whose radius is r (shown in Fig.3.41A). As we know that at any point of equipotential surface, field is perpendicular to the surface.

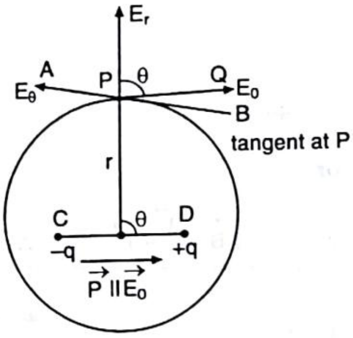


Fig. 3.41A

So, net electric field along tangent (AB) at the point P will be zero.

Since, $\quad \vec{E_0} \parallel \vec{P}$

$\therefore \quad CD \parallel PQ$

$\therefore \quad \angle TPQ = \angle POD = \theta$

$\therefore$ Net electric field along tangent at point P

$= E_t = E_0 \sin\theta - E_Q \quad$ But, $\ E = 0$

$$\therefore \quad E_0 \sin\theta = E_Q \quad \Rightarrow \quad E_0 \sin\theta = \frac{P}{4\pi\varepsilon_0 r^3}$$

$$\text{or} \quad E_0 = \frac{P}{4\pi\varepsilon_0 r^3} \quad \text{or} \quad r^3 = \frac{P}{4\pi\varepsilon_0 E_0}$$

$$r = \sqrt[3]{\frac{P}{4\pi\varepsilon_0 E_0}}$$

YOUR STEP

Obtain the electric field of a point dipole by calculating the gradient of $\phi = \frac{1}{4\pi\varepsilon_0}\frac{P\cdot r}{r^3}$.

$$\left\{ \vec{E} = -\frac{\vec{P}}{4\pi\varepsilon_0 r^3} + \frac{3(\vec{P}\cdot\vec{r})\vec{r}}{4\pi\varepsilon_0 r^5} \right\}$$

§ 3.42

➢ **CONCEPT**

The electric field due to a long charged thread is given by $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ (shown in Fig. 3.42A)

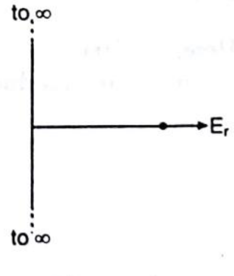


Fig. 3.42A

SOLUTION : From the Fig. 3.42B,

$$r_2 \approx CP = r - \frac{l}{2}\cos\theta \quad \text{and} \quad r_1 = PB \approx PD = r + \frac{l}{2}\cos\theta$$

Here, $\quad E_1 = \frac{\lambda p}{2\pi\varepsilon_0 r_1}$ along $\vec{PB}$, $\quad E_2 = \frac{\lambda}{2\pi\varepsilon_0 r_2}$ along $\vec{AP}$

Since, $\vec{E_1}$ is along $\vec{PB}$. So, unit vector along $\vec{E_1}$ is same as that of along $\vec{PB}$.

$$\therefore \quad \vec{E_1} = \frac{\lambda}{2\pi\varepsilon_0 r_1} \frac{\vec{PB}}{|\vec{PB}|} = \frac{\lambda}{2\pi\varepsilon_0 r_1} \frac{\vec{PB}}{r_1}$$

$$\therefore \quad \vec{E_1} = \frac{\lambda}{2\pi\varepsilon_0 r_1^2} \vec{PB}$$

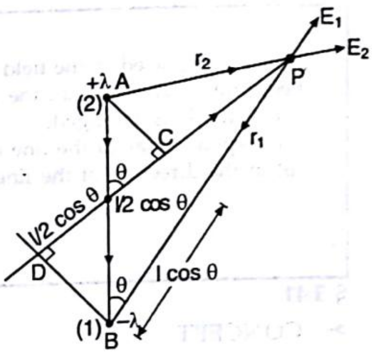


Fig. 3.42A

Similarly,
$$\vec{E_2} = \frac{\lambda}{2\pi\varepsilon_0 r_2^2} \vec{AP}$$

$\therefore$ Resultant electric field is
$$\vec{E} = \vec{E_1} + \vec{E_2}$$

or
$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r_1^2} \vec{PB} + \frac{\lambda}{2\pi\varepsilon_0 r_2^2} \vec{AP}$$

According to addition law of vector
$$\vec{AO} + \vec{OP} = \vec{AP}$$
$$\frac{\vec{l}}{2} + \vec{r} = \vec{AP}$$

Similarly, $\because$
$$\vec{OP} + \vec{PB} = \vec{OB}$$

or
$$\vec{r} + \vec{PB} = \frac{\vec{l}}{2}$$

$$\therefore \quad \vec{PB} = \frac{\vec{l}}{2} - \vec{r} \qquad \therefore \quad \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r_1^2} \vec{PB} + \frac{\lambda}{2\pi\varepsilon_0 r_2^2} \vec{AP}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 \left(r + \frac{l}{2}\cos\theta\right)^2} \vec{PB} + \frac{\lambda}{2\pi\varepsilon_0 \left(r - \frac{l}{2}\cos\theta\right)^2} \vec{AP}$$

$$\therefore \quad \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 \left(r + \frac{l}{2}\cos\theta\right)^2} \left(\frac{\vec{l}}{2} - \vec{r}\right) + \frac{\lambda}{2\pi\varepsilon_0 \left(r - \frac{l}{2}\cos\theta\right)^2} \left(\frac{\vec{l}}{2} + \vec{r}\right)$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \frac{\left(r - \frac{l}{2}\cos\theta\right)^2 \left(\frac{\vec{l}}{2} - \vec{r}\right) + \left(r + \frac{l}{2}\cos\theta\right)\left(\vec{r} + \frac{\vec{l}}{2}\right)}{r^2 - \frac{l^2}{4}\cos^2\theta}$$

But, $r >> l \quad \therefore \quad \vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0} \dfrac{\vec{l}}{r^2} \quad \therefore \quad E = |\vec{E}| = \dfrac{\lambda l}{2\pi\varepsilon_0 r^2}$

Here, $E = f(r)$

Electric potential due to first long wire is

$$\int_0^{\phi_1} d\phi = -\int_{r_0}^{r} E dr$$

$$\int_0^{\phi} d\phi = -\int_{r_0}^{r_1} \frac{\lambda}{2\pi\varepsilon_0 r} dr$$

$$\phi_1 = \frac{-\lambda}{2\pi\varepsilon_0} \ln \frac{r_1}{r_0}$$

Similarly, $$\phi_2 = \frac{-(-\lambda)}{2\pi\varepsilon_0} ln \frac{r_2}{r_0}$$

or $$\phi_2 = \frac{\lambda}{2\pi\varepsilon_0} ln \frac{r_2}{r_0} \quad \therefore \quad \phi = \phi_1 + \phi_2$$

$$= \frac{\lambda}{2\pi\varepsilon_0} [ln\, r_2 - ln\, r_0 - ln\, r_1 + ln\, r_0]$$

$$= \frac{\lambda}{2\pi\varepsilon_0} [ln\, r_2 - ln\, r_1]$$

But, $$r_2 = r_1 + r_2 - r_1 = r_1 + \Delta r$$

$$\phi = \frac{\lambda}{2\pi\varepsilon_0}\left[ln \frac{r_2}{r_1}\right] = \frac{\lambda}{2\pi\varepsilon_0}\left[ln \frac{r_1 + \Delta r}{r_1}\right]$$

$$\phi = \frac{\lambda}{2\pi\varepsilon_0}\left[ln\left(1 + \frac{\Delta r}{r_1}\right)\right]$$

$$\phi = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{\Delta r}{r_1} - \frac{1}{2}\left(\frac{\Delta r}{r_1}\right)^2 + \ldots\right]$$

But Δr is very small.

$$\therefore \quad \phi = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{\Delta r}{r_1}\right]$$

But, $$\Delta r = l \cos\theta$$

and $r_1 \approx r \quad \therefore \quad \phi = \frac{\lambda l \cos\theta}{2\pi\varepsilon_0 r}$

YOUR STEP

1. Two like electric charges of 7×10^{-8} C each are at points A and B (see Fig. 3.42C). Determine the electric field strength at point O which is the apex of the right angle AOB, $AO = BO = 5$ cm.

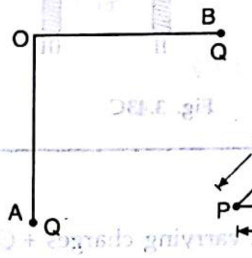


Fig. 3.42C

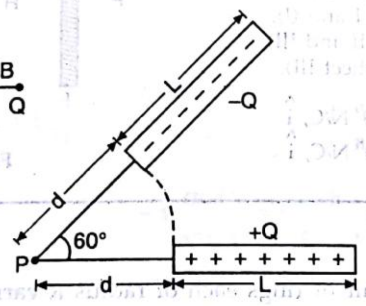


Fig. 3.42D

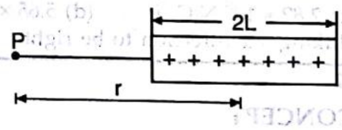


Fig. 3.42E

2. (a) A positive charge Q is distributed uniformly over a thin rod of length $2L$. Calculate the electric field at a point P along the axis of the rod, at a distance r from its centre shown in Fig. 3.42D.
   (b) If two rods each having charge $+Q$ and $-Q$ respectively distributed uniformly over their length's are arranged as shown in Fig. 3.42(E), what is the magnitude of intensity at P?

$$\left\{1.\ \text{About } 3.6 \times 10^5 \text{ V/m} \quad 2.\ (a)\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{(r^2 - L^2)},\ (b)\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{d\,(d + L)}\right\}$$

§ 3.43

➢ **CONCEPT**

The electric potential on the axis of a circular ring is

$$E = \frac{q}{4\pi\varepsilon_0 (R^2 + x^2)^{1/2}}$$

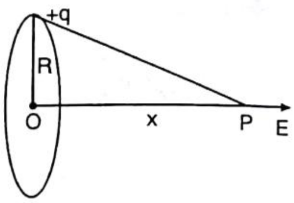


Fig. 3.43A

SOLUTION : The electric potential at point P is

$$\phi = \phi_1 + \phi_2$$

$$= \frac{q}{4\pi\varepsilon_0 \left\{R^2 + \left(\frac{l}{2} + x\right)^2\right\}^{\frac{1}{2}}} - \frac{q}{4\pi\varepsilon_0 \left\{R^2 + \left(\frac{l}{2} - x\right)^2\right\}^{\frac{1}{2}}}$$

On solving, we get $\phi = \frac{ql}{4\pi\varepsilon_0} \frac{x}{(R^2 + x^2)^{3/2}}$

As we know, $E = -\frac{d\phi}{dx} = \frac{-ql}{4\pi\varepsilon_0}\left[\frac{R^2 - 2x^2}{(R^2 + x^2)^{5/2}}\right]$

If $x >> R$ then $\phi = \frac{ql}{4\pi\varepsilon_0 x^2}$ and $E = \frac{ql}{2\pi\varepsilon_0 x^3}$

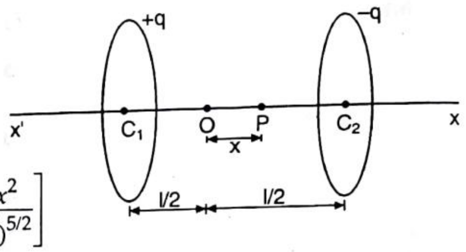


Fig. 3.43B

YOUR STEP

Three large parallel insulating sheets have surface charge densities $+0.0200\ C/m^2$, $+0.0100\ C/m^2$ and $-0.0200\ C/m^2$ (Fig. 3.43C). Adjacent sheets are separated at a distance of 0.300 m from each other. Calculate the net electric field (magnitude and direction) due to all three sheets at
(a) point P (0.150 m to the left of sheet I);
(b) point R (midway between sheets I and II);
(c) point S (midway between sheets II and III);
(d) Point T (0.150 m to the right of sheet III).

(a) 5.65×10^8 N/C, $-\hat{i}$ (b) 1.69×10^9 N/C, $\hat{i}$
(c) 2.82×10^9 N/C, $\hat{i}$ (d) 5.65×10^8 N/C, $\hat{i}$
(Taking $+x$ direction to be right).

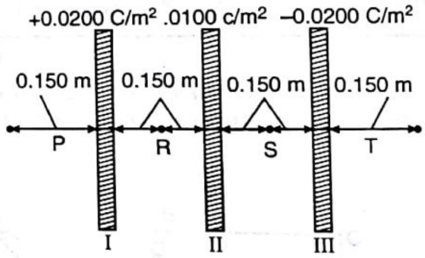


Fig. 3.43C

§ 3.44

➢ **CONCEPT**

The electric potential due to a pair of rings each of radius R carrying charges $+Q$ and $-Q$ at a distance x from the midpoint along the axis is given by

$$\phi = \frac{Qlx}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}}$$

SOLUTION : Let us consider a pair of rings of radius y and thickness dy.

Then $Q =$ electric charge on each ring $= \sigma\, 2\pi y dy$

$$\therefore \quad d\phi = \frac{(2\pi\sigma y dy)\, lx}{4\pi\varepsilon_0 (y^2 + x^2)^{3/2}} = \frac{\sigma l x y dy}{2\varepsilon_0 (y^2 + x^2)^{3/2}} \qquad \therefore \quad \phi = \frac{\sigma l x}{2\varepsilon_0}\int_R^\infty \frac{y dy}{(y^2 + x^2)^{3/2}} = \frac{\sigma l x}{2\varepsilon_0 (R^2 + x^2)^{1/2}}$$

$$\therefore \quad E = \frac{-\partial\phi}{\partial x} = -\frac{\sigma l R^2}{2\varepsilon_0 (R^2 + x^2)^{3/2}}$$

YOUR STEP

A positively charged oil droplet remains stationary in the electric field between two horizontal plates separated by a distance of 1 cm. If the charge on the drop is 9.6×10^{-10} esu and the mass of droplet is 10^{-11} g, what is the potential difference between the plates? Now if the polarity of the plates is reversed what is the instantaneous acceleration of the droplet? [$g = 9.8$ m/s^2].

{19.6 m/sec^2}

§ 3.45

SOLUTION :

Let us consider ring elements radius of r and thickness dr on the round positive plate. The electric potential due to considered element on both rings is

$$d\phi = \frac{lxdq}{4\pi\varepsilon_0 (r^2 + x^2)^{3/2}} \quad \text{(from solu. of previous problem)}$$

The electric charge on considered element is $dq = (2\pi r dr)\,\sigma$

$$\therefore \quad \phi = \int d\phi = \int_0^R \frac{lx\, 2\pi\sigma r dr}{4\pi\varepsilon_0 (r^2 + x^2)^{3/2}} \quad \text{or} \quad \phi = \frac{\sigma\, lx}{4\varepsilon_0} \int_0^R \frac{2rdr}{(r^2 + x^2)^{3/2}}$$

$$= \frac{\sigma l x}{4\varepsilon_0} \left[\frac{-2}{(r^2 + x^2)^{3/2}} \right]_0^R$$

$$\therefore \quad \phi = \frac{\sigma l}{2\varepsilon_0} \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right]$$

This value of potential when upper plate is taken as positively charged. If electric potential is calculated on the side of negative plate, then

$$\phi = \frac{-\sigma l}{2\,\varepsilon_0} \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right] \quad \therefore \quad \phi = \pm \frac{\sigma l}{2\varepsilon_0} \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right] \quad \text{(In general)}$$

If $x >> R$, then

$$\phi = \pm \frac{\sigma l}{2\varepsilon_0} \left[1 - \left(1 + \frac{R^2}{x^2} \right)^{-1/2} \right] = \pm \frac{\sigma l}{2\varepsilon_0} \left[1 - \left(1 - \frac{R^2}{2x^2} \right) \right] \quad \text{(By binomial theorem)}$$

$$\approx \pm \frac{\sigma \pi R^2 l}{4\pi\varepsilon_0 x^2} \approx \pm \frac{P}{4\pi\varepsilon_0 x^2}$$

Here $\quad P = \sigma \pi R^2 l$

Remarks : In the answer of book, read σ as π.

$$E = -\frac{\partial \phi}{dx}$$

$$= \frac{\sigma l R^2}{2\varepsilon_0 (R^2 + x^2)^{3/2}}$$

for $x >> R$, $\quad E = \dfrac{\sigma l R^2}{2\varepsilon_0 x^3} = \dfrac{\pi R^2 \sigma l}{2\pi\,\varepsilon_0\, x^3}$

$$\therefore \quad E = \frac{P}{2\pi\varepsilon_0 x^3} \quad \text{where} \quad P = \pi R^2 \sigma l$$

YOUR STEP

In the region of space between the plates of a parallel plate capacitor there is a uniformly distributed positive charge with a volume density ρ. The plates are connected electrically and their potential is set at zero. Calculate and draw a sketch of the distribution of the potential and electric field strength between the plates.

$$\left\{ E_0 = \frac{-\rho l}{2\varepsilon_0};\ \phi = \frac{\rho}{2\varepsilon_0} x\,(l-x) \right\}$$

Function represents a parabola with a maximum at $x = \frac{l}{2}$

§ 3.46

➢ **CONCEPT**

Electric field due to a long thread is :

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}\ \text{along increasing } r$$

SOLUTION : (a) The arrangement is shown in Fig. 3.46B.

From Fig. 3.46B, the net force on the dipole is $F = qE - qE = 0$

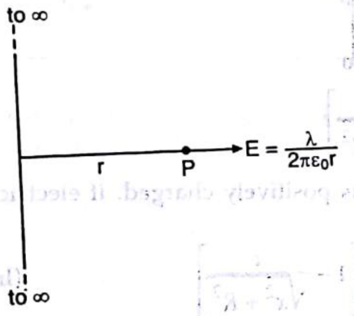


Fig. 3.46A

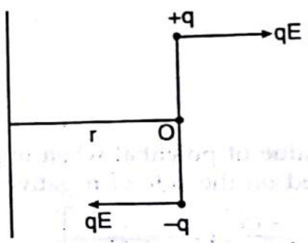


Fig. 3.46B

(b) The electric field at the site of charge $(-q)$ is $E_1 = \frac{\lambda}{2\pi\varepsilon_0 (r-a)}$.

The electric field at the site of charge $(+q)$ is

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0 (r+a)}$$

From Fig. 3.46C, the net force on the dipole is

$$F = qE_1 - qE_2 = q\,(E_1 - E_2)$$

$$F = \frac{q\lambda}{2\pi\varepsilon_0}\left[\frac{1}{r-a} - \frac{1}{r+a}\right]$$

$$= \frac{q\lambda}{2\pi\varepsilon_0}\frac{(2a)}{(r^2-a^2)} = \frac{(q \times 2a)\,\lambda}{2\pi\varepsilon_0 (r^2-a^2)}$$

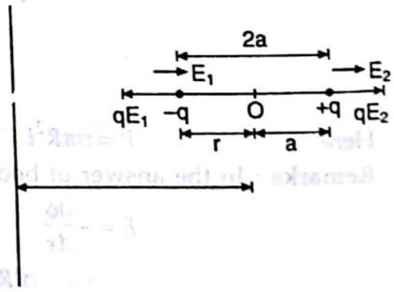


Fig. 3.46C

But, $r >> a$, so, a^2 can be neglected with respect to r^2.

$$\therefore \qquad F = \frac{P\lambda}{2\pi\varepsilon_0 r^2}$$

Since, resultant force on the electric dipole is in the opposite direction of dipole moment.

$$\therefore \qquad \vec{F} = \frac{-\lambda\,\vec{P}}{2\pi\varepsilon_0 r^2} \qquad \text{(In vector form)}$$

(c) In the Fig. 3.46D, the long thread is placed perpendicular to the plane of paper.

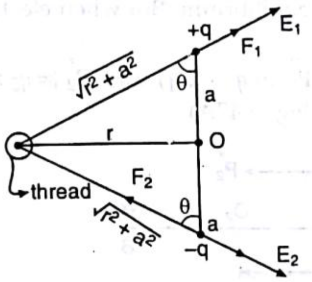


Fig. 3.46D

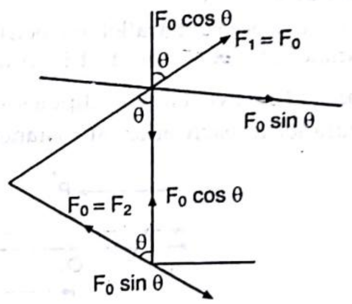


Fig. 3.46E

Here, $E_1 = \dfrac{\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}$ and $E_2 = \dfrac{\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}$

Also, $F_1 = qE_1 = \dfrac{q\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}$ and $F_2 = qE_2 = \dfrac{q\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}$

$\therefore \qquad F_1 = F_2 = F_0 = \dfrac{q\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}$ ...(i)

From force diagram shown in Fig. 3.46E, $F = 2F_0\cos\theta$

Putting the values of F_0 and $\cos\theta$, in Eq. (i) we get

$$F = \frac{2q\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}\cdot\frac{a}{\sqrt{r^2+a^2}}$$

$$= \frac{\lambda(q\times 2a)}{2\pi\varepsilon_0(r^2+a^2)} = \frac{P\lambda}{2\pi\varepsilon_0(r^2+a^2)}$$

But $r >> a$, a^2 may be neglected with respect to r^2

$\therefore F = \dfrac{P\lambda}{2\pi\varepsilon_0 r^2}$ (along the direction of dipole moment)

$$\therefore \qquad \vec{F} = \frac{\lambda}{2\pi\varepsilon_0 r^2}\vec{P}$$

YOUR STEP

An electric dipole is placed at a distance x from centre O on the axis of a charged ring of radius R and charge Q is uniformly distributed over it.

(a) Find the net force acting on the dipole.

(b) What is the work done in rotating the dipole through 180°.

(c) If the dipole is slightly rotated about its equilibrium position.
Find the time period of oscillation. Assume that the dipole is linearly restrained.

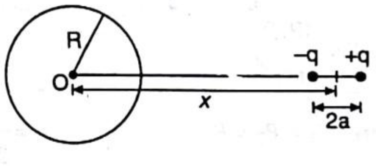


Fig. 3.46F

$$\left\{(a)\ \frac{aqQ}{2\pi\varepsilon_0}\frac{(R^2-2x^2)}{(R^2+x^2)^{5/2}} \quad (b)\ \frac{aqQx}{\pi\varepsilon_0(R^2+x^2)^{3/2}} \quad (c)\ \sqrt{\frac{4\pi^2 m\varepsilon_0 a}{Qx}(R^2+x^2)^{3/2}}\right\}$$

§ 3.47

➢ **CONCEPT**

If electric dipole is parallel to electric field, it is in stable equilibrium. But when electric dipole is antiparallel to electric field, it is in unstable equilibrium.

SOLUTION : Let two electric dipoles of dipole moments $\vec{P_1}$ $(= q_1 \times 2a_1)$ and $\vec{P_2}$ $(= q_2 \times 2a_2)$ are placed parallel to each other at distance l apart (shown in Fig. 3.47A).

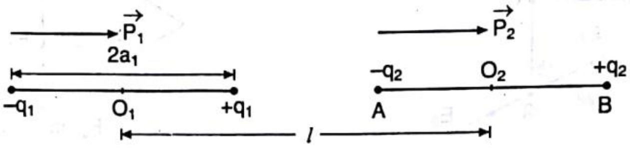


Fig. 3.47A

We have to calculate force of electrostatic interaction between dipoles.

Electric field at point A due to first electric dipole is, $E_1 = \dfrac{P_1}{2\pi\varepsilon_0 (l - a_2)^3}$.

The electric field at point B due to first electric dipole is, $E_2 = \dfrac{P_1}{2\pi\varepsilon_0 (l + a_2)^3}$.

Since, E_1 is greater than E_2. So, electric dipoles attract to each other. The net electric force on the second dipole is.

$$F = q_2 E_1 - q_2 E_2 \qquad \text{(shown in Fig. 3.47B).}$$

$$= q_2 [E_1 - E_2]$$

$$= q_2 \left[\frac{P_1}{2\pi\varepsilon_0 (l - a_2)^3} - \frac{P_1}{2\pi\varepsilon_0 (l + a_2)^3} \right]$$

$$= \frac{q_2 P_1}{2\pi\varepsilon_0} \left[\frac{(l + a_2)^3 - (l - a_2)^3}{(l^2 - a_2)^3} \right]$$

$$= \frac{q_2 P_1}{2\pi\varepsilon_0} \left[\frac{l^3 + a_2^3 + 3a_2^2 l + 3l^2 a_2 - l^3 + a_2^3 - 3a_2^2 l + 3l^2 a_2}{(l^2 - a_2^2)^3} \right]$$

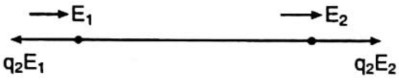


Fig. 3.47B

But $l >>> a_2$, So, a_2^3, and a_2^2 may be neglected.

$$\therefore \quad F = \frac{P_1 q_2}{2\pi\varepsilon_0} \left[\frac{6\, l^2 a_2}{2\pi\varepsilon_0 l^6} \right] \qquad \therefore \quad F = \frac{3P_1 (q_2 \times 2a_2)\, l^2}{2\pi\varepsilon_0 l^6}$$

$$\therefore \quad F = \frac{3P_1 P_2}{2\pi\varepsilon_0 l^4}$$ (along the line joining both electric dipoles)

Here, $P_1 = P_2 = P$ $\quad \therefore \quad F = \dfrac{3P^2}{2\pi\varepsilon_0 l^4} = 2.1 \times 10^{-16}$ N (on putting the values)

Remarks : Electric dipoles interaction as formula :

(a)

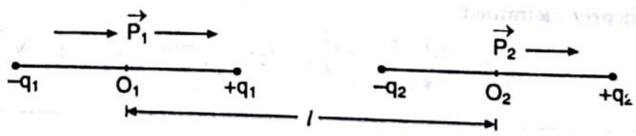


Fig. 3.47C

(i) $F = \dfrac{3P_1P_2}{2\pi\varepsilon_0 l^4}$ is attractive in nature. (ii) electric torque, $\tau = 0$

(iii) Potential energy of interaction, $U = \dfrac{-P_1P_2}{2\pi\varepsilon_0 l^3}$

(b)

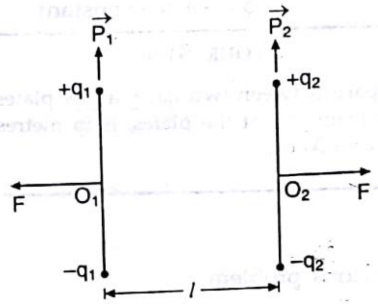


Fig. 3.47D

(i) $F = \dfrac{3P_1P_2}{4\pi\varepsilon_0 l^4}$ is repulsive in nature, (ii) $\vec{\tau} = \vec{0}$, (iii) $U = \dfrac{P_1P_2}{2\pi\varepsilon_0 l^3}$

(c)

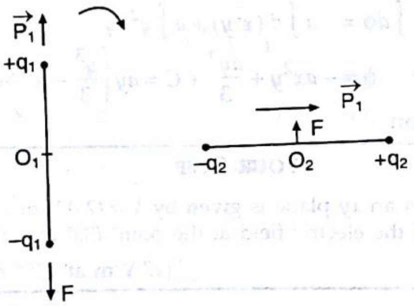


Fig. 3.47E

(i) $F = \dfrac{3P_1P_2}{4\pi\varepsilon_0 l^4}$ (ii) $\tau_1 = \dfrac{P_1P_2}{2\pi\varepsilon_0 l^3}$ clockwise and $\tau_2 = \dfrac{P_1P_2}{2\pi\varepsilon_0 l^3}$ clockwise, (iii) $U = 0$

YOUR STEP

The ammonia molecule NH_3 has a permanent electric dipole moment equal to 1.47D, where, $1D = 1$ debye unit $= 3.34 \times 10^{-30}$ C.m. Calculate the electric potential due to an ammonia molecule at a point 52.0 nm away along the axis of the dipole. (set $V = 0$ at infinity).

[1.63×10^{-5} volt]

§ 3.48

➢ **CONCEPT**

As we known, $d\phi = -\vec{E} \cdot d\vec{r}$

Here, $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$\therefore$ $$d\phi = -(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

or $$\int d\phi = -\int E_x\,dx - \int E_y\,dy - \int E_z\,d_z$$

SOLUTION :

$\because$ $d\phi = -\vec{E}.\vec{dr}$

or $d\phi = -a\,(y\hat{i} + x\hat{j}).(dx\hat{i} + dy\hat{j} + dz\hat{k}) = -aydx - axdy$

or $\int d\phi = -a\int d(xy)$

$\therefore$ $\phi = -axy + \text{constant}$

> **YOUR STEP**
>
> The electric potential V in the space between two flat parallel plates is given by $V = 1500x^2$, where V is in volts if x is the distance from one of the plates, is in metres. Calculate the magnitude and direction of the electric field at $x = 1.3$ cm.
>
> {39 V/m towards first plate}

§ 3.49

➢ **CONCEPT**

The concept is similar to previous problem.

SOLUTION : $\int d\phi = -\int E_x dx - \int E_y\, dy$

$$\int d\phi = -\int 2axdx - a\int (x^2 - y^2)\,dy$$

$$\int d\phi = -\int 2axdx - a\int x^2dy + a\int y^2dy$$

$$\int d\phi = -a\int d\,(x^2y) + a\int y^2dy$$

$\therefore$ $$\phi = -ax^2y + \frac{ay^3}{3} + C = ay\left(\frac{y^3}{3} - x^2\right) + \text{constant}$$

where C is constant of integration.

> **YOUR STEP**
>
> The electric potential at point in an xy plane is given by $V = (2.0\ \text{V/m}^2)\,x^2 - (3.0\ \text{V/m}^2)\,y^2$. What are the magnitude and direction of the electric field at the point (3.0 m, 2.0 m) ?
>
> {17 V/m at 135° counter clockwise from $+x$.}

§ 3.50

➢ **CONCEPT**

The concept is similar to previous problem.

SOLUTION : Here, $E_x = ay$, $E_y = ax + bz$ and $E_z = by$

$\therefore$ $$\int d\phi = -\int E_x\,dx - \int E_y\,dy - \int E_z\,dz$$

or $$\int d\phi = -\int aydx - \int (ax + bz)\,dy - \int bydz$$

or $$\int d\phi = -\int aydx - \int axdy - \int bzdy - \int bydz$$

$$= -\int a\,(ydx + xdy) - b\int (zdy + ydz) = -a\int d\,(xy) - b\int d\,(yz)$$

or $$\phi = -axy - byz + C = -y\,(ax + bz) + \text{constant}$$

where C is constant of integration.

> **YOUR STEP**
>
> The electric potential at any point on the central axis of a uniformly charged disc is given by $V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{Z^2 + R^2} - Z\right]$ starting with this expression, derive an expression for the electric field at any point on the axis of the disc.
>
> $$\left\{E_z = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{Z}{\sqrt{Z^2 + R^2}}\right)\right\}$$

§ 3.51

➢ CONCEPT

A vector field is a vector valued function of three variables. The divergence of a vector field is a scalar field that tells us at each point, the extent to which the electric field explodes or diverges from that point. Divergence of electric field is the net outward flux per unit volume.

It is given by $\vec{\nabla} \cdot \vec{E}$

Here, $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$ and $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

$$\therefore \quad \vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

According to Gauss's law, $\int_c \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$

$$\frac{\int_c \vec{E} \cdot d\vec{S}}{\text{volume}} = \frac{q}{\varepsilon_0 \times \text{volume}} = \frac{\rho}{\varepsilon_0}$$

$$\therefore \quad \frac{\phi_E}{\text{volume}} = \frac{\rho}{\varepsilon_0}$$

$\therefore$ Flux per unit volume $= \frac{\rho}{\varepsilon_0}$

But electric flux per unit volume $= \text{Div } \vec{E} = \vec{\nabla} \cdot \vec{E}$

$$= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \qquad \therefore \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_0}$$

Also, $E_x = \frac{-\partial\phi}{\partial x}$ similarly $E_y = \frac{-\partial\phi}{dy}$ and $E_z = \frac{-\partial\phi}{\partial z}$

SOLUTION : According to problem, $\phi = -ax^3 + b$

$$\therefore \quad E_x = \frac{-\partial\phi}{\partial x} = 3ax^2 \qquad E_y = 0 \quad E_z = 0$$

But $$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_0}$$

or $\frac{\partial (3ax^2)}{\partial x} + 0 + 0 = \frac{\rho}{\varepsilon_0}$ or $6ax = \frac{\rho}{\varepsilon_0}$ $\therefore$ $\rho = 6a\varepsilon_0 x$

YOUR STEP

For the electric field, $\vec{E} = (kx^{1/2})\,\hat{x}$ where k is a constant, compute the electric flux through the face of the cubic region depicted in Fig. 3.51(A). What is the charge confined to the cube?

$$\left\{Q = \frac{\phi}{4\pi} = \frac{kl^{5/2}(\sqrt{2}-1)}{4\pi}\right\}$$

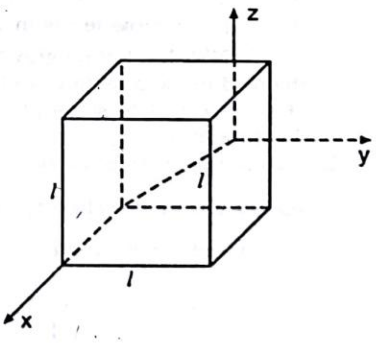


Fig. 3.51

§ 3.52

➢ **CONCEPT**

In this problem, the following formula is applicable.

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{dy}+\frac{dE_z}{\partial z}=\frac{\rho}{\varepsilon_0}$$

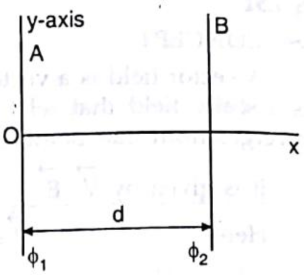


Fig. 3.52A

SOLUTION : Let plates A and B are separated by a distance d apart. The electric potentials of A and B are ϕ_1 and ϕ_2 ($\phi_2<\phi_1$) respectively. Let electric field near plate B is E_0. Here, variation of electric field takes place along x-axis only.

$\therefore$ $$\frac{\partial E_y}{\partial y}=0,\ \frac{\partial E_z}{\partial z}=0$$

Also, $\phi_1-\phi_2=\Delta\phi$ (According to problem).

$\because$ $$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=\frac{\rho}{\varepsilon_0}$$

But electric field is one dimensional (along x-axis).

$\therefore$ $$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=\frac{\partial E_x}{\partial x}=\frac{dE}{dx}$$

$\therefore$ $\dfrac{dE}{dx}=\dfrac{\rho}{\varepsilon_0}$ or $\displaystyle\int_0^E dE=\frac{\rho}{\varepsilon_0}\int_0^x dx$ $\therefore$ $E=\dfrac{\rho x}{\varepsilon_0}$ But, $\int d\phi=-\int E\,dx$

or $\displaystyle\int_{\phi_1}^{\phi_2} d\phi=-\int_0^d \frac{\rho x}{\varepsilon_0}dx$ or $\phi_1-\phi_2=\dfrac{\rho d^2}{2\varepsilon_0}$ or $\Delta\phi=\dfrac{\rho d^2}{2\varepsilon_0}$

$\therefore$ $\rho=\dfrac{2\varepsilon_0\Delta\phi}{d^2}$ $\therefore$ $E=\dfrac{\rho d}{\varepsilon_0}$

According to problem, at the second plate, $x=d$, $E=E_0$

$\therefore$ $$E_0=\frac{\rho d}{\varepsilon_0}=\left(\frac{2\varepsilon_0\,\Delta\phi}{d^2}\right)\frac{d}{\varepsilon_0}$$ (On putting the value of ρ)

$\therefore$ $$E_0=\left(\frac{2\Delta\phi}{d}\right)$$

YOUR STEP

1. Suppose in an insulating medium, having dielectric constant $k=1$, volume density of positive charge varies with y-coordinate according to law $\rho=ay$. A particle of mass m having positive charge q is placed in the medium at point A $(0, y_0)$ and projected with velocity $\vec{v}=v_0\cdot\hat{i}$ as shown in Fig. 3.52(B). Neglecting gravity and frictional resistance of the medium and assuming electric field strength to be zero at $y=0$, calculate slope of trajectory of the particle as a function of y.
2. Two infinite planes are placed at $x=0$ and $x=a$ and carry surface charge densities σ_0 and $\dfrac{-\sigma_0}{2}$ respectively. Calculate electric field everywhere.

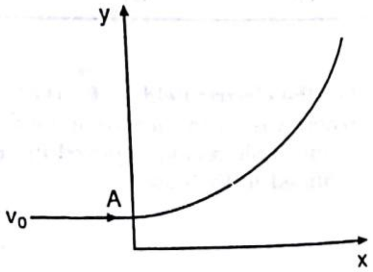


Fig. 3.52B

$\left\{1.\ \sqrt{\dfrac{qa}{3m\varepsilon_0 v_0^2}(y^3-y_0^3)}\quad 2.\ \vec{E}=\pi\sigma_0\hat{x},\ x>a;\ 3\pi\sigma_0\hat{x},\ a>x>0;\ -\pi\sigma_0\hat{x};\ 0>x\right\}$

§ 3.53

➢ **CONCEPT**

According Poisson's equation

$\because$ $$\nabla^2\phi = \frac{-\rho}{\varepsilon_0}$$

or $$\frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} = \frac{-\rho}{\varepsilon_0}$$

SOLUTION : $\because$ $$\phi = ar^2 + b$$

$\therefore$ $$\frac{\partial\phi}{\partial r} = 2ar$$

$\therefore$ $$\frac{\partial^2\phi}{\partial r^2} = 2a$$

$\therefore$ $$\frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} = \frac{-\rho}{\varepsilon_0}$$

or $$2a + \frac{2}{r}(2ar) = \frac{-\rho}{\varepsilon_0}$$

or $$2a + 4a = \frac{-\rho}{\varepsilon_0}$$

$\therefore$ $$\rho = -6\,a\,\varepsilon_0$$

YOUR STEP

1. Find the volume density of charge so that would be produced the Yukawa potential $\phi = \frac{q}{4\pi\varepsilon_0}\frac{e^{-r/a}}{r}$. Why the total charge be zero.
2. The region between the planes $y = 0$ and $y = a$ is filled with uniform space charge density ρ [see Fig. 3.53(A)].

   (a) Compute the electric field everywhere.

   (b) Compute the electric potential everywhere.

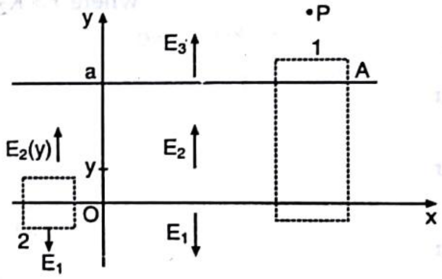


Fig. 3.53A

1. $\rho = \frac{-q}{4\pi a^2}\frac{e^{-r/a}}{r}$,
2. (a) $\vec{E}(y) = c_1\hat{y};\ y < 0\ (4\pi\rho y + c_2)\hat{y};\ 0 \le y \le a;\ c_3\hat{y};$ $y > a$ where, $c_3 = -c_1 = 2\pi\rho a = -c_2$

   (b) $\phi(y) = 2\pi\rho ay;\ y < 0;\ 2\pi\rho\,(ay - y^2);\ 0 \le y \le a;\ 2\pi\rho a\,(a - y);\ y > a$

ELECTRIC CAPACITANCE ENERGY OF AN ELECTRIC FIELD

§ 3.101

➢ CONCEPT

The capacity of a conductor is given by $C = \frac{q}{\phi}$

where q is charge on conductor and ϕ is potential on the surface of conductor.

SOLUTION : Let charge q is supplied to the ball.

The electric field at point A is $E = \frac{q}{4\pi\varepsilon_0 \varepsilon r^2}$ radially outward

Here $R_1 < r < R_2$

Electric field at point B is $E = \frac{q}{4\pi\varepsilon_0 r^2}$

where $r > R_2$

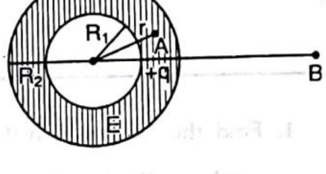


Fig. 3.101A

$\because$ $$-d\phi = Edr$$

or $$-\int_{\phi}^{O} d\phi = \int_{r_1}^{\infty} Edr$$

or $$-\int_{\phi}^{O} d\phi = \int_{R_1}^{R_2} \frac{q}{4\pi\varepsilon_0 \varepsilon r^2} dr + \int_{R_2}^{\infty} \frac{q}{4\pi\varepsilon_0 r^2} dr$$

or $$\phi = \frac{q}{4\pi\varepsilon_0 \varepsilon}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{q}{4\pi\varepsilon_0 R_2}$$

or $$\phi = \frac{q}{4\pi\varepsilon_0 \varepsilon}\left[\left(\frac{\varepsilon - 1}{R_2}\right) + \frac{1}{R_1}\right]$$

$\therefore$ $$C = \frac{q}{\phi} = \frac{4\pi\varepsilon_0 \varepsilon R_1}{(\varepsilon - 1)\frac{R_1}{R_2} + 1}$$

YOUR STEP

Two concentric metallic spheres of radii r_1 and r_2 ($r_1 < r_2$) carry electric charges $+q$ and $-q$ respectively (Shown in fig 3.101B). The space between the spheres is filled with two insulating layers of relative permittivities ε_1 and ε_2 and widths d_1 and d_2 respectively. Calculate the capacity of the system.

$$4\pi\varepsilon_0\left[\frac{1}{\varepsilon_2}\left(\frac{1}{r_1 + d_1} - \frac{1}{r_2}\right) + \frac{1}{\varepsilon_1}\left(\frac{1}{r_1} - \frac{1}{r_1 + d_1}\right)\right]^{-1}$$

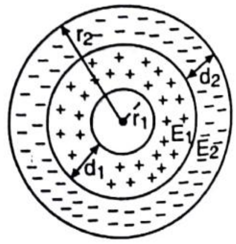


Fig. 3.101B

§ 3.102

➢ **CONCEPT**

The electric field at a point between plates of a parallel plate capacitor is

$$E = \frac{q}{A\varepsilon_0 K}$$

For air or vacuum $K = 1$

SOLUTION : Step I : Draw the circuit before introduction of dielectric material.

In the closed loop $ABCDA$,

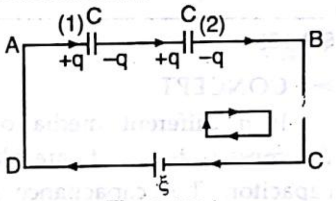


Fig. 3.102A

$$\xi - \frac{q}{C} - \frac{q}{C} = 0$$

$$\xi - \frac{2q}{c} = 0$$

$$\therefore \quad q = \frac{\xi C}{2}$$

The electric field at a point between plates of first capacitor is

$$E_0 = \frac{q}{A\varepsilon_0} = \frac{\xi C}{2A\varepsilon_0}$$

Step II : Draw the circuit after introduction of dielectric medium. When first capacitor is filled with dielectric medium. Then capacitance of first capacitor becomes $C_1 = KC$. In closed loop (2)

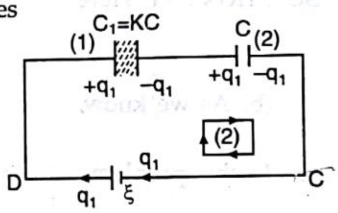


Fig. 3.102B

$$\xi - \frac{q_1}{C_1} - \frac{q_1}{C} = 0 \quad \text{or} \quad \xi - \frac{q_1}{KC} - \frac{q_1}{C} = 0$$

$$\text{or} \quad \xi = \frac{q_1}{C}\left(\frac{1}{K} + 1\right) \quad \text{or} \quad \xi = \frac{q_1}{C}\left(\frac{K+1}{K}\right)$$

$$\therefore \quad q_1 = \left(\frac{KC\xi}{K+1}\right)$$

Now electric field at a point between plates of first capacitor is

$$E = \frac{q}{A\varepsilon_0 K} = \frac{KC\xi}{(K+1)\,A\varepsilon_0 K} \quad \therefore \quad E = \frac{C\xi}{\varepsilon_0 A(K+1)}$$

$$\therefore \quad \frac{E}{E_0} = \frac{\dfrac{C\xi}{(k+1)\,\varepsilon_0 A}}{\dfrac{\xi C}{2A\varepsilon_0}}$$

According to question, $K = \varepsilon$

$$\therefore \quad \frac{E}{E_0} = \frac{2}{K+1} = \frac{2}{\varepsilon + 1}$$

Hence, $$E = \frac{2}{E+1} E_0$$

Hence, the strength of field decreases $\frac{1}{2}(\varepsilon + 1)$ times. Also, electric charge flow through battery is $\Delta q = q_1 - q$.

Putting the values of q_1 and q we get,

$$\Delta q = \frac{1}{2} C\,\xi\,(\varepsilon - 1)/(\varepsilon + 1)$$

YOUR STEP

A parallel plate capacitor with plate area A is arranged horizontally. The lower plate is fixed and upper plate of mass m is hanging with a spring of constant k. A battery of emf V_0 is connected across the plates. In equilibrium condition, separation between plates is d. Find the time period of small oscillation of upper plate. $\left\{2\pi\sqrt{\dfrac{m}{\left(k-\dfrac{\varepsilon_0 A V_0^2}{d^3}\right)}}\right\}$

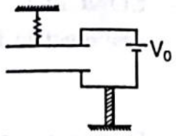


Fig. 3.102C

§ 3.103

> CONCEPT

If n different media of dielectric constant $\varepsilon_1, \varepsilon_2, \ldots \varepsilon_n$ of thickness $t_1, t_2 \ldots t_n$ are placed between plates of a parallel plates capacitor. Then capacitance of the system is given by

$$C=\frac{\varepsilon_0 A}{\dfrac{t_1}{\varepsilon_1}+\dfrac{t_2}{\varepsilon_2}+\ldots+\dfrac{t_n}{\varepsilon_n}}=\frac{\varepsilon_0 A}{\sum\limits_{i=1}^{n}\dfrac{t_i}{\varepsilon_i}}$$

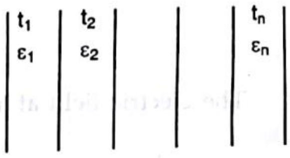


Fig. 3.103A

SOLUTION : (a) Here $$C=\frac{\varepsilon_0 S}{\dfrac{d_1}{\varepsilon_1}+\dfrac{d_2}{\varepsilon_2}}$$

(b) As we know, $$\sigma'=\sigma\left(1-\frac{1}{\varepsilon}\right)$$

$\therefore$ In this problem $$\sigma'=\sigma\left(1-\frac{1}{\varepsilon_1}\right)-\sigma\left(1-\frac{1}{\varepsilon_2}\right)=\sigma\left(\frac{1}{\varepsilon_2}-\frac{1}{\varepsilon_1}\right)$$

But $$q=CV$$

$\therefore$ $$\sigma=\frac{q}{S}=\frac{CV}{S}$$

Putting the values of C, we get $$\sigma=\frac{\varepsilon_0\,\varepsilon_1\,\varepsilon_2\,V}{\varepsilon_2\,d_1+\varepsilon_1\,d_2}$$

$\therefore$ $$\sigma'=\frac{\varepsilon_0\,V\,(\varepsilon_1-\varepsilon_2)}{\varepsilon_2\,d_1+\varepsilon_1\,d_2}$$

YOUR STEP

1. A parallel plate capacitor is made of square plates of side length a. A parabolic cylindrical slab of dielectric constant K is placed between the plates. The equation of parabola is $x^2=ay$ and the vertex of parabola touches the other plate at middle. Find the capacitance.

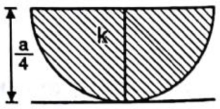


Fig. 3.103B

2. Figure shows two conductors carrying equal and opposite charges. The electric field between conductors due to charges on the conductors varies as Kqx^2, where K is a constant and x is distance from positive plate (from point O). Find the capacitance of the system.

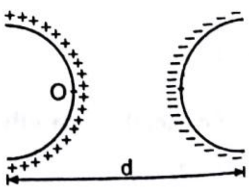


Fig. 3.103C

$\left\{1.\ 4\varepsilon_0 aK\left(\dfrac{\tan^{-1}\sqrt{K-1}}{\sqrt{K-1}}\right)\ 2.\ \dfrac{3}{Kd^3}\right\}$

§ 3.104

➢ CONCEPT

The problem is based upon series combination in continuous medium.

In this case, $\frac{1}{C_{eq}} = \int \frac{1}{dC}$

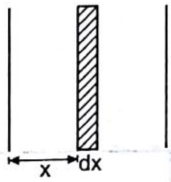


Fig. 3.104A

SOLUTION : We consider an element of thickness dx at distance x from the first plate.

Since, dielectric constant varies linearly.

$$\therefore \quad \varepsilon = mx + K$$

where m and K are constant,

when $x = 0$, $\varepsilon = \varepsilon_1$, when $x = d$, $\varepsilon = \varepsilon_2$

$$\therefore \quad K = \varepsilon_1 \text{ and } m = \left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)$$

$$\varepsilon = \left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right) x + \varepsilon_1$$

The capacitance of considered element is $dC = \varepsilon_0 \varepsilon \frac{S}{dx}$

Since, medium is continuous

$$\therefore \quad \frac{1}{C} = \int \frac{1}{dC}$$

$$\therefore \quad \frac{1}{C} = \int_0^d \frac{dx}{\varepsilon_0 \varepsilon S} = \int_0^d \frac{dx}{\varepsilon_0 S \left\{\frac{\varepsilon_2 - \varepsilon_1}{d}\right\} x + \varepsilon_1}$$

$$= \frac{1}{\varepsilon_0 S}\left(\frac{d}{\varepsilon_2 - \varepsilon_1}\right)\left[ln\left\{\left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right) x + \varepsilon_1\right\}\right]_0^d$$

$$= \frac{d}{\varepsilon_0 S\,(\varepsilon_2 - \varepsilon_1)} ln\left(\frac{\varepsilon_2 - \varepsilon_1 + \varepsilon_1}{\varepsilon_1}\right)$$

$$= \frac{d}{\varepsilon_0 S\,(\varepsilon_2 - \varepsilon_1)} ln \frac{\varepsilon_2}{\varepsilon_1}$$

$$\therefore \quad C = \frac{\varepsilon_0 S\,(\varepsilon_2 - \varepsilon_1)}{d\, ln \frac{\varepsilon_2}{\varepsilon_1}}$$

(b) We consider a thin layer of thickness dx. The bound charge density on the layer is $\sigma_1' = -\sigma\left(1 - \frac{1}{\varepsilon}\right)$. The surface charge density just after this layer is $\sigma_2' = -\sigma\left(1 - \frac{1}{\varepsilon + d\varepsilon}\right)$

The net bound charge density is $\sigma' = \sigma_2' - \sigma_1' = \sigma\left[\frac{-d\varepsilon}{\varepsilon(\varepsilon + d\varepsilon)}\right]$ (Here $\sigma = \frac{q}{S}$)

$$\therefore \quad \sigma' = -\sigma \frac{d\varepsilon}{\varepsilon^2} \qquad (\because d\varepsilon \text{ is very small } \therefore \varepsilon + d\varepsilon \approx \varepsilon.)$$

The volume of considered element is $dV = Sdx$.

The change in dielectric constant for small distance dx is $d\varepsilon = \left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right) dx$

ρ' = The space bound charge density $= \frac{dq'}{dV} = \frac{\sigma' S}{Sdx} = \frac{\sigma'}{dx} = -\sigma \frac{d\varepsilon}{\varepsilon^2\, dx}$

On putting the values of $\sigma = \frac{q}{s}$ and $d\varepsilon$, we get

$$\rho' = \frac{\sigma d\varepsilon}{\varepsilon^2 dx} = -\frac{q}{S}\left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)dx\frac{1}{\varepsilon^2 dx}$$

$$= -\frac{q(\varepsilon_2 - \varepsilon_1)}{Sd\varepsilon^2}$$

YOUR STEP

1. One of the plate of a parallel square plate capacitor of plate area A is tilted relative to the other. The tilt angle is very small (the plates are equipotential). Find the capacity of this configuration shown in fig. 3.104B.

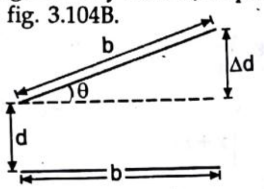


Fig. 3.104B

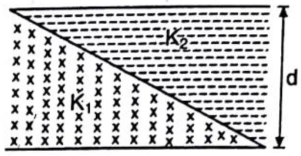


Fig. 3.104C

2. The capacity of a parallel plate capacitor with plate area A and separation d is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2 respectively. Find the capacitance of the resulting capacitor. (see fig. 3.104C).

$$\left\{1.\ \frac{\varepsilon_0 A}{d}\left(1 - \frac{\Delta d}{2d}\right)\ 2.\ \left(\frac{CK_1K_2}{K_2 - K_1}\right) In\frac{K_2}{K_1}\right\}$$

§ 3.105

➢ **CONCEPT**

The capacitance of capacitor

$$= \frac{\text{Charge on positive plate}}{\text{potential difference between positive and negative plates}} = \frac{q}{\phi_+ - \phi_-}$$

SOLUTION : The capacitor is shown in fig. 3.105A. The electric field at point P is

$$E = \frac{q}{4\pi\varepsilon_0 \varepsilon r^2} \qquad \text{Here} \quad \varepsilon = \frac{a}{r}$$

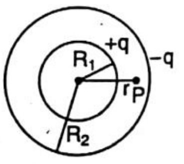


Fig. 3.105A

$$\therefore \quad E = \frac{q}{4\pi\varepsilon_0 \frac{a}{r} r^2} = \frac{q}{4\pi\varepsilon_0 ar}$$

$$\because \quad d\phi = -Edr \quad \text{or} \quad d\phi = -\frac{q}{4\pi\varepsilon_0 ar}dr$$

or $$\int_{\phi_+}^{\phi_-} d\phi = \frac{-q}{4\pi\varepsilon_0 a}\int_{R_1}^{R_2}\frac{dr}{r} \text{ or } -\int_{\phi_+}^{\phi_-} d\phi = \frac{q}{4\pi\varepsilon_0 a} In\frac{R_2}{R_1}$$

$$\therefore \quad C = \frac{q}{\phi_+ - \phi_-} = \frac{q}{\frac{q}{4\pi\varepsilon_0 a} In\frac{R_2}{R_1}}$$

$$\therefore \quad C = \frac{4\pi\varepsilon_0 a}{In\frac{R_2}{R_1}}$$

YOUR STEP

The space between two concentric conductivity spheres is filled on one side of a diametral plane with dielectric constant K_1 and on the other side with dielectric of constant K_2. The inner sphere of radius a has a charge $+q$. Show that the force on it perpendicular to this diametral plane is.

$$\left\{\frac{(K_1 - K_2)\, q^2}{8\pi\varepsilon_0 (K_1 + K_2)^2 a^2}\right\}$$

§ 3.106

➢ **CONCEPT**

For any material there is a maximum electric intensity beyond which damage occurs. This is known as breakdown. At high electric field, the electron gain enough energy in the dielectric to knock other charged particles and make them available for conduction.

SOLUTION : For long cylinder, $E = \dfrac{\lambda}{2\pi\varepsilon_0 \varepsilon r}$

The electric fields at the surfaces of cylindrical layers will be maximum.

$$E_{1m} = \frac{\lambda_1}{2\pi\varepsilon_0\varepsilon_1 R_1}$$

$$\therefore \qquad \lambda_1 = 2\pi\varepsilon_0\varepsilon_1 R_1 E_{1m}$$

and $\quad E_{2m} = \dfrac{\lambda_2}{2\pi\varepsilon_0\varepsilon_2 R_2} \qquad \therefore \qquad \lambda_2 = 2\pi\varepsilon_0\varepsilon_2 R_2 E_{2m}$

Since, break down takes place at maximum electric field.

Hence, for simultaneous breakdown, $\lambda_1 = \lambda_2$. $\quad \therefore \quad 2\pi\varepsilon_0\varepsilon_1 R_1 E_{1m} = 2\pi\varepsilon_0\varepsilon_2 R_2 E_{2m}$

$$\therefore \qquad E_{1m}\,\varepsilon_1 R_1 = E_{2m}\,\varepsilon_2 R_2$$

YOUR STEP

1. The diameter of the outer conductor of a cylindrical capacitor is d_2. What should be the diameter of the core (inner cylinder) d_1 of this capacitor be, so that for a given potential difference between the outer conductor and the core, the electric field strength at the core is minimum.
2. A one core lead sheathed cable has a conductor core of 0.5 cm diameter and the lead seath has inside diameter 1.5 cm. The insulating material is rubber. At what voltage will the insulation breakdown? Given that rubber has a dielectric strength 400kV/cm.

{ 1. $d_1 = \dfrac{d_2}{e}$ 2. 3.3×10^5 volt }

§ 3.107

➢ **CONCEPT**

The concept is similar to previous problem.

SOLUTION :

$$E_1 R_1 \varepsilon_1 < E_2 R_2 \varepsilon_2 \qquad \text{...(i)}$$

But $\quad E_1 = \dfrac{\lambda_1}{2\pi\varepsilon_0\varepsilon_1 R_1}$

$$\therefore \qquad \lambda_1 = 2\pi\varepsilon_0\varepsilon_1 R_1 E_1 \qquad \text{...(ii)}$$

and $\quad E_2 = \dfrac{\lambda_2}{2\pi\varepsilon_0\varepsilon_2 R_2}$

$$\therefore \qquad \lambda_2 = 2\pi\varepsilon_0\varepsilon_2 R_2 E_2 \qquad \text{...(iii)}$$

From Equ. (i), (ii) and (iii), we conclude $\lambda_1 < \lambda_2$

Hence, for breakdown of both dielectric linear charge density should be equal to λ_1.

From (ii), $\qquad \lambda_1 = 2\pi\varepsilon_0\varepsilon_1 R_1 E_1$

$$\therefore \qquad -\int_{\phi_1}^{\phi_2} d\phi = \int_{R_1}^{R_3} E dr$$

$$\text{or} \qquad -\int_{\phi_1}^{\phi_2} d\phi = \int_{R_1}^{R_2} \frac{\lambda_1}{2\pi\varepsilon_0\varepsilon_1 r} dr + \int_{R_2}^{R_3} \frac{\lambda_1}{2\pi\varepsilon_0\varepsilon_2 r} dr$$

$$\therefore \qquad \phi_1 - \phi_2 = \frac{\lambda_1}{2\pi\varepsilon_0}\left[\frac{1}{\varepsilon_1} ln \frac{R_2}{R_1} + \frac{1}{\varepsilon_2} ln \frac{R_3}{R_2}\right]$$

Putting the value of λ_1, breakdown voltage is

$$\phi_1 - \phi_2 = E_1 R_1 \varepsilon_1 \left[\frac{1}{\varepsilon_1} ln \frac{R_2}{R_1} + \frac{1}{\varepsilon_2} ln \frac{R_3}{R_2}\right]$$

$$= E_1 R_1 \left[ln \frac{R_2}{R_1} + \frac{\varepsilon_1}{\varepsilon_2} ln \frac{R_3}{R_2}\right]$$

YOUR STEP

Three concentric thin spherical shells are of radii a, b and c $(a < b < c)$. The first and third are connected by a fine metallic wire through a small hole in the second. The second is connected to earth. Find the capacitance of the system.

$$\left\{4\pi\varepsilon_0\left[\frac{ab}{b-a} + \frac{c^2}{c-b}\right]\right\}$$

§ 3.108

➢ **CONCEPT**

$$\text{Electric capacitance} = \frac{\text{charge on positive plate}}{\text{potential difference between positive and negative plate}}$$

i.e.,
$$C = \frac{q}{\phi_+ - \phi_-}$$

SOLUTION : The capacitor is shown in fig. 3.108 A.

The net electric field at point P is $E = E_1 + E_2$

where E_1 = electric field due to positive plate (cylindrical wire of positive charge) $= \dfrac{\lambda}{2\pi\varepsilon_0 r}$

And, E = electric field due to negative plate (cylindrical wire containing negative charge).

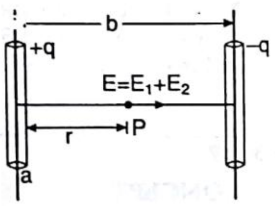


Fig. 3.108A

$$= \frac{\lambda}{2\pi\varepsilon_0 (b-r)}$$

$$\therefore \qquad E = \frac{\lambda}{2\pi\varepsilon_0 r} + \frac{\lambda}{2\pi\varepsilon_0 (b-r)}$$

$$-d\phi = E dr$$

$$\text{or} \qquad -\int_{\phi_+}^{\phi_-} d\phi = \int_a^{b-a}\left(\frac{\lambda}{2\pi\varepsilon_0 r} + \frac{\lambda}{2\pi\varepsilon_0 (b-r)}\right) dr$$

After integrating we get, $\qquad \phi_+ - \phi_- = \dfrac{\lambda}{\pi\varepsilon_0} ln \dfrac{b-a}{a}$.

$$\therefore \qquad C = \frac{q}{\phi_+ - \phi_-} = \frac{q}{\dfrac{\lambda}{\varepsilon_0} ln\left(\dfrac{b-a}{a}\right)} \qquad \text{or} \qquad C = \frac{\pi\varepsilon_0 q}{\lambda\, ln\left(\dfrac{b-a}{a}\right)}$$

$\because$ Capacitance per unit length is $$\frac{C}{l} = \frac{\pi\varepsilon_0 \left(\frac{q}{l}\right)}{\lambda \ln\left(\frac{b-a}{a}\right)}$$

or $$\frac{C}{l} = \frac{\pi\varepsilon_0 \lambda}{\lambda \, ln\left(\frac{b-a}{a}\right)}$$ or $$\frac{C}{l} = \frac{\pi\varepsilon_0}{ln\left(\frac{b-a}{a}\right)} = \frac{\pi\varepsilon_0}{ln\left(\frac{b-a}{a}\right)}$$

But $b >> a$ $\therefore$ $$\frac{C}{l} \approx \frac{\pi\varepsilon_0}{ln\frac{b}{a}}$$

YOUR STEP

A capacitor consists of two very long conducting cylinders of radii a and b, where $a < b$. A Battery of voltage V is connected to the capacitor (Shown in fig. 3.108B).

(a) Calculate the capacitance per unit length of the system.

(b) One adds a third concentric long conducting cylinder of radius c where $c > b$. Battery is then turned off and its positive terminal is reconnected to the new cylinder. What is the capacitance per unit length between the cylinders of radii a and c?

$$\left\{ \text{(a)} \ \frac{2\pi\varepsilon_0}{ln\frac{b}{a}} \ \text{(b)} \ \frac{2\pi\varepsilon_0}{ln\frac{c}{a}} \right\}$$

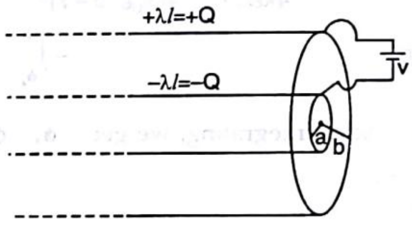


Fig. 3.108B

§ 3.109

➢ **CONCEPT**

The conducting plane may be replaced by taking an oppositely charged wire as a plane mirror image is formed on the other side.

The equivalent system is shown in fig. 3.109A

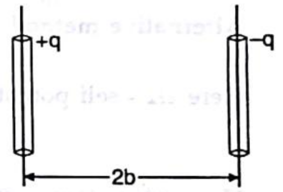


Fig. 3.109A

SOLUTION : The solution is similar to previous problem. In this problem, the distance between wire is $2b$. But in previous problem, distance between wires is b.

Hence, $$C = \frac{2\pi\varepsilon_0}{ln\frac{2b}{a}}$$ $(\because b >> a)$

YOUR STEP

A long conducting cylinder of radius a is oriented parallel and which are at a distance h from an infinite conducting plane. Show that the capacitance of the system per unit length of cylinder is given by

$$\left\{ C = \frac{2\pi\varepsilon_0}{\cosh^{-1}(h/a)} \right\}$$

§ 3.110

➢ **CONCEPT**

Since, $b >> a$, so, both spheres behave as isolated balls

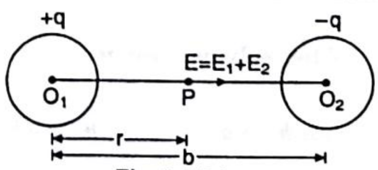


Fig. 3.110A

SOLUTION : The capacitor is shown in fig. 3.110 A.

In this case,

$$C = \frac{q}{\phi_+ - \phi_-}$$

The electric field at point P is $E = E_1 + E_2$

Here E_1 = electric field due to positively charged ball.

$$= \frac{q}{4\pi\varepsilon_0\varepsilon r^2}$$

And E_2 = Electric field due to negatively charged ball.

$$= \frac{q}{4\pi\varepsilon_0\varepsilon (b - r)^2}$$

$\therefore$ $$E = \frac{q}{4\pi\varepsilon_0\varepsilon r^2} + \frac{q}{4\pi\varepsilon_0\varepsilon (b-r)^2} \qquad \therefore \; -d\phi = E dr$$

or $$-\int_{\phi_+}^{\phi_-} d\phi = \int_a^{b-a} \left\{ \frac{q}{4\pi\varepsilon_0\varepsilon r^2} + \frac{q}{4\pi\varepsilon_0\varepsilon (b-r)^2} \right\} dr$$

After integrating, we get $\phi_+ - \phi_- = \frac{q}{4\pi\varepsilon_0\varepsilon}\left[\frac{2}{a} - \frac{2}{b}\right]$

$\therefore$ $$C = \frac{q}{\phi_+ - \phi_-} = \frac{4\pi\varepsilon_0\varepsilon}{\left[\frac{2}{a} - \frac{2}{b}\right]} = \frac{2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}} = \frac{2\pi\varepsilon_0\varepsilon}{\frac{b-a}{ab}}$$

But $b >> a$, $\therefore$ $b - a \approx b$

$\therefore$ $$C \approx \frac{2\pi\varepsilon_0}{\frac{b}{ab}} \qquad \text{or} \qquad C \approx 2\pi\varepsilon_0\varepsilon a$$

Alternative method (on the basis of energy) : Total energy of system is

$$U = U_+ + U_- + U_{+-}$$

Here U_+ = self potential energy of positively charged balls

$$= \frac{q^2}{8\pi\varepsilon_0\varepsilon a}$$

U_- = self potential energy of negatively charged balls

$$= \frac{(-q)^2}{8\pi\varepsilon_0\varepsilon a} = \frac{q^2}{8\pi\varepsilon_0\varepsilon a}$$

And U_{+-} = Mutual potential energy of both balls

$$= \frac{(q)(-q)}{4\pi\varepsilon_0\varepsilon b} = \frac{-q^2}{4\pi\varepsilon_0\varepsilon b}$$

$\therefore$ $$U = \frac{q^2}{4\pi\varepsilon_0\varepsilon}\left[\frac{1}{2a} + \frac{1}{2a} - \frac{1}{b}\right]$$

But in the case of capacitor $U = \frac{q^2}{2c}$

$\therefore$ $$\frac{q^2}{2C} = \frac{q^2}{4\pi\varepsilon_0\varepsilon}\left[\frac{1}{2a} + \frac{1}{2a} - \frac{1}{b}\right]$$

After solving, we get $$C = \frac{2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}} = \frac{2\pi\varepsilon_0\varepsilon}{\left(\frac{b-a}{ab}\right)}$$

But $b >> a$; $\therefore$ $b - a \approx b$

$\therefore$ $$C = 2\pi\varepsilon_0\varepsilon a$$

YOUR STEP

Two equal spheres each of radius a are in contact. Show that the capacity of the conductor so formed is $8\pi\varepsilon_0 a \ln 2$.

§ 3.111

➢ CONCEPT

The conducting plane may be replaced by taking an oppositely charged sphere as a plane mirror image formed on the other side of the conducting plane. The equivalent system is shown in fig 3.111A.

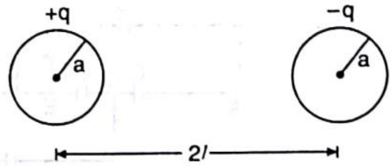


Fig. 3.111A

SOLUTION : The solution is similar to previous problem. In this problem, $b = 2l$.

Also, potential difference between sphere and its image is half of nominal potential difference between sphere and its image.

From previous problem, nominal potential difference is

$$\phi_+ - \phi_- = \frac{q}{4\pi\varepsilon_0}\left[\frac{2}{a} - \frac{2}{b}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{2}{a} - \frac{2}{2l}\right] \quad (\because b = 2l)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{2l-a}{al}\right)$$

But $l >> a$

$\therefore$ $2l - a \approx 2l$

$\therefore$ $$\phi_+ - \phi_- = \frac{q}{4\pi\varepsilon_0}\left[\frac{2l}{al}\right] = \frac{q}{2\pi\varepsilon_0 a}$$

$\therefore$ Effected potential difference between sphere and its image is

$$\Delta\phi = \frac{1}{2}\left(\phi_+ - \phi_-\right) = \frac{q}{4\pi\varepsilon_0 a}$$

$\therefore$ $$C = \frac{q}{\Delta\phi} = 4\pi\varepsilon_0 a \quad \text{for} \quad l >> a$$

YOUR STEP

If the middle conductor of a condenser is formed by a long cylinder (radius b) inside a co-axial shell (internal radius $2a$ and external radius $3a$) with another co-axial cylinder (internal radius d) outside, be charged and the other put to earth, determine the minimum value of d in order that the part of the middle shell may not separate, when it is cut into two parts by a plane through its axis.

$\{3a \cdot 2^{\sqrt{2/3}}\}$

§ 3.112

➢ CONCEPT

For circuit analysis, the points of same potentials are taken as a single point. For parallel combination of capacitors,

$$C_{eq} = C_1 + C_2 + ----$$

SOLUTION : In the given circuit, the potential of points A and E are same.

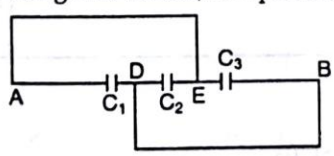


Fig. 3.112A

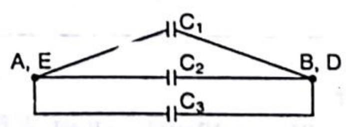


Fig. 3.112B

Also, the potential of points D and B are same. For convenience, the given circuit can be represented as shown in fig. 3.112 B.

From fig. 3.112 B, it is clear that capacitors are in parallel.

∴ The capacitance of the circuit between points A and B is

$$C_{eq} = C_1 + C_2 + C_3$$

(b). The circuit of the problem is shown in fig. 3.112 C. For convenience, the circuit can be represented as shown in fig. 3.112 D.

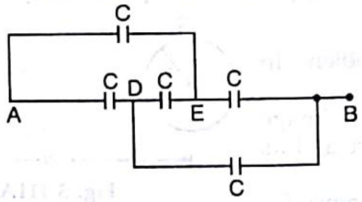


Fig. 3.112C

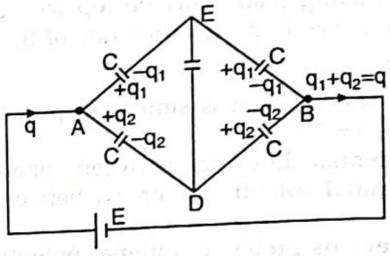


Fig. 3.112D

If a plane mirror is placed along line DE in fig. 3.112 D, then right part of circuit is exact image of left part of the circuit.

So, electric charge on capacitor of branch AE = electric charge in capacitor of branch EB.

The electric charge on capacitor of branch AD = electric charge on capacitor of branch DB

From this point of view, no charge is found on capacitor of branch ED. Hence, the capacitor of branch ED may be removed. The circuit may be represented as shown in fig. 3.112 E.

Hence, equivalent capacitance of the circuit is

$$C_{eq} = C.$$

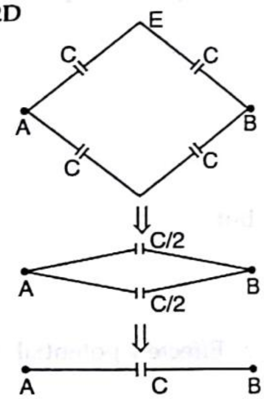


Fig. 3.112E

YOUR STEP

1. Find the equivalent capacitance of the system between points M and N shown in fig. 3.112F.

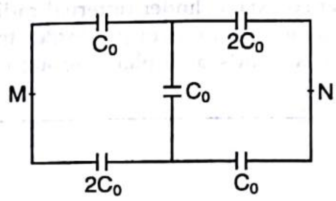


Fig. 3.112F

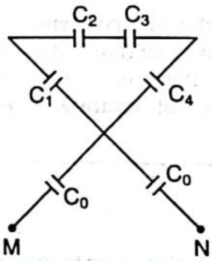


Fig. 3.112G

2. Find the equivalent capacitance between points M and N in the circuit shown in fig. 3.112G.

$$\left\{1.\ \frac{4}{3}Co\ \ 2.\ \frac{C_0}{2}\right\}$$

§ 3.113

➢ **CONCEPT**

SOLUTION : The problem is solved in following steps

(a) Step I : Each plate may be divided into two plates of same area.

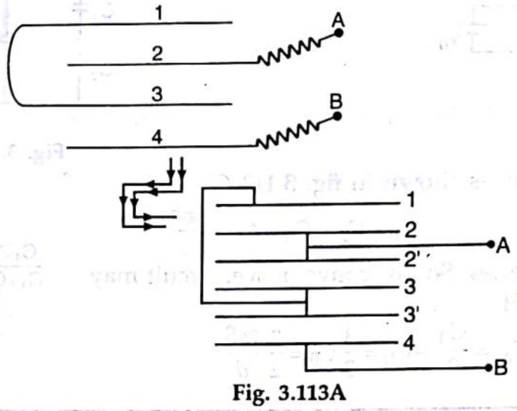


Fig. 3.113A

Step II : A battery of emf V_0 is connected between points A and B.

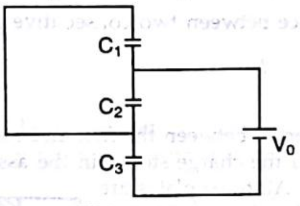


Fig. 3.113B

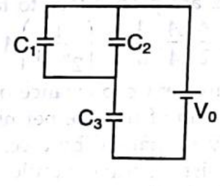


Fig. 3.113C

The circuit in simple form is shown in Fig. 3.113 C.

According to problem, $C_1 = C_2 = C_3 = C_0 = \dfrac{\varepsilon_0 S}{d}$

Since, C_1 and C_2 are in parallel. So, for convenience, circuit may be represented as Fig. 3.113D.

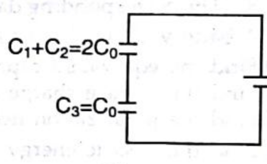


Fig. 3.113D

$\therefore \quad \dfrac{1}{C_{eq}} = \dfrac{1}{2C_0} + \dfrac{1}{C_0} \qquad \therefore \quad C_{eq} = \dfrac{2}{3}C_0 = \dfrac{2\varepsilon_0 S}{3d}.$

(b) Step I : Each plate may be divided into two plates of same area (shown in Fig. 3.113E)

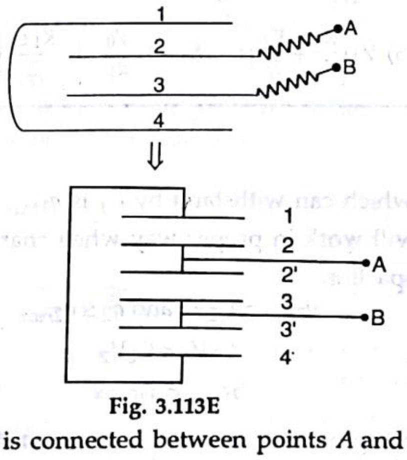


Fig. 3.113E

Step II : A battery of emf V_0 is connected between points A and B.

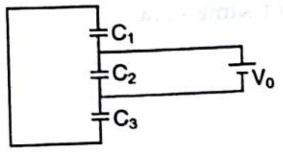


Fig. 3.113F

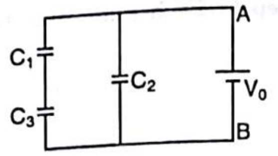


Fig. 3.113G

The circuit in simple form is shown in fig. 3.113 G.

According to problem, $C_1 = C_2 = C_3 = C_0 = \dfrac{\varepsilon_0 S}{d}$

Since, C_1 and C_3 are in series. So, for convenience, circuit may be represented as fig. 3.113 H.

Hence, $C_{eq} = \dfrac{C_0}{2} + C_0 = \dfrac{3}{2}C_0 = \dfrac{3}{2}\dfrac{\varepsilon_0 S}{d}$

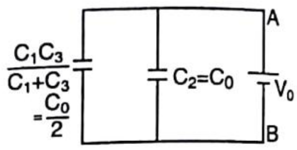


Fig. 3.113H

YOUR STEP

1. n metallic plates are placed face to face. The distance between two consecutive plates is d. Area of plates are $A, \dfrac{A}{2}, \dfrac{A}{4}, \dfrac{A}{8} \cdots \left(\dfrac{1}{2^{n-1}}\right)A$.
   (a) Find the equivalent capacitance of the system.
   (b) A dielectric slab of relative permittivity K is inserted between the first and second plates and the assembly is charged by a cell of emf E. Find the charge stored in the assembly.
2. A capacitor consists of three parallel metallic plates. All three plates are of the same area A. The first pair of plates are placed a distance d_1 apart, and the space between them is filled with a medium of dielectric constant K_1. The corresponding data for the second pair are d_2 and K_2 respectively. A battery of emf V_0 is connected with the plates as shown in fig. 3.113I.
   (a) Find the equivalent capacity of the system.
   (b) Find the surface charge density on the middle plate.
   (c) Find the polarization field in the medium of dielectric constant K_1.
   (d) Find the electric energy density in the medium of dielectric constant K_1.

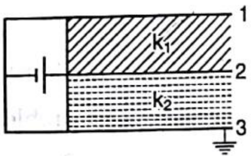


Fig. 3.113I

$$\left\{ \begin{array}{l} \textbf{1. } (a)\ \dfrac{\varepsilon_0 A}{(2^n - 2)d},\quad (b)\ \dfrac{\varepsilon_0 AE}{2d\left[\dfrac{1}{K} + 2^{n-1} - 2\right]} \\[2ex] \textbf{2. } (a)\ \varepsilon_0 A\left(\dfrac{K_1}{d_1} + \dfrac{K_2}{d_2}\right)\ (b)\ V_0\left(\dfrac{K_1}{d_1} + \dfrac{K_2}{d_2}\right)\ (c)\ (K_1 - 1)\dfrac{V_0}{d_1}\ (d)\ \dfrac{K_1\varepsilon_0}{2}\left(\dfrac{V_0}{d_1}\right)^2 \end{array} \right\}$$

§ 3.114

➢ CONCEPT

The maximum charge which can withstand by C_1 is $q_{1max} = C_1V_1$

It means capacitor C_1 will work in proper way when charge on C_1 is $q_1 \le q_{1max}$

Similarly, for second capacitor,

$$q_{2max} = C_2V_2 \text{ and } q_2 \le q_{2max}$$

But $\quad C_1V_1 < C_2V_2 \quad$ (By putting numerical values)

It means $\quad q_{1max} < q_2\text{max}$

SOLUTION : The circuit of the problem is shown in fig. 3.114 A.

Applying loop rule in the circuit (shown in fig. 3.114 A).

$$V_0 - \frac{q}{C_1} - \frac{q}{C_2} = 0$$

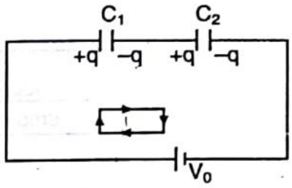


Fig. 3.114A

$\therefore \quad q = \dfrac{C_1C_2V_0}{C_1+C_2} \qquad \therefore \quad q_{\max} = \left(\dfrac{C_1C_2}{C_1+C_2}\right)V_{0max}$ ...(i)

But $q \le q_{1max}$ and $q \le q_{2max}$ Also, $q_{1max} \le q_{2max}$

For satisfying above in equality, $q_{\max} = q_{1max} = C_1V_1$ ...(ii)

From Eqs. (i) and (ii), we get $q_{1max} = q_{\max} = \left(\dfrac{C_1C_2}{C_1+C_2}\right)V_0\max$

or
$$C_1V_1 = \left(\frac{C_1C_2}{C_1+C_2}\right)V_{0max}$$

$\therefore$
$$V_{0max} = \frac{C_1V_1}{\dfrac{C_1C_2}{C_1+C_2}} = \frac{(C_1+C_2)\,V_1}{C_2}$$

$\therefore$
$$V_{0max} = \left(1+\frac{C_1}{C_2}\right)V_1$$

i.e.,
$$V_0 \le \left(1+\frac{C_1}{C_2}\right)V_1$$

Putting the values, we get $V_0 \le 9\text{kV}$

> **YOUR STEP**
>
> A parallel plate capacitor A is filled by a dielectric whose relative permittivity varies with the applied voltage according to law $\varepsilon_r = KV$ where $K = 1\ \text{V}^{-1}$. The same (but containing no dielectric) capacitor B charged to a voltage $V_0 = 156$ V is connected in parallel to the first capacitor. Find the common voltage across each capacitor in steady state.
>
> { 12V }

§ 3.115

➢ CONCEPT

The potential difference between points A and B is

$\phi_{AB} = \phi_A - \phi_B = -$ (algebraic sum of rise up and drop up of voltage through any path).

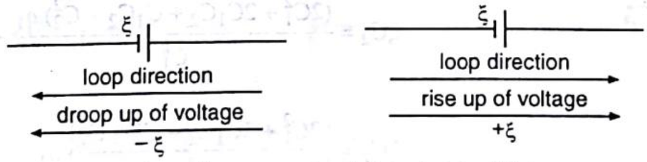
Concept of rise up and drop up of voltage :

For solving circuit problems, two directions come into play..

(i) Loop direction : This direction is not specified. The direction of loop is chosen in comfortable manner.

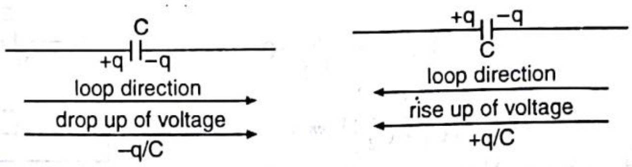
(ii) The direction of flow of charge :

Drop up of voltage : When we go from a point of higher potential to a point of lower potential in loop direction, drop up of voltage takes place. In sign convention, drop up of voltage is taken as negative.



Rise up of voltage : If we go from lower potential point to higher potential point, rise up of voltage takes place. In sign convention, rise up of voltage is taken as positive.

For discussion of circuit problems, two important concepts are involved.



(i) Capacitor circuit obeys conservation principle of charge.

Mathematically, $q = q_1 + q_2$

(shown in fig. 3.115 A).

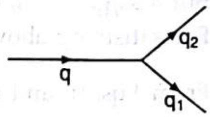


Fig. 3.115A

(ii) In a closed circuit, the algebraic sum of rise up and drop up voltage is zero.

$$\Sigma V = 0 \qquad \text{(for closed circuit)}$$

SOLUTION : Step I : Determine the charges on each capacitor :

The distribution of charges is shown in fig. 3.115 B. In closed loop (1)

$$+\xi - \frac{q}{C_1} - \frac{(q-q_1)}{C_2} = 0 \qquad \text{...(i)}$$

In closed loop (2) $\quad -\dfrac{q_1}{C_1} - \dfrac{q_1}{C_2} + \dfrac{q-q_1}{C_2} = 0$

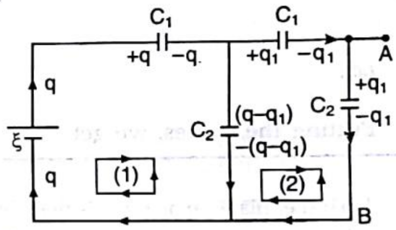


Fig. 3.115B

or $$-\frac{q_1}{C_1} - \frac{q_1}{C_2} - \frac{q_1}{C_2} + \frac{q}{C_2} = 0$$

or $$\frac{q}{C_2} = q_1\left(\frac{2}{C_2} + \frac{1}{C_1}\right)$$

or $$q = \left(\frac{2C_1 + C_2}{C_1}\right) q_1 \qquad \text{...(ii)}$$

From Eq. (i), we get $\xi - \dfrac{q}{C_1} - \dfrac{q}{C_2} + \dfrac{q_1}{C_2} = 0$

or $$\xi + \frac{q_1}{C_2} = q\left(\frac{C_1 + C_2}{C_1 C_2}\right)$$

or $$\xi + \frac{q_1}{C_2} = \left(\frac{2C_1 + C_2}{C_1}\right) q_1 \left(\frac{C_1 + C_2}{C_1 C_2}\right)$$

or $$\xi C_2 + q_1 = \frac{(2C_1 + C_2)(C_1 + C_2)\, q_1}{C_1^2}$$

or $$\xi C_2 = \frac{(2C_1^2 + 2C_1C_2 + C_1C_2 + C_2^2)\, q_1}{C_1^2} - q_1$$

$$\xi C_2 = \left\{\frac{2C_1^2 + 3C_1C_2 + C_2^2 - C_1^2}{C_1^2}\right\} q_1$$

$\therefore$ $$q_1 = \frac{\xi\, C_2 C_1^2}{C_1^2 + 3C_1C_2 + C_2^2}$$

$\therefore$
$$\phi_A - \phi_B = \left|\frac{-q_1}{C_2}\right| = \frac{q_1}{C_2}$$
$$= \frac{\xi C_1^2}{C_1^2 + 3C_1C_2 + C_2^2}$$
$$= \frac{\xi}{1 + 3\dfrac{C_2}{C_1} + \dfrac{C_2^2}{C_1^2}}$$
$$= \frac{\xi}{1 + 3\eta + \eta^2}$$
$$= 10\text{V} \qquad \left(\because \frac{C_2}{C_1} = \eta = 2\right)$$

YOUR STEP

In Fig. shown in 3.115C,

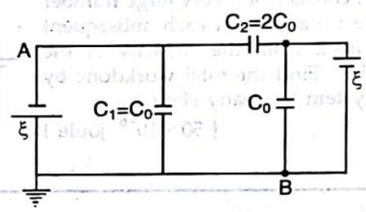


Fig. 3.115C

(a) Find potential of point A and B.
(b) Find energy stored by C_1 and C_2.

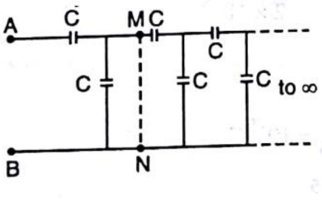
$\{$ (a) $\phi_A = -\xi$, $\phi_B = 0$, (b) $U_1 = \frac{1}{2} C_0 \xi^2$, $U_2 = C_0 \xi^2$ $\}$

§ 3.116

SOLUTION

Let equivalent capacitance between points A and B is C_0.

Since, circuit consists of a large number of capacitors. So equivalent capacitance of the part on right of dotted line MN (shown in Fig. 3.116 A) is also equal to C_0. Now the circuit may be represented as shown in Fig. 3.116 B.



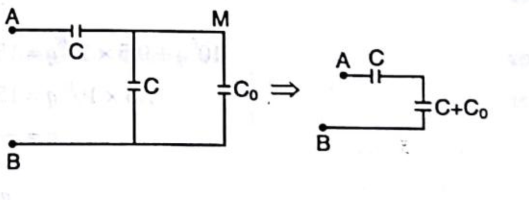


Fig. 3.116A **Fig. 3.116B**

$\therefore$ Equivalent capacitance between points A and B is
$$C_{AB} = C_0$$

From Fig 3.116 B
$$\frac{1}{C_{AB}} = \frac{1}{C} + \frac{1}{C + C_0}$$

$\therefore \qquad C_{AB} = \dfrac{C(C+C_0)}{2C+C_0}$

But $\qquad C_{AB} = C_0$

$\therefore \qquad C_0 = \dfrac{C(C+C_0)}{2C+C_0}$

or $\qquad C_0^2 + C_0C - C^2 = 0$

$\therefore \qquad C_0 = -\dfrac{C \pm \sqrt{C^2+4C^2}}{2}$

But only positive value of C_0 is taken.

$\therefore \qquad C_0 = \dfrac{-C+\sqrt{5}C}{2} = \dfrac{(\sqrt{5}-1)C}{2} = 0.62C$

YOUR STEP

The circuit shown in fig. 3.116C, consists of a very large number of sections. The capacity of the capacitors in each subsequent section differ by a factor of $n = 2$ from the capacity of the capacitors in the previous section. Find the total workdone by the ideal battery to bring the system in steady state.

(50×10^{-6} joule)

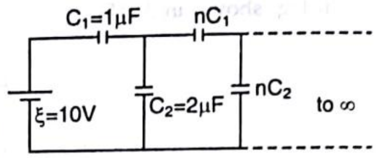


Fig. 3.116C

§ 3.117

➢ **CONCEPT**

The distribution of charges on capacitors is shown in fig. 3.117 A.

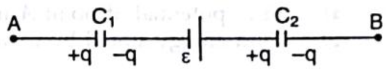


Fig. 3.117A

Here, $\phi_A - \phi_B$

$= -$ {algebraic sum of rise up and drop up voltage}

$$\phi_A - \phi_B = -\left\{-\frac{q}{C_1} + \xi - \frac{q}{C_2}\right\}$$

or $\qquad 5 = \dfrac{q}{C_1} - \xi + \dfrac{q}{C_2}$

or $\qquad 5 = \dfrac{q}{10^{-6}} - 10 + \dfrac{q}{2 \times 10^{-6}}$

or $\qquad 10^6 q + 0.5 \times 10^6 q = 15$

or $\qquad 1.5 \times 10^6\, q = 15$

$\therefore \qquad q = \dfrac{15}{1.5 \times 10^6} = 10^{-5}\text{C}$

$\therefore$ The voltage across C_1 is $\qquad \phi_1 = \dfrac{q}{C_1} = \dfrac{10^{-5}}{10^{-6}} = 10\text{V}$

And the voltage across C_2 is $\qquad \phi_2 = \dfrac{q}{C_2} = \dfrac{10^{-5}}{2 \times 10^{-6}} = 5\text{V}$

YOUR STEP

In the circuit shown in fig. 3.117B, Find
(a) potential of point E
(b) the equivalent capacity between point A and C.

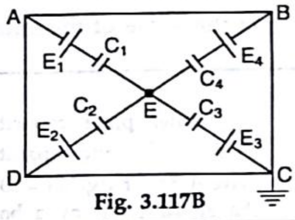


Fig. 3.117B

$$\left\{ \text{(a) } \frac{C_1E_1 + C_2E_2 + C_3E_3 + C_4E_4}{C_1 + C_2 + C_3 + C_4}, \text{ (b) infinity} \right\}$$

§ 3.118

➢ **CONCEPT**

The concept is similar to problem 3.115.

SOLUTION : The distribution of charge is shown in fig. 3.118 A. According to loop rule,

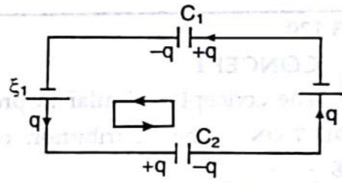


fig. 3.118A

$$\xi_1 - \frac{q}{C_2} - \xi_2 - \frac{q}{C_1} = 0$$

$$\xi_1 - \xi_2 = q\left(\frac{C_1 + C_2}{C_1C_2}\right)$$

$$\therefore \qquad q = (\xi_1 - \xi_2)\left(\frac{C_1C_2}{C_1 + C_2}\right)$$

For C_1,

$$\phi_1 = -\left(\frac{q}{C_1}\right)$$

$$= -\frac{q}{C_1} = -\frac{(\xi_1 - \xi_2)}{C_1}\left(\frac{C_1C_2}{C_1 + C_2}\right)$$

$$= (\xi_2 - \xi_1)\frac{C_2}{C_1 + C_2} = \frac{(\xi_2 - \xi_1)}{\left(1 + \frac{C_1}{C_2}\right)}$$

Similarly, for C_2,

$$\phi_2 = -\left(-\frac{q}{C_2}\right) = \frac{q}{C_2} = \frac{(\xi_1 - \xi_2)\, C_1C_2}{C_2\,(C_1 + C_2)}$$

$$= \frac{(\xi_1 - \xi_2)\, C_1}{(C_1 + C_2)} = \frac{(\xi_1 - \xi_2)}{\left(1 + \frac{C_2}{C_1}\right)}$$

YOUR STEP

In the circuit shown in fig. 3.118B, if potential difference between points A and B is 11 volt, in steady state. Calculate potential difference across C_5.

{ 1.8 volt}

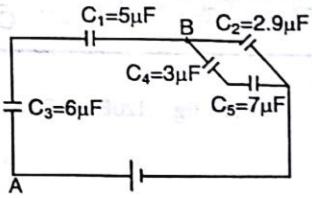


Fig. 3.118B

§ 3.119

SOLUTION

From the solution of 3.118, electric charge on each capacitor is

$$q = (\xi_1 - \xi_2)\left(\frac{C_1C_2}{C_1 + C_2}\right)$$

In the sense of magnitude, $q = |\xi_1 - \xi_2| \left(\frac{C_1C_2}{C_1 + C_2}\right)$

YOUR STEP

A parallel plate capacitor consists of two plates A and B, each of area $S = 5 \times 10^{-3}$ m and separation $d = 8.85 \times 10^{-3}$ m (shown in the figure 3.119A). The plate A has charge $q_A = 10^{-10}$ C and plate B has charge $q_B = 2 \times 10^{-10}$ C. Calculate charge supplied by a battery of emf $V_0 = 10$V when its positive terminal is connected with plate A and negative terminal with plate B.

{10^{-10} C}

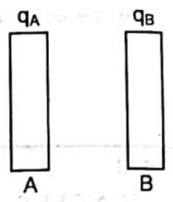


Fig. 3.119A

§ 3.120

➢ **CONCEPT**

The concept is similar to problem 3.115.

SOLUTION : The distribution of charge is shown in Fig. 3.120A.

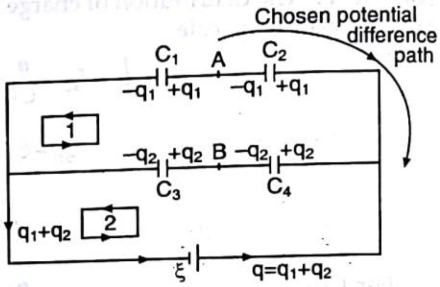


Fig. 3.120A

In closed loop (1),

$$-\frac{q_1}{C_2} - \frac{q_1}{C_1} + \frac{q_2}{C_3} + \frac{q_2}{C_4} = 0$$

In closed loop (2),

$$\xi - \frac{q_2}{C_4} - \frac{q_2}{C_4} = 0$$

$$\therefore \qquad q_2 = \frac{\xi C_3 C_4}{C_3 + C_4}$$

Putting the values of q_2 in equ. (1) we get

$$\text{or} \qquad q_1 = \frac{\xi C_1 C_2}{C_1 + C_2} \qquad \therefore \qquad \phi_A - \phi_B = -\left\{\frac{q_1}{C_2} - \frac{q_2}{C_4}\right\}$$

Putting the values of q_1 and q_2, we get $\phi_A - \phi_B = \frac{\xi(C_2C_3 - C_1C_4)}{(C_1 + C_2)(C_3 + C_4)}$

If $\qquad \phi_A - \phi_B = 0$

Then $\qquad \frac{\xi(C_2C_3 - C_1C_4)}{(C_1 + C_2)(C_3 + C_4)} = 0$

$$\Rightarrow \qquad C_2C_3 = C_1C_4 \qquad \therefore \qquad \frac{C_1}{C_2} = \frac{C_3}{C_4}$$

YOUR STEP

In the fig 3.120B 6 μC charge is added to point A, find charge on capacitor of capacity 3C.

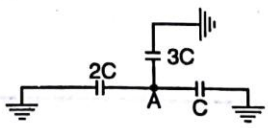


Fig. 3.120B

{ 3μC}

§ 3.121

SOLUTION

The problem may be solved stepwise.

Step I : Determine the charge on C_1 :

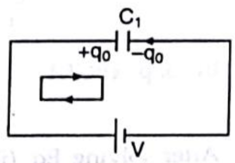


Fig. 3.121A

According to loop rule $\qquad V - \dfrac{q_0}{C_1} = 0$

$\therefore \qquad q_0 = C_1 V \qquad \text{...(1)}$

Step II : Draw the circuit after connecting C_1 to C_2 and C_3 :

Applying loop rule, $\qquad + \dfrac{q_0 - q}{C_1} - \dfrac{q}{C_2} - \dfrac{q}{C_3} = 0$

$$+\frac{q_0}{C_1} - \frac{q}{C_1} - \frac{q}{C_2} - \frac{q}{C_3} = 0$$

or $\qquad +\dfrac{C_1 V}{C_1} - \dfrac{q}{C_1} - \dfrac{q}{C_2} - \dfrac{q}{C_3} = 0$

By solving, $\qquad q = \dfrac{V}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}} = 0.06 \text{ mC}$

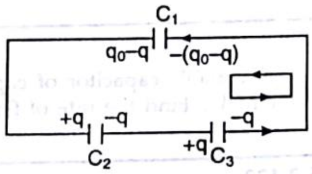
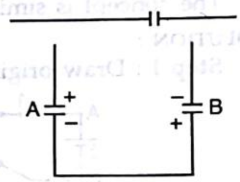


Fig. 3.121B

> **YOUR STEP**
>
> Two capacitors A and B with capacities $3\mu F$ and $2\mu F$ are charged to a potential difference of 100 V and 80 V respectively. the plates of capacitors are connected as shown in fig. 3.121C with one wire from each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged $2\mu F$ capacitor C with lead wires falls on the free ends to complete the circuit. Calculate
>
> **(a)** the final charge on the three capacitors.
>
> **(b)** the amount of electrostatic energy stored in the system before and after the completion of circuit.
>
> { (a) $q_A = 300\mu C$, $q_B = 360\mu C$, $q_c = 210\mu C$, (b) 4.74×10^{-2} J and 18×10^{-3} J }
>
> 
>
>
> **Fig. 3.121C**

§ 3.122

SOLUTION :

In the given circuit of problem (shown in fig. 3.122 A), we have to find the charges which will flow after the shortening of the switch S_w through sections AB and AD in the directions indicated by the arrows.

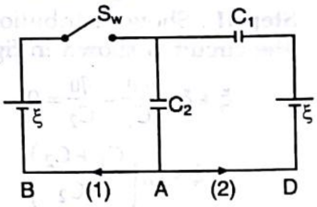


fig. 3.122A

The problem is solved in following steps :

Step I : When switch S_w is opened.

The distribution of charges in this case is shown in fig. 3.122 B.

In closed loop $ADEFA$, $\qquad \xi - \dfrac{q}{C_1} - \dfrac{q}{C_2} = 0$

$\therefore \qquad q = \dfrac{\xi C_1 C_2}{C_1 + C_2} \qquad \text{...(i)}$

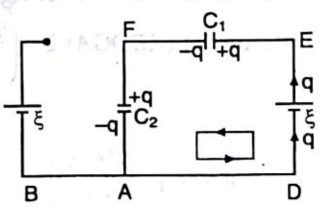


fig. 3.122B

Step II : When switch S is closed :

The circuit is shown in fig. 3.122C.

In loop $FEDAF$,

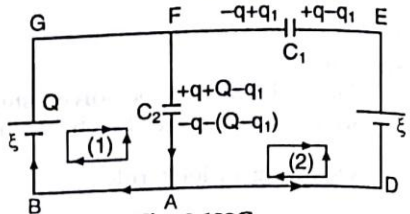


Fig. 3.122C

$$\frac{q-q_1}{C_1}-\xi+\frac{q+Q-q_1}{C_2}=0 \qquad ...(i)$$

In loop $ABGFA$, $\qquad \xi-\frac{q+Q-q_1}{C_2}=0$

$$\therefore \quad q+Q-q_1=\xi C_2 \qquad ...(ii)$$

After solving Eq. (i) and (ii), we get

$$Q=\xi C_2, \quad \text{and} \quad q_1=\frac{\xi C_1 C_2}{C_1+C_2}$$

Hence, flow of charge through path (1) is $Q=\xi C_2$ and flow of charge through (2) is

$$-q_1=\frac{\xi C_1 C_2}{C_1+C_2} \qquad \text{(in indicated direction)}$$

YOUR STEP

A variable capacitor of capacitance $C=\alpha t$ is connected with an ideal battery of emf V_0. Find the rate of flow of charge through capacitor.

$\{V_0\,\alpha\}$

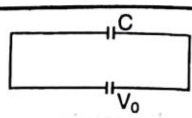


Fig. 3.122D

§ 3.123

➢ **CONCEPT**

The concept is similar to previous problem. The problem is easily solved stepwise.

SOLUTION :

Step I : Draw original circuit.

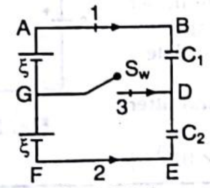


Fig. 3.123A

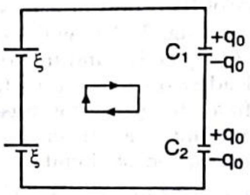


Fig. 3.123B

Step II : Show distribution of charge before closing the switch :

The circuit is shown in fig. 3.123 B. Applying loop rule in the curcuit,

$$\because \quad \xi+\xi-\frac{q_0}{C_1}-\frac{q_0}{C_2}=0 \quad \text{or} \quad 2\xi=q_0\left(\frac{1}{C_1}+\frac{1}{C_2}\right)$$

or $$2\xi=q_0\left(\frac{C_1+C_2}{C_1C_2}\right) \quad \therefore \quad q_0=\frac{2\xi C_1C_2}{C_1+C_2} \qquad ...(i)$$

Step II : Draw the circuit after closing the switch.

The circuit is shown in fig 3.123 C.

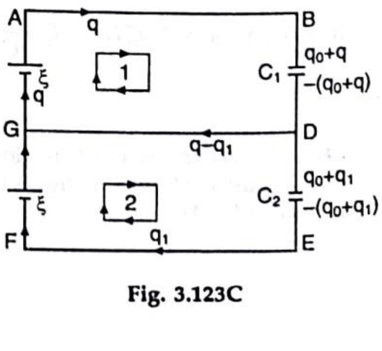


Fig. 3.123C

In loop (1) $(ABDGA)$ $\xi-\left(\frac{q_0+q}{C_1}\right)=0$

$$\therefore \quad q_0+q=\xi C_1$$

$$\therefore \quad q=\xi C_1-q_0$$

$$\therefore \quad q=\xi C_1-\frac{2\xi C_1C_2}{C_1+C_2} \quad \text{(on putting the value of } q_0\text{)}$$

$$=\frac{\xi C_1^2+\xi C_1C_2-2\xi C_1C_2}{C_1+C_2}$$

$$\because \qquad q = \frac{\xi C_1^2 - \xi C_1 C_2}{C_1 + C_2}$$

$$q = \xi C_1 \left(\frac{C_1 - C_2}{C_1 + C_2}\right)$$

In loop (2) (*DEFGD*), $\quad -\left(\frac{q_0 + q_1}{C_2}\right) + \xi = 0$

$$\therefore \qquad q_0 + q_1 = \xi C_2$$

$$\therefore \qquad q_1 = \xi C_2 - q_0$$

$$\therefore \qquad q_1 = \xi C_2 - \frac{2\xi C_1 C_2}{C_1 + C_2}$$

$$= \frac{\xi C_1 C_2 + \xi C_2^2 - 2\xi C_1 C_2}{C_1 + C_2}$$

$$\therefore \qquad q_1 = \frac{\xi C_2^2 - \xi C_1 C_2}{C_1 + C_2} = \xi C_2 \frac{(C_2 - C_1)}{C_1 + C_2}$$

The charge through section (1) in indicated direction is

$$q = \xi C_1 \left(\frac{C_1 - C_2}{C_1 + C_2}\right) = -24\mu C$$

The charge through section (2) in indicated direction is

$$-q_1 = -\xi C_2 \left(\frac{C_2 - C_1}{C_1 + C_2}\right)$$

$$= \xi C_2 \left(\frac{C_1 - C_2}{C_1 + C_2}\right) = -36\mu C$$

The electric charge through section (3) in indicated direction is $-(q - q_1)$

$$= q_1 - q = \xi C_2 \left(\frac{C_2 - C_1}{C_1 + C_2}\right) - \xi C_1 \left(\frac{C_1 - C_2}{C_1 + C_2}\right)$$

$$= \xi (C_2 - C_1) = 60\mu C$$

YOUR STEP

In the circuit shown in Fig 3.123D. Find :

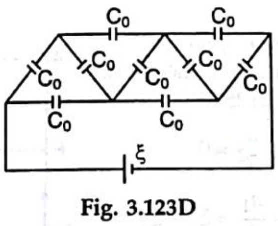


Fig. 3.123D

(a) the total charge supplied by battery.
(b) heat generated in the circuit.
(c) total energy stored on the capacitors.

$$\left\{ \text{(a) } \frac{11}{5} C_0 \xi, \text{ (b) } \frac{11}{30} C_0 \xi^2, \text{ (c) } \frac{11}{30} C_0 \xi^2 \right\}$$

§ 3.124

➢ **CONCEPT**

Potential difference between points A and B is $\phi_A - \phi_B$ $= -$ {Algebraic sum of rise up and drop up of voltage}

SOLUTION : Distribution of electric charge is shown in fig. 3.124A.

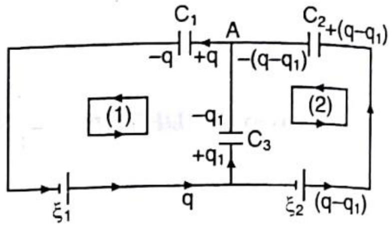


Fig. 3.124A

In loop (1)

$$\xi_1 - \frac{q_1}{C_3} - \frac{q}{C_1} = 0 \quad \text{...(i)}$$

In loop (ii),

$$-\left(\frac{q-q_1}{C_2}\right) + \frac{q_1}{C_3} + \xi_2 = 0 \quad \text{...(ii)}$$

After solving equ. (i) and (ii) we get,

$$q_1 = \frac{-C_3(C_2\xi_2 - C_1\xi_1)}{(C_1 + C_2 + C_3)}$$

$$\therefore \quad \phi_A - \phi_B = -\left\{\frac{q_1}{C_3}\right\} = \left(\frac{C_2\xi_2 - C_1\xi_1}{C_1 + C_2 + C_3}\right)$$

YOUR STEP

Find potential difference between points A and B, B and C in steady state.

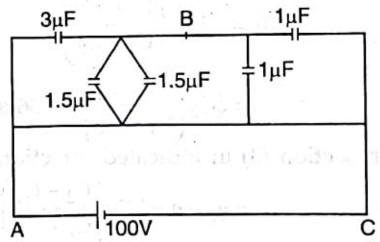


Fig. 3.124B

{25V and 75V}

§ 3.125

➢ **CONCEPT**

The problem is solved by following steps :

Step I : Show the charge distribution in the circuit.

In loop $ABC\,FA$ (shown in fig 3.125A),

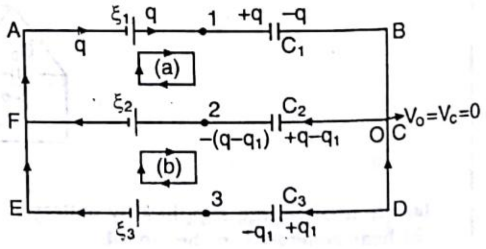


Fig. 3.125A

$$+\xi_1 - \frac{q}{C_1} - \frac{(q-q_1)}{C_2} - \xi_2 = 0$$

$$\therefore \quad \frac{q}{C_1} + \frac{q}{C_2} - \frac{q_1}{C_2} = \xi_1 - \xi_2$$

or $$q\left(\frac{C_1 + C_2}{C_1C_2}\right) = \xi_1 - \xi_2 + \frac{q_1}{C_2}$$

$$\therefore \quad q = \frac{(\xi_1 - \xi_2 + \frac{q_1}{C_2})\, C_1C_2}{(C_1 + C_2)} \quad \text{...(i)}$$

In loop *FCDEF*,

$$+\xi_2 + \frac{q-q_1}{C_2} - \frac{q_1}{C_3} - \xi_3 = 0$$

or $$\xi_2 - \xi_3 = \frac{q_1}{C_2} + \frac{q_1}{C_3} - \frac{q}{C_2} \quad \text{or} \quad \xi_2 - \xi_3 = q_1\left(\frac{C_2+C_3}{C_2C_3}\right) - \frac{q}{C_2} \qquad \text{...(ii)}$$

Putting the value of q in equ. (ii) from equ. (i)

$$\xi_2 - \xi_3 = q_1\left(\frac{C_2+C_3}{C_2C_3}\right) - \frac{\left(\xi_1 - \xi_2 + \frac{q_1}{C_2}\right)}{C_2}\left(\frac{C_1C_2}{C_2+C_2}\right)$$

or $$\xi_2 - \xi_3 = q_1\frac{(C_2+C_3)}{C_2C_3} - \left(\xi_1 - \xi_2 + \frac{q_1}{C_2}\right)\left(\frac{C_1}{C_1+C_2}\right)$$

or $$\xi_2 - \xi_3 = q_1\left(\frac{C_2+C_3}{C_2C_3}\right) - \frac{q_1C_1}{C_2(C_1+C_2)} - (\xi_1-\xi_2)\left(\frac{C_1}{C_1+C_2}\right)$$

or $$\xi_2 - \xi_3 = q_1\left\{\frac{(C_1+C_2)(C_2+C_3) - C_1C_3}{C_2C_3(C_1+C_2)}\right\} - (\xi_1-\xi_2)\frac{C_1}{(C_1+C_2)}$$

or $$\xi_2 - \xi_3 + (\xi_1-\xi_2)\frac{C_1}{(C_1+C_2)} = q_1\left\{\frac{C_1C_2 + C_1C_3 + C_2^2 + C_2C_3 - C_1C_3}{C_2C_3(C_1+C_2)}\right\}$$

or $$\xi_2 - \xi_3 + (\xi_1-\xi_2)\frac{C_1}{(C_1+C_2)} = \left(\frac{C_1C_2 + C_2C_3 + C_2^2}{C_2C_3(C_1+C_2)}\right)q_1$$

or $$(\xi_2-\xi_3)(C_1+C_2) + (\xi_1-\xi_2)C_3 = \left(\frac{C_1C_2 + C_2C_3 + C_2^2}{C_2C_3}\right)q_1$$

$$\therefore \quad q_1 = \frac{\{(\xi_2-\xi_3)(C_1+C_2) + (\xi_1-\xi_2)C_3\}C_2C_3}{C_1C_2 + C_2C_3 + C_2^2}$$

From equ. (i)

$$q = \left(\frac{C_1C_2}{C_1+C_2}\right)\left[\xi_1 - \xi_2 + \frac{1}{C_2}\left\{\frac{(\xi_2-\xi_3)(C_1+C_2) + (\xi_1-\xi_2)C_3}{C_1C_2 + C_2C_3 + C_2^2}\right\}C_2C_3\right]$$

$$= \left(\frac{C_1C_2}{C_1+C_2}\right)\left[\xi_1 - \xi_2 + \frac{1}{C_2^2}\left\{\frac{(\xi_2-\xi_3)(C_1+C_2) + (\xi_1-\xi_2)C_3}{C_1 + C_3 + C_2}\right\}C_2C_3\right]$$

Step II : Calculate potentials of points :

$$\because \quad \phi_1 - \phi_0 = -\left\{-\frac{q}{C_1}\right\} = \frac{q}{C_1}$$

$$\therefore \quad \phi_1 = \frac{q}{C_1} \qquad (\because \phi_0 = 0)$$

Putting the values of q,

$$\therefore \quad \phi_1 = \frac{\xi_2C_2 + \xi_3C_3 - \xi_1(C_2+C_3)}{C_1+C_2+C_3}$$

Similarly, $$\phi_2 - \phi_0 = -\left\{\frac{q-q_1}{C_2}\right\} = \frac{q_1-q}{C_2}$$

$$\therefore \qquad \phi_2 = \frac{q_1 - q}{C_2} \qquad (\because \phi_0 = 0)$$

On putting the values of q and q_1,

$$\phi_2 = \frac{\xi_1 C_1 + \xi_3 C_3 - \xi_2 (C_1 + C_3)}{C_1 + C_2 + C_3}$$

Similarly, $$\phi_3 - \phi_0 = -\left\{\frac{q_1}{C_3}\right\} = \frac{-q_1}{C_3}$$

$$\therefore \qquad \phi_3 = \frac{-q_1}{C_3} \qquad (\because \phi_0 = 0)$$

On putting the value of q_1,

$$\phi_3 = \frac{\xi_1 C_1 + \xi_2 C_2 - \xi_3 (C_1 + C_2)}{(C_1 + C_2 + C_3)}$$

YOUR STEP

In the circuit shown in fig 3.125B, cells are ideal. Find the potentials of points A and B.

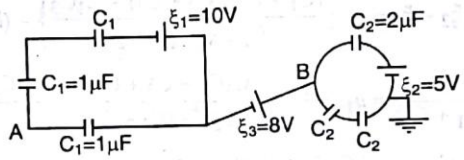


Fig. 3.125B

$$\left\{-8\text{V}, \frac{10}{3}\text{V}\right\}$$

§ 3.126

➢ **CONCEPT**

The given circuit is an example of rotational symmetry. If a circuit is rotated through 180° about a point and no effect is found on the circuit. Then the circuit is in rotational symmetry about the point.

SOLUTION : Step I : If the circuit is rotated through 180° about mid point of CD, given circuit is obtained again without any change.

Due to rotation, point A is replaced by point B. Similarly point C is replaced by the point D. It means, the charge distribution on capacitor in branch AC is same as that of in branch DB. Similarly the charge distribution on capacitor in branch AD is same as that of in branch CB.

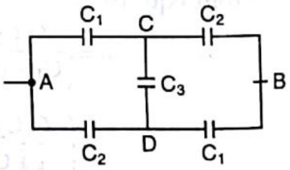


Fig. 3.126A

Step II : When a battery of emf ξ_0 is applied between points A and B (shown in fig. 3.126B).

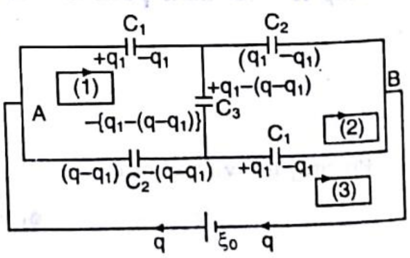


Fig. 3.126B

In loop (1)

$$-\frac{q_1}{C_1} - \left\{\frac{q_1 - (q - q_1)}{C_3}\right\} + \frac{q - q_1}{C_2} = 0$$

or $$-\frac{q_1}{C_1} - \frac{2q_1}{C_3} + \frac{q}{C_3} + \frac{q}{C_2} - \frac{q_1}{C_2} = 0$$

or $$q\left(\frac{1}{C_2} + \frac{1}{C_3}\right) = \frac{q_1}{C_1} + \frac{q_1}{C_2} + \frac{2q_1}{C_3}$$

or
$$q\left(\frac{C_2+C_3}{C_2C_3}\right)=q_1\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{2}{C_3}\right)$$

$\therefore$
$$q=\left(\frac{C_2C_3+C_1C_3+2C_1C_2}{(C_2+C_3)C_1}\right)q_1 \quad \text{...(i)}$$

In loop (3),
$$-\left(\frac{q-q_1}{C_2}\right)-\frac{q_1}{C_1}+\xi_0=0$$

$\because$
$$\xi_0=\left(\frac{C_2C_3+C_1C_3+2C_1C_2}{C_1(C_2+C_3)}\right)\frac{q_1}{C_2}+q_1\left(\frac{C_2-C_1}{C_1C_2}\right) \quad \text{...(ii)}$$

Let equivalent capacitance between points A and B is C_{eq}. The equivalent circuit is shown in Fig. 3.126C.

In loop (4),
$$\xi_0-\frac{q_0}{C_{eq}}=0$$

$\therefore$ $\xi_0=\dfrac{q}{C_{eq}}$ ...(iii) $\quad \because \; q_0=C_{eq}\,\xi_0$

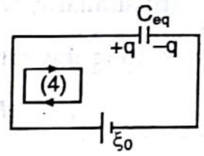


Fig. 3.126C

From Eqs. (ii) and (iii), we get
$$\frac{q}{C_{eq}}=\left(\frac{C_2C_3+C_1C_3+2C_1C_2}{(C_2+C_3)\,C_1}\right)\frac{q_1}{C_2}+q_1\left(\frac{C_2-C_1}{C_1C_2}\right) \quad \text{...(iv)}$$

After solving, (i) and (iv), we get
$$C_{eq}=\frac{2C_1C_2+C_3(C_1+C_2)}{C_1+C_2+2C_3}$$

YOUR STEP

Calculate equivalent capacitance between points A_0 and B_0 of the circuit shown in Fig. 3.126 C

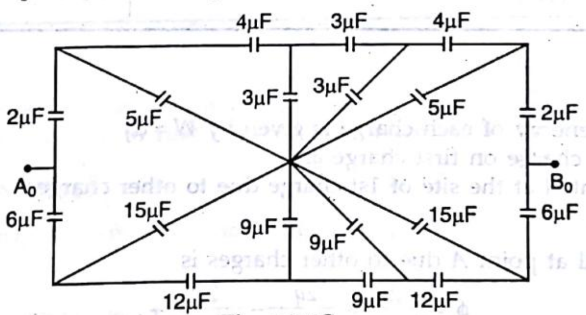


Fig. 3.124C

{3.2μF}

§ 3.127

➢ **CONCEPT**

The number of pair of charges is given by ${}^nC_2$

Also, interaction energy is given by $U=\sum_{\substack{i,j=1\\ i<j}}^{n} U_{ij}$

$$=\frac{1}{4\pi\varepsilon_0}\sum_{\substack{i,j=1\\ i<j}}^{n}\frac{q_iq_j}{r_{ij}}$$

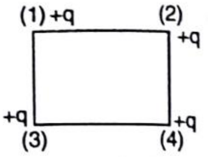


Fig. 3.127A

SOLUTION : (a) The number of pair of charges $= {}^4C_2 = 6$

$$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

How $r_{12} = a, \quad r_{13} = \sqrt{2}, \quad r_{14} = a$

$r_{23} = a, \quad r_{24} = \sqrt{2}a, \quad r_{34} = a$

$$\therefore \qquad U = \frac{q^2}{4\pi\varepsilon_0 r_{12}} + \frac{q^2}{4\pi\varepsilon_0 r_{13}} + \frac{q^2}{4\pi\varepsilon_0 r_{14}} + \frac{q^2}{4\pi\varepsilon_0 r_{23}} + \frac{q^2}{4\pi\varepsilon_0 r_{24}} + \frac{q^2}{4\pi\varepsilon_0 r_{34}}$$

Putting the values, we get $U = \dfrac{(\sqrt{2}+4)\,q^2}{4\pi\varepsilon_0 a}$

(b) similarly, $U = \dfrac{(q)(-q)}{4\pi\varepsilon_0 r_{12}} + \dfrac{q^2}{4\pi\varepsilon_0 r_{13}} + \dfrac{(q)(-q)}{4\pi\varepsilon_0 r_{14}} + \dfrac{(-q)(q)}{4\pi\varepsilon_0 r_{23}} + \dfrac{(-q)(-q)}{4\pi\varepsilon_0 r_{24}} + \dfrac{q(-q)}{4\pi\varepsilon_0 r_{34}}$

Putting the values, we get

$$U = (\sqrt{2} - 4)\frac{q^2}{4\pi\varepsilon_0 a}$$

(c) $$U = \frac{q \times q}{4\pi\varepsilon_0 r_{12}} + \frac{q(-q)}{4\pi\varepsilon_0 r_{13}} + \frac{q(-q)}{4\pi\varepsilon_0 r_{14}} + \frac{q(-q)}{4\pi\varepsilon_0 r_{23}} + \frac{q(-q)}{4\pi\varepsilon_0 r_{24}} + \frac{(-q)(-q)}{4\pi\varepsilon_0 r_{34}} = \frac{-\sqrt{2}\,q^2}{4\pi\varepsilon_0 a}$$

YOUR STEP

Eight point charges of magnitude Q are arranged to form the corners of a cube of side L. The arrangement is made in a manner such that the nearest neighbour of any charge has the opposite sign initially the charges are held at rest. If the system is let free to move, what happens to the arrangement. Does the cube shape shrink or expand? Calculate the velocity of each charge when the side length of the cube changes from L to nL. Assume that the mass of each point charge is m.

$$\left\{\frac{Q^2(n-1)}{\pi\varepsilon_0\sqrt{6}\,nL}(3\sqrt{6} - 3\sqrt{3} - \sqrt{2}), \text{ shrink}\right\}$$

§ 3.128

➢ **CONCEPT**

The interaction energy of each charge is given by $W = \phi q$

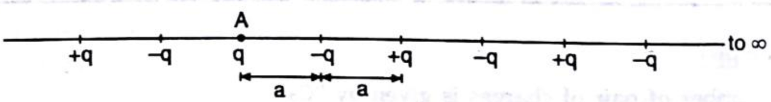
Here q = electric charge on first charge and

ϕ = electric potential at the site of 1st charge due to other charges.

SOLUTION :

The electric potential at point A due to other charges is

$$\phi = \frac{-2q}{4\pi\varepsilon_0 a} + \frac{2q}{4\pi\varepsilon_0 2a} - \frac{2q}{4\pi\varepsilon_0 3a} + \ldots$$



$$\therefore \qquad \phi = \frac{-2q}{4\pi\varepsilon_0 a}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots \text{ upto } \infty\right) = \frac{-2q}{4\pi\varepsilon_0} \ln 2$$

$$\therefore \qquad W = \phi q = \frac{-2q^2}{4\pi\varepsilon_0 a} \ln 2$$

YOUR STEP

The NaCl crystal consists of Na^+ and Cl^- ions. These are arranged at the corners of cubes stacked together in such a way that the ions in the nearest neighbours to any ion have opposite charges. If the side of the cube is R and the electronic charge is $-e$. Calculate the electrostatic energy (i) per ion (ii) pergram of Nacl crystal.

$$\left\{ \text{(i)}\ \frac{-1.7476e^2}{4\pi\varepsilon_0 R}\ \text{(ii)}\ -1.80\times 10^{22}\frac{e^2}{R} \right\}$$

§ 3.129

➢ CONCEPT

The conducting plane may be replaced by taking an opposite charge as a plane mirror image on the other side of the conducting plane. The equivalent system is shown in Fig. 3.129 A.

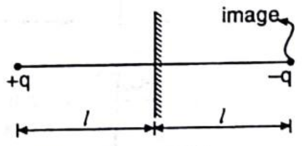


Fig. 3.129A

SOLUTION : The interaction energy is

$$\therefore \quad W = \frac{(+q)(-q)}{4\pi\varepsilon_0(2l)} = \frac{-q^2}{8\pi\varepsilon_0 l}$$

YOUR STEP

Two small metallic balls of radii R_1 and R_2 are in vacuum at a distance considerable exceeding their dimensions and have a certain total charge. Find the ratio $\frac{q_1}{q_2}$ between the charges of the balls at which the energy of the system is minimum. What is the potential difference between the balls in this?

$$\left\{ \frac{R_1}{R_2} > \text{zero} \right\}$$

Potential difference is $\Delta\phi = 0$

§ 3.130

SOLUTION :

We consider a spherical shell of charge of radius r and thickness dr. The potential due to first ball on the surface of considered elements is $\phi = \frac{q_1}{4\pi\varepsilon_0 l}$

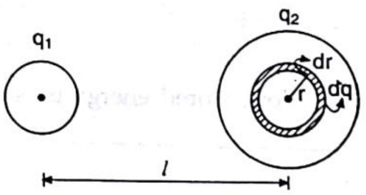


Fig. 3.130

$\therefore$ Interaction energy between spherical shell and ball is $dW = \phi dq$

$$\therefore \text{ Total interaction energy is} \quad W = \int_0^{q_2} \frac{q_1}{4\pi\varepsilon_0 l}\, dq = \frac{q_1 q_2}{4\pi\varepsilon_0 l}$$

YOUR STEP

Small identical balls with equal charges are fixed at the vertices of a right 1977-gon with side a. At a certain instant, one of the ball is released, and a sufficiently long time interval later, the ball adjacent to the first released ball is made free. The kinetic energies of the released balls are found to differ by K at a sufficiently long distance from the polygon. Determine the charge q of each ball.

$$\{q = \sqrt{4\pi\varepsilon_0 K a}\}$$

§ 3.131

SOLUTION :

The problem is solved by following steps.

Step I : Determine charge on capacitor C_1.

According to loop rule, $V - \frac{q_0}{C_1} = 0$

$\therefore$ $q_0 = C_1 V$ ...(ii)

$\therefore$ Energy stored on capacitor is $U_i = \frac{1}{2} C_1 V^2$

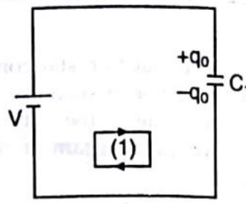


Fig. 3.131A

Step II : Draw the circuit after connecting both capacitors.
At steady state, circuit is shown in Fig. 3.131C.
Applying loop rule in circuit 3.131C.

$\therefore$ $$+\frac{q_0 - q}{C_1} - \frac{q}{C_2} = 0$$

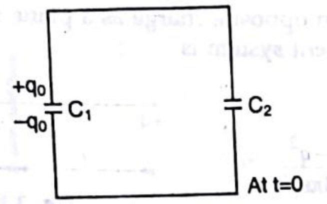


Fig. 3.131B

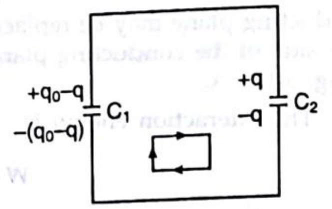


Fig. 3.131C

or $$\frac{q_0}{C_1} - q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 0$$

or $$\frac{q_0}{C_1} = q\left(\frac{C_1 + C_2}{C_1 C_2}\right)$$

or $$\frac{C_1 V}{C_1} = q\left(\frac{C_1 + C_2}{C_1 C_2}\right)$$

$\therefore$ $$q = \frac{C_1 C_2 V}{C_1 + C_2}$$

Now, stored energy on system is $$U_f = U_1 + U_2 = \frac{(q_0 - q)^2}{2C_1} + \frac{q^2}{2C_2}$$

$\therefore$ $$U_f = \frac{\left(C_1 V - \frac{C_1 C_2}{C_1 + C_2} V\right)^2}{2C_1} + \frac{q^2}{2C_2}$$

$\therefore$ $$U_f = \frac{\left(\frac{C_1^2 V + C_1 C_2 V - C_1 C_2 V}{C_1 + C_2}\right)^2}{2C_1} + \frac{q^2}{2C_2}$$

$\therefore$ $$U_f = \frac{C_1^4 V^2}{2C_1 (C_1 + C_2)^2} + \frac{C_1^2 C_2^2 V^2}{2C_2 (C_1 + C_2)^2}$$

$$= \frac{C_1^2 V^2}{2(C_1 + C_2)^2}\left\{\frac{C_1^2}{C_1} + \frac{C_2^2}{C_2}\right\}$$

$$= \frac{C_1^2 V^2}{2(C_1 + C_2)^2}\{(C_1 + C_2)\} = \frac{C_1^2 V^2}{2(C_1 + C_2)}$$

The increament of energy of the system is

$$\Delta U = U_f - U_i$$
$$= \frac{1}{2}\frac{C_1^2V^2}{(C_1+C_2)} - \frac{1}{2}C_1V^2$$
$$= \frac{1}{2}C_1V^2\left\{\frac{C_1}{(C_1+C_2)} - 1\right\}$$
$$= \frac{1}{2}C_1V^2\left\{\frac{C_1 - C_1 - C_2}{C_1+C_2}\right\} = \frac{-C_1C_2V^2}{2(C_1+C_2)}$$

Putting the values, we get $\Delta W = -0.03\text{ mJ}$

YOUR STEP

In the circuit shown in Fig. 3.131D.

The capacity of capacitor A with dielectric medium ($K=2$) is $C_A = 2\mu F$. Air capacitors B and D have capacitance $C_B = 3\mu F$ and $C_D = 6\mu F$ respectively. At $t=0$, switch S_1 is closed and S_2 is opened.

(a) Find energy stored in capacitor A in steady state.

(b) Switch S_1 is now opened and S_2 is closed. Calculate the charge on B in steady state.

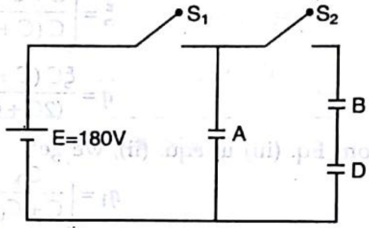


Fig. 3.131D

(c) Now switch S_2 is opened and dielectric medium of capacitor A is replaced by air. Now capacitor B is filled with dielectric medium of constant $K=2$ and then switch S_2 is again closed. Calculate loss of energy during redistribution of charge.

[(a) 0.0324 J, (b) $q_B = q_D = 180\times10^{-6}$C, (c) 5.4×10^{-3} J]

§ 3.132

➢ **CONCEPT**

According to conservation principle of energy,

work done by battery = Heat loss in battery + change in stored energy of capacitor system

Mathematically, $W_b = \Delta H + \Delta U$

$\therefore$ $\Delta H = W_b - \Delta U$

SOLUTION :

The problem is solved in following steps :

Step I : When switch is at position 1. The distribution of charge in this situation is shown in fig. 3.132A.

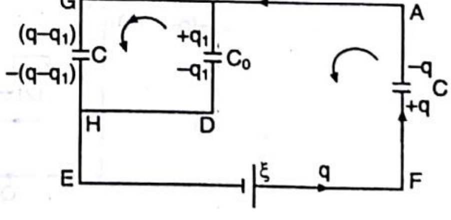


Fig. 3.132A

In the closed loop $ABDHEFA$,

$$-\frac{q_1}{C_0} + \xi - \frac{q}{C} = 0 \qquad \text{...(i)}$$

In the closed $BGHDB$,

$$-\frac{(q-q_1)}{C} + \frac{q_1}{C_0} = 0$$

$$-\frac{q}{C}+\frac{q_1}{C}+\frac{q_1}{C_0}=0$$

$$-\frac{q}{C}+q_1\left(\frac{C+C_0}{CC_0}\right)=0$$

$$q_1\left(\frac{C+C_0}{CC_0}\right)=\frac{q}{C}$$

$$q_1=\frac{qC_0}{C+C_0} \qquad \text{...(ii)}$$

From (i) and (ii) we get,

$$\frac{-q_1}{C_0}+\xi-\frac{q}{C}=0$$

$$\frac{-C_0\,q}{(C+C_0)\,C_0}+\xi-\frac{q}{C}=0$$

$$\xi=\left(\frac{1}{C+C_0}+\frac{1}{C}\right)q$$

$$\xi=\left(\frac{C+C+C_0}{C\,(C+C_0)}\right)q$$

$$q=\frac{\xi C\,(C+C_0)}{(2C+C_0)} \qquad \text{...(iii)}$$

Putting the value of q from Eq. (iii) in equ. (ii), we get

$$q_1=\left(\frac{C_0}{C+C_0}\right)\frac{\xi C\,(C+C_0)}{(2C+C_0)}$$

$$q_1=\frac{\xi CC_0\,(C+C_0)}{(C+C_0)\,(2C+C_0)}$$

The equivalent capacity of the circuit between points E and F is

$$C_{eq}=\frac{(C+C_0)\,C}{C+C_0+C}=\frac{C\,(C+C_0)}{(2C+C_0)}$$

∴ The energy stored on the capacitors system is

$$U_i=\frac{1}{2}\,C_{eq}\,\xi^2=\frac{C\,(C+C_0)\,\xi^2}{2\,(2C+C_0)}$$

Step II : Discuss the circuit at position q :

The distribution of charge is shown in fig. 3.132B.

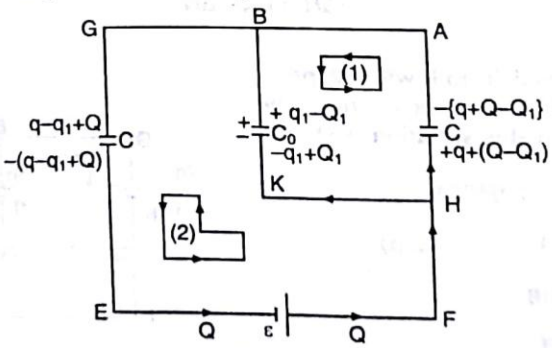


Fig. 3.132B

In loop *ABKHA*, $$-\frac{q_1 - Q_1}{C_0} - \frac{(q + Q - Q_1)}{C} = 0 \quad \text{...(iii)}$$

In loop *FHKBGEF*, $$+\frac{q_1 - Q_1}{C_0} - \frac{(q - q_1 + Q)}{C} + \xi = 0 \quad \text{...(iv)}$$

After solving equ. (iii) and (iv), $$Q = \frac{CC_0 \xi}{C_0 + 2C}$$

Also, equivalent capacity between points E and F is
$$C_{eq} = \frac{(C + C_0)\, C}{2C + C_0}$$

$\therefore$ The energy stored on the system is
$$U_f = \frac{1}{2} C_{eq} \xi^2 = \frac{(C + C_0)\, C}{2\,(2C + C_0)} \xi^2$$

$\therefore$ $$\Delta U = U_f - U_i = 0 \qquad (\because U_i = U_f)$$

Step III : Apply conservation principle of energy :
$$W_{\text{cell}} = \Delta H + \Delta U$$

or $$\varepsilon\phi = \Delta H + 0$$

$\therefore$ $$\Delta H = \xi Q = \frac{CC_0 \xi^2}{(2C + C_0)} \qquad \left(\because Q = \frac{CC_0 \xi}{(2C + C_0)}\right)$$

YOUR STEP

In the circuit shown in fig. 3.132C,
(a) Find charge supplied by battery.
(b) Find total heat generated in the circuit.

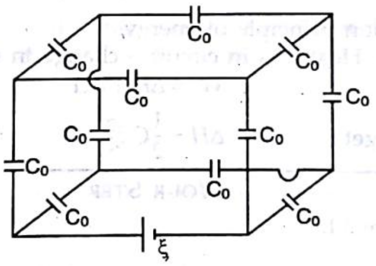


Fig. 3.132C

$$\left\{ \text{(a) } \frac{5}{7} C_0 \xi \qquad \text{(b) } \frac{1}{2} C\xi^2 \text{ where } C = \frac{5}{7} C_0 \right\}$$

§ 3.133

➢ **CONCEPT**

The concept is similar to previous problem.
i.e., $W_b = \Delta H + \Delta U$

SOLUTION : Step I : When switch S is at position 1 :
In this case, circuit is shown in fig. 3.133 B.

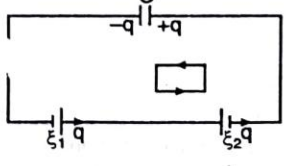


Fig. 3.133A

According to loop rule, $\xi_1 - \xi_2 - \frac{q}{C} = 0$

$\therefore$ $$q = C(\xi_1 - \xi_2) \quad \text{...(i)}$$

The energy stored on capacitor is

$$U_i = \frac{q^2}{2C} = \frac{1}{2}C(\xi_1 - \xi_2)^2$$

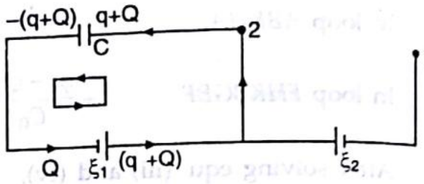


Fig. 3.133B

Step II : When switch S is shifted from position (1) to position (2) :

In this case, the circuit is shown in fig 3.133 B.

According to loop rule, $$\xi_1 - \left(\frac{q+Q}{C}\right) = 0$$

$\therefore$ $$\xi_1 - \frac{q}{C} - \frac{Q}{C} = 0$$

or $$\xi_1 - \frac{C(\xi_1 - \xi_2)}{C} - \frac{Q}{C} = 0$$

or $$\xi_1 - \xi_1 + \xi_2 - \frac{Q}{C} = 0$$

$\therefore$ $$Q = \xi_2 C \quad \text{...(ii)}$$

In this case, battery of emf ξ_2 is useless.

The work done by battery of emf ξ_1 is $W_b = \xi_1 Q = \xi_1 \xi_2 C$

In this step, energy stored on the capacitor is

$$U_f = \frac{(Q+q)^2}{2C} = \frac{\{\xi_2 C + (\xi_1 - \xi_2)C\}^2}{2C}$$

$\therefore$ $$\Delta U = U_f - U_i = \frac{1}{2}C\{2\xi_1\xi_2 - \xi_2^2\}$$

According to conservation principle of energy,

work done by battery = Heat loss in circuit + change in stored energy of capacitor

i.e., $$W_b = \Delta H + \Delta U$$

Putting the values, we get $$\Delta H = \frac{1}{2}C\xi_2^2$$

YOUR STEP

The circuit shown in figure 3.133.

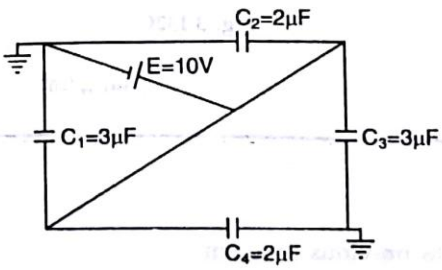


Fig. 3.133C

(a) Find charge on each capacitor in steady state.

(b) Find work done by ideal battery.

(c) Let at $t = 0$, the battery is connected in the circuit. Find total heat generated in the circuit.

[(a) 30μC, 20μC, 30μC, 20μC, (b) 10^{-3} J, (c) 0.5×10^{-3} J]

§ 3.134

➢ CONCEPT

The self potential energy of spherical shell is $W = \int V dq = \int_0^q \frac{q}{4\pi\varepsilon_0 R} dq$

$$\therefore \qquad W = \frac{q^2}{8\pi\varepsilon_0 R}$$

SOLUTION :

$$W_1 = \frac{q_1^2}{8\pi\varepsilon_0 R_1}$$

and

$$W_2 = \frac{q_2^2}{8\pi\varepsilon_0 R_2}$$

The potential on the surface of first shell due to second shell is

$$\phi = \frac{q_2}{4\pi\varepsilon_0 R_2}$$

$$\therefore \qquad W_{12} = q_1 \phi = \frac{q_1 q_2}{4\pi\varepsilon_0 R_2}$$

The total energy is

$$W = W_1 + W_2 + W_{12}$$

$$= \frac{q_1^2}{8\pi\varepsilon_0 R_1} + \frac{q_2^2}{8\pi\varepsilon_0 R_2} + \frac{q_1 q_2}{4\pi\varepsilon_0 R_2}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1^2}{2R_1} + \frac{q_2^2}{2R_2} + \frac{q_1 q_2}{R_2}\right]$$

YOUR STEP

Three conducting balls each of radius a and charge q is placed at the corners of an equilateral triangle of side length l. The side length l is considerably larger than dimensions of the spheres. Find electrical potential energy of system shown in fig. 3.134A

$$\left\{\frac{3q^2}{4\pi\varepsilon_0}\left(\frac{1}{2a} + \frac{1}{l}\right)\right\}$$

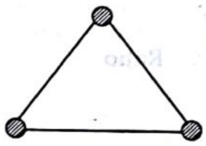


Fig. 3.134A

§ 3.135

➢ CONCEPT

The energy stored per unit volume in an electric field is given by $U = \frac{1}{2}\varepsilon_0 \varepsilon E^2$

$$\therefore \quad \text{Total energy is} \qquad U = \int u dV$$

dV is the elementry considered volume.

SOLUTION : (a) Let at some instant electric charge q is assembled upto radius r. Now we bring dq charge from infinity and put it on this sphere to increase the radius from r to $r + dr$.

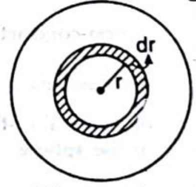


Fig. 3.135A

Here

$$q = \frac{4}{3}\pi r^3 \rho$$

$$\therefore \qquad dq = 4\pi r^2 \rho dr$$

The electric potential on the surface of sphere of radius r is $\phi = \dfrac{q}{4\pi\varepsilon_0 r}$

$$\therefore \quad U = \int \phi dq = \int_0^R \frac{q}{4\pi\varepsilon_0 r} 4\pi\rho r^2 dr$$

$$\therefore \quad U = \frac{4\pi\rho^2 R^5}{15\varepsilon_0}$$

Here $$\rho = \frac{q}{\frac{4}{3}\pi R^3}$$

$$\therefore \quad U = \frac{3q^2}{20\pi\varepsilon_0 R}$$

(b) The electric field out side the sphere is given by

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Here $q = \frac{4}{3}\pi R^3 \rho$ The energy stored outside the sphere is

$$W_2 = \int_{r=R}^{\infty} \frac{1}{2}\varepsilon_0 E^2 \, dV$$

$$= \frac{1}{2}\varepsilon_0 \int_R^{\infty} \left(\frac{q}{4\pi\varepsilon_0 r^2}\right)^2 4\pi r^2 dr$$

$$= \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0}\int_R^{\infty}\frac{r^2\,dr}{r^4} = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{-1}{r}\right]_R^{\infty} = \frac{q^2}{8\pi\varepsilon_0 R}$$

From the result of part (a), total energy is

$$U = W_1 + W_2$$

$$\therefore \quad W_1 = U - W_2$$

$$\therefore \text{ Ratio} \quad \frac{W_1}{W_2} = \frac{U - W_2}{W_2} = \frac{U}{W_2} - 1$$

$$\therefore \text{ Ratio} \quad = \frac{U}{W_2} - 1 = \frac{\dfrac{4\pi\rho^2 R^5}{15\varepsilon_0}}{\dfrac{q^2}{8\pi\varepsilon_0 R}} - 1$$

$$= \frac{4\pi\rho^2 R^5}{15\varepsilon_0} \times \frac{8\pi\varepsilon_0 R}{q^2} - 1$$

$$= \frac{4\pi\rho^2 R^5}{15\varepsilon_0}\,\frac{8\pi\varepsilon_0 R}{\left(\frac{4}{3}\pi R^3 \rho\right)^2} - 1 = \frac{1}{5}$$

YOUR STEP

A non-conducting sphere of radius R has a positive charge which is distributed over its volume with density $\rho = \rho_0\left(1 - \dfrac{x}{R}\right)$ per unit volume, where x is distance from the centre. If dielectric constant of material of the sphere is $K = 1$, calculate energy stored in surrounding space and total self energy of the sphere.

$$\left\{\frac{\pi\rho_0^2 R^5}{72\varepsilon_0}, \ \frac{13\pi\rho_0^2 R^5}{630\varepsilon_0}\right\}$$

§ 3.136

➣ **CONCEPT**

The concept is similar to previous problem.

i.e., $$U = \int u\,dV$$

Here $$u = \frac{1}{2}\varepsilon_0 \varepsilon E^2$$

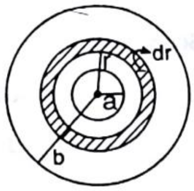


Fig. 3.136

SOLUTION : We consider a concentric spherical shell of radius r and thickness dr inside the dielectric medium.

The volume of considered spherical shell is $dV = 4\pi r^2 dr$

The electric field at a point inside the considered element is $E = \frac{q}{4\pi\varepsilon_0\varepsilon r^2}$

$$\therefore \quad U = \frac{1}{2}\varepsilon_0\,\varepsilon \frac{q^2}{16\pi^2\varepsilon_0^2\,\varepsilon^2 r^4} = \frac{q^2}{32\pi^2\varepsilon_0\varepsilon r^4}$$

$\therefore$ Energy stored in considered element is $d\,U = u dV$

$$\therefore \quad U = \int_{r=a}^{r=b} u dV$$

$$= \int_a^b \frac{q^2}{32\pi^2\varepsilon_0\varepsilon r^4} 4\pi r^2 dr = \int_a^b \frac{q^2}{8\pi\varepsilon_0\varepsilon r^2} dr$$

$$= \frac{q^2}{8\pi\varepsilon_0\varepsilon}\int_a^b r^{-2}\,dr = \frac{q^2}{8\pi\varepsilon_0\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right]$$

Putting the values, we get $U = 27\times 10^{-3}\,\text{J} = 27\,\text{mJ}$

YOUR STEP

1. A solid non-conducting hemisphere of radius R has a uniformly distributed positive charge of density ρ per unit volume. A negatively charged particle having charge q is transferred from centre of its base to inifinity. Calculate work performed in the process. Dielectric constant of material of hemisphere is unity.

2. Why do electrons and not ions cause collision ionisation of atoms although both charges acquire the same kinetic energy $\frac{mv^2}{2} = e\Delta\phi$ (e is the charge of the particles and $\Delta\phi$ is the potential difference) in an accelerating field ? Assume that an atom to be ionized and a particle impinging on it have approximately the same velocity after the collision.

1. $\frac{q\rho R^2}{4\varepsilon_0}$

2. When an atom collides with an ion of mass $m \approx M$, $\frac{mv^2}{2} \approx 2W$ ion, *i.e.*, the energy of the ion required for the ionization must be twice as high as the energy of the electron.

§ 3.137

➣ **CONCEPT**

Work done by electrical forces is given by $A = -\Delta U$

where ΔU = change in electrical potential energy

$$= U_f - U_i$$

SOLUTION :

The initial self potential energy of the spherical shell is $U_i = \dfrac{q^2}{8\pi\varepsilon_0 R_1}$

The final self potential energy of the spherical shell is

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2}$$

$$\therefore \quad \Delta U = U_f - U_i = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

$\because$ Work done by electrical forces is $A = -\Delta U = \dfrac{q^2}{8\pi\varepsilon_0}\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

YOUR STEP

A soap bubble of radius R_0 is slowly given a charge q. Because of mutual repulsion of the surface charges, the radius increases slightly to R. The air pressure inside the bubble drops, because of expansion, to $P\left(\dfrac{V_0}{V}\right)$, when P is the atmospheric pressure, V_0 is the initial volume and V is the final volume. Show that

$$q^2 = 32\pi^2\varepsilon_0 PR\,(R^3 - R_0^3).$$

(Hint : Consider the forces acting on a small area of the charged bubble. These are due to (a) gas pressure, (b) atmospheric pressure, (c) electrostatic stress

§ 3.138

➢ **CONCEPT**

The concept is similar to previous problem.

i.e., $$A = -\Delta U$$

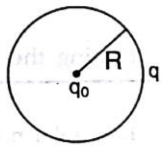


Fig. 3.138A

The electrical potential energy of a point charge q_0 and spherical shell of radius R is (shown in fig 3.138 A)

$U =$ self potential energy of shell + interaction energy of shell and point charge.

$$= \frac{q^2}{8\pi\varepsilon_0 R} + \frac{q\,q_0}{4\pi\varepsilon_0 R}$$

SOLUTION :

Initial potential energy of system is

$$U_i = \frac{q^2}{8\pi\varepsilon_0 R_1} + \frac{q\,q_0}{4\pi\varepsilon_0 R_1}$$

Similarly, final potential energy of system is

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2} + \frac{q\,q_0}{4\pi\varepsilon_0 R_2}$$

$$\therefore \quad A = -\Delta U = -(U_f - U_i)$$

$$\because \quad A = U_i - U_f \qquad ...(i)$$

On putting the values of U_i and U_f, in Eq. (i) we get

$$A = \frac{q\,(q_0 + \frac{q}{2})}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

YOUR STEP

An uncharged, conducting spherical shell of mass m floats with one fourth of its volume submerged in a liquid of dielectric constant K. To what potential must the sphere be charged to float half submerged?

(Hint : Assume the electric field of the half submerged, charged shell to be a purely radial field, and show later that the sum of $\sigma + \sigma p$ over the spherical surface is such as to justify this assumption.)

$$\left[\frac{4mg}{2\pi\varepsilon_0(k-1)}\right]^{\frac{1}{2}}$$

§ 3.139

➢ **CONCEPT**

Since, electrical force is conservative in nature.

$$\therefore \quad F = \frac{-\partial U}{\partial r}$$

SOLUTION :

Here
$$U = \frac{q}{8\pi\varepsilon_0 r}$$

$$\therefore \quad F = \frac{-\partial U}{\partial r} = \frac{-\partial}{\partial r}\left(\frac{q}{8\pi\varepsilon_0 r}\right)$$

or
$$F = \frac{-q}{8\pi\varepsilon_0}\frac{\partial (r^{-1})}{\partial r} = \frac{q^2}{8\pi\varepsilon_0 r^2}$$

$\therefore$ Force per unit area is
$$= \frac{F}{4\pi r^2} = \frac{q^2}{8\pi\varepsilon_0 r^2 \times 4\pi r^2} = \frac{\sigma^2 (4\pi r^2)^2}{8\pi\varepsilon_0 r^2 \times 4\pi r^2} = \frac{\sigma^2}{2\varepsilon_0}$$

YOUR STEP

(a) Calculate the energy density of the electric field at a distance r from an electron (presumed to be a particle) at rest.

(b) Assume now that the electron is not a point but a sphere of radius R over whose surface the electron charge is uniformly distributed. Determine the energy associated with the external electric field in vacuum of the electron as a function of R.

(c) If you now associate this energy with the mass of electron, you can, (using $E_0 = mc^2$), calculate a value of R. Evalaute this radius numerically, it is often called the classical radius of electron.

$$\left\{\text{(a) } \frac{e^2}{32\pi^2\varepsilon_0 r^4}, \text{ (b) } \frac{e^2}{8\pi\varepsilon_0 R}, \text{ (c) } 1.40 \times 10^{-15}\text{ m}\right\}$$

§ 3.140

➢ **CONCEPT**

Work done by external agent is $A_{ext} = \Delta U$

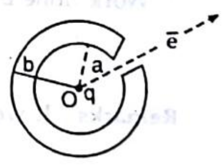


Fig. 3.140A

SOLUTION : Step I : Determine the induced charges on different surfaces : When point charge q is placed at the centre. Then $-q$ and $+q$ charges are induced at inner and outer surface of the conducting layer (shown in fig. 3.140 A).

In other words we can say that when the point charge is at the centre, the layer behaves as two concentric spheres A and B having respective charges $-q$ and $+q$. The equivalent system is shown in fig. 3.140 C.

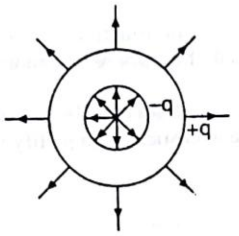


Fig. 3.140B

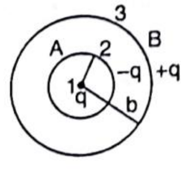


Fig. 3.140C

The total electrical energy of system is

$$U_i = U_{12} + U_{23} + U_2 + U_3$$

Here U_{12} = Electrical interaction energy of point charge on hollow sphere A

$$= \frac{q(-q)}{4\pi\varepsilon_0 a} = \frac{-q^2}{4\pi\varepsilon_0 a}$$

U_{23} = Electrical interaction energy between hollow spheres A and B.

= (Electric potential due to B on A) $\times$ charge on A

$$= \frac{q(-q)}{4\pi\varepsilon_0 a} = \frac{-q^2}{4\pi\varepsilon_0 b}$$

Similarly, $$U_{13} = \frac{q^2}{4\pi\varepsilon_0 b}$$

U_2 = Self potential energy of hollow sphere A.

$$= \frac{(-q)^2}{8\pi\varepsilon_0 Q} = \frac{q^2}{8\pi\varepsilon_0 a}$$

Similarly, $$U_3 = \frac{q^2}{8\pi\varepsilon_0 b}$$

$\therefore$ $$U_i = \frac{-q^2}{4\pi\varepsilon_0 a} - \frac{q^2}{4\pi\varepsilon_0 b} + \frac{q^2}{4\pi\varepsilon_0 b} + \frac{q^2}{8\pi\varepsilon_0 a} + \frac{q^2}{8\pi\varepsilon_0 b}$$

$$= \frac{-q^2}{8\pi\varepsilon_0 a} + \frac{q^2}{8\pi\varepsilon_0 b} = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

Step II : Discuss the situtation, when charge q is shifted at infinity.

When the point charge q is shifted to infinity, induced charges are disappeared.

$\therefore$ Total electric potential energy of system is $U_f = 0$

$\therefore$ Work done by external agent is $A_{ex} = \Delta U = U_f - U_i$

$$= 0 - \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Remarks : It we try to calculate work done by the formula,

$$A_{ext} = q(V_\infty - V_c)$$

where V_c = electric potential at the centre.

V_∞ = electric potential at infinity = 0

The answer would be different and incorrect. The fact behind this incorrectness is that this approach does not take into account the additional work formed by electric forces upon the charge in the configuration of the charge on inner and outer surface of the conducting layer.

YOUR STEP

A small cork ball A of mass m is suspended by a thread of length l. Another ball B is fixed at a distance l from point of suspension and distance $\frac{l}{2}$ from thread when it is vertical, as shown in fig. 3.140D. Balls A and B have charges $(+q)$ each. Ball A is held by an external force such that the thread remains vertical when ball A is released from rest, thread deflects through a maximum angle of $\beta = 30°$, calculate m in terms of other parameters.

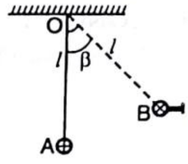


Fig. 3.140D

$$\left\{ m = \frac{q^2}{2\pi\varepsilon_0 g l^2} \frac{(1-\sqrt{2-\sqrt{3}})}{(2-\sqrt{3})^{3/2}} \right\}$$

§ 3.141

➢ **CONCEPT**

(a) When the separation between plates of capacitor is increased, electric charges of plates remain the same (in the absence of battery).

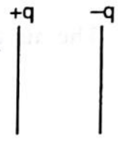


Fig. 3.141A

The initial capacitance of capacitor is $C_i = \frac{\varepsilon_0 S}{x_1}$

The initial energy stored in the capacitor is

$$U_i = \frac{q^2}{2C_i} = \frac{q^2 x_1}{2\varepsilon_0 S}$$

Similarly, final capacitance of capacitor is

$$C_f = \frac{\varepsilon_0 S}{x_2}$$

$$\therefore \quad U_f = \frac{q^2}{2C_f} = \frac{q^2 x_2}{2\varepsilon_0 S}$$

$$\therefore \quad A_{ext} = \Delta U = U_f - U_i = \frac{q^2}{2\varepsilon_0 S}(x_2 - x_1)$$

(b) When the separation between plates of capacitor is increased with battery attached, the potential difference between plates of capacitor remains constant. This potential difference is equal to the emf of the cell.

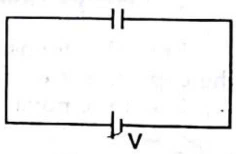


Fig. 3.141B

The initial capacitance is $\quad C_i = \frac{\varepsilon_0 S}{x_1}$

Similarly $\quad C_f = \frac{\varepsilon_0 S}{x_2}$

$$\therefore \quad U_i = \frac{1}{2} C_i V^2$$

and $\quad U_f = \frac{1}{2} C_f V^2$

$$\therefore \quad A_{ext} = \Delta U = U_f - U_i$$

On putting the values, we get $\quad A_{ext} = \frac{\varepsilon_0 S\, V^2 (x_2 - x_1)}{2 x_1 x_2}$

YOUR STEP

A parallel plate capacitor of capacity C is connected with a battery of emf V_0 to charge q. The separation between plates is x. The distance between the plates is changed to $x + dx$

(a) What is change in the capacity?

(b) What is the attraction force between plates?

$$\left\{ \text{(a)} -\frac{C}{x}dx, \text{ (b) } \frac{1}{2}\frac{CV_0^2}{x} \right\}$$

§ 3.142

➢ **CONCEPT**

If the space between plates is filled by a number of media, then

$$C = \frac{\varepsilon_0 S}{\frac{t_1}{\varepsilon_1} + \frac{t_2}{\varepsilon_2} + \ldots}$$

SOLUTION : Step I : When plate is not introduced then

$$C = \frac{\varepsilon_0 S}{d} = 20 \times 10^{-9}\ \text{F}$$

According to problem, the thickness of introducing plate is

$$t_2 = \eta d$$

The air gap is $\quad t_2 = d - \eta d = (1 - \eta)$

$$C_1 = \frac{\varepsilon_0 S}{t_1 + \frac{t_2}{\varepsilon}}$$

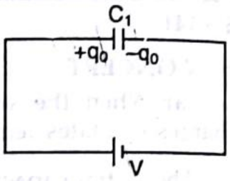


Fig. 3.142A

where ε is dielectric constant of introducing plate.

$$\therefore \quad C_1 = \frac{\varepsilon_0 S}{d\,(1-\eta) + \frac{\eta d}{\varepsilon}} = \frac{\varepsilon_0\, \varepsilon S}{\varepsilon d - \eta \varepsilon d + \eta d}$$

$$= \frac{\varepsilon_0 \varepsilon S}{d\,(\varepsilon - \eta\varepsilon + \eta)} = \frac{C\varepsilon}{(\varepsilon - \eta\varepsilon + \eta)}$$

The corresponding circuit is shown in Fig. 3.142 A.

$$\therefore \quad q_0 = C_1 V$$

The energy stored on capacitor is $U_i = \dfrac{q_0^2}{2C_1}$

Step II : Discuss the problem after removing the battery : When battery is disconnected from the capacitor, electric charge on plates of capacitor remains conserved.

Due to removal of the plate, capacity of capacitor changes. The new capacity of the capacitor is $C = \dfrac{\varepsilon_0 S}{d}$

Now energy stored on the capacitor is $U_f = \dfrac{q_0^2}{2C}$

$$\therefore \quad \Delta U = U_f - U_i = \frac{q_0^2}{2}\left(\frac{1}{C} - \frac{1}{C_1}\right)$$

The work done by external agent is $A_{ext} = \Delta U = \dfrac{q_0^2}{2}\left(\dfrac{1}{C} - \dfrac{1}{C_1}\right)$

On putting the value of C_1, we get $A_{ext} = \dfrac{q_0^2}{2}\left(\dfrac{1}{C} - \dfrac{\varepsilon - \eta\varepsilon + \eta}{\varepsilon C}\right)$

$$= \frac{q_0^2}{2C}\left(\frac{\varepsilon - \varepsilon + \eta\varepsilon - \eta}{\varepsilon}\right)$$

or $$A_{ext} = \frac{q_0^2}{2C}\left(\eta - \frac{\eta}{\varepsilon}\right) = \frac{q_2^2\,\eta}{2c}\left(1 - \frac{1}{\varepsilon}\right)$$

But $$q_0 = C_1 V = \frac{C\varepsilon V}{(\varepsilon - \eta\varepsilon + \eta)}$$

$$q_0 = \frac{CV}{\left(1 - \eta + \frac{\eta}{\varepsilon}\right)}$$

$\therefore$ $$A_{ext} = \frac{C^2V^2}{\left(1 - \eta + \frac{\eta}{\varepsilon}\right)^2} \frac{\eta}{2C}\left(1 - \frac{1}{\varepsilon}\right)$$

$$= \frac{C\eta V^2}{2} \frac{1}{\left(1 - \eta + \frac{\eta}{\varepsilon}\right)^2}\left(1 - \frac{1}{\varepsilon}\right)$$

(a) For conductor, $\varepsilon = \infty$

$\therefore$ $$A_{ext} = \frac{C\eta V^2}{2(1-\eta)^2} = 1.5 \text{ mJ}$$

(b) $\because$ $$A_{ext} = \frac{C\eta V^2}{2\left(1 - \eta + \frac{\eta}{\varepsilon}\right)^2}\left(1 - \frac{1}{\varepsilon}\right) = \frac{CV^2\eta\varepsilon(\varepsilon - 1)}{2[\varepsilon - \eta(\varepsilon - 1)]^2}$$

For glass $\varepsilon = 6$

On putting the values, we get $A_{ext} = 0.8$ mJ

YOUR STEP

A thin conducting plate is introduced in halfway between the plates of a parallel plates capacitor (shown in fig 3.142B).

(a) Calculate the capacity before the plate was introduced.

(b) Find the capacity after the plate was inserted.

(c) What does the value of capacity become when the thin plate and the upper plate are connected by a metallic wire?

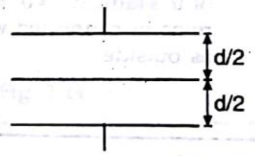


Fig. 3.142B

$$\left\{(a)\ \frac{\varepsilon_0 A}{d},\ (b)\ \frac{2\varepsilon_0 A}{d},\ (c)\ \frac{2\varepsilon_0 A}{d}\right\}$$

§ 3.143

➢ **CONCEPT**

$$F = \frac{dU}{dx} \quad \text{(when battery remains connected)}$$

$$F = -\frac{dU}{dx} \quad \text{(when battery is disconnected)}$$

SOLUTION : Let at an instant a thickness x of gap between plates is filled with water shown in fig. 3.143 A.

Fig. 3.143A

$$C = \frac{\varepsilon_0 S}{\frac{x}{\varepsilon} + (d - x)} = \frac{\varepsilon_0 S}{x\left(\frac{1}{\varepsilon} - 1\right) + d}$$

$\therefore$ $$U = \frac{1}{2}CV^2$$

or $$U = \frac{1}{2}\frac{\varepsilon_0 S\varepsilon V^2}{x(1-\varepsilon) + \varepsilon d}$$

$\therefore$ $$F = \frac{dU}{dx} \quad \text{(since, battery remains connected)}$$

$\therefore$ $$F = -\frac{1}{2}\frac{\varepsilon_0 S\varepsilon V^2 (1-\varepsilon)}{\{x(1-\varepsilon)+\varepsilon d\}^2}$$

When water is filled in the gap between plates of capacitor $x = d$

$\therefore$ $$F = \frac{\frac{1}{2}\varepsilon_0 S\varepsilon V^2(\varepsilon-1)}{\{d(1-\varepsilon)+\varepsilon d\}^2}$$

$$= \frac{\frac{1}{2}\varepsilon_0 S\varepsilon V^2(\varepsilon-1)}{\{d-\varepsilon d+\varepsilon d\}^2} = \frac{\frac{1}{2}\varepsilon_0 S\varepsilon V^2(\varepsilon-1)}{d^2}$$

$\therefore$ Excess pressure is $$\Delta P = \frac{F}{S} = \frac{\varepsilon_0\varepsilon V^2(\varepsilon-1)}{2d^2}$$

For water $\varepsilon = 81$

On putting the values, we get $\Delta p = 7.17 \text{ kPa} = 0.07 \text{ atm}$

YOUR STEP

A parallel plate capacitor is placed in cylindrical tank filled with a liquid of dielectric constant K. The area of cross-section of the tank is A and height of the liquid is equal to the length of the square plate of area l^2. The separation between plates is d. A small hole of area a is opened at the bottom of the tank at $t = 0$. Find the current in the circuit as a function of time. If the capacitor in the process remains connected with a battery of emf E. Assume the level of liquid in the capacitor remains same as outside.

$$\left\{\frac{\varepsilon_0 lE}{Ad}(K-1)\,a\left(\sqrt{2gl}-\frac{a}{A}gt\right)\right\}$$

§ 3.144

➢ CONCEPT

If capacitor is charged upto surface charge density σ and then disconnected from battery. The charge of capacitor will remain constant during whole process. When liquid rises, capacitance of the capacitor changes.

But $$U = \frac{q_0^2}{2C}$$

From this relation, energy stored in capacitor changes without changing charge on the plates of capacitor. When the liquid rises between gap, gravitational potential energy increases but energy stored in capacitor decreases.

At an instant, total energy remains constant.

SOLUTION :

The capacitance of capacitor before rising the liquid is

$$C_0 = \frac{\varepsilon_0 S}{d}$$

Total charge on capacitor is $q = \sigma S$

$\therefore$ Energy stored on capacitor is $$U_i = \frac{q^2}{2C_0}$$

In the final state, the system is equivalent to two capacitors in series.

$$C_1 = \frac{\varepsilon_0 S}{d-h}, \qquad C_2 = \frac{\varepsilon_0\varepsilon S}{h}$$

Here ε is relative permittivity of water.

$\therefore$ Net capacitance after rising the liquid is

$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{\varepsilon_0 \varepsilon S}{h + \varepsilon(d - h)}$$

Total energy of system remains constant.

$\because$ E = Electrical potential energy + gravitational potential energy = constant

$$\because \; E = \frac{q^2}{2C} + mg\frac{h}{2} = \text{constant}$$

$$= \frac{q^2}{2}\left\{\frac{h + \varepsilon(d-h)}{\varepsilon_0 \varepsilon S}\right\} + \frac{\rho S g h^2}{2} = \text{constant}$$

$$\therefore \quad \frac{dE}{dh} = \frac{q^2}{2\varepsilon_0 \varepsilon S}(1 - \varepsilon) + \rho S g h = 0 \quad \text{or} \quad \rho S g h = \frac{q^2}{2\varepsilon_0 \varepsilon S}(\varepsilon - 1)$$

$$\text{or} \quad \rho\, Sgh = \frac{(\sigma' S)^2}{2\,\varepsilon_0\, \varepsilon S}(\varepsilon - 1) \quad \text{or} \quad \rho\, Sgh = \frac{\sigma^2 S^2 (\varepsilon - 1)}{2\varepsilon_0 \varepsilon S} \quad \therefore \quad h = \frac{\sigma^2(\varepsilon - 1)}{2\varepsilon_0 g \rho \varepsilon}$$

YOUR STEP

A large vessel is filled with a liquid of dielectric constant ε and density ρ. Two vertical plates touch the surface of liquid (shown in 3.144A). The dimensions of the plates are a and b, the distance between them is d. The plates have been charged by applying a voltage ϕ_0 and then disconnected from the voltage source. To what height will the liquid rise? Ignore capillary effects. Also, solve the same when voltage source remains connected.

$$\left\{ \text{(a)}\; \frac{-a}{2(\varepsilon - 1)} + \sqrt{\frac{a^2}{4(\varepsilon - 1)^2} + \frac{\varepsilon_0\, a \phi_0^2}{\rho g d^2}}\;, \; \text{(b)}\; \frac{(\varepsilon - 1)\,\varepsilon_0\,\phi_0^2}{\rho g d^2} \right\}$$

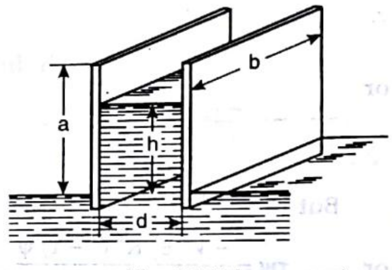


Fig. 3.144A

§ 3.145

➢ CONCEPT

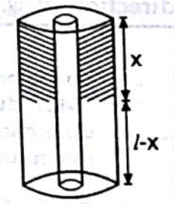


Fig. 3.145A

Step I : Determine the capacity at an instant. Let at an instant, x be length of dielectric inside the capacitor.

$$\therefore \quad C = C_{\text{dielectric}} + C_{\text{air}}$$

$$= \frac{2\pi\varepsilon_0 \varepsilon R x}{d} + \frac{2\pi\varepsilon_0 R}{d}(l - x) \qquad (\because d << r)$$

Step II : Determine the energy of the capacitor.

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{2\pi\varepsilon_0 R}{d}\right)V^2\,[\varepsilon x + (l - x)]$$

But
$$F = \frac{dU}{dx} = \varepsilon_0(\varepsilon - 1)\frac{\pi R V^2}{d}$$

YOUR STEP

In the fig. 3.145B, the capacitors have plate area. $A = l \times b$, and separation d. If the slab is displaced slightly for distance find the time period of oscillation. Is it simple harmonic motion? Given mass of dielectric slab is m.

$$\text{Time period} = \sqrt{\frac{32\, md}{\varepsilon_0\, b\,(K - 1)\,\xi^2}}$$

motion is oscillatory but not S.H.M.

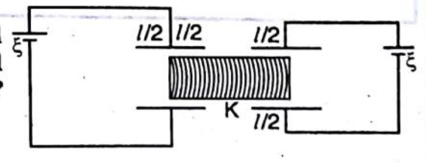


Fig. 3.145B

§ 3.146

➢ **CONCEPT**

In the case under consideration, the system will be equivalent to two capacitors (one air and other containing dielectric slab) in parallel, *i.e.*,

$$C = C_A + C_D$$

$$= \frac{\varepsilon_0 \psi R^2}{2d} + \frac{\varepsilon_0 \varepsilon (\pi - \psi) R^2}{2d}$$

$$= \frac{\varepsilon_0 \psi R^2}{2d} + \frac{\varepsilon_0 \varepsilon \pi R^2}{2d} - \frac{\varepsilon_0 \varepsilon \psi R^2}{2d}$$

$$= \frac{\varepsilon_0 R^2}{2d} \psi (1 - \varepsilon) + \frac{\varepsilon_0 \varepsilon \pi R^2}{2d}$$

Let C_0 be the initial capacitance of the capacitor (dielectric slab is completely inside).

$$C_0 = \frac{\varepsilon \varepsilon_0 (\pi R^2)}{2d}$$

$\therefore$
$$\Delta U = U_f - U_i = \frac{1}{2} C V^2 - \frac{1}{2} C_0 V^2 = \frac{V^2}{2} (C - C_0)$$

or
$$\Delta U = \frac{V^2}{2} \left\{ \frac{\varepsilon_0 R^2 \psi (1 - \varepsilon)}{2d} + \frac{\varepsilon_0 \varepsilon \pi R^2}{2d} - \frac{\varepsilon \varepsilon_0 \pi R^2}{2d} \right\}$$

$$= -\frac{V^2 \varepsilon_0 R^2 (\varepsilon - 1) \psi}{4d}$$

But
$$A_{ext} = \Delta U$$

or $\tau \psi = \dfrac{- V^2 \varepsilon_0 R^2 (\varepsilon - 1) \psi}{4d}$ $\quad \therefore \quad$ $\tau = \dfrac{- V^2 \varepsilon_0 R^2 (\varepsilon - 1)}{4d}$

The negative sign indicates that the moment of the force is acting oppositely to the positive direction of ψ. This moment has a tendency to pull the dielectric slab inside the capacitor.

YOUR STEP

1. The capacity of a variable air capacitor changes linearly from 50 to 364pF during a rotation from 0° to 180°. When set at 75°, a potential difference of 400V is maintained across the capacitor. What is magnitude of electrostatic torque experienced by the capacitor?

2. Two parallel plate capacitors with area A are connected through a conducting spring of natural length l in series shown in fig. 3.146A plate P and S have fixed positions at separation d. Now plates are connected by a battery of emf ξ as shown. If extension in spring in equilibrium is equal to separation between plates. Find spring constant k.

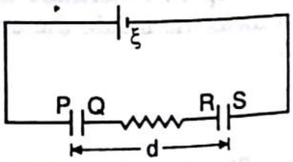


Fig. 3.146A

$$\left\{ \textbf{1. } 13.96 \times 10^{-8} \text{ joule} \quad \textbf{2. } \frac{27 A \varepsilon_0 \xi^2}{8(d - l)^3} \right\}$$

ELECTRIC CURRENT

§ 3.147

➢ **CONCEPT**

The electric field at the surface of a long cylinder is

$$E = \frac{\lambda}{2\pi\varepsilon_0 a} \quad ...(i)$$

where a is the radius of cylinder. Also electric current is

$$I = \frac{dq}{dt}$$

SOLUTION : The electric charge on considered element of length dx is

$$dq = \lambda dx$$

$$\therefore \quad I = \frac{dq}{dt} = \lambda\frac{dx}{dt} = \lambda V \quad ...(ii)$$

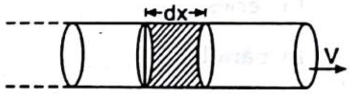


Fig. 3.147A

From equation (i) and (ii), we get $I = 2\pi\varepsilon_0 aEV$

On putting the values, we get, $I = 0.5 \times 10^{-6} A = 0.5\mu A$

YOUR STEP

A metallic sphere of radius R is earthed. A point charge q starts to move towards the metallic sphere with constant speed v_0. Find the electric current in the earthing wire at the instant when the point charge is at distance r_0 ($r_0 > R$) from the centre of the metallic sphere.

$$\left\{\frac{qRv_0}{r_0^2}\right\}$$

§ 3.148

➢ **CONCEPT**

Let at an instant, length x of cylinder is inside the water.
The capacity of capacitor at this instant is

$$C = C_{water} + C_{air}$$

$$= \frac{2\pi\varepsilon_0\varepsilon r x}{d} + \frac{2\pi\varepsilon_0 r(l-x)}{d} \quad (\because d << r)$$

$$\therefore \quad C = \frac{2\pi\varepsilon_0 r}{d}[(\varepsilon - 1)x + l]$$

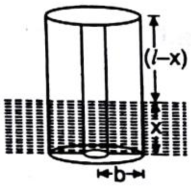


Fig. 3.148A

SOLUION : Draw the circuit

The circuit is shown in Fig. 3.148 B at an instant.

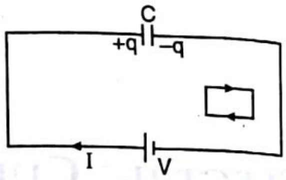
According to loop rule, $V - \frac{q}{C} = 0$

$$\therefore \quad q = CV$$

$$I = \frac{dq}{dt} = \frac{VdC}{dt}$$

$\therefore$
$$= \frac{V\,2\pi\varepsilon_0 r}{d}(\varepsilon - 1)\frac{dx}{dt}$$

$$= \frac{2\pi\varepsilon_0 r V}{d}(\varepsilon - 1)\,v$$



On putting the values, we get $I = 0.11 \times 10^{-6}\ \text{A} = 0.11\mu\text{A}$

Fig. 3.148B

YOUR STEP

Two square metallic plates of side $a = 1$m are placed $d = 8.85$ mm apart, Like a parallel plate capacitor, in air, in such a way that their surfaces are normal to oil surface in a tank filled with that insulating oil ($\varepsilon_r = 11$). The plates are connected to a battery of emf $V = 500$V as shown in fig. 3.148A. The plates are lowered vertically into the oil at a speed of $v_0 = 10^{-3}$ m/s. Neglecting resistance of connecting wires, calculate the current drawn from battery during the process. $\{5 \times 10^{-9}\ \text{A}\}$

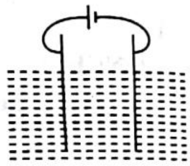


Fig. 3.148C

§ 3.149

➢ **CONCEPT**

In series, $R = R_1 + R_2$

In parallel, $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

Also, $R_t = R_0\{1 + \alpha t\}$

SOLUTION : According to problem, $R_{20} = \eta R_{10}$

Let resistances of first and second conductors at temperature t is R_1 and R_2 respectively.

$\therefore$ $R_1 = R_{10}\{1 + \alpha_1 t\}$

And $R_2 = R_{20}\{1 + \alpha_2 t\}$

(a) $R = R_1 + R_2$

or $R_0(1 + \alpha t) = R_{10}(1 + \alpha_1 t) + R_{20}(1 + \alpha_2 t)$

or $(R_{10} + R_{20})(1 + \alpha t) = R_{10} + R_{20} + R_{10}\alpha_1 t + R_{20}\alpha_2 t$

or $(R_{10} + R_{20}) + (R_{10} + R_{20})\alpha t = (R_{10} + R_{20}) + R_{10}\alpha_1 t + R_{20}\alpha_2 t$

or $(R_{10} + R_{20})\alpha = R_{10}\alpha_1 + R_{20}\alpha_2$

But $R_{20} = \eta R_{10}$

$\therefore$ $(R_{10} + \eta R_{10})\alpha = R_{10}\alpha_1 + \eta R_{10}\alpha_2$

or $(1 + \eta)\alpha = (\alpha_1 + \eta\alpha_2)$

$\therefore$
$$\alpha = \left(\frac{\alpha_1 + \eta\alpha_2}{1 + \eta}\right)$$

(b) In parallel combination $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ or $R = \dfrac{R_1 R_2}{R_1 + R_2}$

Also
$$R_0 = \frac{R_{10}R_{20}}{R_{10} + R_{20}}$$

$\therefore$
$$R = \frac{R_1 R_2}{R_1 + R_2}$$

or
$$R_0(1 + \alpha t) = \frac{R_{10}(1 + \alpha_1 t)\,R_{20}(1 + \alpha_2 t)}{R_{10}(1 + \alpha_1 t) + R_{20}(1 + \alpha_2 t)}$$

or $$(R_{10}+R_{20})(1+\alpha t)=\frac{R_{10}R_{20}(1+\alpha_1 t)(1+\alpha_2 t)}{R_{10}+R_{20}+R_{10}\alpha_1 t+R_{20}\alpha_2 t}$$

But $$R_{20}=\eta R_{10}$$

or $$(1+\alpha t)(R_{10}+\eta R_{10})=\frac{\eta R_{10}^2(1+\alpha_2 t+\alpha_1 t+\alpha_1\alpha_2 t^2)}{(R_{10}+\eta R_{10})+R_{10}\alpha_1 t+\eta R_{10}\alpha_2 t}$$

But $\alpha_1\alpha_2 t^2$ may be neglected $\because$ $$\alpha=\frac{\eta\alpha_1+\alpha_2}{1+\eta}$$

YOUR STEP

Temperature co-efficients of resistance of two wires A and B are $\alpha_1=3\times10^{-3}/°C$ and $\alpha_2=6\times10^{-3}/°C$ respectively and that of their series combination is $\alpha_S=5\times10^{-3}/°C$. Calculate the temperature co-efficient of resistance of a circuit segment consisting of these two wires when they are connected in parallel

$\{4\times10^{-3}/°C\}$

§ 3.150

➢ **CONCEPT**

For circuit problem discussion, points of same potential are taken as a single point.

SOLUTION :

(a) The distribution of current is shown in fig 3.150A.

$$\phi_1-\phi_2=\phi_1-\phi_5=\phi_1-\phi_4$$

$\therefore$ $$\phi_2=\phi_5=\phi_4 \qquad \text{...(i)}$$

Also, $$\phi_7-\phi_6=\phi_7-\phi_8=\phi_7-\phi_3$$

$\therefore$ $$\phi_6=\phi_8=\phi_3 \qquad \text{...(ii)}$$

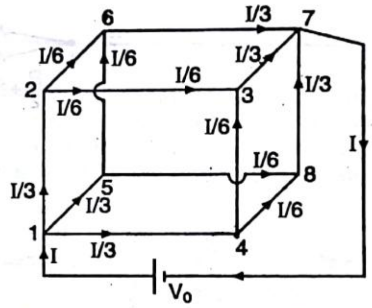


Fig. 3.150A

The circuit may be represented as fig 3.150B.

$\therefore$ $$R_{17}=\frac{R}{3}+\frac{R}{6}+\frac{R}{3}=\frac{2R+R+2R}{6}$$

$\therefore$ $$R_{17}=\frac{5R}{6}$$

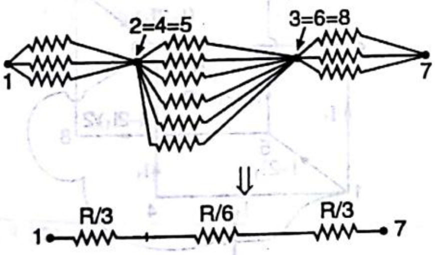


Fig. 3.150B

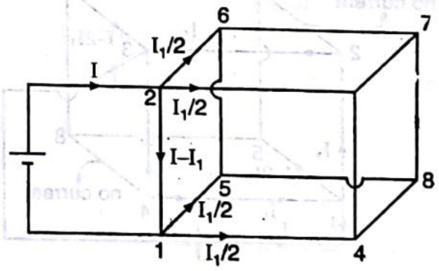


Fig. 3.150C

(b) The distribution of current is shown in fig. 3.150C.

Here $$\phi_2-\phi_6=\phi_2-\phi_3$$

$\therefore$ $$\phi_6=\phi_3$$

Also $$\phi_1-\phi_5=\phi_1-\phi_4$$

$\therefore$ $$\phi_5=\phi_4$$

The circuit may be represented as fig 3.150D.

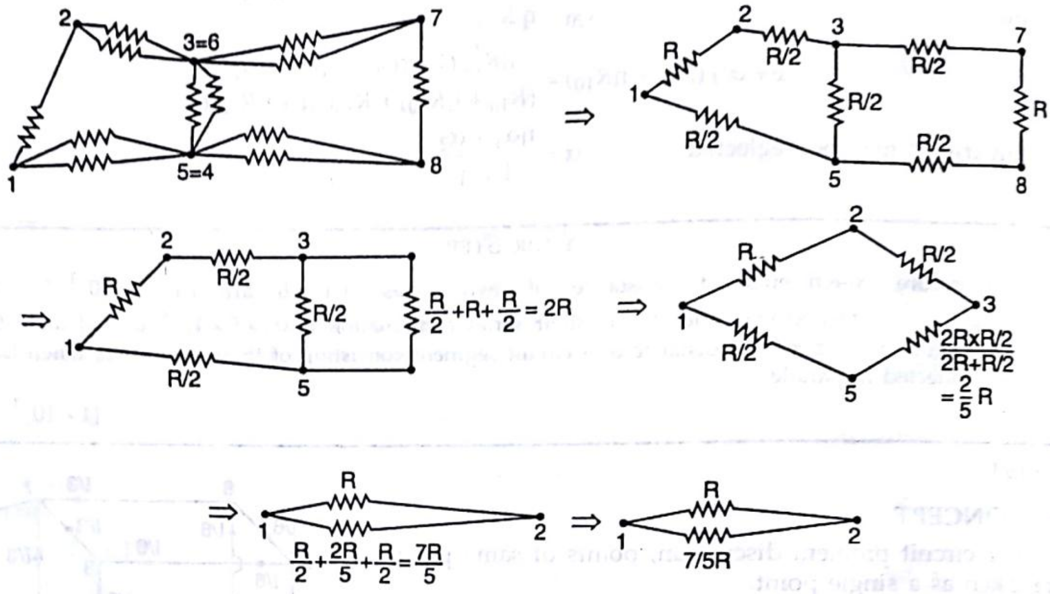


Fig. 3.150D

$$\therefore \qquad R_{12} = \frac{\left(\frac{7}{5}R\right)(R)}{\frac{7}{5}R + R} = \frac{7}{12}R$$

(c) The distribution current is shown in fig. 3.150E.

Here $\qquad \phi_1 - \phi_4 = \phi_1 - \phi_2$

$\therefore \qquad \phi_4 = \phi_2$

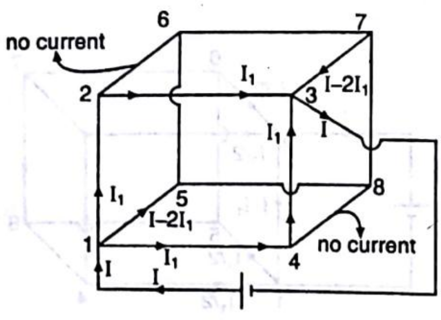


Fig. 3.150E

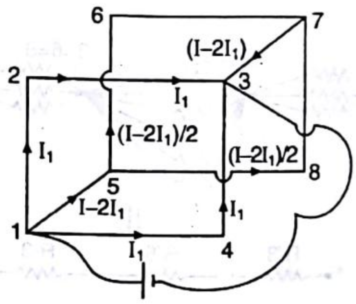


Fig. 3.150F

No current wires may be removed from circuit. Now curcuit is shown in fig. 3.150F.

$$\phi_1 - \phi_4 = \phi_1 - \phi_2$$

$\therefore \qquad \phi_2 = \phi_4$

Also $\qquad \phi_5 - \phi_8 = \phi_5 - \phi_6$

$\therefore \qquad \phi_8 = \phi_6$

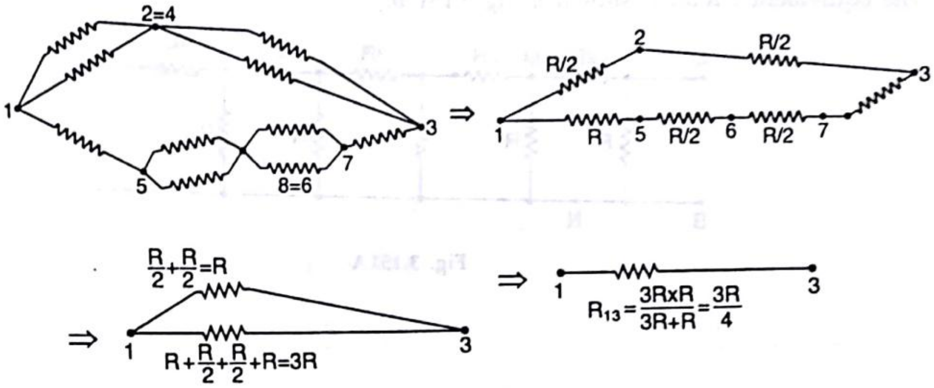


Fig. 3.150G

$$R_{13}=\frac{3R\times R}{3R+R}=\frac{3R}{4}$$

YOUR STEP

1. Twelve straight uniform wires of equal length and equal resistance r are joined to form the edges of a cube of side a. Electric current enters the system at one vertex and leaves from a point on one of the edges meeting at the opposite vertex and at a distance x $(0 \le x < 1)$ from it, the resistance of the system is a quadratic function of x. If x is chosen so as to make the resistance maximum, show that $x=\frac{2}{5}$ and the resistance is $\frac{9r}{10}$.
2. Eight identical resistances r each are connected along edges of a pyramid having square base $ABCD$ as shown in fig 3.150H. Calculate equivalent resistance between (i) A and D (ii) A and O.

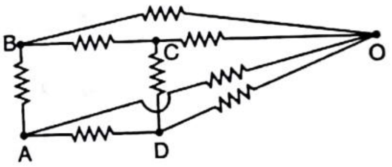


Fig. 3.150H

3. Fourteen identical resistors of each of resistance R_0 are connected as shown in fig. 3.151I. Calculate equivalent resistance between points A and E.

$\left\{2. \text{ (i) } \frac{8r}{15}, \text{ (ii) } \frac{7r}{15}, \ 3.\ 1.2\ R_0\right\}$

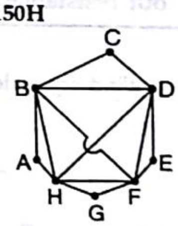


Fig. 3.150I

§ 3.151

➢ CONCEPT

If equivalent resistance between points A and B is $R_{eq}=R_x$, then, the total resistance between point A and B be independent of the number of cells.

SOLUTION : Since, equivalent resistance between points A and B is independent of number of cells. So, equivalent resistance between points A and B is same as that of right part of circuit points between M and N (Shown in fig. 3.151A)

$$\therefore \qquad R_{AB}=R_{MN}=R_x$$

The equivalent circuit is shown in fig 3.151 B.

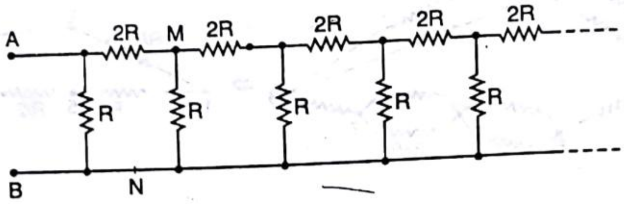
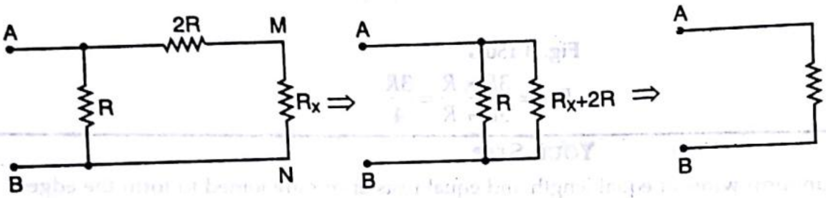


Fig. 3.151A



$$\frac{R(R_x+2R)}{R+(R_x+2R)}=\frac{RR_x+2R^2}{3R+R_x}$$

Fig. 3.151B

$\therefore$ $$R_{AB}=\frac{RR_x+2R^2}{3R+R_x} \quad \text{But } R_{AB}=R_x$$

$\therefore$ $$R_x=\frac{RR_x+2R^2}{3R+R_x} \quad \text{or} \quad R_x^2+2RR_x-2R^2=0$$

or $$R_x=-\frac{2R\pm\sqrt{(2R)^2-4(1)(-2)R^2}}{2}$$

$$=-\frac{2R\pm 2R\sqrt{1+2}}{2}=-R\pm R\sqrt{3}$$

But resistance is always positive. $\therefore$ $R_x=R(\sqrt{3}-1)$

YOUR STEP

1. Find equivalent resistance between point A and B. (Fig. 3.151C)

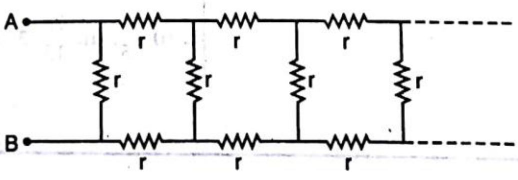


Fig. 3.151C

2. Determine the resistance R_{AB} between point A and B of the frame formed by nine identical wires of resistance R each, shown in fig. 3.151D.

$\left\{1.\ (\sqrt{3}-1)\,r\ \ 2.\ \frac{15}{11}R\right\}$

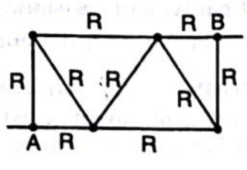


Fig. 3.151D

§ 3.152

➢ **CONCEPT**

The problem is similar to previous problem.

SOLUTION :

Since, circuit consists of infinite number of similar section. So, equivalent resistance of right part of circuit between points M and N is same as that of between point A and B.

The equivalent circuit is shown in fig 3.152 B.

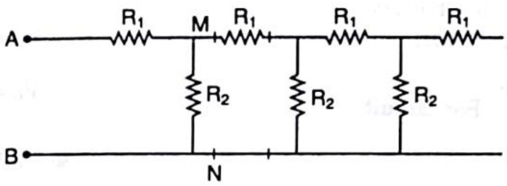


Fig. 3.152A

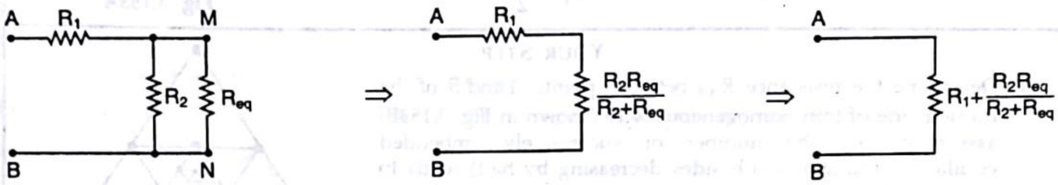


Fig. 3.152B

$\therefore$ $$R_{AB} = R_1 + \frac{R_2 . R_{eq}}{R_2 + R_{eq}}$$

But $$R_{AB} = R_{eq}$$

$\therefore$ $$R_{eq} = R_1 + \frac{R_2 R_{eq}}{R_2 + R_{eq}}$$

or $$R_{eq} = \frac{R_1 R_2 + R_1 R_{eq} + R_2 R_{eq}}{R_2 + R_{eq}}$$

or $$R_2 R_{eq} + R_{eq}^2 = R_1 R_2 + R_1 R_{eq} + R_2 R_{eq}$$

or $$R_{eq}^2 - R_1 R_{eq} - R_1 R_2 = 0$$

$\therefore$ $$R_{eq} = \frac{R_1 \pm \sqrt{R_1^2 + 4R_1 R_2}}{2}$$

But resistance is always positive.

$\therefore$ $$R_{eq} = \frac{R_1}{2}\left(1 + \sqrt{1 + \frac{4R_2}{R_1}}\right) = 6\Omega$$

YOUR STEP

The circuit diagram shown in fig. 3.152C. consists of a very large number of elements. The resistances of the resistors in each subsequent element differ by a factor of $K = \frac{1}{2}$ from the resistances of the resistors in the previous elements. Determine the resistance R_{AB} between points A and B if the resistances of the first element are R_1 and R_2..

$$\left\{ R_{AB} = \frac{R_1 - R_2 + \sqrt{R_1^2 + R_2^2 + 6R_1 R_2}}{2} \right\}$$

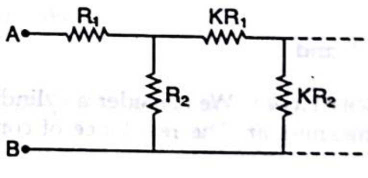


Fig. 3.152C

§ 3.153

➢ **CONCEPT**

We assume that there is a battery of emf V_0 connected between points A and B. If current supplied by battery is I_0 ... from symmetry of figure, current in all wires is the same. This current is equal to half of current supplied by the battery.

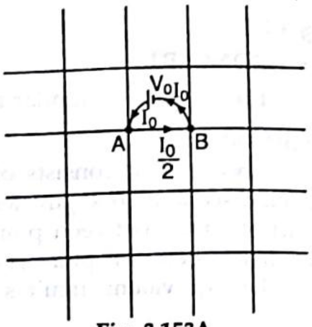


Fig. 3.153A

SOLUTION : $$V_0 = I_0 R_{eq}$$

$\therefore$ For circuit, $$V_0 = \frac{I_0}{2} R_0$$

$\therefore$ $$I_0 R_{eq} = \frac{I_0}{2} R_0$$

$\therefore$ $$R_{eq} = \frac{R_0}{2}$$

YOUR STEP

1. Determine the resistance R_{AB} between points A and B of the frame made of thin homogeneous wire (shown in Fig. 3.153B) assuming that the number of successively embedded equilateral triangles (with sides decreasing by half) tends to infinity. Side AB is equal to a, and the resistance per unit length of the wire is ρ.

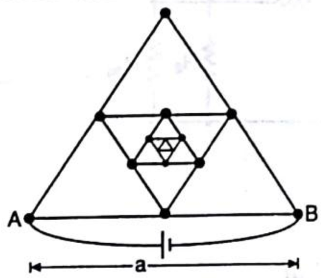


Fig. 3.153B

2. A network is formed of uniform wire in the shape of a rectangle of sides $2a$, $3a$ with parallel wires arranged so as to divide internal space into six squares of side a, the contact at points of intersection being perfect. Show that if a current enters the frame work by one corner and leave it by opposite, the resistance is equivalent to that of length $\frac{121}{69}$ of the wire in Fig. 3.153C.

$$\left\{ 1.\ \frac{a\,\rho(\sqrt{7}-1)}{3} \right\}$$

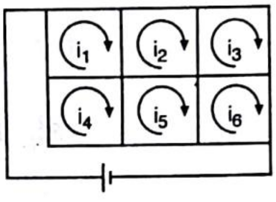


Fig. 3.153C

§ 3.154

➢ **CONCEPT**

The equivalent resistance in continuous medium is given by

$$R_{eq} = \int dR \qquad \text{(in series)}$$

and $$\frac{1}{R_{eq}} = \int \frac{1}{dR} \qquad \text{(in parallel)}$$

SOLUTION : We consider a cylindrical co-axial element of radius r and thickness dr. The resistance of considered element is

$$dR = \rho \frac{dr}{A} = \rho \frac{dr}{2\pi r l}$$

$\therefore$ Total resistance between cylinders is $$R = \int_a^b dR = \int_a^b \frac{dr}{2\pi l r}$$

$\therefore$ $$R = \frac{\rho}{2\pi l} \ln \frac{b}{a}$$

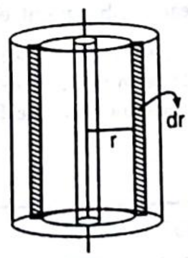


Fig. 3.154A

YOUR STEP

1. Find the resistance of a wire in which the resistivity varies linearly from ρ_1 to ρ_2. The length and the radius of the wire are l and r respectively.
2. A resistor is formed in the shape of a hollow quarter cylinder from a material of resistivity ρ. The inner and outer radii of the cylinder is R_1 and R_2. Find the resistance of this resistor between faces A and B (shown in Fig. 3.154B)

$$\left\{1.\ \frac{(\rho_1+\rho_2)\,l}{2\pi r^2},\ 2.\ \frac{\rho\pi}{2L\,ln\dfrac{R_2}{R_1}}\right\}$$

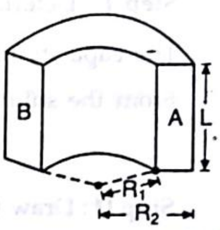


Fig. 3.154B

§ 3.155

➢ **CONCEPT**

The concept is similar to previous problem.

SOLUTION : We consider a concentric spherical element of radius r and thickness dr.

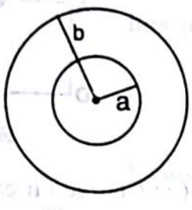


Fig. 3.155A

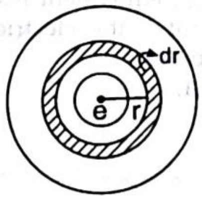


Fig. 3.155B

The resistance of considered element is $dR = \rho\dfrac{dr}{A} = \rho\dfrac{dr}{4\pi r^2}$

The equivalent capacitance between sphere is

$$R = \int dR = \int_a^b \frac{\rho}{4\pi r^2}\,dr = \frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{\rho\,(b-a)}{4\pi ab}$$

When $b \to \infty$ $\qquad R = \dfrac{\rho}{4\pi a}$

YOUR STEP

1. The region between two concentric conducting spheres with radii a and b is filled with a material of resistivity $\rho = \rho_0 r$ where r is distance from centre of sphere. Find equivalent resistance between spheres.
2. A resistor is in the shape of a truncated right circular cone. The end radii are a and b. The altitude is l. If the taper is small, we may assume, that the current density is uniform across any cross-section.
   (a) Calculate the resistance of the object.
   (b) Show that the answer reduced to $\dfrac{\rho l}{A}$ for the special case of zero taper $(a = b)$.

$$\left\{1.\ \frac{\rho_0}{4\pi}\,ln\frac{b}{a}\,\Omega\quad 2.\ \frac{l\rho}{\pi ab}\Omega\right\}$$

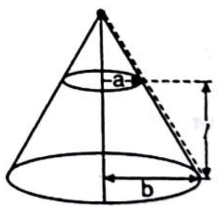


Fig. 3.155

§ 3.156

➢ **CONCEPT**

The problem is based upon leakage current. In this case, equivalent circuit is parallel combination of a capacitor and resistance between spheres.

SOLUTION : The problem is solved by following steps :

Step I : Determine capacity and resistance between spheres :

The capacity of spherical capacitor is $C = \dfrac{4\pi\varepsilon_0 \varepsilon ab}{(b-a)}$

From the solution of problem 3.155, electric resistance between sphere is

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho\,(b-a)}{4\pi ab}$$

Step II : Draw the equivalent circuit before disconnecting voltage circuit.

The circuit is shown in Fig. 3.155A.

In this case, circuit is in steady state.

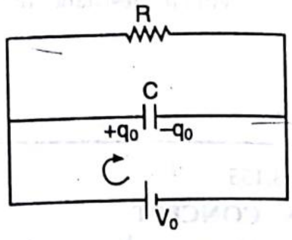


Fig. 3.156A

According to loop rule, $$V_0 - \frac{q_0}{c} = 0$$

$\therefore$ $$q_0 = CV_0 \qquad \text{...(i)}$$

Step III : when voltage source is disconnected, capacitor starts to discharge through equivalent resistance R.

Let at an instant t, the electric charge on capacitor is q and current in the circuit is I.

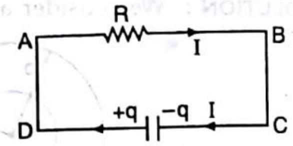


Fig. 3.156B

In loop $ABCDA$,

$$-IR + \frac{q}{C} = 0$$

But $$I = -\frac{dq}{dt} \qquad (\because \text{ Charge on capacitor decreases})$$

$\therefore$ $$-\left(-\frac{dq}{dt}\right)R + \frac{q}{C} = 0$$

or $$RC\int_{q_0}^{q} \frac{dq}{q} = -\int_0^t dt$$

After integrating, we get $$q = q_0 e^{-t/RC}$$

$$q = q_0 e^{-t/RC}$$

or $$\frac{q}{C} = \frac{q_0}{C} e^{-t/RC} \text{ or } V = V_0 e^{-t/RC}$$

According to problem, when $t = \Delta t$, $$V = \frac{V_0}{\eta}$$

$\therefore$ $$\frac{V_0}{\eta} = V_0 e^{-\Delta t/RC}$$

or $$-\ln \eta = -\frac{\Delta t}{RC}$$

$\therefore$ $$\Delta t = RC \ln \eta$$

But $$R = \frac{\rho\,(b-a)}{4\pi ab}$$

$\therefore$ $$\Delta t = RC \ln \eta$$

or $$\Delta t = \frac{C\rho\,(b-a)\ln \eta}{4\pi ab}$$

$\therefore$ $$\rho = \frac{4\pi ab\Delta t}{C\,(b-a)\ln \eta}$$

YOUR STEP

A metal sphere of radius 'a' is surrounded by a concentric metal sphere of radius 'b' ($b > a$). The space between the spheres is filled with a material whose electrical conductivity σ varies with the electric field strength E according to relation $\sigma = KE$. Where K is a constant. A potential difference V is maintained between two spheres. Find the current between the spheres.

$$\left\{\frac{4\pi K V^2}{\left\{\log\left(\frac{b}{a}\right)\right\}^2}\right\}$$

§ 3.157

➢ **CONCEPT**

Electric resistance is $R = \int dR$

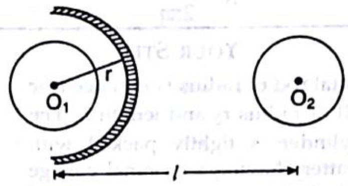


Fig. 3.157A

SOLUTION : We consider a hemispherical shell of radius r and thickness dr whose centre is at O.

The electric resistance of considered element is $dR = \rho \dfrac{dr}{2\pi r^2}$

$\therefore$ The resistance between sphere is

$$R = \int_a^{l-a} \frac{\rho\, dr}{2\pi r^2} = \frac{\rho}{2\pi}\left[-\frac{1}{r}\right]_a^{l-a} = \frac{\rho}{2\pi}\left[\frac{1}{l-a} + \frac{1}{a}\right] = \frac{\rho}{2\pi}\left[\frac{a+l-a}{a(l-a)}\right]$$

$$\therefore \quad R = \frac{\rho}{2\pi}\left[\frac{l}{a(l-a)}\right] \text{ But } l >>> a, \therefore l - a \approx l \therefore R = \frac{\rho}{2\pi a}$$

Alternative method :

➢ **CONCEPT**

Ohm's law in vector method is :

$$\vec{E} = \rho \vec{J}$$

where $\vec{J}$ = current density.

SOLUTION : First of all, spheres are charged by opposite nature of charge each of magnitude q (shown in fig. 3.157 B).

Fig. 3.157B

The electric field at the surface of first ball is $E = \dfrac{q}{4\pi\varepsilon_0 a^2}$

Here second ball is at infinitely large distance from first ball.

So, electric field due to second ball on the surface of first ball will be zero.

The current density $J = \dfrac{E}{\rho}$

$$\therefore \quad J = \frac{q}{4\pi\varepsilon_0 \rho a^2}$$

Electric c… $I = \int \vec{J}\cdot d\vec{S}$

$$= \int J dS \cos 0 = J \int dS = J \times 4\pi a^2$$

$\therefore$ $$I = \frac{q}{4\pi\varepsilon_0 \rho a^2} 4\pi a^2 = \frac{q}{\rho\varepsilon_0}$$

But potential difference between ball is

$$\Delta\phi = \phi_1 - \phi_2$$

$$= \frac{q}{4\pi\varepsilon_0 a} - \left(-\frac{q}{4\pi\varepsilon_0 a}\right) = \frac{q}{2\pi\varepsilon_0 a}$$

$\therefore$ Electric resistance $R = \frac{\Delta\phi}{I}$ (From Ohm's law)

$\therefore$ $$R = \frac{\rho}{2\pi a}$$

YOUR STEP

1. As shown in fig. 3.157B, a metal rod of radius r_1 is concentric with a metal cylindrical shell of radius r_2 and length L. The space between rod and cylinder is tightly packed with material of resistivity ρ. A battery having a terminal voltage V_0 is connected as shown. Neglecting resistances of rod and cylinder find :
   (a) the total current I
   (b) the current density and electric field at any point P between rod and cylinder.
   (c) the resistance R between rod and cylinder.
2. A copper sphere of radius 5cm is lowered into a water filled hemispherical copper vessel of 10cm radius so that the sphere and vessel are concentric. The electrical resistivity of water is 10^3 ohm metre. Find electrical resistance between sphere and vessel.

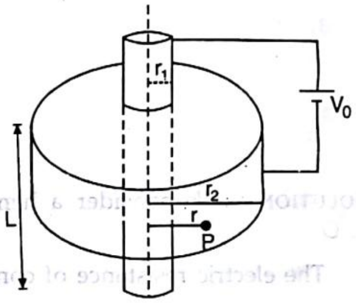


Fig. 3.157B

$$\left\{ \text{1. (a) } I = \frac{2\pi L V_0}{\rho \ln\left(\frac{r_2}{r_1}\right)},\ \text{(b) } J = \frac{V_0}{\rho\, r \ln\frac{r_2}{r_1}} \text{ and } E = \frac{V_0}{r \ln\frac{r_2}{r_1}},\ \text{(c) } \frac{\rho \ln\frac{r_2}{r_1}}{2\pi L}\quad \text{2. } 1590\Omega \right\}$$

§ 3.158

➢ CONCEPT

From the solution of problem 3.111,

$$\Delta\phi = V = \frac{q}{4\pi\varepsilon_0 a}$$

$\therefore$ $$q = 4\pi\varepsilon_0 a V \quad ...(i)$$

(a) The electric field at point P is due to charge q is

$$E_0 = \frac{q}{4\pi\varepsilon_0 r^2}$$

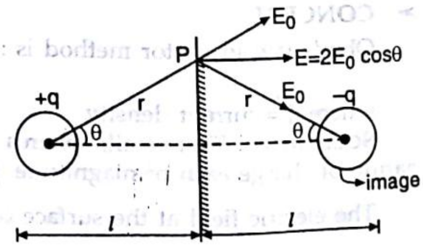


Fig. 3.158A

$\therefore$ Net electric field at point P is $E = 2E_0 \cos\theta$

or $$E = 2\frac{q}{4\pi\varepsilon_0 r^2}\cos\theta$$

or $$E = 2\frac{4\pi\varepsilon_0 a V}{4\pi\varepsilon_0 r^2}\frac{l}{r}$$ (from equ. (i)) or $$E = \frac{2aVl}{r^3}$$

According to ohm's law, $$E = \rho J$$

$\therefore$ $$J = \frac{E}{\rho} = \frac{2alV}{\rho r^3}$$

(b) We consider a hemispherical shell of thickness dx and radius x whose centre is at the centre of the ball.

$\therefore$ $$R = \int dR = \int_0^{2l} \rho \frac{dx}{2\pi x^2} = \frac{\rho}{2\pi}\left[-\frac{1}{x}\right]_a^{2l-a}$$

$$= \frac{\rho}{2\pi}\left[-\frac{1}{2l-a} + \frac{1}{a}\right] = \frac{\rho}{2\pi a} \qquad (\because l >> a)$$

YOUR STEP

1. A material with resistivity ρ is used to make a plate ring of thickness d. The radii of the ring are a and b $(b > a)$. A potential difference is maintained between the outer and inner cylindrical surface of ring. Find the resistance R of the ring in these condition.
2. Electric current flows through a conductor consisting of a sphere of copper of conductivity σ embedded in an infinite mass of iron of conductivity σ_0. The flow at an infinite distance from the sphere is uniform, the current density being $\vec{J_0}$. Prove that the current through the sphere is uniform and that the current density is $3\sigma(2\sigma_0 + \sigma)^{-1}\vec{J_0}$.

$$\left\{1.\ \left(\frac{\rho}{2\pi d}\right) ln\left(\frac{b}{a}\right)\right\}$$

§ 3.159

➢ **CONCEPT**

(a) The electric field at point P at distance r from both wires is

$$E = \frac{Vl}{2r^2\, ln\frac{l}{a}}$$

From ohm's law, $$E = \rho j$$

$\therefore$ $$j = \frac{E}{\rho} = \frac{Vl}{2\rho r^2\, ln\frac{l}{a}}$$

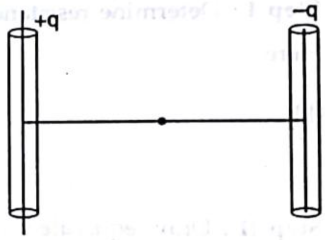


Fig. 3.159A

(b) The electric field near first wire is $$E_0 = \frac{V}{2a\, ln\frac{l}{a}}$$

$\therefore$ $$J = \frac{E_0}{\rho} = \frac{V}{2a\,\rho\, ln\frac{l}{a}}$$

$$I = jS = j\,2\pi a \qquad \text{(per unit length)}$$

$\therefore$ $$R = \frac{V}{I} \qquad \text{(per unit length)}$$

$\therefore$ $$R = \frac{V}{j2\pi a} = \frac{V}{2\pi a \dfrac{V}{2a\,\rho\, ln\frac{l}{a}}} = \frac{\rho\, ln\frac{l}{a}}{\pi}$$

YOUR STEP

1. The resistivity of a cylindrical conductor of area A and l varies as $\rho = \frac{\rho_0\sqrt{x}}{l}$ where x is the distance along the axis of the cylinder from one of its ends (shown in figure 3.159B).
   (a) Calculate the resistance of the system along the cylindrical axis.
   (b) A potential difference V_0 is applied between ends of conductor. What is electric field at each point in the cylinder?
   [Hint : $E = \rho j$]

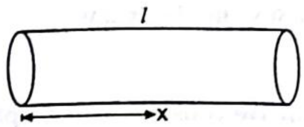


Fig. 3.159B

2. A steady current flows through a homogeneous substance bounded by two infinite cylindrical electrodes where cross-sections are concentric circles of radii a and b $(b > a)$. The conductivity σ at a distance r from the axis of cylinder is given by $\frac{a\sigma_0}{r}$, where σ_0 is constant.
   Show that the resistance R per unit length is given by $\frac{b-a}{2\pi a\sigma_0}$.

$$\left\{ \textbf{1. } \text{(a) } \frac{2\rho_0\sqrt{l}}{3A}, \text{ (b) } \frac{3V_0\sqrt{x}}{2l^{3/2}} \right\}$$

§ 3.160

➢ **CONCEPT**

When the gap between plates of capacitor is filled by poorly conducting material. The system behaves as a parallel combination of a capacitor and resistance between plates of capacitor.

SOLUTION : The problem is solved in following steps :

Step I : Determine resistance and capacity of system separately.

Here $$R = \rho\frac{d}{A}$$

Also, $$C = \varepsilon_0\varepsilon\frac{A}{d}$$

$\therefore$ $$\frac{d}{A} = \frac{\varepsilon_0\varepsilon}{C}$$

Step II : Draw equivalent circuit :

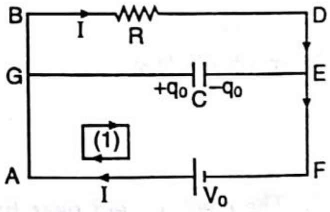


Fig. 3.160A

In loop $AGEFA$, $$V - \frac{q_0}{C} = 0$$

$\therefore$ $$q_0 = CV$$

In loop $ABDFA$, $$-IR + V = 0$$

$\therefore$ $$V = IR$$

$\therefore$ $$I = \frac{V}{R} = \frac{V}{\rho\frac{d}{A}} = \frac{V}{\frac{\rho\varepsilon_0\varepsilon}{C}}$$

$$= \frac{CV}{\rho\varepsilon_0\varepsilon}$$

Here current I through equivalent resistance of the system is known as leakage current.

According to problem, $$C = 4 \times 10^{-9}\text{ F}$$

$$\rho = 100 \times 10^{12}\ \Omega\text{ m}$$

ε = dielectric constant for glass
$= 6$ (see page 377 of the book I.E.Irodov)

Putting the values, we get $$I = 1.5 \times 10^{-6}\text{ A} = 1.5\ \mu\text{A}$$

YOUR STEP

In any leakage experiment a capacitor of capacitance 1μF was first charged to a suitable potential and discharged at once, the throw was 10cm. The capacitor was again charged and was short circuited with a resistor R for 20 sec. and the throw was 7.5 cm when it was discharged through a ballastic galvanometer. Calculate the resistance of resistor.

{69.5×10^6 ohm}

§ 3.161

➢ **CONCEPT**

In this case, ohm's law in vector form is applicable to *i.e.* $\vec{E} = \rho \vec{J}$

SOLUTION : Conductors are oppositely charged by charge q.

The electric field at the surface of conductor is $E = \dfrac{\sigma}{\varepsilon_0}$ in the direction of area.

Here $\sigma = \dfrac{q}{S}$

But $E = \rho J$ (Here $\vec{E} \uparrow\uparrow \vec{J}$) $\therefore$ $J = \dfrac{E}{\rho}$

Since, electric field and area is in same direction so, angle between area vector and current/ density vector should be zero.

$\therefore$ $J = \dfrac{I}{S}$ $\therefore$ $I = JS$ (where S is area)

or $I = \dfrac{E}{\rho} S$ or $V = IR$

(Here V is potential difference between conductors)

or $\dfrac{q}{C} = IR$

$\therefore$ $RC = \dfrac{q}{I} = \dfrac{\sigma S}{\frac{E}{\rho} S} = \dfrac{\sigma\rho}{E} = \dfrac{\sigma\rho}{\frac{\sigma}{\varepsilon_0 \varepsilon}}$ $RC = \varepsilon_0 \varepsilon \rho$

YOUR STEP

Consider a system of three co-axial conducting hollow cylinders shown in fig. 3.161A. The radii are a, b and c where $a < b < c$. The cylinders are all of equal length L, which is much longer than c. (*i.e.*, $L >> c > b > a$) and the electric lines of force and current flow are cylindrically symmetric. The region between cylinders of radii b and c is filled with a medium of conductivity $\sigma = \dfrac{\sigma_0 b}{r}$ where $b < r < c$. An electric charge $-q_0$ is deposited on the internal cylinder and charge $+q_0$ is applied to the middle shell. At $t = 0$, the inner and outer cylinder are connected (the switch S shown in fig. 3.161B) is shut at $t = 0$. Find time dependence charge on inner cylinder.

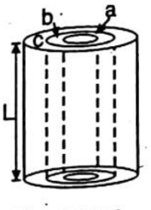


Fig. 3.161A

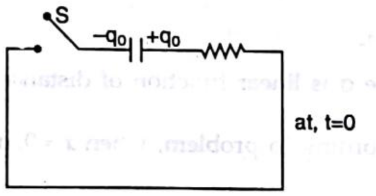


Fig. 3.161B

$$\left\{ \frac{-q_0 \ln\left(\frac{C}{b}\right)}{\ln\left(\frac{C}{a}\right)} e^{-t/RC} \text{Here} : RC = \frac{c-b}{4\pi b \sigma_0 \ln\left(\frac{b}{a}\right)} \right\}$$

§ 3.162

➢ **CONCEPT**

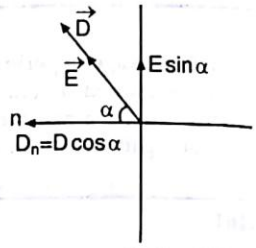


Fig. 3.162A

Surface charge density is $\sigma = D_n = D\cos\alpha$ (normal component)

But $$D = \varepsilon_0 \varepsilon E$$

∴ $$E = \frac{D}{\varepsilon_0 \varepsilon} \text{ at point}$$

The tangential component of electric field inside conductor is

$$E_0 = E\sin\alpha$$

According to ohm's law, $$E_0 = \rho J$$

∴ $$J = \frac{E_0}{\rho} = \frac{E\sin\alpha}{\rho} = \frac{D\sin\alpha}{\varepsilon_0 \varepsilon \rho}$$

YOUR STEP

Two dielectric media with constant permittivities ε_1 and ε_2 are separated by a plane interface. There is no external charge on the interface. A point charge q is embedded in the medium characterised by ε_1 at a distance d from the interface. For convenience we may take the $y-z$ plane through the origin to be the interface, and we locate q on the x-axis at $x = -d$. If

$$r = \sqrt{(x+d)^2 + y^2 + z^2}$$

and $$r' = \sqrt{(x-d)^2 + y^2 + z^2}$$

then it is easily, demonstrated that

$$= \frac{1}{4\pi\varepsilon_1}\left[\frac{a}{r} + \frac{q'}{r'}\right] \text{ satisfies}$$

Laplaces equation at all point in medium (1) except at the position of q. Further more,

$\frac{q''}{4\pi\varepsilon_2 r}$ satisfies Laplace's equation in medium 2. Show that all boundary conditions can be satisfied by these potentials and in doing, so determine q' and q'' (shown in Fig. 3.162B).

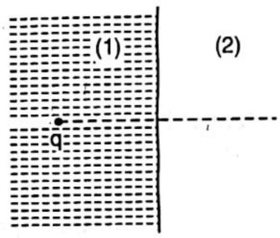


Fig. 3.162B

$$\left\{q' = [(\varepsilon_1 - \varepsilon_2)/(\varepsilon_1 + \varepsilon_2)]\, q,\ q'' = \frac{2\varepsilon_2 q}{\varepsilon_1 + \varepsilon_2}\right\}$$

§ 3.163

➢ **CONCEPT**

In this case, $$R = \int dR$$

Also, $$\rho = \frac{1}{\sigma}$$

Here σ is linear function of distance x.

∴ $$\sigma = mx + K$$

According to problem, when $x = 0$, $\sigma = \sigma_1$

∴ $$\sigma_1 = m \times 0 + K$$

∴ $$K = \sigma_1$$

when $x = d$, $\sigma = \sigma_2$

∴ $$\sigma_2 = md + K$$

or $$\sigma_2 = md + \sigma_1$$

$$\therefore \quad m = \frac{\sigma_2 - \sigma_1}{d}$$

$$\therefore \quad \sigma = \left(\frac{\sigma_2 - \sigma_1}{d}\right)x + \sigma_1$$

The problem is solved by following steps :

Step I : Determine equivalent resistance we consider an element of thickness dx at distance x from first plate.

The electric resistance of considered element is

$$dR = \rho\frac{dx}{S} = \frac{dx}{\sigma S} = \frac{dx}{\left[\left(\frac{\sigma_2 - \sigma_1}{d}\right)x + \sigma_1\right]S}$$

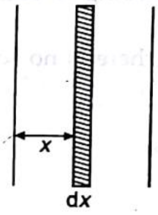


Fig. 3.163A

The equivalent resistance between plates is

$$R = \int dR = \int_0^d \frac{dx}{\left[\left(\frac{\sigma_2 - \sigma_1}{d}\right)x + \sigma_1\right]S}$$

$$= \frac{d \ ln \frac{\sigma_2}{\sigma_1}}{S(\sigma_2 - \sigma_1)}$$

Step II : Draw equivalent circuit of system :

The system behaves as parallel combination of a resistor of resistance R and a capacitor of capacitance C. The circuit is shown in Fig. 3.163B.

In loop $EFABE$, $V - IR = 0$

$$\therefore \quad V = IR$$

$$\therefore \quad I = \frac{V}{R}$$

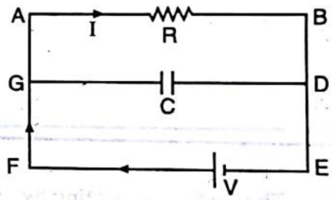


Fig. 3.163B

Putting the value of R_1, we get

$$I = \frac{SV(\sigma_2 - \sigma_1)}{d \ ln \frac{\sigma_2}{\sigma_1}}$$

Putting the values, we get $\quad I = 5\text{nA}$

YOUR STEP

Two long cylindrical shells of metal (radii r_1 and r_2 with $r_2 > r_1$) are arranged co-axially. The plates are maintained at the potential difference $\Delta\phi$.

(a) The region between the shells is filled with a medium of conductivity σ. Calculate the electric current between unit length of shells.

(b) If the region between the shells is filled with a non-conducting medium of permittivity ε, show explicity for this geometry that the product of resistance per unit length and capacity per unit length $= \frac{\varepsilon}{\sigma}$

$$\left\{I = \frac{2\pi\sigma\Delta\phi}{\ln\frac{r_2}{r_1}}\right\}$$

---

§ 3.164

➢ CONCEPT

Electric charge is neither piling up nor disappearing at a boundary, the current toward the boundary on one side must be equal to that away from it on the other side.

According to conservation principle of charge,

$$I_1 = I_2$$

or $$\vec{J_1}\cdot d\vec{S} = \vec{J_2}\cdot d\vec{S}$$

or $$(J_1\cos\theta_1)\,dS = (J_2\cos\theta_2)\,dS$$

or $$J_{in} = J_{2n} \quad \text{...(i)}$$

(normal component of J)

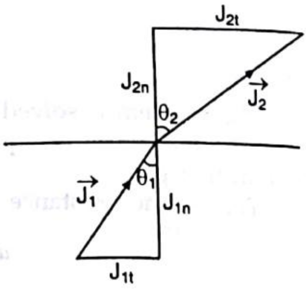


Fig. 3.164

According to ohm's law, $$J = \sigma E$$

$$\sigma_1 E_{1n} = \sigma_2 E_{2n} \quad \text{...(ii)}$$

But there is no source of emf.

$\therefore$ $$\int_C \vec{E}\cdot d\vec{L} = 0$$

Here $$E_{1t} = E_{2t}$$

$\Rightarrow$ $$\frac{J_{1t}}{\sigma_1} = \frac{J_{2t}}{\sigma_2}$$

or $$\frac{J_1\sin\theta_1}{\sigma_1} = \frac{J_2\sin\theta_2}{\theta_2} \quad \text{...(iii)}$$

($\because$ j_{1t} and j_{2t} are horizontal components of J_1 and J_2 respectively)

From equ. (i) and (iii), we get

$$\sigma_1\cot\theta_1 = \sigma_2\cot\theta_2$$

According to problem, $\theta_1 = \alpha_1$ and $\theta_2 = \alpha_2$

$\therefore$ $$\frac{\sigma_1}{\tan\alpha_1} = \frac{\sigma_2}{\tan\alpha_2}$$

$\therefore$ $$\frac{\tan\alpha_2}{\tan\alpha_1} = \frac{\sigma_2}{\sigma_1}$$

YOUR STEP

The surface separating two dielectrics having dielectric constants K_1 and K_2 has surface charge density σ. The electric intensities on the two sides boundary are E_1 and E_2, making angles θ_1 and θ_2 with the common normal, prove that,

$$\left\{K_2\cot\theta_2 = K_1\cot\theta_1\left(1 - \frac{\sigma}{\varepsilon_0 K_1 E_1\cos\theta_1}\right)\right\}$$

§ 3.165

➢ **CONCEPT**

According to ohm's law, $\vec{E} = \rho\,\vec{J}$

The problem is solved by following steps :

Step I : Determine electric field in both conductors.

The current density in first conductor is

$$J_1 = \frac{I}{\pi R^2} \quad \text{(along the length of conductor)}$$

Similarly, $J_2 = \dfrac{I}{\pi R^2}$ along the length of conductor.

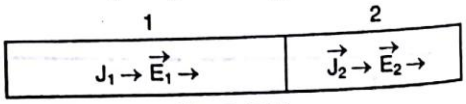


Fig. 3.165A

The electric field inside first conductor is

$$E_1 = \rho_1 J = \frac{\rho_1 I}{\pi R^2} \quad \text{(along the length of conductor)}$$

Similarly, $E_2 = \frac{\rho_2 I}{\pi R^2}$ (along length of conductor)

Step II : Consider small cylindrical region near boundary and apply Gauss's law :

We consider a small cylindrical region at boundary (shown in fig. 3.165B).

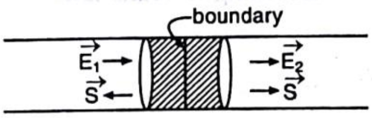


Fig. 3.165B

The area of cylindrical region is

$$S = \pi R^2$$

The net flux through this region is

$$\phi = \phi_1 + \phi_2 = \vec{E_1} \cdot \vec{S} + \vec{E_2} \cdot \vec{S}$$
$$= E_1 S \cos 180° + E_2 S \cos 0 = -E_1 S + E_2 S = (E_2 - E_1) S$$

According to Gauss's Law,

$$\phi = \frac{q}{\varepsilon_0}$$

or $$(E_2 - E_1) = \frac{q}{S\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$\therefore$ The surface charge density at boundary is

$$\sigma = (E_2 - E_1)\,\varepsilon_0$$

Putting the value of E_1 and E_2, we get

$\therefore$ $$\sigma = (\rho_2 j - \rho_1 j) \text{ or } \sigma = (\rho_2 - \rho_1)\, j$$

$\therefore$ $$\sigma = \varepsilon_0 (\rho_2 - \rho_1) \frac{I}{\pi R^2}$$

or $$\sigma \pi R^2 = \varepsilon_0 (\rho_2 - \rho_1) I$$

or $$q = \varepsilon_0 (\rho_2 - \rho_1) I$$

Charge at the boundary $$q = \varepsilon_0 (\rho_2 - \rho_1)\, I$$

YOUR STEP

Two straight parallel cylindrical wires of radii a and b are placed with their centres at a great distance r apart in an infinite medium. Find the resistance per unit length between wires.

$$\left\{\frac{1}{2\pi\sigma} \ln \frac{r^2}{ab}\right\}$$

§ 3.166

➢ CONCEPT

The concept is similar to previous problem.

If electric field is uniform, then $E = \dfrac{\text{potential difference between two points}}{\text{distance between two points}}$

SOLUTION : The problem is solved in following steps :

Step I : Determine electric field in both dielectric separately.

From fig 3.166A

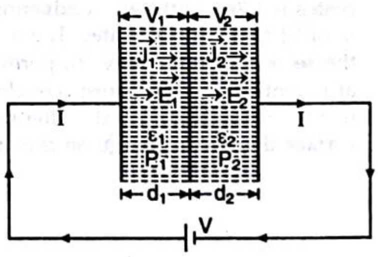


Fig. 3.166A

$\therefore$ $$V = V_1 + V_2 \quad (\because E_1 = \frac{V_1}{d_1} \text{ and } E_2 = \frac{V_2}{d_2})$$

or $$V = E_1 d_1 + E_2 d_2 \qquad \text{...(i)}$$

If leakage current through capacitor is I.

Then $J_1 = \frac{I}{S}$ and $J_2 = \frac{I}{S}$

According to ohms law,

$$E_1 = \rho_1 J_1 \text{ and } E_2 = \rho_2 J_2$$

$\therefore$
$$J_1 = \frac{E_1}{\rho_1} \text{ and } J_2 = \frac{E_2}{J_2}$$

But
$$J_1 = J_2 = \frac{I}{S}$$

$\therefore$
$$\frac{E_1}{\rho_1} = \frac{E_2}{\rho_2}$$

$\therefore$
$$E_2 = \frac{\rho_2}{\rho_1} E_1$$

From equ. (i),
$$E_1 d_1 + E_2 d_2 = V$$

or
$$E_1 d_1 + \frac{\rho_2}{\rho_1} E_1 d_2 = V$$

$\therefore$
$$E_1 = \frac{\rho_1 V}{\rho d_1 + \rho_2 d_2}$$

Similarly,
$$E_2 = \frac{\rho_2 V}{\rho_1 d_1 + \rho_2 d_2}$$

Step II : Apply Gauss's law at the boundary :

We consider a cylindrical region of cross-sectional area dS near the boundary (shown in Fig. 3.166B).

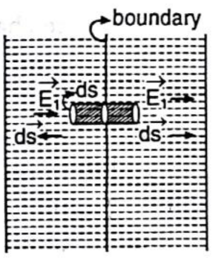


Fig. 2.166B

According to Gauss's Law, $\int \varepsilon \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$

or
$$-\varepsilon_1 E_1 dS + \varepsilon_2 E_2 dS = \frac{q}{\varepsilon_0} \text{ or } \left(\varepsilon_2 E_2 - \varepsilon_1 E_1\right) = \frac{q}{\varepsilon_0 dS}$$

or
$$(\varepsilon_2 E_2 - \varepsilon_1 E_1) = \frac{\sigma}{\varepsilon_0}$$

$\because$
$$\sigma = \varepsilon_0 (\varepsilon_2 E_2 - \varepsilon_1 E_1)$$

Putting the values of E_1 and E_2, we get

$$\sigma = \frac{\varepsilon_0 V}{(\rho_1 d_1 + \rho_2 d_2)} (\varepsilon_2 \rho_2 - \varepsilon_1 \rho_1)$$

For $\sigma = 0$,
$$\varepsilon_2 \rho_2 = \varepsilon_1 \rho_1$$

YOUR STEP

Two infinite, plane, parallel plates of metal are separated by the distance d. The space between the plates is filled with two conducting media, the interface between the media being a plane which is parallel to the metal plates. The first medium (conductivity σ_1, permittivity ε_1) is of thickness a and the second (conductivity σ_2, permittivity ε_2) is of thickness $d - a$. The metal plates are maintained at potentials ϕ_1 and ϕ_2 respectively.

In the steady state, what is the potential of the interface separating the two media. What is the surface density of charge on this interface.

$$\left\{ \begin{aligned} \phi_{int} &= \frac{\phi_1 \sigma_1 (d-a) + \phi_2 \sigma_2 a}{\sigma_2 a + \sigma_1 (d-a)} \\ \text{charge density} &= \frac{(\sigma_1 \varepsilon_2 - \sigma_2 \varepsilon_1)(\phi_1 - \phi_2)}{\sigma_2 a + \sigma_1 (d-a)} \end{aligned} \right\}$$

§ 3.167

➢ CONCEPT

SOLUTION : Since, ρ varies linearly from first plate to second plate.

$$\therefore \quad \rho = mx + K$$

when $x = 0$, $\rho = \rho_1$

$$\therefore \quad \rho_1 = m \times 0 + K$$

$$\therefore \quad K = \rho_1$$

when $x = d$, $\rho = \rho_2$

$$\therefore \quad \rho_2 = md + \rho_1$$

$$\therefore \quad m = \frac{\rho_2 - \rho_1}{d}$$

$$\therefore \quad \rho = \left(\frac{\rho_2 - \rho_1}{d}\right)x + \rho_1$$

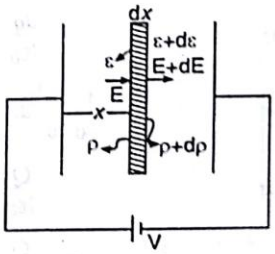


Fig. 3.167A

Similarly,

$$\varepsilon = \left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)x + \varepsilon_1$$

The electric field at left surface of considered element is $E = \rho J$

$$E = \left\{\left(\frac{\rho_2 - \rho_1}{d}\right)x + \rho_1\right\} J$$

$$\therefore \quad E = \rho J$$

$$\because \quad dE = J\, d\rho$$

$$\because \quad \rho = \frac{(\rho_2 - \rho_1)}{d}\, x + \rho_1$$

$$\therefore \quad \frac{d\rho}{dx} = \left(\frac{\rho_2 - \rho_1}{d}\right)$$

$$\therefore \quad d\rho = \left(\frac{\rho_2 - \rho_1}{d}\right)dx$$

$\therefore$ Similarly,

$$d\varepsilon = \left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)dx$$

From the solution of previous problem,

$$\sigma = \varepsilon_0\, \varepsilon_2\, E_2 - \varepsilon_0\, \varepsilon_1\, E_1$$

For considered element, $E_2 = E + dE$

$$\varepsilon_2 = \varepsilon + d\varepsilon$$

$$E_1 = E,$$

$$\therefore \quad d\sigma = \varepsilon_0\, (E + dE)\, (\varepsilon + d\varepsilon) - \varepsilon_0 \varepsilon\, E$$

$$= \varepsilon_0\, E\, \varepsilon + \varepsilon_0\, E\, d\varepsilon + \varepsilon_0\, \varepsilon\, dE + \varepsilon_0\, d\varepsilon\, dE - \varepsilon_0\, \varepsilon E = \varepsilon_0 E d\varepsilon + \varepsilon_0 \varepsilon dE + \varepsilon_0\, dE d\varepsilon$$

But $\varepsilon_0\, dEd\varepsilon$ may be neglected.

$$\therefore \quad d\sigma = \varepsilon_0\, E\, d\, \varepsilon + \varepsilon_0\, \varepsilon\, dE$$

or

$$d\sigma = \varepsilon_0 \left\{\left(\frac{\rho_2 - \rho_1}{d}\right)x + \rho_1\right\} J \left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)dx + \varepsilon_0 \left\{\left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)x + \varepsilon_1\right\} J \left(\frac{\rho_2 - \rho_1}{d}\right)dx$$

But $\quad J = \dfrac{I}{S}$

$$\therefore \quad Sd\sigma = I\, \varepsilon_0 \left\{\left(\frac{\rho_2 - \rho_1}{d}\right)x + \rho_1\right\} \left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)dx + \varepsilon_0\, I \left\{\left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)x + \varepsilon_1\right\} \left(\frac{\rho_2 - \rho_1}{d}\right)dx$$

or $$\frac{dq}{I\varepsilon_0} = \left\{\left(\frac{\rho_2-\rho_1}{d}\right)x+\rho_1\right\}\left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)dx + \left\{\left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)x+\varepsilon_1\right\}\left(\frac{\rho_2-\rho_1}{d}\right)dx$$

or $$\frac{dq}{I\varepsilon_0} = \left(\frac{\rho_2-\rho_1}{d}\right)x+\rho_1\left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)dx + \left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)x+\varepsilon_1\left(\frac{\rho_2-\rho_1}{d}\right)dx$$

or $$\frac{1}{I\varepsilon_0}\int_0^Q dq = \left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)\int_0^d\left\{\left(\frac{\rho_2-\rho_1}{d}\right)x+\rho_1\right\}dx + \left(\frac{\rho_2-\rho_1}{d}\right)\int_0^d\left\{\left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)x+\varepsilon_1\right\}dx$$

or $$\frac{Q}{I\varepsilon_0} = \frac{(\varepsilon_2-\varepsilon_1)}{d}\left\{\left(\frac{\rho_2-\rho_1}{2}\right)\frac{d^2}{2}+\rho_1 d\right\} + \left(\frac{\rho_2-\rho_1}{d}\right)\left\{\left(\frac{\varepsilon_2-\varepsilon_1}{2}\right)\frac{d^2}{2}+\varepsilon_1 d\right\}$$

or $$\frac{Q}{I\varepsilon_0} = \frac{(\varepsilon_2-\varepsilon_1)}{d}\left\{(\rho_2-\rho_1)\frac{d}{2}+\rho_1 d\right\} + \left(\frac{\rho_2-\rho_1}{d}\right)\left\{(\varepsilon_2-\varepsilon_1)\frac{d}{2}+\varepsilon_1 d\right\}$$

$$= \frac{(\varepsilon_2-\varepsilon_1)}{d}(\rho_2-\rho_1)\frac{d}{2} + \left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)\rho_1 d + \left(\frac{\rho_2-\rho_1}{d}\right)(\varepsilon_2-\varepsilon_1)\frac{d}{2} + \left(\frac{\rho_2-\rho_1}{d}\right)\varepsilon_1 d$$

$$= \frac{\varepsilon_2\rho_2}{2} - \frac{\varepsilon_2\rho_1}{2} - \frac{\rho_2\varepsilon_1}{2} + \frac{\rho_1\varepsilon_1}{2} + \rho_1\varepsilon_2 - \rho_1\varepsilon_1$$
$$+\frac{\rho_2\varepsilon_2}{2} - \frac{\rho_2\varepsilon_1}{2} - \frac{\rho_1\varepsilon_2}{2} + \frac{\rho_1\varepsilon_1}{2} + \rho_2\varepsilon_1 - \rho_1\varepsilon_1$$

$$= \rho_2\varepsilon_2 - \rho_1\varepsilon_2 - \rho_2\varepsilon_1 + \rho_1\varepsilon_1 + \rho_1\varepsilon_2 - \rho_1\varepsilon_1 + \rho_2\varepsilon_1 - \rho_1\varepsilon_1$$

or $$\frac{Q}{I\varepsilon_0} = \rho_2\varepsilon_2 - \rho_1\varepsilon_1 \therefore Q = I\varepsilon_0(\rho_2\varepsilon_2 - \rho_1\varepsilon_1)$$

YOUR STEP

A resistor is formed by two square metal plates of edge a, separated by a distance d. The material of resistivity ρ_1 and ρ_2 are filled in the gap as shown in the figure 3.167B. Find the resistance.

$$\left\{\frac{(\rho_1-\rho_2)\,d}{a^2\, ln\left(\frac{\rho_1}{\rho_2}\right)}\right\}$$

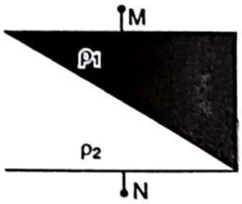


Fig. 3.167B

§ 3.168

➢ **CONCEPT**

According to Gauss's Law,

$$\int_c \vec{E}\cdot d\vec{S} = \frac{q}{\varepsilon_0} \quad \text{or} \quad \frac{\int_c \vec{E}\,d\vec{S}}{\text{Volume}} = \frac{q}{\varepsilon_0 \times \text{volume}} = \frac{\rho_0}{\varepsilon_0}$$

Here ρ_0 is volume charge density.

Divergence of electric field is defined as electric flux per unit volume.

$\because$ $$\vec{\nabla}\cdot\vec{E} = \frac{\int_c \vec{E}\cdot d\vec{S}}{\text{Volume}} = \frac{\rho}{\varepsilon_0}$$

Here $$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

and $$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$\therefore$ $$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_0}$$

If electric field varies along x-axis (one dimensional variation), then $E = E_x$

$$\therefore \qquad \frac{dE}{dx} = \frac{\rho}{\varepsilon_0}$$

SOLUTION : The problem is solved by following steps :

Step I : Determine resistance between plates of capacitor :

Since, resistivity varies linearly from one plate to another.

Then $\qquad \rho = mx + K$

when $x = 0$, $\rho = \rho_1 = \rho_{min}$

$$\therefore \qquad \rho_1 = m \times 0 + K \qquad \therefore\ k = \rho_1$$

when $x = d$, $\rho = \rho_2 = \rho_{max}$

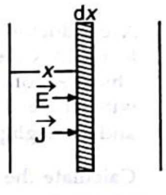


Fig. 3.168A

$$\therefore \qquad \rho_2 = m \times d + \rho_1 \qquad \therefore\ m = \frac{\rho_2 - \rho_1}{d}$$

$$\therefore \qquad \rho = \left\{\left(\frac{\rho_2 - \rho_1}{d}\right)x + \rho_1\right\}$$

According to problem, $\qquad \dfrac{\rho_2}{\rho_1} = \dfrac{\rho_{max}}{\rho_{min}} = \eta$

$$\therefore \qquad \rho_2 = \eta\rho_1 \qquad \therefore \qquad \rho = \frac{(\eta - 1)}{d}\rho_1 x + \rho_1$$

$\therefore$ The electric resistance in considered element (shown in fig. 3.168 A) is

$$dR = \rho\frac{dx}{S} = \left\{\frac{(\eta - 1)\rho_1 x}{d} + \rho_1\right\}\frac{dx}{S}$$

Equivalent resistance between plates is

$$R = \int dR = \int_0^d \left\{\frac{(\eta - 1)\rho_1 x}{d} + \rho_1\right\}\frac{dx}{S} = (\eta + 1)\frac{\rho_1 d}{2S}$$

Step II : Draw equivalent circuit of the problem :

The system behaves as parallel combination of a capacitor and resistance (shown in fig. 3.168B).

According to loop rule, $\qquad V - IR = 0$

$$\therefore \qquad I = \frac{V}{R} \qquad \therefore\ J = \frac{I}{S} = \frac{V}{RS}$$

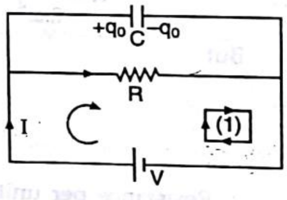


Fig. 3.168B

Putting the value of R, we get

$$J = \frac{V}{(\eta + 1)\dfrac{\rho_1 d}{2S}S} = \frac{2V}{(\eta + 1)\rho_1 d}$$

Step III : Apply ohm's law in vector form :

According to ohm's law, $\qquad \vec{E} = \rho\,\vec{J}$ $\qquad$ (Here $\vec{E}\uparrow\uparrow\vec{J}$)

The electric field at distance x from first plate is

$$E = \left\{\frac{(\eta - 1)}{d}\rho_1 x + \rho_1\right\}J$$

The direction of electric field is shown in fig. 3.168 A.

$$\because \qquad \frac{dE}{dx} = \frac{\rho_0}{\varepsilon_0}$$

$$\text{or} \qquad d\left\{\frac{(\eta - 1)\rho_1 x}{d} + \rho_1\right\}J = \frac{\rho_0}{\varepsilon_0} \text{ or } \frac{(\eta - 1)\rho_1\varepsilon_0 J}{d} = \rho_0$$

Putting the values of J, we get

volume charge density is $\rho_0 = \frac{(\eta - 1)\,\rho_1\,\varepsilon_0}{d}\,\frac{2V}{(\eta+1)\,\rho_1 d} = \frac{2\,(\eta-1)\,\varepsilon_0 V}{(\eta+1)\,d^2}$

YOUR STEP

A conductor of length L has a shape of a semi-cylinder of radius $R\,(<<L)$. Cross-section of the conductor is shown in the Fig. 3.168C. Thickness of conductor is $t\,(t<<R)$. The resistivity of its material varies with angle θ only according to law $\rho = \rho_0 \sec\theta$. If a battery of emf V and of negligible internal resistance is connected across its end faces.

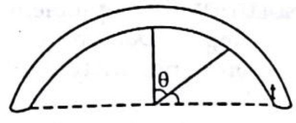


Fig. 3.168C

Calculate the electric current in the circuit.

$$\left\{\frac{2t\rho_0 VR}{L}\right\}$$

§ 3.169

➢ **CONCEPT**

Here $$\frac{1}{R} = \int \frac{1}{dR}$$

SOLUTION : We consider a long co-axial hollow cylinder of radius r and thickness dr (shown in fig. 3.169A). The cross-sectional area of considered element is $dA = 2\pi r dr$

$\therefore$ Electric resistance between ends of conductor of considered element is

$$dR = \rho\frac{l}{dA} = \rho\frac{l}{2\pi r dr} = \frac{\alpha l}{2\pi r^3 dr}$$

The system may be assumed as parallel combination of a number of such types of elementry resistance.

$$\therefore \quad \frac{1}{R} = \int\frac{1}{dR} = \frac{2\pi}{\alpha l}\int_0^a r^3 dr \quad \text{or} \quad \frac{1}{R} = \frac{2\pi}{\alpha l}\left(\frac{a^4}{4}\right)$$

$$\therefore \quad R = \frac{4\alpha l}{2\pi a^4}$$

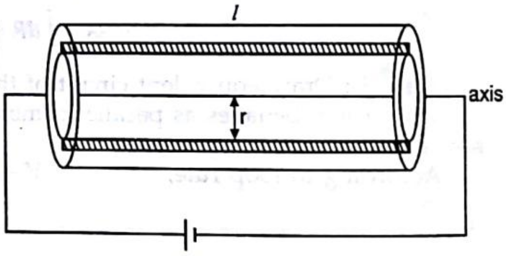


Fig. 3.169A

But $$S = \pi a^2$$

$$\therefore \quad R = \frac{4\alpha l}{2\pi\left(\frac{S}{\pi}\right)^2}$$

$\therefore$ Resistance per unit length is $\frac{R}{l} = \frac{2\pi\alpha}{S^2}$

According to ohm's law $V = IR$

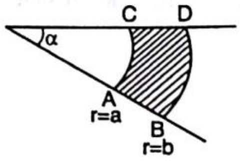
But $E = \frac{V}{l} = \frac{IR}{l}$

$$\therefore \quad E = \left(\frac{R}{l}\right)I = \frac{2\pi\alpha I}{S^2}$$

YOUR STEP

Calculate the resistance of the strip $ABCD$ (shown in fig 3.169B) of conductivity σ between electrodes CD and AB. The thickness of strip is t.

$\left(\text{Hint} : E = \frac{c}{r}\right\}$



$$\left\{R = \frac{\alpha}{\sigma t \ln b/a}\right\}$$

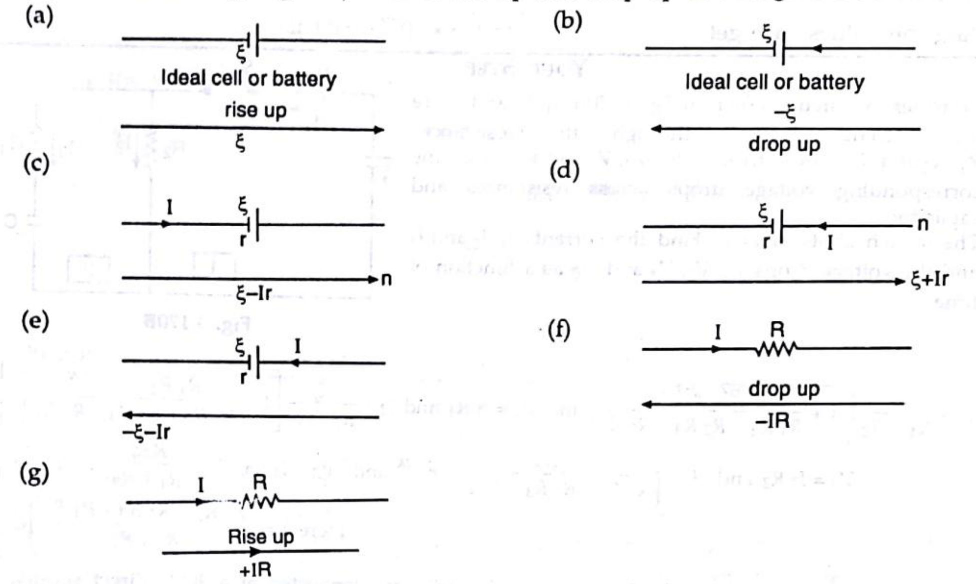
Fig. 3.169B

§ 3.170

➤ **CONCEPT**

In sign convention, rise up voltage is taken as positive and drop up voltage is taken as negative.

(h) In a closed loop, algebraic sum of rise up and drop up of voltage is zero.



SOLUTION : The circuit of problem is shown in Fig. 3.170 A at an instant t.

According to loop rule,

$$V_0 - IR - \frac{q}{C} = 0$$

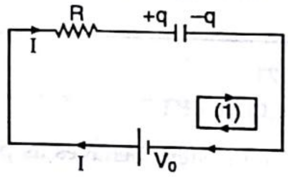


Fig. 3.170A

or $$V_0 - \frac{q}{C} = IR$$

or $$\frac{CV_0 - q}{C} = R\frac{dq}{dt}$$

or $$RC\int_0^q \frac{dq}{CV_0 - q} = \int_0^t dt$$

After integrating, we get

or $$q = CV_0\left(1 - e^{-t/RC}\right) \quad \text{or} \quad \frac{q}{C} = V_0\left(1 - e^{-t/RC}\right)$$

or $$V = V_0\left(1 - e^{-t/RC}\right)$$

∴ $$\frac{V}{V_0} = 1 - e^{-t/RC}$$

or $$1 - \frac{V}{V_0} = e^{-t/RC}$$

Taking logrithm on both sides, we get

$$ln\left[1 - \frac{V}{V_0}\right] = -\frac{t}{RC}\ ln\, e$$

$$ln\left[1-\frac{V}{V_0}\right]=-\frac{t}{RC}$$

$$t=-RC\,ln\left[1-\frac{V}{V_0}\right]$$

Now, $V=0.9\,V_0$

Putting the values, we get $t=0.6\times10^{-6}\text{ s}=0.6\ \mu\text{s}$

YOUR STEP

1. Consider the circuit shown in fig. 3.170B. I_1, I_2, and I_3 are the currents flowing through the resistances R_1, R_2 and R_3 respectively. V_1, V_2, V_3 and V_C are the corresponding voltage drops across resistances and capacitor.
   The switch shuts at $t=0$. Find the currents I_1, I_2 and I_3 and the voltage drops V_1, V_2, V_3 and V_C as a function of time.

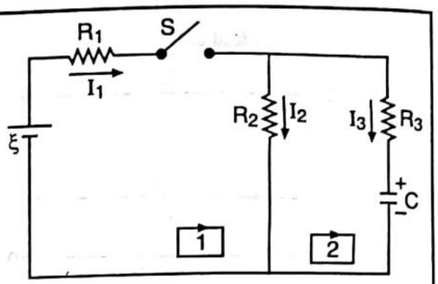


Fig. 3.170B

$$\left\{\begin{array}{l} I_1=\dfrac{\xi}{R_1+R_2}\left[1+\dfrac{R_2^2\;e^{-t/\lambda}}{R_1R_2+R_2R_3+R_3R_1}\right] \text{ and } V_1=I_1R_1 \text{ and } I_2=\dfrac{\xi}{R_1+R_2}\left[1-\dfrac{R_1R_2\quad e^{-t/\lambda}}{R_1R_2+R_2R_3+R_3R_1}\right] \\ V_2=I_2R_2 \text{ and } I_3=\left(\dfrac{\xi R_2}{R_1R_2+R_2R_3+R_3R_1}\right)e^{-t/\lambda} \text{ and } V_3=I_3R_3,\ V_C=\dfrac{R_2\xi}{R_1+R_2}(1-e^{-t/\lambda}) \\ \text{Here } \lambda=\left(\dfrac{R_1R_2+R_2R_3+R_1R_3}{R_1+R_2}\right)C \end{array}\right\}$$

2. A resistor of resistance R and a 2μF capacitor in series are connected at a 200V direct supply. Across the capacitor there is a neon bulb that strikes at 120V. Calculate the value of R to make the bulb strike 5 second after the switch has been closed.

{$2.73\times10^6\ \Omega$}

§ 3.171

➢ CONCEPT

The system behaves as parallel combination of a capacitor $C=\varepsilon_0\,\varepsilon\dfrac{S}{d}$ and a resistance of

$$R=\rho\frac{d}{S}$$

➢ DISCUSSION : At $t=0$, charge on capacitor is q_0 (shown in fig. 3.171A)

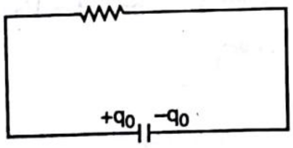


Fig. 3.171A

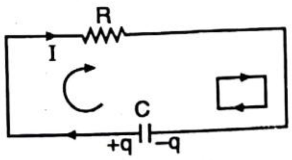


Fig. 3.171B

when $t>0$, charge starts to leak. Due to this, charges on plates start to decrease. At an instant t, the circuit is shown in fig. 3.171B.

SOLUTION : Applying loop rule in circuit shown in fig. 3.171B.

$$+\frac{q}{C}-IR=0$$

But $$I = -\frac{dq}{dt}$$ (Since, charge decreases with increase of time)

$\therefore$ $$\frac{q}{C} + \frac{Rdq}{dt} = 0 \quad \text{or} \quad \frac{dq}{dt} = -\frac{q}{RC}$$

or $$\int_{q_0}^{q} \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

After integrating, $$q = q_0\, e^{-t/RC}$$

Putting the values of R and C, we get $$q = q_0\, e^{-t/\varepsilon_0\varepsilon\rho}$$

According to problem, when $$t = \tau,\ q = \frac{q_0}{2}$$

$\therefore$ $$\frac{q_0}{2} = q_0\, e^{-\tau/\rho\varepsilon_0\varepsilon} \quad \text{or} \quad \frac{1}{2} = e^{-\tau/\rho\varepsilon_0\varepsilon}$$

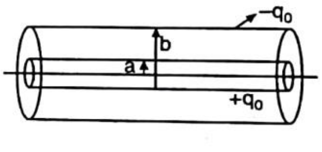
or $$-\ln 2 = -\frac{t}{\rho\varepsilon_0\varepsilon}$$

$\therefore$ $$\rho = \frac{\tau}{\varepsilon_0\varepsilon \ln 2}$$

On putting the values, we get $$\rho = 1.4 \times 10^{13}\ \Omega\,\text{m}$$

YOUR STEP

A capacitor is composed of two long concentric co-axial cylindrical tubes. The tubes are of radii a and b ($a < b$), and of the same length L. It is given also that $b << L$. The outer tube carries a charge $-q_0$ and the inner one carries a charge $+q_0$ (shown in fig. 3.171C). Keeping the potential drop across the capacitor fixed, the space between the tubes is filled with a medium of specific resistivity ρ (and of unit dielectric constant). Compute the net current and the resistance of system.



$$\left\{ I = \frac{2\pi VL}{\rho \ln b/a} \text{ and } R = \frac{\rho \ln \frac{c}{a}}{2\pi L} \right\}$$

Fig. 3.171C

§ 3.172

➢ **CONCEPT**

The concept is similar to previous problem.

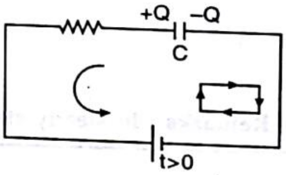


Fig. 3.172A

SOLUTION : The problem is solved by following steps.

Step I : Draw circuit at $t < 0$.

In this situation, circuit is in steady state. So, current in the circuit is zero.

According to loop rule, $$\xi - \frac{Q}{C} = 0$$

$\because$ $$Q = C\xi$$

Step II : Discuss the situation at $t = 0$. Since, at $t = 0$, the capacity of capacitor is abruptly decreased.

i.e., $$C_1 = \frac{C}{\eta}$$

Due to this, voltage of capacitor abruptly increases. this voltage is greater than the emf of cell. So, charge starts to decrease for getting new steady. The direction of flow of charge is shown in fig. 3.171B.

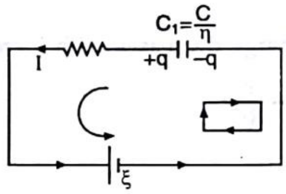


Fig. 3.172B

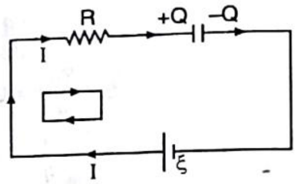


Fig. 3.172C

Step III : Draw the circuit at an instant t (> 0).
At an instant t, the circuit is shown in fig. 3.172 C.

According to loop rule, $$-\frac{q}{C_1} - IR + \xi = 0$$

Here $$I = -\frac{dq}{dt}$$ (since, charge decreases)

$\therefore$ $$\xi - \frac{q}{C_1} = R\frac{dq}{dt}$$

or $$\xi C_1 - q = RC_1\frac{dq}{dt} \quad \text{or} \quad \frac{dq}{\xi C_1 - q} = \frac{dt}{RC_1}$$

or $$\int_Q^q \frac{dq}{\xi C_1 - q} = \frac{1}{RC_1}\int_0^t dt \quad \text{or} \quad \left[-\ln(\xi C_1 - q)\right]_Q^q = \frac{t}{RC_1}$$

or $$-\left[\ln\frac{\xi C_1 - q}{\xi C_1 - Q}\right] = \frac{t}{RC_1}$$

or $$\ln\frac{\xi C_1 - q}{\xi C_1 - Q} = -\frac{t}{RC_1}$$

or $$\xi C_1 - q = (\xi C_1 - Q)\,e^{-t/RC_1}$$

or $$\xi\frac{C}{\eta} - q = \left(\xi\frac{C}{\eta} - C\xi\right)e^{-t\eta/RC}$$

or $$\frac{\xi C}{\eta} - q = \xi C\left(\frac{1-\eta}{\eta}\right)e^{-t\eta/RC}$$

Differentiating both sides with respect to

$$-\frac{dq}{dt} = (\xi C)\left(\frac{1-\eta}{\eta}\right)\left(\frac{-\eta}{RC}\right)e^{-t\eta/RC}$$

or $$I = \frac{\xi}{R}(\eta - 1)\,e^{-t\eta/RC}$$

Remarks : In steady state, capacitor behaves as a resistor of infinite resistance.

YOUR STEP

1. A capacitor of capacity 2μF, resistance $10 \times 10^6\ \Omega$ and a battery of emf 100V are connected in series. The capacitor is initially uncharged. The switch is thrown to the up position for 20 second and then quickly reversed to the down position. How much energy is eventually dissipated in the resistance?
2. A battery is sending current through an external resistance R. The terminals of the battery are connected to a capacitor and the charge communicated is found to be q_1. On repeating the experiment with an infinite external resistance, the charge given to the capacitor is q_2. Find the internal resistance of the battery.

$$\left\{1.\ 10^{-2}\text{ joule} \quad 2.\ \frac{R(q_2 - q_1)}{q_1}\right\}$$

§ 3.173

➢ **CONCEPT**

The resistance of ammeter is zero. But the resistance of voltmeter is not zero. Let the resistance of voltmeter is R_0.

SOLUTION : Draw the circuit before connecting external resistance.

The circuit is shown in fig. 3.173A.

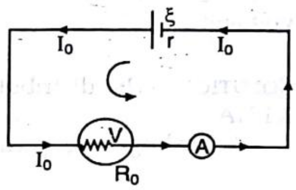


Fig. 3.173A

According to loop rule, $\xi - I_0 r - I_0 R_0 = 0$

$$\therefore \quad I_0 = \frac{\xi}{r + R_0} \quad \text{...(i)}$$

In this case, reading of ammeter is I_0.

Also the reading of voltmeter is

$$V_0 = I_0 R_0$$

Step II : Draw the circuit after connecting external resistance. The circuit is shown in fig. 3.173B.

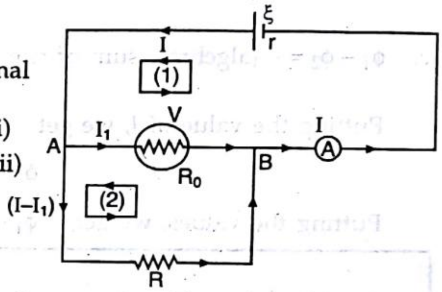


Fig. 3.173B

In loop (1), $\xi - Ir - I_1 R_0 = 0$ ...(ii)

In loop (2) $-(I - I_1) R + I_1 R_0 = 0$ ...(iii)

After solving equ. (ii) and (iii), we get,

$$I = \frac{\xi}{r + \dfrac{R_0 R}{R + R_0}}$$

The reading of voltmeter is V = potential difference between A and B

$$= \xi - Ir = \xi - \frac{\xi r}{r + \dfrac{RR_0}{R + R_0}}$$

According to problem, $V = \dfrac{V_0}{\eta}$

$$\text{or} \quad \xi - \frac{\xi r}{r + \dfrac{RR_0}{R + R_0}} = \frac{I_0 R_0}{\eta} = \frac{\xi R_0}{\eta (r + R_0)} \quad \text{...(iv)}$$

Also, $I = \eta I_0$

$$\text{or} \quad \frac{\xi}{r + \dfrac{R_0 R}{R + R_0}} = \frac{\eta \xi}{r + R_0} \quad \text{...(v)}$$

After solving equ. (iv) and (v), we get

$$\xi - \frac{\xi r}{r + \dfrac{RR_0}{R + R_0}} = \frac{\xi}{\eta + 1}$$

$$\therefore \quad V = \frac{\xi}{\eta + 1} = 2.0 \text{ volt}$$

YOUR STEP

A cell of emf 3.4 volt and internal resistance 3Ω is connected to an ammeter having resistance 2Ω and to an external resistance 100Ω. When a voltmeter is connected across the 100Ω resistance the ammeter reading is 0.04 ampere. Find the voltage reading by the voltmeter and its resistance. Had the voltmeter been an ideal one? What would have been its reading?

[$V = 3.2$ volt, $r = 400\Omega$, $V = 3.23$...

§ 3.174

➢ **CONCEPT**

The potential difference between two points $= \phi_1 - \phi_2 = -$ {algebraic sum of rise up and drop up of voltage}

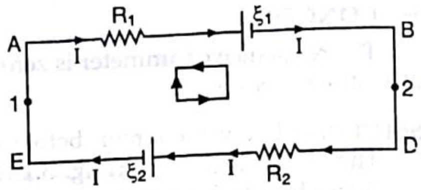


Fig. 3.174A

SOLUTION : The distribution of current is shown in Fig. 3.174A.

According to loop rule, (in loop *ABDEA*)

$$-IR_1 + \xi_1 - IR_2 - \xi_2 = 0$$

$$\therefore \quad I = \frac{\xi_1 - \xi_2}{R_1 + R_2} \qquad \text{...(i)}$$

$\therefore \ \phi_1 - \phi_2 = -$ {algebraic sum of rise and drop up of voltage throw path (1A B2).

$$= -\{-IR_1 + \xi_1\} = IR_1 - \xi_1$$

Putting the value of *I*, we get

$$\phi_1 - \phi_2 = \frac{(\xi_1 - \xi_2) R_1}{R_1 + R_2} - \xi_1$$

Putting the values, we get, $\phi_1 - \phi_2 = -4\text{V}$

YOUR STEP

An electrical circuit is shown in fig. 3.174B. Calculate the potential difference across the resistor of 400Ω as it will be measured by the voltmeter *V* of resistance 400Ω.

$$\left\{\frac{20}{3}\text{V}\right\}$$

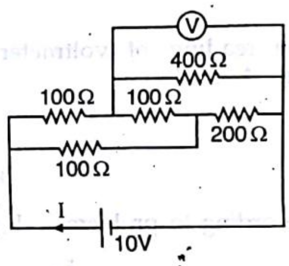


Fig. 3.174B

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§ 3.175

➢ **CONCEPT**

The concept is same as previous problem.

SOLUTION :

The circuit of problem is shown in fig. 3.175A.

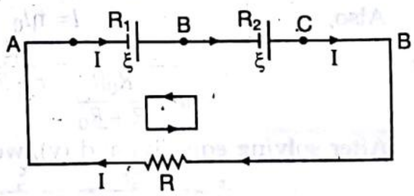

Fig. 3.175A

According to loop rule, $\xi - IR_1 + \xi - IR_2 - IR = 0$

$$\therefore \quad I = \frac{2\xi}{R_1 + R_2 + R}$$

Potential difference between terminals of first cell is

$$\phi_A - \phi_B = -\{\xi - IR_1\}$$

$$\phi_A - \phi_B = IR_1 - \xi = \frac{2\xi R_1}{R_1 + R_2 + R} - \xi$$

problem, $\phi_A - \phi_B = 0$

$$\frac{2\xi R_1}{R_1 + R_2 + R} - \xi = 0$$

$$\frac{2R_1}{+ R_2 + R} = 1$$

or $$2R_1 = R_1 + R_2 + R \quad \text{or } R = R_1 - R_2$$

$\therefore$ $$R_2 > R_1$$

So R is negative, which is phisically not possible.

The potential difference across second cell is $\phi_B - \phi_C = -\{\xi - IR_2\}$

or $$\phi_B - \phi_C = IR_2 - \xi = \frac{2\xi R_2}{R_1 + R_2 + R} - \xi$$

According to problem, $\phi_B - \phi_C = 0$

After solving, we obtain $R = R_2 - R_1$

YOUR STEP

(a) Calculate the current through each source of emf in fig. 3.175B.

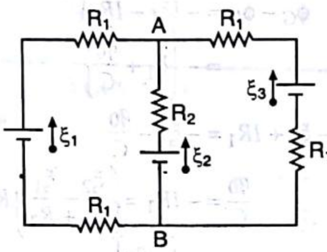

Fig. 3.175B

(b) Calculate $V_B - V_A$. Assume that $R_1 = 1.20\Omega$, $R_2 = 2.30\Omega$, $\xi_1 = 2.00V$, $\xi_2 = 3.80V$, and $\xi_3 = 5.00V$.

{(a) $i_1 = 668$mA, down $i_2 = 85.7$mA, up, $i_3 = 582$mA, up, (b) -3.60 volt}

§ 3.176

➢ **CONCEPT**

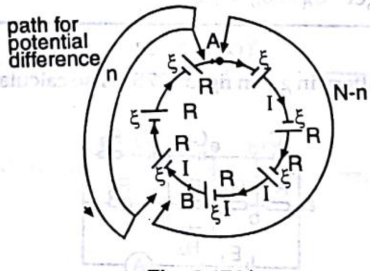

Fig. 3.176A

SOLUTION : According to loop rule, $(\xi - IR) + (\xi - IR) + \ldots +$ up to N terms $= 0$

or $$N\xi - NIR = 0$$

$\therefore$ $$I = \frac{N\xi}{NR} = \frac{\xi}{R} = \frac{\alpha R}{R} = \alpha$$

(b) $\phi_A - \phi_B = -\{(-\xi + IR) + (-\xi + IR) + \ldots$ up to n terms$\}$

$$= n\xi - nIR = n\alpha R - n\alpha R = 0$$

YOUR STEP

n cell each of emf E and internal resistance r are connected as shown in the fig. 3.176B. (a) Calculate the current, (b) Calculate V_{MN}, if they divide the circuit into x and $n - x$ cells, and (c) Calculate current in the resistor which is connected across M and N.

{(a) $\frac{E}{r}$ (b) zero (c) zero}

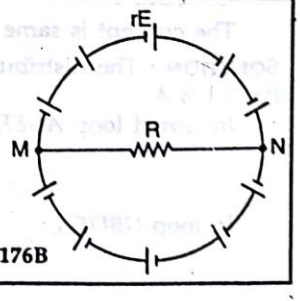

Fig. 3.176B

§ 3.177

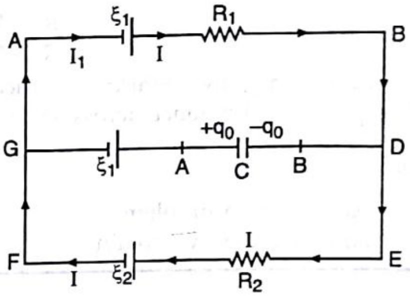

Fig. 3.176A

➢ CONCEPT

In steady state, current throught capacitor branch is zero. Also charge on capacitor is maximum.

SOLUTION : The current distribution in the circuit is shown in Fig. 3.177A.

In closed loop *ABEFA*,

$$\xi_1 - IR_1 - IR_2 - \xi_2 = 0$$

$\therefore$
$$I = \frac{\xi_1 - \xi_2}{R_1 + R_2}$$

$$\phi_G - \phi_D = -\{\xi_1 - IR_2\} \quad \text{(through } GABD\text{)}$$

$$= -\left\{\xi_1 + \frac{q_0}{C}\right\} \quad \text{(through capacitor branch)}$$

or
$$-\xi_1 + IR_1 = -\xi_1 - \frac{q_0}{C}$$

$\therefore$
$$\frac{q_0}{C} = -IR_1 = \left(\frac{\xi_2 - \xi_1}{R_1 + R_2}\right)R_1$$

$\therefore$
$$\phi_A - \phi_B = -\left\{+\frac{q_0}{C}\right\}$$

$$= -\frac{q_0}{C} = \left(\frac{\xi_1 - \xi_2}{R_1 + R_2}\right)R_1$$

On putting the values, we get $\phi_A - \phi_B = -0.5V$

YOUR STEP

Calculate the current through battery in given fig. 3.177B. Also calculate the charge on each condenser.

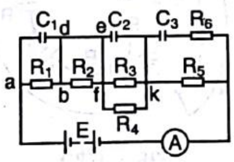

Fig. 3.177B

If $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 4\Omega$, $C_1 = 3\mu F$, $C_2 = 6\mu F$, $C_3 = 6\mu F$ and $E = 20$ volt.

{2 ampere, 24×10^{-6} C, 12×10^{-6} C, 48×10^{-6} C}

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§ 3.178

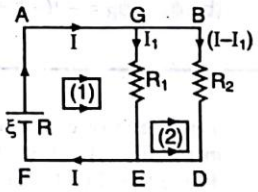


Fig. 3.178A

➢ CONCEPT

The concept is same as of previous problem.

SOLUTION : The distribution of current in the circuit is shown in fig. 3.178 A.

In closed loop *AGEFA*,
$$\xi - IR - I_1 R_1 = 0$$

$\therefore$
$$I = \frac{\xi - I_1 R_1}{R} \quad \text{...(i)}$$

In loop *GBDEG*,
$$-(I - I_1) R_2 + I_1 R_1 = 0 \quad \text{...(ii)}$$

After putting the value of I, we get $I_1 = \dfrac{\xi R_2}{RR_1 + R_1R_2 + RR_2} = 1.2\text{A}$

From equ. (ii), we get $I - I_1 = \dfrac{I_1 R_1}{R_2}$

Putting the value of I_1, we get $I - I_1 = \dfrac{\xi R_1}{RR_1 + R_1R_2 + RR_2}$

Hence, current through R_2 is $I_2 = I - I_1$

$$= \frac{\xi R_1}{RR_1 + R_1R_2 + RR_2}$$

On putting the values, we get $I_2 = 0.8\text{A}$

YOUR STEP

Find the current in branch AB of the circuit.

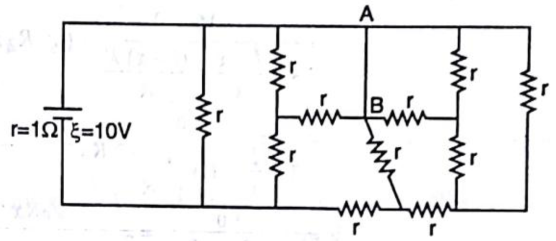


Fig. 3.178B

{7.62A}

§ 3.179

➢ **CONCEPT**

Potentiometer consists of uniform homogeneous wire. So, resistance per unit length of wire (*i.e.*,λ) remains constant.

Here $\lambda = \dfrac{R_0}{l}$

SOLUTION : The corresponding circuit is shown in fig. 3.179A.

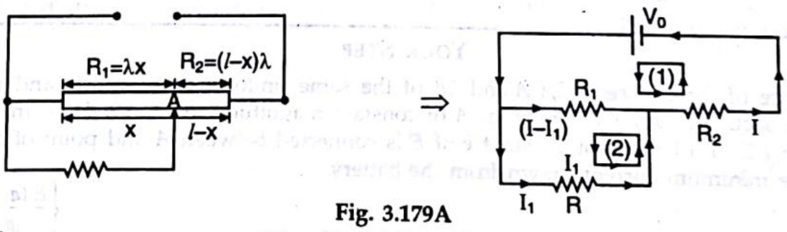


Fig. 3.179A

In loop (1), $V_0 - (I - I_1) R_1 - IR_2 = 0$ ...(i)

In loop (2), $-I_1R + (I - I_1) R_1 = 0$ ...(ii)

Adding equ. (i) and (ii), we get $V_0 - IR_2 - I_1R = 0$

$$\therefore \quad I = \frac{V_0 - I_1R}{R_2}$$

Putting the value of I in equ. (ii), we get

$$-I_1 R + (I - I_1) R_1 = 0$$

or $$-I_1(R+R_1)+IR_1=0$$

$\therefore$ $$IR_1=I_1(R+R_1)$$

or $$\left(\frac{V_0-I_1R}{R_2}\right)R_1=I_1(R+R_1) \text{ or } \frac{V_0R_1}{R_2}-\frac{I_1RR_1}{R_2}=I_1(R+R_1)$$

or $$\frac{(V_0R_1)}{R_2}=I_1\left(\frac{RR_1}{R_2}+R+R_1\right) \text{or } \frac{V_0R_1}{R_2}=I_1\left(\frac{RR_1+RR_2+R_1R_2}{R_2}\right)$$

$\therefore$ $$I_1=\frac{V_0R_1}{RR_1+RR_2+R_1R_2}$$

The voltage fed to device is $$V=I_1R=\frac{V_0R_1R}{RR_1+RR_2+R_1R_2}$$

Putting the value of R_1 and R_2, we get $$V=\frac{V_0}{\frac{RR_1}{RR_1}+\frac{RR_2}{RR_1}+\frac{R_1R_2}{RR_1}}=\frac{V_0}{1+\frac{R_2}{R_1}+\frac{R_2}{R}}$$

$$=\frac{V_0}{1+\frac{l-x}{x}+\frac{(l-x)\lambda}{R}} \quad (\because R_2=(l-x)\lambda \text{ and } R_1=x\lambda)$$

$$=\frac{V_0}{1+\frac{l}{x}-1+\left(\frac{l-x}{R}\right)\frac{R_0}{l}} \quad \left(\because \lambda=\frac{R_0}{l}\right)$$

or $$V=\frac{V_0}{\frac{l}{x}+\frac{R_0}{R}-\frac{R_0x}{Rl}}=\frac{V_0Rx}{\left[Rl+R_0(l-x)\frac{x}{l}\right]}$$

(multiply by Rx to numerator and denominator)

$\therefore$ $$V=\frac{V_0}{\frac{l}{x}+\frac{R_0}{R}-\frac{R_0x}{Rl}}$$

when $R>>R_0$, then $\frac{R_0}{R}$ and $\frac{R_0x}{Rl}$ may be neglected.

$\therefore$ $$V=\frac{V_0}{\frac{l}{x}}=\frac{V_0x}{l} \quad \text{for } R>>R_0$$

YOUR STEP

The resistance of three wires BC, CA and AB of the same uniform cross-section and material are a, b and c respectively, another wire from A of constant magnitude resistance d can make a sliding contact with BC. If a battery of constant emf E is connected between A and point of contact with BC. Find the minimum current drawn from the battery.

$$\left\{\frac{E(a+b+c+4d)}{(a+b+c)d}\right\}$$

§ 3.180

➢ **CONCEPT**

The problem is solved by following steps :

Step I : Draw the circuit :

Step II : Connect the point M and N by an external resistance R :

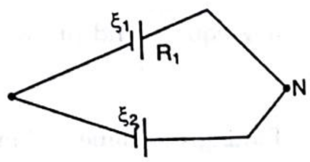


Fig. 3.180A

In loop (1), $\xi_1 - IR_1 - \xi_2 - (I - I_1) R_2 = 0$ ...(i)

In loop (2), $-I_1R + \xi_2 + (I - I_1) R_2 = 0$ ...(ii)

After solving equ. (i) and (ii) we get

$$I_1 = \frac{\xi_1R_2 + \xi_2R_1}{R(R_1 + R_2) + R_1R_2}$$

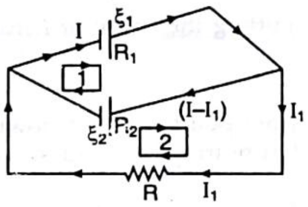


Fig. 3.180B

Dividing Nr. and Dr. by $(R_1 + R_2)$, we get

$$= \frac{\dfrac{\xi_1R_2 + \xi_2R_1}{R_1 + R_2}}{R + \dfrac{R_1R_2}{R_1 + R_2}} \quad ...(iii)$$

Step III : Replace the cells by an equivalent cell of emf ξ and internal resistance r

The corresponding circuit is shown in fig. 3.180C.

According to loop rule, $\xi - I_1r - I_1R = 0$

$$\therefore \quad I_1 = \frac{\xi}{r + R} \quad ...(iv)$$

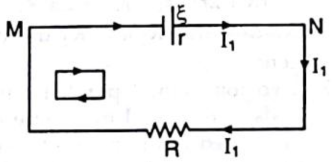


Fig. 3.180C

Step IV : comparing Eq. (iii) and (iv), we get

$$r = \frac{R_1R_2}{R_1 + R_2} \quad \text{and} \quad \xi = \frac{\xi_1R_2 + \xi_2R_1}{R_1 + R_2}$$

YOUR STEP

Find the equivalent emf of the three batteries shown in fig 3.180D.

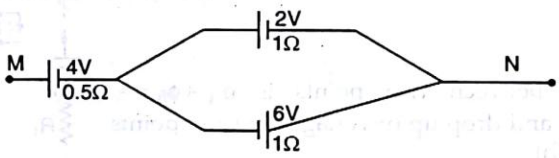


Fig. 3.180D

{ 2V }

§ 3.181

➢ **CONCEPT**

SOLUTION : The distribution of current in the circuit is shown in Fig 3.181 A.

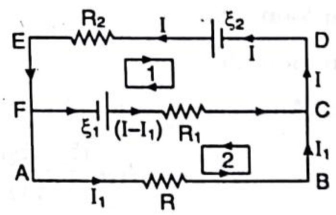


Fig. 3.181A

In loop (1),

$$\xi_2 - IR_2 + \xi_1 - (I - I_1) R_1 = 0$$

or $$\xi_2 + \xi_1 - IR_2 - IR_1 + I_1R_1 = 0$$

or $$\xi_2 + \xi_1 - I(R_1 + R_2) + I_1R_1 = 0$$

$$\therefore \quad I = \frac{\xi_1 + \xi_2}{R_1 + R_2} + \left(\frac{R_1}{R_1 + R_2}\right) I_1 \quad ...(i)$$

In loop (2),

$$-I_1R + (I - I_1) R_1 - \xi_1 = 0$$

or $$-I_1(R + R_1) + IR_1 - \xi_1 = 0$$

$$\therefore \quad I_1 = \frac{\xi - IR_1}{R + R_1}$$

Putting the value of I from Eq. (i), and then solving, we get,

$$I_1 = \frac{\xi_2 R_1 - \xi_1 R_2}{RR_1 + RR_2 + R_1 R_2}$$

from point A to B (shown in Fig. 3.181A.)
On putting the values, we get

$$I_1 = 0.02A$$

from point A to B *i.e.*, from left to right

YOUR STEP

1. If points a and b in fig. 3.181B are connected by a wire of resistance r, show that the current in the wire is $i = \frac{\xi(R_s - R_x)}{(R+2r)(R_s+R_x)+2R_sR_x}$, where ξ is the emf of the battery. Assume that R_1 and R_2 are equal ($R_1 = R_2 = R$) and that R_0 equals zero.

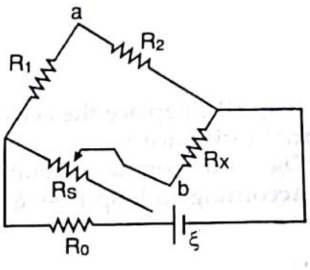


Fig. 3.181B

2. Two long equal parallel wires $AB, A'B'$ of length l, have their ends B, B' joined by a wire of negligible resistance, while A, A' are joined to the poles of a cell whose resistance is equal to that of a length r of the wire. A similar cell is placed as a bridge across the wires at a distance x from A, A'. Show that the effect of second cell is to increase the current in BB' in the ratio

$$\frac{2(2l+r)(2+r)}{\{r(4l+r)+2x(2l-r)-4x^2\}}$$

§ 3.182

➢ CONCEPT

Potential difference between two points is $\phi_A - \phi_B = -$ {algebraic sum of rise up and drop up of voltage between points A and B through any path}

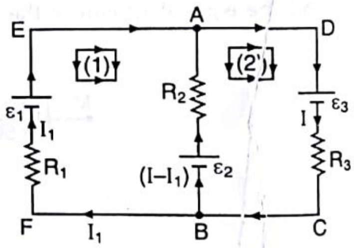


Fig. 3.182A

SOLUTION : The distribution of current in the circuit is shown in fig. 3.182A.

In loop (1), $\quad - I_1 R_1 + \xi_1 + (I - I_1) R_2 - \xi_2 = 0 \quad$ ...(i)

In loop (2), $\quad - \xi_3 - IR_3 + \xi_2 - (I - I_1) R_2 = 0 \quad$ ...(ii)

(a) After solving equ. (i) and (ii), we get

$$I_1 = \frac{(\xi_1 - \xi_2) R_3 + R_2(\xi_1 + \xi_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} - 0.06\text{A} \quad \text{...(iii)}$$

(b) Taking potential difference between point A and B through path $AEFB$,

$$\phi_A - \phi_B = -\{-\xi_1 + I_1 R_1\}$$
$$= \xi_1 - I_1 R_1$$

Putting the value of I_1 from equ. (iii), we get

$$\phi_A - \phi_B = 0.9\text{V}$$

YOUR STEP

1. In the given fig. 3.182B. calculate V_{MN}, if $E_1 = 1V$, $E_2 = 4V$, $E_3 = 3V$, $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, $R_4 = 4\Omega$, $R_5 = R_6 = 5\Omega$

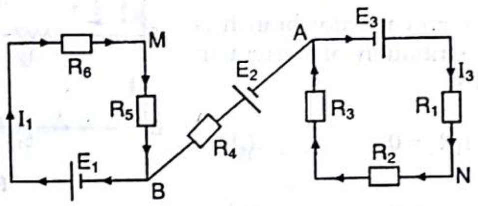


Fig. 3.182B

{ 2 volt }

§ 3.183

➢ **CONCEPT**

For the concept of rise up and drop up of voltage, see the concept of problem 3.170.

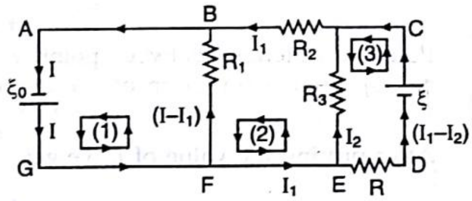


Fig. 3.183A

SOLUTION : The distribution of current in the circuit is shown in Fig. 3.183A.

In loop (1),

$$\xi_0 - (I - I_1)\, R_1 = 0 \qquad \text{...(i)}$$

In loop (2),

$$- I_2R_3 - I_1R_2 + (I - I_1)\, R_1 = 0 \qquad \text{...(ii)}$$

In loop (3),

$$\xi + I_2R_3 - (I_1 - I_2)\, R = 0 \qquad \text{...(iii)}$$

After solving equ. (i), (ii) and (iii), we get

$$I_1 - I_2 = \frac{\xi\,(R_2 + R_3) + \xi_0 R_3}{R\,(R_2 + R_3) + R_2R_3}$$

YOUR STEP

A network of resistance is constructed with R_1 and R_2 as shown in the figure 3.183B. The potential at the points 1, 2, 3, . . . , N are V_1, V_2, V_3, . . . , V_n respectively each having a potential K times smaller than previous one. Find :

(a) $\frac{R_1}{R_2}$ and $\frac{R_2}{R_3}$ in terms of K.

(b) Current that passes through the resistance R_2 nearest to the V_0 in terms V_0, K and R_3.

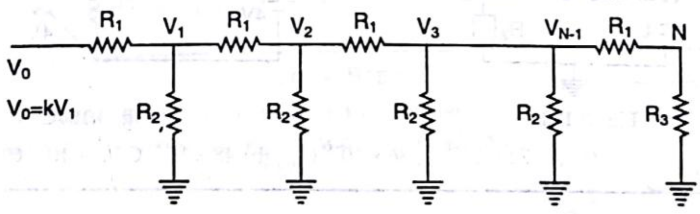


Fig. 3.183B

$\left\{ \text{(a) } \frac{(K-1)^2}{K};\ \frac{K}{K-1} \quad \text{(b) } \left(K - \frac{1}{K^2}\right)\frac{V_0}{R_3} \right\}$

§ 3.184

➢ CONCEPT

The concept of problem is similar to problem 3.177.

SOLUTION : The current through capacitor branch is zero in steady state. The distribution of current in circuit is shown in fig. 3.184A.

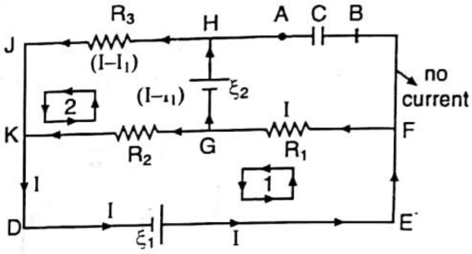


Fig. 3.184A

In loop (1), (DEFGKD)

$$\xi_1 - IR_1 - I_1R_2 = 0 \quad \text{...(i)}$$

In loop (2) (GHJKG)

$$\xi_2 - (I - I_1)R_3 + I_1R_2 = 0 \quad \text{...(ii)}$$

After solving equ. (i) and (ii), we get

$$I = \frac{\xi_1(R_2 + R_3) + \xi_2 R_2}{R_1R_2 + R_1R_3 + R_2R_3}$$

Potential difference between points F and H through paths FGH.

$\phi_F - \phi_H = -$(algebraic sum of rise up and drop up of voltage)

$$= -\{-IR_1 + \xi_q\} = IR_1 - \xi_2$$

After putting the value of I, we get

$$\phi_F - \phi_H = \frac{\xi_1R_1(R_2 + R_3) - \xi_2R_3(R_1 + R_2)}{R_1R_2 + R_1R_3 + R_2R_3}$$

or

$$\phi_H - \phi_F = \frac{\xi_2R_3(R_1 + R_2) - \xi_1R_1(R_2 + R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

From fig., $\phi_F = \phi_B$, $\phi_H = \phi_A$

$\therefore$

$$\phi_B - \phi_A = \phi_H - \phi_F = \frac{\xi_2R_3(R_1 + R_2) - \xi_1R_1(R_2 + R_3)}{R_1R_2 + R_1R_3 + R_2R_3}$$

On putting the values, we get $\phi_A - \phi_B = -1V$

YOUR STEP

1. In the given Fig. 3.184B, calculate the charge on each condenser when the key is (a) open and (b) the key is closed, if $R_1 = 4\Omega$, $R_2 = 2\Omega$, $C_1 = 6\mu F$, $C_2 = 3\mu F$ and $V_{MN} = 12V$
2. In the network shown in the figure 3.184C, calculate the potential difference between A and B.

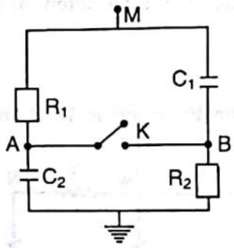


Fig. 3.184B

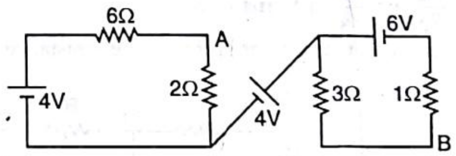


Fig. 3.184C

{1. (a) 72×10^{-6} C, 36×10^{-6} C, (b) 48×10^{-6} C, 12×10^{-6} C, 2. $V_{BA} = 0.5$ volt}

§ 3.185

➢ CONCEPT

The problem is based upon Kirchhoff's current rule (KCL).

Mathematically, $\Sigma i = 0$

~~Also,~~ ohm's law is applicable *i.e.*, $\Delta\phi = IR$

SOLUTION : The distribution of current is shown in fig. 3.185A.

According to *KCL*, $I_1 = I_2 + I_3$ ...(i)

According to ohm's law,

$\phi_1 - \phi_0 = I_1 R_1$ ...(ii)

Similarly, $\phi_0 - \phi_2 = I_2 R_2$ ...(iii)

Similarly, $\phi_0 - \phi_3 = I_3 R_3$ ...(iv)

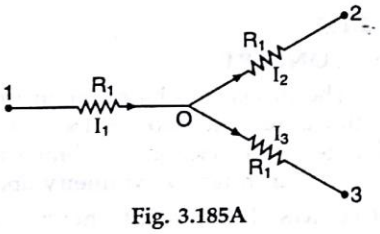


Fig. 3.185A

After solving Eq. (i), (ii), (iii) and (iv), we get

$$i_1 = \frac{R_3(\phi_1 - \phi_2) + R_2(\phi_1 - \phi_3)}{R_1R_2 + R_1R_3 + R_2R_3}$$

On putting the values, we get $i_1 = 0.2\text{A}$

YOUR STEP

In the fig. 3.185B calculate the potential difference between point (a) M and N (b) M and A (c) B and A (d) N and B.

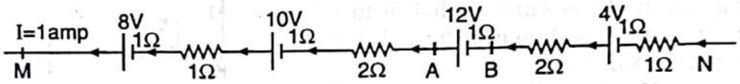


Fig. 3.185B

(a) 6 volt (b) 13 volt (c) – 11 volt (d) 18 volt

§ 3.186

➢ **CONCEPT**

The concept is similar to problem 3.170.

SOLUTION : For the sake of convenience, an ideal battery of emf. $V = 25$ volt is connected between point A and B. The circuit is shown in fig. 3.186 A.

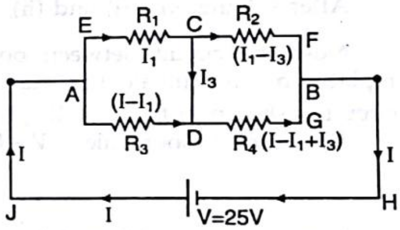


Fig. 3.186A

In loop *HJADGBH*,

$V - (I - I_1) R_3 - (I - I_1 + I_3) R_4 = 0$ ...(i)

In loop *ECDAE*,

$- I_1 R_1 + (I - I_1) R_3 = 0$ ...(ii)

In loop *CFBGDC*,

$- (I_1 - I_3) R_2 + (I - I_1 + I_3) R_2 = 0$ ...(iii)

After solving equ. (i) $$I_3 = \frac{V}{R_2}\left\{\frac{R_1 + R_2}{R_1[1 + R_2R_4(R_1 + R_3)/R_1R_3(R_2 + R_4)]} - 1\right\}$$

On putting the values, we get $I_3 = 1\text{A}$

YOUR STEP

In the circuit shown in fig. 3.186B, V_1 and V_2 are two voltmeters of resistances 3000Ω and 2000Ω respectively in addition $R_1 = 2000\Omega$, $R_2 = 3000\Omega$ and ideal battery of emf $E = 200\text{V}$

(a) Find the reading of V_1 and V_2

(i) when switch S is open

(ii) switch S is closed

(b) current through switch when it is closed

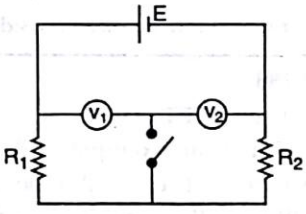


Fig. 3.186B

$\left\{ \text{(a) (i) 120V, 80V (ii) 100V, 100V,} \quad \text{(b) } \frac{1}{60} \text{ A from } D \text{ to } C \right\}$

§ 3.187

➢ CONCEPT

The problem is based upon rotatory symmetry. If a circuit is rotated through 180° about a point O and no change takes place in the dimensions of ciurcuit. Then circuit is in rotatory symmetry about that point.

SOLUTION : If the circuit (shown in fig. 3.187A) is rotated through 180° about middle point O of line CD, then no change takes place in the dimensions of circuit.

So, the circuit is in rotational symmetry about point O.

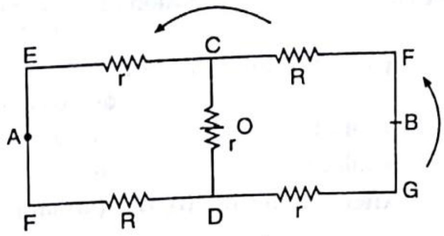


Fig. 3.187A

Due to rotation, branch CE is replaced by DG branch HC is replaced by FD.

So, current in CE is same as that of in DG.

Similarly current in HC is same as that of in FD.

distribution of current is shown in fig. 3.187 B.

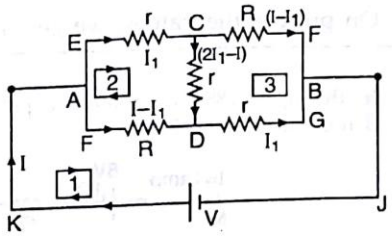


Fig. 3.187B

In loop (1), ($JKAFDGBJ$)

$$V-(I-I_1)R-I_1r=0 \qquad \text{...(i)}$$

In loop (2) ($ECDFAE$)

$$-I_1r-(2I_1-I)r+(I-I_1)R=0 \qquad \text{...(ii)}$$

After solving equ. (i) and (ii), we get $I=\dfrac{V(R+3r)}{r(r+3R)}$ ...(iii)

Now the circuit between points A and B may be replaced by an equivalent resistance R_{AB}. The equivalent circuit is shown in fig. 3.187C.

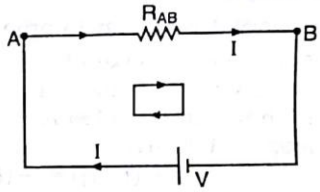


Fig. 3.187C

According to loop rule, $V-IR_{AB}=0$

$$\therefore \qquad I=\frac{V}{R_{AB}} \qquad \text{...(iv)}$$

Comparing equ. (iii) and (iv), we get

$$R_{AB}=\frac{r(r+3R)}{(R+3r)}$$

YOUR STEP

A wire forms a regular hexagon and the angular point are joined to the centre by wries each of which a resistance $\frac{1}{n}$ of the resistance of a side of the hexagon. Show that the resistance to a current entering at one angular point of the hexagon and leaving it by the opposite point is $\dfrac{2(n+3)}{(n+1)(n+4)}$ times the resistance of a side of the hexagon.

§ 3.188

➢ CONCEPT

This is an example of $R-C$ circuit in parallel.

SOLUTION : The distribution of charge and current in circuit at an instant t is shown in fig. 3.188 A.

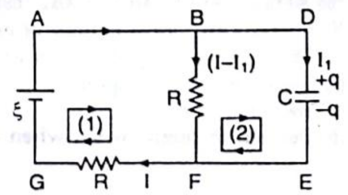


Fig. 3.188A

In loop (1) ($ABFGA$),

$$\xi-(I-I_1)R-IR=0$$

or

$$\xi-IR+I_1R-IR=0$$

or $$\xi - 2IR + I_1R = 0$$

$\therefore$ $$I = \frac{\xi + I_1R}{2R} \quad ...(i)$$

In loop $BDEFB$, $$-\frac{q}{C} + (I - I_1)\,R = 0$$

or $$-\frac{q}{C} + \left(\frac{\xi + I_1R}{2R} - I_1\right)R = 0$$

or $$-\frac{q}{C} + \left(\frac{\xi + I_1R - 2I_1R}{2R}\right)R = 0$$

or $$-\frac{q}{C} + \left(\frac{\xi - I_1R}{2}\right) = 0$$

or $$\frac{\xi}{2} - \frac{q}{C} = \frac{RI_1}{2}$$

But $$I_1 = \frac{dq}{dt}$$

$\therefore$ $$\frac{\xi}{2} - \frac{q}{C} = \frac{R}{2}\frac{dq}{dt}$$

or $$\xi C - 2q = \frac{2RC}{2}\frac{dq}{dt}$$

or $$\int_0^q \frac{dq}{\xi C - 2q} = \int_0^t \frac{dt}{RC}$$

After integrating, we get

$$q = \frac{\xi C}{2}(1 - e^{-2t/RC})$$

or $$\frac{q}{C} = \frac{\xi}{2}(1 - e^{-2t/RC})$$

$\therefore$ $$V = \frac{\xi}{2}(1\ e^{-2t/RC})$$

YOUR STEP

Two capacitors having capacities C_1 and C_2 are arranged as shown in fig. 3.188B and G is galvanometer. The resistances R_3 and R_4 are adjusted so that on closing the battery, no current is flowing through galvanometer G.
Treating the transient charging as steady, show that

$$R_3C_1 = R_4C_2$$

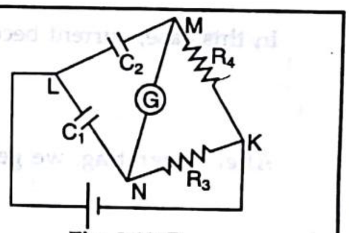


Fig. 3.188B

§ 3.189

➤ **CONCEPT**

Electric current $$I = \frac{dq}{dt}$$

Also, $$q = \int I dt$$

Also, heat generated in circuit is

$$\Delta H = \int I^2 R dt \text{ for variable current}$$

SOLUTION : (a) Since, current decreases uniformly with time. It means current is linear function of time.

$\therefore$ $$I = mt + C$$

when m and C are constant

At $t = 0, I = I_0$ $\therefore$ $$I_0 = m \times 0 + C$$

$\therefore$ $$C = I_0 \quad \therefore \quad I = mt + I_0$$

At $t = \Delta t, I = 0$ $\therefore$ $$0 = m\Delta t + I_0 \quad \therefore \quad m = -\frac{I_0}{\Delta t}$$

$\therefore$ $$I = -\frac{I_0}{\Delta t}t + I_0 \quad \therefore \quad I = I_0 - \frac{I_0}{\Delta t}t$$

But total charge transferred through circuit in time Δt is q.

$\therefore$ $$q = \int_0^{\Delta t} Idt = \int_0^{\Delta t}\left(I_0 - \frac{I_0}{\Delta t}t\right)dt = \left[I_0 t - \frac{I_0}{\Delta t}\frac{t^2}{2}\right]_0^{\Delta t} = I_0\Delta t - \frac{I_0\Delta t}{2} = \frac{I_0\Delta t}{2}$$

$\therefore$ $$I_0 = \frac{2q}{\Delta t}$$

$\therefore$ But $$\Delta H = \int_0^{\Delta t} I^2 Rdt = \int_0^{\Delta t}\left(I_0 - \frac{I_0}{\Delta t}t\right)^2 Rdt = \int_0^{\Delta t}\left(\frac{2q}{\Delta t} - \frac{2q}{\Delta t^2}t\right)^2 Rdt$$

After integrating, we get $$\phi = \Delta H = \frac{4q^2R}{3\Delta t}$$

(b) Let at $t = 0$, current is I_0. $\therefore$ At $t = \Delta t$, $I = \frac{I_0}{2}$

At $t = 2\Delta t$, $I = \frac{\frac{I_0}{2}}{2} = I_0\left(\frac{1}{2}\right)^2$

Similarly, At $t = n\Delta t$, $$I = I_0\left(\frac{1}{2}\right)^n$$

$\therefore$ $$I = I_0\left(\frac{1}{2}\right)^n = I_0\left(\frac{1}{2}\right)^{t/\Delta t} \qquad (\because\ t = n\Delta t)$$

In this case, current becomes zero after infinitely large time.

$$q = \int_0^{t\to\infty} Idt = \int_0^{t\to\infty} I_0\left(\frac{1}{2}\right)^{t/\Delta t} dt$$

After integrating, we get $$q = \frac{I_0\Delta t}{ln 2}$$

$\therefore$ $$I_0 = \frac{q\ln 2}{\Delta t}$$

$\therefore$ $$\phi = \Delta H = \int_0^{t\to\infty} I^2 Rdt = \int_0^{t\to\infty} I_0^2\left(\frac{1}{2}\right)^{2t/\Delta t} Rdt$$

$$= \int_0^{t\to\infty}\left(\frac{q\ln 2}{\Delta t}\right)^2\left(\frac{1}{2}\right)^{2t/\Delta t} Rdt$$

After integrating, we get $$\phi = \Delta H = \frac{q^2R\ln 2}{2\Delta t}$$

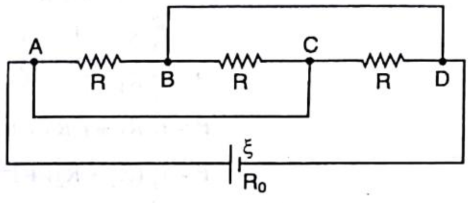
YOUR STEP

The equation for an instantaneous current in a material is given by $i = 5e^{-t}$. Calculate the amount of total charge flown through the material during the time interval $t_1 = 0$ second to $t_2 = \infty$ second

{1C}

§ 3.190

➢ **SOLUTION :**



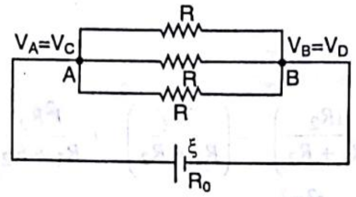
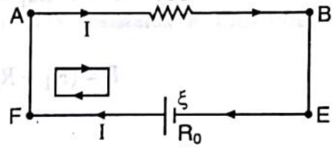
$\Rightarrow$  $\Rightarrow$ 

Fig. 3.190A

Applying loop rule in circuit $ABEFA$,

$$\xi - IR_0 - \frac{IR}{3} = 0$$

$\therefore$
$$I = \frac{\xi}{R_0 + \frac{R}{3}} = \frac{3C}{3R_0 + R}$$

The power in the circuit is
$$P = I^2\left(\frac{R}{3}\right) = \left(\frac{3\xi}{3R_0 + R}\right)^2\left(\frac{R}{3}\right) = \frac{3\xi^2 R}{(3R_0 + R)^2}$$

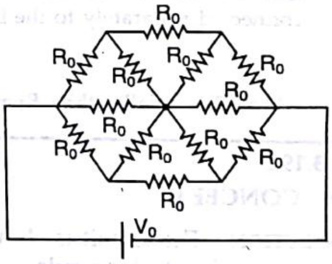
For maximum power generated in the circuit, is
$$\frac{dP}{dR} = 0$$

After solving, we get
$$R_0 = \frac{R}{3}, \quad \therefore\ R = 3R_0$$

YOUR STEP

1. In the given fig. 3.190B, find maximum thermal power developed in internal resistance of the cell.
2. A cell of emf E and internal resistence r is connected to a resistence R. Calculate the maximum power which is delivered in the external resistance.

$\left\{1.\ \frac{5V_0^2}{16R_0}\ 2.\ \frac{E^2}{4R}\right\}$



§ 3.191

➢ **CONCEPT**

The value of resistance is always positive. In parallel combination of two resistance, $i \propto \frac{1}{R}$

i.e.,
$$\frac{i_1}{i_2} = \frac{R_2}{R_1}$$

The thermal power dissipated in resistance is $P = I^2R$

SOLUTION : The circuit is shown in fig. 3.191A.

Total power in the circuit is, $P = I_1^2 R_1 + I_2^2 R_2$

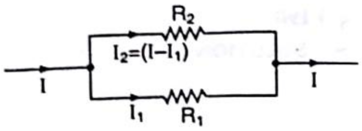


Fig. 3.191A

or $$P = I_1^2 R_1 + (I - I_1)^2 R_2$$

or $$P = I_1^2 R_1 + (I^2 + I_1^2 - 2II_1) R_2$$

or $$P = I_1^2 R_1 + I^2 R_2 + I_1^2 R_2 - 2II_1 R_2$$

or $$P = I_1^2 (R_1 + R_2) + I^2 R_2 - 2II_1 R_2$$

or $$P = I_1^2 (R_1 + R_2) - 2II_1 R_2 + I^2 R_2$$

or $$P = (R_1 + R_2)\left[I_1^2 - \frac{2II_1R_2}{R_1+R_2} + \left(\frac{IR_2}{R_1+R_2}\right)^2 - \left(\frac{IR_2}{R_1+R_2}\right)^2 + \frac{I^2R_2}{R_1+R_2}\right]$$

or $$P = (R_1 + R_2)\left[I_1 - \frac{IR_2}{R_1+R_2}\right]^2 - \frac{I^2R_2^2}{R_1+R_2} + I^2R_2$$

or $$P = (R_1 + R_2)\left[I_1 - \frac{IR_2}{R_1+R_2}\right]^2 - I^2\left(\frac{R_1R_2}{R_1+R_2}\right)$$

For being P minimum, $\quad P = \left(I_1 - \frac{IR_2}{R_1+R_2}\right)^2$ should be minimum

$$\Rightarrow \quad \left(I_1 - \frac{IR_2}{R_1+R_2}\right)^2 = 0$$

$\therefore$ $$I_1 = \frac{IR_2}{R_1+R_2} \quad \text{For minimum power dissipated}$$

> **YOUR STEP**
>
> Coils of resistance R_1 and R_2 are connected in parallel, to a battery of internal resistance r. Show that the rate of expenditure of energy in the two coils will exceed that in either of the coils, if connected separately to the battery, provided
>
> $$\frac{1}{r^2} > \frac{1}{R}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
>
> where R is smaller than R_1 and R_2.

§ 3.192

➢ **CONCEPT**

SOLUTION : The circuit is shown in fig 3.192A

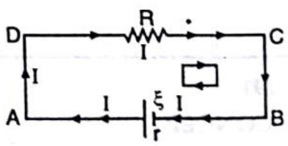


Fig. 3.192A

According to loop rule, $$\xi - Ir - IR = 0$$

$\therefore$ $$I = \frac{\xi}{r+R} \quad \text{...(i)}$$

Also, $$\phi_A - \phi_B = V = -\{-\xi + Ir\}$$

$\therefore$ $$V = \xi - Ir$$

$\therefore$ $$r = \frac{\xi - V}{I} \quad \text{...(ii)}$$

The thermal power generated in case is due to internal resistance.

$\therefore$ $$P = I^2 r = I^2\left(\frac{\xi - V}{I}\right)$$

$\therefore$ $$P = I(\xi - V)$$

On putting the values, we get $P = 0.6$ W

The net power developed by electrical forces is

$$P_1 = -VI$$
$$= -2 \text{ W}$$

YOUR STEP

Current is supplied by a motor working at rate W to two circuits in parallel, one of which containing cell of voltage E and resistance r on charge and the other lamps of total resistance R. Prove that the current through the lamp is

$$E + \frac{\left\{E^2 + 4rW\left(1 + \frac{r}{R}\right)\right\}^{\frac{1}{2}}}{2(R + r)}$$

§ 3.193

➢ **CONCEPT**

In the circuit, motor can be assumed as a resistance.

SOLUTION : The circuit is shown in Fig. 3.193A.

Here V = D.C. applied voltage

R = Resistance of winding armature

R_0 = Resistance of motor

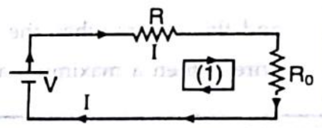


Fig. 3.193A

According to loop rule,

$$V - IR - IR_0 = 0$$

$$\therefore \quad I = \frac{V}{R + R_0} \quad \ldots\text{(i)}$$

$\therefore$ The power dissipated in motor is

$$P = I^2 R_0 = \frac{V^2}{(R + R_0)^2} R_0$$

For maximum power dissipated,

$$\frac{dp}{dR_0} = 0$$

After solving, we get $R_0 = R$

$$\therefore \quad I = \frac{V}{R + R_0} = \frac{V}{2R}$$

$$\therefore \quad P = \frac{V^2}{(R + R_0)^2} R_0$$

$$P_{max} = \frac{V^2}{(R + R)^2} R_0$$

$$\therefore \qquad P_{max} = \frac{V^2}{4R^2} R = \frac{V^2}{4R} \qquad (\because R = R_0)$$

The power supplied by D.C. voltage is

$$P_i = VI = V\left(\frac{V}{R+R_0}\right) = \frac{V^2}{2R} \qquad (\because R = R_0)$$

Percentage efficiency is

$$\eta = \frac{\text{output}}{\text{input}} \times 100 = \frac{P_{max}}{P_i} \times 100$$

$$= \left(\frac{\frac{V^2}{4R}}{\frac{V^2}{2R}}\right) \times 100 = \frac{1}{2} \times 100 = 50\%$$

YOUR STEP

A tram car takes its current from a trolley wire of total resistance r whose both terminals are kept at potential V. If the power taken by the car is assumed constant and equal to H. Show that the minimum potential difference between the trolley and the earth is

$$V' = \frac{1}{2}\left\{V + (V^2 - rH)^{\frac{1}{2}}\right\}$$

and this occurs when the car is midway between the terminals. Show that the waste of power in wire is then a maximum and equal to $H\left(\frac{V}{V'} - 1\right)$

§ 3.194

➤ **CONCEPT**

At steady state, internal energy of filament becomes constant. So, temperature becomes constant. From energy point of view, thermal power developed in filament is lost at heat in environment at steady state.

SOLUTION : According to problem, thermal power Q loss is proportional to surface area A of filament (cylindrical in shape) mathematically,

$$Q \propto A \quad \text{or} \quad Q = kA \quad \text{or} \quad Q = k\, 2\pi rl$$

where r = radius of filament and l = length of filament

The power supplied by applied voltage V is

$$P = \frac{V^2}{R}$$

At steady state,

$$\therefore \qquad P = Q$$

$$\text{or} \qquad \frac{V^2}{R} = k\, 2\pi rl \qquad \ldots\text{(i)}$$

Here R = resistance of filament $= \rho\frac{l}{A} = \frac{\rho l}{\pi r^2}$

From eqn. (i), we get

$$\therefore \qquad \frac{V^2}{R} = k\, 2\pi rl$$

$$\text{or} \qquad \frac{V^2 \pi r^2}{\rho l} = k\, 2\pi rl \qquad \text{or} \qquad V^2 r = 2kl^2\rho$$

$$\text{or} \qquad V^2 r = k_0 \qquad (\text{Here } k_0 = 2kl^2\rho = \text{constant})$$

$$\therefore \qquad V^2 = \frac{k_0}{r}$$

Differentiating both sides with respect to r,

$$2V\frac{dV}{dr} = -\frac{k_0}{r^2}$$

Negative sign indicates that when radius of filament decreases, voltage increases.

$$\therefore \qquad 2V\left(-\frac{dV}{dr}\right) = \frac{k_0}{r^2}$$

or $$2V\frac{\Delta V}{\Delta r} = \frac{k_0}{r^2} \qquad \left(\because -\frac{dV}{dr} = \frac{\Delta V}{\Delta r}\right)$$

or $$2V\frac{\Delta V}{\Delta r} = \frac{V^2}{r} \qquad \left(\because V^2 = \frac{k_0}{r}\right)$$

or $$2\frac{\Delta V}{V} = \frac{\Delta r}{r}$$

or $$2\frac{\Delta V}{V} = \frac{2\Delta r}{2r} = \frac{\Delta D}{D}$$

$$\therefore \qquad \frac{\Delta D}{D} \times 100 = 2\left(\frac{\Delta V}{V} \times 100\right)$$

Hence, percentage decrease in diameter of filament is

$$\frac{\Delta D}{D} \times 100 = 2\left(\frac{\Delta V}{V} \times 100\right) = 2\eta$$

$$= 2 \times 1\% = 2\%$$

YOUR STEP

1. A steady current is passed through a cylindrical conductor of radius r, placed in a vacuum. Show that its steady temperature will be proportional to $r^{-3/4}$, assuming stefan's law of radiation.
2. A circuit contains two lamps, each of resistance R, in parallel on leads each of resistance S. The resistance of the rest circuit, including the battery of constant voltage V is r. Show that if one lamp is broken, the heat emitted in unit time by the other is increased by

$$V^2R\left\{\frac{3r^2 + 2r(R+S)}{(r+R+S)^2(2r+R+S)^2}\right\}$$

§ 3.195

➢ **CONCEPT**

Before steady state, power supplied = Thermal power loss + change in internal energy

➢ **DISCUSSION**

When electrical energy is received by conductor, internal energy of conductor increases. So, the temperature of conductor increases. At $t = 0$, no temperature difference is found between conductor and environment. So at $t = 0$, heat loss is zero.

When $t > 0$, a part of electrical energy increases internal energy (temperature of conductor) and remaining part of electrical energy is lost as heat.

Due to this, temperature difference between conductor and environment increases. Hence, heat lost increases.

After some time, steady state comes into play. At steady state, total electrical power supplied to conductor is lost as heat in environment.

The temperature of conductor becomes constant.

SOLUTION : Before steady state,

Electrical power supplied = Thermal power lost + rate of change in internal energy

or $$\frac{V^2}{R}=k(T-T_0)+\frac{dU}{dt}$$ or $$\frac{V^2}{R}=k(T-T_0)+\frac{d}{dt}(CT)$$

or $$\frac{V^2}{R}=k(T-T_0)+C\frac{dT}{dt}$$ or $$C\int_{T_0}^{T}\frac{dT}{\frac{V^2}{R}-K(T-T_0)}=\int_0^t dt$$

After integrating, we get

$$\therefore \quad T=T_0+\frac{V^2}{kR}(1-e^{-kt/C})$$

YOUR STEP

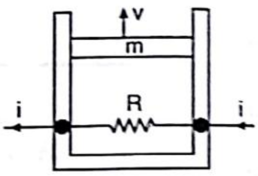


Fig. 3.195A

1. A resistance coil, connected with wire to an external battery, is placed inside an adiabatic cylinder fitted with a frictionless piston and containing an ideal gas. A current $i=240$ mA flows through the coil, which has a resistance $R=550\ \Omega$. At what speed v must the piston, (mass $m=11.8$ kg), move upward in order that the temperature of the gas remains unchanged ?
2. A lead fuse has to be connected to the main circuit formed by copper wires whose cross-sectional area $S_1=5\ \text{mm}^2$ at a temperature $t=25°$ C. The fuse must melt when the temperature of the wires increases by $\Delta t=20°$ C. Find the cross sectional area of the fuse if the specific heats of copper and lead are $c_1=0.36$ kJ/(kg. K) and $c_2=0.12$ kJ/(kg.K), the melting point of lead is $t_m=327°$C, the latent heat of fusion for lead is $\lambda=2.4\times10^4$J/kg, the resistivities of copper and lead are $\rho_1=0.017\ \mu\Omega$m and $\rho_2=0.21\ \mu\Omega$.m, and the densities of copper and lead are $\gamma_1=8.9\times10^3$ kg/m^3 and $\gamma_2=11.4\times10^3$ kg/m^3 respectively.

{1. 27.4 m/sec 2. $S=5.4\ \text{mm}^2$}

§ 3.196

➢ **CONCEPT**

If power developed in resistance R_x is maximum. Then power generated in R_x is practically independent of small variation of resistance. For maximum power developed in R_x is,

$$\frac{dP}{dR_x}=0$$

SOLUTION : The circuit is shown in Fig. 3.196A.

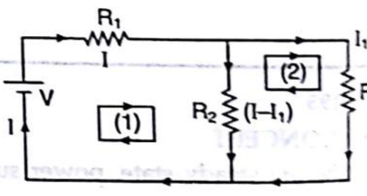


Fig. 3.196A

In loop (1),

$$V-IR_1-(I-I_1)R_2=0 \quad \text{...(i)}$$

In loop (2),

$$-I_1R_x+(I-I_1)R_2=0 \quad \text{...(ii)}$$

After solving Eq. (i) and (ii), we get

$$I_1=\frac{VR_2}{R_1R_2+R_1R_x+R_2R_x}$$

The thermal power generated in the resistance. R_x is

$$P=I_1^2R_x=\left(\frac{VR_2}{R_1R_2+R_1R_x+R_2R_x}\right)^2R_x$$

For maximum value of P, $\frac{dP}{dR_x}=0$

After solving, we get $R_x = \dfrac{R_1R_2}{R_1+R_2}$

On putting the values, we get $R_x = 12\,\Omega$

YOUR STEP

An element with emf ε and internal resistance r is connected across an external resistance R. The maximum power in external circuit is 9 W. The current flowing through the circuit in these conditions is 3A. Find ε and r.

{$\varepsilon = 6\text{V},\ r = 1\Omega$}

§ 3.197

➢ **CONCEPT**

The problem is solved by maximum power transfer theorem.

SOLUTION : The problem is solved by following steps :

Step I : Show the original circuit.

The circuit is shown in fig. 3.197A.

Setp II : Remove the resistor of resistance R from the circuit.

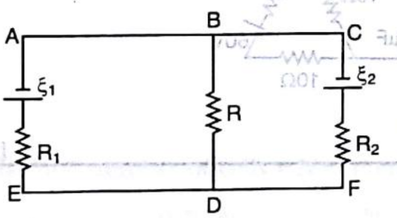


Fig. 3.197A

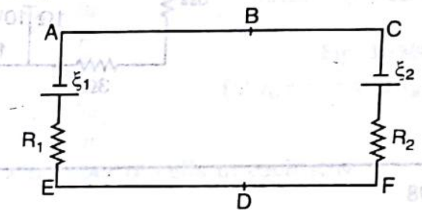


Fig. 3.197B

In this step, we remove that resistance from the circuit in which maximum thermal power is generated.

The circuit is shown in 3.197B.

Step III : Draw the circuit after short circuited the all cells (but internal resistances of cells remains in the circuit) of circuit.

The circuit of this step is shown in Fig. 3.197C.

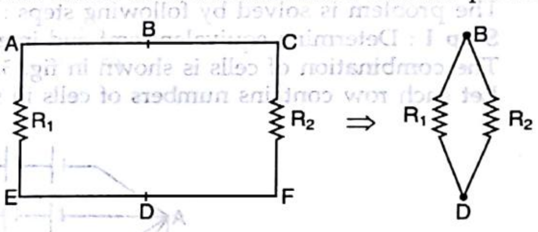


Fig. 3.197C

The equivalent resistance of circuit in Fig. 3.197C between points B and D is

$$R_{BD} = \frac{R_1R_2}{R_1+R_2}$$

For maximum power generated in resistance R, the value of R.

$$= R_{BD} = \frac{R_1R_2}{R_1+R_2}$$

Step IV : Draw original circuit and find current through R.

In loop (1), $\xi_1 - IR_1 - I_1R = 0$ ...(i)

In loop (2)

$$-(I-I_1)R_2 - \xi_2 + I_1R = 0 \quad \text{...(ii)}$$

After solving Eq. (i) and (ii), we get

$$I_1 = \frac{(\xi_1R_2 + \xi_2R_1)}{2R_1R_2}$$

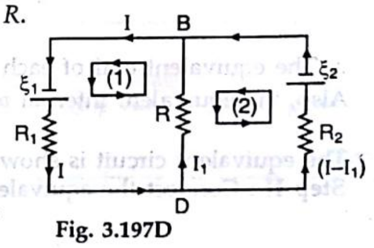


Fig. 3.197D

$\therefore$ Power generated in resistance R is $P = I_1^2 R$

For P_{max}, $$R = R_{BD} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\therefore \quad P_{max} = \frac{(\xi_1 R_2 + \xi_2 R_1)^2}{(2R_1R_2)^2}\left(\frac{R_1R_2}{R_1+R_2}\right)$$

$$= \frac{(\xi_1 R_2 + \xi_2 R_1)^2}{4R_1 R_2 (R_1 + R_2)}$$

Remarks : The problem may be solved as problem 3.190.

YOUR STEP

In the shown circuit. Find the maximum energy stored on the capacitor.

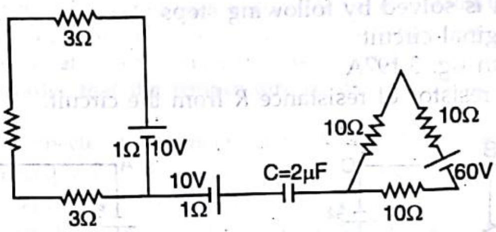


Fig. 3.197E

{Zero}

§ 3.198

➢ **CONCEPT**

The problem is similar to previous problems.
The problem is solved by following steps :
Step I : Determine equivalent emf and internal resistance of the combination of cells :
The combination of cells is shown in fig. 3.198A.
Let each row contains numbers of cells in series.

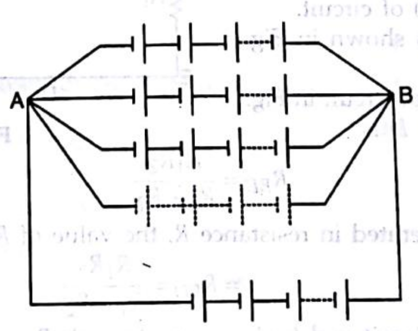


Fig. 3.198A

$\therefore$ The equivalent emf of each row is $\quad \varepsilon_0 = n_0 \varepsilon$

Also, the equivalent internal resistance of each row is
$$r_0 = n_0 r$$

The equivalent circuit is shown in Fig. 3.198B.

Step II : Connect the equivalent cell with external resistance R.

The circuit is shown in fig. 3.198C.

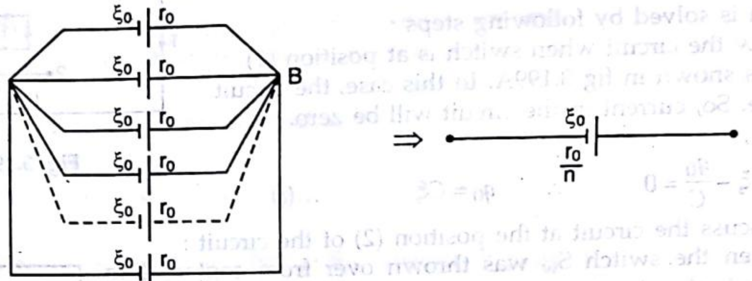


Fig. 3.198B

According to loop rule,

$$\xi_0 - \frac{I\,r_0}{n} - IR = 0$$

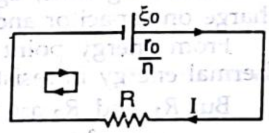


Fig. 3.198C

$\therefore$ $$I = \frac{\xi_0}{R + \dfrac{r_0}{n}}$$

or $$I = \frac{n_0 \xi}{R + \dfrac{n_0 r}{n}}$$ $(\because r_0 = n_0 r \text{ and } \xi_0 = n_0\,\xi)$

The total number of cells $= N =$ number of row $\times$ number of cells in each row

$$N = n n_0 \quad \therefore \quad n_0 = \frac{N}{n}$$

$\therefore$ $$I = \frac{\left(\dfrac{N}{n}\right)\xi}{R + \dfrac{rN}{n^2}} = \frac{nN\xi}{n^2 R + rN}$$

The power generated in external resistance R is

$$P = I^2 R$$

$$= \left(\frac{nN\xi}{n^2 R + rN}\right)^2 R$$

For maximum power, $$\frac{dP}{dn} = 0$$

After solving, we get $$n = \sqrt{\frac{Nr}{R}}$$

On putting the values, we get $n = 3$

Remarks : The problem may be solved as previous problem.

YOUR STEP

200 alkaline accumulators are to be charged by a dynamo generating a voltage of 230 V. The emf of each accumulator is 1.4V, the internal resistance 0.01 Ω the charging current 30A. Suggest the circuit diagram and calculate the resistance of the rheostate.

[1 ohm]

§ 3.199

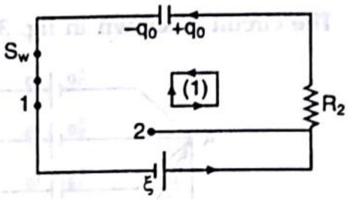


Fig. 3.199A

➢ CONCEPT

The problem is solved by following steps :

Step I : Draw the circuit when switch is at position (1).

The circuit is shown in fig 3.199A. In this case, the circuit is in steady state. So, current in the circuit will be zero.

In loop (1)

$$\xi - \frac{q_0}{C} = 0 \qquad \therefore \qquad q_0 = C\xi \qquad \ldots(i)$$

Step II : Discuss the circuit at the position (2) of the circuit :

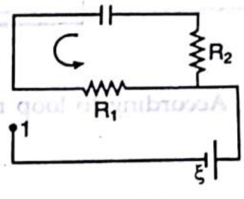


Fig. 3.199B

At $t = 0$, when the switch S_w was thrown over from contact 1 to contact 2, current in the circuit is zero. Now capacitor starts to discharge through resistances R_1 and R_2.

After long time, again circuit comes in steady state. At this state, charge on capacitor and current in circuit both are zero.

From energy point of view energy stored on capacitor appears as thermal energy in resistors R_1 and R_2.

But R_1 and R_2 are in series so, current in both are same.

$$\therefore \quad \Delta H = I^2 Rt \qquad \therefore \qquad \Delta H_1 \propto R_1 \quad \text{and} \quad \Delta H_2 \propto R_2$$

$$\therefore \quad \frac{\Delta H_1}{\Delta H_2} = \frac{R_1}{R_2}$$

$$\therefore \quad \Delta H_2 = \frac{R_2}{R_1} \Delta H_1 \qquad \ldots(ii)$$

Total heat generated is $\qquad \Delta H = \Delta H_1 + \Delta H_2$

According to conservation principle of energy $\Delta H + \Delta U = 0$

$$\text{or} \quad \Delta H + (U_f - U_i) = 0 \qquad \therefore \qquad \Delta H = U_i - U_f$$

$$\therefore \quad \Delta H = \frac{q_0^2}{2C} - 0 \qquad \text{or} \qquad \Delta H_1 + \Delta H_2 = \frac{q_0^2}{2C}$$

$$\Delta H_1 + \frac{R_2}{R_1} \Delta H_1 = \frac{q_0^2}{2C} \qquad \text{(from eqn. (ii))}$$

$$\text{or} \quad \Delta H_1 \left(1 + \frac{R_2}{R_1}\right) = \frac{q_0^2}{2C} \qquad \text{or} \qquad \Delta H_1 \left(\frac{R_1 + R_2}{R_1}\right) = \frac{q_0^2}{2C}$$

$$\therefore \quad \Delta H_1 = \frac{q_0^2 R_1}{(2C)(R_1 + R_2)} = \frac{(C\xi)^2 R_1}{2C(R_1 + R_2)} = \frac{C\xi^2 R_1}{2(R_1 + R_2)} = 60 \text{ mJ}$$

Remarks : The problem may be solved by $\Delta H = \int_{t=0}^{t=\infty} I^2 R_1 dt$

YOUR STEP

Determine the amount of heat liberated in the resistance when switch is changed from position 1 to 2 in fig. 3.199C.

$$\left\{\frac{1}{2} C (E_2 - E_1)^2\right\}$$

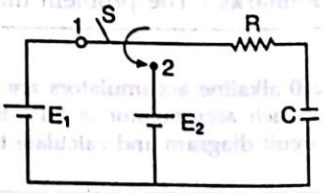


Fig. 3.199C

§ 3.200

➢ **CONCEPT**

The problem is based upon energy conservation principle.
According to conservation principle of energy,

$$W_{ext} + W_{cell} = \Delta U$$

where W_{ext} = work done by external agent

W_{cell} = work done by cell

ΔU = change in energy stored on capacitor

$$= U_f - U_i$$

SOLUTION : The capacity of capacitor in the absence of plate is

$$C = \frac{\varepsilon_0 S}{d}$$

The capacity of capacitor after introduction of metallic plate is

$$C_m = \frac{\varepsilon_0 S}{\frac{t_1}{\varepsilon_1} + \frac{t_2}{\varepsilon_2}}$$

Here, metallic plate, $t_1 = \eta d$, $\varepsilon_1 = \infty$, $t_2 = d - \eta d$, $\varepsilon_2 = 1$

$$\therefore \qquad C_m = \frac{\varepsilon_0 S}{\frac{\eta d}{\infty} + \frac{d - \eta d}{1}} = \frac{\varepsilon_0 S}{d(1-\eta)} = \frac{C}{1-\eta}$$

U_i = initial energy stored on capacitor $= \frac{1}{2} C_m V^2 = \frac{1}{2} \frac{\varepsilon_0 S}{d(1-\eta)} V^2 = \frac{1}{2}\left(\frac{C}{1-\eta}\right) V^2 \qquad \left(\because C = \frac{\varepsilon_0 S}{d}\right)$

U_f = energy stored on capacitor after removal of metallic plate.

$$= \frac{1}{2} CV^2$$

(a) Increment of energy is $\Delta U = \frac{1}{2} CV^2 \left(1 - \frac{1}{1-\eta}\right)$

$$= \frac{1}{2} CV^2 \left(\frac{-\eta}{1-\eta}\right) = -0.15 \text{ mJ}$$

(b) Initial charge on capacitor is $q_i = C_m V = \left(\frac{C}{1-\eta}\right) V$

q_f = final charge stored on capacitor after removal of plate $= CV$

$\therefore$ The charge supplied by battery is

$$\Delta q = q_f - q_i = CV\left(1 - \frac{1}{1-\eta}\right) = -\frac{\eta CV}{(1-\eta)}$$

The work done by the cell is $W_{cell} = V\Delta q = -\frac{V^2 C\eta}{(1-\eta)} = -\frac{V^2 C\eta}{(1-\eta)}$

$$\Delta U = U_f - U_i = \frac{1}{2} CV^2 - \frac{1}{2} C_m V^2$$

$$= \frac{1}{2} CV^2 - \frac{1}{2}\left(\frac{C}{1-\eta}\right) V^2 = \frac{1}{2} CV^2 \left(1 - \frac{1}{1-\eta}\right)$$

$$= \frac{1}{2} CV^2 \left(-\frac{\eta}{1-\eta}\right)$$

According to conservation principle of energy.

$$W_{ext} + W_{cell} = \Delta U$$

$$\therefore \quad W_{ext} = \Delta U - W_{cell}$$

$$= -\frac{1}{2}CV^2\left(\frac{\eta}{1-\eta}\right) + \frac{CV^2\eta}{1-\eta}$$

$$= \frac{CV^2\eta}{2(1-\eta)} = 0.15 \text{ mJ}$$

YOUR STEP

A plane capacitor is filled with a dielectric and a certain potential difference is applied to its plates. The energy of the capacitor is 2×10^{-5} J. After the capacitor is disconnected from the power source, the dielectric is extracted from the capacitor. The work performed against the forces of the electric field in extracting the dielectric is 7×10^{-5} J. Find the dielectric constant (relative permittivity) of the dielectric. {4.5}

§ 3.201

➤ **CONCEPT**

For solving this problem, conservation principle of energy is applicable.
Mathematically,

$$W_{ext} + W_{cell} = \Delta U$$

Here, W_{ext} = The work done by external agent (mechanical work) against electrical forces.

W_{cell} = Work done by cell $= V\Delta q$

$$\Delta U = U_f - U_i$$

SOLUTION : The capacity of capacitor without glass plate

$$= C = \frac{\varepsilon_0 S}{d}$$

Fig. 3.201A

The capacity of capacitor with glass plate is $C_1 = \frac{\varepsilon\varepsilon_0 S}{d} = \varepsilon C$

The initial energy stored on capacitor is

$$U_i = \frac{1}{2}C_1V^2 = \frac{1}{2}\varepsilon CV^2$$

The charge on capacitor is $q_i = C_1V = \varepsilon CV$

The energy stored on capacitor after removal of glass plate is

$$U_f = \frac{1}{2}CV^2$$

The charge on capacitor after removal of plate is

$$q_f = CV$$

$$\therefore \quad \Delta U = U_f - U_i = \frac{1}{2}CV^2(1-\varepsilon)$$

Also, $\Delta q = q_f - q_i = CV - \varepsilon CV = CV(1-\varepsilon) < 0$

$$\therefore \quad W_{cell} = V\Delta q = CV^2(1-\varepsilon) < 0$$

It means work done by cell is negative.
According to conservation of energy,

$$W_{ext} + W_{cell} = \Delta U$$

$$\therefore \quad W_{ext} = \Delta U - W_{cell}$$

$$= \frac{1}{2}CV^2(1-\varepsilon) - CV^2(1-\varepsilon)$$

$$= -\frac{1}{2}CV^2(1-\varepsilon) = \frac{1}{2}CV^2(\varepsilon-1) > 0$$

For glass $\varepsilon = 6$

On putting the values, we get $W_{ext} = 0.5$ mJ

The increment in energy of capacitor is

$$\Delta U = U_f - U_i$$

$$= -\frac{1}{2}(\varepsilon-1)CV^2 = -0.5 \text{ mJ}$$

YOUR STEP

A plane air capacitor in which the plates are spaced 5 mm apart is charged to a potential of 6kV. The area of the capacitor plates is 12.5 cm^2. The plates are moved apart to a distance of 1 cm in one of two ways.

(i) The capacitor remains connected to the power source.

(ii) Before the plates are moved apart the capacitor is disconnected from power source. Find in each these cases.

(a) The change in the capacity of capacitor.

(b) The change in the intensity of flux through the area of the electrode and

(c) the change in the volume density of the energy of the electric field.

(a) the capacity decreased by 1.1 PF,

(b) the intensity flux decreased by 750 V,

(c) the volume density of energy decreased by 4.8×10^{-2} J/m^3

§ 3.202

➢ **CONCEPT**

The capacity of capacitor without water is

$$C_0 = \varepsilon_0\left(\frac{2l\pi R}{d}\right)$$

Let a liquid rises up to height h in the capacitor.

$$\therefore \quad C = C_{air} + C_{liquid}$$

$$= \frac{\varepsilon_0(l-h)2\pi R}{d} + \frac{\varepsilon_0\varepsilon h\, 2\pi R}{d}$$

$$\therefore \quad C = \frac{2\pi\varepsilon_0 R}{d}[l + (\varepsilon-1)h]$$

where ε is dielectric constant of liquid

SOLUTION : The initial energy of capacitor is

$$U_i = \frac{1}{2}C_0V^2$$

The final energy of capacitor is $U_f = \frac{1}{2}CV^2$

$\therefore$ Increase in energy of capacitor is $\Delta U = U_f - U_i = \frac{1}{2}V^2(C - C_0)$

Work done by cell $= V\Delta q = V(q_f - q_i)$

$$= V(CV - C_0V) = V^2(C - C_0)$$

Increase in gravitational potential energy

$$= \Delta U_g = mg\frac{h}{2}$$

$$= \rho \times \text{volume} \times \left(\frac{h}{2}\right) g$$

$$= \rho g\,(2\pi Rhd) \times \frac{h}{2} = \pi \rho g R h^2 d$$

According to conservation principle of energy, a part of work done by cell increases potential energy of capacitor and remaining part of work done by cell increases gravitational potential energy of liquid.

$$\therefore \qquad W_{\text{cell}} = \Delta U + \Delta U_g$$

Putting the values, we get $\quad h = \dfrac{\varepsilon_0 (\varepsilon - 1) V^2}{2\rho g d^2}$

YOUR STEP

A large vessel is filled with dielectric liquid of constant K and resistivity ρ. A parallel plate capacitor having charge q_0 is placed vertically on the surface of liquid. The plates of capacitor are rectangular of length l and breadth b. The separation between the plates is d. Calculate the height upto which the liquid will rise. (Neglect capillary effect).

$$\left\{ h = \frac{q_0^2}{mg}\left(\frac{1}{C_0} - \frac{1}{C}\right),\ C_0 = \frac{\varepsilon_0 lb}{d},\ C = \frac{\varepsilon_0 (l-h)b}{d} + \frac{\varepsilon_0 khb}{d},\ m = \rho bhd \right\}$$

§ 3.203

➢ **CONCEPT**

The system behaves as $R - C$ discharging circuit.

For discussion of problem, the system may be assumed as parallel combination of a resistance and capacitor between capacitor electrodes.

SOLUTION : The problem is solved by following Steps :

Step I : Determine equivalent resistance and capacitance betwen electrodes.

From the soln. of problem 3.155, electric resistance between electrodes is

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho\,(b-a)}{4\pi\, ab}$$

Also, the capacity of spherical capacitor is

$$C = \frac{4\pi\varepsilon_0 \varepsilon ab}{b-a}$$

Step II : Draw equivalent circuit :

At $t = 0$, charge on capacitor is q_0. Now capacitor starts to discharge through resistance R.

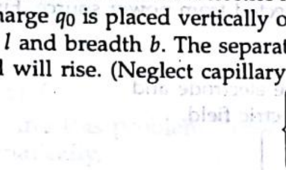


Fig. 3.203A at $t = 0$

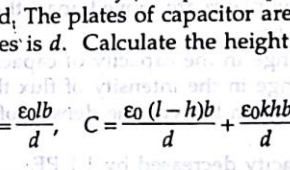


Fig. 3.203B at an instant

Let at an instant t, charge on capacitor is q and current in circuit is I. The circuit at an instant t is shown in figure 3.203B.

Since, charge on capacitor decreases with respect to time.

$$\therefore \qquad I = \frac{\Delta q}{\Delta t} = -\frac{dq}{dt}$$

according to loop rule, $\frac{q}{C} - IR = 0$

or $$\frac{q}{C} + R\frac{dq}{dt} = 0$$

or $$\int_{q_0}^{q} \frac{dq}{q} = -\int_0^t \frac{dt}{RC}$$

After integrating, we get, $q = q_0 e^{-t/RC}$

Putting the values of R and C, we get $q = q_0 e^{-t/\rho\varepsilon_0\varepsilon}$

(b) $$q = q_0 e^{-t/RC}$$

$\therefore$ $$I = -\frac{dq}{dt} = \frac{q_0}{RC} e^{-t/RC}$$

$\therefore$ $$\Delta H = \int_{t=0}^{t=\infty} I^2 R dt$$

$$= \int_0^{\infty} \frac{q_0^2}{(RC)^2} e^{-2t/RC} R dt$$

After integrating, we get $$\Delta H = \frac{q_0^2}{2C} = \frac{q_0^2 (b-a)}{8\pi\varepsilon_0 \varepsilon ab}$$

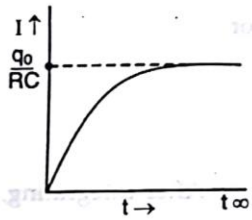


Fig. 3.203C

Remarks : Part (b) may be solved by conservation principle of energy. According to conservation principle of energy, during discharging, total energy stored on capacitor at $t = 0$, appears as heat energy.

i.e., $$\frac{q_0^2}{2C} = \Delta H$$

YOUR STEP

An electric charge $+q_0$ is deposited at $t = 0$ over the surface of a conducting sphere of radius R. The sphere is embedded in atmospheric air which does not act as a perfect insulator, therefore, the charge eventually leaks away from the sphere. Find the time dependence of the charge deposited on the sphere, if the specific conductivity of air is K.

$\{q = q_0 e^{-4\pi K t}\}$

§ 3.204

➢ **CONCEPT**

The problem is similar to previous problem

SOLUTION : The circuit of problem is shown in fig. 3.204 *A* and *B*.

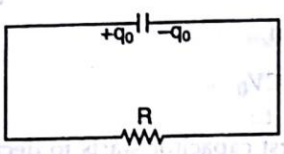


Fig. 3.204A at $t = 0$

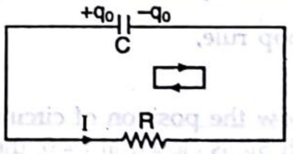


Fig. 3.204B at an instant

Let at an instant, charge on capacitor is q and current in circuit is I.

According to loop rule, $\frac{q}{C} - IR = 0$

or $$\frac{q}{C} - R\left(-\frac{dq}{dt}\right) = 0 \qquad \left(\because I = -\frac{dq}{dt}\right)$$

or $$-RC\int_{q_0}^{q}\frac{dq}{q}=\int_0^t dt \qquad \therefore \quad q=q_0e^{-t/RC}$$

when $t=\tau$, $$q=q_0e^{-\tau/RC}$$

Here q is charge on capacitor at $t=\tau$. The charge flow through circuit is equal to the decrease in charge on capacitor.

$\therefore$ The charge flow through circuit is $Q=q_0-q=q_0\left(1-e^{-\tau/RC}\right)$

On putting the values, we get
$$Q=0.18\text{mC}$$

(b) $\because$ $$q=q_0e^{-t/RC}$$

or $$I=-\frac{dq}{dt}=\frac{q_0}{RC}e^{-t/RC}$$

$\therefore$ $$\Delta H=\int_0^\tau I^2Rdt=\int_0^\tau\left(\frac{q_0}{RC}\right)^2e^{-2t/RC}Rdt$$

After integrating, we get $$\Delta H=\frac{q_0^2}{2C}\left(1-e^{-2\tau/RC}\right)$$

On putting the values, we get $$\Delta H=82\text{ mJ}$$

YOUR STEP

Fig. 3.204C shows the circuit of a flashing lamp, like those attached to barrels at highway construction sites. The fluorescent lamp L is connected in parallel across the capacitor C of an RC circuit. Current passes through the lamp only when the potential across it reaches the breakdown voltage V_L; in this event, the capacitor discharges through the lamp and it flashes for a very short time. Suppose that two flashes per second are needed. Using a lamp with breakdown voltage $V_L=72$V, a 95 – V battery, and a 0.15 – μF capacitor, what should be the resistance R of the resistor?

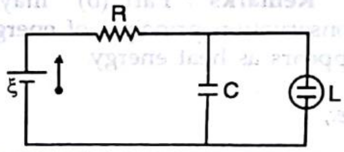


Fig. 3.204C

[2.35MΩ]

§ 3.205

SOLUTION :

The problem is solved by following steps :

Step I : Show the position of circuit before shorted the circuit by means of the switch S_W. The circuit is shown in fig 3.205A.

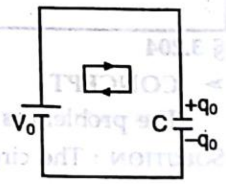


Fig. 3.205A

In this case, the capacitor is in steady state.

Applying loop rule, $$V_0-\frac{q_0}{C}=0$$

$\therefore$ $$q_0=CV_0 \qquad \text{...(i)}$$

Step II : Show the position of circuit at an instant t :

when switch S_W is closed at $t=0$, the charge on first capacitor starts to decrease. But that of

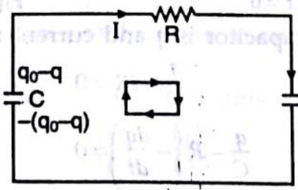


Fig. 3.205B

second capacitor increases. At an instant t, circuit is shown in fig. 3.205B.

According to loop rule, $$+\frac{q_0 - q}{C} - IR - \frac{q}{C} = 0$$

or $$\frac{q_0}{C} - \frac{2q}{C} - R\frac{dq}{dt} = 0 \qquad \left(\because I = \frac{dq}{dt}\right)$$

or $$(q_0 - 2q) = RC\frac{dq}{dt}$$

or $$\int_0^q \frac{dq}{q_0 - 2q} = \int_0^t \frac{dt}{RC}$$

or $$ln\frac{q_0 - 2q}{q_0} = -\frac{2t}{RC}$$

or $$\frac{q_0 - 2q}{q_0} = e^{-2t/RC}$$

$\therefore$ $$q = \frac{q_0}{2}(1 - e^{-2t/RC})$$

(a) $\therefore$ $$I = \frac{dq}{dt} = \frac{q_0}{RC}e^{-2t/RC} = \frac{V_0}{R}e^{-2t/RC}$$

(b) $$\Delta H = \int_0^t I^2 R dt \qquad (\because q_0 = CV_0)$$

$$= \frac{q_0^2 R}{R^2C^2}\int_0^t e^{-4t/RC}dt = \frac{q_0^2}{4C}\left[-e^{-4t/RC} + 1\right]$$

$\therefore$ $$\Delta H = \frac{q_0^2}{4C}\left[1 - e^{-4t/RC}\right]$$

Since, current is flowing for long time

$\therefore$ $t \to \infty$ $\therefore$ $$\Delta H = \frac{q_0^2}{4C} = \frac{1}{4}CV_0^2 \qquad (\because q_0 = CV_0)$$

YOUR STEP

The capacitor C_1 (shown in fig. 3.205C) initially caries a charge q_0. When the S_1 and S_2 are shut, capacitor C_1 is connected in series to a resistance R and a second capacitor C_2 which initially does not carry any charge, compare the electrostatic energies stored in the system before the switches are shut and after a very long time they were shut. Compare the difference between electrostatic energies and the amount of heat produced in resistance R during discharging process.

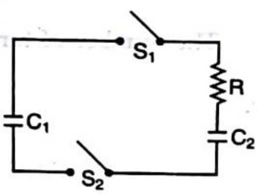


Fig. 3.205C

$$\left\{U_i = \frac{q_0^2}{2C_1}, U_f = \frac{q_0^2}{2(C_1 + C_2)}, \frac{U_i}{U_f} = \left(\frac{C_1 + C_2}{C_1}\right) \text{ and } U_i - U_f = 1\right\}$$

Heat produced = $U_i - U_f$ = loss of energy of capacitor

§ 3.206

➢ **CONCEPT**

According to Faraday's law of electromagnetic induction.

$$\xi = -N\frac{d\phi}{dt}$$

$$IR = -N\frac{d\phi}{dt}$$

$$R\frac{dq}{dt} = -N\frac{d\phi}{dt}$$

$$\therefore \quad R\frac{\Delta q}{\Delta t} = -N\frac{\Delta\phi}{\Delta t}$$

$$\therefore \quad \Delta q = N\frac{|\Delta\phi|}{R}$$

Here $\Delta\phi$ = net charge in magnetic flux, R = total resistance of coil.
Δq = Total charge transferred. Initial flux is $\phi_i = \pi r^2 B$ per turn
final flux is $\phi_F = 0$

$$\therefore \quad \Delta\phi = \phi_f - \phi_i = -\pi r^2 B$$

$$\therefore \quad \Delta q = N\frac{|\Delta\phi|}{R} = \frac{N\pi r^2 B}{R} \qquad \text{...(i)}$$

Since, electron in coil moves on circular path with constant angular velocity ω.
For moving on circular path, magnetic force on electton provides required centripetal force.

$$\therefore \quad evB = \frac{mv^2}{r} \quad \text{or} \quad eB = \frac{mv}{r}$$

$$\therefore \quad B = \frac{mv}{re} = \frac{mr\omega}{re} = \frac{m\omega}{e}$$

Here m = mass of electron, e = charge on electron.

In vector form, $\vec{B} = -\frac{m}{e}\vec{\omega}$ ($\because$ Charge on electron is negative)

$$\therefore \quad \Delta q = \frac{N\pi r^2 B}{R}$$

On putting the value of B, we get

$$<\Delta q> = \frac{q}{2} = \frac{N\pi r^2}{R}\left(\frac{m\omega}{e}\right)$$

($\because$ The average flow of charge is $<\Delta q> = \frac{q}{2}$)

$$\therefore \quad \frac{e}{m} = \frac{2N\pi r^2\omega}{qR}$$

The number of turns $N = \frac{l}{2\pi r}$

$$\therefore \quad \frac{e}{m} = \left(\frac{2l}{2\pi r}\right)\frac{\pi r^2\omega}{qR}$$

$$= \frac{lr\omega}{qR} = 1.8 \times 10^{11} \text{ C/kg}$$

YOUR STEP

A conducting disc of radius R is rotating with an angular velocity ω. Allowing for the fact that electrons each of mass m are the current carriers in a conductor, determine the potential difference between the centre of disc and the edge.

$$\left\{\frac{mR^2\omega^2}{2l}\right\}$$

§ 3.207

➢ CONCEPT

The drift speed is given by $v_d = \frac{J}{ne}$

Here J = current density
n = number of electrons per unit volume
e = charge on electron

SOLUTION : The momentum of each electron is $P_0 = mV_d$

Here m = mass of each electron $= 9.1 \times 10^{-31}$ Kg

If total number of electrons in conductor is N.

Then total momentum of electrons inside the conductor is

$$P = NP_0$$

Let S = cross-sectional area of straight wire (cylindrical wire)

$$\therefore \quad J = \frac{I}{S}$$

Volume of wire is $\quad V = Sl$

$\therefore$ no. of electrons inside the conductor is

$$N = nSl$$

$$\therefore \quad Po = NP_0 = (nSl)\frac{mJ}{ne}$$

$$\therefore \quad P = (nSl)\frac{mI}{Sne} = \frac{Ilm}{e} = 0.40\ \mu\text{Ns}$$

YOUR STEP

An electrical conductor designed to carry large currents has a square cross-section 2 mm on a side and is 12m long. The resistance between ends is 0.0625Ω.
(a) What is the resistivity of the material?
(b) If the electric field in the conductor is 1.28 V/m and material contains 8.5×10^{28} free electrons per m^3. Find drift speed.

{(a) 2.08×10^{-8} Ωm (b) 4.51 m/s}

§ 3.208

➢ **CONCEPT**

Drift speed of electron is negligible with respect to mean speed of thermal motion of electron.

$\therefore$ Distance travelled by electron is $\quad s = \langle v \rangle t$

where $\langle v \rangle$ is mean speed of thermal motion of electron.

SOLUTION : The drift velocity of electron is

$$v_d = \frac{J}{ne}$$

The time taken by electron to displace a distance l is

$$t = \frac{l}{v_d}$$

$$\therefore \quad s = \langle v \rangle t = \langle v \rangle \frac{l}{v_d} = \langle v \rangle \frac{nel}{J}$$

when n is the concentration of free electrons.

On putting the values, we get $s \approx 10^7$

YOUR STEP

Calculate the drift velocity of electrons in a copper conductor carrying 1 ampere current. There are two electrons available for conduction of electricity per atom of copper and the radius of the wire is 1 mm. Atomic weight of copper = 63.5 and Avogadro number = 6.06×10^{23} and density of copper = 7800 kg/m^3.

{1.34×10^{-5} m/sec}

§ 3.209

➢ **CONCEPT**

The concept is similar to previous problem.

SOLUTION : (a) The time taken by electron to displace from one end to another end is

$$t = \frac{l}{v_d} = \frac{l}{\frac{J}{ne}} = \frac{nel}{J} = \frac{nel}{\frac{I}{S}} = \frac{nelS}{I}$$

On putting the values, we get $t = 3\text{ms}$

(b) $F = qE$

Here q = total charge on electron $= Ne$

where N = total number of electrons $= nSl$

where n = number of electrons per unit volume.

According to ohm's law in vector form,

$$E = \rho J$$

where ρ is resistivity of copper. ∴ $F = qE = NeE$

$$F = Ne\rho J = nSle\rho J$$

$$F = nSle\rho\frac{I}{S} \qquad F = nle\rho I$$

On putting the values, we get $F = 1\text{MN}$

YOUR STEP

A voltage of 18V is applied across the ends of a copper wire 200 m long. Determine the mean velocity of the ordered motion of electrons in the conductor if the number density of conduction electrons in it is $3.0 \times 10^{23}\ \text{cm}^{-3}$.

{ 1.1×10^{-4} m/sec}

§ 3.210

➢ **CONCEPT**

For solving the problem, the homogeneous proton beam may be assumed as a long non-conducting cylinder of radius r.

We consider a point P at distance x $(x < r)$ from the axis of cylinder. Let the volume charge density of the beam is ρ. For calculating electric field at point P, we draw an imaginary gaussian co-axial cylinder of length l passing through point P.

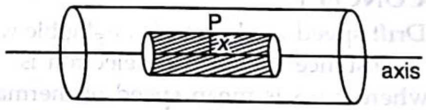


Fig. 3.210A

According to Gauss's law, $\int_C \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$

or $E 2\pi x l = \frac{\rho\pi x^2 l\rho}{\varepsilon_0}$ ∴ $E = \frac{\rho x}{2\varepsilon_0}$

Calculation for ρ :

Let the speed of each proton is u.

∴ $$eV = \frac{1}{2}mu^2$$

where m is mass of proton.

e = charge on proton.

∴ $u = \sqrt{\frac{2eV}{m}}$ ∴ $I = \frac{dq}{dt}$

Here $$dq = \rho dV = \rho\pi r^2 dx$$

$\therefore \quad I = \dfrac{dq}{dt} = \rho\pi r^2 \dfrac{dx}{dt} = \rho\pi r^2 u$

$\therefore \quad \rho = \dfrac{I}{\pi r^2 u}$

Electric field at the surface is $E = \dfrac{\rho x}{2\varepsilon_0} = \dfrac{\rho}{2\varepsilon_0} x$

For surface, $x = r$

$\therefore \quad E = \dfrac{\rho r}{2\varepsilon_0} = \dfrac{Ir}{\pi r^2 u 2\varepsilon_0} \qquad \left(\because \rho = \dfrac{I}{\pi r^2 u}\right)$

$\therefore \quad E = \dfrac{I}{2\pi r\varepsilon_0 u}$

Here $u = \sqrt{\dfrac{2eV}{m}}$

$\therefore \quad E = \dfrac{I}{2\pi\varepsilon_0 r}\sqrt{\dfrac{m}{2eV}} = 32\text{V/m}$

$\because \quad E = \dfrac{\rho x}{2\varepsilon_0}$

$$-\int d\phi = \int E dx$$

$$-\int_{\phi_1}^{\phi_2} = \int_0^r \frac{\rho x}{2\varepsilon_0} dx$$

$\therefore \quad \phi_1 - \phi_2 = \dfrac{\rho r^2}{4\varepsilon_0}$

On putting the value of ρ, we get $\Delta\phi = \phi_1 - \phi_2 = \dfrac{I}{4\pi\varepsilon_0}\sqrt{\dfrac{m}{2eV}}$

On putting the values, we get $\Delta\phi = 0.80\text{V}$

YOUR STEP

(a) The current density across a cylindrical conductor of radius R varies according to equation $J = J_0\left(1 - \dfrac{x}{R}\right)$. Where x is the distance from the axis. Thus, the current density is maximum J_0 at the axis $x = 0$ and decreases linearly to zero at the surface $x = R$. Calculate the current in terms of J_0 and the conductor's cross sectional area $A = \pi R^2$.

(b) Suppose that instead the current density is minimum J_0 at the surface and decreases linearly to zero at the axis. *i.e.*, $J = J_0 \dfrac{x}{R}$. Calculate the current.

$\left\{\text{(a) } \dfrac{J_0 A}{3} \quad \text{(b) } \dfrac{2J_0 A}{3}\right\}$

§ 3.211

➢ CONCEPT

From the concept of problem 3.52,

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_0}$$

Fig. 3.211A

SOLUTION : Here $\phi = ax^{4/3}$

$\sin \phi = f(x)$

$\therefore \qquad E = -\dfrac{\partial\phi}{\partial x} \qquad \therefore \quad E = -\dfrac{4}{3}ax^{1/3}$

As we know, $\dfrac{\partial E_x}{\partial x} + \dfrac{\partial E_y}{\partial y} + \dfrac{\partial E_z}{\partial z} = \dfrac{\rho}{\varepsilon_0}$

Here E is only the function of x.

$\therefore \qquad \dfrac{\partial E_y}{\partial y} = 0, \dfrac{\partial E_z}{\partial z} = 0$

$\therefore \qquad \dfrac{\partial E_x}{\partial x} = \dfrac{dE}{dx} \qquad \because \quad \dfrac{dE}{dx} = \dfrac{\rho}{\varepsilon_0}$

or $\qquad \dfrac{d}{dx}\left(-\dfrac{4}{3}a\,x^{1/3}\right) = \dfrac{\rho}{\varepsilon_0}$

or $\qquad -\dfrac{4}{3}a\dfrac{1}{3}x^{\frac{1}{3}-1} = \dfrac{\rho}{\varepsilon_0}$

$\therefore \qquad \rho = -\dfrac{4\varepsilon_0 a}{9}x^{-2/3}$

(b) We consider an element of thickness dx at distance x from cathode.
The volume of considered element is $dV =$ area $\times$ thickness $= Sdx$
The electric charge in considered element is

$$dq = \rho S dx$$

$\therefore \qquad I = \dfrac{dq}{dt} = \rho S\dfrac{dx}{dt} = \rho S u$

Electric current density is $\qquad J = \dfrac{I}{S} = \rho u$

As we know $\qquad eV = \dfrac{1}{2}mu^2$

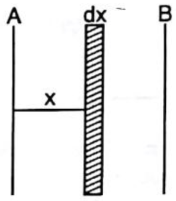


Fig. 3.211B

where m is mass of electron and V is potential difference between point P and cathode.
$\therefore$ The potential of cathode is zero.
$\therefore$ potential difference between point P and cathode is $\phi = ax^{4/3}$

$\therefore \qquad u = \sqrt{\dfrac{2e}{m}ax^{4/3}} \qquad \therefore \ J = \rho u = \dfrac{4}{9}\varepsilon_0 a^{3/2}\sqrt{\dfrac{2e}{m}}$

YOUR STEP

Consider a vacuum tube diode whose cathode and anode are placed a distance d apart and whose common surface is S (shown in fig 3.211C). The anode is held at a potential V_0 relative to the cathode which is grounded. The cathode is heated and it emits electrons into the space between the electrodes. The emission current is of a constant magnitude I. The electrons are emitted at zero velocity from the cathode.

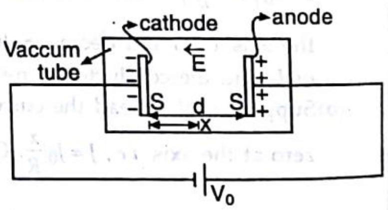


Fig. 3.211C

Find the average electron velocity v_x at the stationary state and the space charge density $\rho(x)$ composed of the electron emitted from the cathode as a function of distance x from the cathode. Neglect the mutual interactions among the electrons in the space charge electron cloud and the effect of the space charge on the homogeneous electric field $\dfrac{V_0}{d}$ which is originally present in the region between the electrodes (This assumption is justified for the case in which the space charge may be viewed as a dilute gas of electrons, in which the average distances between the electrons are negligible).

$$\left\{ v_x = \sqrt{\frac{2eV_0}{md}}\,x, \quad -\rho(x) = \frac{I}{S}\left(\frac{md}{2eV_{0(x)}}\right)^{\frac{1}{2}} \right\}$$

§ 3.212

➢ **CONCEPT**

Mobility of anion is defined as the drift velocity acquired in unit electricfield.

mobility $$u = \frac{v_d}{E}$$

If n_+ and n_- are concentrations of positive and negative ions respectively.

Then, $$n_+ = n_-$$

Also $$v_d = \frac{J}{ne}$$

$\therefore$ $$J = nev_d$$

$\therefore$ J_+ = current density due to positive ion

$$= n_+ ev_d^+ = n_+ eEu_+$$

Similarly $$J_- = n_- eEu_-$$

SOLUTION : Electric current is due to motion of both (negative and positive ions).

$\therefore$ $$I = (J_+ + J_-) S = (n_+ u_+ + n_- u_-) eES$$

But $$n_+ = n_- = n$$

$\therefore$ $$I = neES (u_+ + u_-)$$

But $$E = \frac{V}{d}$$

$\therefore$ $$I = neSV (u_+ + u_-)/d$$

$\therefore$ $$n = \frac{Id}{S(u_+ + u_-) eV}$$

On putting the values, we obtain

$$n = 2.3 \times 10^8/\text{cm}^3$$

YOUR STEP

Find the current in a non-self sustained discharge induced by an ioniser which produces 2×10^6 ion pairs per second. The electron charge $e = 1.6 \times 10^{-19}$ C.

{ 6.4×10^{-13} A}

§ 3.213

➢ **CONCEPT**

mobility $$u = \frac{v_d}{E}$$

$\therefore$ $$v_d = uE$$

SOLUTION :

$\therefore$ $$v_d = uE = u\frac{V}{l} \qquad (\because E = \frac{V}{l})$$

or $$v_d = \frac{u V_0 \sin \omega t}{l}$$

For positive value of v_d, $0 \le \omega t \le \pi$

$\therefore$ The collision of ion with electrode is due to current in the circuuit.

If maximum displacement of an ion is equal to separation l between plates. Then deflection in galvanometer is observed.

For this, $$x_{max} = l \text{ at } \omega = \omega_0$$

At maximum displacement, $v_d = 0$

$\therefore$ $v_d = \frac{uV_0}{l}\sin\omega_0 t = 0 \Rightarrow \omega_0 t = \pi$

$\therefore$ $t = \frac{\pi}{\omega_0}$ $\because$ $v_d = \frac{uV_0 \sin\omega t}{l}$

or $\frac{dx}{dt} = \frac{uV_0}{l}\sin\omega t$

or $\int_0^l dx = \frac{uV_0}{l}\int_0^{\pi/\omega_0} \sin\omega t$

After solving, we get $l = \frac{2uV_0}{l\omega_0}$

$\therefore$ $u = \frac{\omega_0 l^2}{2V_0}$

> **YOUR STEP**
>
> Find the ionization energy for air molecues (in joules and electron volts) if under normal conditions the spark discharge in air takes place at an electric field strength of 3 MV/m, and the mean free path of electrons in air is 1 μm.
>
> { 4.8×10^{-19} J, 3eV }

§ 3.214

➢ **CONCEPT**

At saturation current, all ions produced (positive ions + negative ions) reach at the plate.

SOLUTION :

$\because$ The cause of current is motion of ions

$\therefore$ $I = \frac{q}{t} = \frac{(nV)e}{t}$

$n_i = \frac{n}{t} = \frac{I}{eV}$

On putting the values, we get $n_i = 6 \times 10^9 \text{ cm}^{-3}\text{ s}^{-1}$

(b) The recombination rate of ion is rn^2.

The production rate of ion is $n_i = \frac{I}{eV}$

$\therefore$ Rate of increase of ion is

$\therefore$ $\frac{dn}{dt} = n_i - rn^2$

For balance condition $\frac{dn}{dt} = 0$

$\therefore$ $n = \sqrt{\frac{n_i}{r}}$

On putting the values, we get $n = 6 \times 10^7/\text{cm}^3$

> **YOUR STEP**
>
> A potential differnce of 5V is applied to the electrodes of a discharge tube spaced 10 cm apart. The gas in the tube is singly ionised and the number of ionic pairs in 1 m^3 is 10^8. Also, $u_+ = 3 \times 10^{-2}\ m^2/Vs$ and $u_- = 3 \times 10^2\ m^2/Vs$. Find (a) the current density in the tube (b) the part of the total current transformed by positive ions.
>
> {(a) 2.4×10^{-7} A/m^2 (b) 0.01%}

§ 3.215

➢ **CONCEPT**

From solution of previous problem, $n_0 = \sqrt{\frac{n_i}{r}}$ at $t = 0$

The recombination rate is rn^2.
Since, the ionizer producing is switched off.

$\therefore$ $$\frac{dn}{dt} = -rn^2$$

or $$\int_{\sqrt{n_i/r}}^{\sqrt{n_i/r}/\eta} \frac{dn}{n^2} = -\int_0^t r\,dt$$

After solving, $$t = \frac{\eta - 1}{\sqrt{rn_i}} = 13 \text{ ms}$$

> **YOUR STEP**
>
> The saturation current in a non-self sustained discharge is $I_{sat} = 4.8 \times 10^{-12}$ A. Find the number of ion pairs formed per unit time with the help of an external ionizer. The electron charge $e = 1.6 \times 10^{-19}$ C
>
> $\{15 \times 10^6\}$

§ 3.216

➢ **CONCEPT**

Due to production of ions, charge of capacitor starts to leak. This is caused by decrease of voltage of capacitor.

SOLUTION : $$C = \frac{\varepsilon_0 S}{d}$$

and $I = \frac{q}{t} = \frac{n\,(\text{volume})\,e}{t}$ or $I = \frac{n\,(Sd)\,e}{d}$ $\left(\because \frac{n}{t} = i\right)$

$$I = n_i\,(Sd)\,e = \text{constant}$$

$\therefore$ Flow of charge in time t = decrease in the charge on capacitor in time t.

or $It = CV - CV'$ or $t = \frac{C}{I}(V - V')$ ...(i)

or $$t = \frac{\varepsilon_0 S(V - V')}{d\,n_i(Sd)e} = \frac{\varepsilon_0}{n_i d^2 e}(V - V')$$

According to problem, $\frac{V - V'}{V} = \eta$

$\therefore$ $V - V' = \eta V$

$\therefore$ From equ. (i), we have $$t = \frac{\varepsilon_0 \eta V}{n_i d^2 e}$$

on putting the values, we get $t = 4.6$ days

> **YOUR STEP**
>
> In vacuum diode, the anode is parallel to the cathode at a distance 0.2 cm. If a potential of 100 V is applied to the anode with respect to cathode, calculate the velocity of electrons at the points at the middle of the electrodes.
>
> $\{4.2 \times 10^6$ m/s$\}$

§ 3.217

➢ **CONCEPT**

According to problem, number of new electrons in unit length is $\frac{dv}{dx} = \alpha v$

or $\int_{v_0}^{v} \frac{dv}{v} = \alpha \int_0^x dx$ or $ln \frac{v}{v_0} = \alpha x$

$\therefore$ $\frac{v}{v_0} = e^{\alpha x}$ $\therefore$ $v = v_0 e^{\alpha x}$

For saturation current, $x = d$

$\therefore$ $I = ve = ev_0 e^{\alpha d}$

YOUR STEP

A cathode beam enters the space between the plates of an air parallel plate capacitor in parallel to the plates. Having passed a distance of 5 cm during 5×10^{-9} sec, the beam is deflected by 1 mm. The field strength between the plates of capacitor is 150 V/cm. Find the change in the energy of an electron from the cathode beam per unit time and the angle of deviation of its velocity vector over the specified distance (the angle should be found by two methods). The electron mass and charge are 9.1×10^{-31} kg and 1.6×10^{-19} C respectively.

[4.55×10^{-17} W, 28′]

§ 3.218

➢ **CONCEPT**

From previous problem, $v = v_0 e^{\alpha x}$

Here $v_0 = (n_i\, S dx)$

$\therefore$ $v = (n_i\, S dx)\, e^{\alpha x}$

$\therefore$ $I = \int ev = \int_0^d n_i A e^{\alpha x} dx$

On putting the values, we get $I = e\, n_i\, S \left(\frac{e^{\alpha d} - 1}{\alpha}\right)$

$\therefore$ $J = \frac{I}{S} = \frac{en_i}{\alpha}(e^{\alpha d} - 1)$

YOUR STEP

A cathode beam comprising $n = 10^6$ electrons is emitted with a velocity $v_0 = 10^5$ km/sec into the space between the plates of an air parallel plate capacitor is parallel to them. The potential difference between the plates is $\phi = 400$V, their separation $d = 2$cm, and the area of a plate is $l^2 = 10 \times 10$ cm^2. Find the deflection Δy of the beam and the direction of its outlet velocity. The mass of electron $m = 9.1 \times 10^{-31}$ kg and its charge $e = 1.6 \times 10^{-19}$ C. The relavistic effect should be disregarded.

[$\alpha = 2°$, $v_y = 3.52 \times 10^6$ m/sec]

CONSTANT MAGNETIC FIELD

§ 3.219

➢ **CONCEPT**

The magnetic field at the centre of a circular arc is

$$B = \frac{\mu_0 I \alpha}{4\pi R}$$

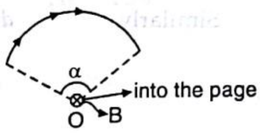


Fig. 3.219A

where I = current in arc

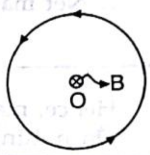
R = Radius of the circular arc.

α = The angle made by the circular arc at its centre.

The arc is shown in fig. 3.219 A.

SOLUTION : (a) For circular loop, $\alpha = 2\pi$

$$\therefore \quad B = \frac{\mu_0 I (2\pi)}{4\pi R} \quad ...(\times)$$



$$\therefore \quad B = \frac{\mu_0 I}{2R} \quad (\times \text{ into the page})$$

Fig. 3.219B

On putting the values, we get

$$B = 6.3\ \mu\text{T}$$

(b) Let the loop is placed in $(y - z)$ plane. For convenience, we present y–z plane in the plane of paper.

The loop is shown in fig. 3.219 C.

We consider a $d\vec{l}$ element at point A on the loop.

The co-ordinates of point A are $(0, R\cos\theta, R\sin\theta)$.

$\therefore$ The position vector of point A is

$$\vec{r}_A = R\cos\theta\,\hat{j} + R\sin\theta\,\hat{k}$$

The position vector of point P is $\vec{r}_P = x\hat{i}$

But $\quad dl = R d\theta$

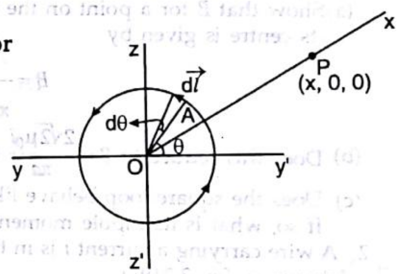


Fig. 3.219C

Also, current element $d\vec{l}$ makes $(90° + \theta)$ with $+y$-axis.

$$\therefore \quad d\vec{l} = dL\cos(90° + \theta)\,\hat{j} + dl\sin(90° + \theta)\,\hat{k}$$

$$= -dl\sin\theta\,\hat{j} + dl\cos\theta\,\hat{k}$$

The magnetic field at point P due to considered element is

$$d\vec{B} = \frac{\mu_0 I\, d\vec{l} \times \vec{r}}{4\pi r^3}$$

$$= \frac{\mu_0 I\,\{-dl\sin\theta\,\hat{j} + dl\cos\theta\,\hat{k}\} \times (\vec{r}_p - \vec{r}_A)}{4\pi\,|\vec{r}_p - \vec{r}_A|^3}$$

or

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\,\frac{\{-dl\sin\theta\,\hat{j} + dl\cos\theta\,\hat{k} \times (x\hat{i} - R\cos\hat{j} - R\sin\theta\,\hat{k})}{|x\hat{i} - R\cos\theta\,\hat{j} - R\sin\theta\,\hat{k}|^3}$$

$$= \frac{\mu_0 I dl \,\{x\sin\theta\,\hat{k} + x\cos\theta\,\hat{j} + R\cos^2\theta\,\hat{i} + R\sin^2\theta\,\hat{i}\}}{4\pi(x^2+R^2)^{3/2}}$$

$$= \frac{\mu_0 I R d\theta}{4\pi(x^2+R^2)^{3/2}}\{x\sin\theta\,\hat{k} + x\cos\theta\,\hat{j} + R\,\hat{i}\} \qquad (\because\ dl = Rd\theta)$$

$$\therefore \qquad dB_x = \frac{\mu_0 I R^2 d\theta}{4\pi(x^2+R^2)^{3/2}}$$

$$\therefore \qquad B_x = \frac{\mu_0 I R^2}{4\pi(R^2+x^2)^{3/2}}\int_0^{2\pi} d\theta = \frac{\mu_0 I R^2}{2\,(R^2+x^2)^{3/2}}$$

Similarly, $$dB_y = \frac{\mu_0 I R x}{4\pi\,(R^2+x^2)^{3/2}}\cos\theta d\theta$$

$$\therefore \qquad B_y = \frac{\mu_0 I R x}{4\pi(R^2+x^2)^{3/2}}\int_0^{2\pi}\cos\theta d\theta = 0$$

Similarly, $$B_z = -\frac{\mu_0 I R x}{4\pi(R^2+x^2)^{3/2}}\int_0^{2\pi}\sin\theta d\theta = 0$$

$\therefore$ Net magnetic field at point P is

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} = B_x\hat{i} = \frac{\mu_0 I R^2}{2\,(R^2+x^2)^{3/2}}\hat{i}$$

Hence, magnetic field B is along axis of the loop.
On putting the value, we get $B = 2.3\mu T$

YOUR STEP

1. A square loop of wire of edge a carries a current i.
   (a) Show that B for a point on the axis of the loop and a distance x from its centre is given by

$$B = \frac{4\mu_0 i a^2}{\pi\,(4x^2+a^2)\,(4x^2+2a^2)^{\frac{1}{2}}}$$

   (b) Does this reduce to $B = \dfrac{2\sqrt{2}\mu_0 i}{\pi a}$ for $x = 0$.
   (c) Does the square loop behave like a dipole for points such that $x >> a$? If so, what is its dipole moment?

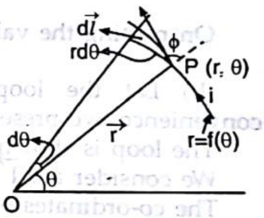


Fig. 3.219D

2. A wire carrying a current i is in the shape of a plane curve $r = f(\theta)$, (r, θ) being polar co-ordinates shown in fig. 3.219D.
   Find the magnetic field at the pole. Deduce also the expression for the field at the focus on elliptical wire carrying a current i.

{1. (b) yes, (c) yes, $P = Ia^2$ 2. $B = \dfrac{\mu_0 i}{2l}$ l where l is the semilatusrectum of ellipse}

§ 3.220

➢ CONCEPT

The magnetic field at point P (shown in fig 3.220 A) is

$$B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

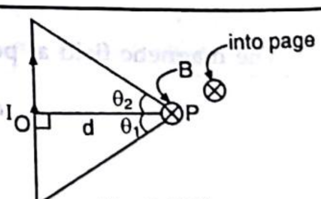


Fig. 3.220A

SOLUTION : From fig. 3.220 B, it is clear that due to each side of polygon the magnetic field is

$$B_1 = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

From fig. 3.220 B, $$\theta_1 = \theta_2 = \frac{\pi}{n}$$

Also, $$\cos\theta_1 = \frac{OD}{OA} = \frac{d}{R} = \cos\theta_2$$

or $$\cos\frac{\pi}{n} = \frac{d}{R}$$

$\therefore$ $$d = R\cos\frac{\pi}{n}$$

$\therefore$ $$B_1 = \frac{\mu_0 I}{4\pi R\cos\frac{\pi}{n}}\left(2\sin\frac{\pi}{n}\right)$$

$$B_1 = \frac{\mu_0 I}{2\pi R}\tan\frac{\pi}{n}$$

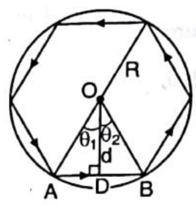


Fig. 3.220B

Since, magnetic field due to all sides are in the same direction (into the page).

$\therefore$ Net magnetic field at point O is $$B = nB_1 = \frac{n\mu_0 I}{2\pi R}\tan\frac{\pi}{n}$$

when $n \to \infty$,

$\therefore$ $$B = \frac{\mu_0 I}{2R}\left(\lim_{n\to\infty}\frac{\tan\pi/n}{\pi/n}\right) = \frac{\mu_0 I}{2R}$$

magnetic field due to circular loop of radius R.

YOUR STEP

A wire shaped to a regular hexagon of side 2 cm carries a current of 2 amp. Find the magnetic induction at the centre of the hexagon.

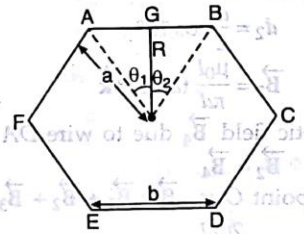


Fig. 3.220C

{ 6.93×10^{-5} wb/m^2}

§ 3.221

➢ **CONCEPT**

The concept is similar to previous problem.

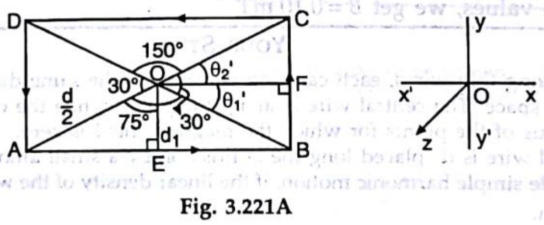


Fig. 3.221A

SOLUTION :

From fig 3.221 A, the magnetic field due to wire AB is equal to that of due to wire CD.

The magnetic field due to wire AD at point O is

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi d_1}(\sin\theta_1 + \sin\theta_2)\hat{k}.$$

Here $\theta_1 = \theta_2 = 75°$

Also, $\cos 75° = \dfrac{d_1}{\frac{d}{2}}$

$\therefore$ $d_1 = \dfrac{d}{2}\cos 75°$

$\therefore$ $\vec{B}_1 = \dfrac{\mu_0 I}{2\pi\frac{d}{2}}\tan 75°\hat{k} = \dfrac{\mu_0 I}{\pi d}\tan 75°\hat{k}$

Similarly magnetic field at O due to wire CD is

$$\vec{B}_3 = \frac{\mu_0 I}{\pi d}\tan 75°\hat{k}$$

The magnetic field at point O due to BC is

$$\vec{B}_2 = \frac{\mu_0 I}{4\pi d_2}(\sin\theta_1' + \sin\theta_2')\hat{k}$$

Here $\theta_1' = \theta_2' = 15°$

In ΔBOF_1 $\cos 15° = \dfrac{OF}{OB} = \dfrac{OF}{\frac{d}{2}}$

$\therefore$ $OF = \dfrac{d}{2}\cos 15°$

$\therefore$ $d_2 = \dfrac{d}{2}\cos 15°$

$\therefore$ $\vec{B}_2 = \dfrac{\mu_0 I}{\pi d}\tan 15°\hat{k}$

From fig. 3.220 A, the magnetic field $\vec{B}_4$ due to wire DA is same as that of BC.

$\therefore$ $\vec{B}_2 = \vec{B}_4$

$\therefore$ The net magnetic field at point O is $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4$

$$\therefore \quad B = \frac{2\mu_0 I}{\pi d}(\tan 75° + \tan 15°)$$

$$= \frac{2\mu_0 I}{\pi d}(\cot 15° + \tan 15°) = \frac{2\mu_0 I}{\pi d}\left(\frac{\cos 15°}{\sin 15°} + \frac{\sin 15°}{\cos 15°}\right)$$

$$= \frac{2\mu_0 I}{\pi d}\left(\frac{\cos^2 15° + \sin^2 15°}{\sin 15° \cos 15°}\right) = \frac{4\mu_0 I}{\pi d \sin 30°} = \frac{4\mu_0 I}{\pi d \sin\phi} \quad (\because \phi = 30°)$$

On putting the values, we get $B = 0.10$ mT

YOUR STEP

Three infinitely long thin wires, each carrying current i in the same direction, are in the x–y plane of a gravity free space. The central wire is along the y-axis while the other two are along $x = \pm d$.

(a) Find the locus of the points for which the magnetic field is zero.

(b) If the central wire is displaced long the z-direction by a small amount and released, show that it will execute simple harmonic motion, if the linear density of the wires is λ_1, find the frequency of oscillation.

$$\left\{ \text{(a) } x = 0 = z, x = \frac{d}{\sqrt{3}}, x = \frac{-d}{\sqrt{3}} (z = 0) \quad \text{(b) } \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}} \right\}$$

§ 3.222

➢ CONCEPT

For solving this problem, the concept of problem 3.219 and 3.220 are applicable.

SOLUTION :

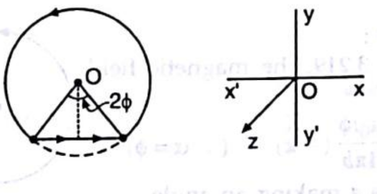


Fig. 3.219A

The given loop (shown in fig. 3.219A) may be devided into two parts :

(i) **circular arc** ⟶This part makes $(2\pi - 2\phi)$ angle at O (shown in fig. 3.219B).

From the concept of problem 3.219, The magnetic field due to an arc at centre O is

$$\vec{B}_1 = \frac{\mu_0 I \alpha}{4\pi R}\hat{k}$$

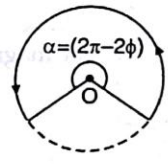


Fig. 3.222B

In this problem, $\alpha = 2\pi - 2\phi$

$$\therefore \quad \vec{B}_1 = \frac{\mu_0 I}{4\pi R}(2\pi - 2\phi)\hat{k}$$

(ii) **Straight line segment :** This part is shown in fig. 3.222 C.

From the concept of problem 3.220, the magnetic field at O due to straight wire is

$$\vec{B}_2 = \frac{\mu_0 I}{4\pi d}(\sin\phi_1 + \sin\phi_2)\hat{k}$$

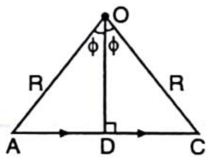


Fig. 3.222C

In this problem, $\phi_1 = \phi_2 = \phi.$

In ΔOAD_1 $\quad \cos\phi = \frac{OD}{R} = \frac{d}{R}$

$$\therefore \quad d = R\cos\phi$$

$$\therefore \quad \vec{B}_2 = \frac{\mu_0 I}{4\pi (R\cos\phi)} 2\sin\phi\hat{k}$$

$$= \frac{\mu_0 I}{2\pi R}\tan\phi\hat{k}$$

Hence, net magnetic field at point O is $\vec{B} = \vec{B}_1 + \vec{B}_2$

$$\therefore \quad \vec{B} = \left\{\frac{\mu_0 I}{4\pi R}(2\pi - 2\phi) + \frac{\mu_0 I}{2\pi R}\tan\phi\right\}\hat{k}$$

$$B = (\pi - \phi + \tan\phi)\frac{\mu_0 I}{2\pi R} = 28\ \mu T$$

This magnetic field is directed perpendicular to the plane of the loop.

YOUR STEP

A current I flows through an infinitely long wire having infinite bends as shown in fig. 3.222D. The radius of the first curved portion is a and the radii of successive curved portions each increase by a factor η Determine the magnetic induction at the point O.

$$\left\{B = \frac{\mu_0 I\theta\eta}{4\pi a(\eta + 1)}\right\}$$

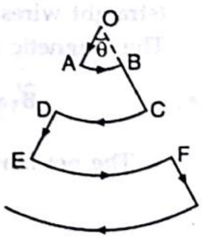


Fig. 3.222D

§ 3.223

➢ **CONCEPT**

The concept is similar to problem 3.219 and 3.220.

The wire shown in fig. 3.223A may be divided into two parts :

(i) Circular arc of radius b :

From concept of problem 3.219, the magnetic field due to this part is

$$\vec{B}_1 = \frac{\mu_0 I\alpha}{4\pi b}(-\hat{k}) = \frac{\mu_0 I\phi}{4\pi b}(-\hat{k}) \qquad (\because \alpha = \phi)$$

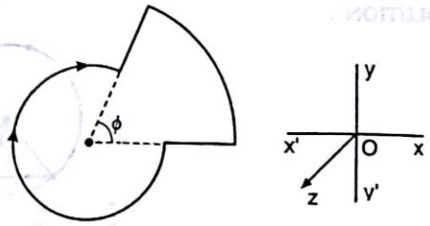


Fig. 3.223A

(ii) A circular arc of radius a making an angle

$$\alpha = (2\pi - \phi)$$

The magnetic field due to this part at O is

$$\vec{B}_2 = \frac{\mu_0 I}{4\pi a}(2\pi - \phi)(-\hat{k})$$

$\therefore$ Net magnetic field at point O is $\vec{B} = \vec{B}_1 + \vec{B}_2$

$$\therefore \quad B = \frac{\mu_0 I}{4\pi}\left(\frac{2\pi - \phi}{a} + \frac{\phi}{b}\right)$$

$$= \left\{\frac{\mu_0 I\phi}{4\pi b} + \frac{\mu_0 I}{4\pi a}(2\pi - \phi)\right\}(-\hat{k})$$

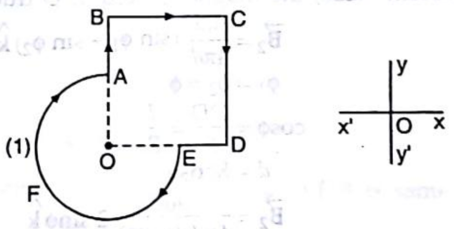


Fig. 3.223B

The diagram of problem is shown in fig. 3.223B. The magnetic fields due to wires AB and DE are zero.

(b) The circuit consists of five parts :

(1) Circular arc : The magnetic field due to this part is

$$\vec{B}_1 = \frac{\mu_0 I\left(2\pi - \frac{\pi}{2}\right)}{4\pi a}(-\hat{k}) = \frac{3\mu_0 I}{8a}(-\hat{k})$$

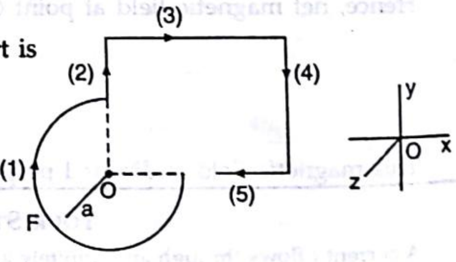


Fig. 3.223C

But $\vec{B}_2 = 0$ and $\vec{B}_5 = 0$

(straight wires are passing through point O)

The magnetic field due to part (3) at point O

i.e., $$\vec{B}_3 = \vec{B}_4 = \frac{\mu_0 i}{4\pi b}\left(\sin 0 + \sin\frac{\pi}{4}\right)(-\hat{k}) = \frac{\sqrt{2}\,\mu_0}{4\pi b}(-\hat{k})$$

$\therefore$ The net magnetic field $\vec{B}$ at point $O = \vec{B}_1 + B_2 + \vec{B}_3 + \vec{B}_4$

$$= \frac{\mu_0 I}{4\pi}\left(\frac{\sqrt{2}}{b} + \frac{3\pi}{2a}\right)(-\hat{k})$$

YOUR STEP

1. A wire bent in the form of an equilateral triangle ABC of side a carries a current i.
   Find the magnetic field at a point having equal distance b from A, B and C in fig. 3.223C.

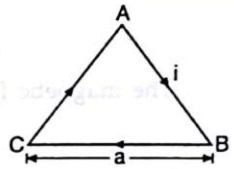


Fig. 3.223C

2. A current loop having two circular arcs joined by two radial lines (shown in fig. 3.223D). Find the magnetic field B at the centre O.

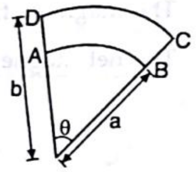


Fig. 3.223D

3. A current of 10A flows around a closed path in a circuit which is in the horizontal plane as shown in the figure. The circuit consists of eight alternating arcs of radii $r_1 = 0.08$ m and $r_2 = 0.12$ m. Each subtends the same angle at the centre.
   (a) Find the magnetic field produced by this circuit at the centre.
   (b) An infinitely long straight wire carrying a current of 10A is passing through the centre of the above circuit vertically with the direction of current being into the plane of the circuit. What is the force acting on the wire at the centre due to the current in the circuit ? What is the force acting on the arc AC and the straight segment CD due to the current at the centre ?

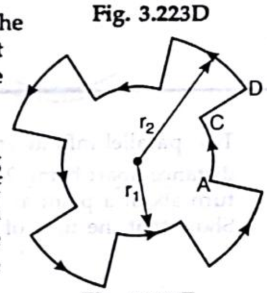


Fig. 3.223E

1. $B = \dfrac{\mu_0 i}{3\sqrt{3}\pi a}$, 2. $\dfrac{\mu_0 i\theta(b-a)}{4\pi ab}$ out of plane

3. (a) 6.54×10^{-4} T
   (vertically upward or outward normal to the paper)
   (b) zero, zero, 8.1×10^{-6} N (inwards)

§ 3.224

➢ CONCEPT

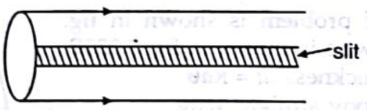


Fig. 3.324A

The magnetic field at a point inside a hollow long tube is zero. The magnetic field due to a long wire is
$$B = \frac{\mu_0 I_0}{2\pi d}$$

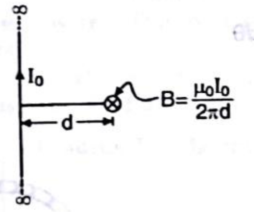


Fig. 3.224B

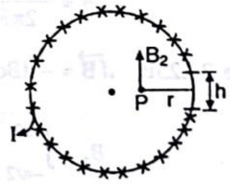


Fig. 3.224C

The top view of tube is shown in Fig. 3.224C.
The cutting slit behaves as a long wire. The current in long slit is

$$I_1 = \frac{Ih}{2\pi R}$$

$\therefore$ The magnetic field due to slit at point P is $B_2 = \frac{\mu_0 I_I}{2\pi r}$

$$\therefore \quad B_2 = \frac{\mu_0}{2\pi r} \times \frac{Ih}{2\pi R} = \frac{\mu_0 Ih}{4\pi^2 rR}$$

The magnetic field due to tube is $\quad B_1 = 0$

The net magnetic field is $\quad \vec{B} = \vec{B_1} - \vec{B_2}$

$$\therefore \quad B = \vec{0} - \vec{B_2} = -\frac{\mu_0 Ih}{4\pi^2 rR}$$

$$\therefore \quad B = |-\vec{B_2}| = B_2$$

$$\therefore \quad B = \frac{\mu_0 Ih}{4\pi^2 rR}$$

YOUR STEP

Two parallel infinite straight wires convey equal currents of strength I in opposite directions, their distance apart being $2a$. A magnetic particle of strength μ and the moment of inertia MK^2 is free to turn about a point at its centre, distance c from each of wires.
Show that the time of small oscillation is that of a pendulum of length l given by

$$\left\{ l = \frac{mgK^2c^2}{4Ia\mu} \quad \text{(In C.G.S.)} \right\}$$

§ 3.225

➢ **CONCEPT**

The magnetic field due to a long straight wire is

$$B = \frac{\mu_0 I}{2\pi d}.$$

SOLUTION : The diagram of problem is shown in fig. 3.225A. The top view of the wire is shown in fig. 3.225B.

We consider a strip of thickness $dt = Rd\theta$.

This strip behaves as a long straight wire.

The current in the considered strip is

$$dI = \frac{I\,(Rd\theta)}{\pi R} = \frac{I\,d\theta}{\pi}$$

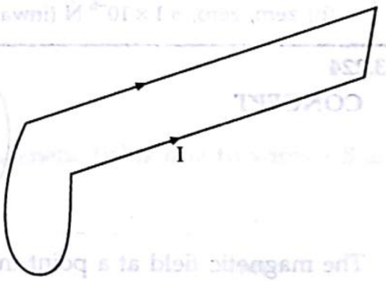


Fig. 3.225A

The magnetic field at O due to considered element is

$$dB = \frac{\mu_0\,(dI)}{2\pi R} = \frac{\mu_0}{2\pi R}\left(\frac{I}{\pi}\,d\theta\right) = \frac{\mu_0 I}{2\pi^2 R}\,d\theta$$

From figure 3.225B, $d\vec{B} = -dB\cos\theta\,\hat{i} + dB\sin\theta\,\hat{j}$

$$\therefore \quad B_x = \int_{-\pi/2}^{\pi/2} -dB\cos\theta$$

$$= \int_{-\pi/2}^{\pi/2} -\frac{\mu_0 I}{2\pi^2 R}\cos\theta d\theta = -\frac{\mu_0 I}{2\pi^2 R}\left[\sin\theta\right]_{-\pi/2}^{\pi/2}$$

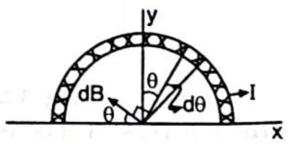


Fig. 3.225B

$$B_x = -\frac{\mu_0 I}{2\pi^2 R}\left[\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right] = -\frac{\mu_0 I}{\pi^2 R}$$

Similarly, $$B_y = \int dB\sin\theta = \int_{-\pi/2}^{\pi/2} \frac{\mu_0 I}{2\pi^2 R}\sin\theta\, d\theta = 0$$

$\therefore$ $$\vec{B} = B_x\,\hat{i} + B_y\,\hat{j}$$

$\therefore$ $\vec{B} = -\frac{\mu_0 I}{\pi^2 R}\,\hat{i} + 0$ $\quad\therefore\quad B = \frac{\mu_0 I}{\pi^2 R}$ (in magnitude)

YOUR STEP

A conductor of length l has shape of a semicylinder of radius R ($<< l$). Cross-section of the conductor is shown in fig 3.225C. Thickness of the conductor is t ($<< R$) and conductivity of its material is $\sigma = \sigma_0 \cos\theta$.

If an ideal battery of emf V is connected across its end faces, calculate the magnetic field at the mid point O of the axis of the semicylinder.

$$\left\{\frac{\mu_0\sigma_0 Vt}{4l}\right\}$$

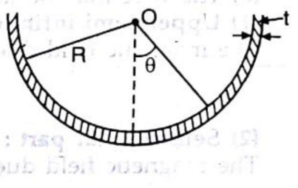


Fig. 3.225C

§ 3.226

➢ **CONCEPT**

The concept is similar to problem 3.219.

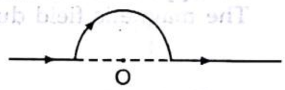


Fig. 3.226A

SOLUTION :

(a) The magnetic field due to semi-infinite wire is $B = \frac{\mu_0 I}{4\pi d}$

The magnetic fields due to straight parts at point O are zero. But the magnetic field due to semicircular part is

$$B = \frac{\mu_0 I\,\pi}{4\pi R} = \frac{\mu_0 I}{4R} \qquad (\because\ \alpha = \pi)$$

(b)

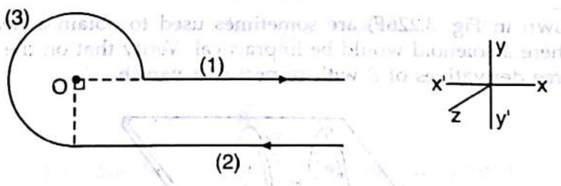


Fig. 3.226B

The given wire may be divided into three parts.

(1) Upper straight wire : This part behaves as semi-infinite wire (shown in fig. 3.226C).

When this part is produced left ward, then it is passing through point O. Hence, magnetic field at O due to this part is zero.

Fig. 3.226C

(2) Circular part of radius R : Making an angle $\alpha = \left(2\pi - \frac{\pi}{2}\right)$ at the centre O.

The magnetic field due to this part is $\vec{B}_1 = \frac{\mu_0 I}{4\pi R}\frac{\alpha}{}(-\hat{k})$

$$= \frac{\mu_0 I}{4\pi R}\left(2\pi - \frac{\pi}{2}\right)(-\hat{k}) = \frac{-3\mu_0 I}{8R}\,\hat{k}$$

(3) **Lower straight wire :** This part behaves as semi-infinite wire (shown in fig. 3.226D).

The magnetic field at point O due to this part is

$$\vec{B}_2 = \frac{\mu_0 I}{4\pi d}(-\hat{k}) = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

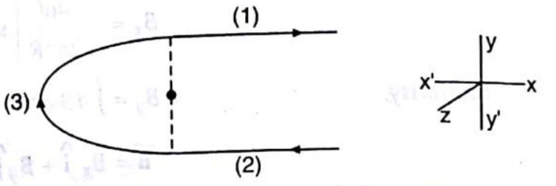


Fig. 3.226D

The net magnetic field at point O is

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$\therefore \quad \vec{B} = \left(\frac{3\mu_0 I}{8R} + \frac{\mu_0 I}{4\pi R}\right)(-\hat{k}) = \frac{\mu_0}{4\pi R}\left(1 + \frac{3\pi}{2}\right) I (-\hat{k})$$

(c) The wire may be divided into three parts :

(1) **Upper semi-infinite wire :** This part is shown in fig. 3.226E.

The magnetic field due to this part is

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

(2) **Semicircular part :**

The magnetic field due to this part is

$$\vec{B}_2 = \frac{\mu_0 I}{4R}(-\hat{k})$$

Fig. 3.226E

(3) **Upper semi-infinite wire :**

The magnetic field due to this part is

$$\vec{B}_3 = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

$$\therefore \quad \vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

On putting the values, we get

$$\vec{B} = \left(\frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R}\right)(-\hat{k})$$

$$= \frac{\mu_0 I}{4\pi R}(2 + \pi)(-\hat{k})$$

YOUR STEP

Helmholtz coils (shown in Fig. 3.226F) are sometimes used to obtain a nearly uniform magnetic field in situations where a solenoid would be impractical. Verify that on the axis, midway between the coils, the first three derivatives of B with respect to x vanish.

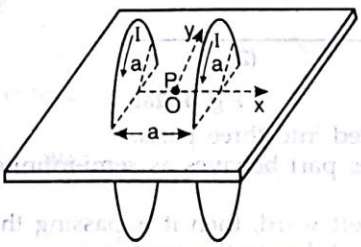


Fig. 3.226F

$\{ B(x) = B_0 + B_4 x^4 \}$

§ 3.227

➢ **CONCEPT**

The problem is similar to previous problem.

SOLUTION : Let the bending point is at origin. The wire is placed as such one part is along x-axis and other is along z-axis (shown in Fig. 3.227A).

Both parts of wire behave as semi-infinite wire.

The magnetic field at point P due to semi-infinite wire placed along x-axis is

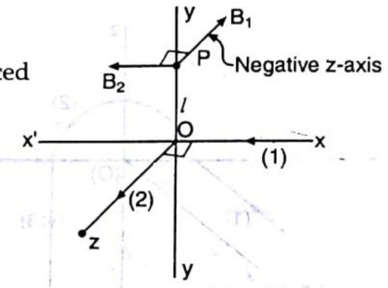


Fig. 3.227A

$$\vec{B_1} = \frac{\mu_0 I}{4\pi l}(-\hat{k})$$

The magnetic field at point P due to semi-infinite wire placed along z-axis is

$$\vec{B_2} = \frac{\mu_0 I}{4\pi l}(-\hat{i})$$

The net magnetic field at point P is

$$\vec{B} = \vec{B_1} + \vec{B_2}$$

$$\therefore \quad \vec{B} = -\frac{\mu_0 I}{4\pi l}\hat{k} - \frac{\mu_0 I}{4\pi l}\hat{i}$$

$$\therefore \quad B = \sqrt{\left(-\frac{\mu_0 I}{4\pi l}\right)^2 + \left(\frac{-\mu_0 I}{4\pi l}\right)^2} = \frac{\mu_0 I\sqrt{2}}{4\pi l} = 2.0\ \mu\text{T}$$

YOUR STEP

A pair of stationary and infinitely long bent wires are placed in the x–y plane shown in fig. 3.227B. The wires carry currents of $i = 10$A each as shown. The segments L and M are along the x-axis. The segments P and Q are parallel to the y-axis such that $OS = OR = 0.02$m. Find the magnitude and direction of magnetic field at the origion O.

{2×10^{-4} T outward the paper}

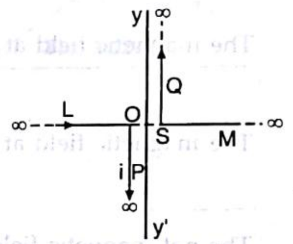


Fig. 3.227B

§ 3.228

➢ **CONCEPT**

The problem is similar to previous problem.

SOLUTION : (a)

The magnetic field due to first semi-infinite wire is

$$\vec{B_1} = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

The magnetic field due to semi-circular part is

$$\vec{B_2} = \frac{\mu_0 I}{4R}(-\hat{i})$$

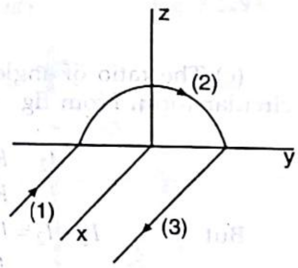


Fig. 3.228A

The magnetic field at O due to third part (semi-infinite wire) is

$$\vec{B_3} = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

$$\because \quad \vec{B} = \mathbf{B_1} + \mathbf{B_2} + \mathbf{B_3}$$

$$\therefore \quad \vec{B} = \frac{\mu_0 I}{4\pi R}(-\hat{k}) + \frac{\mu_0 I}{4R}(-\hat{i}) + \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

$$= \frac{\mu_0 I}{2\pi R}(-\hat{k}) + \frac{\mu_0 I}{4R}(-\hat{i})$$

$$\therefore \quad B = \sqrt{\left(\frac{\mu_0 I}{2\pi R}\right)^2 + \left(\frac{\mu_0 I}{4R}\right)^2} = \frac{\mu_0 I}{4\pi R}\sqrt{4 + \pi^2}$$

On putting the values, we get $\quad B = 0.30\ \mu\text{T}$

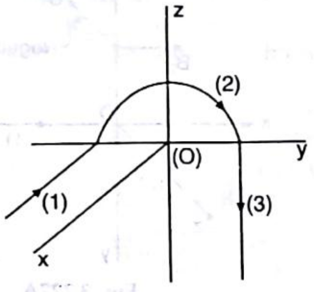


Fig. 3.228B

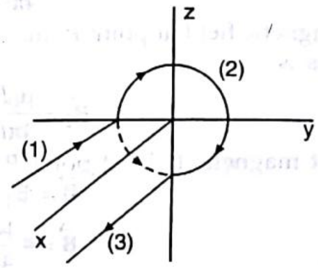


Fig. 3.228C

(b) The magnetic field at point O due to first part (semi-infinite wire) is

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

The magnetic field at point O due to second part (semi-circular part) is

$$\vec{B}_2 = \frac{\mu_0 I}{4R}(-\hat{i})$$

The magnetic field at point O due to third part (semi-infinite wire) is

$$B_3 = \frac{\mu_0 I}{4\pi R}(-\hat{i})$$

The net magnetic field at point O is $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

$$\therefore \quad B = \frac{\mu_0 I}{4\pi R}(-\hat{k}) + \frac{\mu_0 I}{4R}(-\hat{i}) + \frac{\mu_0 I}{4\pi R}(-\hat{i}) = -\frac{\mu_0 I}{4\pi R}[\hat{k} + (\pi + 1)\hat{i}]$$

$$\therefore \quad B = \frac{\mu_0 I}{4\pi R}\sqrt{2 + 2\pi + \pi^2} = 0.34\mu T$$

(c) The ratio of angle should be equal to ratio of resistances in the case of uniform wire in circular form. From fig. 3.228D,

$$\frac{I_2}{I_1} = \frac{R_1}{R_2} = \frac{\frac{3\pi}{2}}{\pi/2} = 3 \qquad \therefore \qquad I_2 = 3I_1$$

But $\quad I_1 + I_2 = I \qquad$ or $\qquad 4I_1 = I$

$$\therefore \quad I_1 = \frac{I}{4} \qquad \text{and} \qquad I_2 = \frac{3I}{4}$$

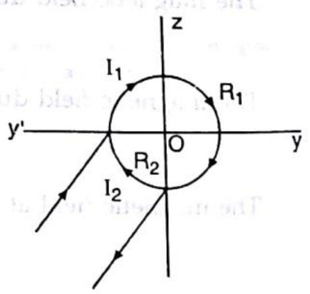


Fig. 3.228D

The magnetic field due to first part (semi-infinite wire) is

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

The magnetic field due to second part at the centre

$$\vec{B}_2 = \frac{\mu_0 I_1}{4\pi R}\left(\frac{3\pi}{2}\right)(-\hat{i}) + \frac{\mu_0 I_2}{4\pi R}\left(\frac{\pi}{2}\right)(\hat{i})$$

$$\therefore \quad \vec{B}_2 = \frac{\mu_0 I}{16\pi R}\left(\frac{3\pi}{2}\right)(-\hat{i}) + \frac{\mu_0 3I}{16\pi R}\left(\frac{\pi}{2}\right)(\hat{i})$$

$$\therefore \quad \vec{B}_2 = \vec{0}$$

The magnetic field at point O due to third part (semi-infinite wire) is

$$\vec{B}_3 = \frac{\mu_0 I}{4\pi R}(-\hat{j})$$

$\therefore$ net magnetic field at point O is $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

$$= \frac{\mu_0 I}{4\pi R}(-\hat{k}) + \frac{\mu_0 I}{4\pi R}(-j)$$

$$\therefore \quad B = |\vec{B}| = \frac{\mu_0}{4\pi}\sqrt{2}\,\frac{I}{R} = 0.11\ \mu T$$

YOUR STEP

Calculate magnetic field at point O if the wire carrying a current I has the shape shown in fig. 3.228E.

$$\left\{\frac{\mu_0 I}{4\pi R}(\sqrt{2}-1)\,\hat{i} - \frac{\mu_0 I}{4\pi R}\,\hat{k}\right\}$$

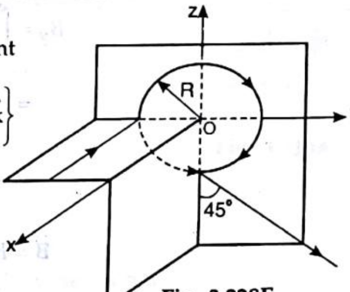


Fig. 3.228E

§ 3.229

➢ **CONCEPT**

The magnetic field near infinite plane is

$$B = \frac{\mu_0 i}{2} \qquad \text{directed parallel to the plane.}$$

The direction of magnetic field is shown in fig 3.229A.

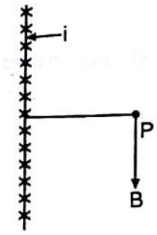


Fig. 3.229A

SOLUTION :

(a) We consider a long strip of thickness dx (shown in Fig 3.229 B)

The considered element behaves as a long wire carry current $di = idx$

where i = Linear current density.

The magnetic field due to considered element at point P is

$$dB = \frac{\mu_0 di}{2\pi r}$$

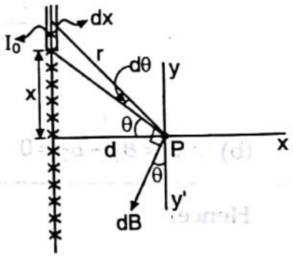


Fig. 3.229B

From fig. 3.229B, $\tan\theta = \frac{x}{d}$

$$\therefore \quad x = d\tan\theta$$

$$\therefore \quad \frac{dx}{d\theta} = d\sec^2\theta$$

$$\therefore \quad dx = d\sec^2\theta\, d\theta$$

similarly, $\sec\theta = \frac{r}{d}$

$$\therefore \quad r = d\sec\theta$$

$$dB = \frac{\mu_0 di}{2\pi r} \qquad (\because di = idx \text{ and } dx = d\sec^2\theta\, d\theta)$$

Putting the values, we get $dB = \frac{\mu_0\, i\, d\sec^2\theta\, d\theta}{2\pi\, d\sec\theta} = \frac{\mu_0\, i\sec\theta\, d\theta}{2\pi}$

From figure $dB = -dB\cos\theta\,\hat{j} - dB\sin\theta\,\hat{i}$

$$\therefore \quad B_x = -\int_{x=-\infty}^{x=\infty} dB\sin\theta$$

$$= -\int_{x=-\infty}^{x=\infty} \frac{\mu_0 I}{2\pi} \sec\theta \sin\theta \, d\theta$$

$$= -\frac{\mu_0 I}{2\pi} \int_{-\pi/2}^{+\pi/2} + \tan\theta \, d\theta = 0$$

$$\therefore \quad B_x = 0$$

$$B_y = \int_{x=-\infty}^{x=+\infty} - dB_y = -\int_{n=-\infty}^{n=+\infty} dB\cos\theta$$

$$= -\int_{-\pi/2}^{+\pi/2} \frac{\mu_0 I \sec\theta \cos\theta \, d\theta}{2\pi}$$

$$= -\frac{\mu_0 I}{2\pi}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = -\frac{\mu_0 I}{2}$$

$$\therefore \quad \vec{B} = B_x \hat{i} + B_y \hat{j} = -\frac{\mu_0 I}{2} \hat{j}$$

$\therefore \quad B = \dfrac{\mu_0 I}{2}$ (directed parallel to the plane and perpendicular to the linear current density)

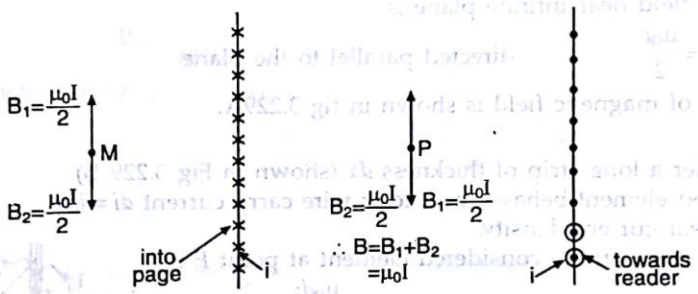


Fig. 3.229C

(b) $\because B = B_1 - B_2 = 0$

Hence, $B = \begin{cases} \mu_0 I \text{ between the plane} \\ 0 \text{ outside} \end{cases}$

YOUR STEP

A conductor consists of an infinite number of adjacent wires, each infinitely long and carrying a current i_0. Show that the lines of B are as represented in fig 3.229C and that B for all points above and below the infinite current sheet is given by

$$B = \frac{1}{2}\mu_0 n i_0$$

where n is the number of wires per unit length. Derive by direct application of Ampere's law.

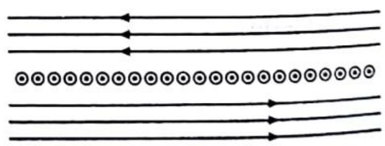


Fig. 3.229C

§ 3.230

➢ **CONCEPT**

We consider an infinite plane element of thickness dr at distance r from median plane (shown in Fig. 3.230A).

The linear charge density in the considered element is

$$i = \frac{I}{l} = \frac{Jldr}{l} = Jdr$$

$\therefore$ The magnetic field at point P due to considered plane is

$$dB = \frac{\mu_0 i}{2} = \frac{\mu_0 j dr}{2} \quad \text{(Here, } < x < d)$$

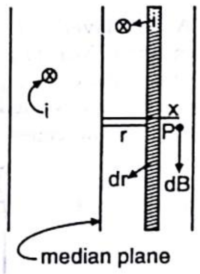


Fig. 3.230A

The net magnetic field at point P is $B = \int_{-x}^{x} \frac{\mu_0 j dr}{2} = \frac{\mu_0 j}{2} [r]_{-x}^{x}$

$$= \frac{\mu_0 j}{2}(x + x) \qquad \text{when} \quad x < d$$

$$= \mu_0 j x \qquad \text{when} \quad x < d$$

when point P is outside the plane, then $x > d$.

$$\therefore \qquad B = \int_{-d}^{+d} \frac{\mu_0 j dr}{2} = \mu_0 j d, \text{ when } x > d$$

YOUR STEP

Two long thin plates of width a are joined to form a section shown in fig. 3.230B. The section carries a total current I along its length. The current density is uniform everywhere in the section. Find the magnitude and direction of magnetic field at the point O.

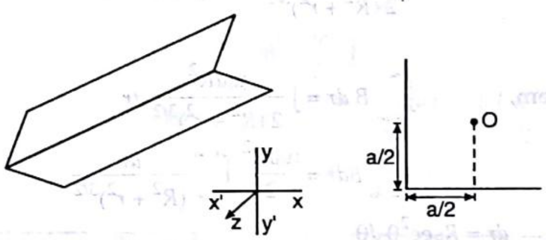


Fig. 3.230B

$$\left\{ \vec{B} = \frac{\mu_0 I}{8a}(\hat{i} + \hat{j}) \right\}$$

§ 3.231

➢ **CONCEPT**

The proble n is based upon ampere's circuital law.

SOLUTION :

For (1)

$$\int_c \vec{B} \cdot d\,\hat{i} = \mu_0 I$$

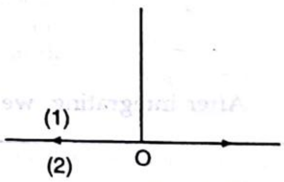


Fig. 3.231A

or

$$B\, 2\pi r = \mu_0 I$$

$$\therefore \qquad B = \frac{\mu_0 I}{2\pi r}$$

For (2),

$$\int_C B \cdot d\vec{l} = 0 \qquad \text{(net current is zero)}$$

$$\therefore \qquad B = 0$$

YOUR STEP

1. A multilayer coil is to be wound on a fixed form with the wires in contact. See the cross-section in fig. 3.231B. The insulation is thin, and the space provided is to be filled. How will the strength of magnetic field produced depend on the diameter d of the wire chosen if the power consumed by the coil is fixed?
2. Each of the eight conductors in fig. 3.231C carries 2A of current into or out of the page. What is the magnetic circulation for the path. (a) at the left and (b) at the right?

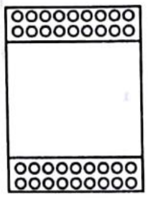


Fig. 3.231B

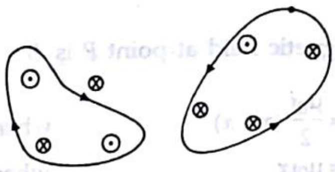


Fig. 3.231C

{1. B is independent of d. 2. (a) $2\mu_0$, (b) $-4\mu_0$}

§ 3.232

➢ **CONCEPT**

The magnetic field on the axis of circular loop is

$$B = \frac{\mu_0 I R^2}{2\,(R^2 + r^2)^{3/2}} \text{ along the axis}$$

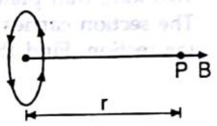


Fig. 3.232A

SOLUTION :

According to problem, $\displaystyle\int_{-\infty}^{\infty} B\,dr = \int \frac{\mu_0 I R^2}{2\,(R^2 + r^2)^{3/2}}\,dr$

$$\therefore \int_{-\infty}^{\infty} Bdr = \frac{\mu_0 I R^2}{2} \int_{-\infty}^{+\infty} \frac{dr}{(R^2 + r^2)^{3/2}}$$

Put $r = R\tan\theta \quad \therefore \quad dr = R\sec^2\theta\, d\theta$

when $r = \infty,\ \theta = \frac{\pi}{2}$, when $r = -\infty,\ \theta = -\frac{\pi}{2}$

$$\therefore \int_{-\infty}^{\infty} Bdr = \frac{\mu_0 I R^2}{2} \int_{-\pi/2}^{\pi/2} \frac{R\sec^2\theta\, d\theta}{R^3 \sec^3\theta}$$

$$= \frac{\mu_0 I}{2} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta = \frac{\mu_0 I}{2}\,[\sin\theta]_{-\pi/2}^{\pi/2} = \mu_0 I$$

After integrating, we get $\displaystyle\int_{-\infty}^{+\infty} Bdr = \mu_0 I$

YOUR STEP

Eight wires cut the page perpendicularly at the points shown in fig. 3.232B. A wire labeled with the integer K ($K = 1, 2, \ldots, 8$) carries the current Ki. For those with odd K, the current is out of the page; for those with even K, it is into the page. Evaluate $\oint \vec{B}\cdot d\vec{S}$ along the closed path in the direction shown.

{ $5\mu_0 \hat{i}$ }

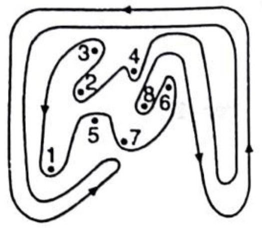


Fig. 3.232B

§ 3.233

➢ **CONCEPT**

The magnetic field at a point outside the hollow cylinder is

$$B = \frac{\mu_0 I}{2\pi r}$$

But the magnetic field at a point inside a hollow long cylinder is zero.

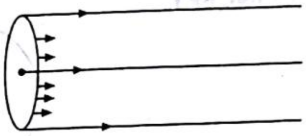


Fig. 3.233A

SOLUTION :

The top view of round uniform wire is shown in fig. 3.233B.

Case I : when point P is at distance $r < R$ from the axis of the wire :

For this, we consider a long hollow cylinder of radius x and thickness dx (shown in fig. 3.233C).

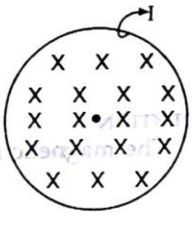


Fig. 3.233B

The cross-sectional area of considered element is

$$dA = 2\pi x dx.$$

The current in the considered element is

$$dI = J\, 2\pi x dx = 2\pi J x dx$$

The magnetic field at point P due to the considered element is

$$dB = \frac{\mu_0\, dI}{2\pi r}$$

$$\therefore \quad B = \int_0^r \frac{\mu_0 2\pi J x dx}{2\pi r} = \frac{\mu_0 J}{r}\left[\frac{r^2}{2}\right]$$

$$\therefore \quad \vec{B} = \frac{\mu_0}{2}(\vec{J} \times \vec{r})$$

When $r > R$, Then $\quad B = \int_0^R \frac{\mu_0 J\, 2\pi x dx}{2\pi r} = \frac{\mu_0 J}{r}\frac{R^2}{2}$

$$\therefore \quad \mathbf{B} = \frac{\mu_0 R^2}{2r^2}(\vec{J} \times \vec{r})$$

Fig. 3.233C

YOUR STEP

1. Two equal circular coils of radius a are placed parallel to one another at a distance $2b$ apart. They carry the same current in the same direction. Show that a point midway between the two centres, the first three differential co-efficients of the field are zero if $2b = a$.
2. Fig. 3.233D shows a cross-section of a long conductor of a type called a co-axial cable of radii a, b and c. Equal but anti parallel uniformly distributed currents i exist in the two conductors. Derive expressions for $B(r)$ in the ranges
(a) $r < c$, (b) $c < r < b$, (c) $b < r < a$ and, (d) $r > a$
(e) Test these expressions for all the special cases that occur to you.
(f) Assume $a = 2.0$ cm, $b = 1.8$ cm, $C = 0.40$ cm and $i = 120$A and plot $B(r)$ over the range $0 < r < 3$ cm.

Fig. 3.233D

3. In the co-axial cable of fig 3.233E, a straight wire of radius a carries a current I_1 along the axis of a metal tube with inner radius b and outer radius c. The tube carries a current I_1 in a direction opposite to that in the wire.
Find B for $a < r < b$. Repeat for $r > c$

2. (a) $\frac{\mu_0 I r}{2\pi c^2}$ (b) $\frac{\mu_0 I}{2\pi r}$ (c) $\frac{\mu_0 i}{2\pi r}\left(\frac{a^2 - r^2}{a^2 - b^2}\right)$ (d) zero

3. $B = \frac{\mu_0 I_1}{2\pi r}$ for $a < r < b$, $\quad B = 0$ for $r > c$

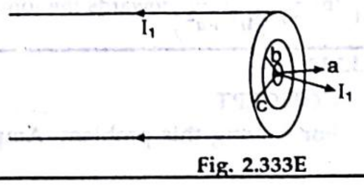


Fig. 2.333E

§ 3.234

➤ **CONCEPT**

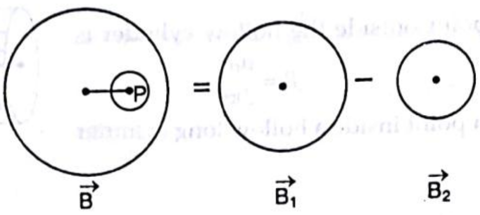


Fig. 3.234A

$\therefore$ $$\vec{B} = \vec{B}_1 - \vec{B}_2$$

SOLUTION :

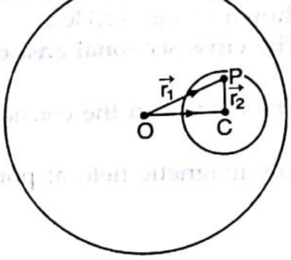


Fig. 3.234B

The magnetic field due to straight wire without cavity is

$$\vec{B}_1 = \frac{\mu_0}{2}(\vec{J} \times \vec{r}_1)$$

$$\vec{B}_1 = \frac{\mu_0}{2}(\vec{J} \times \vec{r}_1)$$

The magnetic field at point P due to cavity is

$$\vec{B}_2 = \frac{\mu_0}{2}(\vec{J} \times \vec{r}_2)$$

$\therefore$ The net magentic field is $$\vec{B} = \vec{B}_1 - \vec{B}_2$$

$$\vec{B} = \frac{\mu_0}{2}\vec{J} \times (\vec{r}_1 - \vec{r}_2)$$

$\because$ According to addition law of vectors,

$$\vec{OC} + \vec{CP} = \vec{OP}$$

or $$\vec{l} + \vec{r}_2 = \vec{r}_1$$

$\therefore$ $$\vec{B} = \frac{\mu_0}{2}(\vec{J} \times \vec{l})$$

Remarks : The magnetic field inside cavity remains constant.

YOUR STEP

A long cylindrical conductor of radius a has two cylindrical cavities of diameter a through its entire length as shown in the cross-section in figure 3.234C. A current I is directed out of the page and is uniform throughout the cross-section of the conductor. Find the magnitude and direction of the magnetic field in terms of μ_0, I, r and a.

(a) at point P_1 and

(b) at point P_2

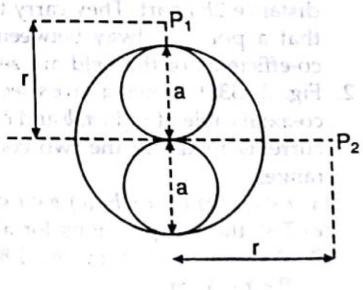


Fig. 3.234C

$$\left\{\begin{array}{l} \text{(a) } \dfrac{\mu_0 I}{\pi r}\left(\dfrac{2r^2 - a^2}{4r^2 - a^2}\right) \text{ to the left} \\ \text{(b) } \dfrac{\mu_0 I}{\pi r}\left(\dfrac{2r^2 + a^2}{4r^2 + a^2}\right) \text{ towards the top of the page} \end{array}\right.$$

§ 3.235

➤ **CONCEPT**

For solving this problem, Ampere's circuital law is applicable.

According to this law, $\int_C \vec{B}\cdot d\vec{l} = \mu_0 I$

SOLUTION :

The current $$I = \int_0^r J\,(2\pi r)\,dr$$

$\because$ $$\int_C \vec{B}\cdot d\vec{L} = \mu_0 I$$

or $$B 2\pi r = \mu_0 \int_0^r J\,(2\pi r)\,dr$$

or $$b r^{\alpha}\, r = \mu_0 \int_0^r J\, r\, dr$$

or $$b r^{\alpha+1} = \mu_0 \int_0^r J\, r\, dr$$

or $$b\frac{d}{dr}(r^{\alpha+1}) = \mu_0 J r$$

or $$b(\alpha+1)\, r^{\alpha} = \mu_0 J r$$

$\therefore$ $$J = \frac{b(\alpha+1)\, r^{\alpha-1}}{\mu_0}$$

YOUR STEP

1. The current density inside a long, solid, cylindrical wire of radius a is in the direction of the axis and varies linearly with radial distance r from the axis according to $J = \frac{J_0 r}{a}$. Find the magnetic field inside the wire. Express your answer in terms of the total current i carried by the wire.
2. Suppose that the current density in the wire varies with r according to $J = Kr^2$, where K is a constant. Find the value of B for (a) $r > a$ and (b) $r < a$ (shown in fig 3.235A).

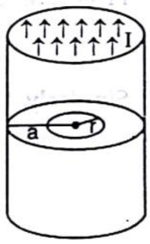
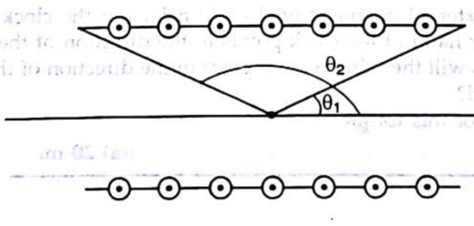


Fig. 3.235A

$\left\{ 1.\ \frac{\mu_0 i r^2}{2\pi a^3} \quad 2.\ \text{(a)}\ \frac{\mu_0 K a^4}{4r},\ \text{(b)}\ \frac{\mu_0 K r^3}{4} \right\}$

§ 3.236

SOLUTION :



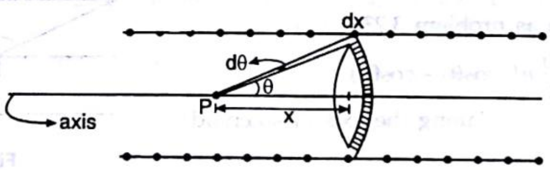


Fig. 3.236A

We consider a circular coil of radius R and thickness dx. The number of turns in the considered element is $dn = ndx$. The magnetic field at point P due to considered element is

$$dB = \frac{dn\mu_0 IR^2}{2(R^2+x^2)^{3/2}} = \frac{\mu_0 IR^2 ndx}{2(R^2+x^2)^{3/2}}$$

or
$$dB = \frac{\mu_0 IR^2 n}{2}\frac{dx}{(R^2+x^2)^{3/2}}$$

Its direction is along the axis of solenoid

From fig. 3.236A, $\cot\theta = \frac{x}{R}$

$\therefore$ $dx = -R\text{cosec}^2\theta d\theta$

$\therefore$ $dB = -\frac{1}{2}\mu_0 nI\sin\theta\, d\theta$

The magnetic field at point P due to entire solenoid is

$$B = \frac{1}{2}\mu_0 nI\int_{\theta_1}^{\theta_2}(-\sin\theta)d\theta = \frac{1}{2}\mu_0 nI(\cos\theta_2 - \cos\theta_1)$$

SOLUTION : If point P is at the centre of coil.

From Fig. 3.236B, $\cos\theta_1 = \frac{l}{2\sqrt{R^2+\frac{l^2}{4}}} = \frac{l}{\sqrt{4R^2+l^2}}$

Similarly, $\cos(180° - \theta_2) = \frac{l}{\sqrt{4R^2+l^2}}$

$\therefore$ $\cos\theta_2 = \frac{-l}{\sqrt{4R^2+l^2}}$

$\therefore$ $$B = \frac{\mu_0 nI}{\sqrt{1+\left(\frac{2R}{l}\right)^2}}$$

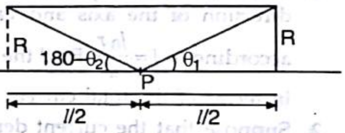


Fig. 3.236B

YOUR STEP

A stationary, circular wall clock has a face with a radius of 15 cm. Six turns of wire are wound around its perimeter, the wire carries a current 2A in the clockwise direction. The clock is located, where there is a constant external magnetic field of 70 mT (but the clock still keeps perfect time) at exactly 1.00 PM, the hour hand of the clock points in the direction of the external magnetic field.

(a) After how many minutes will the minute hand point in the direction of the torque on the winding due to the magnetic field?

(b) What is the magnitude of this torque?

[(a) 20 mi (b) 5.94×10^{-2} N.m]

§ 3.237

➢ **CONCEPT**

The concept is same as problem 3.236.

i.e., $$B = \frac{\mu_0}{2}nI(\cos\theta_2 - \cos\theta_1)$$

(along the axis of solenoid)

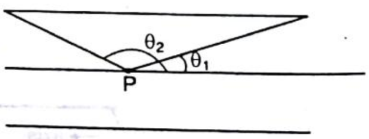


Fig. 3.337A

SOLUTION :

(a) For long solenoid, $\theta_1 = 0$

and $$\cos(180 - \theta_2) = \frac{x}{\sqrt{R^2 + x^2}}$$

$\therefore$ $$\cos\theta_2 = \frac{-x}{(R^2 + x^2)^{1/2}}$$

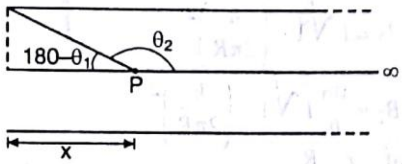


Fig. 3.237B

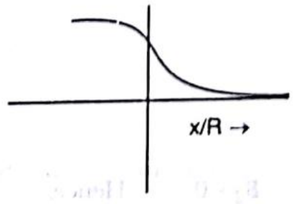


Fig. 3.237C

$\because$ $$B = \frac{\mu_0 nI}{2}(\cos\theta_2 - \cos\theta_1) = \frac{\mu_0 nI}{2}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

(b) According to problem, $$\frac{B_0 - B}{B_0} = 1 - \frac{B}{B_0} = 1 - \eta$$

Here $B = \mu_0 nI$ and $$B_0 = \frac{\mu_0 nI}{2}\left[\frac{1 - x_0}{\sqrt{R^2 + x_0^2}}\right]$$

putting the value of B and B_0, $$1 - \frac{1}{2}\left[1 - \frac{x_0}{\sqrt{R^2 + x_0^2}}\right] = 1 - \eta$$

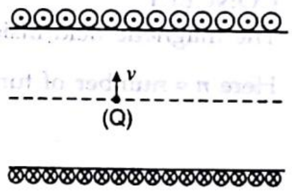
or $$-\frac{x_0}{\sqrt{R^2 + x_0^2}} = 1 - 2\eta$$

or $$x_0^2 = (1 - 4\eta + 4\eta^2)(R^2 + x_0^2)$$

$\therefore$ $$x_0 = \frac{(1 - 2\eta)R}{2\sqrt{\eta(1 - \eta)}} \approx 5R$$

YOUR STEP

A point charge Q with a mass m is placed on the axis of a solenoid in fig. 3.237D. The solonoid has a diameter d with n turns of wire per unit length and carries a current i. What is the maximum possible velocity that can be imparted to the particle perpendicular to the axis of the solenoid if it is to remain in the solenoid at all times.



$$\left\{v_{max} = \frac{Q\mu_0 n i d}{4m}\right\}$$

Fig. 3.237D

§ 3.238

➢ **CONCEPT**

In this case, current is flowing in two mutual perpendicular directions.

(1) A part of current I_1 is flowing along the length of solenoid. Due to this current, the system behaves as a hollow long tube of radius R.

The magnetic field due to long hollow tube is

$$B_1 = 0 \quad \text{at} \quad r < R \qquad B_2 = \frac{\mu_0 I}{2\pi r} \text{ at } r > R$$

(2) Remaining part of current is flowing along the circumference of the coil. Due to this, the system behaves as a long solenoid.

The magnetic field due to this is $B_2 = \mu_0 n I_2$ (where $r < R$)

Here $n = \frac{1}{h}$ (number of turns per unit length)

$\therefore$ $B_2 = \frac{\mu_0}{h} I_2$ (when $r < R$)

Here $I_2 = I\sqrt{1-\left(\frac{h}{2\pi R}\right)^2}$

$\therefore$ $B_2 = \frac{\mu_0}{h} I\sqrt{1-\left(\frac{h}{2\pi R}\right)^2}$

when $r > R$, $B_2 = 0$ Hence, at $r < R$

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$\therefore$ $B = \frac{\mu_0}{h} I\sqrt{1-\left(\frac{h}{2\pi R}\right)^2}$

At $r > R$ $\vec{B} = \vec{B}_1 + \vec{B}_2$

$\therefore$ $B = \frac{\mu_0 I}{2\pi r}$

YOUR STEP

A straight conductor carrying current i splits into identical semicircular arcs as shown in fig. 3.238A. What is the magnetic field at the center C of the resulting circular loop? {zero}

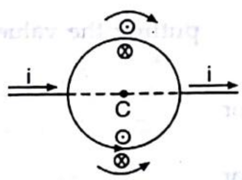


Fig. 3.238A

§ 3.239

➢ **CONCEPT**

The magnetic field inside the core is $B_1 = \mu_0 n I$

Here n = number of turns per unit length $= \frac{N}{2\pi R}$

$\therefore$ $B_1 = \frac{\mu_0 N I}{2\pi R}$

For calculation of magnetic field at the centre of toroid, the system behaves as circular coil,

$\therefore$ $B_2 = \frac{\mu_0 I}{2R}$ (at the centre of circular coil)

$$\eta = \frac{B_1}{B_2} = \frac{N}{\pi}$$

YOUR STEP

A toroidal solenoid has inner radius $r_1 = 0.200$ m and outer radius $r_2 = 0.280$ m. The solenoid has 300 turns and carries a current of 6.20A. What is the magnitude of the magnetic field at each of the following distances from the center of the torus?
(a) 0.150 m (b) 0.240m (c) 0.350m?

{ (a) 0, (b) 1.55 mT, (c) 0}

§ 3.240

➤ **CONCEPT**

Magnetic flux is given by $\phi = \int \vec{B} . d\vec{S}$

The magnetic field at a point inside a long solid wire is

$$\vec{B} = \frac{\mu_0}{2}(\vec{J} \times \vec{r})$$

$$\therefore \quad B = \frac{\mu_0 Jr}{2}$$

SOLUTION : We consider an element of width dr at distance r from the axis of wire.

The area of considered element is

$$dS = ldr \qquad \text{(But } l = 1 \text{ metre)}$$

$$\therefore \quad dS = dr$$

$$\therefore \quad \phi = \int_0^R \frac{\mu_0 Jr}{2} dr \qquad \text{(Here } J = \frac{I}{\pi R^2}\text{)}$$

$$= \frac{\mu_0 J}{2}\frac{R^2}{2} = \frac{\mu_0 IR^2}{4\pi R^2} = \frac{\mu_0 I}{4\pi}$$

YOUR STEP

A long copper wire carries a current of 10A. Calculate the magnetic flux per metre of wire for a plane surface S inside the wire shown in Fig. 3.240A.

$\{10^{-6}$ Wb/metre$\}$

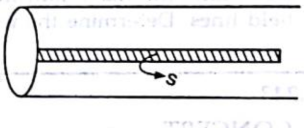


Fig. 3.240A

§ 3.241

➤ **CONCEPT**

At the end of a long solenoid, the magnetic field is $B = \frac{\mu_0 nI}{2}$

SOLUTION :

$$\because \quad \phi = \vec{B} . \vec{S} = B S \cos 0 = \frac{\mu_0 nIS}{2}$$

YOUR STEP

A closed loop of wire consists of a pair of equal semi-circles of radius a lying in mutually perpendicular planes. A uniform magnetic field B is directed perpendicular to the axis AA' and makes an angle 45° with the planes of the semi-circles. Calculate the flux through this closed loop.

$\left\{\frac{\pi Ba^2}{\sqrt{2}}\right\}$

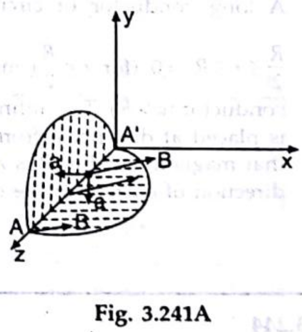


Fig. 3.241A

§ 3.242

➢ **CONCEPT**

The magnetic field at distance r from centre of toroid is

$$B = \mu_0 nI = \frac{\mu_0 NI}{2\pi r}$$

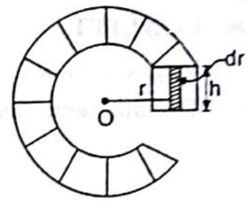


Fig. 3.242A

SOLUTION :

We consider a strip of thickness dr at distance r from centre O.

The area of considered element is

$$dS = hdr$$

$$\therefore \quad d\phi = \vec{B} \cdot d\vec{S} = Bhdr\cos 0$$

$$\therefore \quad \phi = \int_a^b \frac{\mu_0 NI}{2\pi r} hdr = \frac{\mu_0 NIh}{2\pi} ln \frac{b}{a}$$

$$\therefore \quad \phi = \frac{\mu_0 NIh}{2\pi} ln\, \eta$$

> **YOUR STEP**
>
> A rectangular frame made of a wire carrying a current of 2A is in a uniform magnetic field of strength 0.10T. The size of frame is 4×5 cm^2. In a certain position, the magnetic flux through the frame is 0.8×10^{-4} weber.
>
> Determine the maximum magnetic flux in the frame when its plane is perpendicular to magnetic field lines. Determine the work done in rotating the frame to this position.
>
> { 2×10^{-4} weber, 2.4×10^{-4} joule }

§ 3.243

➢ **CONCEPT**

Magnetic moment of a loop is

$$P_m = NIS$$

SOLUTION :

Here $N = 1, \quad S = \pi R^2$

$$\therefore \quad B = \frac{\mu_0 I}{2R} \quad \text{or} \quad I = \frac{2RB}{\mu_0} \quad \therefore \quad P_m = \frac{2\pi BR^3}{\mu_0}$$

> **YOUR STEP**
>
> A long conductor of circular cross-section with radius R has current density $J(r) = \rho_0 \frac{r^2}{R^2}$ (for $\frac{R}{2} \le r \le R) = 0$ (for $r < \frac{R}{2}$) into the plane of paper. There is a point P at distance 'a' from axis of conductor ($a > R$). Two infinitely long thin conducting wires carrying current I_0 in the same direction is placed at distance a from O perpendicular to OP and parallel to conductor at either sides such that magnetic field at P is zero. Find current I_0 in wires and direction of current as compared with direction of current in the conductor.
>
> $\left\{ I_0 = \frac{15\pi\rho_0 R^2}{32} \right\}$

§ 3.244

➢ **CONCEPT**

Magnetic moment is a vector quantity. Its direction is in the direction of area vector.

It is given by $P_m = NIS$

SOLUTION :

We consider an element making angle $d\theta$ at O.

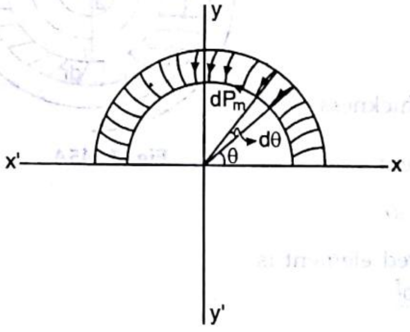


Fig. 3.244A

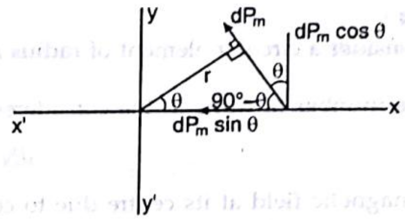


Fig. 3.244B

The number of turns in the considered element is

$$dN = \frac{N}{\pi} d\theta$$

$\therefore$
$$d P_m = dNIS = dNI \frac{\pi d^2}{4}$$

From fig 3.244 B.
$$d \vec{P}_m = d P_m \cos\theta \, \hat{j} - dP_m \sin\theta \, \hat{i}$$

$\therefore$
$$P_{mx} = \int_{\theta=0}^{\theta=\pi} - d P_m \sin\theta$$

$$= -\int_{\theta=0}^{\theta=\pi} dNI \frac{\pi d^2}{4} \sin\theta = -\int_0^{\pi} \frac{NId^2}{4} \sin\theta \, d\theta$$

$$= -\frac{NId^2}{4} \int_0^{\pi} \sin\theta d\theta = \frac{NId^2}{4} [\cos\theta]_0^{\pi}$$

$$= \frac{NId^2}{4} [\cos\pi - \cos 0] = -\frac{NId^2}{2}$$

Similarly,
$$P_{my} = \int_0^{\pi} \frac{NId^2}{4} \cos\theta \, d\theta = \frac{NId^2}{4} [\sin\theta]_0^{\pi} = 0$$

$\therefore$
$$P_m = P_{mx} = \frac{NId^2}{2} = 0.5 \text{A.m}^2$$

(in the sense of magnitude)

YOUR STEP

A wire is bent into three segements each of radius a. Each segment is a quadrant of a circle. The first segment lies in the XY plane, the second in the YZ plane and the third in the XZ plane in fig. 3.244C. If a current i flows in the wire, what is the net dipole moment of the wire?

$$\left\{ \mu = \frac{\pi a^2 i}{4} (\hat{i} + \hat{j} + \hat{k}) \right\}$$

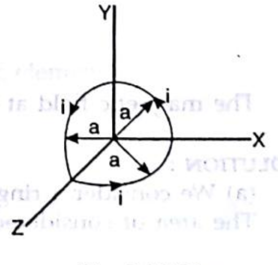


Fig. 3.244C

§ 3.245

➢ **CONCEPT**

The magnetic field due to a circular coil of N turns at its centre is

$$B = \frac{\mu_0 N I}{2R}$$

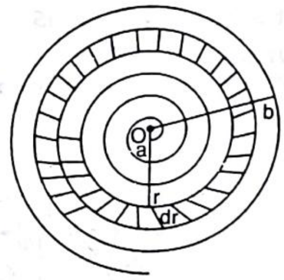


Fig. 3.245A

SOLUTION :

We consider a circular element of radius r and thickness dr.

∴ The number of turns in the considered element is

$$dN = \frac{N}{b-a} dr$$

The magnetic field at its centre due to considered element is

$$dB = \frac{dN \mu_0 I}{2r}$$

∴ $$B = \frac{N\mu_0 I}{2(b-a)} \int_a^b \frac{dr}{r} = \frac{\mu_0 N I}{2(b-a)} ln \frac{b}{a} = 7 \mu\text{T}$$

(b) From the concept of previous problem,

$$dP_m = dNI\pi r^2$$

∴ $$P_m = I\left(\frac{N}{b-a}\right)\int_a^b \pi r^2 dr$$

∴ $$P_m = \frac{\pi NI(b^3 - a^3)}{3(b-a)} = \frac{\pi NI}{3}(b^2 + ab + a^2) = 15 \text{ mA.m}^2$$

> **YOUR STEP**
>
> A wire is in the form of that part of the curve. $2\pi r = a\theta$ where $0 \le \theta \le 2\pi$.
> Prove that when a current I flows in the wire, the magnetic field at a point distant z from O (pole of co-ordinate system) on the normal through O the plane of the wire, has a component along the normal of magnitude,
>
> $$\left\{\frac{\mu_0 I}{2}\left\{\frac{1}{a}\sinh^{-1}\frac{a}{z} - \frac{1}{\sqrt{a^2+z^2}}\right\}\right\}$$

§ 3.246

➢ **CONCEPT**

If a circular ring of radius R is uniformly, charged with the charge q and rotates with constant angular velocity ω about its centre. Then the convection current due to motion of ring is

$$I = \frac{dq}{dt} = \frac{\lambda R d\theta}{dt} = \frac{q}{2\pi R} R \frac{d\theta}{dt}$$

∴ $$I = \frac{\omega q}{2\pi}$$

The magnetic field at the centre of ring is $B = \frac{\mu_0 I}{2R}$

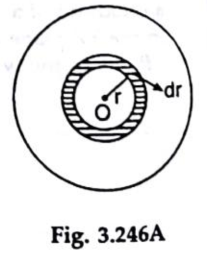


Fig. 3.246A

SOLUTION :

(a) We consider a ring element of radius r and thickness dr.
The area of considered element is

$$dA = 2\pi r dr$$

∴ The electric charge on the considered element is

$$dq = \sigma dA = 2\pi\sigma r dr$$

∴ The convection current due to considered element is

$$dI = \frac{\omega\, dq}{2\pi} = \frac{2\pi\, \sigma\, \omega\, r\, dr}{2\pi} = \sigma\omega r dr$$

∴ The magnetic field at the centre due to considered element is

$$dB = \frac{\mu_0\, dI}{2r} = \frac{\mu_0\, \sigma\, \omega\, r\, dr}{2r}$$

∴ The magnetic field due to disc is

∴ $$B = \frac{\mu_0 \sigma\omega}{2r}\int_0^R dr \qquad \therefore \quad B = \frac{\mu_0}{2}\sigma\omega R$$

(b) The magnetic moment due to considered ring is

$$dP_m = \pi r^2\, dI = \pi r^2 \sigma\omega r dr = \pi\sigma\omega r^3 dr$$

∴ $$P_m = \pi\sigma\omega \int_0^R r^3 dr$$

∴ $$P_m = \pi\sigma\omega \frac{R^4}{4} = \frac{\pi\sigma\omega R^4}{4}$$

YOUR STEP

A non-conducting rod of length L has total charge Q distributed uniformly over its length. The rod rotates in a plane about an axis passing perpendicularly through its end with angular frequency ω. Find the magnetic field at a distance $\sqrt{3}\,L$ from the rod on the axis.

$$\left\{\frac{\mu_0 \lambda\omega}{4\pi}(ln\, 3 - 1)\right\}$$

§ 3.247

➢ **CONCEPT**

From the solution of problem, 3.219, the magnetic field on the axis of a circular ring is

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

SOLUTION :

We consider a circular ring of radius r and thickness $dS = Rd\theta$

So, area of considered ring is

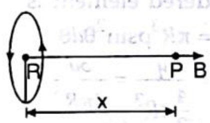


Fig. 3.247A

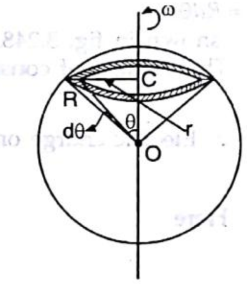


Fig. 3.247B

$$dA = 2\pi r\, dS = 2\pi\,(R\sin\theta)\,(Rd\theta) = 2\pi R^2 \sin\theta\, d\theta$$

∴ $$dq = \sigma dA = 2\pi\sigma R^2 \sin\theta\, d\theta$$

∴ $$dI = \frac{\omega\, dq}{2\pi} = \omega\,(\sigma R^2 \sin\theta\, d\theta) = \sigma\omega R^2 \sin\theta\, d\,\theta$$

∴ The magnetic field at centre O of the sphere due to considered element is

$$dB = \frac{\mu_0\, dI r^2}{2\,(r^2 + x^2)^{3/2}}$$

Here $\quad r = R\sin\theta$

and $\quad x = R\cos\theta$

∴ $$dB = \frac{\mu_0 \sigma\omega R^2 \sin\theta d\theta \left(R^2 \sin^2\theta\right)}{2\,(R^2 \sin^2\theta + R^2 \cos^2\theta)^{3/2}}$$

$$\therefore \qquad B = \int_0^{\pi} \frac{\mu_0 \sigma \omega R}{2} \sin^3 \theta d\theta$$

$$\therefore \qquad B = \int_0^{\pi} \frac{\mu_0 \sigma \omega R}{2} \sin^3 \theta d\theta$$

After integrating, we get $B = \frac{2}{3} \mu_0 \sigma \omega R$

On putting the values, we get $B = 29 \times 10^{-12}\ \mathrm{T} \approx 20\mathrm{pt}$

YOUR STEP

In the fig. shown in 3.247C, the cone is free to rotate about axis OO'. Its curved surface carries a uniform surface charge density σ as shown. The material of the cone is non-conducting. If a time varying magnetic field, $\vec{B} = \vec{K}t$ is switched on at $t = 0$ in the region, where the vector $\vec{K}$ is directed along the axis OO', find the angular velocity ω acquired by the cone due to the electromagnetic forces acting on it as a function of time. The mass of cone is m. Assume density of the cone is uniform.

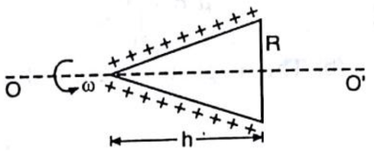


Fig. 3.247C

$$\left\{ \frac{5kR\pi\sigma t\sqrt{R^2 + h^2}}{6m} \right\}$$

§ 3.248

➢ **CONCEPT**

The magnetic moment due to a disc is

$$P_m = \frac{q\omega r^2}{4}$$

SOLUTION : We consider a circular disc of radius r and thickness $dS = Rd\theta$ (shown in fig. 3.248A).

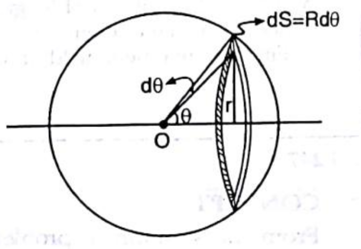


Fig. 3.248A

The volume of considered element is

$$dV = \pi r^2 dS = \pi (R\sin\theta)^2 (Rd\theta) = \pi R^3 \sin^2\theta d\theta \qquad (\because r = R\sin\theta)$$

$\therefore$ Electric charge on considered element is

$$dq = \pi R^3 \rho \sin^2\theta d\theta$$

Here

$$\rho = \frac{q}{\frac{4}{3}\pi R^3} = \frac{3q}{4\pi R^3}$$

$$\therefore \qquad dq = (\pi R^3 \sin^2\theta d\theta) \frac{3q}{4\pi R^3} = \frac{3q}{4} \sin^2\theta d\theta$$

The magnetic moment due to considered element is

$$dP_m = \frac{\omega\, dq r^2}{4} = \frac{\omega\, 3q}{16} \sin^2\theta\, d\theta\, r^2 = \frac{3q\omega}{16} \sin^2\theta\, d\theta\, R^2 \sin^2\theta$$

$\therefore$ The magnetic moment of the sphere is

$$P_m = \frac{3q\omega R^2}{16} \int_0^{\pi} \sin^4\theta d\theta$$

After integrating, we get $P_m = \frac{qR^2\omega}{5}$

The mechanical moment (angular momentum) of the sphere about its diameter is

$$M = I\omega = \frac{2}{5} mR^2\omega \qquad \therefore \qquad \frac{P_m}{M} = \frac{q}{2m}$$

Remarks : $\frac{P_m}{M} = \frac{q}{2m}$ is a general relation between magnetic moment and angular momentum.

YOUR STEP

Q charge is uniformly distributed over the same surface of a right circular cone of semi-vertical angle θ and height h. The cone is uniformly rotated about its axis at angular velocity ω. Calculate the associated magnetic dipole moment.

$$\left\{\frac{Q\omega h^2 \tan^2\theta}{4}\right\}$$

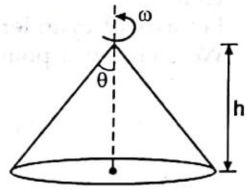


Fig. 3.248B

§ 3.249

➢ **CONCEPT**

Polarisation vector P is numerically equal to surface charge density due to polarisation.

$\therefore$ $$\sigma_P = \alpha R \qquad \text{(at surface, } r = R)$$

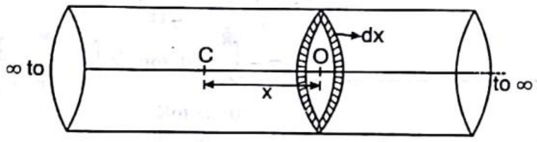


Fig. 3.249A

SOLUTION :

We consider a ring element of thickness dx at distance x from centre of cylinder.

The charge on considered element is

$$dq = (2\pi R dx)\,\sigma_p = 2\pi R^2 \alpha dx$$

$\therefore$ The convection current is $$dI = \frac{\omega\, dq}{2\pi} = \frac{\omega 2\pi R^2 \alpha dx}{2\pi}$$

$\therefore$ $$dI = \omega R^2 \alpha dx$$

The magnetic field at the centre due to considered element is

$$dB = \frac{\mu_0 dI R^2}{2\,(R^2 + x^2)^{3/2}}$$

The magnetic field at the centre due to whole cylinder is

$$B_1 = \int_{-\infty}^{+\infty} \frac{\mu_0 R^2 \omega R^2 \alpha dx}{2\,(R^2 + x^2)^{3/2}}$$

After integrating, we get

Putting $x = R\tan\theta$, $dx = R\sec^2\theta d\theta$

when $x = +\infty$, $\theta = \pi/2$ and when $x = -\infty$, $\theta = -\frac{\pi}{2}$

$\therefore$ $$B_1 = \frac{\mu_0 R^4 \omega\, \alpha}{2} \int_{-\pi/2}^{\pi/2} \frac{R \sec^2\theta\, d\theta}{2R^3 \sec^3\theta}$$

$$= \frac{\mu_0 R^4 \omega\, \alpha}{4R^3} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta = \frac{\mu_0 R \omega \alpha}{4} \Big[\sin\theta\Big]_{-\pi/2}^{\pi/2}$$

$$B_1 = \frac{\mu_0 R\omega\,\alpha}{2}$$

Due to polarisation, volume charge density is given by

$$\rho = -\,div\,\vec{P} = -\frac{\partial P_x}{\partial x} - \frac{\partial P_y}{\partial y} - \frac{\partial P_z}{dz}$$

Here $\vec{P} = \alpha\,\vec{r}$

Let axis of cylinder is along x–axis.

We consider a point $A\,(x, y, z)$ in the cylinder.

$\therefore$ $$\vec{r} = y\hat{i} + z\hat{k}$$

$\therefore$ $$\vec{P} = \alpha y\hat{j} + \alpha z\hat{k} \qquad (\because\ \vec{P} = \alpha\vec{r})$$

$\therefore$ $$\rho = -\alpha\frac{\partial x}{\partial x} - \alpha\frac{\partial z}{\partial z} = -2\alpha$$

We consider a circle of radius r and thickness dr having length dx along the axis of cylinder. The current in considered element is

$$dI = -2\alpha\,dV\frac{\omega}{2\pi} = -2\alpha\,(2\pi\,r\,d\,r\,dx)\frac{\omega}{2\pi} = -2\alpha\,r\,d\,r\,\omega\,dx$$

The magnetic field due to volume charge density is

$$B_2 = \int\!\!\int -\frac{\mu_0\,r^2\,2\pi\,r\,\omega\,dx\,dr}{2\,(r^2 + x^2)^{3/2}}$$

$$= -\int_0^R \alpha\,\mu_0\omega r^3 dr \int_{-\infty}^{\infty} \frac{1}{(r^2 + x^2)^{3/2}}\,dx$$

After integrating $$= -\frac{\mu_0\,\alpha\,\omega R}{2}$$

$\therefore$ $$B = B_1 + B_2$$

On putting the values, we get $$B = 0$$

YOUR STEP

A thin wire is wound very tightly in one layer on the surface of a sphere of paramagnetic material ($\mu_r \approx 1$). The plates of all the turns can be assumed to be perpendicular to the same diameter of the sphere. The turns cover the entire surface of sphere. If the radius of the sphere is R, total number of turns is N and the current in winding is I. Find the magnetic field at the centre of sphere.

$$\left\{B = \frac{\mu_0 NI}{4R}\right\}$$

§ 3.250

➢ CONCEPT

The magnetic force at a point due to moving charge is

$$\vec{B} = \frac{\mu_0\,q_0\,(\vec{v}\times\vec{r})}{4\pi\,r^3}$$

The magnetic force on a charge particle of charge q in a magnetic field $\vec{B}$ is

$$\vec{F}_m = q\,\vec{v}_1 \times \vec{B}$$

SOLUTION :

The magnetic field due to first proton on the site of second proton is

$$\vec{B} = \frac{\mu_0\,e\,(\vec{v}\times\vec{r})}{4\pi r^3}$$

The magnetic force on second proton is

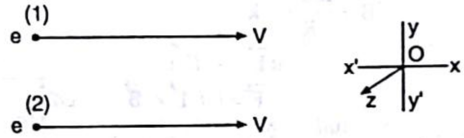


Fig. 3.250A

$$\vec{F}_m = e\,\vec{v} \times \vec{B} = \frac{\mu_0 e\,\vec{v} \times (\vec{v} \times \vec{r})}{4\pi r^3} = -\frac{\mu_0}{4\pi r^3} v^2 \vec{r}$$

The electrical force of interaction between them is

$$\vec{F}_e = \frac{e^2\,\vec{r}}{4\pi\varepsilon_0 r^3}$$

$\therefore \qquad \dfrac{|\vec{F}_m|}{|\vec{F}_e|} = v^2 \mu_0 \varepsilon_0 \qquad$ But $\qquad c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\therefore \qquad \mu_0\,\varepsilon_0 = \dfrac{1}{c^2} \qquad \therefore \qquad \dfrac{|\vec{F}_m|}{|\vec{F}_e|} = \dfrac{v^2}{c^2} = \left(\dfrac{v}{c}\right)^2$

On putting the values, we get $\dfrac{|\vec{F}_m|}{|\vec{F}_e|} = 10^{-6}$

YOUR STEP

A pair of point charges, $q = 5\mu C$ and $q' = -3\mu C$ are moving in a reference frame as shown in fig. 3.250B. At this instant, what are the magnitude and direction of the net magnetic field produced at the origin ?

take $v = 6 \times 10^5$ m/s

and $v' = 8 \times 10^5$ m/s

$\{-4.83 \times 10^{-6}$ tesla $\hat{k}\}$

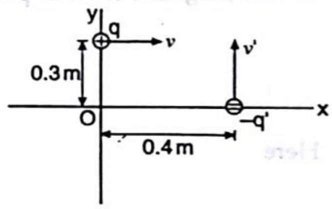


Fig. 3.250B

§ 3.251

➢ **CONCEPT**

The magnetic force on a wire is given by

$$F_m = I d\vec{L} \times \vec{B}$$

$\therefore$ Force per unit length is $\qquad \dfrac{F_m}{dL} = \dfrac{IdLB\sin\theta}{dL} = I\,B\sin\theta$

SOLUTION : (a) The magnetic force at point O due to semi circular wire is

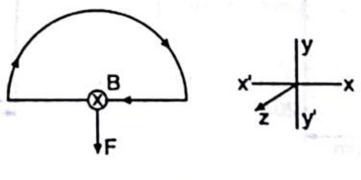


Fig. 3.251A

$$\vec{B} = \frac{\mu_0 I\alpha}{4\pi R}(-\hat{k})$$

Here $\alpha = \pi$ $\therefore$ $\vec{B} = \frac{\mu_0 I}{4R}(-\hat{k})$

From Fig. 3.251A, $d\vec{L} = -dL\hat{i}$

$\therefore$ $\vec{F} = I\,d\,\vec{L} \times \vec{B}$

$$= I\,dL(-\hat{i}) \times \frac{\mu_0 I}{4R}(-\hat{k}) = \frac{IdL\mu_0 I}{4R}(-\hat{j})$$

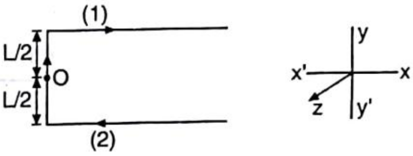


Fig. 3.251B

$\therefore$ $\frac{\vec{F}}{dL} = \frac{\mu_0 I^2}{4R}(-\hat{j})$ $\therefore$ $\left|\frac{\vec{F}}{dL}\right| = \frac{\mu_0 I^2}{4R}$

On putting the values, we get $\left|\frac{\vec{F}}{dL}\right| = 0.2\text{ mN/m}$

(b) Step I : Determine magnetic field at point O :

The magnetic field at point O due to upper semi-infinite wire is

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi\left(\frac{L}{2}\right)}(-\hat{k})$$

The magnetic field at point O due to lower semiinfinite wire is

$$\vec{B}_2 = \frac{\mu_0 I}{4\pi\left(\frac{L}{2}\right)}(-\hat{k})$$

$\therefore$ Net magnetic field at point O is

$$\vec{B} = \left\{\frac{\mu_0 I}{4\pi\left(\frac{L}{2}\right)} + \frac{\mu_0 I}{4\pi\left(\frac{L}{2}\right)}\right\}(-\hat{k}) = \frac{\mu_0 I}{\pi l}(-\hat{k})$$

Here $dL = dL\,\hat{j}$

$\therefore$ $\vec{F}_m = Id\vec{L} \times \vec{B} = IdL\,(\hat{j}) \times \frac{\mu_0 I}{\pi L}(-\hat{k}) = -\frac{\mu_0 I^2 dL}{\pi L}\hat{k}$

$\therefore$ $\frac{F_m}{dL} = \frac{\mu_0 I^2}{\pi L}$

On putting the values, we get $\frac{F_m}{dL} = 0.13 \times 10^{-3}\text{ N/m} = 0.13\text{ mN/m}$

YOUR STEP

1. Consider the three long, straight, parallel wires shown in fig. 3.251C. Find the magnetic force experienced by a 25 cm length of wire C.

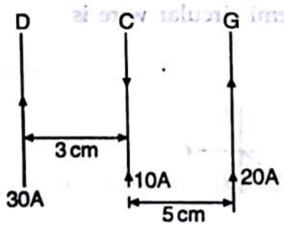


Fig. 3.251C

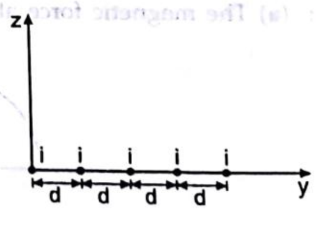


Fig. 3.251D

2. Fig. 3.251D shows five long parallel wires in the xy plane. Each wire carries a current $i = 3A$ in the positive x-direction. The separation between adjacent wires is $d = 8$ cm. In unit vector notation, what is the magnetic force per metre exerted on the first wire by the other wires?

$\left\{ 1.\ 3\times10^{-4}\text{ N towards right.} \quad 2.\ \frac{25}{24}\frac{\mu_0 i^2}{\pi d}\text{ towards left} \right\}$

§ 3.252

➢ **CONCEPT**

The magnetic force on each elemental length dl is the cause of tension in the wire.

For equilibrium of a length dL of wire, tension force balances magnetic force on the element.

We consider a Δl length of wire.

The magnetic force on Δl element is

$$F = I\Delta l B$$

For equlibrium, $F = \Delta\theta$ (shown in fig. 3.252A.)

$$I\,\Delta\,l\,B = T\Delta\theta$$

$$I\,R\,\Delta\,\theta\,B = T\,\Delta\,\theta$$

$$\therefore \quad T = IRB$$

The cross-sectional area of wire is

$$S = \frac{\pi d^2}{4}$$

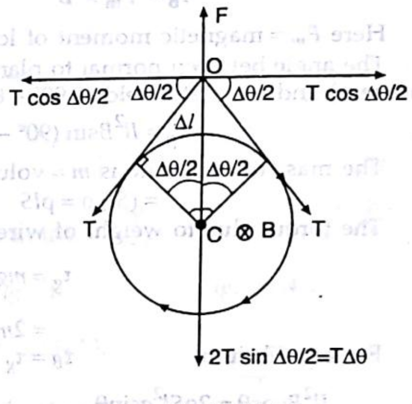


Fig. 3.252A

$$\therefore \quad \sigma = \text{stress} = \frac{T}{S} = \frac{IRB}{S} = \frac{4IRB}{\pi d^2}$$

The breaking stress of wire

$$\sigma = \sigma_m = 0.30\times10^{12}\text{ N/m}^2 \quad \text{(for copper)}$$

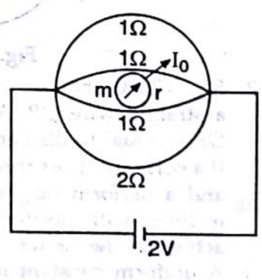
$$\because \quad \sigma_m = \frac{4IRB_{max}}{\pi d^2}$$

$$\therefore \quad B_{max} = \frac{\sigma_m \pi d^2}{4IR}$$

On putting the values, we get $B_{max} = 8\times10^3$ tesla

YOUR STEP

Two circular rings each of radius a are joined together. Such that their planes are perpendicular to each other as shown in the figure 3.252B. The resistance of each half part of the ring is indicated in the figure. A very small loop of mass m and radius r carrying a current I_0, is placed in the plane of the paper at the common centre of each ring. The loop can freely rotate about any of its diameter. If the loop is slightly displaced. Find the time period of oscillation.



$$\left\{T = 2\pi\sqrt{\frac{2ma}{\pi\mu_0 I_0}}\right\}$$

Fig. 3.252B

§ 3.253

➢ **CONCEPT**

In this case, magnetic torque about OO' is balanced by torque due to gravitational torque.

SOLUTION : If OO' is connected by a wire, then the torque due to magnetic force on this wire about OO' is zero. So, for soloving the problem, the system may be assumed as a square loop.

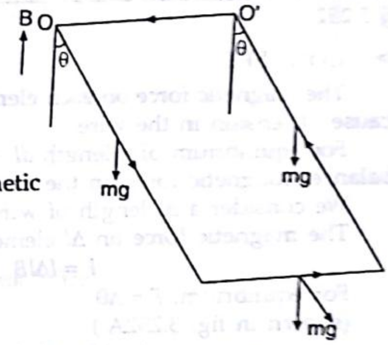


Fig. 3.253A

The area of loop is $A = l^2$

At angle of deflection θ, the magnetic torque is

$$\vec{\tau}_B = \vec{P}_m \times \vec{B}$$

Here P_m = magnetic moment of loop $= IA = Il^2$

The angle between normal to plane (area vector or magnetic moment) and magnetic field is $90° - \theta$.

$$\therefore \quad \tau_B = Il^2 B\sin(90° - \theta) = Il^2 B\cos\theta$$

The mass of each side is m = volume × density

$$= (Sl)\rho = \rho l S$$

The torque due to weight of wires is

$$\tau_g = mg\frac{l}{2}\sin\theta + mg\frac{l}{2}\sin\theta + mgl\sin\theta$$

$$= 2mgl\sin\theta = 2\rho S l^2 g\sin\theta$$

For equilibrium, $\tau_B = \tau_g$

$\therefore \quad Il^2 B\cos\theta = 2\rho S l^2 g\sin\theta \qquad \therefore \quad B = \dfrac{2S\rho g\tan\theta}{I} = 10\text{ mT}$

YOUR STEP

1. A thin wire of length l mass m is bent in the shape of an arc of a circle of radius R ($l < 2\pi R$). The wire is free to swing about the axis OO'. A current I is passed through the wire and a uniform vertical constant magnetic field B is switched on. As a result, the plane of the wire swings out of the vertical plane by an angle θ. Find θ at equilibrium.

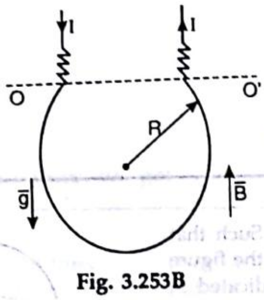


Fig. 3.253B

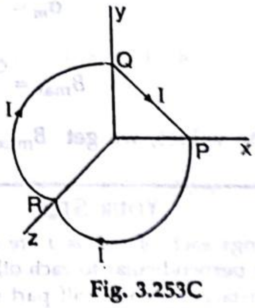


Fig. 3.253C

2. Fig. 3.253C shows a circuit consisting of two semicircular arcs PR and RQ each of radius R and a straight wire QP. Arc PR lies in z–x plane and arc RQ lies in y–z plane such that distance OP is equal to distance OR.

   If a current I flows through the circuit along the direction shown in fig. 3.253C and a uniform magnetic field B exists in the space such that $\vec{B}$ is equally inclined with positive direction of all the three axes, calculate moment $\vec{\tau}$ acting on the circuit.

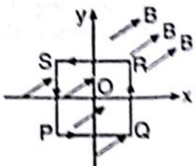


Fig. 3.253D

3. A uniform constant magnetic field B is directed at, an angle of 45° to the x-axis in the x, y plane. $PQRS$ is rigid square wire frame carrying a current I_0, with its centre at the origin O. The frame is initially at rest with its sides

parallel to the x and y-axis as shown in fig. 3.253D. Each side of the frame has a mass m and a length l. Calculate the initial angular acceleration of the frame. Given, the moment of inertia about its axis through O, perpendicular to the plane of the square is $\frac{4}{3}ml^2$.

{ 1. $\theta = \tan^{-1}\left(\frac{IA}{mgr_{cm}}\right)$, Here $r_{cm} = \dfrac{2R\sin\left(\frac{2\pi R - l}{2R}\right)}{\frac{2\pi R - l}{2R}} + \cos\left(\frac{2\pi R - l}{2R}\right)$

$A = \pi R^2 - \frac{R}{2}(2\pi R - l) + \frac{R^2}{2}\sin\left(\frac{2\pi R - l}{R}\right)$

2. $\alpha = \frac{\pi BR^2 I}{2\sqrt{3}}(\hat{i} - \hat{j})$ 3. $\alpha = \frac{3B_0I_0}{2m}$ }

§ 3.254

➢ CONCEPT

This problem is similar to previous problem.

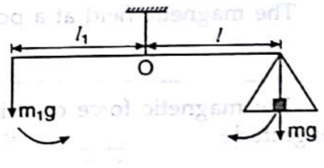


Fig. 3.254A

SOLUTION : The problem is solved in following steps :

Step I : Discuss the problem in the absence of current in the coil.

when current in the coil is zero. then, torque due to magnetic force is zero. In this case, the torque of weight of coil about point O is balanced by the weight of the balance (shown in fig. 3.254A).

$\therefore \qquad m_1gl_1 = mgl \qquad ...(i)$

Step II : Discuss the problem, when current is passing through the coil.

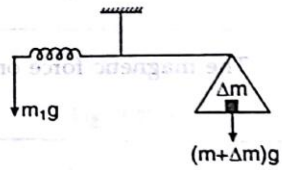


Fig. 3.254B

In this case, torque due to magnetic force is

$$\tau_1 = NISB \text{ Anticlockwise}$$

The torque due to weight of coil is $\tau_2 = m_1gl_1$

The torque due to the weight of the balance + additional weight Δm about O is $\tau_3 = (m + \Delta m)\,gl$

For equilibrium, $\quad \tau_1 + \tau_2 = \tau_3$

$\therefore \qquad m_1gl_1 + NISB = (m + \Delta m)\,gl$

or $\qquad m_1gl_1 + NISB = mgl + \Delta mgl$

or $\qquad NISB = \Delta mgl \qquad$ (from eq. (i)

$\therefore \qquad B = \frac{\Delta mgl}{NIS}$

On putting the values, we get $\qquad B = 0.4$ tesla

YOUR STEP

1. A circular conducting ring of radius a carries a current i_1. It is mounted rigidly on an axle and suspended midway between two cords fig. 3.254C. The mass of the ring is m. If a vertical magnetic field B is present. What will be the tension in the two cords? The distance between the two cords is l.
2. A flat circular coil with 10 loops of wire on it has a diameter of 20 mm and carries a current of 0.5A. It is mounted inside a long solenonid that has 200 loops on its 250 mm length. The current in the solenoid is 2.4A. Calculate the torque required to hold the coil with its axis perpendicular to that of the solenoid.

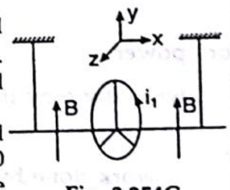


Fig. 3.254C

{ 1. $T_1 = \dfrac{mg - \frac{2\pi a^2 i_1 B}{l}}{2}, T_2 = \dfrac{mg + \frac{2\pi a^2 i_1 B}{l}}{2}$ 2. 3.8×10^{-6} Nm}

§ 3.255

➤ **CONCEPT**

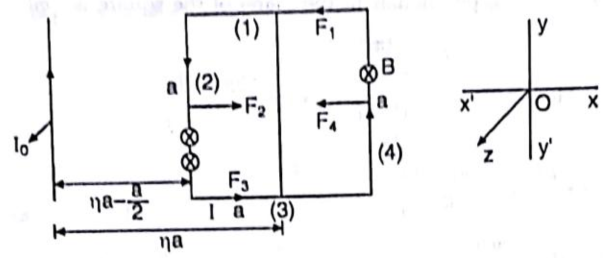


Fig. 3.255A

The magnetic field at a point due to a long wire is

$$B = \frac{\mu_0 I_0}{2\pi r}$$

The magnetic force on wire section (1) and (3) are in opposite directions and are equal in magnitude.

$\therefore$ $$\vec{F}_1 = -\vec{F}_3$$

The magnetic field at the site of wire (2) due to long wire is

$$\vec{B}_1 = \frac{\mu_0 I_0}{2\pi\left(\eta a - \frac{a}{2}\right)}(-\hat{k})$$

The magnetic force on wire section (2) is

$$\vec{F}_2 = IaB_1\,\hat{i} = Ia\frac{\mu_0 I_0}{2\pi\left(\eta a - \frac{a}{2}\right)}\hat{i}$$

Similarly, $$\vec{F}_4 = \frac{Ia\mu_0 I_0}{2\pi\left(\eta a + \frac{a}{2}\right)}(-\hat{i})$$

$\therefore$ Net force on the frame is $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \dfrac{2\mu_0\, i\, i_0}{\pi(4\eta^2 - 1)}\hat{i}$

On putting the values, we get $F = 0.4 \times 10^{-6}\ \text{N} = 0.4\ \mu\text{N}$

(b)

$\because$ $$\varepsilon = -\frac{d\phi}{dt}$$

or power $$P = \varepsilon I = -I\frac{d\phi}{dt}$$

work done by magnetic force is

$$A = Pdt = -I d\phi$$

$\therefore$ $$A = -I\,\Delta\phi$$

$\therefore$ work done by external agent is $A_{ext} = -A$

$\therefore$ $$A_{ext} = I\Delta\phi = I(\phi_f - \phi_i)$$

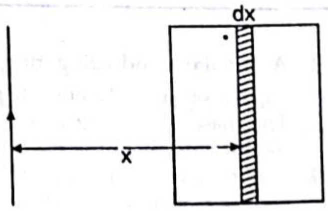


Fig. 3.255B

We consider an element of thickness dx

$\therefore$ $$d\phi = \vec{B}\cdot d\vec{S}$$

$$= -Badx = -\frac{\mu_0 I_0}{2\pi x}dx$$

$\therefore \qquad \phi_i = -\dfrac{\mu_0 I_0}{2\pi}\displaystyle\int_{\eta a - \frac{a}{2}}^{\eta a + \frac{a}{2}} \dfrac{dx}{x}$

$\therefore \qquad \phi_i = -\dfrac{\mu_0 I_0}{2\pi} ln\left(\dfrac{2\eta+1}{2\eta-1}\right)$

when frame is turned through 180°.

$$\phi_f = -\phi_i = \frac{\mu_0 I_0\, a}{2\pi} ln\left(\frac{2\eta+1}{2\eta-1}\right)$$

$\therefore \qquad \Delta\phi = \phi_f - \phi_i = \dfrac{\mu_0 I_0\, a}{\pi} ln\left(\dfrac{2\eta+1}{2\eta-1}\right)$

$\therefore \qquad A_{ext} = I\Delta\phi = \dfrac{\mu_0 I I_0\, a}{\pi} ln\left(\dfrac{2\eta+1}{2\eta-1}\right)$

On putting the values we get, $= 0.10\mu J$

YOUR STEP

For the situation shown in fig. 3.255C, find the force experienced by side *MN* of the rectangular loop. Also find the torque on the loop.

$\{ F = \dfrac{\mu_0 I_1 I_2 L}{2\pi a}$, torque = zero$\}$

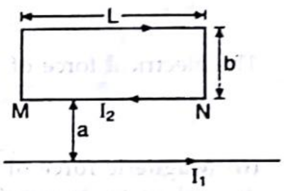


Fig. 3.255C

§ 3.256

➢ **CONCEPT**

The system formed by given long parallel wires is a capacitor.
From the solution of problem 3.108,

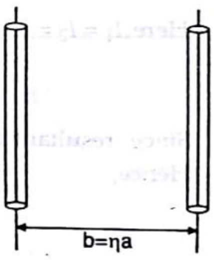


Fig. 3.256A

$$\frac{C}{l} = \frac{2\pi\varepsilon_0}{ln\left(\frac{b-a}{a}\right)}$$

where l is length of each long wire.

or $\qquad C = \dfrac{2\pi\varepsilon_0 l}{ln\,\frac{b}{a}} \qquad (\because b >> a) \qquad = \dfrac{2\pi\varepsilon_0 l}{ln\,\frac{\eta a}{a}} = \dfrac{2\pi\varepsilon_0 l}{ln\,\eta}$

The problem is solved by following steps :

Step I : Draw equivalent circuit of the problem.
The equivalent circuit is shown in Fig. 3.256B.

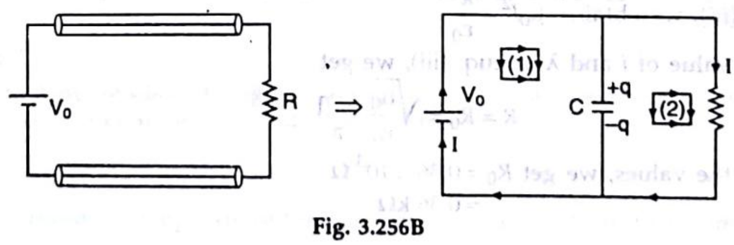


Fig. 3.256B

In loop (1) $$V_0 - \frac{q}{C} = 0$$

$\therefore$ $$q = CV_0 = \frac{2\pi\varepsilon_0 l}{ln\,\eta} V_0$$

$\therefore$ $$\lambda = \frac{q}{l} = \frac{2\pi\varepsilon_0 V_0}{ln\,\eta} \quad ...(i)$$

In loop (2), $$\frac{q}{C} - IR = 0$$

$\therefore$ $$I = \frac{q}{RC} = \frac{V_0}{R} \quad ...(ii) \left(\because \frac{q}{C} = V_0\right)$$

Step : Discuss the problem on the basis of force :

In this case, two forces of interaction between wires comes into play.

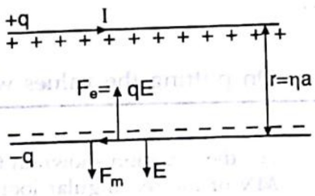


Fig. 3.256C

(i) Electrical force of interaction :

The electric field due to positive plate at the site of negative plate is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{q}{2\pi\varepsilon_0 lr} = \frac{q}{2\pi\varepsilon_0 l\,\eta a}$$

The electrical force of interaction is

$$F_e = qE = \frac{(\lambda l)(\lambda l)}{2\pi\varepsilon_0 l\,\eta a} = \frac{\lambda^2 l}{2\pi\varepsilon_0\,\eta a}$$

(ii) Magnetic force of interaction :

According to Ampere's law of magnetic force, two long parallel wires carrying current in opposite directions repel each other with a magnetic force

$$F_m = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

Here $I_1 = I_2 = I$, $r = \eta a$

$\therefore$ $$F_m = \frac{\mu_0 I^2 l}{2\pi\,\eta a}$$

Since, resultant force of interaction is zero.

Hence, $$F_m = F_e$$

$\therefore$ $$\frac{\mu_0 I^2 l}{2\pi\eta a} = \frac{\lambda^2 l}{2\pi\varepsilon_0\eta a}$$

or $$\mu_0 I^2 = \frac{\lambda^2}{\varepsilon_0} \quad ...(iii)$$

From Eqn. (i) and (ii), we get $I = \frac{V_0}{R}$ and $\lambda = \frac{2\pi\varepsilon_0 V_0}{ln\,\eta}$

From Eqn. (iii), we obtain $\mu_0 I^2 = \frac{\lambda^2}{\varepsilon_0}$

Putting the value of i and λ in euq. (iii), we get

$$R = R_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\frac{ln\,\eta}{\pi}$$

On putting the values, we get $R_0 = 0.36 \times 10^3\ \Omega$
$= 0.36\ k\Omega$

YOUR STEP

Two long co-axial cylindrical conductors of radii a and b $(b > a)$ are maintained at a potential difference of V with the help of a ideal battery shown in fig. 3.256D. A steady current I flows through the circuit. An electron enters into the evacuated space (away from the reader) between the conductors and goes undeviated.

(a) Indicate the polarity of the battery in the direction of current.

(b) Find the velocity of electron.

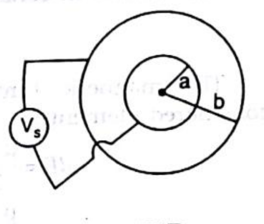


Fig. 3.256D

(a) upper terminal is at higher potential than lower terminal.

(b) $\dfrac{2\pi V}{\mu_0 I \ln\left(\dfrac{b}{a}\right)}$

§ 3.257

➢ **CONCEPT**

From the solution of problem 3.225, the magnetic field due to a long straight conductor whose cross-section is in the form of a thin half ring of radius R is

$$B = \frac{\mu_0 I}{\pi^2 R} \quad \text{(at } O \text{ shown in fig. 3.257A)}$$

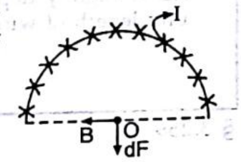


Fig. 3.257A

SOLUTION : The magnetic force on a straight wire of length dl at point O is

$$dF = IdlB\sin 90° = Idl\frac{\mu_0 I}{\pi^2 R} = \frac{\mu_0 I^2 dl}{\pi^2 R}$$

$\therefore$ Force per unit length is $\quad \dfrac{dF}{dl} = \dfrac{\mu_0 I^2}{\pi^2 R}$

YOUR STEP

Two long straight parallel wires are 2m apart, perpendicular to the plane of paper (shown in fig. 3.257B). The wire A carries a current of 9.6A, directed into the plane of the paper. The wire B carries a current such that the magnetic field at the point P at a distance $\frac{10}{11}$ m from the wire B is zero.

Find

(a) The magnitude and direction of the current in B.

(b) The magnitude of magnetic field at the point S.

(c) The magnetic force per unit length on the wire B.

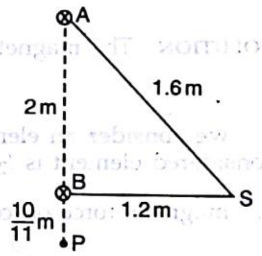


Fig. 3.257B

(a) $i = 3\text{A}$, and directed outward for producing a null point at P.

(b) 13×10^{-7} T

(c) 2.88×10^{-6} N/m

§ 3.258

➢ **CONCEPT**

According to Ampere's law of magnetic force, two long parallel wires carrying current in same direction attract each other with a force

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

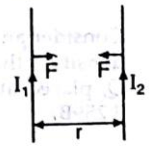


Fig. 3.258A

SOLUTION : We consider a long strip of thickness dr at distance r from first wire.

The current in considered element is

$$dI = \frac{I_2}{b}\,dr$$

The magnetic force of interaction between first wire and considered element is

$$dF = \frac{\mu_0 I_1 dI}{2\pi r}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi b}\frac{dr}{r} \quad \text{(per unit length)}$$

$\therefore$ $$F = \frac{\mu_0 I_1 I_2}{2\pi b}\int_a^{a+b}\frac{dr}{r} \qquad F = \frac{\mu_0 I_1 I_2}{2\pi b}\ln\frac{a+b}{a}$$

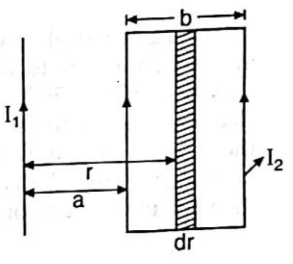


Fig. 3.258B

YOUR STEP

A long horizontal wire AB carries a current i_1. It is free to move in the vertical plane and is in equilibrium at a height h above a second fixed wire CD which carries a current i_2. Show that when AB is slightly depressed, it executes SHM and find the time period of oscillations. The mass per unit length of wire AB is m.

$$\left\{T = 2\pi\sqrt{\frac{2\pi m d^2}{\mu_0 I_1 I_2}}\right\}$$

§ 3.259

➢ **CONCEPT**

For this problem, the solution of problem 3.229 is applicable. Since, magnetic field out side the planes is zero. Hence, linear current density j in both planes are opposite to each other.

Here magnetic field at point P situated between the planes is

$$B = B_1 + B_2 = \frac{\mu_0 i}{2} + \frac{\mu_0 i}{2} = \mu_0 i$$

$\therefore$ $$i = \frac{B}{\mu_0}$$

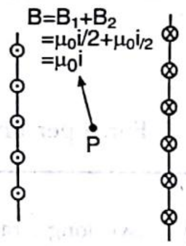


Fig. 3.259A

SOLUTION : The magnetic field due to 1st plane at the site of second plane is

$$B_1 = \frac{\mu_0 i}{2}$$

we consider an element of length l and breadth b in the second plane. The current in the considered element is $I_2 = ib$

$\therefore$ magnetic force on considered element $F = I_2 l B = (ib)\, l\, \frac{\mu_0 i}{2}$

$\therefore$ Force per unit area is $$\frac{F}{lb} = \frac{\mu_0 i^2}{2} = \frac{\mu_0\left(\frac{B}{\mu_0}\right)^2}{2} = \frac{B^2}{2\mu_0}$$

YOUR STEP

Consider an infinite conducting flat strip of width $2D$ which carries a current density $\vec{J}$ that is confined to the strip. Calculate the magnetic field at a point Q, placed at a height h above the symmetry axis of the strip (shown in fig. 3.259B).

$$\left\{B = \frac{\mu_0 J}{\pi}\tan^{-1}\left(\frac{D}{h}\right) \text{ along } x\text{-axis.}\right\}$$

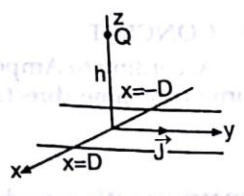


Fig. 3.259B

§ 3.260

➢ **CONCEPT**

when magnetic lines of force in a region is denser, magnetic field is stronger.

SOLUTION :

(a) From figure of problem,

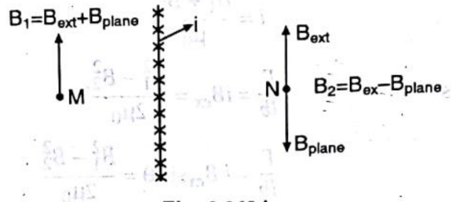


Fig. 3.260A

$$B_1 > B_2$$

The situation of problem is shown in fig. 3.260A

The magnetic field due to plane is $B_{\text{plane}} = \frac{\mu_0 i}{2}$

Here i is linear current density

$$\therefore \quad B_1 = B_{ex} + \frac{\mu_0 i}{2} \quad \text{...(i)}$$

$$\therefore \quad B_{ex} = B_1 - \frac{\mu_0 i}{2}$$

Similarly, $$B_2 = B_{ex} - \frac{\mu_0 i}{2} \quad \text{...(ii)}$$

or $$B_2 = B_1 - \frac{\mu_0 i}{2} - \frac{\mu_0 i}{2} \qquad \therefore \quad i = \frac{B_1 - B_2}{\mu_0}$$

$$\therefore \quad B_{ex} = B_1 - \frac{\mu_0}{2}\left(\frac{B_1 - B_2}{\mu_0}\right)$$

$$= B_1 - \frac{B_1}{2} + \frac{B_2}{2} = \frac{B_1 + B_2}{2}$$

$\therefore$ The magnetic force on an element of length l and breadth b is

$$F = IlB_{ex} = (ib)\, lB_{ex}$$

$\therefore$ Force per unit area is $$\frac{F}{lb} = iB_{ex} = i\left(\frac{B_1 + B_2}{2}\right)$$

$$= \frac{(B_1 - B_2)}{\mu_0}\left(\frac{B_1 + B_2}{2}\right) = \frac{B_1^2 - B_2^2}{2\mu_0}$$

(b) From figure of problem, $B_1 > B_2$

also, $B_{ex} < B_{plane}$

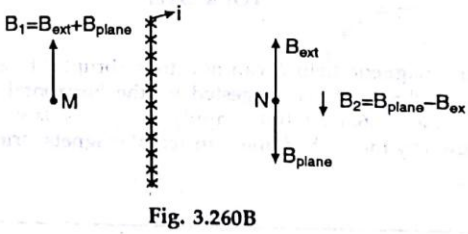


Fig. 3.260B

Here $$B_1 = B_{ex} + \frac{\mu_0 i}{2}$$

Also $$B_2 = \frac{\mu_0 i}{2} - B_{ex}$$

After solving, $$B_{ex} = \frac{B_1 - B_2}{2}$$

and $$i = \frac{B_1 + B_2}{\mu_0}$$

$\because$ Force per unit length is $$\frac{F}{lb} = iB_{ex} = \frac{B_1^2 - B_2^2}{2\mu_0}$$

$\therefore$ Force per unit area is $$\frac{F}{lb} = i\,B_{ex}\sin\theta = \frac{B_1^2 - B_2^2}{2\mu_0}$$

(c) From figure, it is clear that B_{ex} and B_{plane} are inclined to each other shown in fig. 3.260C. Let B_{ex} is directed at an angle θ with x-axis.

From fig 3.260C, $$B_1^2 = (B_{\text{plane}} + B_{ex}\sin\theta)^2 + (B_{ex}\cos\theta)^2$$

and $$B_2^2 = (B_{ex}\cos\theta)^2 + (B_{ex}\sin\theta - B_{\text{plane}})^2$$

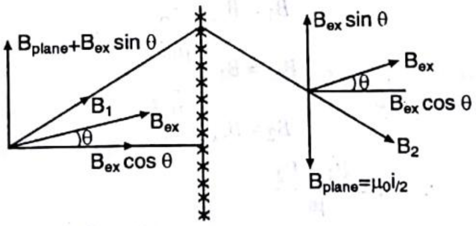


Fig. 3.260C

$$\therefore \quad B_1^2 - B_2^2 = (B_{\text{plane}} + B_{ex}\sin\theta)^2 - (B_{ex}\sin\theta - B_{\text{plane}})^2$$
$$= 4B_{ex}\,B_{\text{plane}}\sin\theta$$

$$\therefore \quad B_1^2 = B_2^2 = 4\left(\frac{\mu_0 i}{2}\right)B_{ex}\sin\theta = 2\mu_0\, i\, B_{ex}\sin\theta$$

$$\therefore \quad B_{ex}\sin\theta = \frac{B_1^2 - B_2^2}{2\mu_0}$$

$\therefore$ Force per unit area is $$\frac{F}{lb} = i\,B_{ex}\sin\theta = \frac{B_1^2 - B_2^2}{2\mu_0}$$

YOUR STEP

1. Show that a uniform magnetic field B can not drop abruptly to zero as one moves at right angles to it, as suggested by the horizontal arrow through point a in fig. 3.260D. (Hint : apply Ampere's law to the rectangular path shown by the dashed lines.) In actual magnets "fringing"

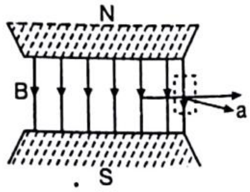


Fig. 3.260D

of the lines of $\vec{B}$ always occurs, which means that B approaches zero in a gradual manner. Modify the $\vec{B}$ lines in the figure to indicate a more realistic situation.

2. In case of magnetic force of interaction of two current elements $I_1 dl_1$ and $I_2 dl_2$ (shown in fig. 3.260E). Show that $dF_{12} \neq -dF_{21}$. But for entire loop, *i.e.*, if the force of one entire circuit on the other is considered, $\vec{F}_{12} = -\vec{F}_{21}$ *i.e.* Newton's third law is obeyed.

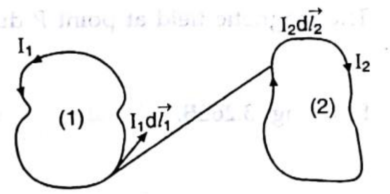


Fig. 3.260E

§ 3.261

➢ **CONCEPT**

We consider a strip of length l and thickness dx.

The current in the strip is

$$I_1 = \frac{I}{a} dx.$$

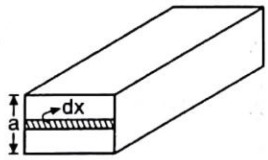


Fig. 3.261A

The area of strip is $dA = ldx$

The magnetic force on the considered element is

$$dF = I_1 lB = \left(\frac{I}{a} dx\right) lB$$

∴ Pressure is $$P = \frac{dF}{dA} = \frac{I}{a}\frac{lBdx}{ldx}$$

∴ The pressure is $$P = \frac{IB}{a} = 0.5 \text{ kPa}$$

YOUR STEP

A wooden cylinder with a mass m, length l and radius r has n turns of wire wrapped around it longitudinally. The plane of the wire contain the axis of the cylinder. The cylinder is placed on an inclined plane with inclination θ and it is in a magnetic field B directed vertically upwards. The plane of the loop is parallel to the inclined plane in fig. 3.261B. What should be the minimum current in the coil to prevent the coil from rolling down the plane.

$$\left\{ i_{min} = \frac{mg}{2Bl} \right\}$$

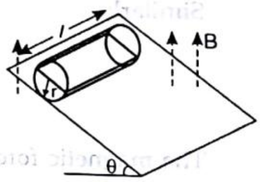


Fig. 3.261B

§ 3.262

➢ **CONCEPT**

The top view of figure is shown in Fig. 3.262B.

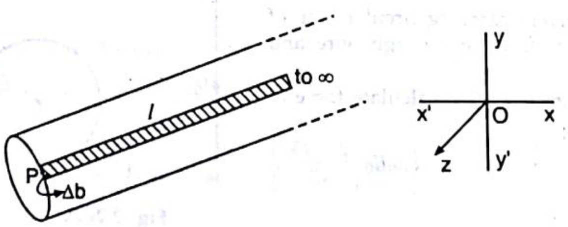


Fig. 3.262A

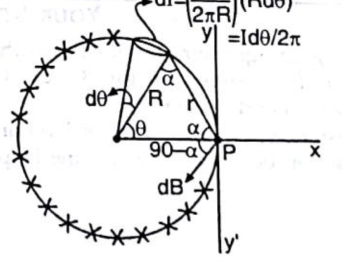


Fig. 3.262B

We consider a long strip of thickness ($Rd\theta$) carrying a current

$$dI = \frac{Id\theta}{2\pi}$$

The magnetic field at point P due to considered element is

$$dB = \frac{\mu_0 dI}{2\pi r} = \frac{\mu_0}{2\pi r}\frac{Id\theta}{2\pi} = \frac{\mu_0 Id\theta}{4\pi^2 r}$$

From fig. 3.262B, $2\alpha + \theta = 180°$

$\therefore$ $$\alpha = 90° - \frac{\theta}{2}$$

Also, $$\cos\theta = \frac{R^2 + R^2 - r^2}{2R^2}$$

$\therefore$ $$r^2 = 2R^2(1 - \cos\theta) = 4R^2 \sin^2\frac{\theta}{2}$$

$\therefore$ $$r = 2R\sin\frac{\theta}{2}$$

$\therefore$ $$dB = \frac{\mu_0 Id\theta}{4\pi^2\left(2R\sin\frac{\theta}{2}\right)} = \frac{\mu_0 I\, d\theta}{8\pi^2 R\sin\frac{\theta}{2}}$$

$\therefore$ $$d\vec{B} = -dB\sin\alpha\,\hat{i} - dB\cos\alpha\,\hat{j}$$

$\therefore$ $$B_x = -\int_{\theta=0}^{\theta=2\pi} dB\sin\alpha = -\int_0^{2\pi}\frac{\mu_0 I\sin\left(90° - \frac{\theta}{2}\right)}{8\pi^2 R\sin\frac{\theta}{2}}d\theta$$

$$= -\frac{\mu_0 I}{8\pi^2 R}\int_0^{2\pi}\cot\theta\, d\theta$$

After integrating, $B_x = 0$

Similarly, $$B_y = -\int_{\theta=0}^{\theta=2\pi} dB\cos\alpha = \int_0^{2\pi}\frac{\mu_0 I\cos\left(90° - \frac{\theta}{2}\right)}{8\pi^2 R\sin\frac{\theta}{2}}d\theta$$

$$= -\frac{\mu_0 I}{8\pi^2 R}\int_0^{2\pi} d\theta = -\frac{\mu_0 I}{8\pi^2 R}2\pi = -\frac{\mu_0 I}{4\pi R}$$

The magnetic force on a strip of thickness Δb and length l is

$$F = I_2\, l\, B_y$$

Here $I_2 = \frac{I}{2\pi R}\Delta b$ $\quad\therefore\quad$ $F = \frac{I}{2\pi R}\Delta b\, l \times \frac{\mu_0 I}{4\pi R}$

$\therefore$ Pressure $$= \frac{F}{\text{area}} = \frac{F}{l\Delta b} = \frac{\mu_0 I^2 l\Delta b}{8\pi^2 R^2} = \frac{\mu_0 I^2}{8\pi^2 R^2}$$

YOUR STEP

A long straight wire is coplanar with a current carrying circular loop of radius R as shown in fig. 3.262C. Current flowing through wire and loop is I_0 and I respectively.
If distance between centre of loop and wire is $r = 2R$, calculate force of attraction between wire and the loop.

$$\left\{\mu_0 I I_0\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right)\right\}$$

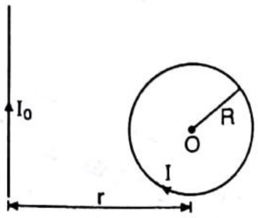


Fig. 3.262C

§ 3.263

➢ **CONCEPT**

The average magnetic field due to long solenoid is

$$B = \frac{1}{2}\mu_0 nI$$

The magnetic force is $$F = (nbI)lB = nbIl\frac{1}{2}\mu_0 nI = \frac{\mu_0 n^2 I^2 lb}{2}$$

$\therefore$ Pressure $$= \frac{F}{l \times b} = \frac{\mu_0 n^2 I^2}{2}$$

YOUR STEP

An interesting (and frustrating) effect occurs when one attempts to confine a collection of electrons and positive ions (a plasma) in the magnetic field of a toroid. Particles whose motion is perpendicular to the B, field will not execute circular path because the field strength varies with radial distance from the axis of the toroid. This, effect, which is shown in fig. 3.263A, causes particles of opposite sign to drift in opposite directions parallel to the axis of the toroid.

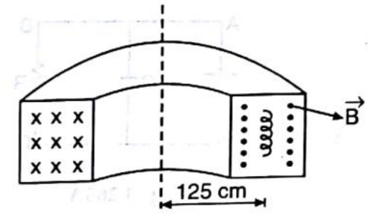


Fig. 3.263A

(a) What is the sign of the charge on the particle whose path is sketched in the figure.

(b) If the particle path has a radius of curvature of 11 cm, when its radial distance from the axis of the toroid is 125 cm, what will be the radius of curvature when the particle is 110 cm from the axis.

{(a) negative (b) 9.7 cm}

§ 3.264

➢ **CONCEPT**

From previous problem, the force on an element Δl is

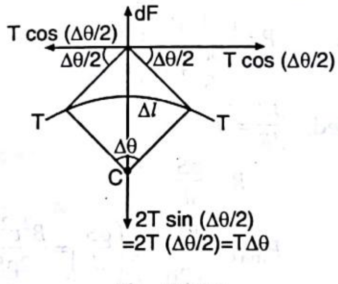


Fig. 3.264A

$$dF = \frac{1}{2}\mu_0 nI^2 \Delta l$$

For equilibrium, $$T\Delta\theta = dF$$

or $$T\Delta\theta = \frac{1}{2}\mu_0 nI^2 \Delta l \quad \text{or} \quad T\Delta\theta = \frac{1}{2}\mu_0 nI^2 R\Delta\theta \qquad (\because \Delta l = R\Delta\theta)$$

$$\therefore \quad I = \sqrt{\frac{2T}{\mu_0 nR}} \qquad \therefore \quad I_{\text{lim}} = \sqrt{\frac{2T_{\text{lim}}}{\mu_0 nR}} = \sqrt{\frac{2F_{\text{lim}}}{\mu_0 nR}} \qquad (\because T_{\text{lim}} = F_{\text{lim}})$$

YOUR STEP

Suppose a very long solenoid carries a (supper conducting) current of 10A and has 1000 turns per centimetre. Find the radial force per unit length F on one turn of the winding. Show that the tension in the wire is $T = Fa$ where a is the radius of the solendid.

$\{F = 6.28 \text{ N/m}\}$

§ 3.265

➢ **CONCEPT**

The system behaves as a parallel combination of a capacitor and resistor. Conducting liquid consists of positive and negative ions. Since, system is placed in an external magnetic field B. So, motional emf is generated between plates of capacitor.

SOLUTION : The equivalent circuit of the problem is shown in fig. 3.265A.

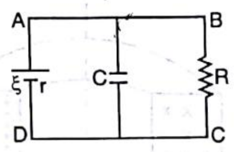


Fig. 3.265A

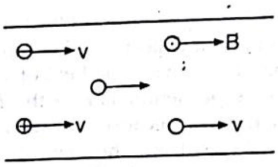


Fig. 3.265B

For equilibrium of ion, $eE = evB$ $\quad\therefore\quad E = vB$

$\therefore$ Motional emf is $\varepsilon = Ed = vBd$

The electric resistance between plates of capacitor is

$$r = \rho\frac{d}{S}$$

Also capacity is $C = \dfrac{\varepsilon_0 S}{d}$

In loop $ABCDA$, $\varepsilon - Ir - IR = 0$

$$\therefore \quad I = \frac{\varepsilon}{r+R} = \frac{vBd}{r+R} \quad \text{or} \quad I = \frac{vBd}{\rho\dfrac{d}{S} + R}$$

$\therefore$ The power generated in external resistance R is

$$P = I^2R \quad \therefore \quad P = \left(\frac{Bvd}{\dfrac{\rho d}{S} + R}\right)^2 R$$

For maximum power generated, $\dfrac{dP}{dR} = 0$

After solving, we get $R = \dfrac{\rho S}{d}$

$$\therefore \quad P_{\max} = \frac{B^2v^2d^2}{\left(\dfrac{2\rho S}{d}\right)^2}\left(\frac{\rho S}{d}\right) = \frac{B^2v^2d^2}{\dfrac{2\rho S}{d}} = \frac{v^2B^2Sd}{4\rho}$$

YOUR STEP

A potential difference of 600V is applied across the plates of a parallel plate capacitor. The separation between the plates is 3 mm. An electron is projected vertically, parallel to the plates with a velocity of 2×10^6 m/s moves undeflected between the plates. Find the magnitude and direction of the magnetic field in the region between the capacitor plates. (Neglect the edge effect).

$\{10^{-1} \text{ weber/m}^2\}$

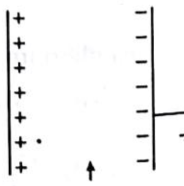


Fig. 3.265A

ELECTROMAGNETIC INDUCTION

§ 3.288

➢ CONCEPT

The motional emf produced in a rod is $\varepsilon = \vec{B}.(\vec{l} \times \vec{v})$. If B, l and v are mutually perpendicular. Then induced emf in the rod is

$$\varepsilon = B\,l\,v$$

SOLUTION : The problem is solved in following steps :

Step I : Determine the effected length of wire :

∵ The length of rod in time t is $\quad l = 2x$

Here $\quad y = ax^2$

∴ $\quad x = \sqrt{\frac{y}{a}} \quad$ ∴ $\quad l = 2\sqrt{\frac{y}{a}}$

But $\quad v^2 = 2wy \quad$ ∴ $\quad v = \sqrt{2wy}$

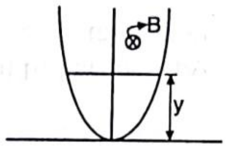


Fig. 3.288A

Step II :

Determine induced emf

∵ $$\varepsilon = Blv = B\,2\sqrt{\frac{y}{a}}\,\sqrt{2wy} = By\sqrt{\frac{8w}{a}}$$

YOUR STEP

1. An electric circuit consists of the three conducting rods MO, ON and PQ as shown in fig. 3.288B. The resistance of the rods per unit length is known to be λ. The rod PQ slides as shown in fig. 3.288B, at a constant velocity v, keeping its tilt angle relative to ON and MO fixed at 45°. At each instance the circuit is closed. The whole system is embedded in a uniform magnetic field B, which is directed perpendicularly into the page. Calculate the time dependence induced electric current.

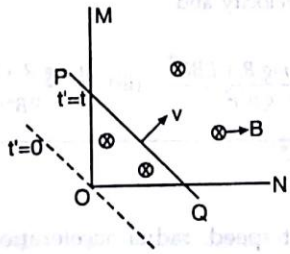


Fig. 3.288B

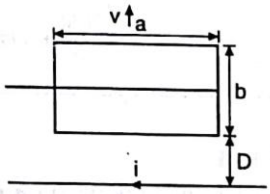


Fig. 3.288C

2. A rectangular loop of wire of length a, width b, and resistance R is placed near an infinitely long wire carrying current i, as shown in fig. 3.288C. The distance from the long wire to the loop is D. Find (a) the magnitude of the magnetic flux through the loop and (b) the current in the loop as it moves away from the long wire with speed v.

$$\left\{ 1.\ I = \frac{Bv}{\lambda(1+\sqrt{2})} \quad 2.\ \text{(a)}\ \frac{\mu_0 i a}{2\pi} \ln\left(1+\frac{b}{D}\right) \quad \text{(b)}\ \frac{\mu_0 i a b v}{2\pi R D (D+b)} \right\}$$

§ 3.289

➢ CONCEPT

The concept is similar to previous problems. The problem is solved by following steps :

Step I : Determine direction and magnitude of induced emf.

The movable rod behaves as a real battery of emf $\xi = Blv$ and internal resistance R (Shown in fig. 3.289 A).

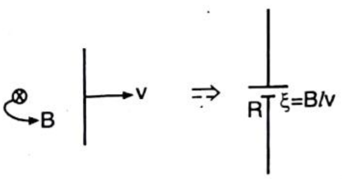


Fig. 3.289A

Step II :

Draw equivalent circuit of the problem :

$= \dfrac{Bvl}{R+R_\mu}$

The circuit of the problem is shown in fig. 3.289 B

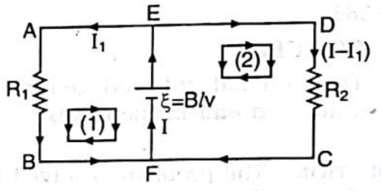


Fig. 3.289B

In loop (1),

$$\xi - IR - I_1R_1 = 0$$

or $$Blv - IR - I_1R_1 = 0 \qquad \text{...(i)}$$

In loop (2) $$\xi - IR - (I - I_1)\, R_2 = 0 \qquad \text{...(ii)}$$

After solving eq (i) and (ii), we get

$$I = \frac{\xi}{R + \dfrac{R_1 R_2}{R_1 + R_2}} = \frac{Bvl}{R + \dfrac{R_1R_2}{R_1 + R_2}} = \frac{Bvl}{R + R_\mu} \qquad \left(\text{where } R_\mu = \frac{R_1R_2}{R_1 + R_2}\right)$$

YOUR STEP

Two long parallel conducting rails are fixed at a distance l apart in a horizontal plane. A rod of mass m and resistance R can slide along the rails. A uniform magnetic field of induction B exists along vertically downward direction and coefficient of friction between rails and rod is μ. If a battery of emf E and negligible internal resistance is connected between the rails at left end, as shown in fig. 3.289C, calculate minimum value, F_{min} of force required to be applied horizontally rightwards on the rod to move it to the right. If $F = 2F_{min}$, calculate :

(i) steady state velocity of the rod,

(ii) power required to keep the rod moving with that steady velocity and

(iii) thermal power generated in the rod.

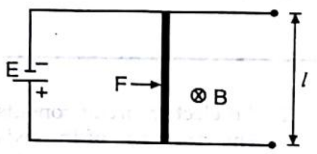


Fig. 3.289C

$$\left\{\left(\frac{EBl}{R} + \mu\, mg\right) \text{(i) } \frac{\mu\, mg\, R + EBl}{B^2l^2} \quad \text{(ii) } \frac{2\,(\mu\, mg\, R + EBl)^2}{RB^2l^2} \quad \text{(iii) } \frac{(\mu\, mg\, R + 2EBl)^2}{RB^2l^2}\right\}$$

§ 3.290

➢ CONCEPT

If a particle in moving on a circular path with constant speed, radial acceleration is acted on the particle radially inwards.

The radial or centripetal acceleration is given by

$$w_n = r\omega^2$$

➢ DISCUSSION

(a) When conducting disc is in rotating motion with constant angular velocity ω about its axis. Every electron moves on a circular path with same angular velocity ω.

SOLUTION : We consider an electron at distance r from centre of disc.

$\therefore$ $$eE = m w_n$$

$\therefore$ $$E = \frac{m r \omega^2}{e}$$ (where m is mass of electron)

But $$-d\phi = E\,dr$$

$$-\int_{\phi_1}^{\phi_2} d\phi = \int_0^a \frac{m r \omega^2}{e}\,dr$$

After integrating, we get $$\phi_1 - \phi_2 = \frac{m\omega^2 a^2}{2e}$$

On putting the values, we get $$\phi_1 - \phi_2 = 3\text{nV}$$

(b) In the presence of magnetic force, the motional emf in an element of thickness dr at distance r from centre is

$$d\varepsilon = B\,(dr)\,v = B(dr)\,(r\omega) = B\,\omega\,r dr$$

$\therefore$ $$\varepsilon = \int_0^a B\omega r dr = \frac{B\omega a^2}{2}$$

On putting the values, we get

$$\phi_1 - \phi_2 = \varepsilon = 20 \times 10^{-3}\ \text{V} = 20\ \text{mV}$$

YOUR STEP

A metal rod AB of length l rotates with a constant angular velocity ω about an axis passing through O and normal to its length. Calculate potential difference between ends A and B if

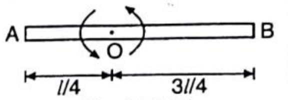


Fig. 3.290A

(i) external magnetic field is absent;

(ii) an external uniform magnetic field of induction B directed parallel to the axis of rotation exists in the space.

$\left\{ \text{(i)}\ \frac{ml^2\omega^2}{4e} \quad \text{(ii)}\ \frac{B\omega l^2}{4} \right\}$

§ 3.291

➢ **CONCEPT**

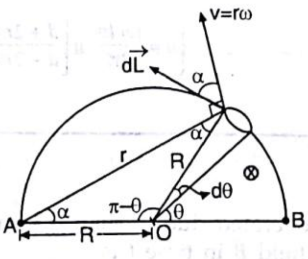


Fig. 3.291A

We consider a $dL = Rd\,\theta$ element at an angle θ with diameter.
The motional emf in the considered element is

$$d\varepsilon = \vec{B}\,.\,(\vec{dL} \times \vec{v}) = \vec{B}\,.\,(dL.v\sin\alpha)\,\hat{n}$$
$$= BdLv\sin\alpha = B\,(Rd\theta)\,(r\omega)\sin\alpha$$

$\therefore$ $$d\varepsilon = B\,R\,r\,\omega\sin\alpha\,d\theta$$

From the fig. 3.291 A,

$$\alpha + \alpha + \pi - \theta = \pi \quad \therefore \quad 2\alpha = \theta \quad \therefore \quad \alpha = \frac{\theta}{2}$$

$\therefore$ $$d\varepsilon = BR r\omega \sin\frac{\theta}{2}\,d\theta \qquad \text{...(i)}$$

Also $$\cos(\pi-\theta)=\frac{R^2+R^2-r^2}{2R^2}$$

$\therefore$ $$2R^2-r^2=-2R^2\cos\theta$$

$\therefore$ $$r^2=2R^2(1+\cos\theta)$$
$$=2R^2\,2\cos^2\frac{\theta}{2}=4R^2\cos^2\frac{\theta}{2}$$

$\therefore$ $$r=2R\cos\frac{\theta}{2}$$

$\therefore$ $$d\varepsilon=BR\omega(2R)\cos\frac{\theta}{2}\sin\frac{\theta}{2}d\theta \quad \text{(from (i))}$$
$$=BR^2\,\omega\sin\theta\,d\theta$$

or $$d\varepsilon=BR^2\,\omega\sin\theta\,d\theta$$

$\therefore$ $$\varepsilon=BR^2\omega\int_0^{\pi}\sin\theta d\theta=BR^2\omega[-\cos\theta]_0^{\pi}$$

$\therefore$ $$\varepsilon=2BR^2\omega$$

$\therefore$ $$-\int_A^C Edr=\varepsilon=2BR^2\omega$$

$\therefore$ $$\int_A^C Edr=-2BR^2\omega$$

$$\int_A^C Edr=-2B\left(\frac{d}{2}\right)^2\omega=-\frac{1}{2}B\,\omega\,d^2=-10\text{ mV} \quad (\because\ d=2R)$$

YOUR STEP

An infinite wire carries a current I. A "S" shaped conducting rod of two semi-circles each of radius r is placed at an angle θ to the wire. The centre of the conductor is at a distance d from the wire. If the rod translates parallel to the wire with a velocity v as shown in fig. 3.291B. Calculate the emf induced across the ends OB of the rod?

$$\left\{v=\frac{\mu_0 Iv}{2\pi}ln\left[\frac{d+2r\cos\theta}{d-2r\cos\theta}\right]\right\}$$

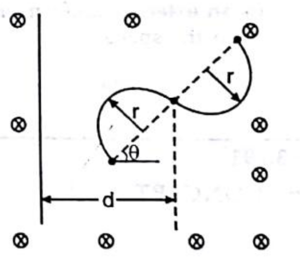


Fig. 3.291B

§ 3.292

➢ CONCEPT

In this case, induced emf is generated due to change in area inside the magnetic field. The area of loop in side the magnetic field B in time t is
$$A=\frac{R^2}{2}\phi$$

The flux passing through loop is $$\phi=BA$$

$\therefore$ $$\xi=\frac{d\varphi}{dt}=B\frac{dA}{dt}=\frac{BR^2}{2}\frac{d\phi}{dt}$$

But $\phi=\frac{1}{2}\beta t^2$ $\quad\therefore\quad \frac{d\phi}{dt}=\beta t$

$\therefore$ $$\xi=\frac{BR^2}{2}\beta t=\frac{BR^2\beta t}{2}$$

when loop enters in the magnetic field current is anti-clockwise. But when loop is coming out from the magnetic field, current is in clockwise direction.

In general,

$$\xi = (-1)^n \frac{BR^2 \beta t}{2}$$

$$\xi = (-1)^n \frac{Ba^2 Bt}{2} \qquad (\because R = a)$$

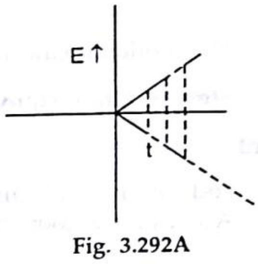


Fig. 3.292A

where n is number of half revolution. The graph is shown in fig. 3.292 A.

Remarks : The answer of the book is wrong. For correctness, we can use homogienity principle of dimension.

YOUR STEP

The space is divided by the line AD into two regions. Region (1) is field free end the region (2) has a uniform magnetic field B directed into the plane of the paper. ACD is semicircular conducting loop of radius r with centre at O (shown in fig. 3.292B); the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and perpendicular to the plane of the paper. The effective resistance of loop is R.

(i) Obtain an expression for the magnitude of induced current in the loop.

(ii) Plot a graph between the induced emf and the time of rotation for two periods of rotation.

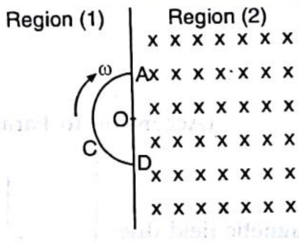


Fig. 3.292B

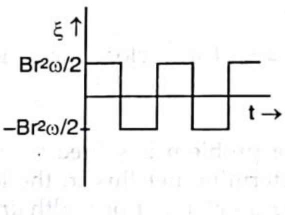
(ii)


Fig. 3.292C

$\left\{ \text{(i) } \frac{Br^2 \omega}{2R} \right\}$

§ 3.293

➢ **CONCEPT**

The magnetic field due to a long straight wire is $B = \frac{\mu_0 I}{2\pi r}$

when B, l and v are mutually perpendicular, then induced emf is $\varepsilon = Blv$

SOLUTION : The problem is solved by following steps.

Step I. Determine motional emf in the rod.

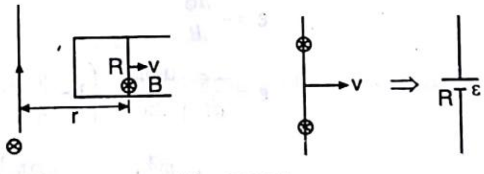


Fig. 3.293A

The magnetic field due to the long straight wire at the site of connector is

$$B = \frac{\mu_0 I}{2\pi r}$$

The motional emf in the rod is $\varepsilon = Blv = \frac{\mu_0 \, lvI}{2\pi r}$ (shown in Fig. 3.293 A)

Step II. Draw equivalent circuit of the system. The connecter behaves as a real battery of emf

$$\varepsilon = \frac{\mu_0 Ilv}{2\pi r}$$

and internal resistance R. The equivalent circuit is shown in fig. 3.293 B.
According to loop rule, $\varepsilon - IR = 0$

$$\therefore \quad I = \frac{\varepsilon}{R} = \frac{\mu_0 Ilv}{2\pi rR}$$

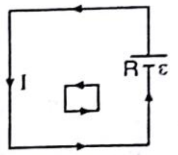


Fig. 3.293B

YOUR STEP

U-frame $ABCD$ and a sliding rod PQ of resistance R, start moving with the velocities v and $2v$ respectively, parallel to a long wire. Carrying currnt i when the distance $AP = l$ at $t = 0$. Determine the current through the inductor of inductance L just before connecting rod PQ loses contact with the U-frame.

$$\left\{ i = \frac{\varepsilon}{R}\left(1 - e^{-lR/Lv}\right) \text{ Here } \varepsilon = \frac{\mu_0 \, I_0 \, v \, ln \, 2}{\pi} \right\}$$

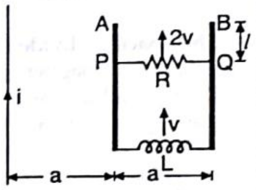


Fig. 3.293C

§ 3.294

➢ **CONCEPT**

The induced emf in a closed loop is

$$\varepsilon = \frac{-d\phi}{dt} \quad \text{(According to Faraday's law)}$$

SOLUTION : The problem is solved by following steps :

Step I : Determine net flux in the loop :

We consider an element of width dr of the loop. The magnetic field due to the long wire in the considered element is

$$B = \frac{\mu_0 I}{2\pi r}$$

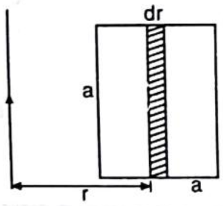


Fig. 3.294A

The magnetic flux in the considered element is

$$d\phi = Badr = \frac{\mu_0 Ia}{2\pi r} dr$$

$$\therefore \quad \phi = \frac{\mu_0 Ia}{2\pi} \int_x^{x+a} \frac{dr}{r} = \frac{\mu_0 Ia}{2\pi} ln\left(1 + \frac{a}{x}\right)$$

Step II : Apply $\varepsilon = \frac{-d\phi}{dt}$

$\because$ induced emf $\varepsilon = -\frac{d\phi}{dt}$

$$\therefore \quad \varepsilon = \frac{-d}{dt}\left\{\frac{\mu_0 Ia}{2\pi} ln\left(1 + \frac{a}{x}\right)\right\} = \frac{\mu_0 Ia}{2\pi\left(1 + \frac{a}{x}\right)} \frac{a}{x^2} \frac{dx}{dt}$$

$$= \frac{\mu_0 Ia^2 v}{2\pi x^2\left(1 + \frac{a}{x}\right)} = \frac{\mu_0 2Ia^2 v}{4\pi x(x + a)}$$

YOUR STEP

A small conducting loop of radius a and resistance r is pulled with velocity v perpendicular to a long straight conductor carrying a current i_0. If a constant power P is dissipated in the loop, find the variation of velocity of the loop as a function of x. Given that $x >> a$.

$$\left\{\frac{2x^2}{\mu_0 i_0 a^2}\sqrt{pr}\right\}$$

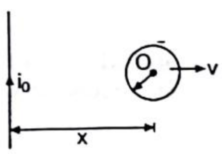


Fig. 3.294A

§ 3.295

➢ CONCEPT

The problem is solved by following steps :

Step I : Determine motional emf in the rod

We consider an element of length dx at distance x from the centre O of the ring.

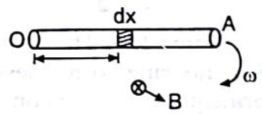


Fig. 3.295A

The motional emf in considered element is

$$d\varepsilon = B\,(dx)\,v = B\,(dx)\,x\omega$$
$$= B\omega x dx$$

∴ Net motional emf in the rod is

$$\varepsilon_{in} = \int_0^a B\omega x dx = \frac{1}{2}Ba^2\omega$$

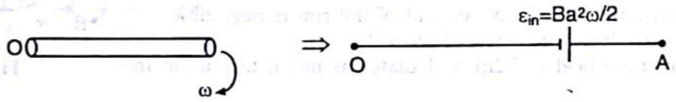


Fig. 3.295B

Step II : Discuss the mechanical condition of rod :

Since, rod is rotating about point O with constant angular velocity ω. So, the net torque on the rod about point O is zero. For this, torque due to magnetic force on rod is balanced by torque due to gravitational force on the rod.

The torque due to gravitational force on rod about point O is

$$\tau_g = mg\frac{a}{2}\sin\omega t \qquad \text{(clockwise direction)}$$

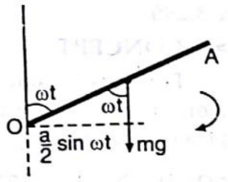


Fig. 3.294C

For balancing gravitational torque by torque due to magnetic force. The magnetic torque should be anticlockwise. This is possible only when the direction of current is from O to A in the rod (shown in fig. 3.294 D). We consider an element dx of rod. The magnetic force on the considered element is

$$dF = I\,(dx)\,B$$

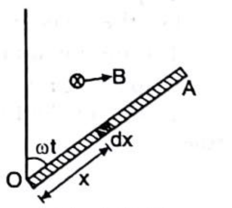


Fig. 3.294D

The torque due to this force dF about O is

$$d\,\tau_B = x\,dF \qquad \text{(anti-clockwise direction)}$$

or $$d\,\tau_B = IBxdx$$

∴ $$\tau_B = IB\int_0^a x\,dx = \frac{IBa^2}{2} \qquad \text{(anti-clockwise)}$$

For equilibrium of rod, $$\tau_B = \tau_g$$

or $$\frac{IBa^2}{2} = mg\,\frac{a}{2}\sin\omega t$$

$$\therefore \qquad = \frac{mg\sin\omega t}{aB} \qquad \text{...(i)}$$

Step III : Draw equivalent circuit of the system :

The equivalent circuit is shown in fig. 3.295 E. According to loop rule,

$$\varepsilon - \varepsilon_{in} - IR = 0$$

$$\therefore \qquad \varepsilon = \varepsilon_{in} + IR$$

$$= \frac{1}{2}Ba^2\omega + \frac{Rmg\sin\omega t}{aB}$$

$$= \frac{1}{2}\frac{(\omega a^3B^2 + 2R\, mg\sin\omega t)}{aB}$$

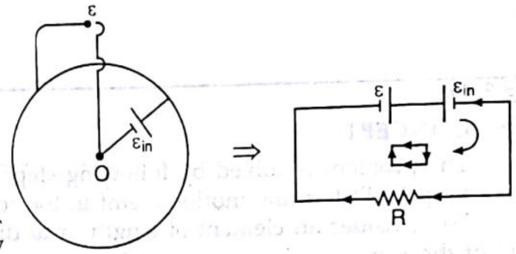


Fig. 3.295E

Remarks : The answer of the book is wrong. For checking correctness, we can use homogienity principle of dimension.

YOUR STEP

A wheel with six spokes is positioned perpendicular to a uniform magnetic field B of magnitude 0.5T. The magnetic field is directed into the plane of the paper and is present over the entire region of the wheel as shown in the fig. 3.296F. When the switch S is closed, there is an initial current of 6A between the axle and the rim; and the wheel begins to rotate. The resistance of the spokes and of the rim is negligible.

(a) What is the direction of rotation of wheel?

(b) The radius of the wheel is 0.2m. Calculate the initial torque on the wheel.

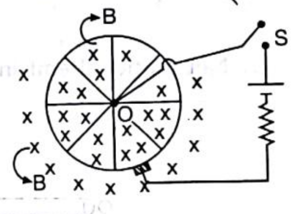


Fig. 3.296F

(a) Anti-clockwise
(b) 0.06 Nm

§ 3.296

➢ CONCEPT

For steady state velocity of rod, magnetic force on the rod is balanced by component of weight of rod along the inclined plane. On the basis of conservation principle of energy, the power supplied by weight of rod appears as thermal power in the circuit.

SOLUTION : The problem is solved by following steps :

Step I : Determine motional emf in rod at steady state.

Let the velocity of rod at steady state is v_0.

The motional emf in the rod is $\varepsilon = B\,l\,v_0$ (shown in fig. 3.296 A)

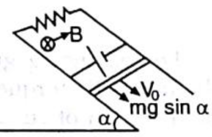


Fig. 3.296A

Step II : Draw the equivalent circuit of the system.

The equivalent circuit is shown in fig. 3.296 B. According to loop rule,

$$\varepsilon - IR = 0 \qquad \therefore \quad I = \frac{\varepsilon}{R} = \frac{B\,l\,v_0}{R}$$

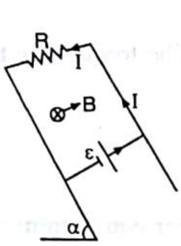


Fig. 3.296B

Step III : Discuss the system on basis of energy conservation principle. The power supplied by weight of the rod is

$$P_g = m\,\vec{g}\,.\,\vec{v}_0 = mgv_0\sin\alpha$$

The thermal power developed in the circuit is

$$P = I^2R = \left(\frac{B\,l\,v_0}{R}\right)^2 R = \frac{B^2l^2\,v_0^2}{R}$$

According to conservation principle of energy,

$$P_g = P$$

$$\therefore \quad mgv_0 \sin\alpha = \frac{B^2 l^2 v_0^2}{R}$$

$$\therefore \quad v_0 = \frac{mgR\sin\alpha}{B^2 l^2}$$

YOUR STEP

A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest, calculate :

(a) the terminal velocity achieved by the rod, and

(b) the acceleration of the mass at an instant when the velocity of the rod is half the terminal velocity.

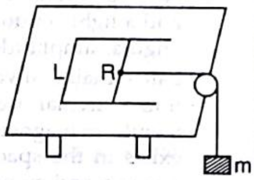


Fig. 3.296C

[(a) $v_T = \frac{mgR}{B^2L^2}$ (b) $a = \frac{g}{2}$]

§ 3.297

➢ **CONCEPT**

The concept is similar to previous problem.

➢ **DISCUSSION**

At $t = 0$, the velocity of rod is zero. But acceleration of rod is

$$w = \frac{mg\sin\alpha}{m} = g\sin\alpha$$

where m is mass of rod.

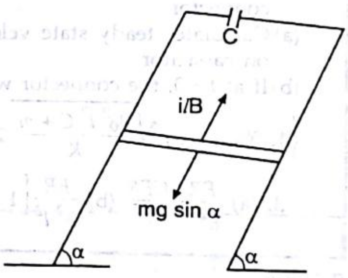


Fig. 3.297A

when $t > 0$, motional emf is founed into the rod which is cause of flow of charge through the circuit. Let at an instant t, current in the circuit is i. The corresponding circuit is shown in fig. 3.297B.

According to loop rode, $\quad \varepsilon - \frac{q}{C} = 0$

$$q = C\varepsilon = BlCv$$

$$\therefore \quad i = \frac{dq}{dt} = B\,l\,C\frac{dv}{dt} = B\,l\,C\,w$$

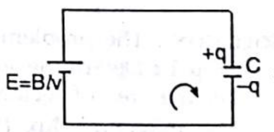


Fig. 3.297B

It means current in circuit depends upon acceleration of the rod.

From free diagram of rod (shown in fig. 3.297 A).

$$mg\sin\alpha - ilB = mw$$

or

$$mg\sin\alpha - (B\,l)\,BlCw = mw$$

or

$$mg\sin\alpha = (m + B^2l^2C)\,w$$

$$\therefore \quad w = \frac{mg\sin\alpha}{m + B^2 l^2 C} = \frac{g\sin\alpha}{1 + \frac{l^2B^2C}{m}}$$

YOUR STEP

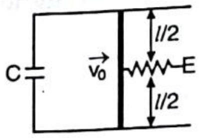


Fig. 3.297C

1. In the arrangement shown in fig. 3.297C, there is a uniform magnetic field B_0 normal to the plane of the paper. The connector is smooth and conducting and it has mass m and length l initially, the spring is relaxed. At time $t = 0$, the connector is suddenly given velocity v_0 as shown. What is the charge acquired by the capacitor at this instant ? What will be the maximum compression of the spring ? The spring is non-conducting, the resistance of the rails is zero and neglect the self inductance.

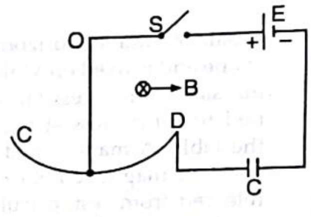


Fig. 3.297D

2. A simple pendulum consists of a small conducting ball of mass m and a light conducting rod of length l. The pendulum oscillates with angular amplitude θ_0 in a vertical plane about axis O. Such that the ball remains always just in contact with a metallic strip CD, bent into a circular arc of radius l as shown in fig. 3.297D. In the space, a uniform magnetic field of inductions B normal to plane of oscillation exists in the space. At the instant $t = 0$, when ball is at its lowest position and moving towards right, the switch S is closed. Neglecting self inductance of the circuit, calculate anticlockwise moment. $\tau(t)$ required to keep the pendulum oscillating as before.

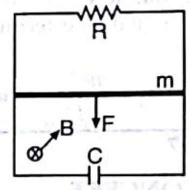


Fig. 3.297E

3. Two long rails are horizontal and parallel to each other. On one end, the rails are connected by a resistance R and on the other end a capacitor of capacitance C is connected as shown in fig. 3.297E. A connector of mass m and length l can slide on the rails without friction. Vertical component of earth's magnetic induction is B (downwards). A constant horizontal force F starts acting on the connector.
(a) Calculate steady state velocity of the connector and steady charge on capacitor.
(b) If at $t = 0$, the connector was at rest. Calculate its velocity as a function of time t.

$\Big\{$1. $X_{max} = v_0 \sqrt{\dfrac{B_0^2 l^2 C + m}{K}}$ 2. $\dfrac{1}{4}(CB^2 l^4 \theta_0 \omega^2 \sin \omega t)$ where $\omega = \sqrt{\dfrac{g}{l}}$

3. (a) $\dfrac{FR}{B^2 l^2}, \dfrac{CFR}{Bl}$ (b) $\dfrac{FR}{B^2 l^2}\left[1 - e^{-B^2 l^2 t/R(m + CB^2 l^2)}\right]\Big\}$

§ 3.298

➢ CONCEPT

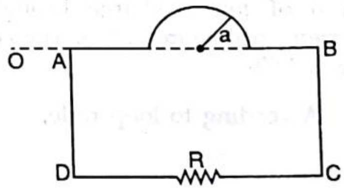


Fig. 3.298A

The induced emf in a loop is

$$\varepsilon = -\frac{d\phi}{dt}$$

SOLUTION : The problem is solved by following steps :

Step I : Determine induced emf in the loop.

Let the area of rectangular part of circuit is A_0.

The magnetic flux through rectangular part $ABCD$ of the circuit is

$$\phi_1 = BA_0$$

At $t = 0$, the area vector of semi circular part, and vector B are in the same direction. At an instant t, the angle between area vector and vector B is

$\theta = \omega t$ (*i.e.* the angular displacement of semicircular part in time t)

So, the magnetie flux through semicircular part of the loop at an instant t is

$$\phi_2 = B\frac{\pi a^2}{2}\cos\theta = \frac{\pi B a^2}{2}\cos\omega t$$

The net flux through the loop at an instant t is

$$\phi = \phi_1 + \phi_2 = BA_0 + \frac{\pi B a^2 \cos\omega t}{2}$$

The induced emf in the loop is

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{BdA_0}{dt} - \frac{\pi B a^2}{2}\frac{d}{dt}(\cos\omega t)$$

$$= 0 + \frac{\pi B a^2 \omega}{2}\sin\omega t$$

Step II : Draw equivalent circuit of the system.
The equivalent circuit of the loop is shown in fig. 3.298 B.

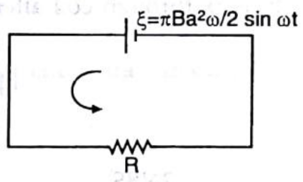


Fig. 3.298B

$$\varepsilon = \frac{\pi B a^2 \omega}{2}\sin\omega t$$

According to loop rule; $\quad I = \dfrac{\varepsilon}{R} = \dfrac{\pi B a^2 \omega \sin\omega t}{2R}$

The instantaneous thermal power in the circuit is

$$P = I^2 R = \frac{\pi^2 B^2 a^4 \omega^2 \sin^2\omega t}{4R^2} R = \frac{\pi^2 B^2 a^4 \omega^2 \sin^2\omega t}{4R}$$

Mean power is

$$\therefore \quad <P> = \frac{\int_0^{T=\frac{2\pi}{\omega}} P\,dt}{\int_0^{t=\frac{2\pi}{\omega}} dt} = \frac{\pi^2 B^2 a^4 \omega^2}{4R}\,\frac{\int_0^{\frac{2\pi}{\omega}} \sin^2\omega t}{\frac{2\pi}{\omega}}$$

After integrating, we get, $\quad <P> = \dfrac{(\pi\omega a^2 B)^2}{2R}$

YOUR STEP

A wire loop in the form of a sector of a circle of radius $a = 20$ cm is located on boundary of a uniform magnetic field of strength $B = 1$ tesla as shown in fig. 3.298C. Angle of sector is $\theta = 45°$ and resistance of the loop is $R = 20\ \Omega$. The loop is rotated about axis O which is parallel to the magnetic field and lies on its boundary, with a constant angular velocity ω. If rate of heat generation in the loop is 3.14 mJ/revolution, calculate.

(i) angular velocity ω and

(ii) average thermal power

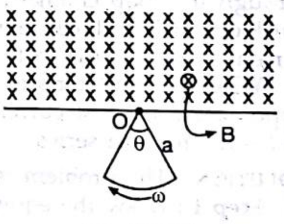


Fig. 3.298C

{(i) 100 rad/sec (ii) 0.05 watt}

§ 3.299

➢ **CONCEPT**

According to Faraday's Law of electromagnetic induction :

$$\varepsilon = \frac{Nd\phi}{dt} \qquad IR = \frac{Nd\phi}{dt}$$

or

$$R\frac{dq}{dt} = N\frac{d\phi}{dt}$$

or $R\frac{\Delta q}{\Delta t} = N\frac{\Delta\phi}{\Delta t}$ or $R\Delta q = N\Delta\phi$

$\therefore$ $$\Delta q = \frac{N\Delta\phi}{R} = \frac{N\,|(\phi_f - \phi_i)|}{R}$$

SOLUTION : The initial flux through the coil in $\phi_i = BS$

when coil is rotated through 180°, the area vector and magnetic field are in opposite direction.

$\therefore$ Flux through coil after 180° rotation of coil is

$$\phi_f = \vec{B}\cdot\vec{S} = BS\cos 180^\circ = -BS$$

$\therefore$ $$|\phi_f - \phi_i| = 2BS$$

$\therefore$ $$\Delta q = \frac{N\,|\phi_f - \phi_i|}{R}$$

$$q = \frac{2NBS}{R} \qquad \therefore \quad B = \frac{qR}{2NS}$$

On putting the values, we get $B = 0.5$ tesla

YOUR STEP

A square current carrying loop made of thin wire and having a mass $m = 10$ g can rotate without friction with respect to the vertical axis OO_1, [Fig. 3.299A], passing through the centre of the loop at right angles to two opposite sides of the loop. The loop is placed in a homogeneous magnetic field with an induction $B = 10^{-1}$ T directed at right angles to the plane of the drawing. A current $I = 2$A is flowing in the loop. Find the period of small oscillations that the loop performs about its position of the stable equilibrium.

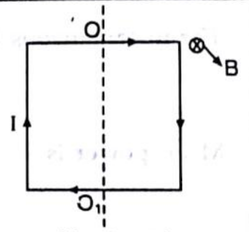


Fig. 3.299A

$$\left\{ T_0 = 2\pi\sqrt{\frac{m}{6\,IB}} = 0.57 \text{ sec} \right\}$$

§ 3.300

➢ **CONCEPT**

During rotation of loop external flux (flux due to magnetic field of long straight wire) crossing through the loop changes. This change in external magnetic flux is caused by induced emf in the loop. This induced emf behaves as external voltage source for the loop. Due to this emf, current starts to flow in the circuit.

When current starts to flow in the circuit, Self induced emf is generated in the circuit which opposes the flow of current. In nut shell the system behaves as $L - R$ circuit in series.

SOLUTION : The problem is solved in following steps :

Step I : Draw the equivalent circuit at an instant t :

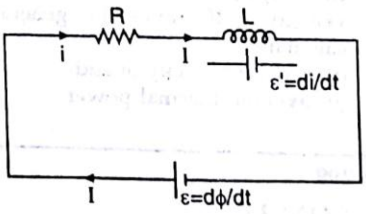


Fig. 3.300A

The equivalent circuit is shown in fig. 3.300 A.

In the shown circuit,

ε = induced emf due to change in external magnetic flux.

ε' = Self induced emf in the loop

According to loop rule,

$\because$ $$\varepsilon - iR - \varepsilon' = 0 \qquad \text{(At an instant } t\text{)}$$

or $$\frac{d\phi}{dt} - Ri - L\frac{di}{dt} = 0$$

or $$\frac{d\phi}{dt} - Ri - L\frac{di}{dt} = 0$$

or $$\frac{d\phi}{dt} - R\frac{dq}{dt} - L\frac{di}{dt} = 0$$

or $$\frac{\Delta\phi}{\Delta t} - R\frac{\Delta q}{\Delta t} - L\frac{\Delta i}{\Delta t} = 0$$

or $$\Delta\phi - R\Delta q - L\Delta i = 0 \quad ...(i)$$

Here $$\Delta i = i_f - i_i$$

Since, initial current in the circuit is zero

$\therefore$ $$i_i = 0$$

After 180° rotation, finally flux becomes constant. Hence, final current in the circuit is

$$I_f = 0$$

$\therefore$ $$\Delta i = i_f - i_i = 0$$

From Eq (i), we get $$\Delta\phi - R\Delta q - 0 = 0$$

$\therefore$ $$\Delta q = \frac{\Delta\phi}{R} \quad ...(ii)$$

Step III. Determine net change in external flux :

At the initial position of loop (frame), magnetic field and area vector are opposite in direction. But after rotation of frame through 180°, the area vector and vector B becomes in the same direction.

$\therefore$ $$\Delta\phi = \phi_2 - (-\phi_1) = \phi_2 + \phi_1$$

$\therefore$ $$\Delta\phi = \phi_2 + \phi_1 = \int_{r=b-a}^{r=b+a} B\,dA$$

$$= \int_{b-a}^{b+a} \frac{\mu_0 i}{2\pi r} a\,dr = \frac{\mu_0 a i}{2\pi} ln\left(\frac{b+a}{b-a}\right) \qquad \left\{B = \frac{\mu_0 I}{2\pi r}, dA = a\,.\,dr\right\}$$

From Eq. (ii), we get $\Delta q = q = \dfrac{\Delta\phi}{R} = \dfrac{\mu_0 a I}{2\pi R} ln\left(\dfrac{b+a}{b-a}\right)$

YOUR STEP

A small conducting loop of mass m, radius r and resistance R is released from rest along a smooth inclined plane. Such that the plane of the loop of perpendicular to a magnetic field which varies along x axis as $B = B_0(1 + ax)$. Here B_0 and a are constant. Find the speed of the ring at time t.

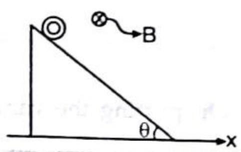


Fig. 3.300B

$$\left\{\frac{g \sin\theta}{K}[1 - e^{-Kt}] \qquad \text{where } k = \frac{\pi^2 r^4 B_0^2 a^2 \cos^2\theta}{mR}\right\}$$

§ 3.301

➢ **CONCEPT**

The problem is based upon motional emf in non-uniform magnetic field.

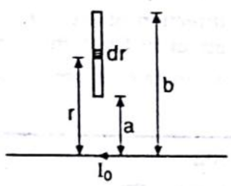


Fig. 3.301A

SOLUTION : (a) The problem is solved by following steps :

Step I : Determine motional emf in connector (rod) :

We consider a dr element of connector at distance r from the straight wire carrying current I_0.

The magnetic field at the site of considered element is

$$B = \frac{\mu_0 I_0}{2\pi r}$$

The motional emf in the considered element is

$$d\varepsilon = B(dr)v = \frac{\mu_0 I_0 v}{2\pi r} dr$$

The net motional emf in the connector is

$$\varepsilon = \int d\varepsilon = \frac{\mu_0 I_0 v}{2\pi} \int_a^b \frac{dr}{r}$$

$$\varepsilon = \frac{\mu_0 I_0 v}{2\pi} ln \frac{b}{a}$$

Step II : Draw equivalent circuit.
The equivalent circuit is shown in fig. 3.301B.

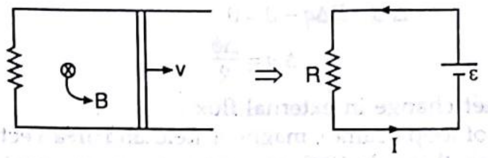


Fig. 3.301B

According to loop rule,

$$\varepsilon - IR = 0 \qquad \therefore \qquad I = \frac{\varepsilon}{R}$$

$$\therefore\ I = \frac{\mu_0 I_0 v}{2\pi R} ln \frac{b}{a} \qquad \text{(anti-clockwise)}$$

Step III : (b) Discuss the circuit on the basis of conservation principle of energy.
According to conservation principle of energy (Lenz's law), the power supplied by an external agent by applying a force F_{ext} appears as thermal power in the circuit.

Mathematically, $F_{ext}\, v = I^2 R$

$$\therefore \qquad F_{ext} = \frac{I^2 R}{v}$$

On putting the value of I, we get $F = F_{ext} = \dfrac{v}{R}\left(\dfrac{\mu_0 I_0}{2\pi} ln \dfrac{b}{a}\right)^2$ (in the direction of v)

> **YOUR STEP**
>
> A wire frame of area $A = 4 \times 10^{-4}$ m^2 and resistance $R = 20\Omega$ is suspended freely by a thread of length $l = 0.40$ m. A uniform horizontal magnetic field of induction $B = 0.8$T exists in the space such that plane of the frame is perpendicular to the magnetic field. At time $t = 0$, the frame is made to oscillate under gravity by displacing it through $a = 2 \times 10^{-2}$m from its initial position along the direction of the magnetic field. The plane of frame is always along the thread and does not rotate about it. Neglecting magnetic field of induced current. Calculate induced emf in the wire frame as a function of time. Calculate also, maximum current in the frame. ($g = 10$m/sec^2).
>
> [-2×10^{-6} sin (10t) volt, 10^{-7} amp]

§ 3.302

➢ **CONCEPT**

The concept is similar to previous problems.

➢ **DISCUSSION**

At $t = 0$, when rod start to move, motional emf is generated in the rod. Due to this a current starts to flow through circuit. According to Lenz's law, the direction of current is as such the magnetic force on the rod opposes the motion of rod. It means magnetic force on the rod is in opposite direction of velocity of rod. Due to this, rod starts to decelerate and finally comes in rest. In the reference of energy conservation principle, total kinetic energy of rod appears of thermal energy in the circuit during the process.

SOLUTION : (a) The motional emf in rod at an instant t is

$$\varepsilon = Blv$$

where v is speed of rod at an insant t.
The current in the circuit at an instant t is

$$I = \frac{\varepsilon}{R} = \frac{Blv}{R}$$

The magnetic force on the root at an instant t is

$$F = IlB = \frac{B^2l^2v}{R}$$

The deceleration in rod is $$w = \frac{F}{m} = \frac{B^2l^2v}{mR}$$

or $$-v\frac{dv}{dS} = \frac{B^2l^2}{mR}v$$

But $$w = -v\frac{dv}{ds}$$

or $$-\int_{v_0}^{0} dv = \frac{B^2l^2}{mR}\int_0^S dS$$

$\therefore$ $$S = \frac{mRv_0}{B^2l^2}$$

(b) Thermal energy generated in the circuit = loss in kinetic energy of rod

$$= \frac{1}{2}mv_0^2$$

YOUR STEP

Fig. 3.302A exhibits a cylinderical region of space occupied by a homogeneous magnetic field B directed perpendicularly into the page. A square conducting frame $MNPQ$, with sides of length a, conductivity σ and mass density ρ falls in the magnetic field as shown in fig. 3.302A. Find the terminal velocity of the frame in the magnetic field. Assume that the frame is small enough, so that it reaches its final falling velocity before leaving the region occupied by the magnetic field.

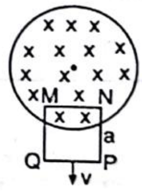


Fig. 3.302A

$$\left\{\frac{16\rho g}{B^2\sigma}\right\}$$

§ 3.303

➢ **CONCEPT**

The concept is similar to problem 3.296.

➢ **DISCUSSION**

At $t = 0$, the velocity of rod is zero. So, no motional emf is induced in the rod. But rod has acceleration $w = \frac{F}{m}$ at $t = 0$. Due to this acceleration, rod starts to accelerate in forward direction. When $t > 0$, speed of rod is greater than zero. So, motional emf is generated in the rod which is equal in the rod is $\varepsilon = Blv$.

Due to this emf, current will be in the circuit. Let current in the circuit at instant t is i and velocity of the rod is v.

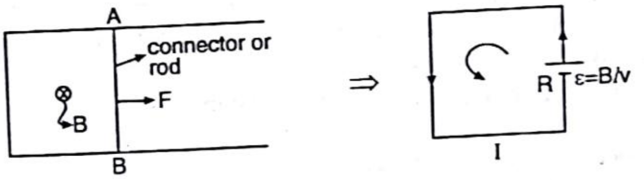


Fig. 3.303A

The equivalent circuit is shown in fig. 3.303A.

According to loop rule, $\varepsilon - IR = 0$

$$Blv - IR = 0$$

$$\therefore \quad I = \frac{Blv}{R} \qquad \text{...(i)}$$

Due to this current, a magnetic force acts on the rod in the opposite direction of applied force F.

SOLUTION :

The force-diagram of rod is shown in fig. 3.303B.

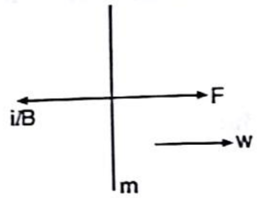


Fig. 3.303B

The acceleration of rod at an instant t is

$$w = \frac{F - ilB}{m}$$

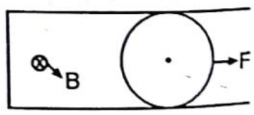
or

$$\frac{dv}{dt} = \frac{F - \frac{B^2l^2v}{R}}{m} \qquad \left(\because i = \frac{Blv}{R}\right)$$

$$\int_0^v \frac{mdv}{F - \frac{B^2l^2v}{R}} = \int_0^t dt$$

After solving we get, $v = \frac{F}{\alpha m}(1 - e^{-\alpha t})$

Here $\alpha = \frac{B^2l^2}{mR}$

> **YOUR STEP**
>
> Two long parallel conducting horizontal rails are connected by a conducting wire at one end. A uniform magnetic field (directed vertically downwards) exists in the region of space. A light uniform ring of diameter d which is practically equal to the separation between the rails, is placed over the rails (fig. 3.303C). If resistance of ring be λ per unit length, calculate the force required to pull the ring with uniform velocity v.
>
> $$\left\{F = \frac{4B^2vd}{\pi\lambda}\right\}$$
>
> 
>
>
> Fig. 3.303C

§ 3.304

➢ **CONCEPT**

The problem is based upon Lenz's law. The problem is an example of statically induced emf.

In author's opinion, the direction of statically induced emf is determined on the basis of magnetic field and the direction of dynamically induced emf is determined on the basis of magnetic force.

➢ DISCUSSION

(a) According to problem, magnetic field B in the loop decreases which is caused by change in magnetic flux in the loop.

According to Lenz's law, the direction of induced emf in the loop should be as such it increases the magnetic field in the loop. This is possible only when magnetic field due to induced current in the loop would be in the direction of external magnetic field B.

SOLUTION :

In fig. (a) of the problem, the loop is divided in two equal semicircular parts. The current in both parts should be clockwise direction for producing magnetic field in the direction of external magnetic field B, (shown in fig. 3.304A).

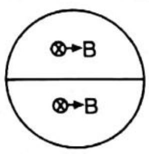


Fig. 3.304A

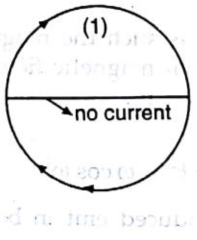


Fig. 3.304B

(b) In fig. (b) of the problem, the net flux in the loop is $\phi = \phi_1 - \phi_2$

Here ϕ_1 = magnetic flux in larger loop

ϕ_2 = magnetic flux in smaller loop.

The direction of induced current depends upon larger loop. So, the direction of magnetic field due to induced current in larger loop should be in the direction of external magnetic field B.

For this the direction of induced current in larger loop should be clockwise (shown in fig. 3.304B).

(c) The reason is same as part (b) The direction of induced emf in both loops (larger and smaller) should be clockwise for producing magnetic field in the direction of external magnetic field B (shown in fig. 3.304C).

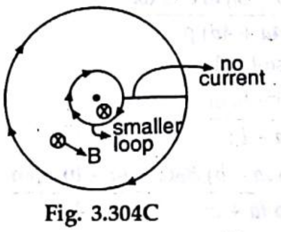


Fig. 3.304C

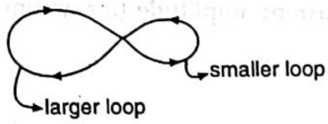


Fig. 3.304D

(d) The reason is same as in part (b)

The direction of induced current is determined on the basis of larger loop. The direction of induced current is shown in fig. 3.304D.

YOUR STEP

A closed loop with the geometry shown in fig. 3.304E. is placed in a uniform magnetic field directed into the plane of paper. If the magnetic field decreases with time, determine the direction of the induced emf in this loop.

(clockwise)

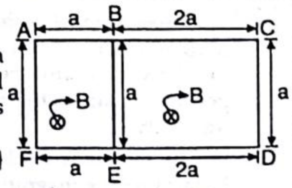


Fig. 3.304E

§ 3.305

➢ **CONCEPT**

In this case, induced emf is generated due to changing magnetic field.

$$\therefore \qquad \varepsilon = \frac{d\,(BA)}{dt} = A\,\frac{dB}{dt}$$

SOLUTION :

The problem is solved by following steps :

Step I : Determine the induced emf in larger loop :

In larger loop,

$$\varepsilon_1 = a^2\frac{dB}{dt} = a^2\frac{d}{dt}(B_0 \sin \omega t) = a^2\omega\, B_0 \cos \omega t$$

Due to this emf, the direction of current is as such the magnetic field due to this induced current should be in opposite direction of external magnetic field B.

Step II : Determine induced emf in smaller loop :

The induced emf in smaller loop is

$$\varepsilon_2 = b^2\frac{d}{dt}(B_0 \sin \omega t) = b^2 B_0\,\omega \cos \omega t$$

According to Lenz's law the direction of induced emf in both loops are opposite to each other.

Step III : Draw equivalent circuit of the system :

The resistance in larger loop is

$$R_1 = \rho\,(4a) = 4a\rho$$

The resistance in smaller loop is

$$R_2 = 4b\rho$$

The equivalent circuit is shown in fig. 3.305A.

According to loop rule,

$$\varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$$

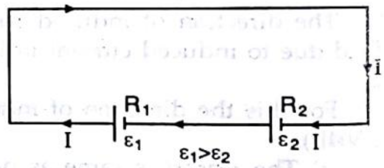


Fig. 3.305A

$$\therefore \qquad I = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2}$$

$$\therefore \qquad I = \frac{(a^2 - b^2)\,B_0\,\omega \cos \omega t}{(4a + 4b)\,\rho}$$

For current amplitude (maximum current), $\cos\omega t = 1$

$$\therefore \qquad I_{max} = \frac{(a^2 - b^2)\,B_0\omega}{4\rho\,(a+b)}$$

$$= \frac{(a+b)\,(a-b)\,B_0\omega}{4\rho\,(a+b)} = \frac{(a-b)\,B_0\omega}{4\rho}$$

On putting the values, we get

$$I_{max} = 0.5\text{A}$$

YOUR STEP

A closed loop of wire consists of a pair of equal semicircles, radius 3.70 cm, lying in mutually perpendicular planes. The loop was formed by folding a circular loop along a diameter until the two halves became perpendicular A uniform magnetic field B of magnitude 760 gauss is directed perpendicular to the fold diameter and makes angle 45° with planes of the semicircles shown in fig. 3.305B.

(a) The magnetic field is reduced at a uniform rate to zero during a time interval 4.50×10^{-3} s. Determine the magnitude of the induced emf and the sense of the induced current in the loop during this time interval.

(b) How would the answer change if $\vec{B}$ is directed as shown in fig. 3.305C, perpendicular to the direction first given for it but still perpendicular, to the fold diameter?

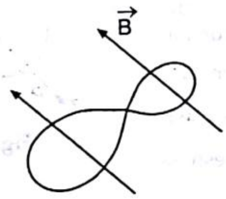


Fig. 3.305B

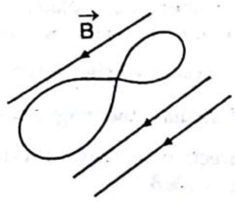


Fig. 3.305C

(a) 5.13×10^{-2} V
(b) Equal in magnitude but opposite in sense, the net emf in complete loop is zero.

§ 3.306

➢ CONCEPT

We consider an element of radius r and thickness dr

$\therefore$ $$dN = \frac{N}{a} dr$$

$\therefore$ The magnetic flux in considered element is $d\phi = B\pi r^2$

The induced emf in considered element is $d\varepsilon = -dN\frac{d\phi}{dt} = dN\,\pi r^2 \frac{dB}{dt}$

$$= dN\pi r^2 B_0\,\omega \sin \omega t$$

$\therefore$ Total emf generated is $$\varepsilon = \pi B_0\,\omega \sin \omega t \int_{r=0}^{r=a} dNr^2$$

$$= \pi B_0\,\omega \sin \omega t \int_0^a \frac{N}{a} r^2 dr$$

$$= \frac{1}{3}\pi B_0\,\omega\, Na^2 \cos \omega t$$

$\therefore$ $$\varepsilon_{max} = \frac{1}{3}\pi B_0\,\omega\, Na^2$$

YOUR STEP

1. A square loop of side l is shown in fig. 3.306A. The loop is placed near a long straight wire carrying current

$$I = \left(\frac{I_0}{t_0}\right)t$$

(a) Find the charge stored on capacitor at any time t.
(b) Find the ratio of heat generated in resistor and the energy stored on capacitor in time t.
(c) If the breakdown voltage of capacitor is V_0, find the time taken the breakdown to take place.

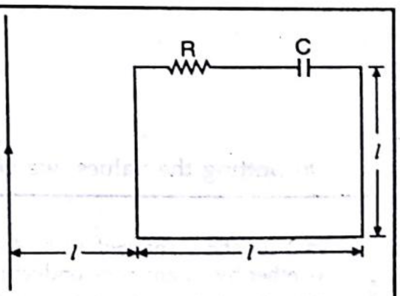


Fig. 3.306A

2. Three identical wires are bent into circular arcs each of radius r such that each arc subtends an angle $\theta = 120°$ at its centre of curvature. These arcs are connected with each other to form a closed mesh such that one of them lies in x–y plane, one in y-z plane and the other in z–x plane as shown in fig. 3.306B. In the region of space a uniform magnetic field of induction $\vec{B} = B_0(\hat{i} + \hat{j})$ exists, whose magnitude increases at a constant rate $\frac{dB}{dt} = \alpha$. Calculate the magnitude of emf induced in the mesh and mark direction of flow of induced current in the mesh as shown in fig. 3.306B.

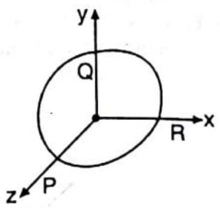


Fig. 3.306B

1. (a) $\frac{\mu_0 l I_0 C \ln 2}{2\pi t_0}(1 - e^{-t/RC})$ (b) $\frac{(1 - e^{-2t/RC})}{(1 - e^{-t/RC})^2}$ (c) $RC \ln\left(\frac{e_t}{e_t - V_0}\right)$ where $e_t = \frac{\mu_0 l I_0 \ln 2}{2\pi t_0}$
2. $\frac{3\sqrt{3}(\sqrt{3} - 1) + 4\pi}{6\sqrt{2}}\alpha r^2$, clockwise.

§ 3.307

➢ **CONCEPT**

In this case emf is induced due to change in area of the loop as well as the change in magnetic field.

Mathematically $$\varepsilon = \frac{d\phi}{dt} = \frac{d(BA)}{dt} = \frac{BdA}{dt} + A\frac{dB}{dt}$$

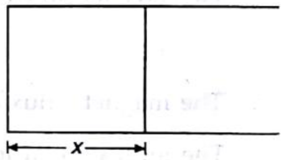


Fig. 3.307A

SOLUTION : The distance travelled by the connected loop in time t is

$$x = \frac{1}{2}wt^2$$

The area of the loop at an instant t is

$$A = xl = \left(\frac{1}{2}wt^2\right)l = \frac{wlt^2}{2}$$

The magnetic field in the loop at instant t is

$$B = \dot{B}t \qquad \therefore \qquad \frac{dB}{dt} = \dot{B}$$

$$\therefore \qquad \varepsilon = B\frac{dA}{dt} + A\frac{dB}{dt}$$

$$= \dot{B}t\frac{d}{dt}\left(\frac{wlt^2}{2}\right) + \frac{wlt^2}{2}\dot{B}$$

$$= \dot{B}tl\,wt + \frac{wlt^2}{2}\dot{B}$$

$$= \dot{B}lwt^2 + \frac{\dot{B}lwt^2}{2} = \frac{3}{2}\dot{B}lwt^2$$

On putting the values, we get $\varepsilon = 12 \times 10^{-3}\,\text{V} = 12\,\text{mV}$

YOUR STEP

Two identical charged balls A and B each of mass m, carrying identical charge q are connected together by a light non-conducting rod of length $l = 1\text{m}$. The system is placed, on a smooth horizontal surface, where there exists a uniform vertical magnetic field of induction 0.2T in a cylindrical region. The axis of the cylindrical region passes through the centre of the rod. A neutral particle C of mass m is placed at a distance $\frac{l}{2}$ from the rod along its perpendicular bisector. The magnetic field decreases to zero at a uniform rate in a time interval of 2×10^{-3} s.

The collision between the balls B and C is perfectly inelastic.
Find
(a) The velocity of centre of mass of A, B and C just after collision.
(b) Angular velocity of the rod about the centre of mass of A, B and C just after collision.
(c) Speed of each ball immediately after collision (given that $\frac{q}{m} = \frac{\pi}{2} \times 10^4$ C/kg)

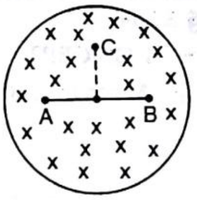


Fig. 3.307B

$$\left\{ \begin{array}{l} \text{(a) zero} \\ \text{(b) 589.05 rad/s} \\ \text{(c) } V_A = 392.7 \text{ m/s}, \ V_B = V_C = 196.35 \text{ m/s} \end{array} \right\}$$

§ 3.308

➢ **CONCEPT**

The magnetic field due to solenoid is $B = \mu_0 nI$

The induced emf in the loop is $\varepsilon = -\frac{d\phi}{dt}$

SOLUTION :

$\therefore$ $$\phi = B\pi a^2 = \mu_0 \, n \, I \, \pi \, a^2 = \pi\mu_0 na^2 I$$

$\therefore$ $$\varepsilon = -\frac{d\phi}{dt} = \pi \, \mu_0 \, n \, a^2 \frac{dI}{dt} = -\pi \, \mu_0 \, a^2 \, n\dot{I}$$

or $$-\int_C \vec{E} \cdot d\vec{t} = -\pi\mu_0 na^2 \dot{I} \qquad \text{(for } r > a\text{)}$$

or $$-E \, 2\pi r = -\pi\mu_0 na^2 \dot{I}$$

$\therefore$ $$E = \frac{\mu_0 na^2 \dot{I}}{2r}$$

when $r < a$, $$-\int_C \vec{E} \cdot dL = -\frac{d\phi}{dt}$$

or $$E \times 2\pi r = \pi r^2 \frac{dB}{dt}$$

$\therefore$ $$E = \frac{r}{2} \frac{d}{dt} (\mu_0 n\dot{I}) = \frac{\mu_0 \, nr \, dl}{2dt} = \frac{\mu_0 nr}{2} \dot{I}$$

YOUR STEP

1. A cylindrically symmetric, time dependent magnetic field confined to a cylindrical region of radius a around the z–axis is given by
$$\vec{B} = C(\tau - t) \, \hat{k}, \rho^2 \le a = 0, \rho^2 > a$$
where c and τ are constants.
A thin, conducting ring is placed in the x–y plane, so that its centre and the origin coincide (shown in the fig. 3.308A).
The radius of the ring is r, its rectangular cross-section is of width d, height l and its specific resistance is σ. Calculate the induced current in the ring in the cases : (a) $r < a$ (b) $r > a$

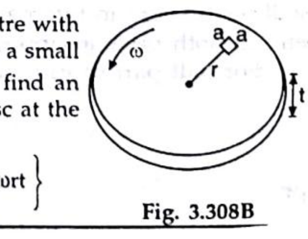


Fig. 3.308A

2. An electromagnetic "eddy current" brake consists of a disk of conductivity σ and thickness t rotating about an axis through its centre with a magnetic field B applied perpendicular to the plane of the disc over a small area a^2 (fig. 3.308B). If the area a^2 is at a distance r from the axis, find an approximate expression for the torque tending to slow down the disc at the instant its angular velocity equals ω.

Fig. 3.308B

$$\left\{ 1. \text{ (a) } I = \frac{Crld}{2\sigma} \text{ for } r < a \quad \text{(b) } I = \frac{a^2 Cld}{2\sigma r} \text{ for } r > a \ 2. \ ((Bar)^2 \ \omega r t \right\}$$

§ 3.309

➢ **CONCEPT**

According to Faraday's law of electron magnetic induction,

$$\varepsilon = \frac{d\phi}{dt}$$

SOLUTION : Here $\phi = BA = \mu_0 nI\pi r^2 = \mu_0 nI\frac{\pi d^2}{4} = \frac{\pi\mu_0 nId^2}{4}$

$\therefore$ $\varepsilon = \frac{d\phi}{dt} = \mu_0 n\pi\frac{d^2}{4}\frac{dI}{dt} = \pi\mu_0 n\frac{d^2}{4}\dot{I}$

The resistance is $R = \frac{\rho l}{S} = \rho\frac{2\pi r}{S} = \rho\frac{2\pi\left(\frac{d}{2}\right)}{S} = \frac{\rho\pi d}{S}$

$\therefore$ The induced current is $I = \frac{\varepsilon}{R} = \frac{\frac{\pi\mu_0 nd^2}{4}\dot{I}}{\frac{\rho\pi d}{S}} = \frac{\mu_0 n\dot{I}Sd}{4\rho}$

On putting the values, we get $I = 2\times 10^{-3}\text{ A} = 2\text{ mA}$

YOUR STEP

In closed circuit shown in fig. 3.309A, AB, BC and CD are straight conductors, each of length R and DEA is a semi circle of radius R. A small circular loop of radius r is coplanar with the circuit and centre of loop coincides with centre of curvature of the semicircle. If current through the circuit increases at a constant rate $\frac{dI}{dt} = \alpha$, calculate emf induced in the loop.

$$\left\{\frac{(\pi + 2\sqrt{3})\,\mu_0\,\alpha\, r^2}{4R}\right\}$$

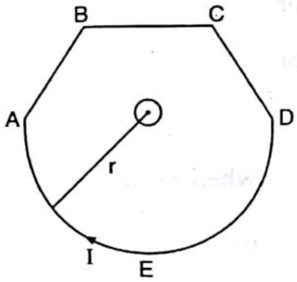


Fig. 3.309A

§ 3.310

➢ **CONCEPT**

The motional emf in the ring is $\xi = \frac{d\phi}{dt} = \frac{d}{dt}(B\,\pi a^2)$ (Here $B = bt$)

$$= \pi a^2\frac{dB}{dt} = \pi\, a^2 b \quad \text{...(i)}$$

Since, induced emf does not depend upon resistance. so, in whole ring induced emf is same.

But $\Delta q = \frac{\Delta\phi}{R}$ $\therefore$ $\Delta q \propto \frac{1}{R}$

From this point of view, transfer of charge depends upon resistance. It means the current will be different in two parts. For symmetry of current, electric field is induced, which produces emf in both parts in opposite directions.

For half part of ring having resistance R_0,

$$\frac{\xi}{2} + \int_0^{\pi a} E.d\vec{L} = IR_0$$

or

$$\frac{\xi}{2} + \int_0^{\pi/a} EdL\cos 180 = IR_0$$

or $$\frac{\xi}{2} - E\pi a = IR_0 \quad ...(ii)$$

Similarly, For second half part, $$\frac{\xi}{2} + E\pi a = \eta IR_0 \quad ...(iii)$$

From equ. (i), (ii) and (iii), we get $$E = \left(\frac{\eta - 1}{\eta + 1}\right)\frac{ab}{2}$$

YOUR STEP

1. A equitaleral triangle of side a is placed in the magnetic field with one side AC along a diameter and its centre coinciding with the centre of the magnetic field [Fig. 3.310A] If the magnetic field varies with time as $B = kt$, Thus,
(a) Show by vectors, the direction of the induced electric field $\vec{E}$ at the three sides of the triangle.
(b) What is the induced emf in side AB?
(c) What is the induced emf in side BC?
(d) What is the induced emf in side CA?
(e) What is the total emf induced in the loop?
(f) Does the above result tally with the induced emf computed using Faraday's law?

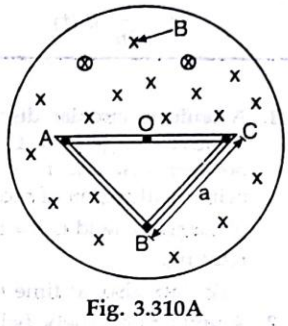


Fig. 3.310A

2. A uniform magnetic field B fills a cylindrical volume of radius R. A metal rod of length L is placed as shown in Fig. 3.310B. If B is changing at the rate $\frac{dB}{dt}$, show that the emf that is produced by the changing magnetic field and that acts between the ends of the rod is given by $\varepsilon = \frac{dB}{dt}\frac{L}{2}\sqrt{R^2 - \left(\frac{L}{2}\right)^2}$

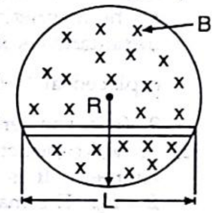


Fig. 3.310B

$$\left\{ \begin{array}{l} 1.\ (b)\ V_{AB} = \frac{\sqrt{3}\,a^2k}{8} \\ (c)\ V_{BC} = \frac{\sqrt{3}\,a^2k}{8} \\ (d)\ V_{CA} = 0 \\ (e)\ V_{\text{loop}} = \frac{\sqrt{3}\,a^2k}{4} \end{array} \right\}$$

§ 3.311

➢ **CONCEPT**

$$\int_C E \cdot d\vec{L} = \pi r^2 \frac{dB}{dt}$$

or $$E\, 2\pi r = \pi r^2 \frac{dB}{dt}$$

$\therefore$ $$E = \frac{r}{2}\frac{dB}{dt} \quad ...(i)$$

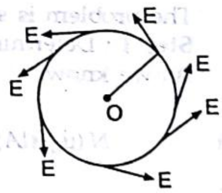


Fig. 3.311A

In vector $$\vec{E} = \frac{1}{2}\left(\vec{r} \times \frac{d\vec{B}}{dt}\right)$$

SOLUTION : The torque due to electric force qE about point O is

$$\tau = qEr$$

or $$mr^2\beta = qEr \qquad (\beta = \text{angular acceleration})$$

or $$mr^2\beta = \frac{qr^2}{2}\frac{dB}{dt} \qquad \text{(from equ. (i))}$$

or $$\beta = \frac{q}{2m}\frac{dB}{dt}$$

But $$\beta = \frac{d\omega}{dt}$$

$\therefore$ $\frac{d\omega}{dt} = \frac{q}{2m} \cdot \frac{dB}{dt}$ or $\int_0^{\omega} d\omega = \frac{q}{2m}\int_0^{B(t)} dB$

or $\omega = \frac{q}{2m} B(t)$ $\therefore$ $\vec{\omega} = -\frac{q}{2m}\vec{B}(t)$ (In vector form)

YOUR STEP

1. A uniform circular disc of mass $m = 0.25$ kg and radius $R = 10$ cm has a uniformly distributed positive charge $q = 4\mu C$. The disc is placed on a rough horizontal floor. The coefficient of friction between disc and floor is $\mu = 0.6$. A uniform vertically downward magnetic field confined in a cylindrical region of radius greater than R and coaxial with the disc exists in the space. Induction of magnetic field is $B = B_0 t^2$ tesla, where t is time in seconds. Calculate time t_0 at which disc starts rotating.
   Calculate also, at time $t\ (> t_0)$, angular velocity of the disc. [$g = 10\ m/s^2$]
2. A uniform magnetic field B is confined to a cylindrical volume of radius r. The magnetic field B is increasing at a constant rate K. Calculate the instantaneous acceleration of a charged particle of mass m and charge q placed at O the centre, B a distance $\frac{r}{2}$ from centre and C a distance $2r$ from the centre. [Fig. 3.311B].

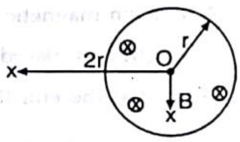


Fig. 3.311B

3. A non-conducting disc of radius R contains a uniformly distributed charge q. It is placed in a uniform magnetic field B which varies as $B = B_0 t$. The magnetic field is oriented perpendicular to the plane of the loop. If the mass of the loop is m, calculate the angular speed of the loop as a function of time.

$$\left\{ \begin{array}{l} 1.\ t_0 = \frac{4\mu\, mg}{3B_0 qR},\ \omega = \frac{B_0 q}{2m}(t^2 - t_0^2) - \frac{4\mu g}{3R}(t - t_0) \\ 2.\ a = \frac{rkq}{4m} \quad 3.\ \omega = \frac{B_0 qt}{2m} \end{array} \right\}$$

§ 3.312

➤ **CONCEPT**

The problem is solved by following steps :

Step I : Determine self inductance of solenoid :

As we know, $$N\phi = LI$$
$$N(BA) = LI$$

or $N(\mu_0 nIA) = LI$ or $N(\mu_0 nI\pi b^2) = LI$

$\therefore$ $$L = \pi\mu_0 Nnb^2 = \pi\mu_0 (nl) nb^2 = \mu_0 n^2 \pi b^2 l$$

Step II : Draw equivalent circuit of solenoid :

The equivalent circuit is shown in fig 3.312 A.

According to loop rule, $$V - \varepsilon' - IR = 0$$
$$V - L\frac{dI}{dt} - IR = 0$$

or $$L\int_0^I \frac{dI}{V - IR} = \int_0^t dt$$

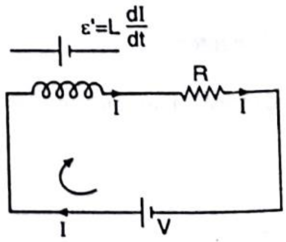


Fig. 3.312A

After solving, we get

$$I = \frac{V}{R}\left(1 - e^{-Rt/L}\right) \quad ...(i)$$

Step III : Determine the induced current in the ring :

The flux associated with ring is $\phi_\rho = B\pi a^2 = \mu_0 n I \pi a^2$

Here B = magnetic field due to solenoid

$$= \mu_0 n I \quad ...(ii)$$

$\therefore$ Induced emf in the ring is $\varepsilon' = \frac{d\phi_r}{dt} = \mu_0 n \pi a^2 \frac{dI}{dt}$

$\therefore$ The current in the ring is $I' = \frac{\varepsilon'}{r} = \frac{\mu_0 n \pi a^2}{r}\frac{dI}{dt}\phi \quad ...(iii)$

From equ. (i), we get

$$\because \quad I = \frac{V}{R}\left(1 - e^{-Rt/L}\right)$$

$$\therefore \quad \frac{dI}{dt} = \frac{V}{L}e^{-Rt/L}$$

$$\therefore \quad I' = \frac{\mu_0 n \pi a^2}{r}\frac{V}{L}e^{-Rt/L} \quad \text{(from equ. (iii))}$$

Step IV : Determine magnetic force on an element of length dl of the ring :

The magnetic force on the considered element is $dF = I'dLB$.

$\therefore$ Force per unit length is $F_0 = \frac{dF}{dL} = I'B$

On putting the value we get, $F_0 = \frac{\mu_0 n \pi a^2 V}{Lr}e^{-Rt/L}(\mu_0 nI)$

$$= \frac{\mu_0^2 n^2 \pi a^2 V}{Lr}e^{-Rt/L}\frac{V}{r}\left(1 - e^{-Rt/L}\right)$$

$$\therefore \quad F_0 = \frac{\mu_0^2 n^2 V^2}{LrR}e^{-Rt/L}\left(1 - e^{-RT/L}\right)$$

For maximum value of F_0, $\frac{dF_0}{dt} = 0$

After solving, we get $F_{0\max} = \left(\frac{dF}{dL}\right)_{\max} = \frac{\mu_0^2 a^2 V^2}{4rRlb^2}$

YOUR STEP

Four uniform wires FG, GH, HI and IF, each of length l and having resistance $R, 2R, 3R$ and $4R$ are connected together to form a square $FGHI$ as shown in fig. 3.312B. The square is placed in a uniform magnetic field whose induction varies with time according to law $B = B_0 t$. Calculate potential difference between F and G.

$$\left\{\frac{3}{20}B_0 l^2\right\}$$

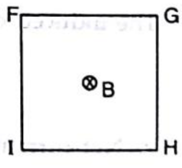


Fig. 3.312B

§ 3.313

➢ **CONCEPT**

The heat generated in resistor is

$$Q = \int I^2 R\,dt$$

SOLUTION : The induced emf is $\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}\{at(\tau - t)\}$

$\therefore$ $\varepsilon = 2at - a\tau$

$\therefore$ $I = \frac{\varepsilon}{R} = \frac{2at - a\tau}{R}$

$\therefore$ $Q = \int_0^{\tau} I^2 R dt = \int_0^{\pi} \frac{(2at - a\tau)^2}{R^2} R\, dt$

After integrating, $Q = \frac{a^2\tau^3}{3R}$

YOUR STEP

A thermocole vessel contains 0.5 kg of distilled water at 30°C. A metal coil of area $5 \times 10^{-3}\ m^2$, number of turns 100, mass 0.06 kg and resistance 1.6 Ω is lying horizontally at the bottom of the vessel. A uniform time varying magnetic field is set up to pass vertically through the coil at time $t = 0$. The field is first increased from zero to 0.8T at a constant rate between 0 and 0.2 sec and then decreased to zero at the same rate between 0.2 and 0.4 sec. The cycle is repeated 12000 times. Make sketches of the current through the coil and the power dissipated in the coil as a function of time for the first two cycles. Clearly indicate the magnitudes of the quantities on the axes. Assume that no heat is lost to the vessel or the surroundings. Determine the final temperature of the water under thermal equilibrium. Specific heat of metal = 500 J/kg-K and the specific heat of water = 4200 J/kg-K. Neglect the inductance of coil.

{θ = 35.6°C}

§ 3.314

SOLUTION : We consider a ring element of radius r and thickness dr (shown in fig. 3.314A).

The area of cross section of considered element is

$$dS = hdr$$

The length of considered element is $l = 2\pi r$

$\therefore$ $dR = \rho \frac{l}{dS} = \rho \frac{2\pi r}{hdr}$

The flux enclosed in considred ring is

$$d\phi = B\pi r^2$$

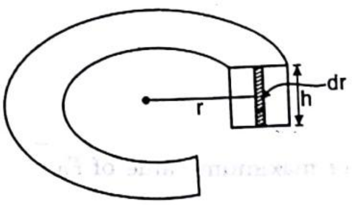


Fig. 3.314A

$\therefore$ Induced emf in the considered ring is

$$d\varepsilon = \pi r^2 \frac{dB}{dt} = \pi r^2 \frac{d}{dt}(\beta t) = \pi r^2 \beta$$

The induced current through considered element is

$$dI = \frac{d\varepsilon}{dR} = \frac{hrdr}{2\rho}\beta$$

$\therefore$ Net current through ring is $I = \frac{h\beta}{2\rho}\int_a^b r\, dr = \frac{h\beta}{4\rho}(b^2 - a^2)$

YOUR STEP

A circular wire of radius a, resistance R is spun in a magnetic field of strength B with angular velocity ω about an axis in the plane of the wire and at right angles to the lines of force. Show that the average rate of dissipation of energy is $\frac{\pi^2 a^4 \omega^2 B^2}{2R}$ (approximately)

§ 3.315

➢ **CONCEPT**

$$N\phi = LI$$

$$\therefore \quad L = \frac{N\phi}{I}$$

SOLUTION : Total length of required wire is

$$l = N\,2\pi r$$

Where N = total number of turns, r = cross-sectional radius of solenoid

But $$n = \frac{N}{l_0}$$

$$\therefore \quad N = nl_0 \quad \therefore \quad l = nl_0\,2\pi r$$

$$\phi = BS = \mu_0\,nI\,\pi r^2$$

$$\therefore \quad N\phi = LI$$

or $$(nl_0)\,\mu_0\,nI\,\pi r^2 = LI$$

$$\therefore \quad n = \frac{1}{r}\sqrt{\frac{L}{\mu_0 l_0 \pi}}$$

$\therefore$ The required length of wire is $l = nl_0\,2\pi r = \sqrt{\dfrac{4\pi\,Ll_0}{\mu_0}}$

On putting the values, we get $l = 100$ m

YOUR STEP

A solenoid 126 cm long is formed from 1870 windings carrying a current of 4.36 A. The core of the solenoid is filled with iron, and the effective permeability constant is 968. Calculate the inductance of the solenoid, assuming that it can be treated as ideal.

{7.87 H}

§ 3.316

➢ **CONCEPT**

The concept is similar to previous problem.

SOLUTION : Let cross-sectional area of wire of solenoid is S.

The total number of turns is $N = nl$

The total length of wire is $l_1 = N\,2\pi r$

$= nl\,2\pi r = 2\pi\,nlr$

The volume of wire is $V = Sl_1$

$$\therefore \quad m = \rho V = \rho S l_1$$

$$\therefore \quad m = \rho S\,(2\pi\,nlr) = 2\pi\,\rho\,Snlr \quad \text{...(i)}$$

Where ρ is density of copper.

$$\therefore \quad R = \rho_0 \frac{l_1}{S}$$ where ρ_0 is resistivity of copper

$$R = \frac{\rho_0\,(2\pi n\,lr)}{S} \quad \text{...(ii)}$$

From concept of previous problem,

$$\because \quad N\phi = LI$$

or $NB\,\pi r^2 = LI$ or $N\,(\mu_0\,nI)\,\pi r^2 = LI$

or $\mu_0 Nn\pi r^2 = L \quad \therefore \quad L = \mu_0 n\pi r^2(nl)$

$$L = \pi\mu_0 n^2 r^2 l \quad \text{...(iii)}$$

From Eq. (i) and (ii), we get $mR = (2\pi \rho S\, nlr) \dfrac{\rho_0 (2\pi\, n\, l\, r)}{S}$

$$n^2 r^2 = \frac{mR}{4\pi^2 \rho \rho_0 l^2}$$

Putting the value of $n^2 r^2$ in Eq. (iii), we get

$$L = \frac{\mu_0\, mR}{4\pi \rho \rho_0 l}$$

> **YOUR STEP**
>
> A solenoid is wound with a single layer of insulated copper wire (diameter, 2.52 mm). It is 4.10 cm in diameter and 2.0 m long. What is the inductance per metre for the solenoid near its centre ? Assume that adjacent wires touch and that insulation thickness is negligble.
>
> [261 μ H/m]

§ 3.317

➢ **CONCEPT**

The problem is an example of rise of current in $L - R$ circuit.

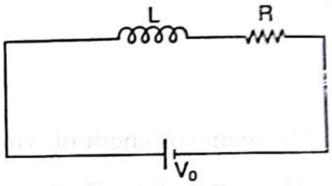


Fig. 3.317A

➢ **DISCUSSION**

At $t = 0$, current in circuit is zero.

When $t > 0$, current starts to rise in circuit. Due to this, inductor opposes rise in current in circuit. After long time, circuit comes in steady state. In steady state, inductor behaves as a resistor of zero resistance. But at $t = 0$, inductor behaves as resistor of infinite resistance.

SOLUTION : At an instant t, the distribution of current in the circuit is shown in fig. 3.317 B.

Accoding to loop rule,

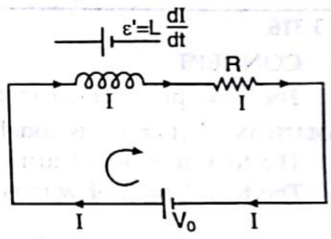


Fig. 3.317B

$$V_0 - L\frac{dI}{dt} - IR = 0$$

or $$L\frac{dI}{dt} = V_0 - IR$$

or $$L\int_0^I \frac{dI}{(V_0 - IR)} = \int_0^t dt$$

or $$-\frac{L}{R}\, ln\, (V_0 - IR) = t$$

or $$I = \frac{V_0}{R}(1 - e^{-tR/L}) \quad \text{or} \quad I = I_0 (1 - e^{-t/\tau}) \qquad ...(i)$$

Here τ = time constant $= \dfrac{L}{R}$

And I_0 = maximum current or steady state current in the circuit.

According to problem, $I = \eta I_0$

From eqn (i), we get $\eta I_0 = I_0 (1 - e^{-t/\tau})$

or $e^{-t/\tau} = 1 - \eta$ or $-\dfrac{t}{\tau} = ln\,(1 - \eta)$

$\therefore$ $$t = \tau\, ln \frac{1}{1-\eta} = \frac{L}{R}\, ln \frac{1}{(1-\eta)}$$

On putting the values, we get $t = 1.49$ second ≈ 1.5 second

YOUR STEP

A metal rod OA mass m and length r is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction $\vec{B}$ is applied perpendicular and into the plane of rotation as shown in fig 3.317C. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.

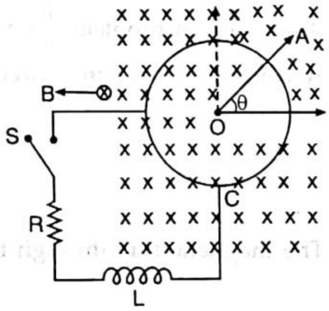


Fig. 3.317C

(a) What is the induced emf across the terminals of the switch ?
(b) The switch S is closed at time $t = 0$,
(i) Obtain an expression for the current as a function of time.
(ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed. Given that the rod OA was along the positive x-axis at $t = 0$.

$$\left\{\begin{array}{l} \text{(a) } e = \dfrac{B\omega r^2}{2} \\ \text{(b) (i) } i = \dfrac{B\omega r^2}{2R}[1 - e^{-(R/L)t}] \\ \quad \text{(ii) } \tau_{ext} = \dfrac{B^2\omega r^4}{4R} + \dfrac{mgr}{2}\cos\omega t \ \{\text{anti-clockwise}\} \end{array}\right\}$$

§ 3.318

➢ CONCEPT

From the discussions of previous problem, time contant is $\tau = \dfrac{L}{R}$

SOLUTION : From the solution of problem 3.316,

$$L = \frac{\mu_0}{4\pi}\frac{mR}{l\rho\rho_0} \qquad \therefore \ \tau = \frac{L}{R} = \frac{\mu_0 m}{4\pi l\rho\rho_0}$$

On putting the vlaues, we get $\tau = 0.7 \times 10^{-3}$ s $= 0.7$ ms

YOUR STEP

1. A uniform magnetic field B is changing in magnitude at a constant rate dB/dt. You are given a mass m of copper which is to be drawn into a wire of radius r and formed into a circular loop of radius R. Show that the induced current in the loop does not depend on the size of the wire or of the loop and, assuming B perpendicular to the loop, is given by $i = \dfrac{m}{4\pi\rho\delta}\dfrac{dB}{dt}$, where ρ is the resistivity and δ the density of copper.
2. Show that the inductive time constant τ_L can also be defined as the time that would be required for the current in an LR circuit to reach its equilibrium value if it continued to increase at its initial rate.

§ 3.319

➢ CONCEPT

For calculation of inductance, the relation $\phi = LI$ is applicable.

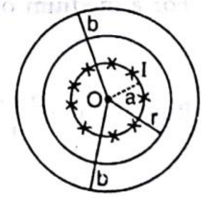


Fig. 3.319A

SOLUTION :

According to problem, $\frac{b}{a} = \eta$

According to Ampere's circuital law, $\int_C \vec{B}\cdot d\vec{l} = \mu_0 \mu I$

$$B \times 2\pi r = \mu_0 \mu I$$

$\therefore$
$$B = \frac{\mu_0 \mu I}{2\pi r}$$

The magnetic flux through thickness dr and length l (along the length of the cable) is

$$d\phi = BdS = Bldr = \frac{\mu_0 \mu I l dr}{2\pi r}$$

The net flux is
$$\phi = \int_a^b \frac{\mu_0 \mu I l dr}{2\pi r} = \frac{\mu_0 \mu I l}{2\pi}[ln r]_a^b$$

$$= \frac{\mu_0 \mu I l}{2\pi} ln \frac{b}{a}$$

$\therefore$
$$L = \frac{\phi}{I} = \frac{\mu_0 \mu \, l \, ln \frac{b}{a}}{2\pi}$$

Hence, inductance per unit length is
$$\frac{L}{l} = \frac{\mu_0 \mu \, ln \frac{b}{a}}{2\pi}$$

$$\frac{L}{l} = \frac{\mu_0 \mu}{2\pi} ln \, \eta$$

On putting the values, we get
$$\frac{L}{l} = 0.26 \times 10^{-6} = 0.26 \, \mu H / m$$

YOUR STEP

A long wire carries a current i uniformly distributed over a cross section of the wire.

(a) Show that the magnetic energy of a length l stored within the wire equals $\frac{\mu_0 i^2 l}{16\pi}$.

(b) Show that the inductance for a length l of the wire associated with the flux inside the wire is $\frac{\mu_0 l}{8\pi}$

§ 3.320

➢ **CONCEPT**

The concept is similar to previous problem. The shape of doughnut solenoid is shown in Fig. 3.65 of the problem 3.242 of the book, I.E. Irodov (see page 139).

SOLUTION : From the solution of problem 3.242 of the book,

$$\phi = \frac{\mu_0 (2I Nh) \, ln \eta}{4\pi}$$

For a medium of relative magnetic permeability μ,

$$\phi = \frac{\mu_0 \, \mu \, (2I \, Nh) \, ln \, \eta}{4\pi}$$

But $\eta = \frac{b+a}{b} = 1 + \frac{a}{b}$ and $h = a$

$\therefore$
$$\phi = \frac{\mu_0 \, \mu \, (2I \, Na) \, ln\left(1 + \frac{a}{b}\right)}{4\pi}$$

$\because \quad N\phi = LI \quad \therefore \quad L = \dfrac{N\phi}{I}$

$$\therefore \quad L = \frac{\mu_0 \mu N^2 a \ln\left(1+\dfrac{a}{b}\right)}{2\pi}$$

YOUR STEP

Show that the self inductance L of a torus of radius R containing N loops each of radius r is

$$L \approx \frac{\mu_0 N^2 r^2}{2R}$$

If $R >> r$. Assume the field is uniform inside the torus ; is this actually true? Is this result consistent with L for a solenoid ? Should it be? (Yes)

§ 3.321

➢ CONCEPT

The magnetic field due to large plane carrying linear current density j is $\dfrac{\mu_0 i}{2}$.

Also energy stored per unit volume in a magnetic field is

$$u = \frac{B^2}{2\mu_0}$$

SOLUTION : The shape of system is just like parallel plate capacitor (shown in fig 3.321).
The magnetic field between plates is

$$B = \frac{\mu_0 i}{2} + \frac{\mu_0 i}{2} = \mu_0 i$$

The width of plate is b.

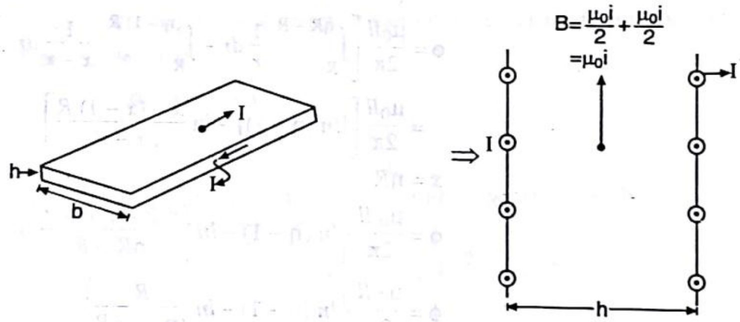


Fig. 3.321A

$\therefore$ Linear density of current is $\quad i = \dfrac{I}{b}$

$$\therefore \quad B = \frac{\mu_0 I}{b}$$

The energy stored per unit volume between planes of tapeline is

$$u = \frac{B^2}{2\mu_0} = \frac{\mu_0^2 I^2}{2b^2 \mu_0} = \frac{\mu_0 I^2}{2b^2}$$

Let the length of each plate is l.

$\therefore$ Volume of space between plates is $V = bhl$

$\therefore$ Total energy stored between plates is $U = uV = \dfrac{\mu_0 I^2}{2b^2}(bhl) = \dfrac{\mu_0 I^2 hl}{2b}$

But energy stored in inductor is $\quad U = \dfrac{1}{2} LI^2$

$\therefore \quad \dfrac{1}{2} LI^2 = \dfrac{\mu_0 I^2 hl}{2b} \qquad \therefore \quad L = \dfrac{\mu_0 h}{b} l$

$\therefore \quad \dfrac{L}{l}$ = inductance per unit length $= \dfrac{\mu_0 h}{b}$

On putting the values, $\quad \dfrac{L}{l} = 25$ nH/m

YOUR STEP

Show that the inductive time constant τ can also be defined as the time would be required for the current in an LR circuit to reach its equilibrium value if it continued to increase at its initial rate.

§ 3.322

➢ **CONCEPT**

The concept is similar to previous problem.
The magnetic field at P (shown in fig. 3.322 A) is

$$B = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2\pi (x - r)}$$

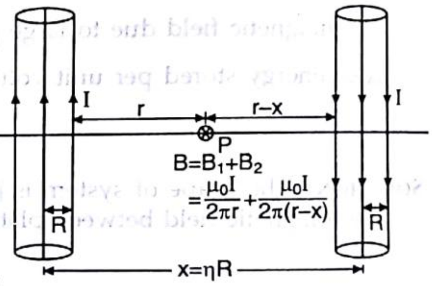


Fig. 3.322A

$\therefore$ Energy density is $\quad u = \dfrac{B^2}{2\mu_0}$

$$\therefore \quad d\phi = \left\{\frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2\pi (x - r)}\right\} l dr$$

$$\therefore \quad \phi = \frac{\mu_0 Il}{2\pi}\left[\int_R^{\eta R - R} \frac{1}{r} dr + \int_R^{(\eta - 1)R} \frac{1}{x - r} dr\right]$$

$$= \frac{\mu_0 Il}{2\pi}\left[[ln(\eta - 1)] - ln\frac{x - (x-1)R}{x - R}\right]$$

But $\quad x = \eta R$

$$\therefore \quad \phi = \frac{\mu_0 Il}{2\pi}\left\{ln(\eta - 1) - ln\left(\frac{\eta R - \eta R + R}{\eta R - R}\right)\right\}$$

$$\therefore \quad \phi = \frac{\mu_0 Il}{2\pi}\left\{ln(\eta - 1) - ln\frac{R}{(\eta - 1)R}\right\}$$

$$= \frac{\mu_0 Il}{2\pi}\left\{ln\,\eta - ln\frac{1}{\eta}\right\} \qquad (\because \eta >>> 1)$$

$$= \frac{\mu_0 Il}{2\pi}\{ln\,\eta + ln\,\eta\} = \frac{\mu_0 Il}{2\pi} 2 ln\,\eta = \frac{\mu_0 Il\, ln\,\eta}{\pi}$$

But $\quad \phi = LI$

$$\therefore \quad L = \frac{\phi}{I} = \frac{\mu_0 l ln\eta}{\pi}$$

$\therefore$ Inductance per unit length is $\quad \dfrac{L}{l} = \dfrac{\mu_0\, ln\,\eta}{\pi}$

YOUR STEP

Show that self inductance for a length l of a long wire associated with the flux inside the wire only is $\frac{\mu_0 l}{8\pi}$, independent of the wire diameter.

§ 3.323

➢ CONCEPT

The resistance of super conducting wire is zero since, ring is made of super conducting material. So, resistance of the ring is zero. Hence, ring behaves as a pure inductor.

➢ DISCUSSION

Initially the external magnetic field B and area vector of ring are in same direction. So maximum flux (*i.e.* $\phi_i = B\pi a^2$) is passing through ring. When ring starts to rotate, so external flux due to external magnetic field starts to decrease. Due to this emf generated in the circuit. This emf behaves as an external voltage source. Due to this emf, current starts to increase in the ring.

When current starts to increase in the circuit, self emf is induced in the circuit which opposes the increase in current in the circuit.

SOLUTION : (a) The system behaves as a circuit of pure inductor having a voltage source, (induced emf due to external magnetic field).

The equivalent circuit at an instant t is shown in figure, 3.323A

Applying loop rule in the circuit at an instant t,

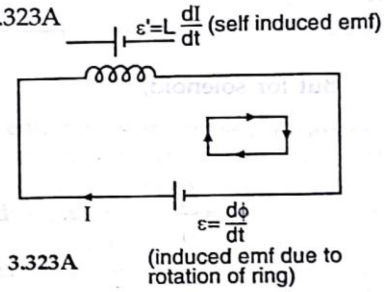


Fig. 3.323A

$$\varepsilon - \varepsilon' = 0$$

$$\frac{d\phi}{dt} - L\frac{dI}{dt} = 0$$

or $\quad \frac{d\phi}{dt} = L\frac{dI}{dt} \quad$ or $\quad \frac{\Delta\phi}{\Delta t} = L\frac{\Delta I}{\Delta t}$

or $\quad \Delta\phi = L\,\Delta I \quad$ or $\quad |\phi_f - \phi_i| = L\,(I_f - I_i)$

Hence, $\quad \phi_i = B\pi a^2 \quad I_i = 0$

Find area vector of ring becomes perpendicular to external magnetic field B.

$\therefore \quad \phi_f = B\pi a^2 \cos 90° = 0 \quad \therefore \quad I_f = I$

$$\therefore \quad |\phi_f - \phi_i| = L\,(I_f - I_i)$$

or $\quad |0 - B\pi a^2| = L\,(I - 0) \quad \therefore \quad I = \frac{B\pi a^2}{L}$

(b) According to conservation principle of energy, work done by external agent should be equal to increase in energy of inductor.

$$\therefore \quad A_{ext} = U_f - U_i = \frac{1}{2}LI_f^2 - \frac{1}{2}LI_i^2$$

$$= \frac{1}{2}LI^2 - 0 = \frac{1}{2}LI^2$$

On putting the values of L and I, we get

$$A_{ext} = \frac{\pi^2 a^4 B^2}{2L}$$

YOUR STEP

A copper disc A in fig. 3.323B has a radius of $r = 5$ cm and its plane is perpendicular to the direction of a magnetic field with an induction of $B = 0.2$ T. A current of $I = 5$A passes along radius ab of the disc (a and b are sliding contacts). The disc rotates with a frequency of $\nu = 3$ rev/s.

Find

(a) the power of such a motor

(b) The direction of rotation of the disc if the magnetic field is directed towards us from the drawing
(c) the torque acting on the disc.

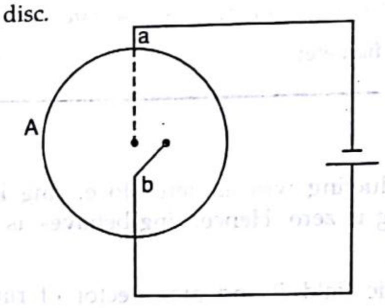


Fig. 3.323B

{(a) 2.36×10^{-2} W (b) Anti-clockwise (c) 12.5×10^{-4} Nm}

§ 3.324

➢ CONCEPT

In this problem, flux does not change

$\therefore$ $\phi = LI = \text{constant}$

$\therefore$ $L_1 I_1 = L_2 I_2$

But for solenoid, $L = \dfrac{\mu_0 \mu N^2 s}{l}$

$\therefore$ $L \propto \dfrac{1}{l}$ (l = length of solenoid)

$\therefore$ $\dfrac{L_1}{L_2} = \dfrac{l_2}{l_1}$ $\therefore$ $L_1 I_1 = L_2 I_2$

$\therefore$ $I_2 = \dfrac{L_1 I_1}{L_2} = \dfrac{l_2}{l_1} I_1$

According to problem, $l_2 = (l_1 + \eta l_1) = (\eta + 1)\, l_1$

and $I_1 = I_0$

$\therefore$ $I_2 = \dfrac{(\eta + 1)\, l_1}{l_1} I_0 = (\eta + 1)\, I_0 = 2A$

YOUR STEP

An iron core solenoid of length l and cross-sectional area A having N turns on it is connected to a battery to a resistance as shown in the figure 3.324A. At instant $t = 0$, the iron rod of permeability μ from the core is abruptly removed.
Find the current as function of time.

$$\left\{\frac{\varepsilon}{R}\left[1 + (\mu - 1)\, e^{-tR/L}\right]\right\}$$

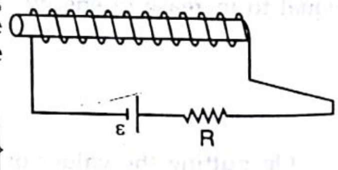


Fig. 3.324A

§ 3.325

➢ CONCEPT

When ring comes in super conducting state, the resistance of ring becomes zero. But no effect is found on the magnetic flux.

SOLUTION : Here $\dfrac{d\phi}{dt} = L\dfrac{dI}{dt}$

or $d\phi = L\, dI$

$\therefore \qquad \Delta\phi = L\Delta I$

Here $\quad \phi_i = B\pi a^2 \quad$ (when field is switched off) $\quad \phi_f = 0$

$\therefore \quad |\Delta\phi| = B\pi a^2 \qquad I_i = 0, \qquad I_f = I.$

$\therefore \qquad \Delta\phi = L\Delta I$

$\therefore \qquad B\pi a^2 = I\mu_0 a\left(ln\dfrac{8a}{b} - 2\right)$

$\therefore \qquad I = \dfrac{\pi a B}{\mu_0\left(ln\dfrac{8a}{b} - 2\right)} = 50\text{A}$

YOUR STEP

A toroidally shaped inductor has a mean radius of R and a cross-sectional area A. The shape is such that $B \approx \dfrac{\mu_0 I}{2\pi R}$ is approximately constant throughout the volume of inductor. It would with two coils, one with N_1, turns and the other on top of the first with N_2 turns. Find the mutual inductance between the two windings.

$\left\{\dfrac{\mu_0 N_1 N_2 A}{2\pi R}\right\}$

§ 3.326

> **CONCEPT**

For discussion of problem, magnetic flux is analogous to momentum ($P = mv$) mass is measure of inertia of a body. But inductance is measure of inertia of electric circuit. So, inductance L is analogous to mass. Similarly, current plays same role in electric circuit as speed in mechanics. In a mechanical problem, if mass of a body is abruptly decreased η times. For maintaining constant momentum, velocity of body abruptly increases by the same factor. In similar fashion, if inductance was abruptly decreased η times, current was abruptly increased η times for maintaining constant magnetic flux. Mathematically,

$$\phi = LI_1 = \frac{L}{\eta} I_0 \qquad \therefore \quad I_0 = \eta I_1$$

SOLUTION : The problem is solved by following steps.

Step I : Discuss the circuit before changing inductance :

From the statement of problem, before changing inductance of the inductor, the circuit of choke coil is in steady state. In steady state, current I_1 in the circuit is constant and no induced emf is generated in the inductor (shown in fig 3.326A). According to loop rule,

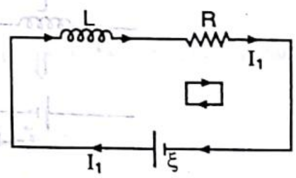


Fig. 3.326A

$$\xi - I_1 R = 0 \quad \therefore \qquad I_1 = \frac{\xi}{R} \qquad (1)$$

Step II : Discuss the circuit after changing the inductance :

At $t = 0$, when inductance was abruptly decreased η times, current in the circuit is $I_0 = \eta I_1 = \dfrac{\eta\varepsilon}{R}$.

But after long time, again circuit comes in steady state. When circuit comes in steady state, inductor bechaves as short circuit. At this state current in circuit again becomes $I_1 = \dfrac{\xi}{R}$

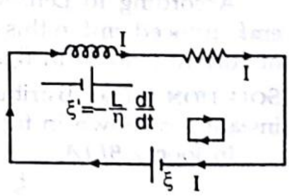


Fig. 3.326B

From this discussion, we can say that the current starts to decrease from $I_0 = \dfrac{\eta\varepsilon}{R}$ (at $t = 0$) to $I_1 = \dfrac{\varepsilon}{R}$ (at $t \to \infty$)

The distribution of current in circuit at an instant t is shown in the fig. 3.326B.

In the circuit, $$\varepsilon' = -\frac{L}{\eta}\frac{dI}{dt} \quad \left(\text{since, current decreases, } \therefore \frac{\Delta I}{\Delta t} = -\frac{dI}{dt}\right)$$

According to loop rule, $$\xi + \xi' - IR = 0$$

$$\varepsilon - \frac{L}{\eta}\frac{dI}{dt} - IR = 0$$

or $$\frac{L}{\eta}\frac{dI}{dt} = \varepsilon - IR$$

or $$\frac{L}{\eta}\int_{I_0 = \frac{\eta\varepsilon}{R}\xi}^{I} \frac{dI}{\xi - IR} = \int_0^t dt$$

After integrating, we get $$I = \frac{\xi}{R}\left\{1 + (\eta - 1)\, e^{-\eta Rt/L}\right\}$$

YOUR STEP

A Wheatstone bridge is adjusted so that there is no permanent current through the galvanometer. One of the arms whose resistance is P contains a coil of inductance L. If the resistance of the galvanometer is negligible, prove that when the circuit of battery is broken. A quantity $\xi\, Q\, L/[(P+Q)\{(P+Q)B + (P+R)Q\}]$of electricity will flow through the galvanometer.

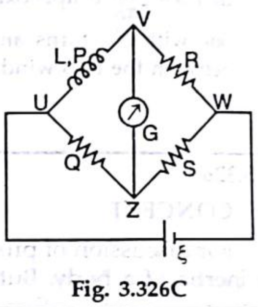


Fig. 3.326C

§ 3.327

➢ **CONCEPT**

If current passing through an inductor increases, emf induced in inductor opposes the current (shown in fig 3.327 A). It means induced emf is in opposite direction of flow direction of current.

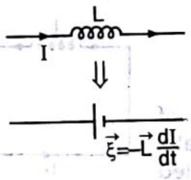


Fig. 3.327A

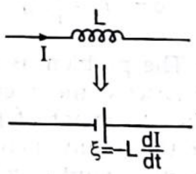


Fig. 3.327B

If current passing through an inductor decreases with variation of time.

Then $$\lim_{\Delta t \to 0} \frac{\Delta I}{\Delta t} = -\frac{dI}{dt}.$$

According to Lenz's law, the direction of emf. induced emf in this case is in the direction of current (shown in fig. 3.327 B).

SOLUTION : The distribution of current at an instant t is shown in fig 3.327C.

In loop $ABEFA$,

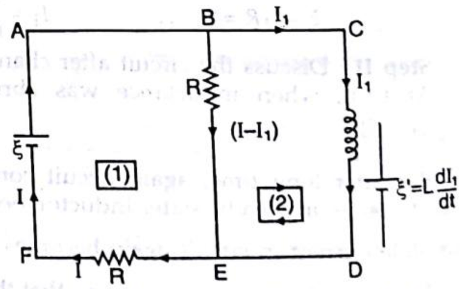


Fig. 3.327C

$$\xi - (I - I_1)R - IR = 0$$

or $$\xi - IR - I_1R - IR = 0$$

$$\therefore \qquad I = \frac{\xi - I_1 R}{2R} \qquad ...(1)$$

In loop $BCDEB$, $\quad -\xi' + (I - I_1)R = 0$

or $\quad -L\dfrac{dI_1}{dt} + IR - I_1 R = 0$

or $\quad -L\dfrac{dI_1}{dt} + \left(\dfrac{\xi - I_1 R}{2R}\right)R - I_1 R = 0$

or $\quad -L\dfrac{dI_1}{dt} + \dfrac{\xi}{2} - \dfrac{I_1}{2}R - I_1 R = 0$

or $\quad -L\dfrac{dI_1}{dt} + \dfrac{\xi}{2} - \dfrac{3I_1 R}{2} = 0$

or $\quad \dfrac{\xi - 3I_1 R}{2} = L\dfrac{dI_1}{dt}$

or $\quad 2L\displaystyle\int_0^{I_1} \frac{dI_1}{\xi - 3I_1 R} = \int_0^t dt$

After integrating, we get $\quad I_1 = \dfrac{\xi}{R}(1 - e^{-tR/2L})$

YOUR STEP

A coil of resistance R and self inductance L is joined to a battery of emf ε. Show that the current after a time b is $(1 - x)\dfrac{\varepsilon}{R}$ and that if contact be then broken, the current after a further time t is $\dfrac{(x - x^2)\,\varepsilon}{R}$, where $L \ln x + Rt = 0$

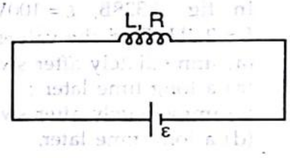


Fig. 3.327D

§ 3.328

➢ **CONCEPT**

For the sake of convenience, the circuit can be represented into two parts (shown in fig. 3.328A).

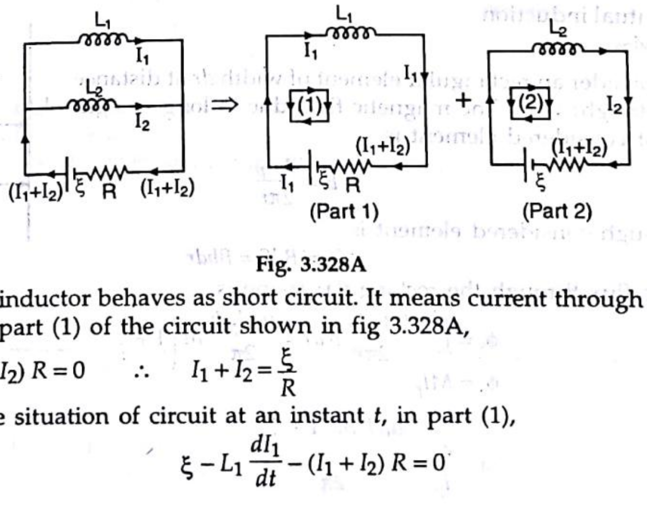


Fig. 3.328A

In steady state, inductor behaves as short circuit. It means current through inductor becomes constant. From the part (1) of the circuit shown in fig 3.328A,

$$\xi - (I_1 + I_2)R = 0 \qquad \therefore \qquad I_1 + I_2 = \frac{\xi}{R} \qquad ...(i)$$

We consider the situation of circuit at an instant t, in part (1),

$$\xi - L_1\frac{dI_1}{dt} - (I_1 + I_2)R = 0$$

$$\therefore \quad L_1 \frac{dI_1}{dt} = \xi - (I_1 + I_2) R \quad ...(ii)$$

Similarly, in part (2),

$$\xi - L_2 \frac{dI_2}{dt} - (I_1 + I_2) R = 0$$

$$\therefore \quad L_2 \frac{dI_2}{dt} = \xi - (I_1 + I_2) R \quad ...(iii)$$

From eqn (ii) and (iii), we get

$$L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt}$$

or

$$L_1 \int_0^{I_1} dI_1 = L_2 \int_0^{I_2} dI_2$$

$$\therefore \quad L_1 I_1 = L_2 I_2 \quad ...(iv)$$

After solving eqn (i) and (iv), we get, $I_1 = \dfrac{\xi L_2}{R(L_1 + L_2)}$

and $I_2 = \dfrac{\xi L_1}{R(L_1 + L_2)}$

YOUR STEP

In fig. 3.328B, $\varepsilon = 100\text{V}$, $R_1 = 10\Omega$, $R_2 = 20\Omega$, $R_3 = 30\Omega$ and $L = 2.0\text{H}$. Find the values of i_1 and i_2

(a) immediately after switch S is closed ;
(b) a long time later ;
(c) immediately after switch S is opened again ;
(d) a long time later,

{(a) $i_1 = i_2 = 3.33$ A (b) $i_1 = 4.55$ A, $i_2 = 2.73$ A
(c) $i_1 = 0$, $i_2 = 1.82$ A (d) $i_1 = i_2 = 0$}

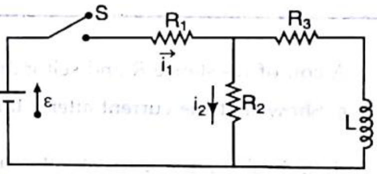


Fig. 3.328B

§ 3.329

➢ **CONCEPT**

$$\phi_s = MI_p$$

whre M = Mutual induction
Also, $M_{12} = M_{21}$.

SOLUTION : We consider an rectangular element of width dr at distance r from the long straight wire. The magnetic field due to long straight wire at the site of considered element is

$$B = \frac{\mu_0 I_p}{2\pi r}$$

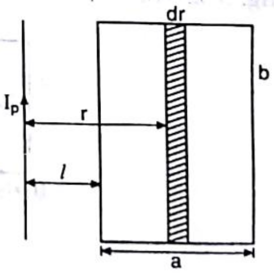


Fig. 3.329A

The flux through considered element is

$$d\phi = BdS = Bbdr$$

The magnetic flux through the rectangular frame is

$$\phi_s = \int_l^{l+a} \frac{\mu_0 I_p}{2\pi r} b\, dr = \frac{\mu_0 I_p b}{2\pi} ln\left(1 + \frac{a}{l}\right)$$

$$\therefore \quad \phi_s = MI_p$$

$$M = \frac{\phi_s}{I_p} = \frac{\mu_0 b\, ln\left(1 + \frac{a}{l}\right)}{2\pi}$$

YOUR STEP

Calculate the mutual inductance between the toroid and rectangular loop described in fig 3.329B.

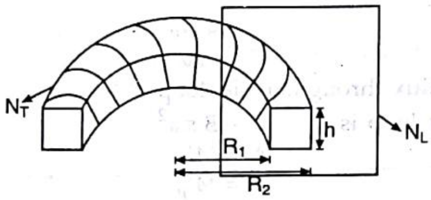


Fig. 3.329B

$$\left\{\frac{\mu_0 N_T N_L h \ln\left(\frac{R_2}{R_1}\right)}{2\pi}\right\}$$

§ 3.330

➢ **CONCEPT**

$N\phi_s = MI_p$ formula is applicable for solving the problem.

SOLUTION : We consider an element of thickness dr of the coil at distance r from the long wire (from axis of the coil).

The area of considered element is $dS = hdr$

$\therefore \quad d\phi = Bds = \frac{\mu_0 \mu I_p}{2\pi r} hdr$

$\therefore \quad \phi_s = \frac{\mu_0 \mu I_p h}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 \mu I_p h}{2\pi} \ln \frac{b}{a}$

$\because \quad N\phi_s = MI_p$

$\therefore \quad M = \frac{N\phi_s}{I_p} = \frac{\mu_0 \mu Nh \ln \frac{b}{a}}{2\pi}$

Hence $\quad M_{12} = M_{21} = M$

YOUR STEP

The solenoids S_1 and S_2 are co-axial as shown in fig 3.330A. S_1 is very long and contains N_1 circular coils per unit length. Its radius is r_1. S_2 is very short in comparison with S_1, and has a total of N_2 tightly wound circular coils, where $N_2 << N_1$.

(a) calculate the mutual inductance if $r_1 > r_2$.
(b) calculate the mutual inductance if $r_2 > r_1$.
(c) calculate the self inductance per unit length of the long solenoid S_1.

$$\left\{\begin{array}{l}\text{(a) } M_{21} = \mu_0 \pi r_2^2 N_1 N_2 \\ \text{(b) } M_{21} = \mu_0 \pi r_1^2 N_1 N_2 \\ \text{(c) } \mu_0 \pi r_1^2 N_1^2\end{array}\right\}$$

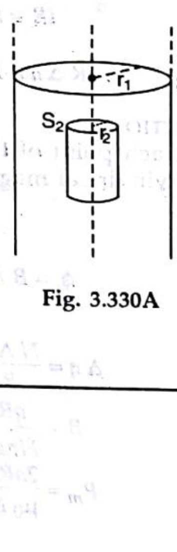


Fig. 3.330A

§ 3.331

➢ **CONCEPT**

The problem is similar to previous problem.

SOLUTION : The problem is solved by following steps :

Step I : Determine magnetic field due to larger loop at the centre of smaller loop : Now magnetic field due to larger loop at centre of smaller loop is

$$B = \frac{\mu_0 I_p}{2b}$$

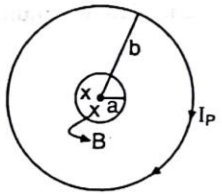


Fig. 3.331A

Step II : Determine the flux through smaller loop :

The flux through smaller loop is $\phi_s = B\,\pi\,a^2$

Step III : Use $\phi_s = MI_p$

$\therefore$ $\phi_s = MI_p$

$$B\,\pi\,a^2 = MI_p$$

or $\frac{\mu_0 I_p}{2b}\pi a^2 = MI_p$ $\quad\therefore\quad M = \frac{\pi\,\mu_0\,a^2}{2b}$

(a) Here $M_{12} = M_{21} = M = L_{12} \approx \frac{\pi\mu_0 a^2}{2b}$

(b) $\phi_{21} = MI = \frac{\pi\mu_0 a^2 I}{2b}$

YOUR STEP

There are two coils co-axially displaced by a distance x. The radius of the coils are a and b respectively ($b >> a$ and $x > b$). In the coil (2), a current i was flown for a moment. Find the charge flow through the coil (1) in that time. The resistance per unit length of coil (1) is λ.

$$\left\{\frac{\mu_0\, i\, ab^2}{4\lambda\,(b^2 + x^2)^{3/2}}\right\}$$

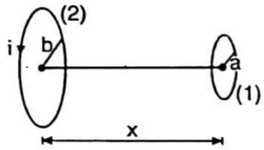


Fig. 3.331B

§ 3.332

➢ **CONCEPT**

According to Faraday's law,

$$\varepsilon = \frac{N d\phi}{dt}$$

or $\quad iR = N\frac{d\phi}{dt} \quad$ or $\quad R\frac{dq}{dt} = N\frac{d\phi}{dt}$

or $\quad R\,\Delta q = N\,\Delta\phi \quad \therefore \quad \Delta q = \frac{N\,\Delta\phi}{R}$

SOLUTION :

Each point of the coil is on the axial position of the magnet. So, the magnetic field due to the cylindrical magnet on the surface of coil is

$$B = \frac{\mu_0 P_m}{2\pi a^3}$$

$\therefore \quad \phi_i = B\,\pi a^2 \qquad \phi_f = 0 \qquad$ (since, magnet is shifted to infinity)

$\therefore \quad \Delta\phi = |\phi_f - \phi_i| = B\,\pi a^2$

$\therefore \quad \Delta q = \frac{N\,\Delta\phi}{R} \quad$ or $\quad q = \frac{NB\,\pi\,a^2}{R}$

$\therefore \quad B = \frac{qR}{N\pi a^2} \quad$ or $\quad \frac{\mu_0 P_m}{2\pi\,a^3} = \frac{qR}{N\,\pi a^2}$

$\therefore \quad P_m = \frac{2aRq}{\mu_0 N}$

YOUR STEP

An infinitesimally small bar magnet of dipole moment $\vec{M}$ is pointing and moving with the speed v in the x-direction. A small closed circular conducting loop of radius a and negligible self inductance lies in the $y-z$ plane with its centre at $x=0$ and its axis coinciding with the x-axis. Find the force opposing the motion of the magnet, if the resistance of loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a.

$$\left\{\frac{9\mu_0^2 M^2 a^4 v}{4x^8 R}\right\}$$

§ 3.333

➢ **CONCEPT**

The magnetic field due to 1st coil at the centre of second coil is

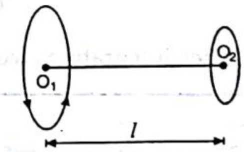


Fig. 3.333A

$$B = \frac{\mu_0 I a^2}{2(l^2 + a^2)^{3/2}} \quad \text{(from solution of problem 3.319)}$$

For $\quad l >> a,\ l^2 + a^2 \approx l^2$

$$\therefore \quad B = \frac{\mu_0 a^2 I}{2l^3}$$

SOLUTION : Magnetic flux through second coil is

$$\phi = B\,\pi a^2$$

But $\quad \phi = MI \qquad \therefore\ M = \frac{\phi}{I}$

On putting the values, we get $M_{12} = M_{21} = M = \dfrac{\mu_0 \pi a^4}{2l^3}$.

YOUR STEP

Two long cylindrical coils with uniform windings of the same length and nearly the same radius have inductances L_1 and L_2. The coils are co-axially inserted into each other and connected to a current source, as shown in fig. 3.333B. The directions of the current in the circuit and in the turns are shown by arrows. Determine the inductance L of such a composite coil.

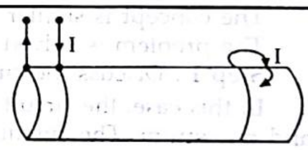


Fig. 3.333B

$$\{ L = L_1 + L_2 + 2\sqrt{L_1 L_2} \}$$

§ 3.334

➢ **CONCEPT**

The problem is solved by following steps :

Step I : Determine mutually induced emf due to first loop in the second loop is

$$\varepsilon_{21} = M_{21}\frac{dI_1}{dt}$$

$$= L_{21} = \frac{d(\alpha t)}{dt} = L_{21}\,\alpha = L_{12}\,\alpha \qquad (\because\ L_{12} = L_{21})$$

Emf ε_{21} behaves as external source for second loop.

Step II : Draw equivalent circuit for second loop : Due to mutually induced emf ε_{21}, current starts to flow through second loop. When current in second loop increases, self emf is induced in second loop due to self inductance of the second loop. This induced emf opposes the flow of current in the loop.

The equivalent circuit at an instant t is shown in fig 3.334A.

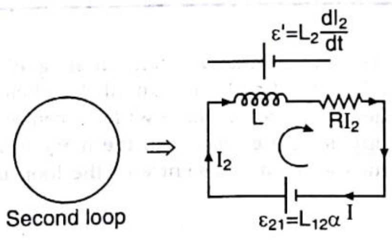


Fig. 3.334A

According to loop rule,

$$\varepsilon_{21} - \varepsilon' - I_2 R = 0$$

or
$$L_{12}\alpha - L_2 \frac{dI_2}{dt} - RI_2 = 0$$

or
$$L_{12}\alpha - I_2 R = L_2 \frac{dI_2}{dt}$$

or
$$L_2 \int_0^{I_2} \frac{dI_2}{L_{12}\alpha - I_2 R} = \int_0^t dt$$

After integrating, we get, $I_2 = \frac{L_{12}\alpha}{R}(1 - e^{-Rt/L_2})$

YOUR STEP

Two inductors of self inductances L_1 and L_2 and of resistances R_1 and R_2 (not shown here) respectively, are connected in the circuit as shown in the fig. 3.334B. At the instant $t = 0$, key k is closed, obtain an expression for which the galvanometer will show zero deflection at all times after the key is closed.

$$\left\{\frac{L_1}{L_2} = \frac{R_3}{R_4}\right\}$$

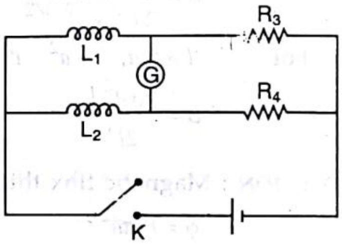


Fig. 3.334B

§ 3.335

➢ **CONCEPT**

The concept is similar to problem 3.327.

The problem is solved by following steps :

Step I : Discuss the circuit before connection of resistance R_0 :

In this case, the circuit is in steady state and current is constant and maximum. The circuit is shown in fig. 3.335A.

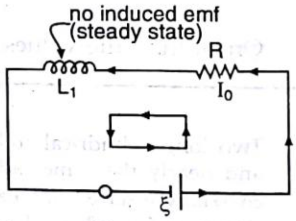


Fig. 3.335A

According the loop rule, $\xi - I_0 R = 0$

$\therefore$
$$I_0 = \frac{\xi}{R}$$

Step II : Discuss after connection of resistance R_0. In this case, battery is disconnected. So, current starts to decrease through inductor. After long time ($t \to \infty$), current becomes zero. On the basis of energy conservation principle, total magnetic energy stored on capacitor appears as thermal energy in resistors R of resistances R and R_0 (in series).

Mathematically,
$$\frac{1}{2} L I_0^2 = \Delta H$$

or
$$\Delta H = \frac{1}{2} L \left(\frac{\xi}{P}\right)^2 = \frac{L\xi^2}{2R^2} \quad \text{...(i)}$$

But
$$\Delta H = \Delta H_1 + \Delta H_2 \quad \text{...(ii)}$$

Here ΔH_1 = Heat generated in resistance R (in the coil)

and ΔH_2 = Heat generated in resistance R_0.

Since, resistances R and R_0 are in series. So, current through R and R_0 are same.

But,
$$\Delta H = I^2 R \, dt$$

$\therefore \quad \Delta H_1 \propto R \quad$ and $\quad \Delta H_2 \propto R_0$

$\therefore \quad \dfrac{\Delta H_1}{\Delta H_2} = \dfrac{kR}{kR_0} = \dfrac{R}{R_0} \quad \therefore \quad \Delta H_2 = \dfrac{\Delta H_1 R_0}{R}$ ...(iii)

$\because \quad \Delta H = \Delta H_1 + \Delta H_2 \quad$ or $\quad \Delta H = \Delta H_1 + \dfrac{\Delta H_1 R_0}{R}$

or
$$\frac{L\varepsilon^2}{2R^2} = \Delta H_1\left(1 + \frac{R_0}{R}\right)$$

$\therefore$
$$\Delta H_1 = \frac{L\xi^2}{2R^2\left(1+\dfrac{R_0}{R}\right)} = \frac{L\xi^2}{2R\,(R+R_0)}$$

On putting the values, $\quad \Delta H_1 = 3 \times 10^{-6}\ \text{J} = 3\ \mu\text{J}$

YOUR STEP

In the circuit shown in fig 3.335B, if the switch S is suddenly shifted to position (2) from (1) at $t = 0$, find the current in the circuit as a function of time. Assume, initially the circuit is in steady state condition.

$$\left\{\frac{\varepsilon}{R}\left[1 + \frac{L_2}{L_1} e^{-Rt/L_1}\right]\right\}$$

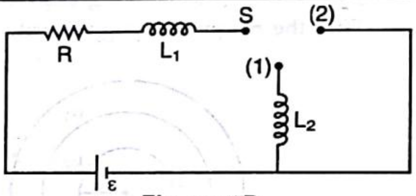


Fig. 3.335B

§ 3.336

➢ **CONCEPT**

Magnetic energy stored in magnetic field is

$$U = \frac{1}{2} LI^2$$

SOLUTION : We know,

$$N\phi = LI \qquad \therefore \qquad L = \frac{N\phi}{I}$$

$\therefore$
$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{N\phi}{I}\right)I^2 = \frac{N\phi I}{2}$$

On putting the values, we get $\quad U = 0.5\ \text{J}$

YOUR STEP

Assuming the Earth's magnetic field averages about 0.50×10^{-4} T near the surface of the earth, estimate the total energy stored in this field in the first 10 km above the earth's surface.

$\{ 5.1 \times 10^{15}\ \text{J} \}$

§ 3.337

➢ **CONCEPT**

The total energy stored in the core is

$$W = \frac{1}{2}(\vec{B}\,.\,\vec{H})\,V$$

SOLUTION : Here, the average magnetic field is

$$B = \frac{\mu_0 NI}{2\pi b}$$

The volume of iron core is $V = (2\pi b)\, S = (2\pi b)\, \pi a^2$

$\therefore$ $B = \mu_0 H$ $\quad\therefore\quad H = \dfrac{B}{\mu_0}$

$$\therefore \quad W = \frac{1}{2} BH\,(2\pi b)\, \pi a^2 = BH\, \pi^2 a^2 b$$

where $$H = \frac{B}{\mu_0} = \frac{\mu_0 NI}{2\pi b \mu_0} = \frac{NI}{2\pi b}$$

On putting the values, we get $W = 2$ joule

YOUR STEP

1. Consider a torus of radii $a < b$ and a uniform rectangular cross-sction.
The torus carries N tightly bound coils covering its surface through which a current I flows. The core of torus is composed of two paramagnetic substances of permeabilities μ_1 $(a \le r < c)$ and μ_2 $(c \le r \le b)$ (shown in fig 3.337A).
Find the magnetic energy density inside the torus.

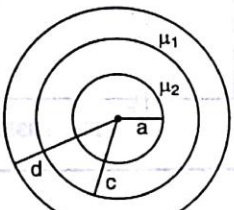


Fig. 3.337A

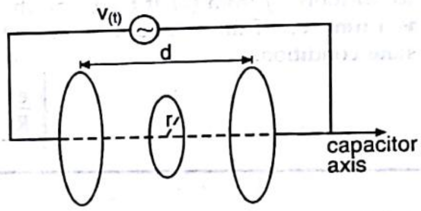


Fig. 3.337B

2. A parallel plate capacitor is made of two discs of radius a that are a distance d apart. The space between the plates is occupied by a conducting medium whose specific conductivity is σ. An alternating voltage $V = V_0 \sin \omega t$ is applied to the capacitor. Find the induced magnetic field at a distance r from the axis of the capacitor (shown in fig. 3.337B. Assume $r << a$ and neglect all boundary effects.

{1. $\dfrac{\mu_1}{2\pi}\left(\dfrac{NI}{rc}\right)^2, a < r < c$ $\quad \dfrac{\mu_2}{2\pi}\left(\dfrac{NI}{rc}\right)^2, c < r < b$

2. $\vec{B}(r) = \dfrac{\mu_0 V_0 r}{8\pi d}(4\pi\sigma \sin\omega t + \omega \cos \omega t)\,\hat{\phi}$ where $\hat{\phi}$ is the unit tangent to the circle of radius r whose orientation is fixed by the right hand law.}

§ 3.338

➢ **CONCEPT**

From Ampere's circuital law,

$$\int_c \vec{H} \cdot d\vec{L} = NI$$

$$\because \quad d >> b$$

$$\therefore \quad H\, 2\pi r + \frac{B}{\mu_0} b = NI \qquad \text{...(i)}$$

The relation between $\vec{H}$ and $\vec{B}$ is $\quad \vec{B} = \mu \mu_0 \vec{H}$ ...(ii)

From Eqn (i) and (ii), we get, $$\frac{B}{\mu \mu_0} = \frac{NI}{2\pi r + \mu b}$$

$\therefore$ $$B = \mu\mu_0 \frac{NI}{\pi d + \mu b} \quad ...(iii) \quad (\because 2r = d)$$

(a) The volume of gap is $V_1 = Sb$

$\therefore$ Energy stored in gap is $$W_g = \frac{B^2}{2\mu_0} V_1 = \frac{B^2}{2\mu_0} Sb$$

Similarly $$W_{magnetic} = \frac{B^2}{2\mu\mu_0} S\,(2\pi r) = \frac{B^2}{2\mu\,\mu_0} S\pi d$$

$\therefore$ $$\frac{W_g}{W_{magnetic}} = \frac{\mu b}{\pi d} = 3.0$$

(b) Magnetic flux is $$\phi = \int \vec{B} \cdot d\vec{S} = \mu\,\mu_0 \frac{NI}{\pi d + \mu b} \int dS$$

$$= \mu\,\mu_0 \frac{NI}{(\pi d + \mu b)} S = \frac{\mu\mu_0\, SN\,I}{(\pi d + \mu b)}$$

But $N\phi = LI$

$\therefore$ $$L = \frac{N\phi}{I} = \frac{\mu\mu_0\, SN^2}{(\pi d + \mu b)}$$

$$= \frac{\mu_0 S\, N^2}{\frac{\pi d}{\mu} + \frac{\mu b}{\mu}} = \frac{\mu_0\, SN^2}{\left(\frac{\pi d}{\mu} + b\right)}$$

By energy method $U = \frac{1}{2} LI$

From part (a), $$U = (W_{gap} + W_{mag}) = \frac{B^2}{2\mu_0} Sb + \frac{B^2\, S\pi d}{2\mu\mu_0}$$

$\therefore$ $$\frac{B^2}{2\mu_0}\left(\frac{\pi d}{\mu} + b\right) S = \frac{1}{2} LI^2$$

After putting the value of B from Eqn (iii), we get

$$L = \frac{\mu_0\, SN^2}{\left(\frac{\pi d}{\mu} + b\right)} = 0.15 \text{ H}$$

YOUR STEP

Figure 3.338A shows a toroidal coil S_1 of a rectangular cross-section containing N loops.Its inner and outer radii are a and b, respectively. The height of the torus is also a. A thin conducting ring, S_2, encircles the torus and carries a time dependent current, $I = I_0 e^{-t/\tau}$ where I_0 and τ are constants.

(a) Calculate the mutual inductance between the ring and the coil.

(b) Calculate the induced time dependent current $I_1(t)$ in the coil.

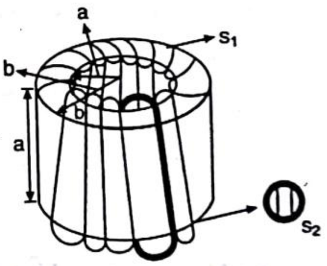


Fig. 3.338A

$$\left\{ \begin{array}{l} \text{(a) } M_{12} = \frac{\mu_0}{2\pi} Na\, ln\left(\frac{b}{a}\right) \\ \text{(b) } I_1(t) = \frac{M_{12}}{\tau} \cdot \frac{I_0}{R} e^{-t/\tau} \end{array} \right\}$$

§ 3.339

➢ CONCEPT

In this case, convection current due to mechanic transfer of charge is along the surface of cylinder perpendicular to the length of the cylinder.

We consider a ring element of thickness dx of the cylinder.

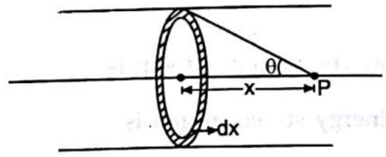


Fig. 3.339A

The charge on the considered element is

$$dq = \lambda\, dx$$

∴ The convection current is due to motion of considered element is

$$dI = \frac{dq}{T} = \frac{dq}{\frac{2\pi}{\omega}}$$

$$= \frac{\omega\, dq}{2\pi} = \frac{\omega\, \lambda dx}{2\pi} \quad \text{...(i)}$$

The magnetic field at point p dueto considered ring element is

$$B = \frac{\mu_0\, dI\, a^2}{2\,(a^2+x^2)^{3/2}} \quad \text{(From solution of problem 3.219)}$$

or $$B = \frac{\mu_0\, a^2}{2\,(a^2+x^2)^{3/2}}\, \frac{\omega\lambda}{2\pi}\, dx = \frac{\mu_0\, a^2 \omega\lambda}{4\pi\,(a^2+x^2)^{3/2}}\, dx$$

∴ $$d\phi = \frac{\mu_0\, a^2\, \omega\lambda\, \pi a^2}{4\pi} \cdot \frac{dx}{(a^2+x^2)^{3/2}}$$

∴ $$\phi = \frac{\mu_0\, a^4\, \omega\lambda}{4} \int_{-\infty}^{+\infty} \frac{dx}{(a^2+x^2)^{3/2}}$$

Putting $x = a \tan\theta$ ∴ $$\sin\theta = \frac{x}{\sqrt{a^2+x^2}}$$

$$dx = a \sec^2 d\theta$$

∴ $$\phi = \frac{\mu_0\, a^4\, \omega\lambda}{4} \int_{n=-\infty}^{+\infty} \frac{a \sec^2\theta\, dv}{(a^2+a^2\tan^2\theta)^{3/2}}$$

or $$\phi = \frac{\mu_0\, a^4\, \omega\lambda}{4a^2} [\sin\theta]_{x=-\infty}^{x=+\infty}$$

$$= \frac{\mu_0\, a^2 \omega\lambda}{4} \left[\frac{x}{\sqrt{a^2+x^2}}\right]_{-\infty}^{+\infty} = \frac{\mu_0\, a^2\, \omega\lambda}{4} [1+1]$$

$$= \frac{\mu_0\, a^2\, \omega\lambda}{2}$$

∴ $\phi = LI$ ∴ $LI^2 = \phi\, I$

∴ Magnetic energy is $$U = \frac{1}{2} LI^2 = \frac{1}{2} (LI)\, I = \frac{1}{2} \phi\, I$$

$$= \frac{\phi\, I}{2} = \frac{\mu_0\, a^2\, \omega\lambda}{4}\, I$$

$\therefore$ Magnetic energy per unit length is $\dfrac{U}{l} = \dfrac{\mu_0 a^2 \omega \lambda I}{4l}$

But $$\frac{I}{l} = \frac{dI}{dx} = \frac{\omega\lambda}{2\pi}\frac{dx}{dx} = \frac{\omega\lambda}{2\pi} \qquad \text{(from Eqn. (i))}$$

$\therefore$ $$\frac{U}{l} = \frac{\mu_0 a^2 \omega\lambda}{4}\frac{\omega\lambda}{2\pi} = \frac{\mu_0 a^2 \omega^2 \lambda^2}{8\pi}$$

YOUR STEP

A small electric car overcomes a 250 N frictional force when traveling 30km/hr. The electric motor is powered by ten 12-V batteries connected in sereis and is coupled directly to the wheels whose diameters are 50 cm. The 300 armature coils are rectangular, 10 cm × 15cm, and rotate in *a* 0.6 T magnetic field. What percent of the input power is used to drive the car?

{75%}

§ 3.340

➢ **CONCEPT**

The volume energy density in an electric field E is

$$w_E = \frac{1}{2}\varepsilon_0 E^2.$$

The volume energy density in a magnetic field is

$$w_B = \frac{B^2}{2\mu_0}$$

SOLUTION : According to problem,

$$u_E = u_B$$

or $$\frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0} \qquad \left(\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

$\therefore$ $$E = \frac{B}{\sqrt{\mu_0 \varepsilon_0}}$$

$\therefore$ $$E = cB = 3\times 10^8 \times 1 = 3\times 10^8 \text{ N/C}$$

YOUR STEP

A long, straight copper rod of 1 cm diameter carries a current of 16.0A. Assume the current density within the bar is uniform and compute the total energy stored in the magnetic field within the volume of a 1 m length of the bar (that is, within a 1m long region that is bounded by the surface of the bar).

{6.40×10^{-6} J}

§ 3.341

➢ **CONCEPT**

From the solution of problem 3.9,

$$E = \frac{qr}{4\pi\varepsilon_0 (a^2 + l^2)^{3/2}}$$

From the solution of problem 3.219,

$$B = \frac{\mu_0 I a^2}{2(a^2 + l^2)^{3/2}} \qquad \text{Here} \qquad I = \frac{q\omega}{2\pi}$$

$$\therefore \qquad B = \frac{\mu_0 \left(\frac{q\omega}{2\pi}\right)^2 a^2}{2\,(a^2 + l^2)^{3/2}}$$

SOLUTION : From the solution of previous problem,

$$w_E = \frac{1}{2}\varepsilon_0 E^2 \quad \text{and} \quad w_B = \frac{B^2}{2\mu_0}$$

$$\therefore \qquad \frac{w_B}{w_E} = \frac{\frac{B^2}{2\mu_0}}{\frac{1}{2}\varepsilon_0 E^2}$$

After putting the values of E and B, we get

$$\frac{w_B}{w_E} = \frac{\varepsilon_0 \mu_0 \omega^2 a^4}{l^2} = 1.1 \times 10^{-15}$$

YOUR STEP

Consider a long insulating cylinder of radius R and height b. The cylindrical surface carries a charge Q, which is distributed uniformly. A string is coiled around the cylinder, at the end of which a point like particle of mass m is hung (shown in fig. 3.341A). The mass m falls, causing the cylinder to revolve about its axis.

(a) Calculate the magnetic flux inside the cylinder.

(b) Find the falling acceleration of m (Neglect the moment of inertia of the cylinder).

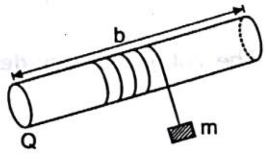


Fig. 3.341A

$$\left\{ \begin{array}{l} \text{(a) } \dfrac{\mu_0 Q\omega}{2b} R^2 \\ \text{(b) } \dfrac{g}{1 + \dfrac{\mu_0 Q^2}{4\pi m b}} \end{array} \right\}$$

§ 3.342

➢ **CONCEPT**

SOLUTION : The total energy inside the magnetic field is

$$U = \int w_B\, dV = \int \frac{B^2}{2\mu_0\mu}\, dV$$

$$= \frac{1}{2}\int \vec{B} \cdot \frac{\vec{B}}{\mu_0 \mu}\, dV$$

$$= \frac{1}{2}\int (\vec{B} \cdot \vec{H})\, dV \qquad (\because \vec{B} = \mu_0 \mu \vec{H})$$

Here $$\vec{H} = \left(\frac{\vec{B}}{\mu_0} - \vec{J}\right)$$

$$\therefore \qquad W = \frac{1}{2}\int \vec{B} \cdot \left(\frac{\vec{B}}{\mu_0} - \vec{J}\right) dV$$

$$= \frac{1}{2\mu_0}\int B^2\, dV - \frac{1}{2}\int \vec{B} \cdot \vec{J}\, dV \qquad \text{...(i)}$$

But total magnetic energy in vacuum is $w_{vacuum} = \int \frac{B^2}{2\mu_0}$

After magnetisation,
total magnetic energy is W = magnetic energy in vaccum + energy of magnetication.

i.e. $$W = W_{vacuum} + W_{magnetisation} \quad ...(ii)$$

$\therefore$ From Eq. (i), we have $$W = W_{vacuum} - \frac{1}{2}\int \vec{B}\cdot\vec{J}dV \quad ...(iii)$$

From equation (ii) and (iii), we get $$= -\frac{1}{2}\int \vec{B}\cdot\vec{J}dV$$

$$\therefore \quad W_{magnetization} = -\frac{1}{2}\int \vec{B}\cdot\vec{J}dV$$

$$\therefore \quad A = W_{magnetization} \text{ per unit volume} = -\frac{1}{2}\vec{B}\cdot\vec{J} = -\frac{1}{2}\vec{J}\cdot\vec{B}$$

YOUR STEP

Calculate the force between two co-axial solenoids carrying constant currents i_1 and i_2, of one of which extends a part of it within the other. Assume that area of cross-section of solenoids is S and n_1 and n_2 are the number of turns per unit length on the two solenoids.

{ $\mu_0 n_1 n_2 i_1 i_2 S$ }

§ 3.343

➢ **CONCEPT**

If isolated inductors are in series, then $L_{eq} = L_1 + L_2 + ...$

If isolated inductors are in parallel, $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + ...$

SOLUTION : (a) $$L_{series} = L + L = 2L$$

(b) $$\frac{1}{L_{parallel}} = \frac{1}{L} + \frac{1}{L} = \frac{2}{L}$$

$$\therefore \quad L_{parallel} = \frac{L}{2}$$

YOUR STEP

As is seen from fig 3.343A

I_1 and I_2 can be independently increased, decreased or held constant by varrying resistance R_1 and R_2. Let L_1 = 50 mH, L_2 = 40 mH, M = 15 mH.

(a) I_1 is made to increase at the rate of 120 A/s; I_2 is held constant. Calculate the voltage V_1 and V_2 induced in each coil.

(b) I_1 is made to decrease at the rate of 120 A/s, I_2 is held constant. Calculate V_1 and V_2

(c) I_1 is increased at the rate of 120 A/s and I_2 is decreased at the rate of 200 A/s. Calculate V_1 and V_2.

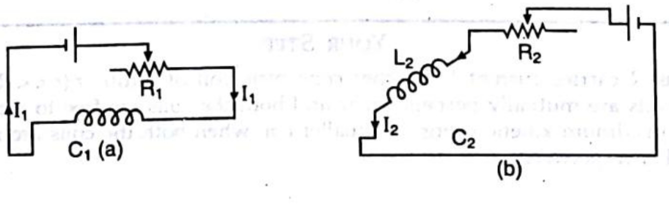


Fig. 3.343A

{(a) – 1.8 V (b) + 1.8 V (c) – 3 V and + 6.2 V}

§ 3.344

➢ **CONCEPT**

If total flux due to one coil in passing through other, then co-efficient of coupling, $k = 1$.
As we know,

$$M \propto \sqrt{L_1 L_2} \qquad \therefore \qquad M = K\sqrt{L_1 L_2}$$

where K is co-efficient of coupling. In this case, $K = 1$

$$\therefore \qquad M = \sqrt{L_1 L_2}$$

YOUR STEP

Two coils are wound on the same iron rod so that the flux generated by one passes through the other also. The primary coil has N_p loops on it and when a current of 2A flows through it, the flux in it is 2.5×10^{-4} wb. Determine the mutual inductance of the two coils if the secondary coil has N_s loops on it.

{ $(1.25 \times 10^{-4} N_s)$ H}

§ 3.345

➢ **CONCEPT**

The magnetic energy is given by $\frac{dW_B}{dt} = \frac{1}{\mu_0}\int B \cdot \frac{\partial \vec{B}}{\partial t}\, dT$

or $$dW_B = \frac{1}{\mu_0}\int [B_1 (dB_1)\, T_1 + B_2\, dB_2\, T_2 + B_1 (dB_2)\, Sl + (dB_1)\, B_2 Sl + B_1 B_2\, SdL] \qquad \text{...(i)}$$

Since, total flux linkage for first coil is constant, *i.e.*,

$$B_1 n_1 S + B_2 n_1 \left(\frac{l}{l_1}\right) S = \text{constant}$$

$$\therefore \qquad dB_1 + dB_2 \frac{l}{l_1} + B_2 \frac{l}{l_1}\, dl = 0 \qquad \text{...(ii)}$$

Similarly for second coil, $$dB_2 + dB_1 \frac{l}{l_2} + B_1 \frac{l}{l_2}\, dl = 0 \qquad \text{...(iii)}$$

Solving these equations, we get

$$dB_1 = (lB_1 - l_2 B_2)(l_1 l_2 - l^2)^{-1}\, dl \qquad \text{...(iv)}$$

and $$dB_2 = (lB_2 - l_1 B_1)(l_1 l_2 - l^2)^{-1}\, dl \qquad \text{...(v)}$$

From eqs. (i), (iv) and (v), we get

$$dW_B = \frac{1}{\mu_0} B_1 B_2\, Sdl = \frac{1}{\mu_0} \vec{B}_1 \cdot \vec{B}_2 \cdot dV$$

$$W_B = \frac{1}{\mu_0}\int \vec{B}_1 \cdot \vec{B}_2\, dV$$

YOUR STEP

A coil of radius R carries current I. Another concentric coil of radius $r\,(r << R)$ carries current i. Planes of two coils are mutually perpendicular and both the coils are free to rotate about common diameter. Find maximum kinetic energy of smaller coil when both the coils are released, masses of coils are M and m respectively.

$$\left\{\frac{\mu_0 \pi I i M R r^2}{2(MR^2 + mr^2)}\right\}$$

§ 3.346

➢ **CONCEPT**

The dipole moment of the smaller loop is $P = I_1 \pi a^2 \hat{n}$

The magnetic field due to second loop is $B = \dfrac{\mu_0 I_2}{2b}$ (at centre)

SOLUTION : $\because$ $W = -\vec{P} \cdot \vec{B}$

$$= -PB \cos (180 - \theta)$$

$\therefore$ $W = PB \cos \theta$

$$= \frac{\mu_0 I_1 I_2 \pi a^2 \cos \theta}{2b}$$

YOUR STEP

A toroid of 0.5m circumference and 480mm^2 cross-sectional area has 2500 turns bearing a current of 0.6 A. It is wound on an iron ring with relative permeability of 350. Find the magnetising field H, the magnetic induction B, the flux ϕ, inductance L and the energy stored in the magnetic field.

{ $H = 3000$ A/m, $\phi = 633$ μwb, $B = 1.32$ T, $L = 2.64$ H and $U = 0.475$ J}

Motion Of Charged Particles In Electric And Magnetic Fields

§ 3.372

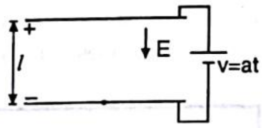


Fig. 3.372A

➢ **CONCEPT**

For a given instant of time, the electric field at every point between plates remains constant. The electric field is given by

$$E = \frac{V}{l} = \frac{at}{l}$$

The direction of electric field is from positive plates to negative plate.

➢ **DISCUSSION**

At $t = 0$, the velocity of electron is zero. The electron is situated at A at $t = 0$. Due to electric field E, an electric force

$$F = eE \qquad \text{(in opposite direction of electric field)}$$

This force acts on the electron. Due to force, electron starts to accelerate with an acceleration of

$$w = \frac{eE}{m}$$

Due to this acceleration electron starts to move towards positive plate of the capacitor.

SOLUTION : Since, acceleration $w = \frac{eE}{m}$ of the electron is not constant. So, kinematic equation for solving the problem is applicable.

Here ∴ $$w = \frac{eE}{m} = \frac{eV}{ml}$$

or $$w = \frac{e\,at}{ml} \quad \text{or} \quad \frac{dv}{dt} = \frac{e\,at}{ml}$$

or $$\int_0^v dv = \frac{ea}{ml}\int_0^t t\,dt \quad \text{or} \quad v = \frac{e\,at^2}{2ml} \qquad \text{...(i)}$$

∵ $$v = \frac{e\,a\,t^2}{2ml} \quad \text{or} \quad \frac{dx}{dt} = \frac{ea}{2ml}t^2$$

or $$\int_0^l dx = \frac{ea}{2ml}\int_0^{t_0} t^2\,dt \quad \text{or} \quad l = \frac{e\,a\,t_0^3}{6ml}$$

∴ $$t_0 = \left(\frac{6\,ml^2}{ea}\right)^{1/3} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $$v = \frac{ea}{2\,ml}\left(\frac{6ml^2}{ea}\right)^{2/3}$$

$$= \left(\frac{9}{2}\frac{a\,l\,e}{m}\right)^{1/3} = 16\text{km/s}$$

YOUR STEP

An electron is projected with an initial speed $v_0 = 4 \times 10^6$ m/sec into the uniform field between the parallel plates in fig. 3.372B. The direction of the field is vertically downward, and the field is zero except in the space between the plates. The electron enters the field at a point midway between the plates.

(a) If the electron just misses the upper plate as it emerges from the field, find the magnitude of the electric field.

(b) Suppose that in fig.3.372B the electron is replaced by a proton with the same intial speed v_0. Would the proton hit one of the plate?

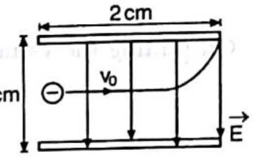


Fig. 3.372B

$\left\{ \begin{array}{l} \text{(a) } 2.27 \times 10^3 \text{ N/C} \\ \text{(b) No.} \end{array} \right\}$

§ 3.373

➢ **CONCEPT**

Here eV = Change in kinetic energy of proton

$$= \frac{1}{2} mu^2 - 0 = \frac{1}{2} mu^2$$

➢ **DISCUSSION**

Let the velocity of proton just before entering into plate capacitor is u.

Since, proton is accelerated through a voltage V before entering into the region of parallel plate capacitor.

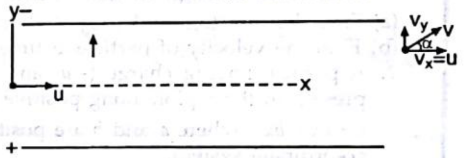


Fig. 3.373A

$$\therefore \qquad eV = \frac{1}{2} mu^2$$

$$\therefore \qquad u = \sqrt{\frac{2eV}{m}}$$

where e = charge on proton, V = Potential difference, m = mass of proton.

At $t = 0$, proton is at origin and has a velocity $u\,\hat{i}$. The electric force is

$$\vec{F} = eE\,\hat{j}$$

This force starts to act on the proton. Due to this, acceleration of proton is

$$\vec{w} = \frac{eE}{m}\hat{j} = \frac{e\,a\,t}{m}\hat{j},\ E = \frac{at}{l} \text{ or } E = at \text{ per unit length}$$

Hence, resultant motion of proton is two dimensional motion.

SOLUTION : The time taken by proton to travel a distance l along x-axis. (shown in fig. 3.373 A) is t_0.

$$\therefore \qquad l = u\,t_0 \qquad (\because \text{ no acceleration along } x\text{-axis})$$

$$\therefore \qquad t_0 = \frac{l}{u} \qquad ...(i)$$

i.e. The acceleration of proton along y-axis is

$$w = \frac{e\,a\,t}{m} \qquad \text{(along } y\text{-axis)}$$

or

$$\frac{dv_y}{dt} = \frac{ea}{m} t$$

or

$$\int_0^{v_y} d\,v_y = \frac{ea}{m} \int_0^{t_0} t\,dt$$

or $$v_y = \frac{e\,a t_0^2}{2m}$$

On putting the value of t_0, we get

$\therefore$ $$v_y = \frac{e\,a}{2m}\left(\frac{l}{u}\right)^2 = \frac{e\,a\,l^2}{2mu^2}$$

The velocity of proton is $$\vec{v} = v_x\,\hat{i} + v_y\,\hat{j} = u\,\hat{i} + \frac{e\,a\,l^2}{2mu^2}\,\hat{j}$$

$\therefore$ $$\tan\alpha = \frac{v_y}{v_x} = \frac{e\,a\,l^2}{2mu^3}$$

On putting the value of $$u = \sqrt{\frac{2eV}{m}}$$

We get, $$\tan\alpha = \frac{a\,l^2}{4}\sqrt{\frac{m}{2eV^3}}$$

YOUR STEP

1. A particle of charge q and mass m which is initially at rest at the origin of coordinate system is accelerated through an electric field $\vec{E} = E_0 \sin \omega t\ \mathrm{i} + E_0 \sin \omega t\ \mathrm{j}$, where E_0 and ω are constant.
(a) Find the direction and magnitude of acceleration of the charged particle at $t = 0$
(b) Find the velocity of particle at time t.
2. A particle having charge $(-q)$ and mass m moves in $x-y$ plane A constant elecric field E is present in the region along positive y-direction. The equation of motion of the particle has form $y = ax - bx^2$, where a and b are positive constant . Find the speed of the particle at the origin of co-ordinate system.

$$\left\{\begin{array}{l} 1.\ (a)\ a = 0 \quad (b)\ v = \dfrac{q\,\epsilon_0}{m\omega}(1 - \cos\omega t)\,(\hat{i} + \hat{j}) \\ 2.\ v_0 = \sqrt{\dfrac{(1+a)^2\,qE}{2mb}} \end{array}\right\}$$

§ 3.374

➢ **CONCEPT**

When $$w = f(x)$$

Thus $$w = v\frac{dv}{dx}$$

SOLUTION :

(a) The electric force on the charged particle is

$$w = \frac{F}{m} = \frac{qE}{m}$$

or $$w = \frac{q}{m}(E_0 - ax)$$

or $$v\frac{dv}{dx} = \frac{q}{m}(E_0 - ax)$$

or $$\int_0^0 v\,dv = \frac{q}{m}\int_0^{x_0}(E_0 - ax)\,dx$$

or $$\left[\frac{v^2}{2}\right]_0^0 = \frac{q}{m}\left[E_0\,x_0 - \frac{a x_0^2}{2}\right]$$

or $$0 = \frac{q}{m}\left[E_0\,x_0 - \frac{a x_0^2}{2}\right]$$

or $$\frac{a x_0^2}{2} = E_0 x_0$$

$\therefore$ $$x_0 = \frac{2E_0}{a}$$

(b) $$w = \frac{q}{m}[E_0 - ax]$$

At $$x = x_0 = \frac{2E_0}{a}$$

$$w = \frac{q}{m}\left(E_0 - a\,\frac{2E_0}{a}\right)$$

$$w = -\frac{q\,E_0}{m}$$

In the series of magnitude, $$w = \frac{qE_0}{m}$$

YOUR STEP

The electric potential at point $P(x, 0, 0)$ is $V = 1.8 \times 10^4\left[\frac{8}{\sqrt{\frac{27}{2}+x^2}} - \frac{1}{\sqrt{\frac{3}{2}+x^2}}\right]$ volt. A particle of mass 6×10^{-4} kg and charge 10^{-7} coulomb moves along the x-direction. Its speed at $x = \infty$ is v_0. Find the minimum value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assum~ that space is gravity free.

{3m/s and 3×10^{-4} joule}

§ 3.375

➢ **CONCEPT**

The rest mass energy of a body of mass m_0 (at rest) is

$$W_0 = m_0 c^2$$

If a body is moving with speed v ($v < c$), then total energy

$$W = mc^2$$

where m = mass of body at speed v.

Total energy at speed v = rest mass energy + kinetic energy

$\therefore$ $$mc^2 = m_0c^2 + \text{kinetic energy} \quad \text{...(i)}$$

The relavistic variation of mass is $$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{...(ii)}$$

SOLUTION : According to work-energy theorem, $A = \Delta T$

Workdone by electrical force is

$$A = qEx$$

Here $$\Delta T = T_f - T_i$$

$$= \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2$$

$\therefore$ $$A = \Delta T$$

$\therefore$ $$qEx = \frac{1}{2}mv^2 \quad \text{...(iii)}$$

From Eqn. (i), we get $$mc^2 = m_0c^2 + eEx$$

or $$\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}=m_0c^2+eEx$$

or $$\sqrt{1-\frac{v^2}{c^2}}=\frac{m_0c^2}{m_0c^2+eEx}$$

or $$\frac{v}{c}=\frac{\sqrt{(m_0c^2+eEx)^2-m_0^2c^4}}{m_0c^2+eEx}$$

or $$\frac{1}{c}\frac{dx}{dt}=\frac{\sqrt{(m_0c^2+eEx)^2-m_0^2c^4}}{m_0c^2+eEx}$$

or $$\frac{1}{c}\int_0^v\frac{(m_0c^2+eEx)}{\sqrt{(m_0c^2+eEx)^2-m_0^2c^4}}dx=\int_0^t dt$$

After solving, we get, $$ct=\sqrt{(m_0c^2+eEx)^2-m_0^2c^4} \quad \text{...(iv)}$$

Accroding to problem,
Final kinetic energy is $$T=eEx \quad \text{...(v)}$$

After putting $eEx=T$ in equ. (iv), we get

$$t=\frac{\sqrt{T(T+2m_0c^2)}}{ceE}$$

On putting the values, we get $t=3\times10^{-9}$ sec $=3$ ns

YOUR STEP

Find the velocity of a meson if its total energy is 10 times greater than its rest energy.
{ $v=2.985\times10^8$ m/sec}

§ 3.376

➢ **CONCEPT**

As we know $$F=\frac{dp}{dt}$$

$$F=\frac{d(mv)}{dt}=m\frac{dv}{dt}+v\frac{dm}{dt}$$

Here $$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

∴ $$\frac{dm}{dt}=\frac{1}{2}\frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}2v\frac{dv}{dt}=\frac{m_0v\omega}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

SOLUTION : Here $$F=eE$$

$$F=m\frac{dv}{dt}+v\frac{dm}{dt}=mw+v\frac{dm}{dt}$$

∴ $$F=\frac{m_0w}{\sqrt{1-\frac{v^2}{c^2}}}+\frac{m_0v^2w}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

or
$$F = \frac{m_0 w}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{m_0 v^2 \omega}{\left[1-\frac{v^2}{c^2}\right]^{3/2}}$$

But
$$\frac{1}{2} m v^2 = T$$

$\therefore$
$$F = \frac{m_0 w}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{2T\omega}{\left(1-\frac{v^2}{c^2}\right)}$$

But
$$F = eE$$

$\therefore$
$$eE = \frac{m_0 w}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{2T w}{\left(1-\frac{v^2}{c^2}\right)}$$

or
$$eE = w\left(\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{2T}{1-\frac{v^2}{c^2}}\right)$$

or
$$eE = w\frac{(T+m_0 c^2)^3}{m_0^2 c^6}$$

$\therefore$
$$w = \frac{eE}{m_0}\left(1+\frac{T}{m_0 c^2}\right)^{-3}$$

YOUR STEP

Making use of the principle of relativity prove that the intensity of the transverse electric field of moving charges exceeds that of their coulomb field.

§ 3.377

➢ **CONCEPT**

As we know,
$$F_x = \frac{dP_x}{dt},\quad F_y = \frac{dP_y}{dt} \quad \text{and} \quad F_z = \frac{dP_z}{dt}$$

Also,
$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

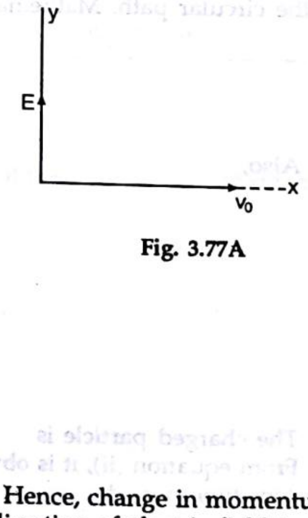


Fig. 3.77A

SOLUTION :

Here $P_x = m v_x$ But $F_x = 0$ Also, $P_y = m v_y$, $F_y = eE$

$\because$
$$F = \frac{dP_x}{dt}$$

or
$$\frac{dP_x}{dt} = 0$$

(a) The initial momentum of proton is $P_{\phi_x} = \frac{m_0 v_0}{\sqrt{1-\frac{v_0^2}{c^2}}}$

Since, no force is present in the direction of initial momentum. Hence, change in momentum in the direction of initial velocity is zero. The momentum in the direction of electric field is

$$P_y = \frac{mv_y}{\sqrt{1-\dfrac{v_y^2}{c^2}}}$$

But $$\frac{dP_y}{dt} = eE$$

or $$\int d\left\{\frac{mv_y}{\sqrt{1-\dfrac{v^2}{c^2}}}\right\} = eE\int dt$$

$\therefore$ $$P_y = eEt$$

$\therefore$ $$\tan\theta = \frac{P_y}{P_x} = \frac{eEt}{\dfrac{m_0 v_0}{\sqrt{1-v_0^2/c^2}}} = \frac{eEt\sqrt{1-v_0^2/c^2}}{m_0 v_0}$$

(b) The projection v_x vector $\vec{v}$ along x-axis is $v_x = v_0 \cos\alpha = \dfrac{v_0}{\sqrt{1+\left(1-(v_0^2/c^2)\right)\left(\dfrac{eEt}{m_0 c^2}\right)^2}}$

YOUR STEP

Show that the scalar product E.B. is unchanged by a Lorentz transformation. Show the same for $E^2 - c^2 B^2$.

§ 3.378

➢ **CONCEPT**

It a charged particle is projected in a region of uniform magnetic field B acting perpendicular to the direction of velocity.

Then the speed of charged particle remains constant. Also, the charged particle is moving on a circular path whose plane is perpendicular to the direction of magnetic field.

In this case, magnetic force provides required centripetal force for moving charged particle on the circular path. Mathematically,

$$\frac{mv^2}{R} = qvB$$

or $$R = \frac{mv}{qB} \quad ...(i)$$

Also, $$v = R\omega$$

$\therefore$ $$R = \frac{mR\omega}{qB}$$

$\therefore$ $$\omega = \frac{qB}{m}$$

or $$\frac{2\pi}{T} = \frac{qB}{m}$$

$\therefore$ $$T = \frac{2\pi m}{qB} \quad ...(ii)$$

The charged particle is

From equation (ii), it is obvious that the charged particle is moving on the circular path with constant time period.

SOLUTION :

Let the velocity of proton just before entering inside the magnetic field is v_0.

$$\therefore \quad eV = \frac{1}{2} m v_0^2$$

$$\therefore \quad v_0 = \sqrt{\frac{2eV}{m}}$$

The trajectory of proton (*i.e.*, circular arc) is shown in fig. 3.378A.

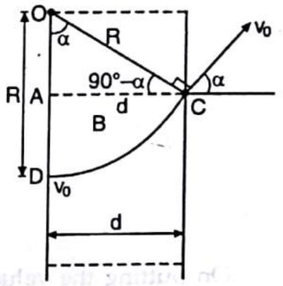


Fig. 3.378A

Here
$$R = \frac{mv_0}{eB}$$

From fig. 3.378 A.

$$\sin\alpha = \frac{d}{R} = \frac{d}{\frac{mv_0}{eB}}$$

or
$$\sin\alpha = \frac{eBd}{mv_0} = \frac{eBd}{m\sqrt{\frac{2eV}{m}}}$$

or
$$\sin\alpha = Bd\sqrt{\frac{e}{2mV}}$$

Here e = charge on proton $= 1.6 \times 10^{-19}$ C

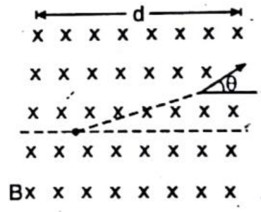
$$\therefore \quad \alpha = \sin^{-1}\left\{Bd\sqrt{\frac{e}{2mV}}\right\}$$

On putting the values, we get $\alpha = 30°$

YOUR STEP

A particle of mass m and charge q is projected into a region having a perpendicular magnetic field B. Find the angle of deviation in fig. 3.378B of the particle as it comes out of the magnetic field, if the width d of the region is very slightly smaller than

(a) $\frac{mV}{qB}$ (b) $\frac{mV}{2qB}$ (c) $\frac{2mV}{qB}$



{ (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c)

Fig. 3.378B

§ 3.379

➢ **CONCEPT**

The concept of problem 3.378 is applicable

SOLUTION : (a) In this case, magnetic force provides required centripetal force for moving charged particle on the circular path.

i.e.
$$\frac{mv^2}{r} = qvB$$

$$\therefore \quad v = \frac{qBr}{m} = \frac{eBr}{m} = 100 \text{ km/s} \qquad (\because q = e)$$

or
$$r\omega = \frac{qBr}{m}$$

or
$$\frac{2\pi}{T} = \frac{qB}{m}$$

$$\therefore \quad T = \frac{2\pi m}{qB} = \frac{2\pi m}{eB} = 6.5\ \mu\text{s} \qquad (\because q = e)$$

(b) $\therefore$ $$T = \frac{2\pi m}{qB}$$

But $$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$T_{\text{relavistic}} = \frac{2\pi\, m_0}{qB\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{2\pi\, m_0}{eB\sqrt{1 - \frac{v^2}{c^2}}} \qquad (\because\ q = e)$$

On putting the values, $$T_{relavistic} = 4.1 \text{ ns}$$

Also, $$\frac{mv^2}{r} = qvB$$

$\therefore$ $$\frac{mv}{r} = qB$$

or $$mv = qBr$$

or $$\frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} = qBr$$

or $$\frac{m_0\, c\left(\frac{v}{c}\right)}{\sqrt{1 - \frac{v^2}{c^2}}} = qBr$$

or $$m_0^2\, c^2\left(\frac{v^2}{c^2}\right) = q^2 B^2 r^2\left(1 - \frac{v^2}{c^2}\right)$$

or $$\frac{v^2}{c^2} = \left(\frac{qBr}{m_0\, c}\right)^2\left(1 - \frac{v^2}{c^2}\right)$$

or $$1 - \frac{v^2}{c^2} = 1 - \left(\frac{eBr}{m_0 c}\right)^2\left(1 - \frac{v^2}{c^2}\right) \qquad (\because\ q = e)$$

or $$\frac{1}{1 - \frac{v^2}{c^2}} = \left(\frac{qrB}{m_0 c}\right)^2 + 1$$

or $$1 - \frac{v^2}{c^2} = \frac{1}{\left(\frac{qrB}{m_0 c}\right)^2 + 1}$$

or $$\frac{v^2}{c^2} = 1 - \frac{1}{\left\{\left(\frac{qrB}{m_0 c}\right)^2 + 1\right\}}$$

or $$\frac{v^2}{c^2} = \frac{\left(\frac{qrB}{m_0 c}\right)^2}{\left\{\left(\frac{qrB}{m_0 c}\right)^2 + 1\right\}}$$

or $$\frac{v^2}{c^2} = \frac{1}{1 + \left(\frac{m_0 c}{qrB}\right)^2}$$

$\therefore$ $$v = \frac{c}{\sqrt{1 + \left(\frac{m_0 c}{qrB}\right)^2}} = \frac{c}{\sqrt{1 + (m_0 c / reB)^2}} \qquad (\because\ q = e)$$

On putting the values, $v = 0.51$ c

YOUR STEP

A neutral particle is at rest in a uniform magnetic field of magnitude B. At time $t = 0$ it decays into two charged particles each of mass m. (a) If the charge of one of the particles is $+q$, what is the charge of the other ?

$\{-q\}$

§ 3.380

➢ **CONCEPT**

Linear momentum is given by $\vec{p} = m\vec{v}$

SOLUTION : (a) From the solution of previous problem,

$$\frac{mv^2}{r} = qvB$$

$\therefore$ $$mv = qBr$$

$\therefore$ The magnitude of momentum is $P = mv = qBr$

Remarks : Electric charge is relavistic invarient quantity.

$\because$ $$p = qrB$$

(b) From equation (1.8 *h*) on page 68 of the book "I. E. Irodov".

$$pc = \sqrt{T(T + 2m_0 c^2)}$$

or $$qBrC = \sqrt{T(T + 2m_0 c^2)}$$

Squaring both sides we get $$q^2 B^2 r^2 c^2 = T(T + 2m_0 c^2)$$

or $$T^2 + 2m_0 c^2 T - q^2 B^2 r^2 c^2 = 0$$

After solving, we get $$T = m_0 c^2 \left[\sqrt{1 + \left(\frac{qrB}{m_0 c}\right)^2} - 1\right]$$

(c) $\therefore$ $$F = qvB$$

or $$\frac{d(mv)}{dt} = qvB$$

or $$v\frac{dm}{dt} + m\frac{dv}{dt} = qvB$$

Here $$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\therefore \qquad \frac{dm}{dt} = \frac{m_0 v}{c^2\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \frac{dv}{dt}$$

Also $$\frac{dv}{dt} = w$$

Putting the values in equ. (i), we get $$\frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \frac{dv}{dt} = qvB$$

$$\therefore \qquad \frac{dv}{dt} = \frac{qvB\left(1 - \frac{v^2}{c^2}\right)^{3/2}}{m_0}$$

From the solution of 3.379 (b), $$v = \frac{c}{\sqrt{1 + \left(\frac{m_0 c}{qrB}\right)^2}}$$

$$\therefore \qquad w = \frac{dv}{dt} = \frac{qB}{m_0}\left(\frac{c}{\sqrt{1 + \left(\frac{m_0 c}{qrB}\right)^2}}\right)\left\{1 - \frac{1}{c^2} \frac{c^2}{1 + \left(\frac{m_0 c}{qrB}\right)^2}\right\}$$

After solving, we get $$w = \frac{c^2}{r\left[1 + \left(\frac{m_0 c}{qrB}\right)^2\right]}$$

YOUR STEP

A magnetic field with an induction of $B = 5$ gauss is perpendicular to an electric field with an intensity of $E = 10$ V/cm. A beam of electron flies with a certain velocity v into the space where these fields are present, the velocity of the electrons being perpendicular to the plane in which the vector E and B lie. Find :

(a) the velocity v of the electrons if the electron beam is not deflected when both field act simultaneously.

(b) the radius of curvature of the trajectory of the electrons when only the magnetic field is switched on.

[(a) $v = 2 \times 10^6$ m/sec. (b) $R = 2.3 \times 10^{-2}$ m]

§ 3.381

➢ **CONCEPT**

For non relavistic charged particle, $$T = \frac{2\pi m}{qB}$$ (from solution of 3.379 (a))

and $$T_{rel} = \frac{2\pi m_0}{qB\sqrt{1 - \frac{v^2}{c^2}}}$$ (from solution of 3.379 (b))

$$\therefore \qquad T_{rel} = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or
$$\frac{T_{rel}-T}{T}=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1$$

or
$$\frac{\Delta T}{T}=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1$$

or
$$\eta=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1$$

∴ The kinetic energy is
$$T_0=mc^2-m_0c^2$$

or
$$T_0=\frac{m_0\,c^2}{\sqrt{1-\frac{v^2}{c^2}}}-m_0\,c^2=m_0\,c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)$$
$$=m_0\,c^2\,\eta=\eta\,m_0\,c^2$$

On putting the values, for electron we get
$$T_0=5\text{ keV}$$

For proton
$$T_0=9\text{ MeV}$$

YOUR STEP

An ion of mass M and charge q is accelerated by potential difference V and allowed to enter a field of magnetic induction B. In the field it moves in a semi-circle, striking a photographic plate at a distance x from the slit. Show that the mass M is given by
$$\left\{M=\frac{B^2\,q\,x^2}{8V}\right\}$$

§ 3.382

➢ **CONCEPT**

When a charged particle is projected at an angle α with the direction of magnetic field B. The resultant path of particle is helical. Here is component of velocity perpendicular to magnetic field is responsible for circular motion in a plane perpendicular to magnetic field.

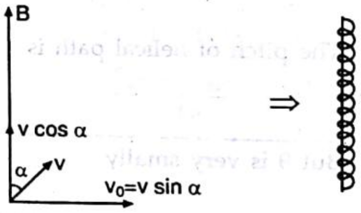


Fig. 3.382A

∴
$$\frac{mv_0^2}{r}=qv_0\,B$$

∴
$$r=\frac{m\,v_0}{qB}=\frac{m\,r\,\omega}{qB}$$

∴
$$\omega=\frac{qB}{m}$$

or
$$\frac{2\pi}{T}=\frac{qB}{m} \qquad \therefore \quad T=\frac{2\pi m}{qB}$$

Since, magnetic force is perpendicular to the direction of magnetic field. So, the component of velocity $v\cos\alpha$ in the direction of magnetic field remains constant.
$$\text{pitch}=v\cos\alpha\,T$$

$$= v\cos\alpha\,\frac{2\pi m}{qB} = \frac{2\pi\, mv\cos\alpha}{qB}$$

But $$eV = \frac{1}{2}mv^2$$

$\therefore$ $$v = \sqrt{\frac{2eV}{m}}$$

$\therefore$ $$\text{pitch} = \frac{2\pi\, m\cos\alpha}{qB}\,\frac{\sqrt{2eV}}{\sqrt{m}}$$

$\therefore$ $$\text{pitch} = \Delta l = 2\pi\sqrt{\frac{2mV}{eB^2}}\cos\alpha$$

On putting the values, we get $\Delta l = 2.0$ cm

YOUR STEP

An electron accelerated by a potential difference of $V = 3000$ V flies into the magnetic field of a solenoid at an angle of $\alpha = 30°$ to its axis. The number of solenoid ampere turns is 5000 and its length is 25 cm. Find the pitch of the helical trajectory of the electron in the magnetic field of the solenoid.

{ $l = 3.94 \times 10^{-2}$ m $= 3.94$ cm}

§ 3.383

➢ **CONCEPT**

The magnetic due to solenoid is along the axis of solenoid.

Let charged particles enter inside the magnetic field with velocity v_0 at an angle θ with direction of magnetic field. So, the motion of charged particles will be on helical path.

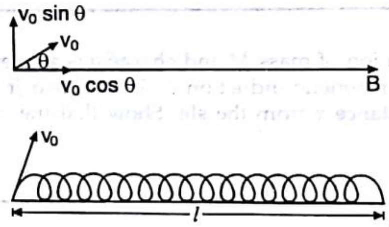


Fig. 3.383A

As we know,

$$qV = \frac{1}{2}m v_0^2$$

$\therefore$ $$v_0 = \sqrt{\frac{2qV}{m}}$$

The pitch of helical path is $$\Delta x = v_0\cos\theta \times T = v_0\cos\theta\,\frac{2\pi m}{qB}$$

$$= \frac{2\pi\, m\, v_0\cos\theta}{qB}$$

But θ is very smally.

$\therefore$ $$\cos\theta = 1$$

$\therefore$ $$\Delta x = \frac{2\pi\, m\, v_0}{qB}$$

Let for θ first magnetic field B_1 the number of turns of helical path is n.

$\therefore$ For another value of magnetic field B_2, the number of turns of helical path is $(n+1)$.

According to problem,

$$l = n\,\Delta x_1 = (n+1)\,\Delta x_2$$

or $$l = \frac{n\, 2\pi\, m\, v_0}{qB_1} = (n+1)\frac{2\pi\, m\, v_0}{qB_2} \quad ...(i)$$

or $$l = \frac{2\pi\, m\, v_0}{q}\,\frac{n}{B_1} = \frac{2\pi\, m\, v_0}{q}\,\frac{(n+1)}{B_2}$$

or $$\frac{l}{v_0} = \frac{2\pi\, m\, n}{qB_1} = \frac{2\pi\, m\,(n+1)}{qB_2}$$

or $$\frac{lq}{2\pi\, m\, v_0} = \frac{n}{B_1} = \frac{(n+1)}{B_2}$$

$\therefore$ $$n = \frac{lqB_1}{2\pi\, m\, v_0} \qquad \text{...(ii)}$$

Also $$n+1 = \frac{lqB_2}{2\pi\, m\, v_0}$$

or $$\frac{lqB_1}{2\pi\, mv_0} + 1 = \frac{lqB_2}{2\pi\, mv_0}$$

or $$\frac{lqB_2}{2\pi\, mv_0} - \frac{lqB_1}{2\pi\, mv_0} = 1$$

or $$\frac{q}{m}\left(B_2 - B_1\right)\frac{l}{2\pi\, v_0} = 1$$

or $$\frac{q}{m} = \frac{2\pi\, v_0}{l\,(B_2 - B_1)}$$

or $$\frac{q}{m} = \frac{2\pi}{l\,(B_2 - B_1)}\sqrt{\frac{2q\,V}{m}}$$

or $$\frac{q^2}{m^2} = \frac{4\pi^2}{l^2\,(B_2 - B_1)^2}\,\frac{2qV}{m}$$

or $$\frac{q^2}{m^2} = \frac{8\pi^2\, qV}{l^2\,(B_2 - B_1)^2\, m}$$

$\therefore$ $$\frac{q}{m} = \frac{8\pi^2\, V}{l^2\,(B_2 - B_1)^2}$$

YOUR STEP

A particle of mass m and charge q is lying at the origin in a uniform magnetic field B directed along x axis. At time $t = 0$, it is given a velocity v_0 at an angle θ with the y-axis in the xy plane. Find the coordinates of the particle after one revolution.

$$\left[\frac{2\pi\, m\, v_0 \sin\theta}{Bq}, 0, 0\right]$$

§ 3.384

➢ **CONCEPT**

The magnetic field due to solenoid is along the axis of the solenoid.
The charged particle enters in magnetic field with speed v.

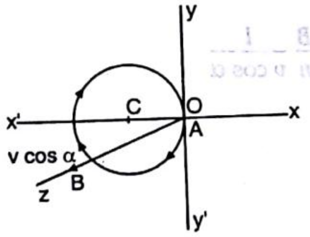


Fig. 3.384A

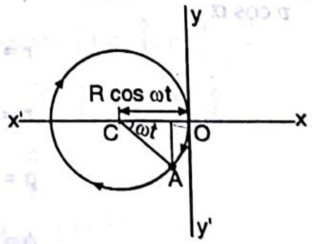


Fig. 3.384B

➢ DISCUSSION

Let the axis of the solenoid is along the z-axis. The charged particle enters into magnetic field at an angle α with the axis of solenoid.

The magnetic force $(q\,\vec{v}\times\vec{B})$ on the charged particle should be in a plane perpendicular to magnetic field. Since, magnetic field is along negative z-axis. So, magnetic force on the charged particle is in $x-y$ plane.

The component of velocity in $x-y$ plane is $v_0 = v\sin\alpha$. Since, magnetic force is perpendicular to v_0. So, the path of particle in $x-y$ plane is circular. The path of charged particle is shown in fig. 3.384 B. The position of charged particle at an instant t is $D\,(x, y, z)$.

The radius of circular path is

$$R = \frac{m\,v_0}{qB} = \frac{m\,v\sin\alpha}{qB}$$

and

$$\omega = \frac{qB}{m}$$

$$x = -(R - R\cos\omega t)$$

$\therefore$ $$x = R(\cos\omega t - 1) \qquad \text{...(i)}$$

$$y = -R\sin\omega t \qquad \text{...(ii)}$$

$\therefore$ $$v_z = \frac{dy}{dt} = -R\,\omega\cos\omega t$$

Also, $$v_z = v_0\cos\alpha$$

$\therefore$ $$z = v_z\,t = vt\cos\alpha$$

But screen is perpendicular to z-axis (*i.e.* in *x-y* plane).

Here position of screen is $$z = l$$

$\therefore$ $$l = v\,t_0\cos\alpha$$

$\therefore$ $$t_0 = \frac{l}{v\cos\alpha}$$

Let the particle strikes on screen at point $p\,(x_0, y_0, l)$, from eq. (i) and (ii)

$\therefore$ $$r = \sqrt{x_0^2 + y_0^2}$$

$$= \sqrt{\{R(\cos\omega t_0 - 1)\}^2 + (-R\sin\omega t_0)^2}$$

$$= R\sqrt{1 + \cos^2\omega t_0 - 2\cos\omega t_0 + \sin^2\omega t_0}$$

$$= R\sqrt{2 - 2\cos\omega t_0} = 2R\sin\frac{\omega t_0}{2}$$

Here, $$R = \frac{m\,v\sin\alpha}{qB}$$

$\omega = \frac{qB}{m}$, $t_0 = \frac{l}{v\cos\alpha}$ and $q = e$

$\therefore$ $$r = 2\frac{m\,v\sin\alpha}{eB}\sin\frac{eB}{m}\frac{l}{v\cos\alpha}$$

$$r = 2\rho\sin\frac{\phi}{2}$$

Here $$\rho = \frac{m\,v}{eB}\sin\alpha$$

$$\phi = \frac{leB}{m\,v\cos\alpha}$$

YOUR STEP

A beam of electrons whose kinetic energy is K emerges from a thin-foil "window" at the end of an accelerator tube. There is a metal plate a distance d from this window and at right angles to the direction of the emerging beam. See fig. 3.383C.

Show that we can prevent the beam from hitting the plate if we apply a magnetic field B such that

$$B \geq \sqrt{\frac{2mK}{e^2 d^2}}$$

in which m and e are the mass and charge of electron

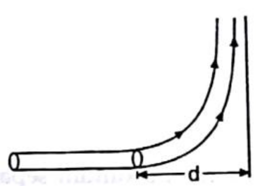


Fig. 3.384C

§ 3.385

➢ **CONCEPT**

This is an example of motion of charged particle in non-uniform magnetic field

Here $\vec{F} = q\,\vec{v} \times \vec{B}$

Let the wire is placed along y-axis.

The charged particle is projected from point A $(a, 0, 0)$ with velocity v_0 along x-axis. The magnetic field due to wire at point A is $B = \frac{\mu_0 I}{2\pi a}$ directed along positive z-axis. Due to this magnetic field, a magnetic force acts on the electron along y-axis at point A. So, the resultant motion of electron is on a curve path in x-y plane. At every point during motion of electron, magnetic force is perpendicular to the velocity of electron. Hence, the magnitude of velocity v_0 remains constant.

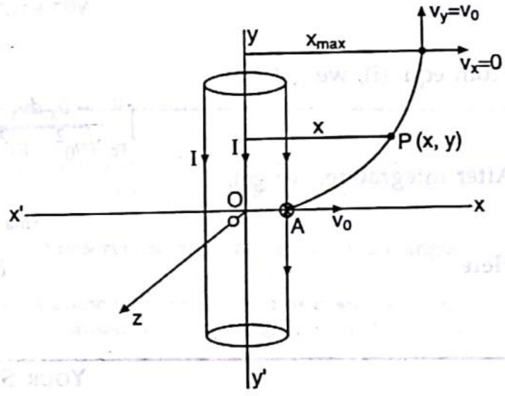


Fig. 3.385A

SOLUTION : Let at an instant t, the electron is at point $P(x, y)$.

The magnetic field at the point P is

$$B = \frac{\mu_0 I}{2\pi x}(\hat{k})$$

The velocity of electron at point p is

$$\vec{v} = v_x\,\hat{i} + v_y\,\hat{j}$$

Here $$v_x^2 + v_y^2 = v_0^2 = \text{constant}$$

$\therefore$ $$v_y^2 = v_0^2 - v_x^2$$

The magnetic force on the electron at point P is

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

$$= (-e)(v_x\,\hat{i} + v_y\,\hat{j}) \times \frac{\mu_0 I}{2\pi x}\hat{k}$$

$$= \frac{e\,v_x\,\mu_0 I}{2\pi x}\hat{j} - \frac{e\,v_y\,\mu_0 I}{2\pi x}\hat{i}$$

$\therefore$ $$F_x = -\frac{e\,\mu_0 I\,v_y}{2\pi x}$$

or $$w_x = \frac{F_x}{m} = -\frac{e\,\mu_0 I\,v_y}{2\pi m x}$$

or
$$w_x = -\frac{e\,\mu_0\,I}{2\pi\,m\,x}\sqrt{(v_0^2 - v_x^2)}$$

or
$$v_x\frac{dv_x}{dx} = -\frac{e\,\mu_0\,I}{2\pi\,m\,x}\sqrt{v_0^2 - v_x^2}$$

or
$$\frac{-v_x\,dv_x}{\sqrt{v_0^2 - v_x^2}} = \frac{e\,\mu_0\,I}{2\pi\,m}\frac{dx}{x} \qquad \text{...(i)}$$

At the maximum separation, velocity v_0 should be perpendicular to x-axis.

At the maximum separation

$x = x_{max}$, $u_x = 0$,

$\therefore$
$$\sqrt{v_x^2 + v_y^2} = v_0^2$$

or
$$\sqrt{0^2 + v_y^2} = v_0^2$$

$\therefore$
$$v_y = v_0$$

From equ. (i), we get
$$\int_{v_0}^{0}\frac{-v_x\,dv_x}{\sqrt{v_0^2 - v_x^2}} = \frac{e\,\mu_0\,I}{2\pi\,m}\int_a^{x_{max}}\frac{dx}{x}$$

After integrating, we get,
$$x_{max} = a\,e^{2\pi\,m\,v_0/\mu_0\,I\,e} = a\,e^{v_0/b}$$

Here
$$b = \frac{\mu_0\,I\,e}{2\pi\,m}$$

YOUR STEP

A particle of charge q and mass m is projected from the origin with velocity $\vec{v} = v_0\,\hat{i}$ in a non-uniform magnetic field $\vec{B} = -B_0\,x\,\hat{k}$. Here v_0 and B_0 are positive constants of proper dimensions. Find the maximum positive x coordinate of the particle during its motion.

$$\left\{x_{max} = \sqrt{\frac{2m\,v_0}{B_0\,q}}\right\}$$

---

§ 3.386

➢ **CONCEPT**

Electric potential at a point between plates of capacitor is
$$\phi = \frac{V\,ln\,r/a}{ln\,b/a}$$

$\therefore$
$$E = -\frac{d\,\phi}{dr} = \frac{-V}{r\,ln\,b/a}$$

SOLUTION : Since, charged particle is moving on a circular path;

$\therefore$
$$\frac{m\,v^2}{r} = qE$$

or
$$\frac{mv^2}{r} = \frac{qV}{r\,ln\,b/a}$$

$\therefore$
$$mv^2 = \frac{qV}{ln\,b/a} \qquad \text{...(i)}$$

But due to magnetic force,
$$qvB = \frac{mv^2}{r}$$

$$\therefore \qquad r = \frac{mv}{qB}$$

$$\therefore \qquad mv = qBr \qquad \text{...(ii)}$$

Dividing equ. (i) by (ii), we get

$$\therefore \qquad \frac{mv^2}{mv} = \frac{\dfrac{qV}{\ln\dfrac{b}{a}}}{qBr}$$

$$\therefore \qquad v = \frac{V}{B\,r\ln\dfrac{b}{a}}$$

$$\therefore \qquad mv = qBr$$

$$\therefore \qquad \frac{q}{m} = \frac{v}{Br} = \frac{V}{B^2 r^2 \ln\dfrac{b}{a}}$$

YOUR STEP

In Bohr's theory of the hydrogen atom the electron can be thought of as moving in a circular orbit of radius r about the proton. Suppose that such an atom is placed in a magnetic field, with the plane of the orbit at right angles to B.

(a) If the electron is circulating clockwise, as viewed by an observer sighting along B, will the angular frequency increase or decrease.

(b) What if the electron is circulating counter clockwise. Assume that the orbit radius does not change.

(c) Show that the change in frequency of revolution caused by the magnetic field is given approximately by $\Delta v = \pm \dfrac{Be}{4\pi m}$

§ 3.387

➢ CONCEPT

Experiments show that the force $\vec{F}$ acting on a point q depends on both position and velocity. The net electro-magnetic force on the charged particle is given by

$$\vec{F} = \vec{F_e} + \vec{F_m}$$

$$\therefore \qquad \vec{F} = q\,\vec{E} + q\,\vec{v}\times\vec{B}$$

Here $\vec{F}$ is known as Lorentz force. The above expression for Lorentz force is valid for constant as well as variable fields. A distinctive feature of Lorentz force is that in the non-relavistic approximation it does not depend upon choice of the inertial reference.

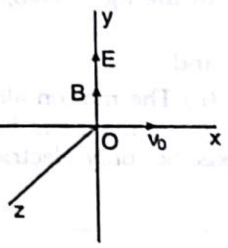


Fig. 3.387A

➢ DISCUSSION

In the case of three dimensional motion discussion of motion of particle along axis are physically independent to each other.

The motion of charged particle is divided into two parts :

(1) Motion in x-y plane :

The motion of charged particle in xz-plane is physically independent to y-axis. In this case, magnetic field is along y-axis. So, magnetic force will be in x-z plane.

This force is always perpendicular to component of velocity in x-z plane. So, the motion of charged particle is x-z plane is on a circular path with constant speed v_0.

In x-z plane :

$$qv_0 B = \frac{mv_0^2}{r}$$

or
$$qB = \frac{mv_0}{r}$$

or
$$qB = \frac{mr\,\omega}{r}$$

or
$$\omega = \frac{qB}{m}$$

$\therefore$
$$T = \frac{2\pi m}{qB}$$

The time taken by charged particle to cross the y-axis n times is

$$t_0 = nT = \frac{2\pi mn}{qB} \quad \text{...(i)}$$

The motion of charged particle is x-z plane, is shown in fig. 3.387 B.

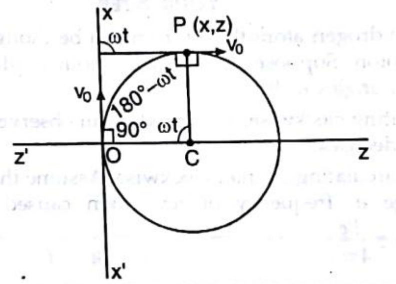


Fig. 3.387B

In the fig. 3.387B, the position of particle is P (only in x-z plane). From the figure 3.387 B.

$$v_x = v_0 \cos \omega t \quad \text{...(ii)}$$

and
$$v_z = v_0 \sin \omega t \quad \text{...(iii)}$$

(ii) The motion along y-axis

Since, magnetic field is along y-axis. So, no component of magnetic force is formed along y-axis. So, only electrical force $F_y = qE$ is present along y-axis.

$\therefore$
$$w_y = \frac{F_y}{m} \qquad \phi = \frac{qE}{m} \text{ (constant)}$$

$\therefore$
$$v_y = u_y + \omega_y t$$

But
$$u_y = 0$$

$\therefore$
$$v_y = w_y t = \frac{qE}{m} t \quad \text{...(iv)}$$

Also,
$$y = \frac{1}{2} w_y t^2 = \frac{qE}{2m} t^2 \quad \text{...(v)}$$

The resultant path of the particle is helical but pitch increases continuously. The approximate path is shown in Fig. 3.387 C.

SOLUTION : (a) From Equ. (v)

$$y = \frac{qE}{2m} t^2$$

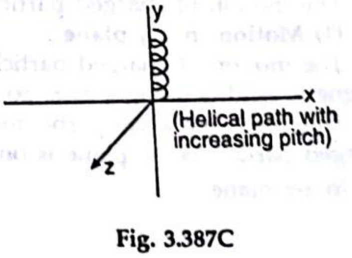


Fig. 3.387C

$\therefore \qquad y_n = \dfrac{qE}{2m} t_0^2 = \dfrac{qE}{2m}\left(\dfrac{2\pi\, mn}{qB}\right)^2$

$\therefore \qquad y_n = \dfrac{2\pi^2\, m\, En^2}{qB^2}$

(b) The velocity of particle is $\vec{v} = v_x\,\hat{i} + v_y\,\hat{i} + v_z\,\hat{k}$

$$\vec{v} = v_0 \cos \omega t\,\hat{i} + v_0 \sin \omega t\,\hat{k} + \frac{qE}{m} t\,\hat{j}$$

$\therefore \qquad v = \sqrt{(v_0 \sin \omega t)^2 + (v_0 \cos \omega t)^2 + \left(\dfrac{qE}{m} t\right)^2}$

Let the angle between velocity and y-axis is α

$\because \qquad \vec{v}\cdot\hat{j} = |\vec{v}|\,|\hat{j}| \cos\alpha$

or $\qquad v_y = v \cos\alpha$

$\therefore \qquad \cos\alpha = \dfrac{\dfrac{qE}{m} t}{\sqrt{v_0^2 \sin^2 \omega t + v_0^2 \cos^2 \omega t + \left(\dfrac{qE}{m} t\right)^2}}$

or $\qquad \cos\alpha = \dfrac{\dfrac{qE}{m} t}{\sqrt{v_0^2 + \left(\dfrac{qE}{m}\right)^2 t^2}}$

or $\qquad \tan\alpha = \dfrac{\sqrt{v_0^2 + \left(\dfrac{qE}{m}\right)^2 t^2 - \left(\dfrac{qE}{m}\right)^2 t^2}}{\dfrac{qE}{m} t} = \dfrac{v_0}{\dfrac{qE}{m} t} = \dfrac{mv_0}{qEt}$

Here $\qquad t = t_0 = \dfrac{2\pi\, mn}{qB}$

On putting the value of t, we get

$$\tan\alpha = \frac{v_0 B}{2\pi E n}$$

YOUR STEP

In a region of space, a uniform electric field strength E and a uniform magnetic field strength B exist. Both the fields are directed along positive direction of x-axis. A particle having mass m and positive charge q enters into the region. From origin at $t = 0$ at that instant velocity of the particle was equal to v_0 towards positive direction of y-axis. Neglecting gravity, calculate velocity v of the particle at time 't'.

$$\left\{ \frac{qEt}{m}\,\hat{i} + v_0 \cos\left(\frac{qBt}{m}\right)\hat{j} - v_0 \sin\left(\frac{qBt}{m}\right)\hat{k} \right\}$$

§ 3.388

➢ **CONCEPT**

The concept is similar to previous problem.
From previous problem,

$$v_x = v_0 \cos \omega t$$

or $$\frac{dx}{dt} = v_0 \cos \omega t$$

or $$\int_0^x dx = v_0 \int_0^t \cos \omega t \, dt$$

or $$x = \frac{v_0}{w} [\sin \omega t]_0^t$$

$\therefore$ $$x = \frac{v_0}{\omega} \sin \omega t \quad \text{...(i)}$$

From equ. (v) of previous problem,

$$y = \frac{qE}{2m} t^2 \quad \text{...(ii)}$$

From equation (iii) of previous problem,

$$v_z = v_0 \sin \omega t$$

$\therefore$ $$z = \int_0^t v_z \, dt = \int_0^t v_0 \sin \omega t \, dt$$

or $$z = -\frac{v_0}{\omega} [\cos \omega t]_0^t$$

$\therefore$ $$z = \frac{v_o}{\omega} [1 - \cos \omega t] \quad \text{...(iii)}$$

According to problem, $$x = l$$

or $$\frac{v_0}{\omega} \sin \omega t = l \quad \text{(from equ. (i))}$$

or $$\sin \omega t = \frac{\omega l}{v_0}$$

or $$\sin \omega t = \frac{qBl}{mv_0} \quad \left(\because \ \omega = \frac{qB}{m}\right) \quad \text{...(iv)}$$

$\therefore$ $$y = \frac{qE}{2m} t^2$$

or $$t = \sqrt{\frac{2my}{qE}}$$

or $$\omega t = \omega \sqrt{\frac{2my}{qE}}$$

or $$\omega t = \frac{qB}{m} \sqrt{\frac{2my}{qE}} \quad \left(\because \ \omega = \frac{qB}{m}\right)$$

or $$\omega t = \sqrt{\frac{2yqB^2}{mE}} \quad \text{...(v)}$$

From equ. (iii), we get $$z = \frac{v_0}{\omega} [1 - \cos \omega t]$$

or $$z = \frac{mv_0}{qB} [1 - \cos \omega t]$$

or $$\frac{qBz}{mv_0} = [1 - \cos \omega t] \quad \text{...(vi)}$$

Dividing equ. (vi) by (vi), we get

$$\frac{q\,B\,z\,m\,v_0}{m\,v_0\,q\,B\,l} = \frac{1 - \cos \omega t}{\sin \omega t}$$

or $$\frac{z}{l} = \frac{1-\cos\omega t}{\sin\omega t} = \frac{2\sin^2\frac{\omega t}{2}}{\sin\omega t}$$

or $$\frac{z}{l} = \frac{2\sin^2\frac{\omega t}{2}}{2\sin\frac{\omega t}{2}\cos\frac{\omega t}{2}}$$

$\because$ $$\frac{z}{l} = \tan\frac{\omega t}{2}$$

or $$\frac{z}{l} = \tan\frac{\sqrt{\frac{2qB^2y}{mE}}}{2}$$ (from Eqn. (v))

$\therefore$ $$z = l\tan\sqrt{\frac{qB^2y}{2mE}}$$

But $$z << l,$$

$\therefore$ $$\tan\theta = \theta$$

$\Rightarrow$ $$\tan\frac{\omega t}{2} = \frac{\omega t}{2}$$

$\therefore$ $$\frac{\omega t}{2} = \frac{z}{l}$$

or $$\omega t = \frac{2z}{l}$$

or $$\sqrt{\frac{2yqB^2}{mE}} = \frac{2z}{l}$$ (from Eqn. (v))

Squaring both sides we get

or $$\frac{2yqB^2}{mE} = \frac{4z^2}{l^2}$$

or $$z^2 = \frac{2yql^2B^2}{4mE}$$

or $$z^2 = \frac{yql^2B^2}{2mE}$$

$\therefore$ $$y = \frac{2mEz^2}{ql^2B^2}$$

This is an equation of parabola.

YOUR STEP

A magnetic field with an intensity of $H = 8\times10^3$ A/m and an electric field with an intensity of $E = 10$ W/cm are directed similarly. An electron flies into such an electromagnetic field with a velocity of $v = 10^5$ m/sec. Find the normal a_n, tangential a_t and total a accelerations of the electron. Solve the problem when
(a) the velocity of the electron is directed parallel to the force lines and
(b) the velocity of the electron is directed perpendicular to the force lines of the field.

{(a) $a_n = 0$, $a_t\, a = 1.76\times10^{14}$ m/sec^2 (b) $a_t = 0$, $a = a_n = 2.5\times10^{14}$ m/sec^2}

§ 3.389

➢ **CONCEPT**

Since, proton is moving without deviation. This is possible only when electrical force is balanced by magnetic force.

Mathematically,

$$evB = eE$$

$\therefore$ $$v = \frac{E}{B}$$

SOLUTION : $\therefore$ $$I = \frac{dq}{dt} = \frac{d(ne)}{dt}$$

or $$I = e\frac{dn}{dt}$$

where $\frac{dn}{dt}$ = number of protons strike with the target per second.

$\therefore$ $$\frac{dn}{dt} = \frac{I}{e}$$

But $$F = \frac{dp}{dt} = \frac{d(nmv)}{dt} = mv\frac{dn}{dt}$$

$$= mv\frac{I}{e} = \frac{mI}{e}\frac{E}{B}$$

$\therefore$ $$F = \frac{mEI}{eB}$$

Here m = mass of proton $= 1.67 \times 10^{-27}$ kg

e = charge on proton

On putting the values, we get $F = 20\ \mu\text{N}$

YOUR STEP

A charged dielectric sphere of negligible mass is connected to a conducting square shaped loop of total resistance R and side 'l' by means of a light and non-conducting thread, which passes over two fixed pulleys as shown in fig. 3.389A. The sphere carries a charge q and is kept in a constant and uniform electric field of strength E acting vertically downwards. The mass of the loop is m where $mg < qE$. The loop is kept in a uniform and constant horizontal magnetic field of induction B perpendicular to the plane of the loop. The system is released from rest at $t = 0$.

Find the acceleration of the loop

Take $g = 10\ \text{m/sec}^2$, $\frac{qE}{m} = 20\ \text{m/sec}^2$ and $\frac{B^2 l^2}{mR} = 2\text{N} - \text{s/m}^2$

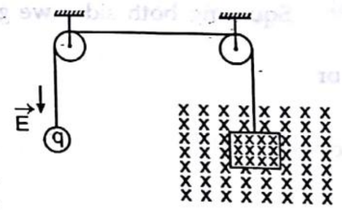


Fig. 3.389A

{ $a = 10\ \text{m/sec}^2$ }

§ 3.390

➢ **CONCEPT**

The problem is solved in two steps.

Step I : when both electric field and magnetic field are present.

In this case, acceleration of proton is zero.

$\therefore$ $$eE = evB \sin(90° - \phi)$$

$\because$ $$v = \frac{E}{B\cos\phi} \quad \text{...(i)}$$

Step II : When electric field is switched off.

In this case proton is moving on a helical path.

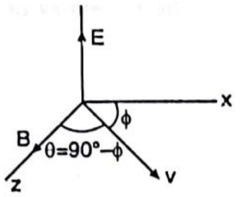


Fig. 3.390A

The time period is $$T=\frac{2\pi m}{eB}$$

The component of velocity along z-axis is

$$v_z = v\cos\theta = v\cos(90°-\phi)$$
$$= v\sin\phi$$

∴ Pitch is $$\Delta z = v_z = T$$
$$=(v\sin\phi)\frac{2\pi m}{eB}=\frac{2\pi mv\sin\phi}{eB}$$

On putting the values, we get

∴ $$\Delta z=\frac{2\pi m\sin\phi}{eB}\left(\frac{E}{B\cos\phi}\right)=\frac{2\pi mE}{eB^2}\tan\phi\left(\because\ v=\frac{E}{B\cos\phi}\right)$$

On putting the values, we get $$\Delta z=\Delta l = 6\text{ cm}$$

YOUR STEP

A positively charged particle, having charge q, is accelerated by a potential difference V. This particle moving along the x-axis enters a region where an electric field E exists. The direction of the electric field is along positive y-axis. The electric field exists in the region bonded by the lines $x=0$ and $x=a$. beyond the line $x=a$ there exists a magnetic field of strength B, directed along the positive y-axis. Find at which point does the particle meet the line $x=a$.

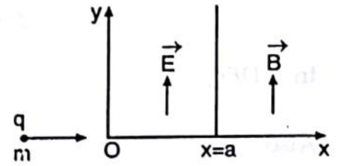


Fig. 3.390B

$$\left\{y=\frac{Ea^2}{4V}\right\}$$

§ 3.391

➢ **CONCEPT**

The problem is solved by following steps :

Step I : Discuss the motion in the presence of both electric field and magnetic field :

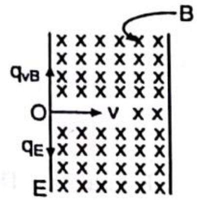


Fig. 3.391A

Since, the charged particle moves without deviation. So, magnetic force is balanced by electric force

∴ $$qE = qvB$$
$$v=\frac{E}{B} \quad \text{...(i)}$$

Step II : Discuss the motion of charged particle in region A in the absence of electric field.

Since, velocity of charged particle is perpendicular to magnetic field. So, the motion of charged particle in region A is on a circular path.

In this case, magnetic force provides required centripetal force.

∵ $$qvB=\frac{mv^2}{r}$$

∴ $$r=\frac{mv}{qB} \quad \text{...(ii)}$$

or $$r=\frac{mr\omega}{qB}$$

∴ $$\omega=\frac{qB}{m}$$

or $$\frac{2\pi}{T}=\frac{qB}{m} \quad \therefore \quad T=\frac{2\pi m}{qB} \quad \text{...(iii)}$$

The trajectory of charged particle in region A is shown in fig. 3.391 B.

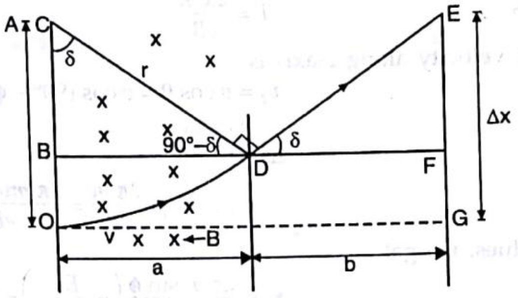


Fig. 3.391B

In triangle CBD, $\sin\delta = \dfrac{BD}{CD} = \dfrac{a}{r}$

$\therefore$ $\tan\delta = \dfrac{a}{\sqrt{r^2 - a^2}}$

In $\Delta\, DFE$, $\tan\delta = \dfrac{EF}{DF} = \dfrac{EF}{b}$

$\therefore$ $EF = b\tan\delta$

Also, $GF = OB = OC - CB = r - r\cos\delta$

$\therefore$ $\Delta x = GE = GF + EF$

$$= (r - r\cos\delta) + b\tan\delta$$

$\therefore$ $\sin\delta = \dfrac{a}{r}$

$\therefore$ $\cos\delta = \dfrac{\sqrt{r^2 - a^2}}{r}$

$\therefore$
$$\Delta x = r - r\frac{\sqrt{r^2 - a^2}}{r} + \frac{b\,a}{\sqrt{r^2 - a^2}}$$
$$= r - \sqrt{r^2 - a^2} + \frac{ab}{\sqrt{r^2 - a^2}}$$

Remarks : The answer of this problem in the book is wrong. For checking the correctness, use theHomogienity principle of dimensions.

But $r = \dfrac{mv}{qB} = \dfrac{mE}{qB^2}$ $\qquad (\because\ v = E/B)$

On putting the value of r, we get $\dfrac{q}{m} = \dfrac{2\,E\Delta x}{aB^2\,(a + 2b)}$

YOUR STEP

(a) The electron beam in the device shown in fig. 3.391C is deflected upwards by a transverse magnetic field. The field is effective along a length $l = 20$ mm the distance of the deflection system from the screen being $L = 175$ mm. The magnetic induction is 10^{-3} T, the anode voltage is 500 V. Find the deflection of the electron beam on the screen.

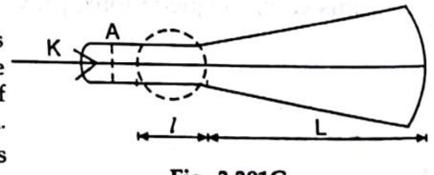


Fig. 3.391C

(b) What electric field should be set up in the device discussed in the previous problem to return the electrons to the centre of the screen?

[(a) 51 mm (b) 1.38×10^4 V/m]

§ 3.392

➢ **CONCEPT**

The concept of problem is similar to 3.387.

➢ **DISCUSSION**

At $t = 0$, the charged particle is at rest. So, magnetic force is zero but electric force acts along y-axis, which accelerates the charged particle along y-axis. When $t > 0$, charged particle picks up speed, variable magnetic force starts to act which has a tendency to make curve the path of particle back around towards x-axis.

Let the velocity of charged particle at point P is

$$\vec{\mathbf{v}} = v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}}$$

So, Lorentz force on the charged particle at point P is

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

or $$\vec{\mathbf{F}} = qE\hat{\mathbf{j}} + q(v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}} \times B\hat{\mathbf{k}}$$

or $$\vec{\mathbf{F}} = qE\hat{\mathbf{j}} - qv_x B\hat{\mathbf{j}} + qv_y B\hat{\mathbf{i}}$$

$\because$ $$F_x = qv_y B$$

$\therefore$ $$w_x = \frac{qB}{m} v_y \quad \text{...(i)}$$

Also, $$F_y = qE - q v_x B$$

or $$w_y = \frac{qE - qv_x B}{m}$$

or $$\frac{dw_y}{dt} = -\frac{qB}{m}\frac{dv_x}{dt} = -\frac{qB}{m} w_x$$

or $$\frac{dw_y}{dt} = -\frac{qB}{m}\left(\frac{qB}{m} v_y\right) \quad \text{(from equ. (i))}$$

or $$\frac{dw_y}{dt} = -\frac{q^2 B^2}{m^2} v_y$$

or $$\frac{d^2 v_y}{dt^2} = -\frac{q^2 B^2}{m^2} v_y \quad \text{...(ii)}$$

In the case of simple harmonic motion,

$$\omega \alpha - y$$

or $$w = -\omega^2 y$$

or $$\frac{dv}{dt} = -\omega^2 y$$

or $$\frac{d^2 y}{dt^2} = -\omega^2 y$$

$\therefore$ $$y = A \sin(\omega t + \phi) \quad \text{...(iii)}$$

The equation (ii) is similar to equation (iii) comparing Eqn. (ii) and (iii), we get

$$\omega = \frac{qB}{m}$$

$\therefore$ $$v_y = A \sin(\omega t + \phi) \quad \text{...(iv)}$$

According to problem, at $\quad t = 0$

$w_x = 0, w_y = \frac{qE}{m}, \qquad v_x = 0, \; v_y = 0$

From equ. (iv)

$$v_y = A \sin(\omega t + \phi)$$

or $$0 = A \sin(\omega \times 0 + \phi)$$

$\Rightarrow$ $$\phi = 0$$

$\therefore$ $$v_y = A \sin \omega t$$

or $$\frac{dv_y}{dt} = A\,\omega \cos \omega t$$

or $$w_y = A\,\omega \cos \omega t$$

But at $t = 0$, $$w_y = \frac{qE}{m}$$

$\therefore$ $$\frac{qE}{m} = A\,\omega$$

$\therefore$ $$A = \frac{qE}{m\,\omega}$$

$\therefore$ $$v_y = \frac{qE}{m\,\omega} \sin\omega t$$

or $$v_y = \frac{qE}{m\frac{qB}{m}} \sin\omega t$$

$\therefore$ $$v_y = \frac{E}{B} \sin\omega t \qquad \text{...(v)}$$

$\therefore$ $$y = \int_0^t v_y\, dt = \frac{E}{B\,\omega}[1 - \cos \omega t]$$

$\therefore$ where $$\omega = \frac{qB}{m} \qquad \text{...(vi)}$$

For x-component : $$w_x = \frac{qBv_y}{m}$$

or $$w_x = \frac{qB}{m}\frac{E}{B} \sin \omega t = \frac{qE}{m} \sin \omega t$$

or $$\frac{dv_x}{dt} = \frac{qE}{m} \sin \omega t$$

or $$\int_0^{v_x} d\,v_x = \frac{qE}{m} \int_0^t \sin \omega t\, dt$$

$\therefore$ $$v_x = \frac{qE}{m\,\omega}[1 - \cos \omega t]$$

or $$v_x = \frac{E}{B}[1 - \cos \omega t] \qquad \left(\because \; \omega = \frac{qB}{m}\right)$$

$\therefore$ $$x = \int_0^t v_x\, dt = \frac{E}{B\,\omega}\left[t - \frac{\sin \omega t}{\omega}\right]_0^t$$

$\therefore$ $$x = \frac{E}{B}\left[t - \frac{\sin\omega t}{\omega}\right]$$

$\therefore$ $$x = \frac{E}{B\,\omega}[\omega t - \sin\omega t]$$

$$\therefore \qquad x = \frac{mE}{qB^2}[\omega t - \sin\omega t]$$

$$x = a(\omega t - \sin\omega t) \qquad \text{...(vii)}$$

Here $\quad a = \dfrac{mE}{qB^2}$ and $\omega = \dfrac{qB}{m}$

Discussion of trajectory :

$$\therefore \qquad x = \frac{E}{B\omega}[\omega t - \sin\omega t] \qquad \text{...(viii)}$$

and $\quad y = \dfrac{E}{B\omega}[1 - \cos\omega t\,] \qquad \text{...(ix)}$

or $\quad Y = \dfrac{mE}{qB^2}(1 - \cos\,\omega t) \qquad \left(\because\ \omega = \dfrac{qB}{m}\right)$

or $\quad y = a\,(1 - \cos\omega t)$

From equ. (viii) $\quad \sin\omega\,t = \omega\,t - \dfrac{B\,\omega\,x}{E}$

From equ. (ix) $\quad \cos\omega\,t = 1 - \dfrac{B\,\omega\,y}{E}$

$$\therefore \qquad \sin^2\omega\,t + \cos^2\omega\,t = 1$$

or $\quad \left(\dfrac{B\,\omega\,x}{E} - \omega\,t\right)^2 + \left(\dfrac{B\,\omega\,y}{E} - 1\right)^2 = 1$

or $\quad \dfrac{B^2\,\omega^2}{E^2}\left(x - \dfrac{E}{B}t\right)^2 + \dfrac{B^2\,\omega^2}{E^2}\left(y - \dfrac{E}{B\,\omega}\right)^2 = 1$

or $\quad \left(x - \dfrac{E}{B}t\right)^2 + \left(y - \dfrac{E}{B\,\omega}\right)^2 = \dfrac{E^2}{B^2\,\omega^2} \qquad \text{...(x)}$

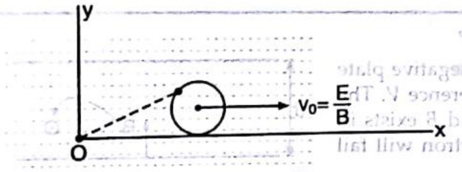


Fig. 3.392A

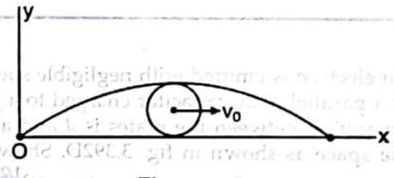


Fig. 3.392B

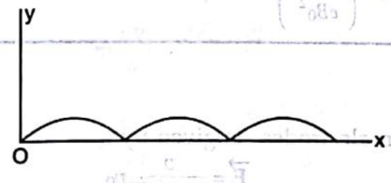


Fig. 3.392C

This is an equation of circle of radius $R = \dfrac{E}{B\,\omega}$

whose centre moves with constant velocity $v_0 = \dfrac{E}{B}$

The actual path of particle is cycloid (shown in fig. 3.392 C) defined as the path generated by a point on the circumference of a circle of radius $R = \frac{E}{B\omega}$. The motion of charged particle is similar to a point on the rim of a wheel of radius R rolling with constant speed v_0 along x-axis (shown in fig. 3.392 A). From the fig. 3.392 B, it is clear that maximum displacement of charged particle along the y-axis is equal to the diameter of rolling circle.

$$\left(i.e. = 2R = \frac{2E}{B\omega}\right)$$

(b) $\therefore$ From discussion, after one time periodic time, velocity becomes zero.

$$\because \quad \omega = \frac{qB}{m}$$

$$\therefore \quad T = \frac{2\pi m}{qB} \qquad \left(\because \ \omega = \frac{2\pi}{T}\right)$$

The instantaneous speed of charged particle is

$$v = \sqrt{v_x^2 + v_y^2}$$

Putting the values S of v_x and v_y we get

$$v = \frac{E}{B}\sqrt{2 - 2\cos\omega t} = \frac{E}{B}\sqrt{2(1 - \cos\omega t)}$$

$$= \frac{E}{B} 2 \sin\frac{\omega t}{2} = \frac{2E}{B}\sin\frac{\omega t}{2}$$

$$\therefore \quad s = \int_0^T v\, dt = \int_0^{t = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}} \frac{2E}{B} Sm \frac{\omega t}{2}$$

$$\therefore \quad s = \frac{8mE}{qB^2}$$

(c) From figure shown in 3.392 A, drift speed is equal to speed of centre of the circle

$$i.e. \quad \langle v_x \rangle = \frac{E}{B}$$

YOUR STEP

An electron is emitted with negligible speed from the negative plate of a parallel plate capacitor charged to a potential difference V. The separation between the plates is d and a magnetic field B exists in the space as shown in fig. 3.392D. Show that the electron will fail to strike the upper plate if $d > \left(\frac{2m_e V}{eB_0^2}\right)^{1/2}$

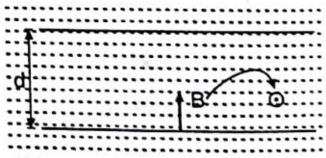


Fig. 3.392D

§ 3.393

➢ **CONCEPT**

The electric field between electrodes is given by

$$\vec{E} = \frac{v}{x\, ln\left(\frac{b}{a}\right)} v_0 \qquad \left(\text{where } \frac{V}{ln\frac{b}{a}} = V_0\right)$$

$$\therefore \quad E = \frac{V_0}{x} \qquad \text{(from anode to cathode)}$$

Electron originates from point A (at the surface of cathode)

$$\therefore \quad \vec{E} = \frac{-V_0}{x}\hat{i}$$

The magnetic field due to long wire is

$$\vec{B} = \frac{\mu_0 I}{2\pi x}\hat{j} = \frac{B_0}{x}\hat{j}$$

Here

$$B_0 = \frac{\mu_0 I}{2\pi}$$

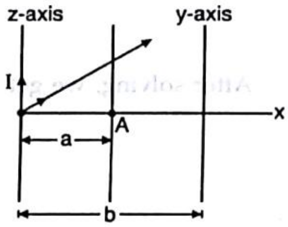


Fig. 3.393A

SOLUTION : Lorentz force on electron is

$$\vec{F} = q\,\vec{E} + q\,\vec{v}\times\vec{B}$$

Since, magnetic force and electrical force are present in x-z plane.

$\therefore$ $$\vec{v} = v_x\,\hat{i} + v_z\,\hat{k}$$

$\therefore$ $$\vec{F} = (-e)\frac{V_0}{x}(-\hat{i}) - e\,(v_x\,\hat{i} + v_z\,\hat{k})\times\frac{B_0}{x}\hat{j}$$

or $$\vec{F} = \frac{eV_0}{x}\hat{i} - \frac{e\,v_x\,B_0}{x}\hat{k} + \frac{e\,B_0\,v_z}{x}\hat{i}$$

or $$F_x = \frac{eV_0}{x} + \frac{eB_0\,v_z}{x}$$

$$m\frac{dv_x}{dt} = \frac{eV_0}{x} + \frac{eB_0\,v_z}{x} \quad \text{...(i)}$$

Also, $$F_z = -\frac{e\,B_0}{x}v_x$$

or $$m\frac{dv_z}{dt} = -\frac{e\,B_0}{x}v_x \quad \text{...(ii)}$$

or $$m\frac{dv_z}{dx}\frac{dx}{dt} = -\frac{e\,B_0}{x}v_x$$

or $$m\frac{dv_z}{dx}v_x = \frac{-e\,B_0}{x}v_x$$

or $$m\int_0^{v_z} dv_z = -e\,B_0\int_a^x \frac{dx}{x}$$

or $$mv_z = -eB_0\,ln\frac{x}{a}$$

$\therefore$ $$v_z = \frac{-e\,B_0}{m}ln\frac{x}{a} \quad \text{...(iii)}$$

From equation (i) $$m\frac{dv_x}{dt} = e\frac{V_0}{x} + \frac{eB_0\,v_z}{x}$$

or $$m\frac{dv_x}{dt} = \frac{eV_0}{x} + \frac{eB_0}{x}\left(-\frac{eB_0}{m}ln\frac{x}{a}\right)$$

or $$m\frac{dv_x}{dt} = \frac{eV_0}{x} - \frac{e^2 B_0^2\,ln\frac{x}{a}}{mx}$$

or $$mv_x\frac{dv_x}{dx} = \frac{e}{x}\left(V_0 - \frac{B_0^2 e}{m}ln\frac{x}{a}\right)$$

or $$m\int_0^{v_x} v_x\,dv_x = \int_a^x \frac{e}{x}\left(V_0 - \frac{B_0^2 e}{m}ln\frac{x}{a}\right)dx$$

Since, electron just reaches at the anode,

$\therefore$ $v_x = 0$, when $x = b$

$$\therefore \quad m\int_0^0 v_x\,dv_x = \int_a^b \frac{e}{x}\left(V_0 - \frac{B_0^2 e}{m}\ln\frac{x}{a}\right)dx$$

After solving, we get

$$V_0 = \frac{2e}{m}B_0^2 \ln\left(\frac{a}{b}\right)$$

$$\therefore \quad V_0 = \frac{2e}{m}\left(\frac{\mu_0 I}{4\pi}\right)^2 \ln\left(\frac{a}{b}\right) \qquad \left(\because B_0 = \frac{\mu_0 I}{4\pi}\right)$$

YOUR STEP

A charged particle of mass m and charge Q has acquired a certain velocity by passing through a potential difference U_0. With this velocity it flies into the field of a parallel plate capacitor, with the distance between the plates being l, the potential difference being U. The velocity of the particle is directed parallel to the plates the particle move along a straight line in the capacitor be directed and what should its value be (the induction B)?

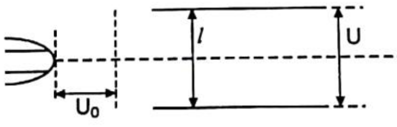


Fig. 3.393B

$$\left\{B = \frac{U}{l}\sqrt{\frac{m}{2QU_0}}\right\}$$

§ 3.394

➢ **CONCEPT**

If electron is accelerated through a potential difference V.

$$\therefore \quad eV = \frac{1}{2}mv^2$$

$$\therefore \quad v = \sqrt{\frac{2eV}{m}} \qquad \text{...(i)}$$

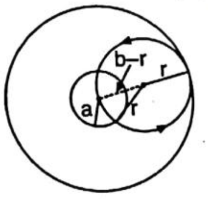


Fig. 3.394A

SOLUTION : The magnetic field maintains circular path for electron of radius r. This circle touches the anode. From fig. 3.394 A,

$$a^2 + r^2 = (b-r)^2$$

or

$$b^2 - a^2 = 2br$$

$$\therefore \quad r = \frac{b^2 - a^2}{2b} \qquad \text{...(ii)}$$

For circular motion

$$evB = \frac{mv^2}{r}$$

$$\therefore \quad r = \frac{mv}{eB} \qquad \text{...(iii)}$$

From equation (i), (ii) and (iii), we get

$$B = \left(\frac{2b}{b^2 - a^2}\right)\sqrt{\frac{2mV}{e}}$$

YOUR STEP

Deuterons are accelerated in a fixed frequency cyclotron to a maximum dee orbit radius of 88 cm. The magnetic field is 1.4 tesla. Calculate the energy of the emerging deuteron beam and the frequency of the dee voltage. What change in magnetic flux density is necessary if doubly ionised helium ions are accelerated?

{36.3 MeV, 10.67 cycles/sec, 1.39 tesla}

§ 3.395

➢ **CONCEPT**

From the solution of 3.392

$$w_x = \frac{qB}{m} v_y \quad \text{...(i)}$$

$$w_y = \frac{qE - q\, v_x B}{m}$$

Here $E = E_m \cos \omega t$

$\therefore$ $$w_y = \frac{q\, E_m}{m} \cos \omega t - \frac{qB}{m} v_x$$

or $$\frac{d\, w_y}{dt} = \frac{-q\, E_m\, \omega}{m} \sin\omega t - \frac{qB}{m} w_x$$

or $$\frac{d\, w_y}{dt} = \frac{-q\, E_m\, \omega}{m} \sin \omega t - \frac{qB}{m}\frac{qB}{m} v_y$$

or $$\frac{d\, w_y}{dt} = \frac{-q\, E_m\, \omega}{m} \sin\omega t - \frac{q^2 B^2}{m^2} v_y$$

or $$\frac{d^2 v_y}{dt^2} + \frac{q\, E_m\, \omega}{m} \sin\omega t + \frac{q^2 B^2}{m^2} v_y = 0$$

Here $$\omega = \frac{qB}{m}$$

After solving these differential equation we get,

$$v_y = \frac{E_m}{2B} \sin \omega t + \frac{E_m}{2B} \omega\, t \cos \omega\, t.$$

Put the value of v_y in equation (i) we get the value of w_x.

Again, $$w_x = \frac{dv_x}{dt}$$

So integrating we get v_x *i.e.* $$v_x = \frac{E_m}{2B} \omega\, t \sin \omega\, t$$

Now, $$x = \int_0^t v_x\, dt \quad \text{and} \quad y = \int_0^t v_y\, dt$$

on solving, we get $$x = \frac{a}{2\omega^2} (\sin \omega\, t - \omega\, t \cos \omega\, t)$$

and $$y = \frac{a}{2\, \omega} t \sin \omega\, t$$

YOUR STEP

A particle of mass $m = 0.01$ kg and having negative charge $q = 0.1$ C is at rest at the origin of a co-ordinate system. In the space, a uniform magnetic field $\vec{B} = 5\hat{i}$ tesla exists. A sharp impulse $\vec{j} = 0.025$ Ns $\hat{j}$ is applied on the particle at $t = 0$. At $t = \frac{\pi}{100}$ second, it collides with another particle of equal mass, having no charge and moving with velocity $\vec{v} = 2\hat{i}$ m/s and gets stick with it calculate the co-ordinates of point at which combined particle strike a screen placed at $x = 4\pi$ cm. Neglect gravity.

{ (4π cm, -5 cm, 5 cm)}

§ 3.396

➢ **CONCEPT**

In the case of cyclotron,

$$T = \frac{2\pi m}{eB}$$

∴ Cyclotron frequency is $\nu = \frac{1}{T} = \frac{eB}{2\pi m}$

SOLUTION : ∴ $\nu = \frac{eB}{2\pi m}$

∴ $B = \frac{2\pi m\nu}{e}$ ...(i)

Also, $\frac{mv^2}{r} = evB$

∴ $r = \frac{mv}{eB}$

Similarly, $r + \Delta r = \frac{m(v + \Delta v)}{eB}$

∴ $2eV$ = change in kinetic energy $= \frac{1}{2} m (v + \Delta v)^2 - \frac{1}{2} mv^2$

On putting the value of B, we get $2eV = \frac{1}{2} m [2\pi\nu (r + \Delta r)]^2 - \frac{1}{2} m(2\pi \nu r)^2$

Here Δr is very small. So, Δr^2 may be neglected.

or $$2eV = \frac{1}{2} m \, 4\pi^2 \nu^2 (r^2 + 2r\,\Delta r - r^2)$$

∴ $$V = \frac{2\pi^2 \nu^2 mr\Delta r}{e}$$

On putting the values, we get

$$V = 0.10 \text{ MV}$$

YOUR STEP

The university of Pittsburgh cyclotron is normally adjusted to accelerate dueterons.

(a) What energy of protons could it produce, using the same oscillator frequency as that used for dueterons?

(b) What magnetic field would it require?

(c) What energy of protons could be produced if the magnetic field was left at the value used for deutrons?

(d) What oscillator frequency could then be required?

(a) 8.5 MeV
(b) 0.80 tesla
(c) 34 meV
(d) 24 m cycle/sec

§ 3.397

➢ **CONCEPT**

The concept is similar to previous problem

SOLUTION : (a) Here $R = \frac{mv}{eB}$

∴ $$v = \frac{eRB}{m}$$

∵ $$\text{K.E.} = T = \frac{1}{2} mv^2$$

$\therefore$ $$T_{max} = \frac{1}{2} m \cdot v_{max}{}^2$$

$$= \frac{1}{2} m \left(\frac{e R_{max} B}{m} \right)^2$$

According to problem $R_{max} = r$

$\therefore$ $$T_{max} = \frac{(erB)^2}{2m}$$

On putting the values, we get $T_{max} = 12$ MeV

(b) $$T = \frac{1}{2} m v^2$$

$\therefore$ $$v = \sqrt{\frac{2T}{m}}$$

or $$r \omega = \sqrt{\frac{2T}{m}}$$

or $$R \, 2\pi \nu = \sqrt{\frac{2T}{m}}$$

or $$\nu = \frac{\sqrt{\frac{2T}{m}}}{2\pi R}$$

or $$\nu_{min} = \frac{\sqrt{2T/m}}{2\pi R_{max}}$$

or $$\nu_{min} = \frac{\sqrt{2T/m}}{2\pi r} \qquad (\because R_{max} = r)$$

or $$\nu_{min} = \frac{1}{\pi r} \sqrt{\frac{T}{2m}}$$

On putting the values, we get $\nu_{min} = 20$ MHz

YOUR STEP

In a synchrocyclotron producing 400 MeV protons what must the ratio be of the oscillator frequency at the begining of an accelerating cycle to that at the end ? Such a proton has a speed of 0.7 c, where c is the speed of light.

{1.4}

§ 3.398

➢ **CONCEPT**

SOLUTION : (a) Let n be the total number of transits across the gap and Δr be the increase in radius in each transit.

$\therefore$ $$t = \frac{nT}{2} \text{ and } n = \frac{R}{\Delta r}$$

$\therefore$ $$t = \frac{RT}{2\Delta r}$$

But $$T = \frac{1}{\nu}$$

$\therefore$ $$t = \frac{R}{2\nu \, \Delta r}$$

From the solution of problem 3.396

$$V = \frac{2\pi^2 \nu^2 mr\,\Delta r}{e}$$

$$\therefore \quad t = \frac{\pi^2 \nu\, m\, r^2}{eV}$$

On putting the values, $t = 17\ \mu s$

(b) Here $v_n = \sqrt{\frac{2evn}{m}}$

When v_n be the velocity of the nth passage.

$$\therefore \quad S = \Sigma\, v_n \frac{T}{2}$$

$$\therefore \quad S = \Sigma \sqrt{2\frac{Ven}{m}}\, \frac{T}{2}$$

But $n = \frac{R}{\Delta r} = \frac{2\pi^2 R^2 \nu^2 m}{eV}$

$$\therefore \quad S = \sqrt{\frac{eV}{2m}}\, T\, \Sigma_1^n \sqrt{n}$$

By using integration $S = \sqrt{\frac{eV}{2m}}\, T \int_0^n \sqrt{n}\, dx = \sqrt{\frac{eV}{2m}} \cdot \frac{1}{\nu}\left(\frac{2}{3}\right) n^{3/2}$

On putting the value of n $S = \frac{4\pi^3 \nu^2 m r^2}{3eV} = 0.74\ \text{km}$

YOUR STEP

Estimate the total path length transversed by a deutron in the university of Pittsburgh cyclotron during the acceleration process. Assume an accelerating potential between the dees of 80000 volt.

{232 metre}