Note for Social Force Model for Pedestrain Dynamics

Formular of Social Force Model

1. Pedestrains with Pedestrains

(Note that α, β means the different pedestrains.)

Someone wants to reach a certain destination \vec{r}_{α}^0 as comfortable as possible, therefore usually taking the shortest possible way having the shape of a polygon with edges $\vec{r}_{\alpha}^0 := \vec{r}_{\alpha}^1, \dots, \vec{r}_{\alpha}^n$. The desired direction:

$$ec{e}_lpha(t) = rac{ec{r}_lpha^k - ec{r}_lpha(t)}{||ec{r}_lpha^k - ec{r}_lpha(t)||}$$

 $ec{r}_lpha(t) :=$ actual position of pedestrain lpha at time t.

 $\vec{r}_{\alpha}^{k}(t):=$ steer for the nearest point at time t.

If the pedestrains is not deiturbed, he or she will walk to the desired direction with a desired speed v_{α}^0 . Then, the actual velocity $\vec{v}_{\alpha}^0(t)=v_{\alpha}^0\vec{e}_{\alpha}$. The acceleration term:

$${ec F}_lpha^0({ec v}_lpha, v_lpha^0{ec e}_lpha) = rac{v_lpha^0{ec e}_lpha - {ec v}_lpha}{ au_lpha}$$

If the motion of the pedestrain is influenced by other pedestrains, he or she will keeps a certain distance with the privat sphere. The vectorial quantities repulsive effects of other β (monotonic decrease):

$$ec{f}_{~lphaeta}(ec{r}_{lphaeta}) = - riangledown_{ec{r}_{lphaeta}} V_{lphaeta}[b(ec{r}_{lphaeta})],~where \ V_{lphaeta}(b(ec{r}_{lphaeta})) = V_{lphaeta}^0 e^{-rac{b}{\sigma}}$$

 $b(\vec{r}_{\alpha\beta})$ denotes the semi-minor axis of the ellipse given by:

$$b(ec{r}_{lphaeta}) = rac{\sqrt{(||ec{r}_{lphaeta}|| + ||ec{r}_{lphaeta} - v_eta\Delta t ec{e}_eta||)^2 - (v_eta\Delta t)^2}}{2}, \ where \ ec{r}_{lphaeta} = ec{r}_lpha - ec{r}_eta$$

 $v_{\beta}\Delta t:=$ order of the step width of pedestrain β .

2. Pedestrains with Space Objects

A pedestrian will keeps a certain distance from borders of buildings, walls, obstacles to avoid the danger of getting hurt. So, a border evokes a repulsive effect (monotonic decrease):

$$ec{F}_{lpha B}(ec{r}_{lpha B}) = - riangledown_{ec{r}_{lpha B}} U_{lpha B}(||ec{r}_{lpha B}||), ~where ~ec{r}_{lpha B} = ec{r}_{lpha} - ec{r}_{lpha B}^{(min)} \ U_{lpha B}(||ec{r}_{lpha B}||) = U_{lpha B}^0 e^{-rac{||ec{r}_{lpha B}||}{R}}$$

 $B := \mathsf{the}\;\mathsf{border}$

 $ec{r}_{\alpha B}^{(min)} :=$ the point of B that is nearest to pedestrain lpha.

3. Pedestrains with Attractive Objects

Pedestrians are sometimes attracted by other persons or objects. The attractive effects (monotonic decrease):

$$ec{f}_{lpha i}(||ec{r}_{lpha i}||,t) = - riangledown_{ec{r}_{lpha i}} W_{lpha i}(||ec{r}_{lpha i}||,t), \ where \ ec{r}_{lpha i} = ec{r}_{lpha} - ec{r}_{i}$$

4. Field of Sight Processing

The formula for repulsive and attractive effects only hold for β , B, i in the sight of α . Therefore,

$$s(ec{e},ec{f}) := \left\{ egin{aligned} 1, \ if \ ec{e} \cdot ec{f} \geq ||ec{f}|| cos\phi \ c, \ otherwise. \end{aligned}
ight.$$

Now, the repulsive and attractive effects are given by

$$ec{F}_{lphaeta}(ec{e}_lpha,ec{r}_{lphaeta}) = s(ec{e}_lpha,-ec{f}_{lphaeta})ec{f}_{lphaeta}(ec{r}_{lphaeta})$$

$$ec{F}_{lpha i}(ec{e}_lpha,ec{r}_{lpha i},t)=s(ec{e}_lpha,ec{f}_{lpha i})ec{f}_{lpha i}(ec{r}_{lpha i},t)$$

5. Social Force Model

$$ec{F}_lpha(t) = {ec{F}}_lpha^0(ec{v}_lpha, v_lpha^0 ec{e}_lpha) + \sum_eta ec{F}_{lphaeta}(ec{e}_lpha, ec{r}_{lphaeta}) + \sum_B ec{F}_{lpha B}(ec{r}_{lpha B}) + \sum_i ec{F}_{lpha i}(ec{e}_lpha, ec{r}_{lpha i}, t).$$

Then, the social force model is given by:

$$rac{dec{w}_{lpha}}{dt} = ec{F}_{lpha}(t) + fluctuations ext{(random value)}.$$

6. Maximum Acceptable Speed

Since the actual speed is limited by a pedestrian's maximal acceptable speed $v_{\alpha}^{(max)}$. Hence, the realized motion is given by:

$$egin{aligned} rac{dec{r}_lpha}{dt} &= ec{v}_lpha(t) = ec{w}_lpha(t) g(rac{v_lpha^{(max)}}{||ec{w}_lpha||}), \ where \ g(rac{v_lpha^{(max)}}{||ec{w}_lpha||}) &:= egin{cases} 1, \ if \ rac{v_lpha^{(max)}}{||ec{w}_lpha||} &\geq 1 \ rac{v_lpha^{(max)}}{||ec{w}_lpha||} &> i = min\{1, rac{v_lpha^{(max)}}{||ec{w}_lpha||} \} \end{cases} \ ec{g}(rac{v_lpha^{(max)}}{||ec{w}_lpha||}) &:= min\{1, rac{v_lpha^{(max)}}{||ec{w}_lpha||} \} \end{aligned}$$

Implementation of SFM

Enviornment

C++ (Programming Language) Visual Studio 2019 (IDE)

Given Parameters

The followings are the given parameters in the paper:

Parameters	Values
$V^{0}_{lphaeta}$	2.1
σ	0.3
ϕ	100
c	0.5
$U_{lpha B}^{0}$	10
R	0.2
$v_{lpha}^{(max)}$	$1.3v_{lpha}^{0}$
Δt	2
$ au_lpha$	0.5

Result

Link: https://github.com/BoCyuanLin/Social_Force_Model_Exercise

(https://github.com/BoCyuanLin/Social Force Model Exercise)

Reference

[1] Helbing, D. & Mulnár, P. Social force model for pedestrian dynamics. Phys. Rev. E 51, 4282–4286 (1995).

[2] https://github.com/svenkreiss/socialforce) (Python SFM using Numpy)