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**HO CHI MINH CITY UNIVERSITY OF TECHNOLOGY**

**OFFICE FOR INTERNATIONAL STUDY PROGRAMS**

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Description automatically generatedFACULTY OF TRANSPORTATION**

**Capstone Project**

**Design the model of the resistance moment on the electric power steering system of the VIOS and simulate its effects by using MATLAB software.**

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**Class:** CC19OTO1

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December 27th, 2022, Ho Chi Minh City, Vietnam

**🙦 Semester 222**



1. **Student’s name**: Hồ Bình Minh - Student ID: 1852169
2. **Major**: Automotive Engineering - Class: CC19OTO1
3. **Thesis title**: Modeling and simulation the resistance torque for specific wheel alignment in the Electric Power Steering system by using Matlab/Simulink and its application.
4. **Content:**

* Find out how wheel alignment can affect the resistance torque in the steering mechanism especially in the EPS system.
* Fully understanding knowledge about the resistance torque between the tire force and road surface in steering mechanism especially in the EPS system.
* Build the complete model of the resistance torque between the tire forces and road by using Matlab/Simulink.

1. **Result**: Learning how the tire forces and wheel alignment can affect to the steering mechanism through the resistance moment and illustrating its effects which is showed in the result diagram from Matlab/Similunk.
2. **Product**:
   * Presentation report.
   * Poster.
   * Result diagrams
3. **Assigned day**: 23 December 2022.
4. **Finished day**: 22 May 2023.

The content and requirements of the thesis is already approved by the Head of Department of Automotive Engineering.

HCMC, day….... month…… year 2023 HCMC, day… . month…… year 2023

**Head of Department**  **Instructor**

**ACKNOWLEDGEMENT**

First and foremost, I want to express my gratitude to my family, who have always been by my side, accompanying, supporting, and assisting me in any way possible so that I can get to where I am now. I want to thank the teachers at Bach Khoa University in general and the Department of Automotive Engineering for their efforts. The knowledge I have gained from teachers over the last four years has assisted me in being brave enough to complete this project.

Sincere thanks to PhD. Ngo Dac Viet, PhD. Tran Dang Long created conditions for me to study, practice, and conduct field surveys.

Finally, I want to thank the reviewer and department lecturers for sharing their knowledge and providing me with feedback and suggestions so that I could finish this project.

Wishing health to parents, family, lecturers in the Faculty of Transportation Engineering as well as lecturers in the Department of Automotive Engineering and all of my friends in class CC19OTO1.

**ABSTRACT**

Vehicle steering dynamics is an essential topic in development of safety driving systems. These complex and integrated control units require precise information about vehicle steering dynamics, especially, tire/road contact forces. In the term of interaction between tyre force and road surface, we are going to primarily focus on the total resistance moment which is the factor torque that urges the tyres to steer. This resistance moment that causes this will be described in below when considering the slip angle, the mass of vehicle, wheel alignment, lateral force generation, longitudinal force generation and normal force generation. Through this capstone project, this torque will be fully showed with the theoretically corresponding equation and combine with the result diagrams by using Matlab/Simulink software.

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# I/ Introduction:

1. **Objective:**

During the past years, steering mechanism and its relevant fields has been researched and developed in gradually increasing numbers. The reason for this point is the steering mechanism plays an important role in vehicle control. To get fully understanding knowledge about this system, at first, we need to know the interaction between the tire force and road surface especially the aligning moment of the tire apply on the steering mechanism.

The focus of this project is that we can get a deep study as much as possible on the resistance moment on the steering mechanism and its dependence on the difference of tire forces. Besides, I will choose the Electric Power Steering system which is built based on the Ackerman steering mechanism for modelling and analyzing.

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Figure 1: Electric Power Steering system structure

1. **Scope of implementation:**

The model is developed to present vehicle behaviour when driving in normal condition of roads and cars, so it can not be reliable in non-linear conditions (When the vehicle is driven up to its limits). The model developed in this project does not represent the steering condition in parking situations. The model is developed by assuming that the wheels are in contact with the road surface. So, the wheel lift phenomenon is assumed negligible in this model. Besides, the resistance moment that acting on the wheel withstands a lot of types of force: longitudinal force, lateral force, and wheel alignment angle: kingpin angle, caster angle, camber angle, etc. In this project, this resistance torque is going to be fully considered in the effect of tire forces with the specific wheel alignment.

1. **Working condition:**

Continuously change to adapt with variable driving conditions.

1. **Technical requirement:**

Working normally in above condition.

1. **Mission summary:**

As I mentioned on the scope of implementation, this project is mainly going to be focus on the resistance torque under the effect of wheel alignments and other specific factors such as vehicle mass, steering angle and so on. Base on this scope, I will divide the whole objective into 2 main missions: The first one is get fully understanding knowledge about the resistance torque between the tire force and road surface in steering mechanism especially in the EPS system and the second one is how wheel alignment and other factors can affect to the resistance torque in steering mechanism especially in the EPS system.

**Summary table of the resistant torque acting on the EPS steering system for wheel alignments**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Camber angle | Caster angle | Kingpin angle | Equation for the resistant torque | Illustration |
| Longitudinal forces |  | X | X | ]  While:  *“DESIGN OF STEERING WHEEL FORCE FEEDBACK SYSTEM WITH FOCUS ON LANE KEEPING ASSISTANCE APPLIED IN DRIVING SIMULATOR”* | Diagram  Description automatically generated |
| Lateral forces | X | X | X | While: , and trail  *“DESIGN OF STEERING WHEEL FORCE FEEDBACK SYSTEM WITH FOCUS ON LANE KEEPING ASSISTANCE APPLIED IN DRIVING SIMULATOR”* | Diagram, engineering drawing  Description automatically generated |
| Normal force |  | X | X | *“DESIGN OF STEERING WHEEL FORCE FEEDBACK SYSTEM WITH FOCUS ON LANE KEEPING ASSISTANCE APPLIED IN DRIVING SIMULATOR”* | Chart, radar chart  Description automatically generated with medium confidence |

# II/ Theoretical basics:

* 1. **General Steering Theory:**
     1. **Coordinate systems**:

In the following section, the basic concepts of the coordinate systems used in this project will be presented. In this model, the ISO coordinate systems are used. They are based on the seven coordinate systems as following:

* Earth (X, Y, Z)

The global coordinate system describes the entire environment of the model. It is used as the position reference for the vehicle because of the global coordinate system which does not move.

* Vehicle (x, y, z)

The Center of Gravity (COG) coordinate system describes the position of COG during simulation. In this coordinate system, the x-axis is parallel to the longitudinal movement of the vehicle and points to the front of the vehicle. The yaxis is parallel to the lateral movement of the vehicle and the Z axis is parallel to the vertical movement of the vehicle.

* Wheel (xw, yw, zw)

The wheel coordinate system is in the center of each wheel. In this coordinate system, the x-axis points to the heading of the wheel.

* Path (xp, yp, zp)

The velocity coordinate system is fixed to the center of gravity of the vehicle. The difference of the center of gravity positions follows the velocity vector of the vehicle such as: longitudinal velocity (in x axis direction), Lateral velocity (in y axis direction), vertical velocity. (in z axis direction)

* Yaw (ψ)

Yaw is the rotation around the vertical axis (z-axis) through the center of gravity of the vehicle. The yaw can be felt in skidding or spin movement.

* Pitch (φ)

Pitch is the rotation around the lateral axis (y-axis) through the center of gravity of the vehicle. It can be felt in acceleration or braking movement around (y-axis) of vehicle.

* Roll (ϴ)

Roll is the rotation around the longitudinal axis (x-axis) through the center of gravity of the vehicle. This rotation can be felt during lateral acceleration (side-to-side movement) of the vehicle.

The overall scheme of ISO coordinate system is shown in Figure 2.1.

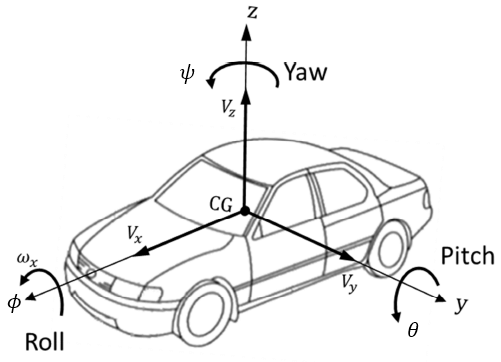


Figure 2: Overall scheme of ISO coordinate system for vehicle

* + 1. **: Model terminology**

In this part, vehicle dynamics terminology used in this project is shown in Figure 2.2 and described respectively:

A picture containing diagram, text, map, plan

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Figure 3: Vehicle dynamics terminology used in this project

As we can see in Figure 2.2, the tire is the main component interacting with the road. The performance of a vehicle is mainly influenced by the characteristics of its tires. Tires affect a vehicle’s handling, traction, ride comfort, and fuel consumption. To understand its importance, it is enough to remember that a vehicle can maneuver only by longitudinal, vertical, and lateral force systems generated under the tires. Steering process mainly depend on the interaction between below forces, moment, angles:

* Normal force it is vertical force, normal to the ground plane. The resultant normal force > 0 if it is upward. Normal force is also called the vertical force or wheel load.
* Longitudinal force : It is a force acting along the x-axis. The resultant longitudinal force > 0 if the car is accelerating, and < 0 if the car is braking. Longitudinal force is also called forward force.
* Lateral force It is a force, tangent to the ground and orthogonal to both and . The resultant lateral force > 0 if it is in the y direction.
* Yaw moment It is an upward moment about the z-axis. The resultant yaw moment Mz > 0 if it tends to turn the tire about the z-axis. The yaw moment is also called the *aligning moment, self -aligning moment, or bore torque*.
* Side-slip angle is the angle between the velocity vector *v* and the *x*-axis measured about the z-axis. This angle has a big influence on the steering because it directly affects on the magnitude of the lateral force
* Caster angle is the angle to which the steering pivot axis is tilted forward or rearward from vertical, as viewed from the Figure 3. This is one of the most important factors that effects on the resistance torque .
* Kingpin angle is the angle between the kingpin axis and the vertical axis  
  of the tire. The kingpin axis is the line between the lower and upper ball joints of the wheel’s hub.
  1. **Steering System Modeling and Wheel Alignment theory:**

In this part, the steering system used in this thesis is described. As mentioned before, steering system modeling is one of the most important issues in driving simulation. The high fidelity of steering system simulation is useful to achieve high reality steering feel for the driver during driving simulation. The steering system modeled during this project consists of two main parts: steering geometry and steering wheel feedback torque. Steering geometry is created to transmit the steering wheel angle applied by the driver as an input to virtual wheels angles as output. Steering wheel feedback torque has the main purpose of transmitting the torque created in a tire (self-aligning torque, friction torque…) to the steering wheel. In other words, steering system model receives the steering wheel position which is applied by the driver as input and provides the steering wheel feedback torque as output. Besides, wheel alignment angle that are necessary for only this project will also be mentioned in this part.

* + 1. **Steering system overview:**

The steering system transfers the steering wheel angle to the wheels through a mechanical system composed by a series of rods and pivots linkages. In this case when the driver turns the steering wheel, the steering wheel’s rotation is transmitted through the steering column (steering shaft) to the pinion, the pinion converts the rotation to the linear displacement through the rack and pinion. The created linear movement is transferred to the uprights through the tie roads. The created linear movement at upright generates the steering angle in the wheels. The steering mechanism between the steering box and the steering angle in the wheels presents a transmission rate which is called steering ratio. It is important to notice that the steering wheel angle and wheel angle relates via a steering ratio coefficient. Rack and pinion steering system is commonly used in conventional cars. In this project, the power steering assistance system is used as well as the rack and pinion system. A power steering assist system helps drivers by decreasing the driver’s effort in the steering wheel. The power steering assistance system is comprised of a DC motor and a control unit, so that the control unit calculates if a steering assistance is required for the driver. The rack and pinion steering system is shown in Figure 2.3 and steering box is shown in Figure 2.4 respectively.

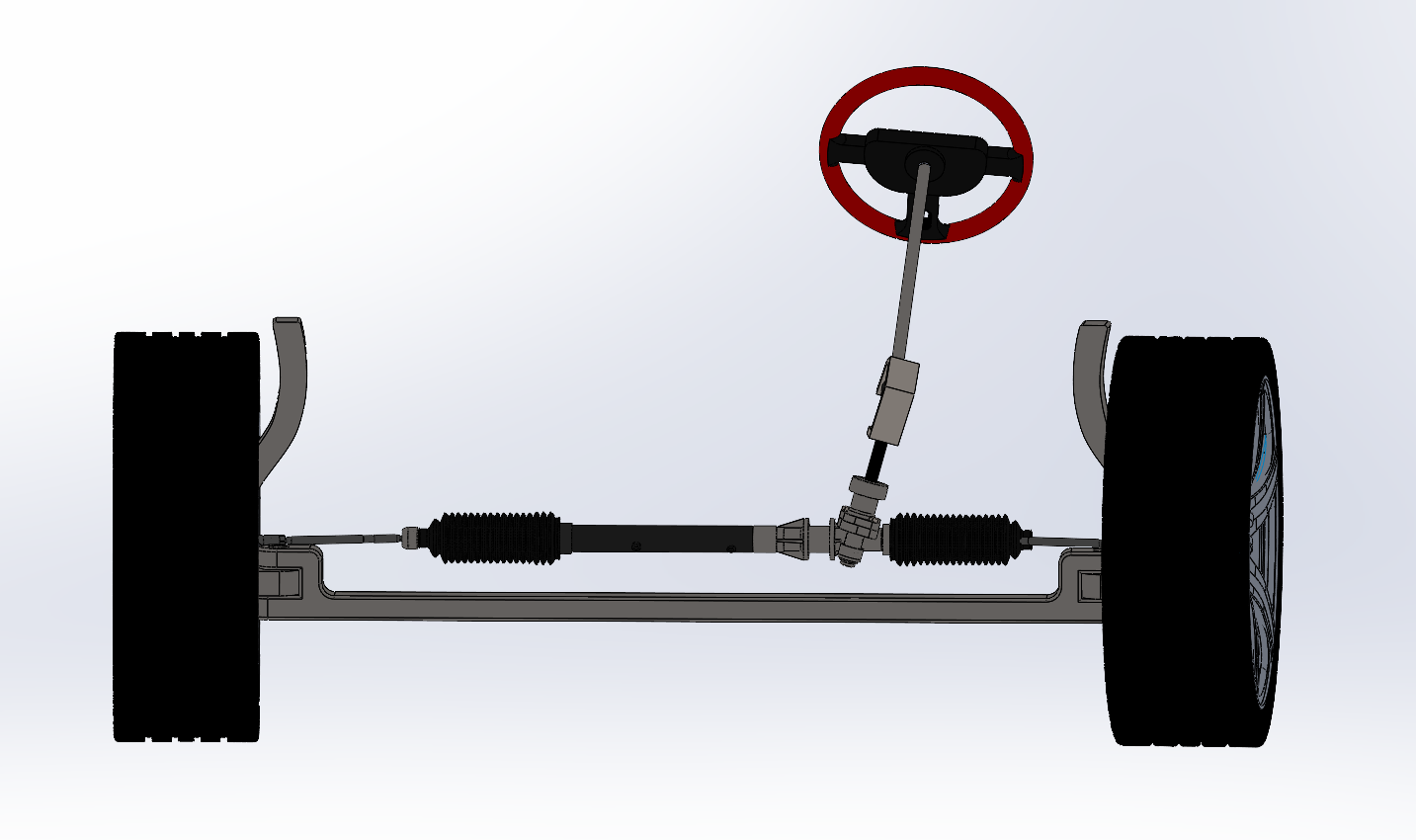


Figure 4: Steering systems (rack and pinion)

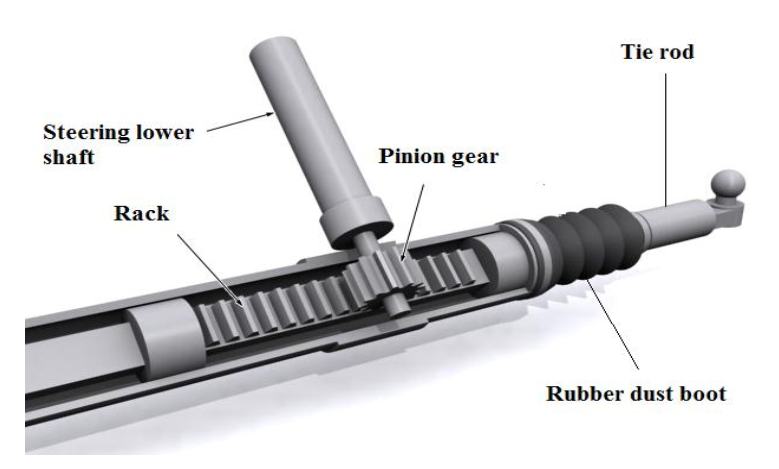


Figure 5: Steering gear schematic

This mechanical linkage between the steering box and the wheels usually conforms to the required condition. When the vehicle is moving very slowly, there is a kinematic condition between the inner and outer wheels that allows them to turn slip-free. It is called as Ackerman condition and expressed by:

**(\*)**

where is the steer angle of the inner wheel, is the steer angle of the outer wheel, The distance between the steer axes of the steerable wheels is called the track and is shown by . The distance between the front and real axles is called the wheelbase and is shown by . Track w and wheelbase are considered as kinematic width and length of the vehicle. Ackerman steering geometry is the term used to describe the behavior of the front wheel when the vehicle is driven around a corner. In the corner when the front tires turn, the inner wheels radius is smaller than the outer wheels and that means the steering wheel is needed to generate the wheel angle for the inner wheels which are larger than the outer wheels, otherwise the inner wheel tends to slide over the road. The Ackerman geometry neglects the effect of road on tire, so it is not completely suitable for modern cars. The wheels behavior interface corner turning can be seen in Figure 2.5.

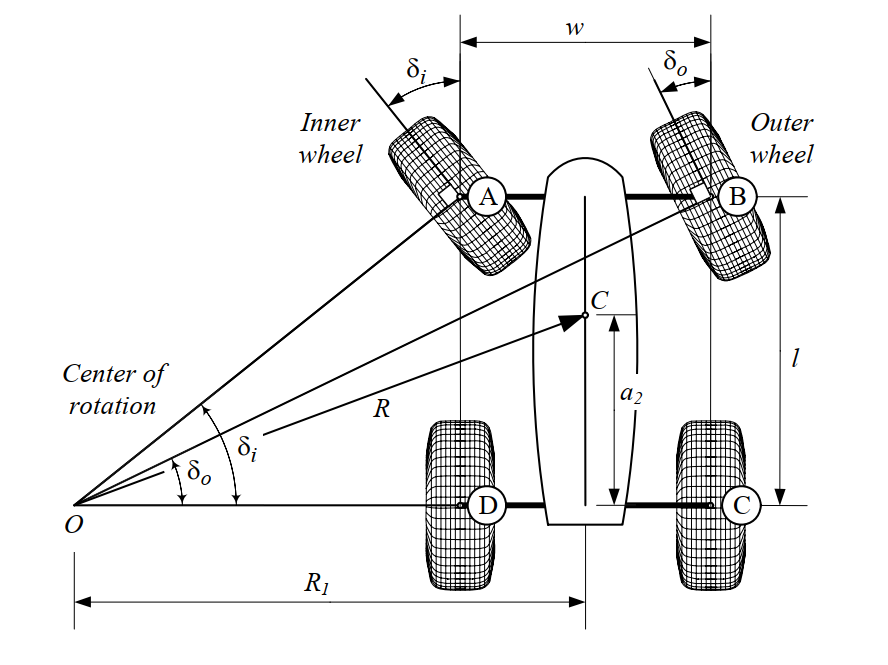


Figure 6: A front-wheel-steering vehicle and steer angles of the inner and

As can be seen in the Figure 2.5, the inner wheel angle is larger than the outer wheel, when the vehicle turns around a circle:

It is important to notice that the wheels behavior analysis is a very important point to accurately simulate tire forces. For this reason, all the parameters which can affect the tires must consider in tire modeling.

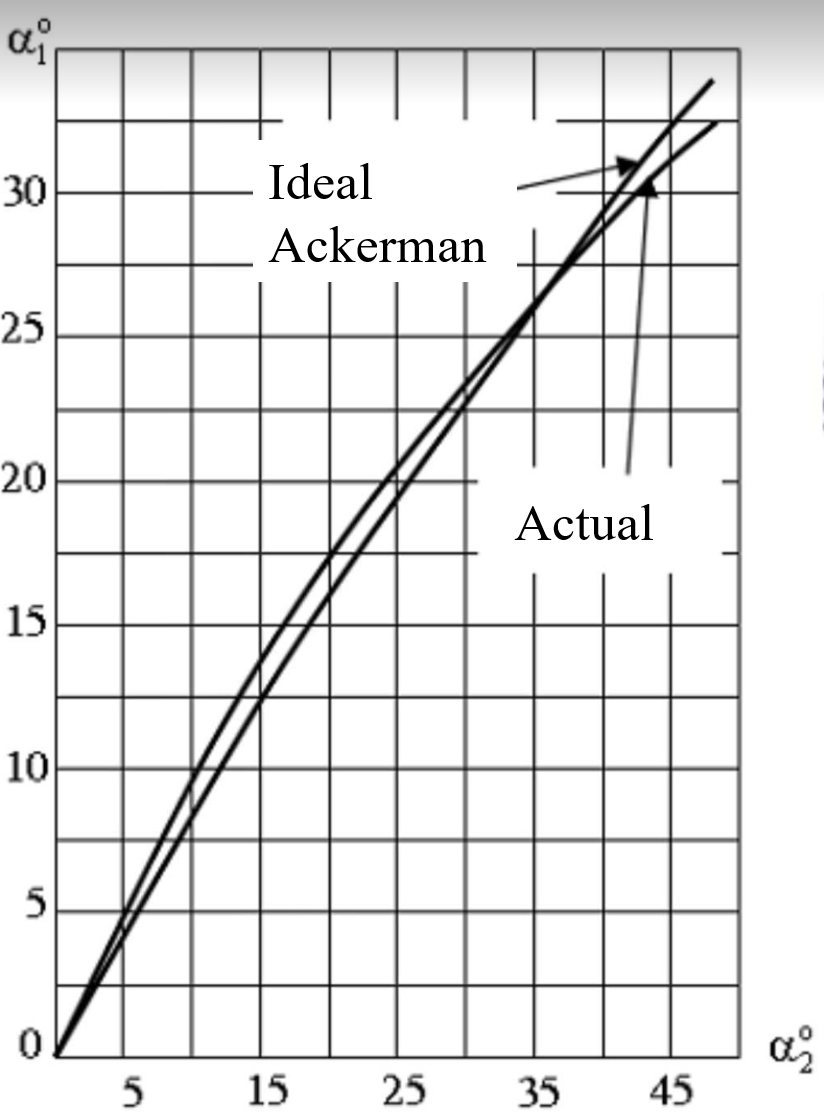


Figure 7: The diagram comparison about the inner and outer angle between actual condition and Ackerman condition

From the Figure 2.6, we can see that the ideal Ackerman condition and actual inner and outer steering angle have the minimum error only in the range from 0 to 35 degrees. That is the reason why in this project, the steering angle of the wheel is just under 35 degrees.

### 2.2.2. Wheel Alignment:

**Caster angle** affects the steering feel by creating a self-centering torque to reduce the toughness of steering. For example, when the caster angle is positive and the wheel is steered, the lateral forces will create a torque around the steering axis and will increase the self-aligning torque of the tire. Increasing of self-aligning torque causes the steering wheel to align quickly. Furthermore, positive caster improves the stability of vehicle in a turn and reduces under-steering situation of the vehicle when the vehicle is exiting from a turn. Positive caster angle will increase handling of the vehicle when the vehicle is turning but it causes the steering wheel to be tougher to move. When the caster angle is negative the lateral forces will produce a torque that helps steering. (Figure 2.7)

Diagram, schematic

Description automatically generated

Figure 8: Overall in Caster angle

**Kingpin angle** has the effects which is usually discussed in terms of the scrub radius offset which determines the value of the self-aligning torque when the wheels are turned. For the zero-scrub radius, no reaction will transmit to the steering wheel and the driver is not able to perceive the change of the vehicle lateral offset. In case of the positive scrub radius (many conventional cars have a positive scrub radius offset) the wheels are returned to the straight position quickly. In case of the negative scrub radius (some modern cars have a negative scrub radius offset) the longitudinal forces will generate a moment that increases the steering of the wheels in a longitudinal direction. For this reason, the vehicle becomes more oversteering when the scrub radius offset is negative, thus the driver is not able to sense the self-aligning torque effect correctly. (Figure 2.8)

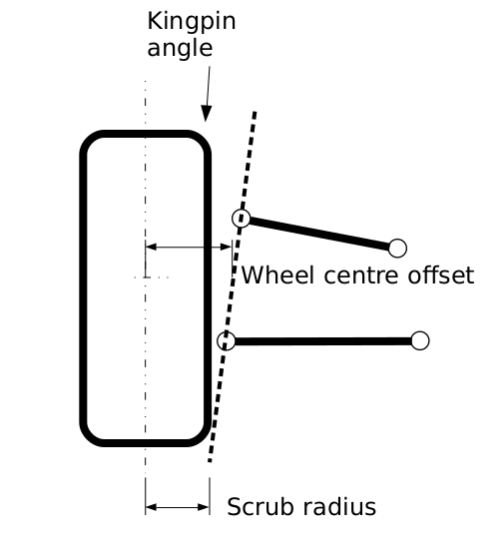


Figure 9: Kingpin angle and its scrub radius

**Camber angle** is the tilting angle of tire about the longitudinal x-axis. Figure 2.9 illustrates a front view of a cambered tire and generated camber force . Camber angle is assumed positive γ > 0, when it is in the positive direction of the x-axis, measured from the z-axis to the tire. A positive camber angle generates a camber force along the −y-axis. Itis directly influence on the magnitude of the lateral force which the most important factor in vehicle’s steering.

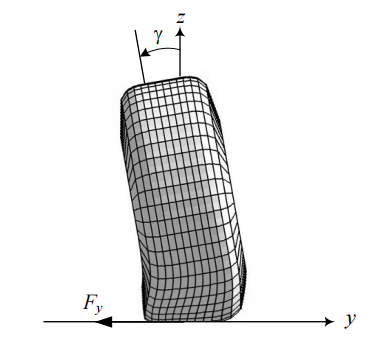


Figure 10: : A front view of a cambered tire and the generated camber force.

In this project, only Kingpin angle and Caster angle will be considered for the resistance moment which is going to be mentioned in the next part.

**III/ VEHICLE STEERING EQUATIONS:**

**3.1) Mathematical modelling of tire force:**

* **Normal force :** has an influence on a lateral displacement between the contact point of the point of application of this force and the centre plane of the wheel. In this project, the driving situation is the vehicle speed is constant that means we have no acceleration for the normal force calculation. The Electric Power Steering system in this project is set up for the front wheel drive, so the equation of each front tire is:

**(1)**

While:

: the normal force (N)

m: mass of the vehicle (kg)

g = 9.8: gravitational acceleration (m/

the distance from the center of vehicle mass to rear axle respectively (m)

* **Lateral force** : when a turning tire is under a vertical force and a lateral force , its path of motion makes an angle α with respect to the tire plane. Basically, this force is the friction force to the centrifugal force. The angle is called side-slip angle and is proportional to the lateral force:

**(2)**

While:

Lateral force (N)

: Cornering stiffness of the tire (N/rad)

side-slip angle (rad or degree)

At the maximum lateral force, the wheel will start sliding laterally and its value will be calculated by:

**(3)**

While:

Lateral force (N)

: Normal force at the contact point of the tire and the road surface (N)

: Lateral friction coefficient

The slip angle α always increases by increasing the lateral force . However, the sliding line moves toward the tail at first and then moves forward by increasing the lateral force . Slip angle α and lateral force work as action and reaction. A lateral force generates a slip angle, and a slip angle generates a lateral force. Hence, we can steer the tires of a car to make a slip angle and produce a lateral force to turn the car. In this project, the lateral force is only considered by the effects of the sideslip angle and cornering stiffness of the tire.

* **The rolling resistance force:** In vehicle dynamics, rolling resistance force refers to the force that opposes the motion of a vehicle's wheels as they roll on the road surface. This force is caused by the deformation of the tire and the road surface, as well as other factors such as tire design, inflation pressure, and load. The rolling resistance force is calculated by equation:

**(4)**

While:

: mass of vehicle

= 9.81

= 0.011 : rolling resistance coefficient

Rolling resistance force (N)

* **Longitudinal force** : The longitudinal forces are generated between tire and road, due to the difference in velocity between road and tire, when accelerating and braking. The force is proportional to the normal force:

**(5)**

While:

Longitudinal force (N)

: Normal force at the contact point of the tire and the road surface (N)

: Longitudinal friction coefficient.

One of the most important factors that effects the magnitude of the longitudinal force is longitudinal slip ratio . This slip ratio illustrates the difference between the rotational speed of tire and vehicle longitudinal speed. The slip rate can be calculated as follows:

While:

the rotational speed of tire (rpm)

radius of tire (inch)

vehicle speed (m/s)

Increasing the slip of tire causes increasing of force as well, on the other hand, the longitudinal force is gengerated mostly depending on the construction of tire, the road condition and the vertical force applied on the tire. The main reason of force increasing is that the thread element of tire will be deformed and create the longitudinal force. The slip has linear relation with force for low slip rates, so the slope of this curve is called longitudinal tire stiffness. The longitudinal force decreases because the thread elements become saturated and unable to generate more force and the tire is locked in this condition such as Figure 3.1 illustrates below.

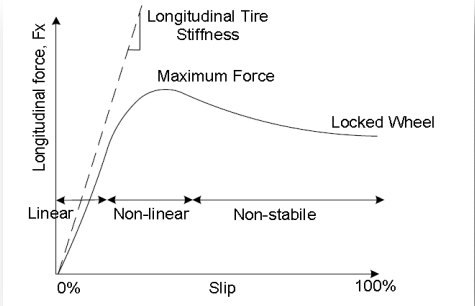


Figure 11: Longitudinal forces vs slip in the tire coordinate system

**3.2) Mathematical modelling of tire moments:**

In the modeling of the steering wheel feedback torque, resistance moment will be considered from the three forces of forces and wheel alignment. They are described as follows:

### 3.2.1. The resistance torque by longitudinal force:

These forces create a torque in the tire when the vehicle accelerates or brakes. The created torque in the tire due to a longitudinal force is the product of the longitudinal forces and the moment arm. The moment arm in this case is the scrub radius caused by the longitudinal forces effect, which would be sensed in the steering wheel. The total moment generated around the steering axis by can be calculated starting from Figure 3.2.1:

Diagram, engineering drawing

Description automatically generated

Figure 12: Scheme used to calculate the resistant torque generated by FX

* **Calculation diagram**:

Diagram

Description automatically generated

The Figure 3.2 and the above diagram show the total resistance moment generated around the steering axis due to FX can be computed from:

**(5)**

While:

aligning moment caused by longitudinal force (N.m)

Longitudinal force (N)

Caster angle (degree or rad)

Kingpin angle (degree or rad)

normal scrub radius of the Kingpin angle (m)

tire radius (m)

### 3.2.2. The resistance moment by lateral force:

* **Only Caster angle:**
* Self-resistance moment in the case of pneumatic trail:

A picture containing sketch, diagram, drawing

Description automatically generated

Figure 13: Tire print deflection and resistance moment with pneumatic trail

Figure 3.3 shows that pneumatic trail is a measure of how a tire's footprint or contact patch changes as it rolls. Pneumatic trail is caused by the progressive build-up of lateral force along the length of the contact patch, such that lateral forces are greater towards the rear of the contact patch (though less so when the rear of the contact patch begins sliding).

Pneumatic trail explains how tires can help you keep your stability and control while you drive. This effect occurs regardless of the steered direction of the tires and can result in a surf-like sensation that occurs when traveling at higher speeds. This force develops and is applied to the length of the contact patch with the rear of the contact patch experiencing the greatest pressure force. This lateral force causes the tire to rotate somewhat, which results in physical force known as self-resistance moment.

**(6)**

While:

: The resistance moment caused by pneumatic trail (N.m)

Resultant lateral force (N)

Pneumatic trail (m)

* Self-resistance moment in the case of mechanical trail:

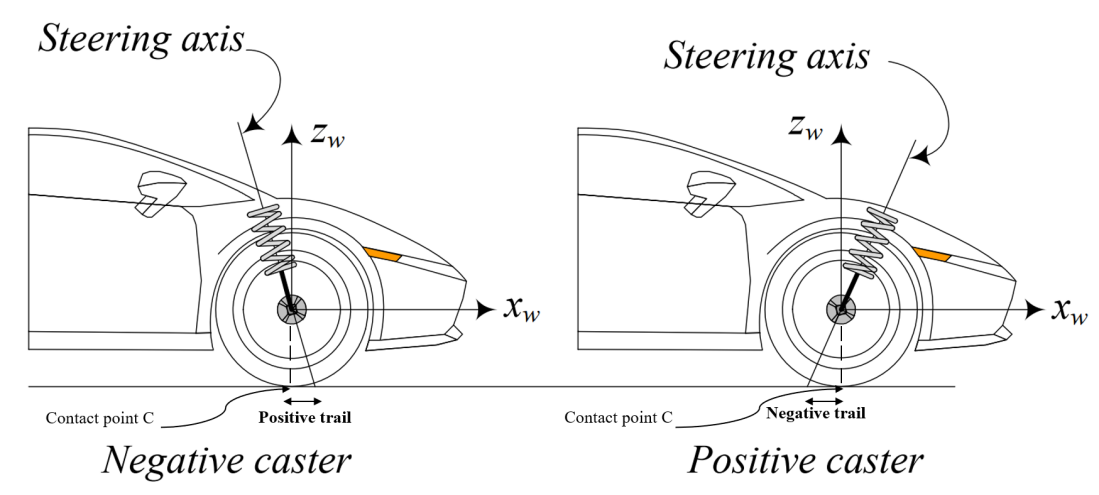


Figure 14: A positive and negative caster on front wheel of a car

Mechanical trail is the horizontal distance between the point where the steering axis of the front wheel intersects the ground and the point where the front tire contacts the ground which is show in Figure 3.4. In this sense, the contact patch of the tire “trails” behind the steering axis. The greater this distance, the “higher” the trail and the lower the distance the “lower” the trail. This factor also decides how much caster angle can affect on the steering feeling and how returnability of the vehicle through the resistance moment

**(7)**

While:

The resistance moment caused by mechanical trail (N.m)

Lateral force (N)

Mechanical trail (m)

As we can see in the Figure 3.5, since the trail is positive, friction force F generates a moment that tends to align the front wheel. The straightening moment is proportional to the value of the normal trail. Small positive trail values generate small aligning moments of the lateral friction force. Higher value of the trail (obtained with high value of the caster angle). If the value of the trail were negative (the contact point in front of the intersection point of the steering head axis with the road plane) and considering that friction force F is always in the opposite direction of the velocity of slippage, a moment around the steering head axis that would tend to increase the rotation to the left would be generated

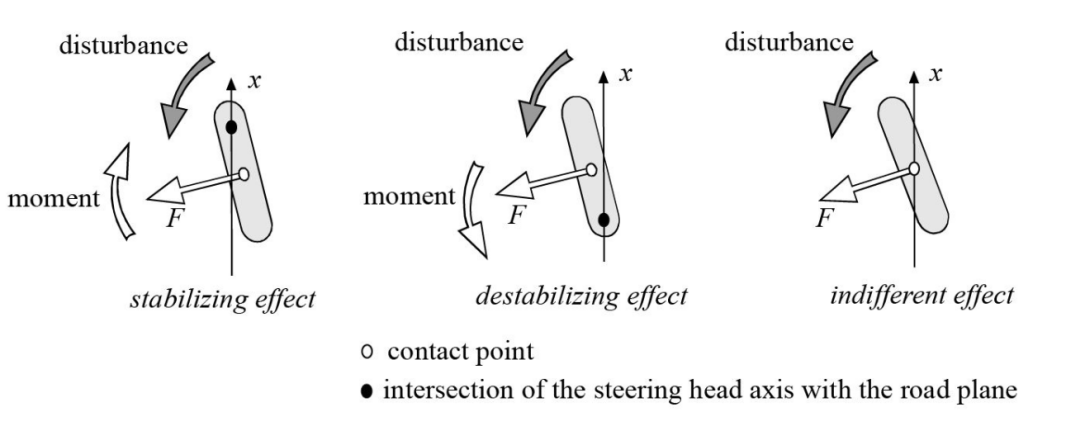


Figure 15: Summary of the effect of trail during forward movement

* Self-resistance moment in the collaboration of both trails

This moment tends to turn the tire about the z-axis and make the x-axis align with the velocity vector v. The resistance moment always tends to reduce α. It calculated by the equation:

**()**

While:

The resistance moment (N.m)

The resistance moment caused by pneumatic trail (N.m)

The resistance moment caused by mechanical trail (N.m)

Lateral force (N)

Pneumatic trail (m)

Mechanical trail (m)

As we can see in above equation, the self-aligning moment depend on the lateral force and the magnitude of the total trail (the sum of mechanical and pneumatic trail).

* **Kingpin and Caster angle collaboration:**

**Diagram

Description automatically generated**

Figure 16: Scheme used to calculate the resistance torque generated by FY

* **Calculation diagram:**

**Diagram

Description automatically generated**

Figure 3.6 and the diagram show the the caster, KPI and sideslip angle effect on the lateral forces of tire. So, the generated moment due to them around the steering axis can be determined from:

**(8)**

Where:

The resistance moment caused by lateral force (N.m)

Lateral force (N)

Caster angle (degree)

Kingpin angle (degree)

tire radius (m)

### 3.2.3. The resistance moment by normal force:

The resistance torque caused by normal force is one of three resistance components that oppose the steering effort of the driver. In this torque section, the main force affect to the aligning moment is the normal force which is calculated by the equation (1). The force and the moment arm will be illustrated in the Figure 3.7 below:

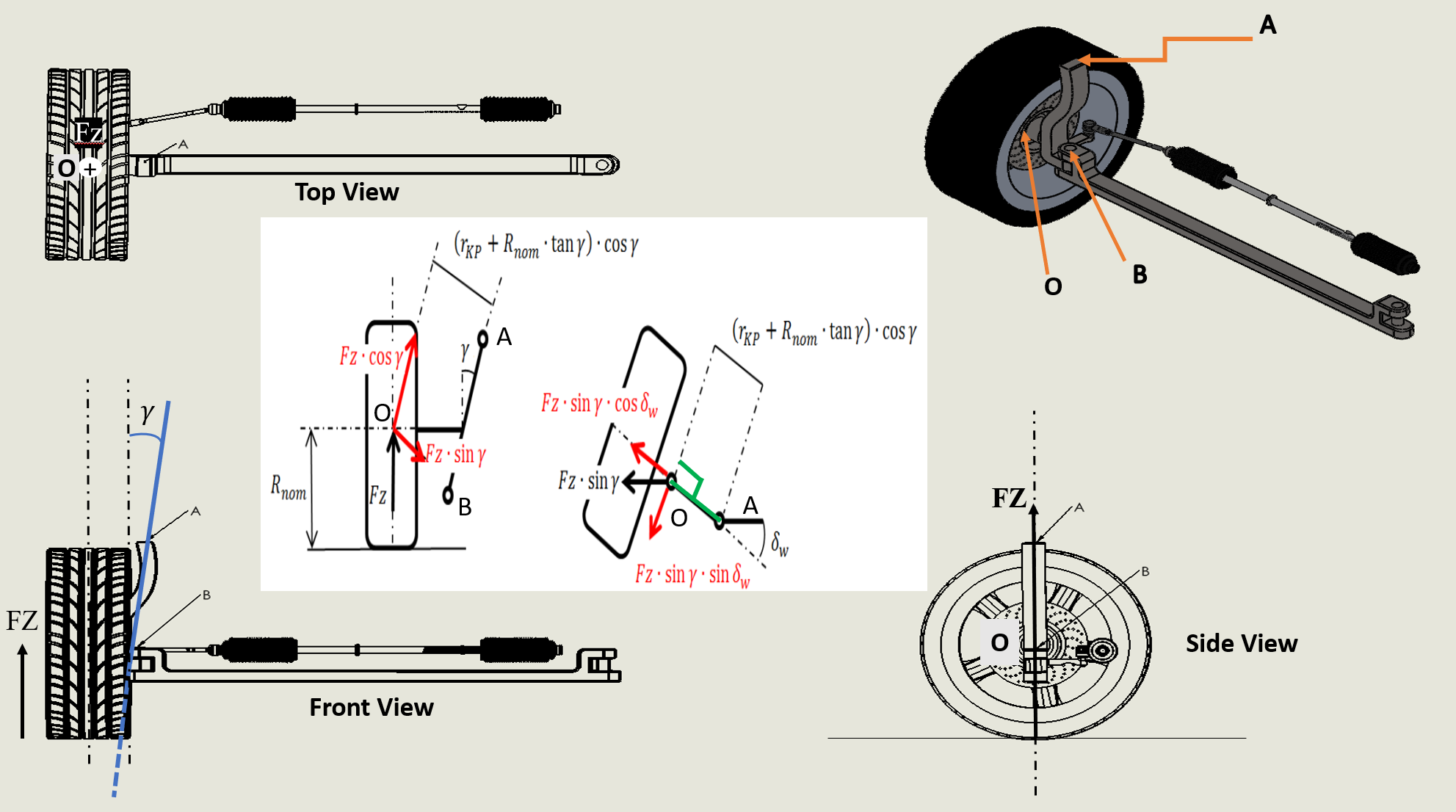


Figure 17: Scheme used to calculate the resistance torque generated by FZ

* **Calculation diagram:**

Diagram

Description automatically generated

Figure 3.7 and the diagram show how the Caster angle, Kingpin angle and the wheel steering angle can influence on the resistance torque caused by the normal force and this torque can be calculated by the equation:

**(9)**

Where:

The resistance moment caused by lateral force (N.m)

Normal force (N)

Caster angle (degree)

Kingpin angle (degree)

tire radius (m)

: Wheel steering angle (degree)

: Scrub radius (m)

From the 3.2.1, 3.2.2, 3.2.3 sections, the total resistance torque generated around the steering axis by longitudinal force, lateral force and normal force can be calculated as:

The resistance torque by FX:

Total resistance torque

The resistance torque by FY:

The resistance torque by FZ:

**IV/ MATLAB Software:**

**4.1) Introduction MATLAB software:**

MATLAB is a digital computing software and programming language widely used in many fields, including science, engineering, finance, and business. MATLAB stands for "MATrix LABoratory" and focuses on matrix calculus and arithmetic in it. MATLAB provides a wide range of tools and functions to process and analyze data, plot graphs, and perform arithmetic and digital operations on data. It also allows users to create and run MATLAB programs to perform complex tasks. MATLAB is developed by MathWorks and is available on multiple platforms, including Windows, Linux, and macOS. MATLAB also has a wide range of auxiliary tools and toolboxes to support specific applications, including signal processing, control, computer vision, and deep learning.

Because of the wide learning fields of MATLAB, I decided to use this software to support and carry out this project simulation.

**4.2) MATLAB blocks:**

In my project, the used blocks list includes:

* Constant block: The Constant block generates a real or complex constant value signal. Use this block to provide a constant signal input.



* Gain: The Gain block multiplies the input by a constant value.



* Inport and Outport block: Provide an input and output port for a subsystem or model.

**Graphical user interface, application

Description automatically generated** A picture containing icon

Description automatically generated

* Degree to Radian block: Conversion from Degrees to Radians.

**Shape

Description automatically generated with low confidence**

* Velocity Convension block: Convert unit of input signal to desired output unit.

**Shape, arrow

Description automatically generated**

* Sum block: Add or subtract inputs.

**Diagram

Description automatically generated**

* Sin and cos block: Trigonometric and hyperbolic functions.

**Shape, arrow

Description automatically generated**

* Product block: Multiply or divide inputs.

**Diagram, schematic

Description automatically generated**

* Vehicle Body 3DOF Dual Track: Implements a 3 DOF rigid two-axle vehicle body model to calculate longitudinal, lateral, and yaw motion. Accounts for body mass, aerodynamic drag, and weight distribution between the axles due to acceleration and steering.

**Diagram

Description automatically generated**

In term of lateral force calculation, I will use the longitudinal velocity, lateral velocity and yaw rate which are taken from this block to calculate the sideslip angle at the center of gravity of the vehicle as the equation (10) below:

**(11)**

Where:

sideslip angle at center of vehicle

lateral velocity at center of vehicle

longitudinal velocity at center of vehicle

After that, I will use the sideslip at center of gravity to calculate the front sideslip angle of each tire by using below equation:

**(12)**

Where:

δ: front wheel steer angle

ξ: is the angle between the X-axis and the velocity at the mid point of the front shaft.

β: sideslip angle at the center of gravity of the vehicle.

: is the distance from front tire to the vehicle’s center of mass.

: yaw rate of the vehicle’s center of mass.

: longitudinal velocity of the vehicle at center of mass.

* To workspace block: Write input to specified timeseries, array, or structure in a workspace. For menu-based simulation, data is written in the MATLAB base workspace.

A picture containing table

Description automatically generated

* Rolling Resistance Force Cal block:

A picture containing text, screenshot, diagram, rectangle

Description automatically generated

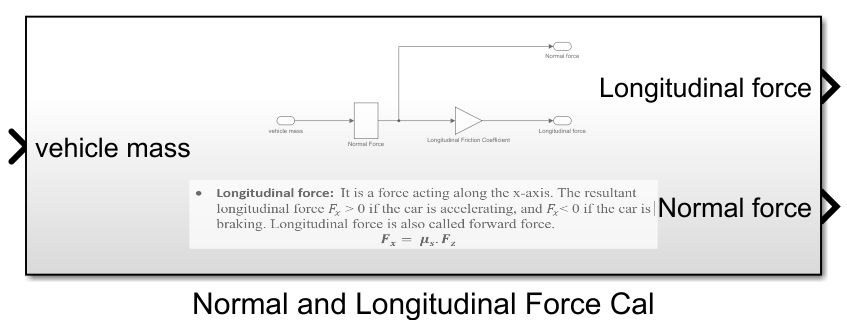
Base on the equation (4), I can build the block to calculate the rolling resistance force of the vehicle. In this block, we will have the input port to provide the vehicle mass for the calculation and output port to take out the rolling resistance force at current driving state.

Modelling it into Matlab/Simulink:

A picture containing text, diagram, line, screenshot

Description automatically generated

* Normal and Longitudinal Force Cal block:



This block is used for normal force and longitudinal force, which is the outputs, calculation base on the equation (1) and (5) by providing the vehicle mass as the input.

Modelling in Matlab/Simulink:

A picture containing diagram, text, line, plan

Description automatically generated

A picture containing diagram, line, text, technical drawing

Description automatically generated

* Lateral Force Cal block:

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Description automatically generated

In term of lateral force calculation, there is the Lateral Force Cal block which is mainly depend on the equations (2) and (11). In this block, the inputs are the steering angle left and right collab in collaboration with the vehicle speed and Cornering stiffness. From those input data, we can calculate the lateral force of the left and right wheel. Besides, we also can take out the both wheels sideslip angle for the other calculation.

Modelling into Matlab/Simulink:

A picture containing diagram, plan, technical drawing, schematic

Description automatically generated

* Wheel Alignment Input block:

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Description automatically generated

This block is used for receiving the wheel alignment constants: Caster angle and Kingpin angle. Morever, we can add with cos block, sine block and tan block of Matlab/Simulink which is mentioned above to send the output as the sin Kingpin, cosine Kingpin, tan Kingpin, cosine Caster, sine Caster and tan Caster.

* The Pneumatic trail Cal block:

A close-up of a sign

Description automatically generated with low confidence

This block is going to be used to calculate the pneumatic trail of the left and right wheel. In the Pneumatic trail input port, there are the longitudinal force signal, left and right sideslip signal, Cornering stiffness signal. From those signals, we can calculate the pneumatic trail of the left and the right wheel base on the below equation:

Where:

Pneumatic trail (m)

Pneumatic trail at zero slip angle (m)

Front sideslip angle (rad)

Cornering Stiffness (N/rad)

Longitudinal frictionn coefficient = 0.35

From the above equation, we can model it into Matlab/Simulink:

A picture containing diagram, text, plan, line

Description automatically generated

* The Mechanical trail block:

A diagram of a mechanical trail cal

Description automatically generated with medium confidence

This block will use the constant scrub radius and tire radius following the Michelin 175/65R14 as the inputs to calculate the mechanical trail by the following equation:

Where:

Mechanical trail (m)

Tire radius (m)

Caster angle (rad)

And modelling it into Matlab/Simulink:

A picture containing text, diagram, screenshot, plan

Description automatically generated

* The Resistant Torque Calulation by Normal Force block:

A picture containing text, screenshot, display, diagram

Description automatically generated

This block is used for the resistance torque caused by the normal force . From the equation (9), this resistance components include: wheel alignment in cosine, sine and tan; the scrub radius and tire radius; left and right steer angle in sine and finally the normal force. From those input signals, this block can calculate the resistance moment of the left and right front wheel.

Modelling in Matlab/Simulink:

A screenshot of a computer

Description automatically generated with medium confidence

* The Resistance Moment by Longitudinal Force block:

A picture containing text, screenshot, diagram, parallel

Description automatically generated

This block is used for the resistance torque calculation caused by the longitudinal forces . Base on the equation (5), this block need many input parameters include: sine and cosine of wheel alignment (Kingpin and Caster angle), longitudinal force, rolling resistance force, left and right lateral force, tire radius and scrub radius, sine and cosine of left and right steer angle. All parameters are mentioned in the previous subsystems. By using specific factors, the resistance torque or moment caused by longitudinal forces of the left and right wheel.

Modelling into Matlab/Simulink:

A diagram of a flowchart

Description automatically generated with low confidence

* The Total Resistance Torque by Lateral Force FY block:

A picture containing text, screenshot, parallel, font

Description automatically generated

This block is used for the resistance torque calculation caused by the lateral forces . Base on the equation (5), this block need many input parameters include: sine and cosine of wheel alignment (Kingpin and Caster angle), longitudinal force, rolling resistance force, left and right lateral force, tire radius and scrub radius, sine and cosine of left and right steer angle, left and right pneumatic trail, mechanical trail. All parameters are mentioned in the previous subsystems. By using specific factors, the resistance torque or moment caused by lateral forces of the left and right wheel.

Modelling into Matlab/Simulink:

A picture containing diagram, sketch, plan, technical drawing

Description automatically generated

* From all the blocks mentioned above, we can model the total resistance torque of both front wheels into by using equation (10):

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# V/ Result and Discussion:

* *The result will be calculated from the mean value of the resistance moment of two front tyres of VIOS.*

All the parameters[[1]](#footnote-1) will be taken from the table below:

|  |  |  |
| --- | --- | --- |
| **Symbols** | **Value** | **Name** |
|  | 0.03 [m] | Pneumatic trail at zero slip angle |
| g | 9.81 [m/] | Gravity of earth |
|  | [rad] | Caster angle |
|  | 0.011 | Rolling resistance coefficient |
|  | [rad] | Kingpin angle |
|  | N/rad | Cornering stiffness of tire |
|  | 0.35-0.4 | Friction coefficient |
|  | [m] | Nominal radius of tire |
|  | [m] | Wheelbase |
|  | [m] | Distance from the center of vehicle to rear axle |
|  | [rad] | Front side slip angle |
|  | [m] | Pneumatic trail |
|  | [m] | Mechanical trail |
|  | [m] | Total trail |
|  | [m] | Scrub radius |
|  | [rad] | Wheel steer angle |
|  | [N] | Longitudinal force |
|  | [N] | Lateral force |
|  | [N] | Rolling resistance force |
|  | [N] | Normal force |

Table 2: Model Parameters

Before going into the survey cases, we need to prove the accuracy of the built model. In the “Study on low-speed steering resistance torque of vehicles considering friction between tire and pavement”11 article, they surveyed the total resistance torque at different steering wheel angle as below Figure shows:

A picture containing diagram, colorfulness, screenshot, design

Description automatically generated

Figure 18: 3D surface plot of the low-speed steering resistance torque for example project

Figure 18 shows the The low-speed steering resistance torque increased with the steering wheel angle at a certain speed. When the vehicle speed increases, the steering resistance torque almost changed linearly decreased. Besides, the steering resistance torque varied increasingly when the steering wheel angle increased or the driver put more effort (more steering torque). The steering resistance torque is peak at about 200 N.m for no steering wheel angle and about 273 N.m for 600 degrees steering wheel angle.

Figure 19: 3D surface plot of the low-speed steering resistance torque for project model

Moving to the Figure 19 which is plotted by the project model, we can see that the total resistance result is almost similar to the Figure 18 which is an example model. In comparison with the example model, the total resistance torque at 0 degree of steering wheel angle is about 205 N.m and the total resistance torque at 600 degrees of steering wheel angle is about 270.3 N.m , which is almost exactly the same with the total resistance torque example (200N.m at 0 degree steering wheel angle and 273N.m at 600 degree steering wheel angle respectively). From the given data, we can conclude that the project model is similar to the example model with an acceptable error rate and use this model to convey different situations to test effects of the specific factors on the total resistance torque.

* **Vehicle mass effects on the total resistance torque:**

Figure 20: Vehicle mass effects on the total resistance torque

In this section, the vehicle mass effects on the self-resistance torque are going to be surveyed among at different vehicle mass levels: from 1100kg to 1500kg. From the given data, we can observe that the total resistance torque is increasing proportionally at constant steering torque, vehicle speed and wheel alignment. This is expected, as a heavier vehicle would experience more resistance and the reason is when the vehicle mass changes it will causes the change in the moment components such as: the resistance torque caused by longitudinal forces, resistance torque caused by lateral forces and even the resistance torque caused by the normal force. To get further information, we can look at the Figure 5.2 below:

Figure 21: The component torque distribution

The data shows that the total resistance torque formed by the three forces of longitudinal forces , lateral forces and normal force increases gradually as the impact force increases. However, the distribution of torque for each force is different. The force creates the smallest resistance, while the force creates the greatest resistance. In addition to the points mentioned above, we can also observe that the resistance torque generated by and forces increases steadily and continuously with the increase of the acting force. However, the drag due to the force increases more slowly than the other two. In conclusion, the mass can affect a lot on the resistance torque caused by .

Besides, the vehicle mass effects on the total resistance torque can alse conveyed at different vehicle speed ranges as the Figure 5.3 shows:

Figure 22: Vehicle mass effects on the steering resistance torque at different speed

To test the accuracy of the model, it is necessary to demonstrate the impact of changes in weight across different speed ranges, as the effect of weight on a vehicle's performance is not limited to a specific speed range. In this situation, we will convey the total steering resistance torque at different vehicle speed in collaboration with varying in vehicle mass. For example, at a speed of 20km/h, the total resistance force increases as the weight of the vehicle is changed from 1100kg (the estimated weight of the vehicle) to 1500kg (the weight of the vehicle with full passengers and luggage). There is a similar trend in comparision with the total resistance torque at 100km/h, 140km/h and 180km/h. From the given data, we can observe that the total resistance torque also increases as the speed increases. That is the reason why in the next section we will conduct the survey for chaging vehicle speed.

* **Vehicle speed effects on the total resistance torque:**

Figure 23: Effect of the vehicle velocity on the steering resistance torque

From the given data in the figure, it can be observed that there is a decreasing trend as we increase the speed of the vehicle while keeping all other factors constant. As shown in the Figure 23, we investigate the total resistance force while steering with a steering force of 20 N.m, a vehicle weight of 1100kg, and wheel angles of Kingpin = 9 degrees and Caster = 5 degrees. After the investigation, it is found that the steering resistance force decreases rapidly in the lower speed ranges (as shown in the figure from 0 to 40 km/h) and the values quickly drop from 132 N.m to 106.2 N.m. After that, the resistance torque gradually decreases (from 101.7 N.m to 92.9 N.m). To get further information, we are going to look at the resistance torque distribution for each torque component in the Figure 24 below.

Figure 24: The resistance torque distribution on vehicle speed

The graph above shows the distribution of resistance torque components include total resistance torque by longitudinal forces , lateral forces and normal force . At first, it can observe that the total resistance torque by normal force account for 0% in total resistance torque among vehicle speed ranges (which is constant at 14.55 N.m). Besides, the influence of the total resistance torque by lateral forces will gradually decrease as we increase the vehicle velocity (at 0 km/h the resistance torque by forces will have the greatest value as 57.45 N.m and much more lower at 180 km/h). The reason is lateral force will be calculated by the multiplication of Cornering stiffness and sideslip angle which is mainly depend on the longitudinal velocity (in equation **(11)** and **(12)**). Those equations indicate that if we increase the velocity and keep other factors as constant, the value of sideslip angle will be smaller. Finally, the resistance torque distribution by longitudinal forces has the biggest influence on the total resistance torque when we increase the vehicle speed.

Figure 25: Vehicle velocity effect on total resistance torque by changing vehicle mass

From the given Figure, it is obviously that the vehicle speed impact on the total self-resistance torque also happen when we change the vehicle weight. For example, at 0km/h and 1300kg (this means one driver, no passengers and luggage), the driver need to spend more effort to steer because the resistance torque is biggest as 155 N.m and this value is going to be down when we increase the vehicle speed. It is similar to 1500 kg vehicle weight (driver, passengers and luggage) case. In this situation, at 0 km/h, the total resistance moment is 185.2 N.m.

* **Wheel alignment impacts on the steering resistance torque:**

In the previous cases, we tested and conveyed how vehicle mass and vehicle velocity can affect to the total resistance moment and there are two opposite trends when we increase both ones. In this section, we are going to investigate the effect of the wheel alignment: Kingpin angle and Caster angle on the steering resistance torque.

* Kingpin angle:

Figure 26: Kingpin angle impact on the total resistance torque

This figure illustrates how kingpin angle can affect to the resistance moment. In this investigation, the kingpin angle will be changed from 0 degrees to 9 degrees (Kingpin angle value of VIOS). It is obvious that when we change the value of this angle, the resistance torque increases gradually from 37.65 N.m to 106.2 N.m. This means that lower Kingpin angle lower resistance torque produced. We can also get more information about this effect in the Figure 27.

Figure 27: Torque component distribution on Kingpin effects

From the given data, it is easily seen that the distribution of the resistance torque by normal force is least in comparision with others. Besides, there is an almost constant trend for the distribution of the lateral forces in the total resistance steering which is approximately about 19 N.m. Finally, the steering resistance moment caused by longitudinal forces has the biggest distribution on the total resistance torque in the exception of zero Kingpin angle and this value will go up if we increase the Kingpin angle (from 18.6 N.m to 72.7 N.m).

Figure 28: Kingpin effects on resistance torque at different vehcle mass

When we look at the given figure, it is easily concluded that kingpin angle effects also happen in the different range of vehicle mass (1100kg, 1300kg and 1500kg) and there is an almost exactly increasing trend among them. However, at heavier vehicle mass, the difference value when we increase the Kingpin angle is bigger than at lighter vehicle mass (106.2-37.65 N.m in comparison with 151.5 - 60.21 N.m).

* Caster angle

Figure 29: Caster angle effects on steering resistance torque

In comparison with the Kingpin angle, the Caster angle effects have the similarity with the King angle effects that means when we increase the Caster angle, the value of total resistance torque also increases. However, the Caster angle’s increased torque value is smaller than the Kingpin’s angle increased torque value. That is the reason why we can conclude that the Kingpin’s angle has bigger influence on the steering resistance torque value if we change the value of both at same condtion.  
 To see this assumption clearly, we should look at the Figure 29 below:

Figure 30: The torque distribution of Caster angle effects

From the given data, we can see that the trend is slightly increasing in overall resistance torque. However, in comparison with the torque component distribution of the Kingpin angle in Figure 27, there are 2 differences. Firstly, while the total resistance torque by lateral forces for Kingpin effects is almost constant, the total resistance torque by lateral forces for Caster effects has the increasing trend at bigger Caster angle. Secondly, the total resistance torque by normal force and longitudinal forces in Caster effects remain almosst constant, otherwise, this values rapidly fluctuated at the same convey situations.

Lastly, when we conduct the survey about the total resistance torque by varying Caster angle at different vehicle mass, the similar result happen in the case of Kingpin angle effects: bigger resistance torque at bigger vehicle mass value and bigger Caster angle values as an Figure 31 illustrates.

Figure 31: Caster angle effects on the steering resistance torque at many vehicle mass value

# V/ CONCLUSION AND FUTURE PLAN:

All the steering resistance torque conveys are clearly mentioned in this Capstone project. Moreover, a total steering resistance torque model of a power steering system is built exactly by applying above knowledges: numerical equations and relevant factors. The model can be used for performance evaluation and can be easily adapted to fit in a larger vehicle handling model. It can also be used for the design of other power steering systems, thanks to the resistance torque model to calculate the appropriate assist torque in different driving conditions.

The main conclusion obtained in this Capstone project is how the wheel alignment especially Caster angle and Kingpin angle affect to the resistance moment in collaboration with specific factors such as vehicle mass, vehicle velocity,… Through all the figures mentioned above, we can conclude that all wheel alignment has the huge impact on the total steering resistance torque in collaboration with vehicle mass and vehicle speed.

In the future, I wil continue to develop this steering mechanism model and going further to simulate different situations by assist simulator such as Matlab/simulink base on all relevant theories that mentioned in this Capstone project to provide the exact results in ccomparision with reality.

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