is usually enough variability between images that, even if global thresholding is a suitable approach, an algorithm capable of estimating automatically the threshold value for each image is required. The following iterative algorithm can be used for this purpose:

1. Select an initial estimate for the global threshold, T.
2. Segment the image using T in Eq. (10.3-1). This will produce two groups of pixels: G_1 consisting of all pixels with intensity values $> T$, and G_2 consisting of pixels with values $\leq T$.
3. Compute the average (mean) intensity values m_1 and m_2 for the pixels in G_1 and G_2, respectively.
4. Compute a new threshold value:
$$T = \frac{1}{2}(m_1 + m_2)$$
5. Repeat Steps 2 through 4 until the difference between values of T in successive iterations is smaller than a predefined parameter ΔT.

This simple algorithm works well in situations where there is a reasonably clear valley between the modes of the histogram related to objects and background. Parameter ΔT is used to control the number of iterations in situations where speed is an important issue. In general, the larger ΔT is, the fewer iterations the algorithm will perform. The initial threshold must be chosen greater than the minimum and less than maximum intensity level in the image (Problem 10.28). The average intensity of the image is a good initial choice for T.

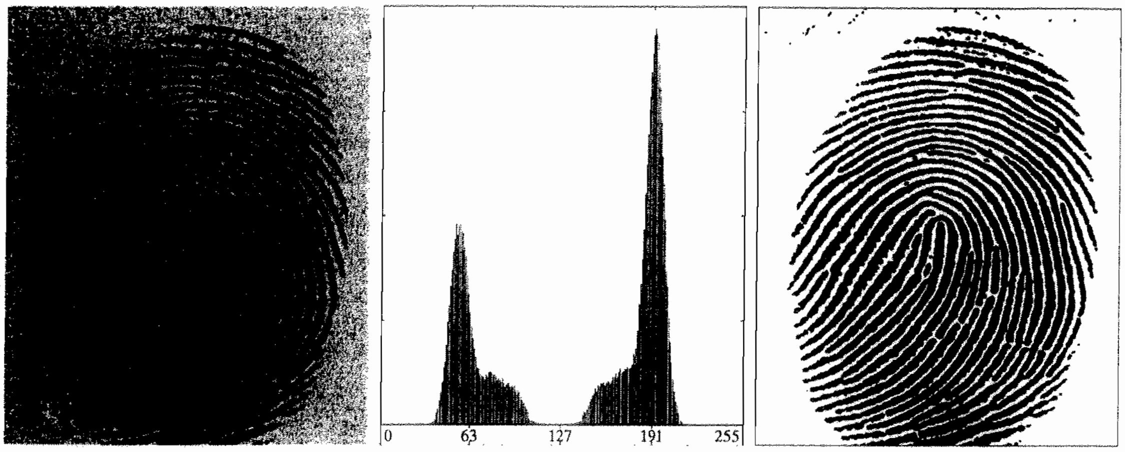
EXAMPLE 10.15: Global thresholding.

■ Figure 10.38 shows an example of segmentation based on a threshold estimated using the preceding algorithm. Figure 10.38(a) is the original image, and Fig. 10.38(b) is the image histogram, showing a distinct valley. Application of the preceding iterative algorithm resulted in the threshold $T = 125.4$ after three iterations, starting with $T = m$ (the average image intensity), and using $\Delta T = 0$. Figure 10.38(c) shows the result obtained using $T = 125$ to segment the original image. As expected from the clear separation of modes in the histogram, the segmentation between object and background was quite effective. ■

The preceding algorithm was stated in terms of successively thresholding the input image and calculating the means at each step because it is more intuitive to introduce it in this manner. However, it is possible to develop a more efficient procedure by expressing all computations in the terms of the image histogram, which has to be computed only once (Problem 10.26).

10.3.3 Optimum Global Thresholding Using Otsu's Method

Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups (also called *classes*). This problem is known to have an elegant closed-form solution known as the *Bayes decision rule* (see Section 12.2.2). The solution is based on only two parameters: the probability density function (PDF) of the intensity levels of each class and the probability that each class occurs in a given application. Unfortunately, estimating PDFs is not a trivial



a b c

FIGURE 10.38 (a) Noisy fingerprint. (b) Histogram. (c) Segmented result using a global threshold (the border was added for clarity). (Original courtesy of the National Institute of Standards and Technology.)

matter, so the problem usually is simplified by making workable assumptions about the form of the PDFs, such as assuming that they are Gaussian functions. Even with simplifications, the process of implementing solutions using these assumptions can be complex and not always well-suited for practical applications.

The approach discussed in this section, called *Otsu's method* (Otsu [1979]), is an attractive alternative. The method is optimum in the sense that it maximizes the ***between-class variance***, a well-known measure used in statistical discriminant analysis. The basic idea is that well-thresholded classes should be distinct with respect to the intensity values of their pixels and, conversely, that a threshold giving the best separation between classes in terms of their intensity values would be the best (optimum) threshold. In addition to its optimality, Otsu's method has the important property that it is based entirely on computations performed on the histogram of an image, an easily obtainable 1-D array.

Let $\{0, 1, 2, \dots, L - 1\}$ denote the L distinct intensity levels in a digital image of size $M \times N$ pixels, and let n_i denote the number of pixels with intensity i. The total number, MN, of pixels in the image is $MN = n_0 + n_1 + n_2 + \dots + n_{L-1}$. The normalized histogram (see Section 3.3) has components $p_i = n_i/MN$, from which it follows that

$$\sum_{i=0}^{L-1} p_i = 1, \qquad p_i \geq 0 \tag{10.3-3}$$

Now, suppose that we select a threshold $T(k) = k$, $0 < k < L - 1$, and use it to threshold the input image into two classes, C_1 and C_2, where C_1 consists of all the pixels in the image with intensity values in the range $[0, k]$ and C_2 consists of the pixels with values in the range $[k + 1, L - 1]$. Using this threshold, the probability, $P_1(k)$, that a pixel is assigned to (i.e., thresholded into) class C_1 is given by the cumulative sum

$$P_1(k) = \sum_{i=0}^{k} p_i \tag{10.3-4}$$

Viewed another way, this is the probability of class C_1 occurring. For example, if we set $k = 0$, the probability of class C_1 having any pixels assigned to it is zero. Similarly, the probability of class C_2 occurring is

$$P_2(k) = \sum_{i=k+1}^{L-1} p_i = 1 - P_1(k) \tag{10.3-5}$$

From Eq. (3.3-18), the mean intensity value of the pixels assigned to class C_1 is

$$\begin{aligned} m_1(k) &= \sum_{i=0}^{k} iP(i/C_1) \\ &= \sum_{i=0}^{k} iP(C_1/i)P(i)/P(C_1) \\ &= \frac{1}{P_1(k)} \sum_{i=0}^{k} ip_i \end{aligned} \tag{10.3-6}$$

where $P_1(k)$ is given in Eq. (10.3-4). The term $P(i/C_1)$ in the first line of Eq. (10.3-6) is the probability of value i, given that i comes from class C_1. The second line in the equation follows from Bayes' formula:

$$P(A/B) = P(B/A)P(A)/P(B)$$

The third line follows from the fact that $P(C_1/i)$, the probability of C_1 given i, is 1 because we are dealing only with values of i from class C_1. Also, $P(i)$ is the probability of the ith value, which is simply the ith component of the histogram, p_i. Finally, $P(C_1)$ is the probability of class C_1, which we know from Eq. (10.3-4) is equal to $P_1(k)$.

Similarly, the mean intensity value of the pixels assigned to class C_2 is

$$\begin{aligned} m_2(k) &= \sum_{i=k+1}^{L-1} iP(i/C_2) \\ &= \frac{1}{P_2(k)} \sum_{i=k+1}^{L-1} ip_i \end{aligned} \tag{10.3-7}$$

The cumulative mean (average intensity) up to level k is given by

$$m(k) = \sum_{i=0}^{k} ip_i \tag{10.3-8}$$

and the average intensity of the entire image (i.e., the *global* mean) is given by

$$m_G = \sum_{i=0}^{L-1} ip_i \tag{10.3-9}$$

The validity of the following two equations can be verified by direct substitution of the preceding results:

$$P_1 m_1 + P_2 m_2 = m_G \tag{10.3-10}$$

and

$$P_1 + P_2 = 1 \tag{10.3-11}$$

where we have omitted the ks temporarily in favor of notational clarity.

In order to evaluate the "goodness" of the threshold at level k we use the normalized, dimensionless metric

$$\eta = \frac{\sigma_B^2}{\sigma_G^2} \tag{10.3-12}$$

where σ_G^2 is the *global variance* [i.e., the intensity variance of all the pixels in the image, as given in Eq. (3.3-19)],

$$\sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2 p_i \tag{10.3-13}$$

and σ_B^2 is the *between-class variance*, defined as

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 \tag{10.3-14}$$

This expression can be written also as

$$\begin{aligned} \sigma_B^2 &= P_1 P_2 (m_1 - m_2)^2 \\ &= \frac{(m_G P_1 - m)^2}{P_1(1 - P_1)} \end{aligned} \tag{10.3-15}$$

The second step in Eq. (10.3-15) makes sense only if P_1 is greater than 0 and less than 1, which, in view of Eq. (10.3-11), implies that P_2 must satisfy the same condition.

where m_G and m are as stated earlier. The first line of this equation follows from Eqs. (10.3-14), (10.3-10), and (10.3-11). The second line follows from Eqs. (10.3-5) through (10.3-9). This form is slightly more efficient computationally because the global mean, m_G, is computed only once, so only two parameters, m and P_1, need to be computed for any value of k.

We see from the first line in Eq. (10.3-15) that the farther the two means m_1 and m_2 are from each other the larger σ_B^2 will be, indicating that the between-class variance is a measure of *separability* between classes. Because σ_G^2 is a constant, it follows that η also is a measure of separability, and maximizing this metric is equivalent to maximizing σ_B^2. The objective, then, is to determine the threshold value, k, that maximizes the between-class variance, as stated at the beginning of this section. Note that Eq. (10.3-12) assumes implicitly that $\sigma_G^2 > 0$. This variance can be zero only when all the intensity levels in the image are the same, which implies the existence of only one class of pixels. This in turn means that $\eta = 0$ for a constant image since the separability of a single class from itself is zero.

Reintroducing k, we have the final results:

$$\eta(k) = \frac{\sigma_B^2(k)}{\sigma_G^2} \tag{10.3-16}$$

and

$$\sigma_B^2(k) = \frac{\left[m_G P_1(k) - m(k)\right]^2}{P_1(k)\left[1 - P_1(k)\right]} \tag{10.3-17}$$

Then, the optimum threshold is the value, k^*, that maximizes $\sigma_B^2(k)$:

$$\sigma_B^2(k^*) = \max_{0 \le k \le L-1} \sigma_B^2(k) \tag{10.3-18}$$

In other words, to find k^* we simply evaluate Eq. (10.3-18) for all *integer* values of k (such that the condition $0 < P_1(k) < 1$ holds) and select that value of k that yielded the maximum $\sigma_B^2(k)$. If the maximum exists for more than one value of k, it is customary to average the various values of k for which $\sigma_B^2(k)$ is maximum. It can be shown (Problem 10.33) that a maximum always exists, subject to the condition that $0 < P_1(k) < 1$. Evaluating Eqs. (10.3-17) and (10.3-18) for all values of k is a relatively inexpensive computational procedure, because the maximum number of integer values that k can have is L.

Once k^* has been obtained, the input image $f(x, y)$ is segmented as before:

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > k^* \\ 0 & \text{if } f(x, y) \le k^* \end{cases} \tag{10.3-19}$$

for $x = 0, 1, 2, \ldots, M - 1$ and $y = 0, 1, 2, \ldots, N - 1$. Note that all the quantities needed to evaluate Eq. (10.3-17) are obtained using only the histogram of $f(x, y)$. In addition to the optimum threshold, other information regarding the segmented image can be extracted from the histogram. For example, $P_1(k^*)$ and $P_2(k^*)$, the class probabilities evaluated at the optimum threshold, indicate the portions of the areas occupied by the classes (groups of pixels) in the thresholded image. Similarly, the means $m_1(k^*)$ and $m_2(k^*)$ are estimates of the average intensity of the classes in the original image.

The normalized metric η, evaluated at the optimum threshold value, $\eta(k^*)$, can be used to obtain a quantitative estimate of the separability of classes, which in turn gives an idea of the ease of thresholding a given image. This measure has values in the range

Although our interest is in the value of η at the optimum threshold, k^*, this inequality holds in general for any value of k in the range $[0, L - 1]$.

$$0 \le \eta(k^*) \le 1 \tag{10.3-20}$$

The lower bound is attainable only by images with a single, constant intensity level, as mentioned earlier. The upper bound is attainable only by 2-valued images with intensities equal to 0 and $L - 1$ (Problem 10.34).

Otsu's algorithm may be summarized as follows:

1. Compute the normalized histogram of the input image. Denote the components of the histogram by p_i, $i = 0, 1, 2, \ldots, L - 1$.
2. Compute the cumulative sums, $P_1(k)$, for $k = 0, 1, 2, \ldots, L - 1$, using Eq. (10.3-4).
3. Compute the cumulative means, $m(k)$, for $k = 0, 1, 2, \ldots, L - 1$, using Eq. (10.3-8).
4. Compute the global intensity mean, m_G, using (10.3-9).
5. Compute the between-class variance, $\sigma_B^2(k)$, for $k = 0, 1, 2, \ldots, L - 1$, using Eq. (10.3-17).
6. Obtain the Otsu threshold, k^*, as the value of k for which $\sigma_B^2(k)$ is maximum. If the maximum is not unique, obtain k^* by averaging the values of k corresponding to the various maxima detected.
7. Obtain the separability measure, η^*, by evaluating Eq. (10.3-16) at $k = k^*$.

The following example illustrates the preceding concepts.

EXAMPLE 10.16: Optimum global thresholding using Otsu's method.

■ Figure 10.39(a) shows an optical microscope image of polymersome cells, and Fig. 10.39(b) shows its histogram. The objective of this example is to segment the molecules from the background. Figure 10.39(c) is the result of using the basic global thresholding algorithm developed in the previous section. Because the histogram has no distinct valleys and the intensity difference between the background and objects is small, the algorithm failed to achieve the desired segmentation. Figure 10.39(d) shows the result obtained using Otsu's method. This result obviously is superior to Fig. 10.39(c). The threshold value computed by the basic algorithm was 169, while the threshold computed by Otsu's method was 181, which is closer to the lighter areas in the image defining the cells. The separability measure η was 0.467.

Polymersomes are cells artificially engineered using polymers. Polymorsomes are invisible to the human immune system and can be used, for example, to deliver medication to targeted regions of the body.

As a point of interest, applying Otsu's method to the fingerprint image in Example 10.15 yielded a threshold of 125 and a separability measure of 0.944. The threshold is identical to the value (rounded to the nearest integer) obtained with the basic algorithm. This is not unexpected, given the nature of the histogram. In fact, the separability measure is high due primarily to the relatively large separation between modes and the deep valley between them. ■

10.3.4 Using Image Smoothing to Improve Global Thresholding

As noted in Fig. 10.36, noise can turn a simple thresholding problem into an unsolvable one. When noise cannot be reduced at the source, and thresholding is the segmentation method of choice, a technique that often enhances performance is to smooth the image prior to thresholding. We illustrate the approach with an example.

Figure 10.40(a) is the image from Fig. 10.36(c), Fig. 10.40(b) shows its histogram, and Fig. 10.40(c) is the image thresholded using Otsu's method. Every black point in the white region and every white point in the black region is a