Assignment A03

ENSE 885AY Application of Deep Learning in Computer Vision

Student Name: Behnam Moradi

Camera Calibration and Fundamental Matrix Estimation with RANSAC

This is the third assignment of our wonderful course on **deep learning applications in computer vision** at the University of Regina. We the students are required to come up with unique and fast algorithms to estimate the followings:

1. Camera Projection Matrix
2. Fundamental Matrix
3. Fundamental Matrix with RANSAC

In this report, we will be going through these wonderful algorithms one after another. So, let's get started.

Camera Projection Matrix

Simply, a camera projection matrix is a set of parameters to map a 2D image to a 3D coordinate. Mathematically, we will be using the following algebraic equation to find the projection matrix:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \cong \begin{pmatrix} u*s \\ v*s \\ s \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

(u, v) → points location in 2D image
M matrix → projection matrix
(X, Y, Z) → points location in 3D world

Programming algorithm to find the projection matrix:

$$
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\
0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\
 & & & & & & \vdots & & & & \\
X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n
\end{bmatrix}
\begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \end{bmatrix}
=
\begin{bmatrix} u_1 \\ v_1 \\ \vdots \\ u_n \\ v_n \end{bmatrix}
$$

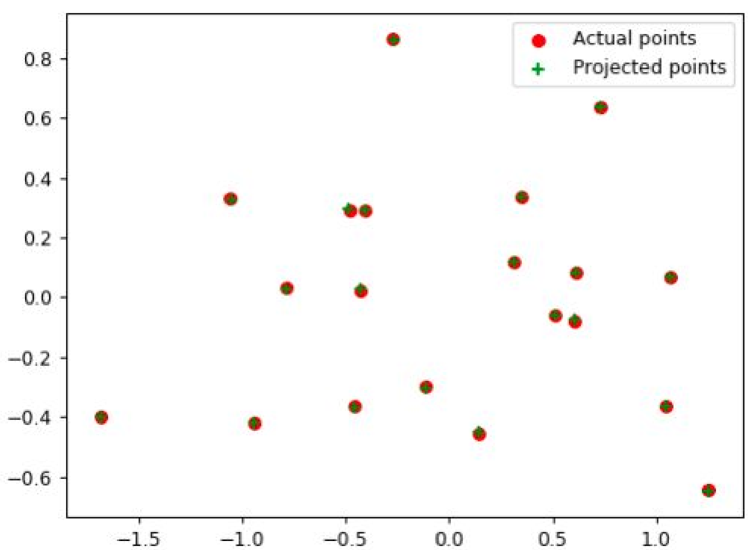
I will be solving the this linear equation directly by inverting A matrix:

$$AM=B \Rightarrow A.A^T.M = A^T.B \Rightarrow M = (A.A^T)^{-1}A^T.B$$

$$M = \begin{bmatrix} 0.7678 & -0.4938 & -0.02339 & 0.0067 \\ -0.085 & -0.091 & -0.9065 & -0.87 \\ 0.1826 & 0.2988 & -0.0741 & 1.00 \end{bmatrix}$$

$$total\ residual = 0.044535$$

The following figure illustrates the actual and projected points all together in one:

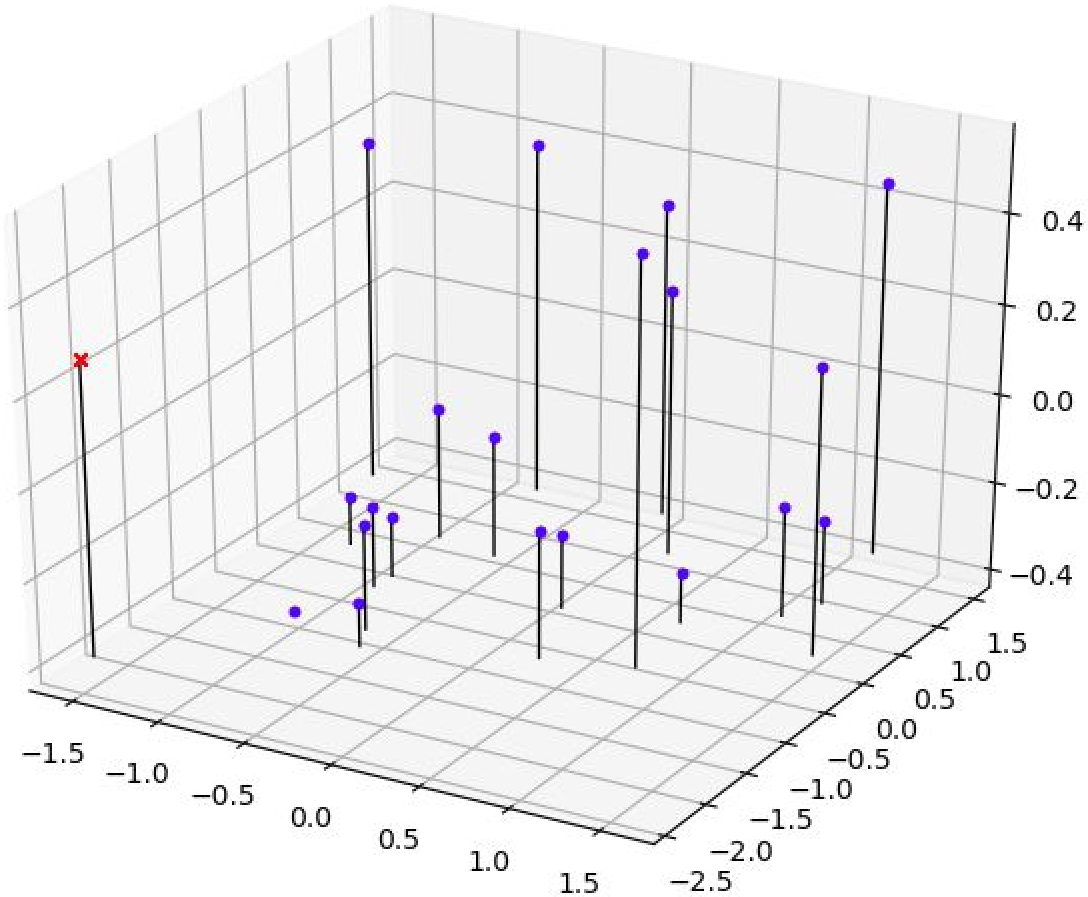


Camera center can simply be calculated by the following algebraic equation:

$$CC = -Q^{-1}M_4$$

$$M_4 \rightarrow M(:,\ 0{:}3)$$

Camera Center = [-1.5126, -2.3517, 0.2827]
The following figure illustrates the camera center in a 3D coordinate system.



Fundamental Matrix

In stereo version systems, two cameras provide the system with two images at the same time. The Fundamental Matrix helps us to find identical points in these two images. The mathematical idea behind it is very simple. As it can be seen from the following linear system, we have 2D points from both images involved in this linear equation:

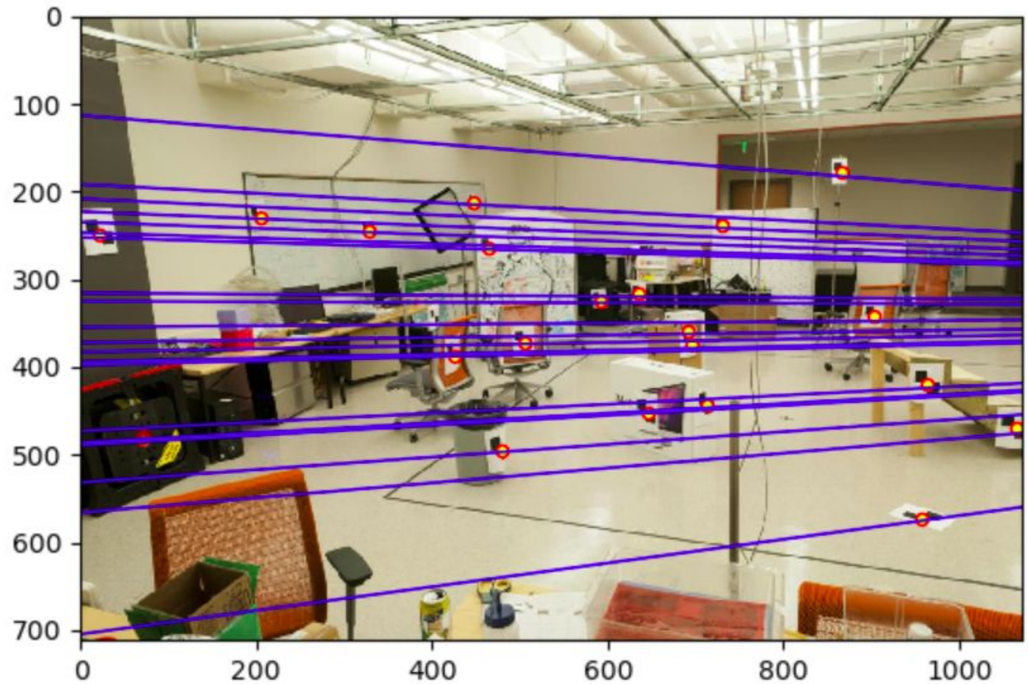
$$\begin{pmatrix} u' & v' & 1 \end{pmatrix} \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = 0$$

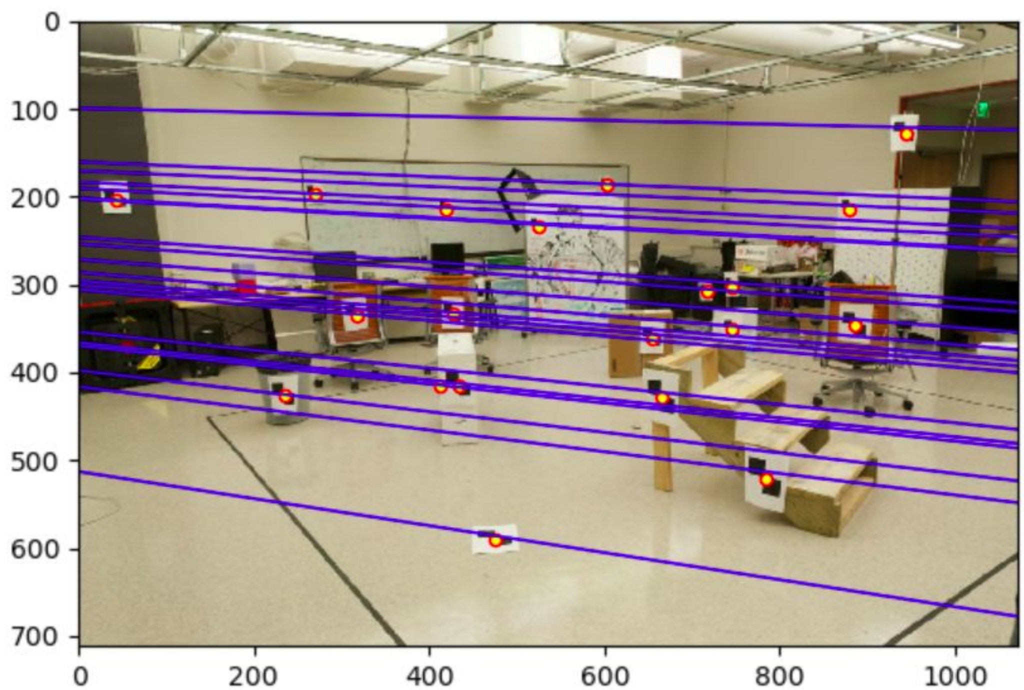
The goal is to find the “f” scalers in the “F” matrix by solving the simplified version on the system:

$$\begin{bmatrix} u_1 u_1{}' & u_1 v_1{}' & u_1 & v_1 u_1{}' & v_1 v_1{}' & v_1 & u_1{}' & v_1{}' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_n u_v' & u_n v_n{}' & u_n & v_n u_n{}' & v_n v_n{}' & v_n & u_n{}' & v_n{}' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = \mathbf{0}$$

Using the algorithm we wrote in Part #1, we will be using SVD to estimate a second ranked matrix by forcing the lowest singular value to 0 while using SVD for the second time. Once the fundamental matrix “F” is calculated, we can simply plot the epipolar lines. The following figures illustrate the epipolar lines in both images.

```
The fundamental matrix is calculated successfully!
[[ 7.20248917e-06 -9.67102416e-05  2.53206006e-02]
 [-6.04123213e-05  1.84673970e-05 -1.91377377e-01]
 [ 3.38104691e-04  2.59523165e-01 -5.80819930e+00]]
```



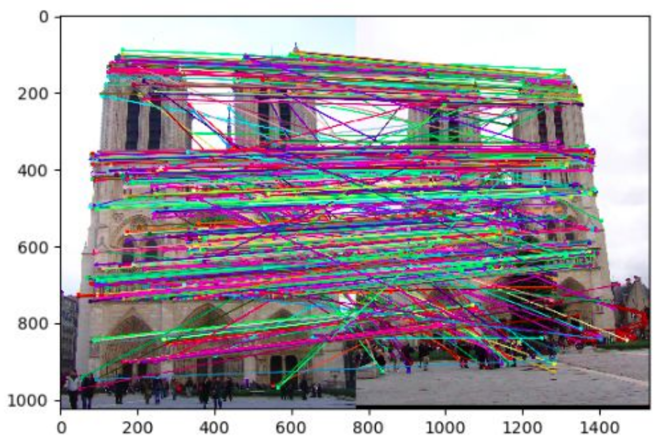


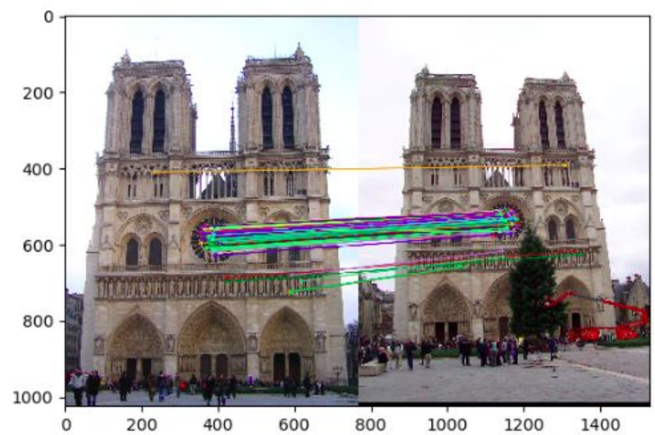
Fundamental Matrix with RANSAC

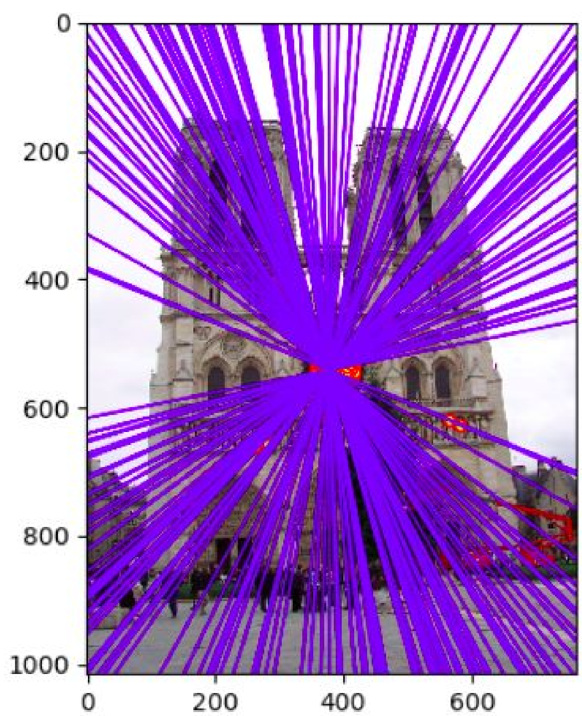
We assume that we have possible matching points in 2 images and they are calculated by SIFT in project #2. We will be using the concept of fundamental matrix with RANSAC. Here is my algorithm:

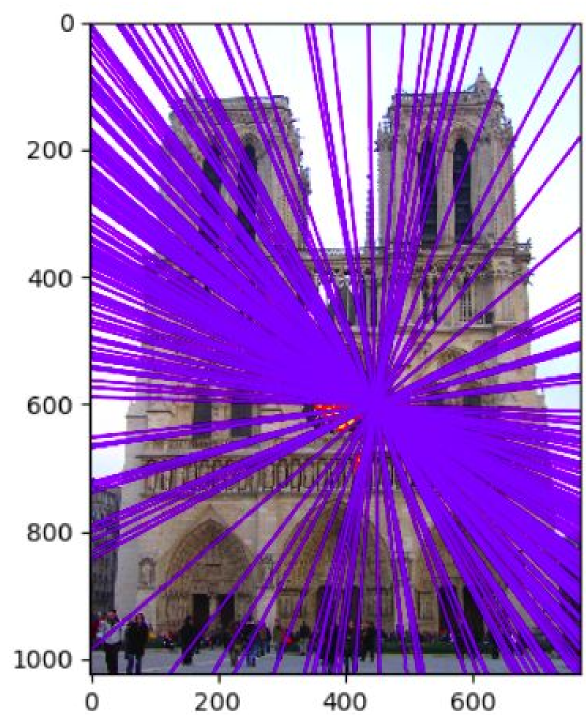
1. Randomly, select 8 matches and calculate the fundamental matrix
2. Calculate the deviation from zero and select the inliers.
3. Check the number of inliers with the previous step.
4. If the number of inliers are increased, then update the fundamental matrix until you get the final fundamental matrix.

Results: Notre dome

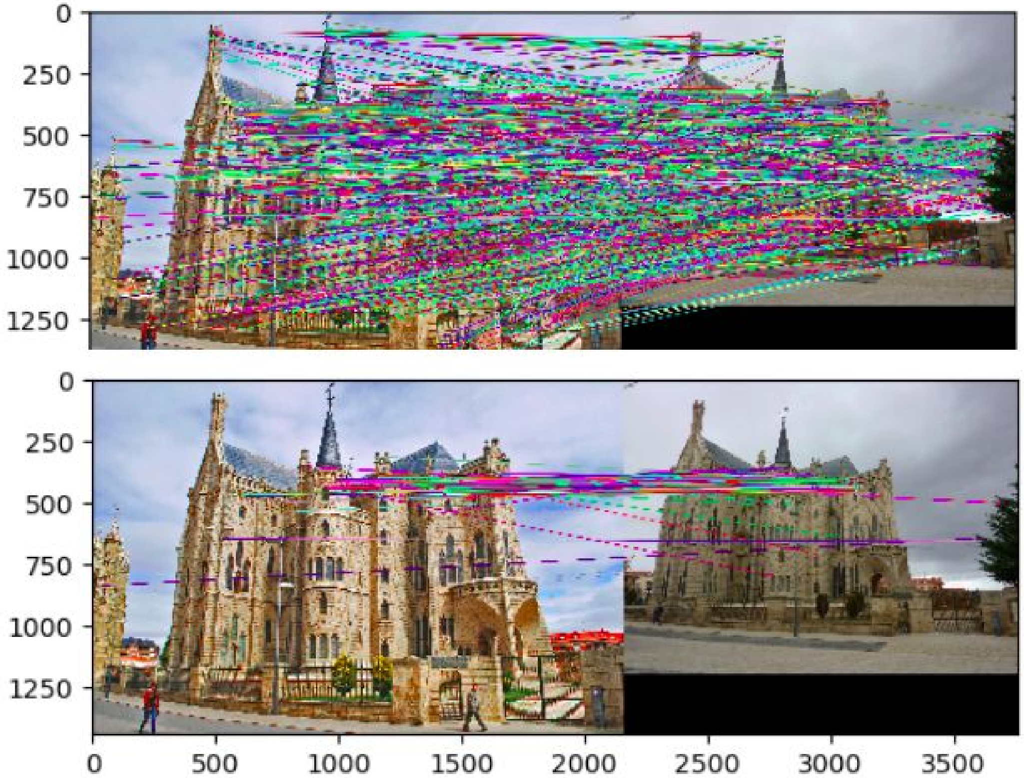


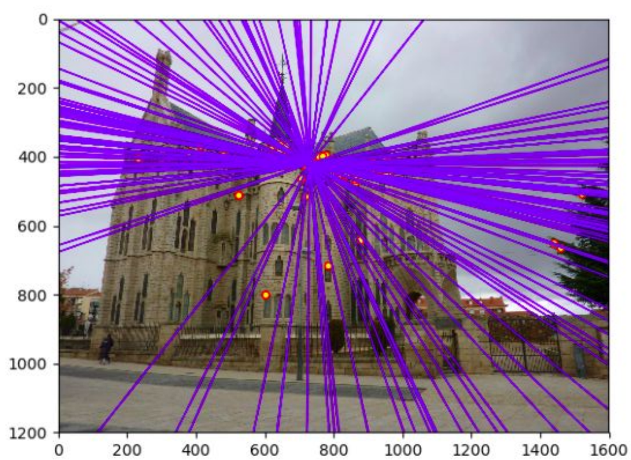


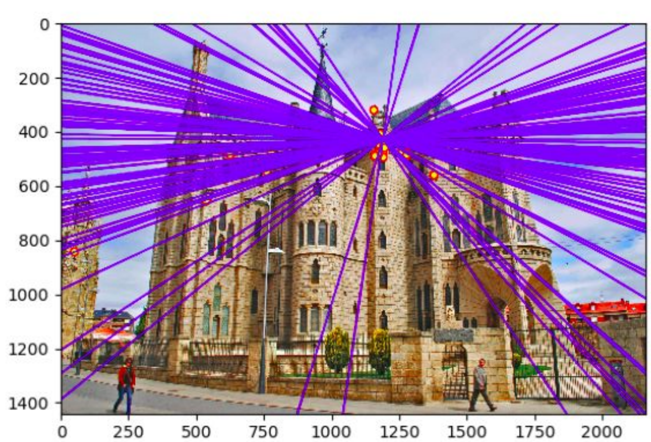




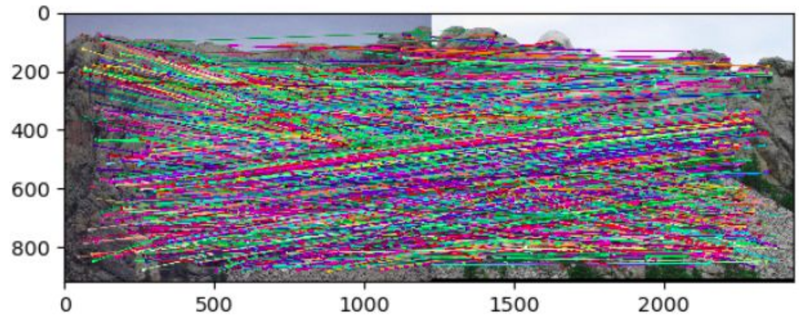
Episcopal Palase:

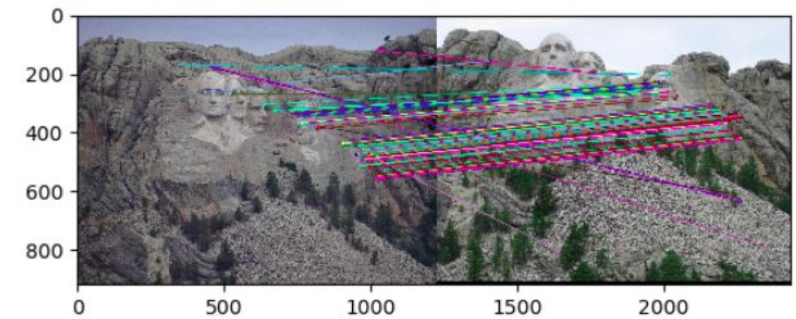


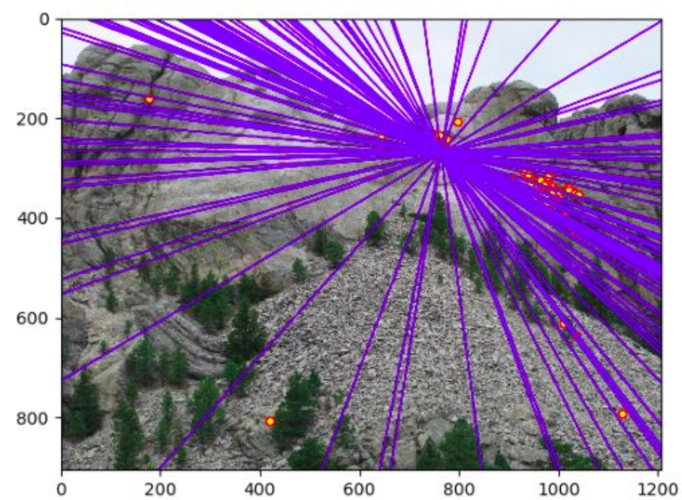


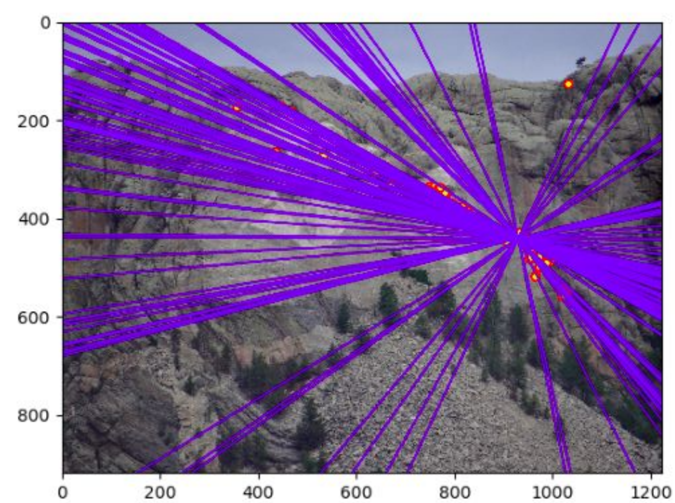


Mount Rushmore

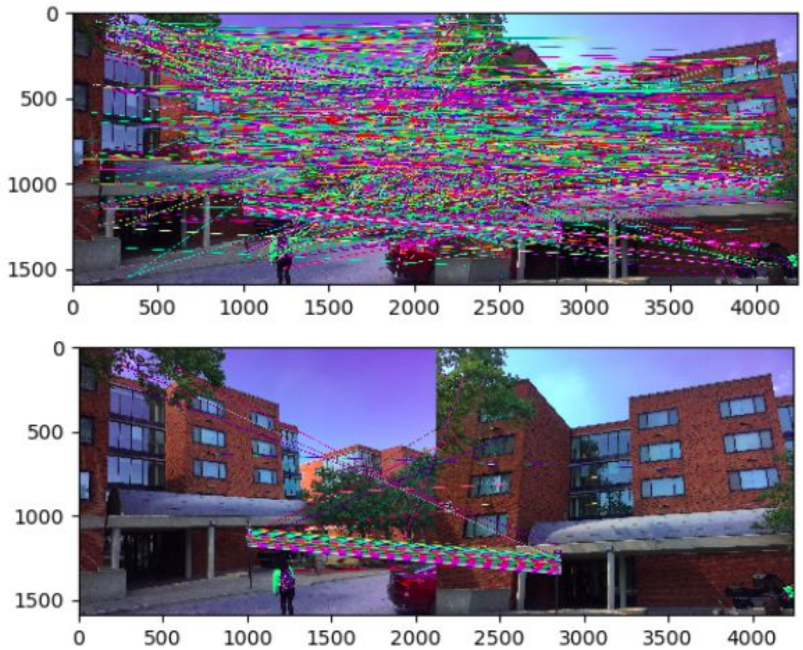


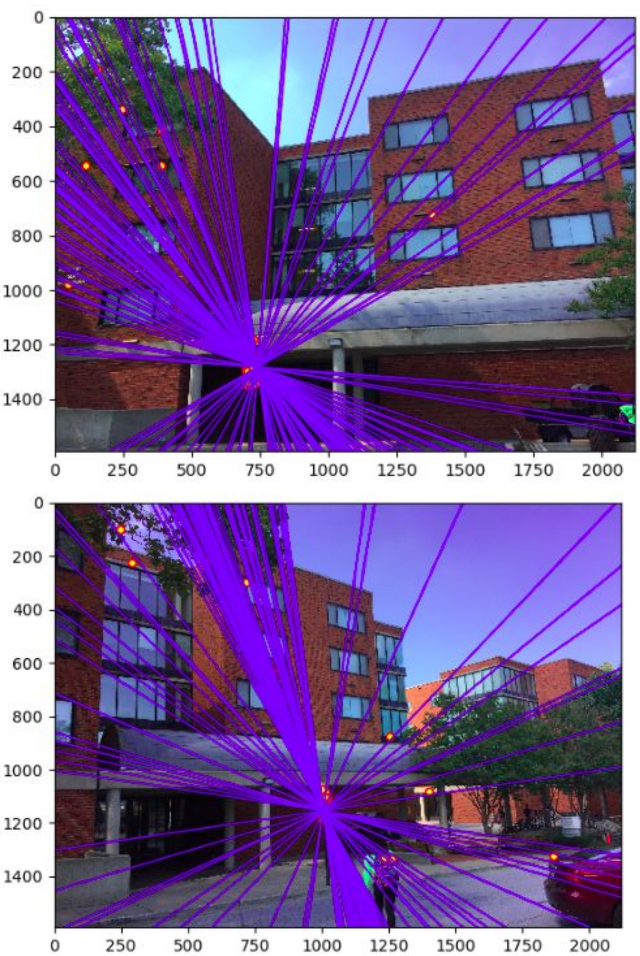






Woodruff Dorm:





Conclusion

In this project, the projection matrix was extracted to map a 2D set of points into its corresponding 3D.
An algorithm was developed to calculate the fundamental matrix in stereo version systems.
The aforementioned fundamental matrix was used for improving feature matching algorithms and RANSAC was used to deal with noisy data. An algorithm has been developed to perform RANSAC.

The End
March 08, 2021