Contents

1 Basic 1
- 1.1 vimrc . . . 1
- 1.2 default code . . . 1
- 1.3 state machine . . . 1

2 Flow 2
- 2.1 Dinic . . . 2
- 2.2 GomoryHu tree 全點對最小割 DC 法 . . . 2
- 2.3 min cost flow . . . 3
- 2.4 SW mincut 全點對最小割 . . . 3

3 Matching 3
- 3.1 Hungarian . . . 3
- 3.2 KM . . . 4
- 3.3 Matching.txt . . . 4
- 3.4 Maximum General Matching . . . 4
- 3.5 Minimum General Weighted Matching . . . 5

4 Graph 5
- 4.1 BCC edge . . . 5
- 4.2 Dijkstra . . . 6
- 4.3 Domination.txt . . . 6
- 4.4 LCA . . . 6
- 4.5 max clique . . . 6
- 4.6 min mean cycle . . . 7
- 4.7 SSSP related concepts . . . 7
- 4.8 Tarjan.cpp . . . 7
- 4.9 2-SAT . . . 8
- 4.10平面圖判定 . . . 8

5 Math 10
- 5.1 ax+by=gcd(a,b) . . . 10
- 5.2 FFT . . . 10
- 5.3 NTT . . . 10
- 5.4 GaussElimination . . . 11
- 5.5 inverse . . . 11
- 5.6 Miller-Rabin . . . 11
- 5.7 Mobius . . . 11
- 5.8 pollardRho . . . 12
- 5.9 SG . . . 12
- 5.10theorem . . . 12

6 Geometry 12
- 6.1 2D point template . . . 12
- 6.2 circumcentre . . . 12
- 6.3 ConvexHull . . . 13
- 6.4 3D ConvexHull . . . 13
- 6.5 half plane intersection . . . 13
- 6.6 Intersection of two circle . . . 13
- 6.7 Intersection of two lines . . . 14
- 6.8 Smallest Circle . . . 14

7 String 14
- 7.1 AC automaton . . . 14
- 7.2 KMP . . . 14
- 7.3 palindromic tree . . . 14
- 7.4 SAM . . . 15
- 7.5 smallest rotation . . . 15
- 7.6 suffix array . . . 15
- 7.7 Z value . . . 16
- 7.8 BWT (Burrows-Wheeler Transform) . . . 16

8 Data structure 16
- 8.1 2D range tree . . . 16
- 8.2 ext heap . . . 16
- 8.3 KD tree . . . 16
- 8.4 Link-Cut tree . . . 17
- 8.5 Treap . . . 18

9 Other 18
- 9.1 count spanning tree . . . 18
- 9.2 C++11 random . . . 19
- 9.3 Digit Counting . . . 19
- 9.4 DP optimization . . . 19
- 9.5 DP 1D/1D . . . 20
- 9.6 Manhattan MST.cpp . . . 20
- 9.7 stable marriage . . . 21
- 9.8 Mo's algorithm . . . 21
- 9.9 Parser . . . 22
- 9.10python cheat sheet . . . 22

1 Basic

1.1 vimrc

```
=== .vimrc ===

syntax on
syntax enable
set et nu cin ls=2 ts=4 sw=4 sts=4 ttm=100
set number t_Co=256 mouse=a cursorline
colorscheme torte
hi CursorLine cterm=none ctermbg=DarkBlue ctermfg=none

nn <F4> :w ! cat -n \| lpr <CR>
nn <F7> :w <bar> :!vim %<_.in<left><left><left>
nn <F8> :w <bar> :!g++ % -o %< -std=c++11
\ -fsanitize=undefined -Wall -Wextra -Wshadow -DBANANA
    &&
\ for i in %<_*.in; do echo == && ./%< < $i; done <CR>
nn <F9> :w <bar> :!g++ % -o %< -std=c++11
\ -fsanitize=undefined -Wall -Wextra -Wshadow -DBANANA
    &&
\ echo == && ./%<
```

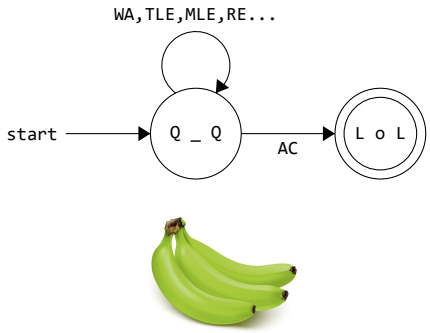
1.2 default code

```cpp
#pragma GCC optimize("Ofast")
#include <bits/stdc++.h>
#include <sys/time.h>
#include <sys/resource.h>
using namespace std;

void setstack(){
    // Set soft limit and hard limit to max
    const rlimit tmp {RLIM_INFINITY,RLIM_INFINITY};
    setrlimit(RLIMIT_STACK,&tmp);
}

int main(){
    #define name ""
    #ifndef BANANA
    // fopen to read/write
    freopen(name".in","r",stdin);
    freopen(name".out","w",stdout);
    // ifstream to read/write
    ifstream cin(name".in");
    ofstream cout(name".out");
    #endif
    static_assert(strlen(name));
    ios::sync_with_stdio(0);
    cin.tie(0), cout.tie(0);
}
```

1.3 state machine



2 Flow

2.1 Dinic

```
(a) Bounded Maxflow Construction:
1. add two node ss, tt
2. add_edge(ss, tt, INF)
3. for each edge u -> v with capacity [l, r]:
        add_edge(u, tt, l)
        add_edge(ss, v, l)
        add_edge(u, v, r-l)
4. see (b), check if it is possible.
5. answer is maxflow(ss, tt) + maxflow(s, t)
-----------------------------------------------------
(b) Bounded Possible Flow:
1. same construction method as (a)
2. run maxflow(ss, tt)
3. for every edge connected with ss or tt:
        rule: check if their rest flow is exactly 0
4. answer is possible if every edge do satisfy the rule
    ;
5. otherwise, it is NOT possible.
-----------------------------------------------------
(c) Bounded Minimum Flow:
1. same construction method as (a)
2. answer is maxflow(ss, tt)
-----------------------------------------------------
(d) Bounded Minimum Cost Flow:
* the concept is somewhat like bounded possible flow.
1. same construction method as (a)
2. answer is maxflow(ss, tt) + (∑ l * cost for every
    edge)
-----------------------------------------------------
(e) Minimum Cut:
1. run maxflow(s, t)
2. run cut(s)
3. ss[i] = 1: node i is at the same side with s.
-----------------------------------------------------
```

```cpp
const long long INF = 1LL<<60;
struct Dinic {  //O(VVE), with minimum cut
    static const int MAXN = 5003;
    struct Edge{
        int u, v;
        long long cap, rest;
    };

    int n, m, s, t, d[MAXN], cur[MAXN];
    vector<Edge> edges;
    vector<int> G[MAXN];

    void init(){
        edges.clear();
        for ( int i = 0 ; i < n ; i++ ) G[i].clear();
        n = 0;
    }

    // min cut start
    bool side[MAXN];
    void cut(int u) {
        side[u] = 1;
        for ( int i : G[u] ) {
            if ( !side[ edges[i].v ] && edges[i].rest )
                cut(edges[i].v);
        }
    }
    // min cut end

    int add_node(){
        return n++;
    }

    void add_edge(int u, int v, long long cap){
        edges.push_back( {u, v, cap, cap} );
        edges.push_back( {v, u, 0, 0LL} );
        m = edges.size();
        G[u].push_back(m-2);
        G[v].push_back(m-1);
    }

    bool bfs(){
        fill(d,d+n,-1);
        queue<int> que;
        que.push(s); d[s]=0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (int ei : G[u]){
                Edge &e = edges[ei];
                if (d[e.v] < 0 && e.rest > 0){
                    d[e.v] = d[u] + 1;
                    que.push(e.v);
                }
            }
        }
        return d[t] >= 0;
    }

    long long dfs(int u, long long a){
        if ( u == t || a == 0 ) return a;
        long long flow = 0, f;
        for ( int &i=cur[u]; i < (int)G[u].size() ; i++
            ) {
            Edge &e = edges[ G[u][i] ];
            if ( d[u] + 1 != d[e.v] ) continue;
            f = dfs(e.v, min(a, e.rest) );
            if ( f > 0 ) {
                e.rest -= f;
                edges[ G[u][i]^1 ].rest += f;
                flow += f;
                a -= f;
                if ( a == 0 )break;
            }
        }
        return flow;
    }

    long long maxflow(int _s, int _t){
        s = _s, t = _t;
        long long flow = 0, mf;
        while ( bfs() ){
            fill(cur,cur+n,0);
            while ( (mf = dfs(s, INF)) ) flow += mf;
        }
        return flow;
    }
} dinic;
```

2.2 GomoryHu tree 全點對最小割 DC 法

```
Construct of Gomory Hu Tree

1. make sure the whole graph is clear
2. set node 0 as root, also be the parent of other
    nodes.
3. for every node i > 0, we run maxflow from i to
    parent[i]
4. hense we know the weight between i and parent[i]
5. for each node j > i, if j is at the same side with i
    ,
   make the parent of j as i
-----------------------------------------------------
```

```cpp
int e[MAXN][MAXN], p[MAXN];
Dinic D; // original graph

void gomory_hu() {
    fill(p, p+n, 0);
    fill(e[0], e[n], INF);
    for ( int s = 1 ; s < n ; s++ ) {
        int t = p[s];
        Dinic F = D;
        int tmp = F.max_flow(s, t);

        for ( int i = 1 ; i < s ; i++ )
            e[s][i] = e[i][s] = min(tmp, e[t][i]);
        for ( int i = s+1 ; i <= n ; i++ )
            if ( p[i] == t && F.side[i] ) p[i] = s;
    }
}
```

2.3 min cost flow

```
typedef long long lol;
#define MAX 100010

struct Edge {
    int x, y;
    lol c;
    bool operator < (const Edge& rhs) const {
        return c < rhs.c;
    }
};

vector<Edge> v, g;
int p[MAX], s[MAX], n, m;
lol b[MAX], ans_min, ans_smi;

void init() {
    memset(b, 0, sizeof(b));
    for (int i = 1; i < MAX; i++) {
        p[i] = i;
        s[i] = 1;
    }
    ans_min = 0;
    ans_smi = 10000000000000000;
    v.clear();
    g.clear();
}

int findset(int x) {
    return p[x] == x ? x : findset(p[x]);
}

void unio(int x, int y, lol c) {
    if (s[x] > s[y]) {
        p[y] = x;
        s[x] += s[y];
        b[y] = c;
    } else {
        p[x] = y;
        s[y] += s[x];
        b[x] = c;
    }
}

lol find(int x, int y) {
    lol now = 0;
    while (x != y) {
        if (s[x] < s[y]) {
            now = max(now, b[x]);
            x = p[x];
        } else {
            now = max(now, b[y]);
            y = p[y];
        }
    }
    return now;
}

void solve() {
    for (int i = 0; i < m; i++) {
        Edge e = v[i];
        int x = findset(e.x), y = findset(e.y);
        if (x == y) g.push_back(e);
        else {
            ans_min += e.c;
            unio(x, y, e.c);
        }
    }
    for (int i = 0; i < g.size(); i++) {
        Edge e = g[i];
        int x = e.x, y = e.y;
        lol now = find(x, y);
        ans_smi = min(ans_smi, e.c - now);
    }
}

int main() {
    cin.tie(0), cout.sync_with_stdio(false);
    init();
    cin >> n >> m;
    for (int i = 1; i <= m; i++) {
        int x, y; lol c;
        cin >> x >> y >> c;
        v.push_back((Edge){x, y, c});
    }
    sort(v.begin(), v.end());
    solve();
    cout << ans_min << ' ' << ans_smi + ans_min << '\n'
        ;
    return 0;
}
```

2.4 SW mincut 全點對最小割

```
// all pair min cut
// global min cut
struct SW{ // O(V^3)
  static const int MXN = 514;
  int n,vst[MXN],del[MXN];
  int edge[MXN][MXN],wei[MXN];
  void init(int _n){
    n = _n; FZ(edge); FZ(del);
  }
  void addEdge(int u, int v, int w){
    edge[u][v] += w; edge[v][u] += w;
  }
  void search(int &s, int &t){
    FZ(vst); FZ(wei);
    s = t = -1;
    while (true){
      int mx=-1, cur=0;
      for (int i=0; i<n; i++)
        if (!del[i] && !vst[i] && mx<wei[i])
          cur = i, mx = wei[i];
      if (mx == -1) break;
      vst[cur] = 1;
      s = t; t = cur;
      for (int i=0; i<n; i++)
        if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
    }
  }
  int solve(){
    int res = 2147483647;
    for (int i=0,x,y; i<n-1; i++){
      search(x,y);
      res = min(res,wei[y]);
      del[y] = 1;
      for (int j=0; j<n; j++)
        edge[x][j] = (edge[j][x] += edge[y][j]);
    }
    return res;
  }
}graph;
```

3 Matching

3.1 Hungarian

```
// Maximum Cardinality Bipartite Matching
// Worst case O(nm)

struct Graph{
    static const int MAXN = 5003;
    vector<int> G[MAXN];
    int n, match[MAXN], vis[MAXN];

    void init(int _n){
        n = _n;
        for (int i=0; i<n; i++) G[i].clear();
    }

    bool dfs(int u){
        for (int v:G[u]){
            if (vis[v]) continue;
            vis[v]=true;
            if (match[v]==-1 || dfs(match[v])){
                match[v] = u;
                match[u] = v;
```

```
                return true;
            }
        }
        return false;
    }

    int solve(){
        int res = 0;
        memset(match,-1,sizeof(match));
        for (int i=0; i<n; i++){
            if (match[i]==-1){
                memset(vis,0,sizeof(vis));
                if ( dfs(i) ) res++;
            }
        }
        return res;
    }
} graph;
```

3.2 KM

```
const int MAXN = 400 + 10;
const long long INF64 = 0x3f3f3f3f3f3f3f3fll;
int nl, nr;
int pre[MAXN];
long long slack[MAXN];
long long W[MAXN][MAXN];
long long lx[MAXN], ly[MAXN];
int mx[MAXN], my[MAXN];
bool vx[MAXN], vy[MAXN];
void augment(int u) {
    if(!u) return;
    augment(mx[pre[u]]);
    mx[pre[u]] = u;
    my[u] = pre[u];
}
void match(int x) {
    queue<int> que;
    que.push(x);
    while(1) {
        while(!que.empty()) {
            x = que.front(); que.pop();
            vx[x] = 1;
            for (int i=1; i<=nr; i++) {
                if(vy[i]) continue;
                long long t = lx[x] + ly[i] - W[x][i];
                if(t > 0) {
                    if(slack[i] >= t) slack[i] = t, pre
                        [i] = x;
                    continue;
                }
                pre[i] = x;
                if(!my[i]) {
                    augment(i);
                    return;
                }
                vy[i] = 1;
                que.push(my[i]);
            }
        }
        long long t = INF64;
        for (int i=1; i<=nr; i++) if(!vy[i]) t = min(t,
             slack[i]);
        for (int i=1; i<=nl; i++) if(vx[i]) lx[i] -= t;
        for (int i=1; i<=nr; i++) {
            if(vy[i]) ly[i] += t;
            else slack[i] -= t;
        }
        for (int i=1; i<=nr; i++) {
            if(vy[i] || slack[i]) continue;
            if(!my[i]) {
                augment(i);
                return;
            }
            vy[i] = 1;
            que.push(my[i]);
        }
    }
}
int main() {
    int m;
    cin >> nl >> nr >> m;
    nr = max(nl, nr);
    while(m--) {
        int u, v;
        long long w;
        cin >> u >> v >> w;
        W[u][v] = w;
        lx[u] = max(lx[u], w);
    }
    for (int i=1; i<=nl; i++) {
        for (int x=1; x<=nl; x++) vx[x] = 0;
        for (int y=1; y<=nr; y++) vy[y] = 0, slack[y] =
             INF64;
        match(i);
    }
    long long ans = 0;
    for (int i=1; i<=nl; i++) ans += W[i][mx[i]];
    cout << ans << '\n';
    for (int i=1; i<=nl; i++) {
        if (i > 1) cout << ' ';
        cout << (W[i][mx[i]] ? mx[i] : 0);
    }
    cout << '\n';
}
```

3.3 Matching.txt

```
最大匹配 + 最小邊覆蓋 = V
最大獨立集 + 最小點覆蓋 = V
最大匹配 = 最小點覆蓋
最小路徑覆蓋數 = V - 最大匹配數
```

3.4 Maximum General Matching

```
// Maximum Cardinality Matching

struct Graph {
  vector<int> G[MAXN];
  int pa[MAXN], match[MAXN], st[MAXN], S[MAXN], vis[
      MAXN];
  int t, n;

  void init(int _n) {
    n = _n;
    for ( int i = 1 ; i <= n ; i++ ) G[i].clear();
  }
  void add_edge(int u, int v) {
    G[u].push_back(v);
    G[v].push_back(u);
  }
  int lca(int u, int v){
    for ( ++t ; ; swap(u, v) ) {
      if ( u == 0 ) continue;
      if ( vis[u] == t ) return u;
      vis[u] = t;
      u = st[ pa[ match[u] ] ];
    }
  }
  void flower(int u, int v, int l, queue<int> &q) {
    while ( st[u] != l ) {
      pa[u] = v;
      if ( S[ v = match[u] ] == 1 ) {
        q.push(v);
        S[v] = 0;
      }
      st[u] = st[v] = l;
      u = pa[v];
    }
  }
  bool bfs(int u){
    for ( int i = 1 ; i <= n ; i++ ) st[i] = i;
    memset(S, -1, sizeof(S));
    queue<int>q;
    q.push(u);
    S[u] = 0;
    while ( !q.empty() ) {
```

```
      u = q.front(); q.pop();
      for ( int i = 0 ; i < (int)G[u].size(); i++) {
        int v = G[u][i];
        if ( S[v] == -1 ) {
          pa[v] = u;
          S[v] = 1;
          if ( !match[v] ) {
            for ( int lst ; u ; v = lst, u = pa[v] ) {
              lst = match[u];
              match[u] = v;
              match[v] = u;
            }
            return 1;
          }
          q.push(match[v]);
          S[ match[v] ] = 0;
        } else if ( !S[v] && st[v] != st[u] ) {
          int l = lca(st[v], st[u]);
          flower(v, u, l, q);
          flower(u, v, l, q);
        }
      }
    }
    return 0;
  }
  int solve(){
    memset(pa, 0, sizeof(pa));
    memset(match, 0, sizeof(match));
    int ans = 0;
    for ( int i = 1 ; i <= n ; i++ )
      if ( !match[i] && bfs(i) ) ans++;
    return ans;
  }
} graph;
```

3.5 Minimum General Weighted Matching

```
// Minimum Weight Perfect Matching (Perfect Match)

struct Graph {
    static const int MAXN = 105;
    int n, e[MAXN][MAXN];
    int match[MAXN], d[MAXN], onstk[MAXN];
    vector<int> stk;
    void init(int _n) {
        n = _n;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                e[i][j] = 0;
    }
    void add_edge(int u, int v, int w) {
        e[u][v] = e[v][u] = w;
    }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.push_back(u);
        onstk[u] = 1;
        for ( int v = 0 ; v < n ; v++ ) {
            if (u != v && match[u] != v && !onstk[v] )
                {
                int m = match[v];
                if ( d[m] > d[u] - e[v][m] + e[u][v] )
                    {
                    d[m] = d[u] - e[v][m] + e[u][v];
                    onstk[v] = 1;
                    stk.push_back(v);
                    if (SPFA(m)) return true;
                    stk.pop_back();
                    onstk[v] = 0;
                }
            }
        }
        onstk[u] = 0;
        stk.pop_back();
        return false;
    }
    int solve() {
        for ( int i = 0 ; i < n ; i += 2 ) {
            match[i] = i+1;
            match[i+1] = i;
        }
        while (true){
            int found = 0;
            for ( int i = 0 ; i < n ; i++ )
                onstk[ i ] = d[ i ] = 0;
            for ( int i = 0 ; i < n ; i++ ) {
                stk.clear();
                if ( !onstk[i] && SPFA(i) ) {
                    found = 1;
                    while ( stk.size() >= 2 ) {
                        int u = stk.back(); stk.
                            pop_back();
                        int v = stk.back(); stk.
                            pop_back();
                        match[u] = v;
                        match[v] = u;
                    }
                }
            }
            if (!found) break;
        }
        int ret = 0;
        for ( int i = 0 ; i < n ; i++ )
            ret += e[i][match[i]];
        ret /= 2;
        return ret;
    }
} graph;
```

4 Graph

- Maximum Independent Set
  - General: [NPC] maximum clique of complement of G
  - Bipartite Graph: [P] Maximum Cardinality Bipartite Matching
  - Tree: [P] dp
- Minimum Dominating Set
  - General: [NPC]
  - Bipartite Graph: [NPC]
  - Tree: [P] DP
- Minimum Vertex Cover
  - General: [NPC] (?)maximum clique of complement of G
  - Bipartite Graph: [P] Maximum Cardinality Bipartite Matching
  - Tree: [P] Greedy, from leaf to root
- Minimum Edge Cover
  - General: [P] V - Maximum Matching
  - Bipartite Graph: [P] Greedy, strategy: cover small degree node first.
  - (Min/Max)Weighted: [P]: Minimum/Minimum Weight Matching

4.1 BCC edge

邊雙連通

任意兩點間至少有兩條不重疊的路徑連接，找法：
1. 標記出所有的橋
2. 對全圖進行 DFS，不走橋，每一次 DFS 就是一個新的邊雙連通

```
// from BCW

struct BccEdge {
  static const int MXN = 100005;
  struct Edge { int v,eid; };
  int n,m,step,par[MXN],dfn[MXN],low[MXN];
  vector<Edge> E[MXN];
  DisjointSet djs;
  void init(int _n) {
    n = _n; m = 0;
    for (int i=0; i<n; i++) E[i].clear();
    djs.init(n);
  }
  void add_edge(int u, int v) {
    E[u].PB({v, m});
```

```
    E[v].PB({u, m});
    m++;
  }
  void DFS(int u, int f, int f_eid) {
    par[u] = f;
    dfn[u] = low[u] = step++;
    for (auto it:E[u]) {
      if (it.eid == f_eid) continue;
      int v = it.v;
      if (dfn[v] == -1) {
        DFS(v, u, it.eid);
        low[u] = min(low[u], low[v]);
      } else {
        low[u] = min(low[u], dfn[v]);
      }
    }
  }
  void solve() {
    step = 0;
    memset(dfn, -1, sizeof(int)*n);
    for (int i=0; i<n; i++) {
      if (dfn[i] == -1) DFS(i, i, -1);
    }
    djs.init(n);
    for (int i=0; i<n; i++) {
      if (low[i] < dfn[i]) djs.uni(i, par[i]);
    }
  }
}graph;
```

4.2 Dijkstra

```
struct Edge{
    int v; long long len;
    bool operator < (const Edge &b)const { return len>b
        .len; }
};

const long long INF = 1LL<<60;

void Dijkstra(int n, vector<Edge> G[], long long d[],
    int s, int t=-1){
    static priority_queue<Edge> pq;
    while ( pq.size() )pq.pop();
    for (int i=1; i<=n; i++)d[i]=INF;
    d[s]=0; pq.push( {s,d[s]} );
    while ( pq.size() ){
        auto x = pq.top(); pq.pop();
        int u = x.v;
        if (d[u]<x.len)continue;
        if (u==t)return;
        for (auto &e:G[u]){
            if (d[e.v] > d[u]+e.len){
                d[e.v] = d[u]+e.len;
                pq.push( {e.v,d[e.v]} );
            }
        }
    }
}
```

4.3 Domination.txt

```
Maximum Independent Set
General: [NPC] maximum clique of complement of G
Tree: [P] Greedy
Bipartite Graph: [P] Maximum Cardinality Bipartite
    Matching
-----------------------------------------------------
Minimum Dominating Set
General: [NPC]
Tree: [P] DP
Bipartite Graph: [NPC]
-----------------------------------------------------
Minimum Vertex Cover
General: [NPC] (?)maximum clique of complement of G
Tree: [P] Greedy, from leaf to root
Bipartite Graph: [P] Maximum Cardinality Bipartite
    Matching
-----------------------------------------------------
Minimum Edge Cover
General: [P] V - Maximum Matching
Bipartite Graph: [P] Greedy, strategy: cover small
    degree node first.
(Min/Max)Weighted: [P]: Minimum/Minimum Weight Matching
```

4.4 LCA

```
#define MAX 200000
vector<int> v[MAX];
int MAX_LOG, dep[MAX], par[(int)log2(MAX)+1][MAX], n,q;

void init() {
    for (int i = 1; i <= n; i++) v[i].clear();
    memset(dep, 0, sizeof(dep));
    memset(par, 0, sizeof(par));
    MAX_LOG = log2(n) + 1;
}

void dfs(int now, int fa, int d) {
    par[0][now] = fa, dep[now] = d;
    for (int i = 0; i < v[now].size(); i++)
        if (v[now][i] != fa) dfs(v[now][i], now, d + 1)
            ;
}

void build() {
    dfs(1, -1, 0);

    for (int i = 0; i < MAX_LOG; i++) {
        for (int j = 1; j <= n; j++) {
            if (par[i][j] <= 0) par[i + 1][j] = -1;
            else par[i + 1][j] = par[i][par[i][j]];
        }
    }
}

int lca(int x, int y) {
    if (dep[x] > dep[y]) swap(x, y);
    for (int i = 0; i < MAX_LOG; i++)
        if ((dep[x] - dep[y]) >> i & 1) y = par[i][y];

    if (x == y) return x;

    for (int i = MAX_LOG - 1; i >= 0; i--)
        if (par[i][x] != par[i][y]) x = par[i][x], y =
            par[i][y];

    return par[0][x];
}
```

4.5 max clique

```
const int MAXN = 105;
int best;
int n;
int num[MAXN];
int path[MAXN];
int G[MAXN][MAXN];

bool dfs( int *adj, int total, int cnt ){
    int t[MAXN];
    if (total == 0){
        if( best < cnt ){
            best = cnt;
            return true;
        }
        return false;
    }
    for(int i = 0; i < total; i++){
        if( cnt+(total-i) <= best ) return false;
        if( cnt+num[adj[i]] <= best ) return false;
        int k=0;
        for(int j=i+1; j<total; j++)
            if(G[ adj[i] ][ adj[j] ])
                t[k++] = adj[j];
        if (dfs(t, k, cnt+1)) return true;
```

```
    }
    return false;
}
int MaximumClique(){
    int adj[MAXN];
    if (n <= 0) return 0;
    best = 0;
    for(int i = n-1; i >= 0; i--){
        int k=0;
        for(int j = i+1; j < n; j++)
            if (g[i][j]) adj[k++] = j;
        dfs( adj, k, 1 );
        num[i] = best;
    }
    return best;
}
```

4.6 min mean cycle

```
// from BCW

/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
  int v,u;
  double c;
};
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
  for(int i=0; i<n; i++) d[0][i]=0;
  for(int i=0; i<n; i++) {
    fill(d[i+1], d[i+1]+n, inf);
    for(int j=0; j<m; j++) {
      int v = e[j].v, u = e[j].u;
      if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
        d[i+1][u] = d[i][v]+e[j].c;
        prv[i+1][u] = v;
        prve[i+1][u] = j;
      }
    }
  }
}
double karp_mmc() {
  // returns inf if no cycle, mmc otherwise
  double mmc=inf;
  int st = -1;
  bellman_ford();
  for(int i=0; i<n; i++) {
    double avg=-inf;
    for(int k=0; k<n; k++) {
      if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])
          /(n-k));
      else avg=max(avg,inf);
    }
    if (avg < mmc) tie(mmc, st) = tie(avg, i);
  }
  for(int i=0; i<n; i++) vst[i] = 0;
  edgeID.clear(); cycle.clear(); rho.clear();
  for (int i=n; !vst[st]; st=prv[i--][st]) {
    vst[st]++;
    edgeID.PB(prve[i][st]);
    rho.PB(st);
  }
  while (vst[st] != 2) {
    int v = rho.back(); rho.pop_back();
    cycle.PB(v);
    vst[v]++;
  }
  reverse(ALL(edgeID));
  edgeID.resize(SZ(cycle));
  return mmc;
}
```

4.7 SSSP related concepts

最短路問題分類：

三個工具 Bellman-Ford, Floyd, Dijkstra，

1. 可以把 Dijkstra Priority Queue 裡面存的東西想成「狀態」，他可以拿來統計甚至轉移。
2. 當遇到邊權會扣掉走的人的血量（或油量之類的），當不能有負值的時候，就要使用 Bellman-Ford 來做，

一開始可以把起點設為最初的血量（油量），拿去做 Bellman-Ford，當做了 n-1 次之後，還能轉移，那就是有負環或正環（端看如何轉移 Bellman-Ford，這部分的轉移式很自由可以依照題目敘述亂改。）

3. 特別注意如果要判到某一個點的長度是不是無限小，可在做了 n-1 次之後，發現 u->v 可以更新，那我可以去看 v 是否可以到另一點 k，如果是聯通的，代表 k 這個點的長度是無限小。

4.8 Tarjan.cpp

割點
點 u 為割點 if and only if 滿足 1. or 2.
1. u 爲樹根，且 u 有多於一個子樹。
2. u 不爲樹根，且滿足存在 (u,v) 爲樹枝邊（或稱父子邊，即 u 爲 v 在搜索樹中的父親），使得 DFN(u) <= Low(v)。

橋
一條無向邊 (u,v) 是橋 if and only if (u,v) 爲樹枝邊，且滿足 DFN(u) < Low(v)。

```
// 0 base
struct TarjanSCC{
  static const int MAXN = 1000006;
  int n, dfn[MAXN], low[MAXN], scc[MAXN], scn, count;
  vector<int> G[MAXN];
  stack<int> stk;
  bool ins[MAXN];

  void tarjan(int u){
    dfn[u] = low[u] = ++count;
    stk.push(u);
    ins[u] = true;

    for(auto v:G[u]){
      if(!dfn[v]){
        tarjan(v);
        low[u] = min(low[u], low[v]);
      }else if(ins[v]){
        low[u] = min(low[u], dfn[v]);
      }
    }

    if(dfn[u] == low[u]){
      int v;
      do {
      v = stk.top();
      stk.pop();
      scc[v] = scn;
      ins[v] = false;
      } while(v != u);
      scn++;
    }
  }

  void getSCC(){
    memset(dfn,0,sizeof(dfn));
    memset(low,0,sizeof(low));
    memset(ins,0,sizeof(ins));
    memset(scc,0,sizeof(scc));
    count = scn = 0;
    for(int i = 0 ; i < n ; i++ ){
      if(!dfn[i]) tarjan(i);
    }
  }
```

```
}SCC;
```

4.9 2-SAT

```
const int MAXN = 2020;

struct TwoSAT{
    static const int MAXv = 2*MAXN;
    vector<int> GO[MAXv],BK[MAXv],stk;
    bool vis[MAXv];
    int SC[MAXv];

    void imply(int u,int v){ // u imply v
        GO[u].push_back(v);
        BK[v].push_back(u);
    }
    int dfs(int u,vector<int>*G,int sc){
        vis[u]=1, SC[u]=sc;
        for (int v:G[u])if (!vis[v])
            dfs(v,G,sc);
        if (G==GO)stk.push_back(u);
    }
    int scc(int n=MAXv){
        memset(vis,0,sizeof(vis));
        for (int i=0; i<n; i++)if (!vis[i])
            dfs(i,GO,-1);
        memset(vis,0,sizeof(vis));
        int sc=0;
        while (!stk.empty()){
            if (!vis[stk.back()])
                dfs(stk.back(),BK,sc++);
            stk.pop_back();
        }
    }
}SAT;

int main(){
    SAT.scc(2*n);
    bool ok=1;
    for (int i=0; i<n; i++){
        if (SAT.SC[2*i]==SAT.SC[2*i+1])ok=0;
    }
    if (ok){
        for (int i=0; i<n; i++){
            if (SAT.SC[2*i]>SAT.SC[2*i+1]){
                cout << i << endl;
            }
        }
    }
    else puts("NO");
}

void warshall(){
    bitset<2003> d[2003];
    for (int k=0; k<n; k++){
        for (int i=0; i<n; i++) if (d[i][k]) {
            d[i] |= d[k];
        }
    }
}
```

4.10 平面圖判定

```
//skydog
#include <bits/stdc++.h>
using namespace std;

typedef long long ll;
typedef pair<int, int> ii;
typedef pair<ll, ll> l4;

#define mp make_pair
#define pb push_back
#define debug(x) cerr << #x << " = " << x << " "

const int N=400+1;

struct Planar{
    int n,m,hash[N],fa[N],deep[N],low[N],ecp[N];
    vector<int> g[N],son[N];
    set< pair<int,int> > SDlist[N],proots[N];
    int nxt[N][2],back[N],rev[N];
    deque<int> q;
    void dfs(int u){
        hash[u]=1; q.pb(u);
        ecp[u]=low[u]=deep[u];
        int v;
        for (int i = 0; i < g[u].size(); ++i)
            if(!hash[v=g[u][i]]){
                fa[v]=u;
                deep[v]=deep[u]+1;
                dfs(v);
                low[u]=min(low[u],low[v]);
                SDlist[u].insert(mp(low[v],v));
            }
            else ecp[u]=min(ecp[u],deep[v]);
        low[u]=min(low[u],ecp[u]);
    }

    int visited[N];

    void addtree(int u,int t1,int v,int t2){
        nxt[u][t1]=v; nxt[v][t2]=u;
    }

    void findnxt(int u,int v,int& u1,int& v1){
        u1=nxt[u][v^1];
        if(nxt[u1][0]==u) v1=0;
        else v1=1;
    }

    void walkup(int u,int v){
        back[v]=u;
        int v1=v,v2=v,u1=1,u2=0,z;
        for (;;){
            if(hash[v1]==u || hash[v2]==u) break;
            hash[v1]=u;hash[v2]=u; z=max(v1,v2);
            if(z>n){
                int p=fa[z-n];
                if(p!=u){
                    proots[p].insert(mp(-low[z-n], z));
                    v1=p,v2=p,u1=0,u2=1;
                }
                else break;
            }
            else{
                findnxt(v1,u1,v1,u1);
                findnxt(v2,u2,v2,u2);
            }
        }
    }

    int topstack;
    pair<int,int> stack[N];

    int outer(int u,int v){
        return ecp[v]<deep[u] || (SDlist[v].size() &&
            SDlist[v].begin()->first<deep[u]);
    }

    int inside(int u,int v){
        return proots[v].size()>0 || back[v]==u;
    }

    int active(int u,int v){
        return inside(u,v) || outer(u,v);
    }

    void push(int a,int b){
        stack[++topstack]=mp(a,b);
    }

    void mergestack(){
        int v1,t1,v2,t2,s,s1;
        v1=stack[topstack].first;t1=stack[topstack].
            second;
        topstack--;
        v2=stack[topstack].first;t2=stack[topstack].
            second;
```

```
        topstack--;

        s=nxt[v1][t1^1];
        s1=(nxt[s][1]==v1);
        nxt[s][s1]=v2;
        nxt[v2][t2]=s;

        SDlist[v2].erase( make_pair(low[v1-n],v1-n) );
        proots[v2].erase( make_pair(-low[v1-n],v1) );
    }

    void findnxtActive(int u,int t,int& v,int& w1,int S
        ){
        findnxt(u,t,v,w1);
        while(u!=v && !active(S,v))
            findnxt(v,w1,v,w1);
    }

    void walkdown(int S,int u){
        topstack=0;
        int t1,v=S,w1,x2,y2,x1,y1,p;
        for(t1=0;t1<2;++t1){
            findnxt(S,t1^1,v,w1);
            while(v!=S){
                if(back[v]==u){
                    while(topstack>0) mergestack();
                    addtree(S,t1,v,w1); back[v]=0;
                }
                if(proots[v].size()){
                    push(v,w1);
                    p=proots[v].begin()->second;
                    findnxtActive(p,1,x1,y1,u);
                    findnxtActive(p,0,x2,y2,u);
                    if(active(u,x1) && !outer(u,x1))
                        v=x1,w1=y1;
                    else if(active(u,x2) && !outer(u,x2
                        ))
                        v=x2,w1=y2;
                    else if(inside(u,x1) || back[x1]==u
                        )
                        v=x1,w1=y1;
                    else v=x2,w1=y2;
                    push(p,v==x2);
                }
                else if(v>n || ( ecp[v]>=deep[u] && !
                    outer(u,v) ))
                    findnxt(v,w1,v,w1);
                else if(v<=n && outer(u,v) && !topstack
                    ){
                    addtree(S,t1,v,w1); break;
                }
                else break;
            }
        }
    }

    int work(int u){
        int v;
        for (int i = 0; i < g[u].size(); ++i)
            if(fa[v=g[u][i]]==u){
                son[u].push_back(n+v);
                proots[n+v].clear();
                addtree(n+v,1,v,0);
                addtree(n+v,0,v,1);
            }
        for (int i = 0; i < g[u].size(); ++i)
            if(deep[v=g[u][i]]>deep[u]+1)
                walkup(u,v);
        topstack=0;
        for (int i = 0; i < son[u].size(); ++i)
            walkdown(son[u][i], u);
        for (int i = 0; i < g[u].size(); ++i)
            if(deep[v=g[u][i]]>deep[u]+1 && back[v])
                return 0;
        return 1;
    }

    void init(int _n){
        n = _n;
        m = 0;
        for(int i=1;i<=2*n;++i){
            g[i].clear();
            SDlist[i].clear();
            son[i].clear();
            proots[i].clear();
            nxt[i][0]=nxt[i][1]=0;
            fa[i]=0;
            hash[i]=0;low[i]=ecp[i]=deep[i]=back[i]=0;
            q.clear();
        }
    }
    void add(int u, int v){
        ++m;
        g[u].pb(v); g[v].pb(u);
    }
    bool check_planar(){
        if(m>3*n-5) return false;
        for(int i=1;i<=n;++i)
            if(!hash[i]){
                deep[i]=1;
                dfs(i);
            }
        memset(hash,0,sizeof(hash));
        // originally only looks at last n element
        assert(q.size() == n);
        while (!q.empty()){
            if (!work(q.back()))
                return false;
            q.pop_back();
        }
        return true;
    }
} base, _new;
vector<ii> edges;
int n, m;
inline void build(int n, Planar &_new){
    _new.init(n);
    for (auto e : edges)
        _new.add(e.first, e.second);
}
void end(){
    puts("-1");
    exit(0);
}
bool vis[N];
const int maxp = 5;
int path[maxp], tp=0;
void dfs(int cur){
    vis[cur] = true;
    path[tp++] = cur;
    if (tp == maxp){
    auto it = lower_bound(base.g[cur].begin(), base.g[
        cur].end(), path[0]);
        if ( it != base.g[cur].end() && *it == path[0])
            {
            //a cycle
            int x = n+1;
            for (int i = 0; i < 5; ++i) edges.pb(mp(x,
                path[i]));
            build(x, _new);
            if (_new.check_planar()){
                for (int i = 0; i < maxp; ++i) printf("
                    %d%c", path[i], i==maxp-1?'\n':' ')
                    ;
                exit(0);
            }
            for (int i = 0; i < 5; ++i) edges.pop_back
                ();
        }
    }
    else{
        for (auto e : base.g[cur]) if (!vis[e]) dfs(e);
    }
    vis[cur] = false;
    --tp;
}
int main(){
    scanf("%d %d", &n, &m);
    if (n <= 4) {
    assert(false);
    puts("0");
    return 0;
    }
    for (int i = 0; i < m; ++i){
```

```
        int u, v; scanf("%d %d", &u, &v);
        edges.pb(mp(u, v));
    }
    build(n, base);
    if (!base.check_planar()) end();
    for (int i = 1; i <= n; ++i)
        sort(base.g[i].begin(), base.g[i].end());
    for (int i = 1; i <= n; ++i)
        dfs(i);
    end();
}
```

5 Math

- Stirling number of second kind
  $S(n,m)$: n 個相異球，放到 m 個相同的相子，每個箱子至少 1
  $= m \times S(n-1,m) + S(n-1,m-1)$
  $= \frac{1}{m!}\sum_{j=0}^{m}\binom{m}{j}(m-j)^n(-1)^j$

- Stirling number of first kind
  $s(n,m)$: n 個相異球，分配到 m 個有向環，每個環至少 1
  $s(n+1,m) = n \times s(n,m) + s(n,m-1)$
  $s(n,m) \equiv \binom{\lfloor n/2 \rfloor}{m-\lfloor n/2 \rfloor} \mod 2$

- Pick's Theorem (Bangkok regional 2016 pD)
  多邊形頂點都在整數點上
  多邊形面積 = 內部整數點個數 + 邊上格子點個數/2 - 1
  $A = i + b/2 - 1$

5.1 ax+by=gcd(a,b)

```
pair<int,int> extgcd(int a, int b){
    if (b==0) return {1,0};
    int k = a/b;
    pair<int,int> p = extgcd(b,a-k*b);
    return { p.second, p.first - k*p.second };
}
/* ¨D¥Xax+by=cªº x, y¬°¤]¦³
  ¦³¦¸¤è: g is gcd(a,b)
  x = (x+b/g)*(c/g), x = (x%b/g+b/g)%b/g;
  y = (c-a*x)/b; */
```

5.2 FFT

```
#define MAXN 262144
#define cplx complex<long double>
const long double PI = acos(-1);
const cplx I(0, 1);

cplx w[MAXN];

void pre_fft() {
  for (int i = 0; i < MAXN; i++)
    w[i] = exp(PI * i * 2 / MAXN * I);
}

int reverse_add(int x) {
  for (int l = (1 << 17); (x ^= l) < l; l >>= 1);
  return x;
}

void bit_reverse(cplx a[], int n) {
  for (int i = 0, j = 0; i < n; i++) {
    if (i > j) swap(a[i], a[j]);
    j = reverse_add(j);
  }
}

void fft(cplx a[], int n) {
  bit_reverse(a, n);
  for (int i = 2; i <= n; i <<= 1) {
    int m = i >> 1;
    for (int j = 0; j < n; j += i) {
      for (int k = 0; k < m; k++) {
        cplx z = w[n / i * k] * a[j + m + k];
        a[j + m + k] = a[j + k] - z;
        a[j + k] += z;
      }
    }
  }
}

void ifft(cplx a[], int n) {
  fft(a, n);
  vector<cplx> v(n);
  for (int i = 0; i < n; i++) v[i] = a[(n - i) % n] / (
      long double) n;
  for (int i = 0; i < n; i++) a[i] = v[i];
}

cplx g[MAXN], h[MAXN], x[MAXN];
int ans[MAXN], r;

void init() {
  memset(ans, 0, sizeof(ans));
  memset(g, 0, sizeof(g));
  memset(h, 0, sizeof(h));
  memset(x, 0, sizeof(x));
  r = 0;
}

int main() {
  cin.tie(0), cout.sync_with_stdio(false);
  pre_fft();
  string s1, s2;
  while (cin >> s1 >> s2) {
    init();
    for (int i = 0; i < s1.length(); i++)
      g[s1.length() - i - 1] = cplx(s1[i] - '0', 0);
    for (int i = 0; i < s2.length(); i++)
      h[s2.length() - i - 1] = cplx(s2[i] - '0', 0);

    fft(g, MAXN);
    fft(h, MAXN);

    for (int i = 0; i < MAXN; i++)
      x[i] = g[i] * h[i];

    ifft(x, MAXN);

    for (int i = 0; i < MAXN; i++)
      ans[i] = (int)(real(x[i]) + 0.5);

    for (int i = 0; i < MAXN; i++) {
      ans[i] = (ans[i] + r);
      r = ans[i] / 10;
      ans[i] %= 10;
    }

    int pos = MAXN - 1;
    while (pos > 0 && ans[pos] == 0) pos--;
    for (int i = pos; i >= 0; i--)
      cout << ans[i];
    cout << '\n';
  }
  return 0;
}
```

5.3 NTT

```
// Remember coefficient are mod P
// {n, 2^n, p, a, root}   Note: p = a*2^n+1
// {16, 65536, 65537, 1, 3}
// {20, 1048576, 7340033, 7, 3}

template < LL P, LL root, int MAXN > // (must be 2^k)
struct NTT {
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
            if (b & 1) res = (res * bs) % P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if (a == 1) return 1;
        return (((LL)(a - inv(b % a, a)) * b + 1) / a)
            % b;
    }
    LL omega[MAXN + 1];
```

```
    NTT() {
        omega[0] = 1;
        LL r = bigmod(root, (P - 1) / MAXN);
        for (int i = 1; i <= MAXN; i++)
            omega[i] = (omega[i - 1] * r) % P;
    }
    // n must be 2^k
    void tran(int n, LL a[], bool inv_ntt = false) {
        int basic = MAXN / n, theta = basic;
        for (int m = n; m >= 2; m >>= 1) {
            int mh = m >> 1;
            for (int i = 0; i < mh; i++) {
                LL w = omega[i * theta % MAXN];
                for (int j = i; j < n; j += m) {
                    int k = j + mh;
                    LL x = a[j] - a[k];
                    if (x < 0) x += P;
                    a[j] += a[k];
                    if (a[j] > P) a[j] -= P;
                    a[k] = (w * x) % P;
                }
            }
            theta = (theta * 2) % MAXN;
        }
        int i = 0;
        for (int j = 1; j < n - 1; j++) {
            for (int k = n >> 1; k > (i ^= k); k >>= 1)
                ;
            if (j < i) swap(a[i], a[j]);
        }
        if (inv_ntt) {
            LL ni = inv(n, P);
            reverse(a + 1, a + n);
            for (i = 0; i < n; i++)
                a[i] = (a[i] * ni) % P;
        }
    }
};
const LL P=2013265921, root=31;
const int MAXN=4194304; // MAXN 的因數也可以跑
NTT<P, root, MAXN> ntt;
```

5.4 GaussElimination

```
// by bcw_codebook

const int MAXN = 300;
const double EPS = 1e-8;

int n;
double A[MAXN][MAXN];

void Gauss() {
  for(int i = 0; i < n; i++) {
    bool ok = 0;
    for(int j = i; j < n; j++) {
      if(fabs(A[j][i]) > EPS) {
        swap(A[j], A[i]);
        ok = 1;
        break;
      }
    }
    if(!ok) continue;

    double fs = A[i][i];
    for(int j = i+1; j < n; j++) {
      double r = A[j][i] / fs;
      for(int k = i; k < n; k++) {
        A[j][k] -= A[i][k] * r;
      }
    }
  }
}
```

5.5 inverse

```
const int MAXN = 1000006;
int inv[MAXN];
void invTable(int bound, int p){
    inv[1] = 1;
    for (int i=2; i<bound; i++){
        inv[i] = (long long)inv[p%i] * (p-p/i) %p;
    }
}

int inv(int b, int p){
    if (b==1) return 1;
    return (long long)inv(p%b,p) * (p-p/b) %p;
}
```

5.6 Miller-Rabin

```
typedef long long LL;

inline LL bin_mul(LL a, LL n,const LL& MOD){
  LL re=0;
  while (n>0){
    if (n&1) re += a;
    a += a; if (a>=MOD) a-=MOD;
    n>>=1;
  }
  return re%MOD;
}

inline LL bin_pow(LL a, LL n,const LL& MOD){
  LL re=1;
  while (n>0){
    if (n&1) re = bin_mul(re,a,MOD);
    a = bin_mul(a,a,MOD);
    n>>=1;
  }
  return re;
}

bool is_prime(LL n){
  //static LL sprp[3] = { 2LL, 7LL, 61LL};
  static LL sprp[7] = { 2LL, 325LL, 9375LL,
    28178LL, 450775LL, 9780504LL,
    1795265022LL };
  if (n==1 || (n&1)==0 ) return n==2;
  int u=n-1, t=0;
  while ( (u&1)==0 ) u>>=1, t++;
  for (int i=0; i<3; i++){
    LL x = bin_pow( sprp[i]%n, u, n);
    if (x==0 || x==1 || x==n-1)continue;

    for (int j=1; j<t; j++){
      x=x*x%n;
      if (x==1 || x==n-1)break;
    }
    if (x==n-1)continue;
    return 0;
  }
  return 1;
}
```

5.7 Mobius

```
void mobius() {
    fill(isPrime, isPrime + MAXN, 1);
    mu[1] = 1, num = 0;
    for (int i = 2; i < MAXN; ++i) {
        if (isPrime[i]) primes[num++] = i, mu[i] = -1;
        static int d;
        for (int j = 0; j < num && (d = i * primes[j])
            < MAXN; ++j) {
            isPrime[d] = false;
            if (i % primes[j] == 0) {
                mu[d] = 0; break;
            } else mu[d] = -mu[i];
        }
    }
}
```

5.8 pollardRho

```
// from PEC
// does not work when n is prime
Int f(Int x, Int mod){
  return add(mul(x, x, mod), 1, mod);
}
Int pollard_rho(Int n) {
  if ( !(n & 1) ) return 2;
  while (true) {
    Int y = 2, x = rand()%(n-1) + 1, res = 1;
    for ( int sz = 2 ; res == 1 ; sz *= 2 ) {
      for ( int i = 0 ; i < sz && res <= 1 ; i++) {
        x = f(x, n);
        res = __gcd(abs(x-y), n);
      }
      y = x;
    }
    if ( res != 0 && res != n ) return res;
  }
}
```

5.9 SG

```
#include<bits/stdc++.h>
using namespace std;

int mex(set S) {
  // find the min number >= 0 that not in the S
  // e.g. S = {0, 1, 3, 4} mex(S) = 2
}

state = []
int SG(A) {
  if (A not in state) {
    S = sub_states(A)
    if( len(S) > 1 ) state[A] = reduce(operator.xor, [
        SG(B) for B in S])
    else state[A] = mex(set(SG(B) for B in next_states(
        A)))
  }
  return state[A]
}
```

5.10 theorem

```
/*
Lucas's Theorem
  For non-negative integer n,m and prime P,
  C(m,n) mod P = C(m/M,n/M) * C(m%M,n%M) mod P
  = mult_i ( C(m_i,n_i) )
  where m_i is the i-th digit of m in base P.
-----------------------------------------------------
Kirchhoff's theorem
  A_{ii} = deg(i), A_{ij} = (i,j) \in E ? -1 : 0
  Deleting any one row, one column, and cal the det(A)
-----------------------------------------------------
Nth Catalan recursive function:
C_0 = 1, C_{n+1} = C_n * 2(2n + 1)/(n+2)
-----------------------------------------------------
Mobius Formula
u(n) = 1         , if n = 1
       (-1)^m    , 若 n 無平方數因數，且 n = p1*p2*p3
           *...*pk
       0         , 若 n 有大於 1 的平方數因數
- Property
1. (積性函數) u(a)u(b) = u(ab)
2. Σ_{d|n} u(d) = [n == 1]
-----------------------------------------------------
Mobius Inversion Formula
if      f(n) = Σ_{d|n} g(d)
then    g(n) = Σ_{d|n} u(n/d)f(d)
             = Σ_{d|n} u(d)f(n/d)
- Application
the number/power of gcd(i, j) = k
- Trick
分塊, O(sqrt(n))
-----------------------------------------------------
Chinese Remainder Theorem (m_i 兩兩互質)

  x = a_1 (mod m_1)
  x = a_2 (mod m_2)
  ....
  x = a_i (mod m_i)

construct a solution:

  Let M = m_1 * m_2 * m_3 * ... * m_n
  Let M_i = M / m_i

  t_i = 1 / M_i
  t_i * M_i = 1 (mod m_i)

  solution x = a_1 * t_1 * M_1 + a_2 * t_2 * M_2 + ...
      + a_n * t_n * M_n + k * M
  = k*M + Σ a_i * t_i * M_i, k is positive integer.

  under mod M, there is one solution x = Σ a_i * t_i *
      M_i
-----------------------------------------------------
Burnside's lemma
|G| * |X/G|  = sum( |X^g| ) where g in G
總方法數: 每一種旋轉下不動點的個數總和 除以 旋轉的方法
    數
*/
```

6 Geometry

6.1 2D point template

```
typedef double Double;
struct Point {
  Double x,y;

  bool operator < (const Point &b)const{
    //return tie(x,y) < tie(b.x,b.y);
    //return atan2(y,x) < atan2(b.y,b.x);
    assert(0 && "choose compare");
  }
  Point operator + (const Point &b)const{
    return {x+b.x,y+b.y};
  }
  Point operator - (const Point &b)const{
    return {x-b.x,y-b.y};
  }
  Point operator * (const Double &d)const{
    return {d*x,d*y};
  }
  Point operator / (const Double &d)const{
    return {x/d,y/d};
  }
  Double operator * (const Point &b)const{
    return x*b.x + y*b.y;
  }
  Double operator % (const Point &b)const{
    return x*b.y - y*b.x;
  }
  friend Double abs2(const Point &p){
    return p.x*p.x + p.y*p.y;
  }
  friend Double abs(const Point &p){
    return sqrt( abs2(p) );
  }
};
typedef Point Vector;

struct Line{
  Point P; Vector v;
  bool operator < (const Line &b)const{
    return atan2(v.y,v.x) < atan2(b.v.y,b.v.x);
  }
};
```

6.2 circumcentre

```cpp
#include "2Dpoint.cpp"

Point circumcentre(Point &p0, Point &p1, Point &p2){
  Point a = p1-p0;
  Point b = p2-p0;
  Double c1 = abs2(a)*0.5;
  Double c2 = abs2(b)*0.5;
  Double d = a % b;
  Double x = p0.x + ( c1*b.y - c2*a.y ) / d;
  Double y = p0.y + ( c2*a.x - c1*b.x ) / d;
  return {x,y};
}
```

6.3 ConvexHull

```cpp
#include "2Dpoint.cpp"

// retunr H, 第一個點會在 H 出現兩次
void ConvexHull(vector<Point> &P, vector<Point> &H){
    int n = P.size(), m=0;
    sort(P.begin(),P.end());
    H.clear();

    for (int i=0; i<n; i++){
        while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
            <0)H.pop_back(), m--;
        H.push_back(P[i]), m++;
    }

    for (int i=n-2; i>=0; i--){
        while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
            <0)H.pop_back(), m--;
        H.push_back(P[i]), m++;
    }
}
```

6.4 3D ConvexHull

```cpp
// return the faces with pt indexes
int flag[MXN][MXN];
struct Point{
  ld x,y,z;
  Point operator - (const Point &b) const {
    return (Point){x-b.x,y-b.y,z-b.z};
  }
  Point operator * (const ld &b) const {
    return (Point){x*b,y*b,z*b};
  }
  ld len() const { return sqrtl(x*x+y*y+z*z); }
  ld dot(const Point &a) const {
    return x*a.x+y*a.y+z*a.z;
  }
  Point operator * (const Point &b) const {
    return (Point){y*b.z-b.y*z,z*b.x-b.z*x,x*b.y-b.x*y
        };
  }
};
Point ver(Point a, Point b, Point c) {
  return (b - a) * (c - a);
}
vector<Face> convex_hull_3D(const vector<Point> pt) {
  int n = SZ(pt);
  REP(i,n) REP(j,n)
    flag[i][j] = 0;

  vector<Face> now;
  now.push_back((Face){0,1,2});
  now.push_back((Face){2,1,0});
  int ftop = 0;
  for (int i=3; i<n; i++){
    ftop++;
    vector<Face> next;
    REP(j, SZ(now)) {
      Face& f=now[j];
      ld d=(pt[i]-pt[f.a]).dot(ver(pt[f.a], pt[f.b], pt
          [f.c]));
      if (d <= 0) next.push_back(f);
      int ff = 0;
      if (d > 0) ff=ftop;
      else if (d < 0) ff=-ftop;
      flag[f.a][f.b] = flag[f.b][f.c] = flag[f.c][f.a]
          = ff;
    }
    REP(j, SZ(now)) {
      Face& f=now[j];
      if (flag[f.a][f.b] > 0 and flag[f.a][f.b] != flag
          [f.b][f.a])
        next.push_back((Face){f.a,f.b,i});
      if (flag[f.b][f.c] > 0 and flag[f.b][f.c] != flag
          [f.c][f.b])
        next.push_back((Face){f.b,f.c,i});
      if (flag[f.c][f.a] > 0 and flag[f.c][f.a] != flag
          [f.a][f.c])
        next.push_back((Face){f.c,f.a,i});
    }
    now=next;
  }
  return now;
}
```

6.5 half plane intersection

```cpp
bool OnLeft(const Line& L,const Point& p){
  return Cross(L.v,p-L.P)>0;
}
Point GetIntersection(Line a,Line b){
  Vector u = a.P-b.P;
  Double t = Cross(b.v,u)/Cross(a.v,b.v);
  return a.P + a.v*t;
}
int HalfplaneIntersection(Line* L,int n,Point* poly){
  sort(L,L+n);

  int first,last;
  Point *p = new Point[n];
  Line *q = new Line[n];
  q[first=last=0] = L[0];
  for(int i=1;i<n;i++){
    while(first < last && !OnLeft(L[i],p[last-1])) last
        --;
    while(first < last && !OnLeft(L[i],p[first])) first
        ++;
    q[++last]=L[i];
    if(fabs(Cross(q[last].v,q[last-1].v))<EPS){
      last--;
      if(OnLeft(q[last],L[i].P)) q[last]=L[i];
    }
    if(first < last) p[last-1]=GetIntersection(q[last
        -1],q[last]);
  }
  while(first<last && !OnLeft(q[first],p[last-1])) last
      --;
  if(last-first<=1) return 0;
  p[last]=GetIntersection(q[last],q[first]);

  int m=0;
  for(int i=first;i<=last;i++) poly[m++]=p[i];
  return m;
}
```

6.6 Intersection of two circle

```cpp
vector<Point> interCircle(Point o1, Double r1, Point o2
    , Double r2) {
  Double d2 = abs2(o1 - o2);
  Double d = sqrt(d2);
  Point u = (o1+o2)*0.5 + (o1-o2)*(r2*r2-r1*r1)/(2.0*d2
      );
    if (abs((r1+r2)*(r1+r2) - d2) < 1e-6) return {u};
  if (d < fabs(r1-r2) || r1+r2 < d) return {};
  Double A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d) *
      (-r1+r2+d));
  Point v = Point{o1.y-o2.y, -o1.x+o2.x} * A / (2.0*d2)
      ;
  return {u+v, u-v};
```

```
}
```

6.7 Intersection of two lines

```
Point interPnt(Point p1, Point p2, Point q1, Point q2,
    bool &res){
  Double f1 = cross(p2, q1, p1);
  Double f2 = -cross(p2, q2, p1);
  Double f = (f1 + f2);

  if(fabs(f) < EPS) {
    res = false;
    return {};
    }

  res = true;
  return (f2 / f) * q1 + (f1 / f) * q2;
}
```

6.8 Smallest Circle

```
#include "circumcentre.cpp"
pair<Point,Double> SmallestCircle(int n, Point _p[]){
  Point *p = new Point[n];
  memcpy(p,_p,sizeof(Point)*n);
  random_shuffle(p,p+n);

  Double r2=0;
  Point cen;
  for (int i=0; i<n; i++){
    if ( abs2(cen-p[i]) <= r2)continue;
    cen = p[i], r2=0;
    for (int j=0; j<i; j++){
      if ( abs2(cen-p[j]) <= r2)continue;
      cen = (p[i]+p[j])*0.5;
      r2 = abs2(cen-p[i]);
      for (int k=0; k<j; k++){
        if ( abs2(cen-p[k]) <= r2)continue;
        cen = circumcentre(p[i],p[j],p[k]);
        r2 = abs2(cen-p[k]);
      }
    }
  }

  delete[] p;
  return {cen,r2};
}
// auto res = SmallestCircle(,);
```

7 String

7.1 AC automaton

```
// remember make_fail() !!!
// notice MLE

const int sigma = 62;
const int MAXC = 200005;

inline int idx(char c){
    if ('A'<= c && c <= 'Z')return c-'A';
    if ('a'<= c && c <= 'z')return c-'a' + 26;
    if ('0'<= c && c <= '9')return c-'0' + 52;
    assert(false);
}

struct ACautomaton{
    struct Node{
        Node *next[sigma], *fail;
        int cnt; // dp
        Node() : next{}, fail{}, cnt{}{}
    } buf[MAXC], *bufp, *ori, *root;

    void init(){
        bufp = buf;
        ori = new (bufp++) Node();
        root = new (bufp++) Node();
    }

    void insert(char *s){
        Node *ptr = root;
        for (int i=0; s[i]; i++){
            int c = idx(s[i]);
            if (!ptr->next[c])
                ptr->next[c] = new (bufp++) Node();
            ptr = ptr->next[c];
        }
        ptr->cnt=1;
    }

    Node* trans(Node *o, int c){
        if (o->next[c]) return o->next[c];
        return o->next[c] = trans(o->fail, c);
    }

    void make_fail(){
        static queue<Node*> que;

        for (int i=0; i<sigma; i++)
            ori->next[i] = root;
        root->fail = ori;

        que.push(root);
        while ( que.size() ){
            Node *u = que.front(); que.pop();
            for (int i=0; i<sigma; i++){
                if (!u->next[i])continue;
                u->next[i]->fail = trans(u->fail,i);
                que.push(u->next[i]);
            }
            u->cnt += u->fail->cnt;
        }
    }
} ac;
```

7.2 KMP

```
template<typename T>
void build_KMP(int n, T *s, int *f){ // 1 base
  f[0]=-1, f[1]=0;
  for (int i=2; i<=n; i++){
    int w = f[i-1];
    while (w>=0 && s[w+1]!=s[i])w = f[w];
    f[i]=w+1;
  }
}

template<typename T>
int KMP(int n, T *a, int m, T *b){
  build_KMP(m,b,f);
  int ans=0;

  for (int i=1, w=0; i<=n; i++){
    while ( w>=0 && b[w+1]!=a[i] )w = f[w];
    w++;
    if (w==m){
      ans++;
      w=f[w];
    }
  }
  return ans;
}
```

7.3 palindromic tree

```
// remember init()      !!!
// remember make_fail() !!!
// insert s need 1 base !!!
// notice MLE
const int sigma = 62;
const int MAXC = 1000006;
inline int idx(char c){
    if ('a'<= c && c <= 'z')return c-'a';
    if ('A'<= c && c <= 'Z')return c-'A'+26;
```

```
    if ('0'<= c && c <= '9')return c-'0'+52;
}
struct PalindromicTree{
    struct Node{
        Node *next[sigma], *fail;
        int len, cnt; // for dp
        Node(){
            memset(next,0,sizeof(next));
            fail=0;
            len = cnt = 0;
        }
    } buf[MAXC], *bufp, *even, *odd;

    void init(){
        bufp = buf;
        even = new (bufp++) Node();
        odd  = new (bufp++) Node();
        even->fail = odd;
        odd->len = -1;
    }

    void insert(char *s){
        Node* ptr = even;
        for (int i=1; s[i]; i++){
            ptr = extend(ptr,s+i);
        }
    }

    Node* extend(Node *o, char *ptr){
        int c = idx(*ptr);
        while ( *ptr != *(ptr-1-o->len) )o=o->fail;
        Node *&np = o->next[c];
        if (!np){
            np = new (bufp++) Node();
            np->len = o->len+2;
            Node *f = o->fail;
            if (f){
                while ( *ptr != *(ptr-1-f->len) )f=f->
                    fail;
                np->fail = f->next[c];
            }
            else {
                np->fail = even;
            }
            np->cnt = np->fail->cnt;
        }
        np->cnt++;
        return np;
    }
} PAM;
```

7.4 SAM

```
// par : fail link
// val : a topological order ( useful for DP )
// go[x] : automata edge ( x is integer in [0,26) )

struct SAM{
  struct State{
    int par, go[26], val;
    State () : par(0), val(0){ FZ(go); }
    State (int _val) : par(0), val(_val){ FZ(go); }
  };
  vector<State> vec;
  int root, tail;

  void init(int arr[], int len){
    vec.resize(2);
    vec[0] = vec[1] = State(0);
    root = tail = 1;
    for (int i=0; i<len; i++)
      extend(arr[i]);
  }
  void extend(int w){
    int p = tail, np = vec.size();
    vec.PB(State(vec[p].val+1));
    for ( ; p && vec[p].go[w]==0; p=vec[p].par)
      vec[p].go[w] = np;
    if (p == 0){
      vec[np].par = root;
    } else {
      if (vec[vec[p].go[w]].val == vec[p].val+1){
        vec[np].par = vec[p].go[w];
      } else {
        int q = vec[p].go[w], r = vec.size();
        vec.PB(vec[q]);
        vec[r].val = vec[p].val+1;
        vec[q].par = vec[np].par = r;
        for ( ; p && vec[p].go[w] == q; p=vec[p].par)
          vec[p].go[w] = r;
      }
    }
    tail = np;
  }
};
```

7.5 smallest rotation

```
string mcp(string s){
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n){
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}
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```

7.6 suffix array

```
/*he[i]保存了在後綴數組中相鄰兩個後綴的最長公共前綴長度
 *sa[i]表示的是字典序排名為i的後綴是誰（字典序越小的排
     名越靠前）
 *rk[i]表示的是後綴我所對應的排名是多少  */

const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX];
int sa[MAX], tsa[MAX], tp[MAX][2];
void suffix_array(char *ip){
  int len = strlen(ip);
  int alp = 256;
  memset(ct, 0, sizeof(ct));
  for(int i=0;i<len;i++) ct[ip[i]+1]++;
  for(int i=1;i<alp;i++) ct[i]+=ct[i-1];
  for(int i=0;i<len;i++) rk[i]=ct[ip[i]];
  for(int i=1;i<len;i*=2){
    for(int j=0;j<len;j++){
      if(j+i>=len) tp[j][1]=0;
      else tp[j][1]=rk[j+i]+1;
      tp[j][0]=rk[j];
    }
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][1]+1]++;
    for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++) tsa[ct[tp[j][1]]++]=j;
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][0]+1]++;
    for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++)
      sa[ct[tp[tsa[j]][0]]++]=tsa[j];
    rk[sa[0]]=0;
    for(int j=1;j<len;j++){
      if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
        tp[sa[j]][1] == tp[sa[j-1]][1] )
        rk[sa[j]] = rk[sa[j-1]];
      else
        rk[sa[j]] = j;
    }
  }
  for(int i=0,h=0;i<len;i++){
```

```
    if(rk[i]==0) h=0;
    else{
      int j=sa[rk[i]-1];
      h=max(0,h-1);
      for(;ip[i+h]==ip[j+h];h++);
    }
    he[rk[i]]=h;
  }
}
```

7.7 Z value

```
z[0] = 0;
for ( int bst = 0, i = 1; i < len ; i++ ) {
  if ( z[bst] + bst <= i ) z[i] = 0;
  else z[i] = min(z[i - bst], z[bst] + bst - i);
  while ( str[i + z[i]] == str[z[i]] ) z[i]++;
  if ( i + z[i] > bst + z[bst] ) bst = i;
}
```

```
// 回文版

void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
        centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        if (z[b]+b >= i) z[i] = min(z[2*b-i], b+z[b]-i)
            ;
        else z[i] = 0;
        while (i+z[i]+1 < len && i-z[i]-1 >= 0 &&
                s[i+z[i]+1] == s[i-z[i]-1]) z[i] ++;
        if (z[i]+i > z[b]+b) b = i;
    }
}
```

7.8 BWT (Burrows-Wheeler Transform)

```
string BWT(string); // by suffix array

string iBWT(string &s, int start=0){
    int n = (int) s.size();
    string ret(n,' ');
    vector<int> next(n,0), box[256];

    for (int i=0; i<n; i++) // bucket sort
        box[ (int)s[i] ].push_back(i);

    for (int i=0, j=0; i<256; i++)
        for (int x:box[i])
            next[j++] = x;

    for (int i=0, p=start; i<n; i++)
        ret[i] = s[ p=next[p] ];

    return ret;
}
```

8 Data structure

8.1 2D range tree

```
// remember sort x !!!!!
typedef int T;
const int LGN = 20;
const int MAXN = 100005;

struct Point{
    T x, y;
    friend bool operator < (Point a, Point b){
        return tie(a.x,a.y) < tie(b.x,b.y);
    }
};
struct TREE{
    Point pt;
    int toleft;
}tree[LGN][MAXN];
struct SEG{
    T mx, Mx;
    int sz;
    TREE *st;
}seg[MAXN*4];

vector<Point> P;

void build(int l, int r, int o, int deep){
    seg[o].mx = P[l].x;
    seg[o].Mx = P[r].x;
    seg[o].sz = r-l+1;;

    if(l == r){
        tree[deep][r].pt = P[r];
        tree[deep][r].toleft = 0;
        seg[o].st = &tree[deep][r];
        return;
    }
    int mid = (l+r)>>1;
    build(l,mid,o+o,deep+1);
    build(mid+1,r,o+o+1,deep+1);

    TREE *ptr = &tree[deep][l];
    TREE *pl = &tree[deep+1][l], *nl = &tree[deep+1][
        mid+1];
    TREE *pr = &tree[deep+1][mid+1], *nr = &tree[deep
        +1][r+1];

    int cnt = 0;
    while(pl != nl && pr != nr) {
        *(ptr) = pl->pt.y <= pr->pt.y ? cnt++, *(pl++):
             *(pr++);
        ptr -> toleft = cnt; ptr++;
    }
    while(pl != nl) *(ptr) = *(pl++), ptr -> toleft =
        ++cnt, ptr++;
    while(pr != nr) *(ptr) = *(pr++), ptr -> toleft =
        cnt, ptr++;

}
int main(){
    int n; cin >> n;
    for(int i = 0 ;i < n; i++){
        T x,y; cin >> x >> y;
        P.push_back((Point){x,y});
    }
    sort(P.begin(),P.end());
    build(0,n-1,1,0);
}
```

8.2 ext heap

```
#include <bits/extc++.h>
typedef __gnu_pbds::priority_queue<int> heap_t;
heap_t a,b;

int main() {
  a.clear();
  b.clear();
  a.push(1);
  a.push(3);
  b.push(2);
  b.push(4);
  assert(a.top() == 3);
  assert(b.top() == 4);
  // merge two heap
  a.join(b);
  assert(a.top() == 4);
  assert(b.empty());

  return 0;
}
```

8.3 KD tree

```
// from BCW

const int MXN = 100005;

struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int id,f;
    Node *L, *R;
  }tree[MXN];
  int n;
  Node *root;

  long long dis2(int x1, int y1, int x2, int y2) {
    long long dx = x1-x2;
    long long dy = y1-y2;
    return dx*dx+dy*dy;
  }
  static bool cmpx(Node& a, Node& b){ return a.x<b.x; }
  static bool cmpy(Node& a, Node& b){ return a.y<b.y; }
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
    for (int i=0; i<n; i++) {
      tree[i].id = i;
      tree[i].x = ip[i].first;
      tree[i].y = ip[i].second;
    }
    root = build_tree(0, n-1, 0);
  }
  Node* build_tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2;
    tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
        cmpy : cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;

    tree[M].L = build_tree(L, M-1, dep+1);
    if (tree[M].L) {
      tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    }

    tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
      tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    }

    return tree+M;
  }
  int touch(Node* r, int x, int y, long long d2){
    long long dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
        r->y2+dis)
      return 0;
    return 1;
  }
  void nearest(Node* r, int x, int y, int &mID, long
      long &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    long long d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 || (d2 == md2 && mID < r->id)) {
      mID = r->id;
      md2 = d2;
    }
    // search order depends on split dim
    if ((r->f == 0 && x < r->x) ||
        (r->f == 1 && y < r->y)) {
      nearest(r->L, x, y, mID, md2);
      nearest(r->R, x, y, mID, md2);
    } else {
      nearest(r->R, x, y, mID, md2);
      nearest(r->L, x, y, mID, md2);
    }
  }
  int query(int x, int y) {
    int id = 1029384756;
    long long d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
  }
}tree;
```

8.4 Link-Cut tree

```
// from bcw codebook

const int MXN = 100005;
const int MEM = 100005;

struct Splay {
  static Splay nil, mem[MEM], *pmem;
  Splay *ch[2], *f;
  int val, rev, size;
  Splay () : val(-1), rev(0), size(0) {
    f = ch[0] = ch[1] = &nil;
  }
  Splay (int _val) : val(_val), rev(0), size(1) {
    f = ch[0] = ch[1] = &nil;
  }
  bool isr() {
    return f->ch[0] != this && f->ch[1] != this;
  }
  int dir() {
    return f->ch[0] == this ? 0 : 1;
  }
  void setCh(Splay *c, int d) {
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  }
  void push() {
    if (rev) {
      swap(ch[0], ch[1]);
      if (ch[0] != &nil) ch[0]->rev ^= 1;
      if (ch[1] != &nil) ch[1]->rev ^= 1;
      rev=0;
    }
  }
  void pull() {
    size = ch[0]->size + ch[1]->size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
  }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;

void rotate(Splay *x) {
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(); x->pull();
}

vector<Splay*> splayVec;
void splay(Splay *x) {
  splayVec.clear();
  for (Splay *q=x;; q=q->f) {
    splayVec.push_back(q);
    if (q->isr()) break;
  }
  reverse(begin(splayVec), end(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir()==x->f->dir()) rotate(x->f),rotate
        (x);
    else rotate(x),rotate(x);
  }
}

Splay* access(Splay *x) {
```

```
  Splay *q = nil;
  for (;x!=nil;x=x->f) {
    splay(x);
    x->setCh(q, 1);
    q = x;
  }
  return q;
}
void evert(Splay *x) {
  access(x);
  splay(x);
  x->rev ^= 1;
  x->push(); x->pull();
}
void link(Splay *x, Splay *y) {
//  evert(x);
  access(x);
  splay(x);
  evert(y);
  x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
//  evert(x);
  access(y);
  splay(y);
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
}

int N, Q;
Splay *vt[MXN];

int ask(Splay *x, Splay *y) {
  access(x);
  access(y);
  splay(x);
  int res = x->f->val;
  if (res == -1) res=x->val;
  return res;
}
int main(int argc, char** argv) {
  scanf("%d%d", &N, &Q);
  for (int i=1; i<=N; i++)
    vt[i] = new (Splay::pmem++) Splay(i);
  while (Q--) {
    char cmd[105];
    int u, v;
    scanf("%s", cmd);
    if (cmd[1] == 'i') {
      scanf("%d%d", &u, &v);
      link(vt[v], vt[u]);
    } else if (cmd[0] == 'c') {
      scanf("%d", &v);
      cut(vt[1], vt[v]);
    } else {
      scanf("%d%d", &u, &v);
      int res=ask(vt[u], vt[v]);
      printf("%d\n", res);
    }
  }

  return 0;
}
```

8.5 Treap

```
struct Node {
  Node *ls, *rs;
  int val, sum, siz;

  Node (int val) {
    this -> val = val;
    this -> sum = val;
    siz = 1;
    ls = rs = NULL;
  }
}*rot;

int SZ(Node* now) {
  return now == NULL ? 0 : now -> siz;
}

int getSum(Node* now) {
  return now == NULL ? 0 : now -> sum;
}

void maintain(Node* now) {
  now -> siz = 1 + SZ(now -> ls) + SZ(now -> rs);
  now -> sum = now -> val + getSum(now -> ls) + getSum(
      now -> rs);
}

void split(Node* now, int k, Node* &lef, Node* &rig) {
  if (now == NULL) {
    lef = rig = NULL;
    return;
  }
  if (k <= SZ(now -> ls)) {
    rig = now;
    split(now -> ls, k, lef, rig -> ls);
  } else {
    lef = now;
    split(now -> rs, k - SZ(now -> ls) - 1, lef -> rs,
        rig);
  }
  maintain(now);
}

Node* merge(Node* lef, Node* rig) {
  if (lef == NULL) return rig;
  if (rig == NULL) return lef;

  if (rand() % (SZ(lef) + SZ(rig)) < SZ(lef)) {
    lef -> rs = merge(lef -> rs, rig);
    maintain(lef);
    return lef;
  } else {
    rig -> ls = merge(lef, rig -> ls);
    maintain(rig);
    return rig;
  }
}

int32_t main() {
  cin.tie(0), cout.sync_with_stdio(false);

  int t; cin >> t;
  while (t--) {
    rot = NULL;

    int n, m; cin >> n >> m;
    for (int i = 1; i <= n; i++) {
      int x; cin >> x;
      rot = merge(rot, new Node(x));
    }

    Node *lef, *now, *rig, *tmp;
    while (m--) {
      int x, y; cin >> x >> y;

      split(rot, y, tmp, rig);
      split(tmp, x - 1, lef, now);

      int ans = getSum(now);
      cout << ans << '\n';

      rot = merge(merge(lef, new Node(ans)), rig);
    }
  }
  return 0;
}
```

9 Other

9.1 count spanning tree

新的方法介绍
下面我们介绍一种新的方法——Matrix-Tree定理(Kirchhoff矩阵-树定理)。

Matrix-Tree定理是解决生成树计数问题最有力的武器之一。它首先于1847年被Kirchhoff证明。在介绍定理之前，我们首先明确几个概念：
1、G的度数矩阵D[G]是一个n*n的矩阵，并且满足：当i≠j时，dij=0；当i=j时，dij等于vi的度数。
2、G的邻接矩阵A[G]也是一个n*n的矩阵， 并且满足：如果vi、vj之间有边直接相连，则aij=1，否则为0。
我们定义G的Kirchhoff矩阵(也称为拉普拉斯算子)C[G]为C[G]=D[G]-A[G]，
则Matrix-Tree定理可以描述为：G的所有不同的生成树的个数等于其Kirchhoff矩阵C[G]任何一个n-1阶主子式的行列式的绝对值。
所谓n-1阶主子式，就是对于r(1≤r≤n)，将C[G]的第r行、第r列同时去掉后得到的新矩阵，用Cr[G]表示。

生成树计数
算法步骤：
1、 构建拉普拉斯矩阵
Matrix[i][j] =
degree(i) , i==j
-1，i-j有边
0，其他情况
2、 去掉第r行，第r列 (r任意)
3、 计算矩阵的行列式

```
#include <bits/stdc++.h>
using namespace std;
const double eps = 1e-8;
const int MAXN = 110;
int sgn(double x)
{
    if(fabs(x) < eps)return 0;
    if(x < 0)return -1;
    else return 1;
}
double b[MAXN][MAXN];
double det(double a[][MAXN],int n)
{
    int i, j, k, sign = 0;
    double ret = 1;
    for(i = 0;i < n;i++)
    for(j = 0;j < n;j++) b[i][j] = a[i][j];
    for(i = 0;i < n;i++)
    {
        if(sgn(b[i][i]) == 0)
        {
            for(j = i + 1; j < n;j++)
            if(sgn(b[j][i]) != 0) break;
            if(j == n)return 0;
            for(k = i;k < n;k++) swap(b[i][k],b[j][k]);
            sign++;
        }
        ret *= b[i][i];
        for(k = i + 1;k < n;k++) b[i][k]/=b[i][i];
        for(j = i+1;j < n;j++)
        for(k = i+1;k < n;k++) b[j][k] -= b[j][i]*b[i][
            k];
    }
    if(sign & 1)ret = -ret;
    return ret;
}
double a[MAXN][MAXN];
int g[MAXN][MAXN];
int main()
{
    int T;
    int n,m;
    int u,v;
    scanf("%d",&T);
    while(T--)
    {
        scanf("%d%d",&n,&m);
        memset(g,0,sizeof(g));
        while(m--)
        {
            scanf("%d%d",&u,&v);
            u--;v--;
            g[u][v] = g[v][u] = 1;
        }
        memset(a,0,sizeof(a));
        for(int i = 0;i < n;i++)
        for(int j = 0;j < n;j++)
        if(i != j && g[i][j])
        {
            a[i][i]++;
            a[i][j] = -1;
        }
        double ans = det(a,n-1);
        printf("%.0lf\n",ans);
    }
    return 0;
}
```

9.2 C++11 random

```
void init(){
    std::random_device rd;
    std::default_random_engine gen( rd() );
    std::uniform_int_distribution <unsigned long long>
        dis(0,ULLONG_MAX);

    for (int i=0; i<MAXN; i++){
        h[i] = dis(gen);
    }
}
```

9.3 Digit Counting

```
int dfs(int pos, int state1, int state2 ....., bool
    limit, bool zero) {
    if ( pos == -1 ) return 是否符合條件;
    int &ret = dp[pos][state1][state2][....];
    if ( ret != -1 && !limit ) return ret;
    int ans = 0;
    int upper = limit ? digit[pos] : 9;
    for ( int i = 0 ; i <= upper ; i++ ) {
        ans += dfs(pos - 1, new_state1, new_state2,
            limit & ( i == upper), ( i == 0) && zero);
    }
    if ( !limit ) ret = ans;
    return ans;
}

int solve(int n) {
    int it = 0;
    for ( ; n ; n /= 10 ) digit[it++] = n % 10;
    return dfs(it - 1, 0, 0, 1, 1);
}
```

9.4 DP optimization

```
Monotonicity & 1D/1D DP & 2D/1D DP
-------------------------------------------------------
Definition xD/yD
1D/1D DP[j] = min(0≤i<j) { DP[i] + w(i, j) }; DP[0] = k
2D/1D DP[i][j] = min(i<k≤j) { DP[i][k - 1] + DP[k][j] }
     + w(i, j); DP[i][i] = 0
-------------------------------------------------------
Monotonicity
       c       d
  -----------------
a | w(a, c) w(a, d)
b | w(b, c) w(b, d)

Monge Condition
Concave(凹四邊形不等式): w(a, c) + w(b, d) >= w(a, d) +
     w(b, c)
Convex (凸四邊形不等式): w(a, c) + w(b, d) <= w(a, d) +
     w(b, c)

Totally Monotone
Concave(凹單調): w(a, c) <= w(b, d) -----> w(a, d) <= w
    (b, c)
```

```
Convex (凸單調): w(a, c) >= w(b, d) -----> w(a, d) >= w
    (b, c)
-------------------------------------------------------
1D/1D DP O(n^2) -> O(nlgn)
**CONSIDER THE TRANSITION POINT**
Solve 1D/1D Concave by Stack
Solve 1D/1D Convex by Deque
-------------------------------------------------------
2D/1D Convex DP (Totally Monotone) O(n^3) -> O(n^2)
h(i, j − 1) ≤ h(i, j) ≤ h(i + 1, j)
```

9.5 DP 1D/1D

```
#include<bits/stdc++.h>

int t, n, L;
int p;
char s[MAXN][35];
ll sum[MAXN] = {0};
long double dp[MAXN] = {0};
int prevd[MAXN] = {0};

long double pw(long double a, int n) {
    if ( n == 1 ) return a;
    long double b = pw(a, n/2);
    if ( n & 1 ) return b*b*a;
    else return b*b;
}
long double f(int i, int j) {
//    cout << (sum[i] - sum[j]+i-j-1-L) << endl;
    return pw(abs(sum[i] - sum[j]+i-j-1-L), p) + dp[j];
}
struct INV {
    int L, R, pos;
};
INV stk[MAXN*10];
int top = 1, bot = 1;
void update(int i) {
    while ( top > bot && i < stk[top].L && f(stk[top].L
        , i) < f(stk[top].L, stk[top].pos) ) {
        stk[top - 1].R = stk[top].R;
        top--;
    }
    int lo = stk[top].L, hi = stk[top].R, mid, pos =
        stk[top].pos;
    //if ( i >= lo ) lo = i + 1;
    while ( lo != hi ) {
        mid = lo + (hi - lo) / 2;
        if ( f(mid, i) < f(mid, pos) ) hi = mid;
        else lo = mid + 1;
    }
    if ( hi < stk[top].R ) {
        stk[top + 1] = (INV) { hi, stk[top].R, i };
        stk[top++].R = hi;
    }
}

int main() {
    cin >> t;
    while ( t-- ) {
        cin >> n >> L >> p;
        dp[0] = sum[0] = 0;
        for ( int i = 1 ; i <= n ; i++ ) {
            cin >> s[i];
            sum[i] = sum[i-1] + strlen(s[i]);
            dp[i] = numeric_limits<long double>::max();
        }
        stk[top] = (INV) {1, n + 1, 0};
        for ( int i = 1 ; i <= n ; i++ ) {
            if ( i >= stk[bot].R ) bot++;
            dp[i] = f(i, stk[bot].pos);
            update(i);
//            cout << (ll) f(i, stk[bot].pos) << endl;
        }
        if ( dp[n] > 1e18 ) {
            cout << "Too hard to arrange" << endl;
        } else {
            vector<PI> as;
            cout << (ll)dp[n] << endl;
        }
    }
    return 0;
}
```

9.6 Manhattan MST.cpp

```
#include <bits/stdc++.h>
using namespace std;

const int MAXN = 100005;
const int OFFSET = 2000; // y-x may < 0, offset it, if
    y-x too large, please write a unique function
const int INF = 0xFFFFFFF;
int n;
int x[MAXN], y[MAXN], p[MAXN];

typedef pair<int, int> pii;
pii bit[MAXN]; // [ val, pos ]

struct P {
    int x, y, id;
    bool operator<(const P&b ) const {
        if ( x == b.x ) return y > b.y;
        else return x > b.x;
    }
};
vector<P> op;

struct E {
    int x, y, cost;
    bool operator<(const E&b ) const {
        return cost < b.cost;
    }
};
vector<E> edges;

int find(int x) {
    return p[x] == x ? x : p[x] = find(p[x]);
}

void update(int i, int v, int p) {
    while ( i ) {
        if ( bit[i].first > v ) bit[i] = {v, p};
        i -= i & (-i);
    }
}

pii query(int i) {
    pii res = {INF, INF};
    while ( i < MAXN ) {
        if ( bit[i].first < res.first ) res = {bit[i].
            first, bit[i].second};
        i += i & (-i);
    }
    return res;
}

void input() {
    cin >> n;
    for ( int i = 0 ; i < n ; i++ ) cin >> x[i] >> y[i
        ], op.push_back((P) {x[i], y[i], i});
}

void mst() {
    for ( int i = 0 ; i < MAXN ; i++ ) p[i] = i;
    int res = 0;
    sort(edges.begin(), edges.end());
    for ( auto e : edges ) {
        int x = find(e.x), y = find(e.y);
        if ( x != y ) {
            p[x] = y;
            res += e.cost;
        }
    }
    cout << res << endl;
}

void construct() {
    sort(op.begin(), op.end());
    for ( int i = 0 ; i < n ; i++ ) {
```

```
        pii q = query(op[i].y - op[i].x + OFFSET);
        update(op[i].y - op[i].x + OFFSET, op[i].x + op
            [i].y, op[i].id);
        if ( q.first == INF ) continue;
        edges.push_back((E) {op[i].id, q.second, abs(x[
            op[i].id]-x[q.second]) + abs(y[op[i].id]-y[
            q.second]) });
    }
}

void solve() {

    // [45 ~ 90 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,
        INF};
    construct();

    // [0 ~ 45 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,
        INF};
    for ( int i = 0 ; i < n ; i++ ) swap(op[i].x, op[i
        ].y);
    construct();
    for ( int i = 0 ; i < n ; i++ ) swap(op[i].x, op[i
        ].y);

    // [-90 ~ -45 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,
        INF};
    for ( int i = 0 ; i < n ; i++ ) op[i].y *= -1;
    construct();

    // [-45 ~ 0 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,
        INF};
    for ( int i = 0 ; i < n ; i++ ) swap(op[i].x, op[i
        ].y);
    construct();

    // mst
    mst();

}

int main () {
    input();
    solve();
    return 0;
}
```

9.7 stable marriage

```
// normal stable marriage problem
// input:
//3
//Albert Laura Nancy Marcy
//Brad Marcy Nancy Laura
//Chuck Laura Marcy Nancy
//Laura Chuck Albert Brad
//Marcy Albert Chuck Brad
//Nancy Brad Albert Chuck

#include<bits/stdc++.h>
using namespace std;
const int MAXN = 505;

int n;
int favor[MAXN][MAXN]; // favor[boy_id][rank] = girl_id
    ;
int order[MAXN][MAXN]; // order[girl_id][boy_id] = rank
    ;
int current[MAXN]; // current[boy_id] = rank; boy_id
    will pursue current[boy_id] girl.
int girl_current[MAXN]; // girl[girl_id] = boy_id;

void initialize() {
  for ( int i = 0 ; i < n ; i++ ) {
    current[i] = 0;
    girl_current[i] = n;
    order[i][n] = n;
  }
}

map<string, int> male, female;
string bname[MAXN], gname[MAXN];
int fit = 0;

void stable_marriage() {

  queue<int> que;
  for ( int i = 0 ; i < n ; i++ ) que.push(i);
  while ( !que.empty() ) {
    int boy_id = que.front();
    que.pop();

    int girl_id = favor[boy_id][current[boy_id]];
    current[boy_id] ++;

    if ( order[girl_id][boy_id] < order[girl_id][
        girl_current[girl_id]] ) {
      if ( girl_current[girl_id] < n ) que.push(
          girl_current[girl_id]); // if not the first
          time
      girl_current[girl_id] = boy_id;
    } else {
      que.push(boy_id);
    }
  }

}

int main() {
  cin >> n;

  for ( int i = 0 ; i < n;  i++ ) {
    string p, t;
    cin >> p;
    male[p] = i;
    bname[i] = p;
    for ( int j = 0 ; j < n ; j++ ) {
      cin >> t;
      if ( !female.count(t) ) {
        gname[fit] = t;
        female[t] = fit++;
      }
      favor[i][j] = female[t];
    }
  }

  for ( int i = 0 ; i < n ; i++ ) {
    string p, t;
    cin >> p;
    for ( int j = 0 ; j < n ; j++ ) {
      cin >> t;
      order[female[p]][male[t]] = j;
    }
  }

  initialize();
  stable_marriage();

  for ( int i = 0 ; i < n ; i++ ) {
    cout << bname[i] << " " << gname[favor[i][current[i
        ] - 1]] << endl;
  }

}
```

9.8 Mo's algorithm

```
int l = 0, r = 0, nowAns = 0, BLOCK_SIZE, n, m;
int ans[];
struct QUE{
    int l, r, id;
    friend bool operator < (QUE a, QUE b){
        if(a.l / BLOCK_SIZE != b.l / BLOCK_SIZE)
            return a.l / BLOCK_SIZE < b.l / BLOCK_SIZE;
        return a.r < b.r;
    }
```

```
}querys[];

inline void move(int pos, int sign) {
    // update nowAns
}

void solve() {
    BLOCK_SIZE = int(ceil(pow(n, 0.5)));
    sort(querys, querys + m);
    for (int i = 0; i < m; ++i) {
        const QUE &q = querys[i];
        while (l > q.l) move(--l, 1);
        while (r < q.r) move(r++, 1);
        while (l < q.l) move(l++, -1);
        while (r > q.r) move(--r, -1);
        ans[q.id] = nowAns;
    }
}
```

9.9 Parser

```
using LL = long long;
const int MAXLEVEL = 2;
//  binary operators
const vector<char> Ops[MAXLEVEL] = {
    {'+', '-'}, // level 0
    {'*', '/'}  // level 1
};
//  unary operators
const vector<pair<char,int>> Op1s = {
    {'-', 0} // operator negative works on level 0
};
struct Node{
    ~Node(){ delete L; delete R; }
    enum { op, op1, num } type;
    LL val;
    Node *L, *R;
} *root;
char getOp1(int LEVEL, istream& is){
    is >>ws;
    for (auto& x : Op1s){
        auto& op = x.first;
        auto& lev = x.second;
        if (LEVEL == lev && is.peek() == op)
            return is.get();
    }
    return 0;
}
template <int LEVEL> void parse(Node*& x, istream& is){
    char op1 = getOp1(LEVEL, is);
    parse<LEVEL+1>(x, is);
    if (op1) x = new Node{Node::op1, op1, x, nullptr};
    auto& ops = Ops[LEVEL];
    while (is>>ws && count(ops.begin(), ops.end(), is.
        peek())){
        x = new Node{Node::op, is.get(), x, nullptr};
        parse<LEVEL+1>(x->R, is);
    }
}
template <> void parse<MAXLEVEL>(Node*& x, istream& is)
    {
    char op1 = getOp1(MAXLEVEL, is);
    is>>ws;
    if (is.peek()>='0' && is.peek()<='9'){
        LL t; is >>t;
        x = new Node{Node::num, t, nullptr, nullptr};
    } else if (is.peek() == '('){
        is.get();
        parse<0>(x, is);
        is>>ws;
        if (is.get()!=')') throw 0;
    } else throw 0;
    if (op1) x = new Node{Node::op1, op1, x, nullptr};
}
//  throw when error occur !!!!!
void build(istream& is){
    parse<0>(root, is);
    if ((is>>ws).peek() != EOF) throw 0;
}
```

9.10 python cheat sheet

```
#!/usr/bin/env python3

# import
import math
from math import *
import math as M
from math import sqrt

# input
n = int( input() )
a = [ int(x) for x in input().split() ]

# EOF
while True:
    try:
        solve()
    except:
        break;

# output
print( x, sep=' ')
print( ''.join( str(x)+' ' for x in a ) )
print( '{:5d}'.format(x) )

# sort
a.sort()
sorted(a)

# list
a = [ x for x in range(n) ]
a.append(x)

# Basic operator
a, b = 10, 20
a/b  # 0.5
a//b # 0
a%b  # 10
a**b # 10^20

# if, else if, else
if a==0:
    print('zero')
elif a>0:
    print('postive')
else:
    print('negative')

# loop
while a==b and b==c:
for i in LIST:

# stack          # C++
stack = [3,4,5]
stack.append(6) # push()
stack.pop()     # pop()
stack[-1]       # top()
len(stack)      # size() O(1)

# queue          # C++
from collections import deque
queue = deque([3,4,5])
queue.append(6) # push()
queue.popleft() # pop()
queue[0]        # front()
len(queue)      # size() O(1)

# random
from random import *
randrange(L,R,step) # [L,R) L+k*step
randint(L,R) # int from [L,R]
choice(list) # pick 1 item from list
choices(list,k) # pick k item
shuffle(list)
Uniform(L,R) # float from [L,R]

# Decimal
from fractions import Fraction
from decimal import Decimal, getcontext
getcontext().prec = 250 # set precision
```

```
itwo = Decimal(0.5)
two = Decimal(2)

N = 200
def angle(cosT):
  """given cos(theta) in decimal return theta"""
  for i in range(N):
    cosT = ((cosT + 1) / two) ** itwo
  sinT = (1 - cosT * cosT) ** itwo
  return sinT * (2 ** N)
pi = angle(Decimal(-1))

# file IO
r = open("filename.in")
a = r.read() # read whole content into one string

w = open("filename.out", "w")
w.write('123\n')

# IO redirection
import sys
sys.stdin = open('filename.in')
sys.stdout = open('filename.out', 'w')
```