Note for Social Force Model for Pedestrain Dynamics

Formular of Social Force Model

1. Pedestrains with Pedestrains

(Note that α, β means the different pedestrains.)
Someone wants to reach a certain destination $\vec{r}^0_\alpha$ as comfortable as possible, therefore usually taking the shortest possible way having the shape of a polygon with edges $\vec{r}^0_\alpha := \vec{r}^1_\alpha, \ldots, \vec{r}^n_\alpha$. The desired direction:

$$\vec{e}_\alpha(t) = \frac{\vec{r}^k_\alpha - \vec{r}_\alpha(t)}{||\vec{r}^k_\alpha - \vec{r}_\alpha(t)||}$$

$\vec{r}_\alpha(t) :=$ actual position of pedestrain α at time t.
$\vec{r}^k_\alpha(t) :=$ steer for the nearest point at time t.

If the pedestrains is not deiturbed, he or she will walk to the desired direction with a desired speed v^0_α. Then, the actual velocity $\vec{v}^0_\alpha(t) = v^0_\alpha \vec{e}_\alpha$. The acceleration term:

$$\vec{F}^0_\alpha(\vec{v}_\alpha, v^0_\alpha \vec{e}_\alpha) = \frac{v^0_\alpha \vec{e}_\alpha - \vec{v}_\alpha}{\tau_\alpha}$$

If the motion of the pedestrain is influenced by other pedestrains, he or she will keeps a certain distance with the privat sphere. The vectorial quantities repulsive effects of other β (monotonic decrease):

$$\vec{f}_{\alpha\beta}(\vec{r}_{\alpha\beta}) = -\nabla_{\vec{r}_{\alpha\beta}} V_{\alpha\beta}[b(\vec{r}_{\alpha\beta})],\ where$$
$$V_{\alpha\beta}(b(\vec{r}_{\alpha\beta})) = V^0_{\alpha\beta} e^{-\frac{b}{\sigma}}$$

$b(\vec{r}_{\alpha\beta})$ denotes the semi-minor axis of the ellipse given by:

$$b(\vec{r}_{\alpha\beta}) = \frac{\sqrt{(||\vec{r}_{\alpha\beta}|| + ||\vec{r}_{\alpha\beta} - v_\beta \Delta t \vec{e}_\beta||)^2 - (v_\beta \Delta t)^2}}{2},\ where\ \vec{r}_{\alpha\beta} = \vec{r}_\alpha - \vec{r}_\beta$$

$v_\beta \Delta t :=$ order of the step width of pedestrain β.

2. Pedestrains with Space Objects

A pedestrian will keeps a certain distance from borders of buildings, walls, obstacles to avoid the danger of getting hurt. So, a border evokes a repulsive effect (monotonic decrease):

$$\vec{F}_{\alpha B}(\vec{r}_{\alpha B}) = -\nabla_{\vec{r}_{\alpha B}} U_{\alpha B}(||\vec{r}_{\alpha B}||),\ where\ \vec{r}_{\alpha B} = \vec{r}_\alpha - \vec{r}^{(min)}_{\alpha B}$$
$$U_{\alpha B}(||\vec{r}_{\alpha B}||) = U^0_{\alpha B} e^{-\frac{||\vec{r}_{\alpha B}||}{R}}$$

$B :=$ the border
$\vec{r}_{\alpha B}^{(min)} :=$ the point of B that is nearest to pedestrain α.

3. Pedestrains with Attractive Objects

Pedestrians are sometimes attracted by other persons or objects. The attractive effects (monotonic decrease):

$$\vec{f}_{\alpha i}(||\vec{r}_{\alpha i}||, t) = -\nabla_{\vec{r}_{\alpha i}} W_{\alpha i}(||\vec{r}_{\alpha i}||, t),\ where\ \vec{r}_{\alpha i} = \vec{r}_{\alpha} - \vec{r}_i$$

4. Field of Sight Processing

The formula for repulsive and attractive effects only hold for β, B, i in the sight of α. Therefore,

$$s(\vec{e}, \vec{f}) := \begin{cases} 1,\ if\ \vec{e} \cdot \vec{f} \geq ||\vec{f}||cos\phi \\ c,\ otherwise. \end{cases}$$

Now, the repulsive and attractive effects are given by

$$\vec{F}_{\alpha\beta}(\vec{e}_{\alpha}, \vec{r}_{\alpha\beta}) = s(\vec{e}_{\alpha}, -\vec{f}_{\alpha\beta})\vec{f}_{\alpha\beta}(\vec{r}_{\alpha\beta})$$
$$\vec{F}_{\alpha i}(\vec{e}_{\alpha}, \vec{r}_{\alpha i}, t) = s(\vec{e}_{\alpha}, \vec{f}_{\alpha i})\vec{f}_{\alpha i}(\vec{r}_{\alpha i}, t)$$

5. Social Force Model

$$\vec{F}_{\alpha}(t) = \vec{F}_{\alpha}^{0}(\vec{v}_{\alpha}, v_{\alpha}^{0}\vec{e}_{\alpha}) + \sum_{\beta} \vec{F}_{\alpha\beta}(\vec{e}_{\alpha}, \vec{r}_{\alpha\beta}) + \sum_{B} \vec{F}_{\alpha B}(\vec{r}_{\alpha B}) + \sum_{i} \vec{F}_{\alpha i}(\vec{e}_{\alpha}, \vec{r}_{\alpha i}, t).$$

Then, the social force model is given by:

$$\frac{d\vec{w}_{\alpha}}{dt} = \vec{F}_{\alpha}(t) + fluctuations(\text{random value}).$$

6. Maximum Acceptable Speed

Since the actual speed is limited by a pedestrian's maximal acceptable speed $v_{\alpha}^{(max)}$. Hence, the realized motion is given by:

$$\frac{d\vec{r}_{\alpha}}{dt} = \vec{v}_{\alpha}(t) = \vec{w}_{\alpha}(t)g(\frac{v_{\alpha}^{(max)}}{||\vec{w}_{\alpha}||}),\ where$$
$$g(\frac{v_{\alpha}^{(max)}}{||\vec{w}_{\alpha}||}) := \begin{cases} 1,\ if\ \frac{v_{\alpha}^{(max)}}{||\vec{w}_{\alpha}||} \geq 1 \\ \frac{v_{\alpha}^{(max)}}{||\vec{w}_{\alpha}||},\ otherwise. \end{cases} \Leftrightarrow g(\frac{v_{\alpha}^{(max)}}{||\vec{w}_{\alpha}||}) := min\{1, \frac{v_{\alpha}^{(max)}}{||\vec{w}_{\alpha}||}\}$$

Implementation of SFM

Enviornment

C++ (Programming Language)
Visual Studio 2019 (IDE)

Given Parameters

The followings are the given parameters in the paper:

Parameters	Values
$V_{\alpha\beta}^{0}$	2.1
σ	0.3
ϕ	100
c	0.5
$U_{\alpha B}^{0}$	10
R	0.2
$v_{\alpha}^{(max)}$	$1.3v_{\alpha}^{0}$
Δt	2
τ_{α}	0.5

Result

Link: https://github.com/BoCyuanLin/Social_Force_Model_Exercise (https://github.com/BoCyuanLin/Social_Force_Model_Exercise)

Reference

[1] Helbing, D. & Mulnár, P. Social force model for pedestrian dynamics. Phys. Rev. E 51, 4282–4286 (1995).
[2] https://github.com/svenkreiss/socialforce (https://github.com/svenkreiss/socialforce) (Python SFM using Numpy)