Spark Spread Option Pricing: kirk Approximation

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Abstract

This paper presents a pricing model for spark spread options written on electricity and gas forward prices. The Margrabe exchange option formula is employed for cases where the strike price is zero, and Kirk's approximation is used for non-zero strike prices. We derive Kirk's approximate closed-form solution for the spread option price directly from Margrabe's exchange option formula, clarifying the link between these two models and addressing prior concerns about the absence of this explicit derivation.

1 Model Assumptions for Spark Spread Options

In the context of energy markets, a spark spread option is a derivative based on the forward price difference between electricity and natural gas. The payoff is typically based on the $\tilde{T}$-forward price of electricity, denoted by $F(T, \tilde{T})$, and a scaled version of the $\tilde{T}$-forward price of natural gas, given by $h \cdot G(T, \tilde{T})$, where h is the heat rate representing the efficiency of converting gas into electricity.

In this analysis, since the maturity of the forward contracts does not explicitly impact the pricing calculations, we simplify the notation by setting:

- $F(t) = F(t, \tilde{T})$
- $G(t) = G(t, \tilde{T})$

This allows us to streamline the formulation without loss of generality.

We assume that the forward prices $F(T)$ and $G(T)$ follow stochastic processes driven by Wiener Process with zero drift under the risk-neutral probability measure Q.
Specifically, the dynamics of the forward prices are modeled as:

$$dF(t) = \sigma_F F(t) dW_F(t) \tag{1}$$

$$dG(t) = \sigma_G G(t) dW_G(t) \tag{2}$$

which solutions are given by :

$$F(t) = F(0) \exp\left(-\frac{1}{2}\sigma_F^2 t + \sigma_F W_F(t)\right)$$

$$G(t) = G(0) \exp\left(-\frac{1}{2}\sigma_G^2 t + \sigma_G W_G(t)\right)$$

where σ_F and σ_G are the volatilities of the forward prices of electricity and gas, respectively, and $W_F(t)$ and $W_G(t)$ are correlated Wiener Process with correlation coefficient ρ.

The payoff of the spark spread option at maturity T is given by:

$$C_T = (F(T) - h \cdot G(T) - K)^+ \tag{3}$$

Where K is the strke price.

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Under the risk-neutral probability measure :

$$C_0 = e^{-rT}\mathbb{E}^Q[(F(T) - h \cdot G(T) - K)^+] \tag{4}$$

2 The Special Case of Zero Strike Price

We begin by considering the simplest case of a spark spread option, where the strike price is set to zero ($K = 0$). In this scenario, the payoff at maturity simplifies to:

$$C_T = (F(T) - h \cdot G(T))^+.$$

This is analogous to a standard exchange option, where one asset (electricity) is exchanged for another (natural gas, scaled by h).

The expression of C_0 in (4) become:

$$\begin{aligned} C_0 &= e^{-rT}\mathbb{E}^Q(F(T) - h \cdot G(T))^+] \\ &= e^{-rT}\mathbb{E}^Q\left[G(T)(\frac{F(T)}{G(t)} - h)^+\right] \end{aligned}$$

Applying a probability measure changing using Radon-Nikodym derivative

$$\frac{dQ'}{dQ} = \exp\left(-\frac{1}{2}\sigma_G^2 t + \sigma_G W_G(t)\right)$$

$$\begin{aligned} C_0 &= e^{-rT}\mathbb{E}^{Q'}\left[G(T)\left(\frac{F(T)}{G(t)} - h\right)^+ \frac{dQ}{dQ'}\right] \\ C_0 &= e^{-rT}G(0)\mathbb{E}^{Q'}\left[\left(\frac{F(T)}{G(t)} - h\right)^+\right] \end{aligned} \tag{5}$$

let $X(t) := \frac{F(T)}{G(t)}$, Using Ito's Lemma and Eqs (1) and (2) :

$$\begin{aligned} dX(t) &= \frac{1}{G(t)}dF(t) + \frac{-F(t)}{G(t)^2}dG(t) + \frac{-1}{G(t)^2}F(t)G(t)\rho\sigma_F\sigma_G dt + \frac{2F(t)}{G(t)^3}\frac{G(t)^2\sigma_G^2}{2}dt \\ &= X(t)\left(\sigma_F dW_F - \sigma_G dW_G - \rho\sigma_G\sigma_F dt + \sigma_G^2 dt\right) \\ &= X(t)\left(\sigma_F(\rho dW_G + \sqrt{1-\rho^2}dW^\perp) - \sigma_G dW_G - \rho\sigma_G\sigma_F dt + \sigma_G^2 dt\right) \\ &= X(t)\left((\rho\sigma_F - \sigma_G)(dW_G - \sigma_G dt) + \sqrt{1-\rho^2}\sigma_F dW^\perp\right) \end{aligned} \tag{6}$$

where $W^\perp$ is a Wiener Process independent of W_G
Girsanov's Theorem gives us :

$W_{\tilde{G}} = W_G - \sigma_G t$ a Q' Wiener Process.

eq (6) becomes:

$$dX(t) = X(t)\left((\rho\sigma_F - \sigma_G)dW_{\tilde{G}} + \sqrt{1-\rho^2}\sigma_F dW^\perp\right) \tag{7}$$

Notice that since the independence of $W^\perp$ and W_G, this measure changing doesnt impact it, also $W_{\tilde{G}}$ and $W^\perp$ are independent.
Taking

$$W := \frac{(\rho\sigma_F - \sigma_G)W_{\tilde{G}} + \sqrt{1-\rho^2}\sigma_F W^\perp}{\sqrt{\sigma_F^2 + \sigma_G^2 - 2\rho\sigma_F\sigma_G}} \quad \text{(by definition a Wiener Process)}$$

Finally, replacing in (7) :

$$dX(t) = X(t)\sqrt{\sigma_F^2 + \sigma_G^2 - 2\rho\sigma_F\sigma_G}dW = X(t)\sigma \cdot dW$$

with

$$\sigma := \sqrt{\sigma_F^2 + \sigma_G^2 - 2\rho\sigma_F\sigma_G}$$

As can be sen, $X(t)$ is a lognormal process, so we can use the Black & Scholes closed formula to solve eq (5), this result is the Margrabe[1] closed formula for spread option.

$$C_0 = e^{-rT}\left(F(0)N(d) - G(0)hN(d-\sigma\sqrt{T})\right) \quad , \quad d = \frac{ln\left(\frac{F(0)}{G(0)}\right) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \tag{8}$$

3 Pricing with a Non-Zero Strike Price

In the previous section, we analyzed the spark spread option for the case where the strike price is zero. Now, we extend this to the more general case where the strike price is non-zero ($K \neq 0$).

Ewan Kirk[2], in 1995, introduced a simple but powerful approximation technique for pricing such options by extending the exchange option model to include a strike price.

We begin by expressing the payoff at maturity T as:

$$C_T = \max\{F(T) - h \cdot G(T) - K, 0\}.$$

To tackle this problem, Kirk proposed an approximation based on modifying the Margrabe exchange option formula. Let us define an adjusted forward price for electricity:

$$\tilde{G}(t) := hG(t) + K$$

The payogg become:

$$C_T = \max\{F(T) - \tilde{G}(T), 0\}.$$

While it might seem tempting to apply Margrabe's formula directly, the complication arises because $\tilde{G}(T)$ is not a lognormal process.

To deal with this, we apply Ito's Lemma to compute the dynamics of $\tilde{G}(t)$:

$$\begin{aligned} d\,\tilde{G}(t) &= h\,dG(t) \\ &= \tilde{G}(t)\,\underbrace{\frac{h\,G(t)}{h\,G(t)+K}\,\sigma_G}_{\tilde{\sigma}_G}\,dW_G \end{aligned}$$

The idea is that when $hG(t) >> K$, $\tilde{G}$ is supposed to be a driftless lognormal process:

$$\frac{d\tilde{G}(t)}{\tilde{G}(t)} \approx \tilde{\sigma}_G dW_G$$

where

$$\tilde{\sigma}_G := \frac{hG(0)}{hG(0)+K}\,\sigma_G$$

[1] Margrabe, W. (1978). The Value Of An Option To Exchange One Asset For Another. The Journal of Finance, 33(1)

[2] Kirk, E. (1995). Correlation in the Energy Markets. In, Robert Jameson (Ed.), Managing Energy Price Risk (pp. 71-78). London: Risk Publications and Enron Capital & Trade Resources

Thus, the approximate price of the spark spread option at time $t = 0$ is given by the Margrabe Formula in (8):

$$\boxed{C_0 = e^{-rT} \left[F(0)\, N(\tilde{d}) - (h\, G(0) + K)\, N(\tilde{d} - \tilde{\sigma}\sqrt{T}) \right]} \tag{9}$$

where

$$\tilde{d} = \frac{\ln\left(\frac{F(0)}{h\, G(0)+K}\right) + \frac{1}{2}\tilde{\sigma}^2 T}{\tilde{\sigma}\sqrt{T}}, \tag{10}$$

and $\tilde{\sigma}$ is the effective volatility, as before:

$$\tilde{\sigma} = \sqrt{\sigma_F^2 + \tilde{\sigma}_G^2 - 2\sigma_F \tilde{\sigma}_G \rho} \tag{11}$$

This approximation provides a practical way to price spark spread options in the presence of a non-zero strike price. By assuming the volatility $\tilde{\sigma}$ remains approximately constant, Kirk's formula delivers a computationally efficient method that works well in many real-world energy market scenarios.

3.1 Numerical Results

K	ρ	**Kirk**	**Monte-Carlo**	**Error (%)**
5	0.7	3.9906	3.9935	0.0741
	0.8	3.2254	3.2241	0.0396
	0.9	2.3412	2.3393	0.0807
	0.999	1.2735	1.2629	0.8324
10	0.7	2.5702	2.5689	0.0505
	0.8	1.9403	1.9380	0.1203
	0.9	1.2618	1.2554	0.5140
	0.999	0.5560	0.5358	3.7635
20	0.7	0.9785	0.9790	0.0534
	0.8	0.6401	0.6349	0.8228
	0.9	0.3358	0.3252	3.2405
	0.999	0.1052	0.0884	18.9102

Table 1: Comparison of Kirk approximation and Monte-Carlo simulation

Comments:

- When K is small (0.7, 0.8, 0.9) and ρ is not close to 1 (0.7, 0.8, 0.9), the Kirk approximation appears to be quite accurate. The percentage error is relatively low, ranging from 0.13% to 0.33%.
- However, as K increases to 10 and 20, and ρ approaches 1, the percentage error starts to grow significantly:

This indicates that the Kirk approximation starts to break down and become less accurate when the strike price (K) is not small and the correlation (ρ) is close to 1. The key takeaway is that the Kirk approximation, while simple and useful for certain parameter values, may not be reliable when the strike price is relatively high, and the correlation between the underlying assets is close to 1.

3.2 Modified Kirk's Approximation.

In this section, we will present the modified Kirk's Approximation [3], that can be applied to spark spread option pricing when K is not small and ρ is close to 1. The new Approximation is defined this way:

$$\boxed{C_0 = e^{-rT}\left[F(0)\;N(\tilde{d}_I) - (h\;G(0)+K)\;N(\tilde{d}_I - I\sqrt{T})\right]} \tag{12}$$

where

$$\tilde{d}_I := \frac{\ln\left(\frac{F(0)}{h\;G(0)+K}\right) + \frac{1}{2}I^2T}{I\sqrt{T}}$$

$$I := \tilde{\sigma} + \frac{1}{2}\cdot(\tilde{\sigma}_G - \rho\sigma_F)^2 \cdot \frac{1}{\tilde{\sigma}^3}\cdot\sigma_G^2\cdot\frac{hG(0)K}{(hG(0)+K)^2}\cdot ln\left(\frac{F(0)}{hG((0)+K}\right)$$

and $\tilde{\sigma}$ is the effective volatility, as before:

$$\tilde{\sigma} = \sqrt{\sigma_F^2 + \tilde{\sigma}_G^2 - 2\sigma_F\tilde{\sigma}_G\rho}$$

Numerical results : Modified Kirk's Approximation

K	ρ	Kirk	Mod Kirk	Monte-Carlo (95% CI)	Error (%)	Mod Err (%)
5	0.7	3.9906	3.9905	3.9935 (3.9900, 3.9971)	0.0741	0.0753
	0.8	3.2254	3.2249	3.2241 (3.2211, 3.2272)	0.0396	0.0258
	0.9	2.3412	2.3392	2.3393 (2.3369, 2.3417)	0.0807	0.0055
	0.999	1.2735	1.2642	1.2629 (1.2614, 1.2645)	0.8324	0.0959
10	0.7	2.5702	2.5699	2.5689 (2.5659, 2.5718)	0.0505	0.0389
	0.8	1.9403	1.9385	1.9380 (1.9356, 1.9404)	0.1203	0.0281
	0.9	1.2618	1.2555	1.2554 (1.2536, 1.2572)	0.5140	0.0095
	0.999	0.5560	0.5360	0.5358 (0.5348, 0.5369)	3.7635	0.0353
20	0.7	0.9785	0.9771	0.9790 (0.9772, 0.9808)	0.0534	0.1948
	0.8	0.6401	0.6354	0.6349 (0.6335, 0.6363)	0.8228	0.0844
	0.9	0.3358	0.3251	0.3252 (0.3243, 0.3262)	3.2405	0.0464
	0.999	0.1052	0.0882	0.0884 (0.0880, 0.0889)	18.9102	0.2287

Table 2: Comparison of Kirk approximation, Modified Kirk, and Monte-Carlo simulation

The authors have modified the volatility parameter in the Kirk's Approximation formula to achieve a more accurate approximation that remains easy and simple to apply. This modified version of the Kirk's Approximation formula is a highly improved technique for pricing spark spread options, particularly when the strike price (K) is not small and the correlation (ρ) is close to 1. The improvement offered by this modified formula yields significantly lower errors compared to the original Kirk's formula.

[3] Alòs, E., & León, J. A. (2015). On the short-maturity behaviour of the implied volatility skew for random strike options and applications to option pricing approximation. Quantitative Finance, 16(1), 31-42.

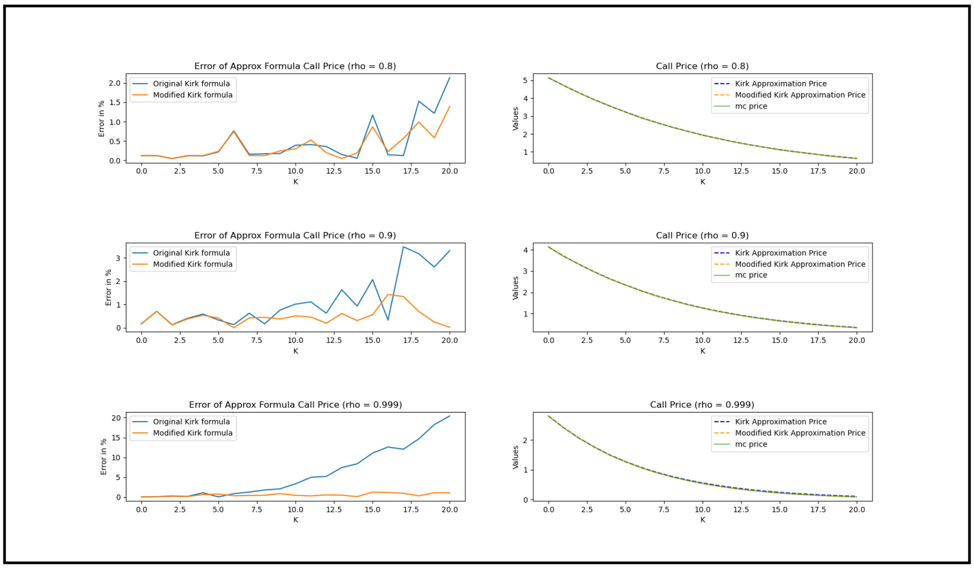

Figure 1: Kirk Approximation vs Modified Kirk Approximation

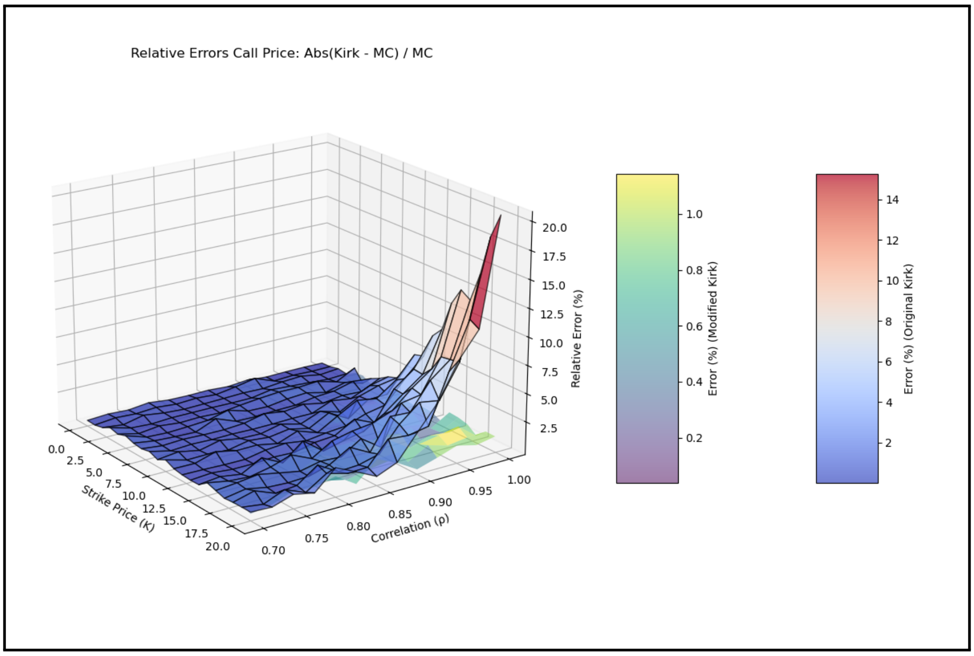

Figure 2: Surface of Errors (%) : Kirk Approximation vs Modified Kirk Approximation

4 Delta Hedging in Spark Spread Options

Delta hedging is a fundamental risk management strategy used by option traders to mitigate the sensitivity of an option's price to small changes in the price of the underlying asset. Specifically, the **delta** of an option represents the rate of change of the option's price with respect to a change in the price of the underlying asset. For a spark spread option, the delta can be defined with respect to both the forward price of electricity $F(T)$ and the forward price of gas $G(T)$, as the option's payoff depends on the difference between these two.

4.1 Kirk Approximation : Delta of a Spark Spread Option

For a spark spread option, we define two deltas: one with respect to the electricity price (Δ_F) and one with respect to the gas price (Δ_G). These deltas measure how the price of the option changes with small movements in the respective forward prices:

$$\Delta_F = \frac{\partial C_0}{\partial F(0)}, \quad \Delta_G = \frac{\partial C_0}{\partial G(0)}.$$

- Δ_F:
  Using Kirk's approximation in eq (12), the delta with respect to the forward electricity price is:

$$\boxed{\Delta_F = e^{-rT} \cdot N(\tilde{d})} \tag{13}$$

Proof:

$$\frac{\partial \tilde{d}}{\partial F(0)} = \frac{1}{F(0)\tilde{\sigma}\sqrt{T}}$$

Define

$$n(x) := N'(x) = \frac{\exp(-x^2/2)}{\sqrt{2\pi}}$$

A straightforward calculation yields:

$$n(\tilde{d} - \tilde{\sigma}\sqrt{T}) = n(d) \cdot \frac{F(0)}{hG(0) + K} \tag{14}$$

Therefore, we can express:

$$\frac{\partial C_0}{\partial F(0)} = e^{-rT} \cdot \left[N(\tilde{d}) + F(0)\frac{1}{F(0)\tilde{\sigma}\sqrt{T}}n(\tilde{d}) - (hG(0) + K)\frac{1}{F(0)\tilde{\sigma}\sqrt{T}}n(\tilde{d} - \tilde{\sigma}\sqrt{T}) \right]$$

Applying eq (14), this simplifies to:

$$\frac{\partial C_0}{\partial F(0)} = e^{-rT} N(\tilde{d})$$

- Δ_G:
  And the delta with respect to the forward gas price is:

$$\boxed{\Delta_G = e^{-rT} \left[-hN(\tilde{d} - \tilde{\sigma}\sqrt{T}) + (hG(0) + K) \cdot n(\tilde{d} - \tilde{\sigma}\sqrt{T}) \cdot \sqrt{T}\xi_G \right]} \tag{15}$$

with :

$$\xi_G := \frac{\partial \tilde{\sigma}}{\partial G(0)}$$

Proof:
Starting from the expression for $\tilde{\sigma}$ and $\tilde{\sigma}_G$, we derive:

$$\xi_G = \frac{\partial \tilde{\sigma}}{\partial G(0)} = \frac{\sigma_G K}{\tilde{\sigma}} \cdot \frac{\tilde{\sigma}_G - \rho\sigma_F}{(hG(0) + K)^2}$$

Next, consider the partial derivatives:

$$\frac{\partial \tilde{d}}{\partial G(0)} = -\frac{\xi_G}{\tilde{\sigma}} \cdot (\tilde{d} - \tilde{\sigma}\sqrt{T}) - \frac{h}{hG(0)+K} \cdot \frac{1}{\tilde{\sigma}\sqrt{T}}$$

Additionally, we find:

$$\frac{\partial (\tilde{d} - \tilde{\sigma}\sqrt{T})}{\partial G(0)} = -\frac{\xi_G}{\tilde{\sigma}} \cdot \tilde{d} - \frac{h}{hG(0)+K} \cdot \frac{1}{\tilde{\sigma}\sqrt{T}}$$

Finally, the delta with respect to the forward gas price is given by:

$$\frac{\partial C_0}{\partial G(0)} = e^{-rT}\left[-hN(\tilde{d} - \tilde{\sigma}\sqrt{T}) + (hG(0)+K) \cdot n(\tilde{d} - \tilde{\sigma}\sqrt{T}) \cdot \sqrt{T}\xi_G\right]$$

To implement delta hedging, a trader adjusts the portfolio continuously by holding a quantity of electricity and gas forward contracts proportional to the deltas, thereby offsetting the risk arising from changes in the prices of these forwards.

4.2 Modified Kirk Approximation : Delta of a Spark Spread Option

Some similar calculations as the previous section leads to: Modified Approximation to Δ_F

$$\boxed{\Delta_F = e^{-rT}\left[\cdot N(\tilde{d}) + F(0)\sqrt{T}\frac{\partial I}{\partial F(0)}\right]} \tag{16}$$

with :

$$\frac{\partial I}{\partial F(0)} = \frac{1}{2} \cdot (\tilde{\sigma}_G - \rho\sigma_F)^2 \cdot \frac{1}{\tilde{\sigma}^3} \cdot \sigma_G^2 \cdot \frac{hG(0)K}{(hG(0)+K)^2} \cdot \frac{1}{F(0)}$$

and for Δ_G:

$$\boxed{\Delta_G = e^{-rT}\left(-N(\tilde{d}_I - I\sqrt{T}) + (hG(0)+K) \cdot N(\tilde{d}_I - I\sqrt{T}) \cdot \sqrt{T}\frac{\partial I}{\partial G(0)}\right)} \tag{17}$$

with:

$$\frac{\partial I}{\partial G(0)} = \xi_G + 0.5(Add_1 + Add_2 + Add_3 + Add_4)$$

$$Add_1 = \frac{K}{(hG(0)+K)^2}\sigma_G(\tilde{\sigma}_G - \rho\sigma_F)\frac{1}{\tilde{\sigma}^3}\tilde{\sigma}_G\frac{\sigma_G K}{hG(0)+K}(X_t - Y_t)$$

$$Add_2 = \xi_G \cdot \frac{-3}{\tilde{\sigma}^4}(\tilde{\sigma}_G - \rho\sigma_F)^2\tilde{\sigma}_G\frac{\sigma_G K}{hG(0)+K}(X_t - Y_t)$$

$$Add_3 = \left(\frac{K}{(hG(0)+K)^2} - \frac{2hG(0)K}{(hG(0)+K)^3}\right)(\tilde{\sigma}_G - \rho\sigma_F)^2\frac{1}{\tilde{\sigma}^3}\sigma_G^2(X_t - Y_t)$$

$$Add_4 = -\frac{1}{hG(0)+K}(\tilde{\sigma}_G - \rho\sigma_F)^2\frac{1}{\tilde{\sigma}^3}\tilde{\sigma}_G\frac{\sigma_G K}{hG(0)+K}$$

4.3 Numerical Results

4.3.1 Delta F

K	ρ	Kirk	Modified Kirk	Monte-Carlo	Error (%)	Modified Error (%)
5	0.7	0.3993	0.3993	0.3991 (0.3988, 0.3993)	0.0712	0.0733
	0.8	0.3769	0.3770	0.3766 (0.3764, 0.3769)	0.0821	0.1024
	0.9	0.3423	0.3427	0.3425 (0.3422, 0.3427)	0.0337	0.0595
	0.999	0.2755	0.2766	0.2760 (0.2758, 0.2762)	0.1918	0.2091
10	0.7	0.2892	0.2892	0.2892 (0.2889, 0.2894)	0.0015	0.0076
	0.8	0.2567	0.2568	0.2565 (0.2563, 0.2567)	0.0714	0.1038
	0.9	0.2101	0.2102	0.2100 (0.2098, 0.2102)	0.0315	0.1010
	0.999	0.1356	0.1348	0.1345 (0.1343, 0.1347)	0.7877	0.2268
20	0.7	0.1338	0.1338	0.1335 (0.1333, 0.1337)	0.2428	0.2153
	0.8	0.1032	0.1030	0.1029 (0.1027, 0.1030)	0.2933	0.0793
	0.9	0.0677	0.0668	0.0666 (0.0665, 0.0668)	1.5577	0.2292
	0.999	0.0297	0.0269	0.0268 (0.0268, 0.0269)	10.5586	0.2897

Table 3: Comparison of Kirk approximation, Modified Kirk, and Monte-Carlo simulation for Delta F

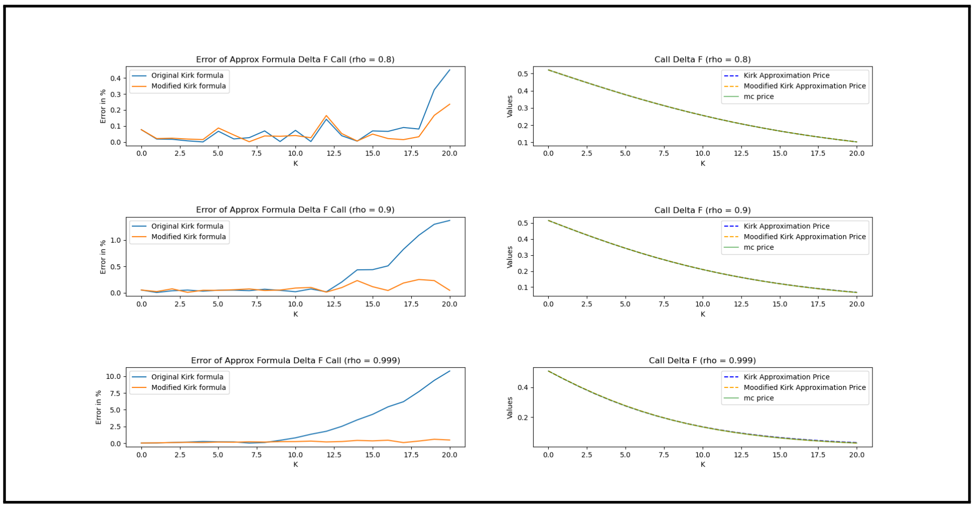

Figure 3: Delta F: Kirk Approximation vs Modified Kirk Approximation

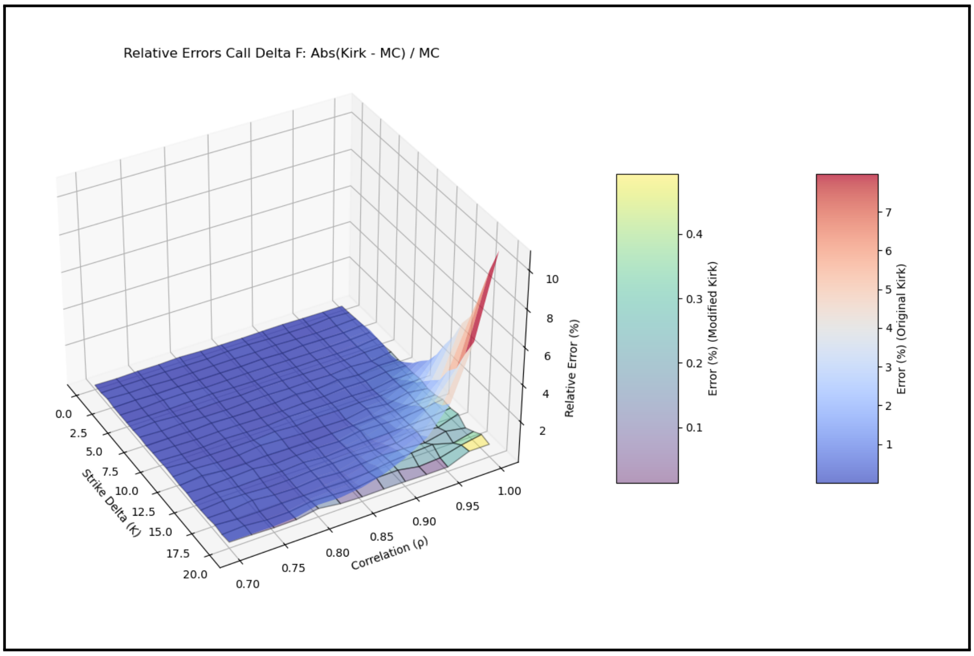

Figure 4: Delta F: Surface of Errors (%) : Kirk Approximation vs Modified Kirk Approximation

4.3.2 Delta G

K	ρ	Kirk	Modified Kirk	Monte-Carlo	Error (%)	Modified Error (%)
5	0.7	-0.3425	-0.3426	-0.3426 (-0.3428, -0.3424)	0.0089	0.0067
	0.8	-0.3289	-0.3290	-0.3288 (-0.3290, -0.3286)	0.0284	0.0504
	0.9	-0.3050	-0.3053	-0.3051 (-0.3054, -0.3049)	0.0599	0.0398
	0.999	-0.2523	-0.2534	-0.2534 (-0.2536, -0.2532)	0.4212	0.0011
10	0.7	-0.2400	-0.2400	-0.2399 (-0.2401, -0.2397)	0.0380	0.0441
	0.8	-0.2169	-0.2169	-0.2170 (-0.2172, -0.2168)	0.0760	0.0422
	0.9	-0.1814	-0.1815	-0.1815 (-0.1816, -0.1813)	0.0437	0.0308
	0.999	-0.1207	-0.1200	-0.1198 (-0.1199, -0.1196)	0.8106	0.2692
20	0.7	-0.1042	-0.1042	-0.1041 (-0.1043, -0.1040)	0.0490	0.0139
	0.8	-0.0820	-0.0818	-0.0818 (-0.0819, -0.0817)	0.2409	0.0083
	0.9	-0.0552	-0.0544	-0.0545 (-0.0546, -0.0544)	1.3001	0.0611
	0.999	-0.0252	-0.0228	-0.0227 (-0.0228, -0.0226)	10.8119	0.5431

Table 4: Comparison of Kirk approximation, Modified Kirk, and Monte-Carlo simulation for Delta F

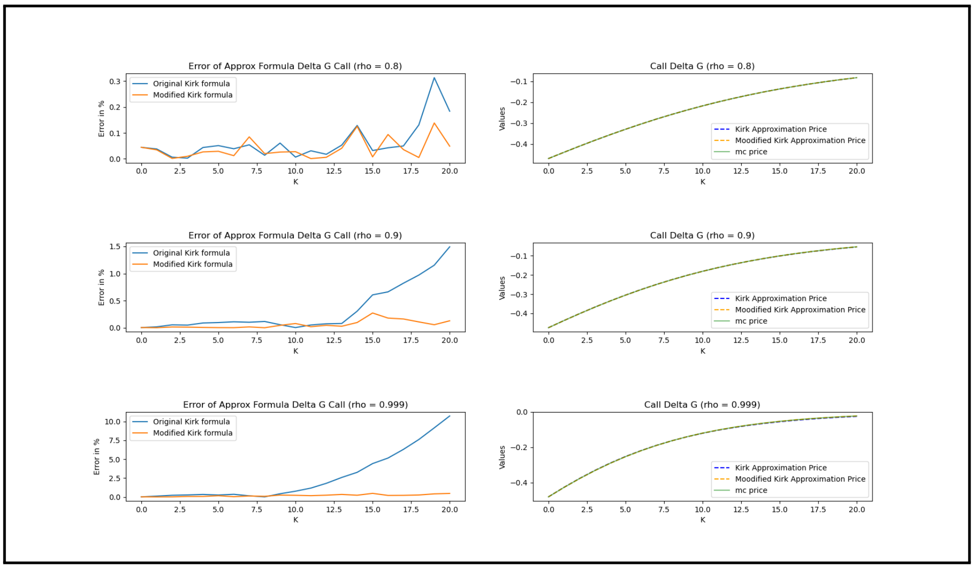

Figure 5: Delta G: Kirk Approximation vs Modified Kirk Approximation

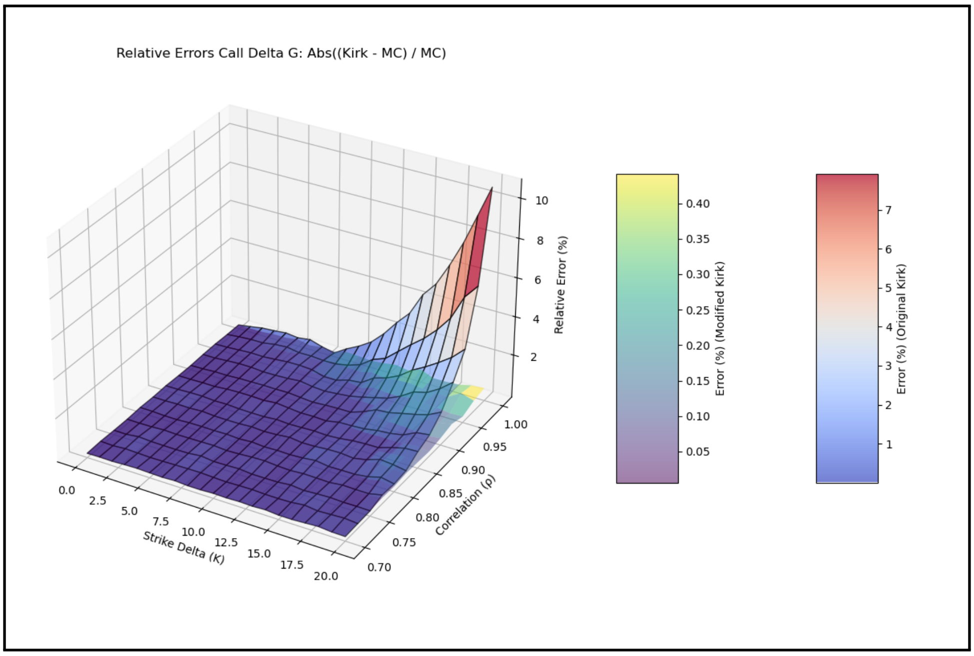

Figure 6: Delta G: Surface of Errors (%) : Kirk Approximation vs Modified Kirk Approximation

4.4 Delta Hedging Strategy

A typical delta hedging strategy for a spark spread option might involve dynamically adjusting positions in electricity forwards and gas forwards to remain delta-neutral. If the price of electricity rises, the value of the option increases, so the trader would sell some electricity forwards to offset this gain. Conversely, if the price of gas rises, the value of the option decreases, so the trader would buy some gas forwards to counterbalance the loss.

This dynamic process of buying and selling underlying assets (electricity and gas forwards) continues throughout the life of the option. The idea is to maintain a position that is locally risk-free with respect to small movements in the underlying prices, ensuring that the portfolio's value does not fluctuate significantly due to price changes in electricity and gas.

4.5 Real-World Limitations of Delta Hedging

While delta hedging is theoretically sound and widely used in financial markets, its application in the real world, particularly in energy markets like electricity and gas, can present several significant challenges:

1. Market Liquidity: Energy markets, especially electricity markets, can suffer from low liquidity. Electricity is not storable like other commodities, which can lead to periods of highly volatile prices and illiquid markets, especially during peak demand or supply constraints. In such conditions, executing the necessary trades for delta hedging may become difficult, expensive, or even impossible. Additionally, thinly traded gas markets in certain regions can also pose problems, particularly when trying to hedge large positions.

2. Transaction Costs: Delta hedging requires frequent adjustments to the portfolio to maintain a neutral position. Each adjustment incurs transaction costs, which can erode the profitability of the hedging strategy. In markets with high bid-ask spreads or other forms of friction, these costs can significantly reduce the benefits of delta hedging, especially when dealing with large quantities of electricity or gas forwards.

3. Price Jumps and Spikes: Electricity prices, in particular, are prone to sudden spikes due to unexpected changes in demand or supply, such as plant outages, extreme weather conditions, or regulatory interventions. These large, sudden price movements can cause the delta of the option to change abruptly, making it difficult to hedge effectively in real time. Standard delta hedging assumes continuous price movements, which is a poor assumption in markets with frequent jumps and discontinuities.

4. Correlation Breakdown: The correlation between electricity prices and gas prices, which is a key factor in spark spread options, is not constant over time. Periods of stress in energy markets, such as fuel shortages or extreme weather, can lead to a breakdown in the historical correlation between these two prices. When correlation deviates from its expected behavior, the effectiveness of delta hedging may be reduced, as the model assumptions used to calculate the deltas no longer hold.

5. Model Risk: Delta hedging relies on the accuracy of the option pricing model, such as Kirk's approximation, to compute the appropriate hedge ratios. However, these models are based on simplifications and assumptions that may not fully capture the complex dynamics of energy prices. If the model is not well-calibrated or fails to account for certain risk factors (e.g., seasonality, regulatory impacts, or non-linear dependencies), the hedging strategy may be suboptimal or even increase risk rather than mitigate it.