

$$\int_0^2 \int_0^{\sqrt{y}} \exp(-x^5 - y^4) dx dy =$$

A) $\int_0^4 \int_{x^2}^4 \exp(-x^5 - y^4) dy dx ;$ B) $\int_0^2 \int_{\sqrt{x}}^{\sqrt{2}} \exp(-x^5 - y^4) dy dx ;$

C) $\int_0^{\sqrt{2}} \int_{x^2}^2 \exp(-x^5 - y^4) dy dx ;$ D) $\int_0^2 \int_{x^2}^2 \exp(-x^5 - y^4) dy dx .$

Evaluate $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-x^2-y^2} dx dy = \underline{\hspace{1cm}} (9) \hspace{1cm} .$

Let $D = \{(x, y) \mid x^2 + y^2 \leq 3, y \geq 0\}$ be a lamina with density $\rho(x, y) = y$. Then, the center of mass $(\bar{x}, \bar{y}) = \underline{\hspace{1cm} (5) \hspace{1cm}}$.

$$\int_0^2 \frac{\cos x}{\sin x + \sin (2 - x)} dx = ?$$

Let $x = r \cos \theta$ and $y = r \sin \theta$. Then $\frac{\partial x}{\partial r} \cdot \frac{\partial y}{\partial \theta} - \frac{\partial x}{\partial \theta} \cdot \frac{\partial y}{\partial r} = \underline{\hspace{2cm}} (2)$

已知 $\lim_{x \rightarrow 0} [\alpha \arctan \frac{1}{x} + (1 + |x|)^{\frac{1}{x}}]$ 存在，求 α 的值.

$$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right) \cdots \left(1 - \frac{1}{n^2}\right) \cdots = ?$$

令 $f(x, y) = (x + y) \sin \frac{1}{x} \sin \frac{1}{y}$ 。討論以下兩極限是否存在; 若存在, 則求其值:

(a) $\lim_{x \rightarrow 0} \left(\lim_{y \rightarrow 0} f(x, y) \right)$, 答: (2a) 。

(b) $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$, 答: (2b) 。

- (a) Given $\varepsilon = 0.1$, find a number $\delta > 0$ such that if $0 < |x - 3| < \delta$, then $|1/(x + 1) - 1/4| < 0.1$. (*4 points*)
- (b) Use ε, δ languages to show that $\lim_{x \rightarrow 3} 1/(1 + x) = 1/4$. (*8 points*)

If $f(x) = e^{x^3}$, then $f^{(3n)}(0) =$

- (A) 0; (B) $n!$; (C) $(3n)!$; (D) $\frac{(3n)!}{n!}$.

(4) 设函数 $f(x)$ 在区间 $[0,1]$ 上连续, 则 $\int_0^1 f(x)dx =$

(A) $\lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(\frac{2k-1}{2n}\right) \frac{1}{2n}.$

(B) $\lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(\frac{2k-1}{2n}\right) \frac{1}{n}.$

(C) $\lim_{n \rightarrow \infty} \sum_{k=1}^{2n} f\left(\frac{k-1}{2n}\right) \frac{1}{n}.$

(D) $\lim_{n \rightarrow 0} \sum_{k=1}^{2n} f\left(\frac{k}{2n}\right) \cdot \frac{2}{n}.$

$$\int_0^x (\int_0^u f(t) dt) du = \int_0^x f(u) (x - u) du.$$

已知曲线 $C: \begin{cases} x^2 + 2y^2 - z = 6 \\ 4x + 2y + z = 30 \end{cases}$ ，求 C 上的点到 xoy 坐标面距离的最大值.

(6 %) Find the slope of tangent to the curve $x^3 + y^3 - 9xy = 0$ at the point $(2, 4)$.

Find the Maclaurian series for the function $f(x) = x \sin(x^2 + 1)$ and also indicate the interval of convergence. (10 points)

由方程組 $\begin{cases} u = x^2 + xy - y^2 \\ v = 2xy + y^2 \end{cases}$, 定義求在 $(x, y) = (2, -1)$ 的 $(\frac{\partial x}{\partial u})_v$ 及 $(\frac{\partial x}{\partial v})_u$