$$\sqrt{\pi}\ \frac{(4k)!}{2^{4k}(2k)!\,k!}\ ,\qquad k! \approx \sqrt{2\pi k}\left(\frac{k}{e}\right)^k$$

$$= \sqrt{\pi}\ \frac{1}{2^{4k}}\ \sqrt{\cancel{2\pi}}\sqrt{4k}\left(\frac{4k}{e}\right)^{4k}\frac{1}{\sqrt{\cancel{2\pi}}\sqrt{2k}}\left(\frac{e}{2k}\right)^{2k}\frac{1}{\sqrt{2\pi}\sqrt{k}}\left(\frac{e}{k}\right)^{k}$$

$$= \cancel{\sqrt{\pi}}\ \frac{1}{\cancel{\sqrt{2\pi}}}\ \frac{1}{2^{4k}}\ \underbrace{\frac{\sqrt{4k}}{\sqrt{2k}}}_{\cancel{\sqrt{2}}}\ \frac{1}{\sqrt{k}}\ e^{2k+k-4k}\ k^{4k-2k-k}\ \underset{\substack{\| \\ 2^{8k}}}{4^{4k}}\ 2^{-2k}$$

$$= \frac{1}{2^{4k}}\ \frac{1}{\sqrt{k}}\ e^{-k}\ k^k\ \underset{2^{2k}}{2^{6k}}$$

$$= 4^k\frac{1}{\sqrt{k}}\underbrace{\left(\frac{k}{e}\right)^k}_{\frac{1}{\sqrt{2\pi k}}k!} = \frac{1}{\sqrt{2\pi}}\ 4^k\frac{1}{k}\ k! = \frac{1}{\sqrt{2\pi}}\ 4^k (k-1)!$$

not sure why
I don't get k!