Lectures on Resurgence and Transseries

University of Alabama

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Abstract

These are lectures for the HEP/AdS-CFT/Hydro group at the University of Alabama.

Contents

Chapter 1

Introduction

References

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1.1 Divergence of Perturbation Theory in QED

Dyson paper [6]

1.2 What is Non-Perturbative?

1.3 Instantons in Quantum Mechanics

1.4 What are Transseries?

[8]

3 important steps

1.5 Asymptotics

Definition

Example: Stirling Formula
asdf

Example: Exponential function
asdf

Example: Exponential Integral
asdf

1.6 Optimal Truncation

blablup

Example: ϕ^4-Integral
asdf

Chapter 2

Mathematical Methods

2.1 Analytic Functions and Analytic Continuation

2.2 Saddle Point Method

2.3 Borel Summation

Chapter 3

ϕ^4-Integral

3.0.1 Perturbation Theory Similar to QFT

3.0.2 Method of Steepest Descent and Instantons

3.0.3 As Differential Equation