

Contents

1 Basic	1
1.1 vimrc	1
1.2 default code	1
1.3 state machine	1
2 Flow	2
2.1 Dinic	2
2.2 GomoryHu tree 全點對最小割 DC 法	2
2.3 min cost flow	3
2.4 SW mincut 全點對最小割	3
3 Matching	3
3.1 Hungarian	3
3.2 KM	3
3.3 Matching.txt	4
3.4 Maximum General Matching	4
3.5 Minimum General Weighted Matching	4
4 Graph	5
4.1 BCC edge	5
4.2 Dijkstra	5
4.3 Domination.txt	6
4.4 LCA	6
4.5 max clique	6
4.6 min mean cycle	6
4.7 SSSP related concepts	7
4.8 Tarjan.cpp	7
4.9 2-SAT	8
4.10 平面圖判定	8
5 Math	10
5.1 ax+by=gcd(a,b)	10
5.2 FFT	10
5.3 NTT	10
5.4 GaussElimination	11
5.5 inverse	11
5.6 Miller-Rabin	11
5.7 Mobius	11
5.8 pollardRho	11
5.9 SG	12
5.10 theorem	12
6 Geometry	12
6.1 2D point template	12
6.2 circumcentre	12
6.3 ConvexHull	13
6.4 3D ConvexHull	13
6.5 half plane intersection	13
6.6 Intersection of two circle	13
6.7 Intersection of two lines	14
6.8 Smallest Circle	14
7 String	14
7.1 AC automaton	14
7.2 KMP	14
7.3 palindromic tree	14
7.4 SAM	15
7.5 smallest rotation	15
7.6 suffix array	15
7.7 Z value	16
7.8 BWT (Burrows-Wheeler Transform)	16
8 Data structure	16
8.1 2D range tree	16
8.2 ext heap	16
8.3 KD tree	16
8.4 Link-Cut tree	17
8.5 Treap	18
8.6 Treap Lin	18
9 Other	19
9.1 count spanning tree	19
9.2 C++11 random	20
9.3 Digit Counting	20
9.4 DP optimization	20
9.5 DP 1D/1D	20
9.6 Manhattan MST.cpp	21
9.7 stable marriage	22
9.8 Mo's algorithm	22
9.9 Parser	22
9.10 python cheat sheet	23

1 Basic

1.1 vimrc

```

=== .vimrc ===

set et nu cin ls=2 ts=4 sw=4 sts=4 ttm=100
syntax on

nn <F4> :w ! cat -n \| lpr <CR>
nn <F7> :w <bar> :!vim %<_<left><left><left>
nn <F8> :w <bar> :!g++ % -o %< -std=c++11
\ -fsanitize=undefined -Wall -Wextra -Wshadow -DFOX &&
\ for i in %<_*.in; do echo == && ./%< < $i; done <CR>
nn <F9> :w <bar> :!g++ % -o %< -std=c++11
\ -fsanitize=undefined -Wall -Wextra -Wshadow -DFOX &&
\ echo == && ./%<

```

1.2 default code

```

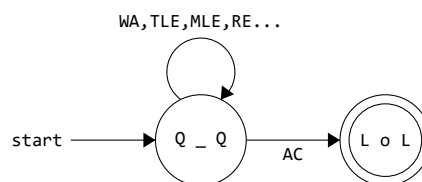
#pragma GCC optimize("Ofast")
#include <bits/stdc++.h>
#include <sys/time.h>
#include <sys/resource.h>
using namespace std;

void setstack(){
    // Set soft limit and hard limit to max
    const rlimit tmp {RLIM_INFINITY, RLIM_INFINITY};
    setrlimit(RLIMIT_STACK, &tmp);
}

int main(){
    #define name ""
    #ifndef FOX
    // fopen to read/write
    freopen(name".in", "r", stdin);
    freopen(name".out", "w", stdout);
    // ifstream to read/write
    ifstream cin(name".in");
    ofstream cout(name".out");
    #endif
    static_assert(strlen(name));
    ios::sync_with_stdio(0);
    cin.tie(0), cout.tie(0);
}

```

1.3 state machine



2 Flow

2.1 Dinic

(a) Bounded Maxflow Construction:

1. add two node ss, tt
2. add_edge(ss, tt, INF)
3. **for** each edge u -> v with capacity [l, r]:
 add_edge(u, tt, l)
 add_edge(ss, v, l)
 add_edge(u, v, r-l)
4. see (b), check **if** it is possible.
5. answer is maxflow(ss, tt) + maxflow(s, t)

(b) Bounded Possible Flow:

1. same construction method as (a)
2. run maxflow(ss, tt)
3. **for** every edge connected with ss **or** tt:
 rule: check **if** their rest flow is exactly 0
4. answer is possible **if** every edge **do** satisfy the rule
5. otherwise, it is NOT possible.

(c) Bounded Minimum Flow:

1. same construction method as (a)
2. answer is maxflow(ss, tt)

(d) Bounded Minimum Cost Flow:

- * the concept is somewhat like bounded possible flow.
1. same construction method as (a)
 2. answer is maxflow(ss, tt) + ($\sum l * \text{cost}$ **for** every edge)

(e) Minimum Cut:

1. run maxflow(s, t)
2. run cut(s)
3. ss[i] = 1: node i is at the same side with s.

```
const long long INF = 1LL<<60;
struct Dinic { //O(VVE), with minimum cut
    static const int MAXN = 5003;
    struct Edge{
        int u, v;
        long long cap, rest;
    };

    int n, m, s, t, d[MAXN], cur[MAXN];
    vector<Edge> edges;
    vector<int> G[MAXN];

    void init(){
        edges.clear();
        for ( int i = 0 ; i < n ; i++ ) G[i].clear();
        n = 0;

        // min cut start
        bool side[MAXN];
        void cut(int u) {
            side[u] = 1;
            for ( int i : G[u] ) {
                if ( !side[ edges[i].v ] && edges[i].rest )
                    cut(edges[i].v);
            }
        }
        // min cut end

        int add_node(){
            return n++;
        }

        void add_edge(int u, int v, long long cap){
            edges.push_back( {u, v, cap, cap} );
            edges.push_back( {v, u, 0, 0LL} );
            m = edges.size();
            G[u].push_back(m-2);
            G[v].push_back(m-1);
        }

        bool bfs(){
```

```
        fill(d,d+n,-1);
        queue<int> que;
        que.push(s); d[s]=0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (int ei : G[u]){
                Edge &e = edges[ei];
                if (d[e.v] < 0 && e.rest > 0){
                    d[e.v] = d[u] + 1;
                    que.push(e.v);
                }
            }
        }
        return d[t] >= 0;
    }

    long long dfs(int u, long long a){
        if ( u == t || a == 0 ) return a;
        long long flow = 0, f;
        for ( int &i=cur[u]; i < (int)G[u].size() ; i++ ) {
            Edge &e = edges[ G[u][i] ];
            if ( d[u] + 1 != d[e.v] ) continue;
            f = dfs(e.v, min(a, e.rest) );
            if ( f > 0 ) {
                e.rest -= f;
                edges[ G[u][i]^1 ].rest += f;
                flow += f;
                a -= f;
                if ( a == 0 ) break;
            }
        }
        return flow;
    }

    long long maxflow(int _s, int _t){
        s = _s, t = _t;
        long long flow = 0, mf;
        while ( bfs() ){
            fill(cur,cur+n,0);
            while ( (mf = dfs(s, INF)) ) flow += mf;
        }
        return flow;
    }
} dinic;
```

2.2 GomoryHu tree 全點對最小割 DC 法

Construct of Gomory Hu Tree

1. make sure the whole graph is clear
2. set node 0 as root, also be the parent of other nodes.
3. **for** every node i > 0, we run maxflow from i to parent[i]
4. hence we know the weight between i and parent[i]
5. **for** each node j > i, **if** j is at the same side with i, make the parent of j as i

```
int e[MAXN][MAXN], p[MAXN];
Dinic D; // original graph

void gomory_hu() {
    fill(p, p+n, 0);
    fill(e[0], e[n], INF);
    for ( int s = 1 ; s < n ; s++ ) {
        int t = p[s];
        Dinic F = D;
        int tmp = F.max_flow(s, t);

        for ( int i = 1 ; i < s ; i++ )
            e[s][i] = e[i][s] = min(tmp, e[t][i]);
        for ( int i = s+1 ; i <= n ; i++ )
            if ( p[i] == t && F.side[i] ) p[i] = s;
    }
}
```

2.3 min cost flow

```
typedef long long lol;
#define MAX 100010

struct Edge {
    int x, y;
    lol c;
    bool operator < (const Edge& rhs) const {
        return c < rhs.c;
    }
};

vector<Edge> v, g;
int p[MAX], s[MAX], n, m;
lol b[MAX], ans_min, ans_smi;

void init() {
    memset(b, 0, sizeof(b));
    for (int i = 1; i < MAX; i++) {
        p[i] = i;
        s[i] = 1;
    }
    ans_min = 0;
    ans_smi = 10000000000000000;
    v.clear();
    g.clear();
}

int findset(int x) {
    return p[x] == x ? x : findset(p[x]);
}

void unio(int x, int y, lol c) {
    if (s[x] > s[y]) {
        p[y] = x;
        s[x] += s[y];
        b[y] = c;
    } else {
        p[x] = y;
        s[y] += s[x];
        b[x] = c;
    }
}

lol find(int x, int y) {
    lol now = 0;
    while (x != y) {
        if (s[x] < s[y]) {
            now = max(now, b[x]);
            x = p[x];
        } else {
            now = max(now, b[y]);
            y = p[y];
        }
    }
    return now;
}

void solve() {
    for (int i = 0; i < m; i++) {
        Edge e = v[i];
        int x = findset(e.x), y = findset(e.y);
        if (x == y) g.push_back(e);
        else {
            ans_min += e.c;
            unio(x, y, e.c);
        }
    }
    for (int i = 0; i < g.size(); i++) {
        Edge e = g[i];
        int x = e.x, y = e.y;
        lol now = find(x, y);
        ans_smi = min(ans_smi, e.c - now);
    }
}

int main() {
    cin.tie(0), cout.sync_with_stdio(false);
    init();
    cin >> n >> m;
```

```
for (int i = 1; i <= m; i++) {
    int x, y; lol c;
    cin >> x >> y >> c;
    v.push_back((Edge){x, y, c});
}
sort(v.begin(), v.end());
solve();
cout << ans_min << ' ' << ans_smi + ans_min << '\n';
return 0;
}
```

2.4 SW mincut 全點對最小割

```
// all pair min cut
// global min cut
struct SW{ // O(V^3)
    static const int MXN = 514;
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n){
        n = _n; FZ(edge); FZ(del);
    }
    void addEdge(int u, int v, int w){
        edge[u][v] += w; edge[v][u] += w;
    }
    void search(int &s, int &t){
        FZ(vst); FZ(wei);
        s = t = -1;
        while (true){
            int mx=-1, cur=0;
            for (int i=0; i<n; i++)
                if (!del[i] && !vst[i] && mx<wei[i])
                    cur = i, mx = wei[i];
            if (mx == -1) break;
            vst[cur] = 1;
            s = t; t = cur;
            for (int i=0; i<n; i++)
                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    int solve(){
        int res = 2147483647;
        for (int i=0, x, y; i<n-1; i++){
            search(x, y);
            res = min(res, wei[y]);
            del[y] = 1;
            for (int j=0; j<n; j++)
                edge[x][j] = (edge[j][x] += edge[y][j]);
        }
        return res;
    }
}graph;
```

3 Matching

3.1 Hungarian

```
// Maximum Cardinality Bipartite Matching
// Worst case O(nm)

struct Graph{
    static const int MAXN = 5003;
    vector<int> G[MAXN];
    int n, match[MAXN], vis[MAXN];

    void init(int _n){
        n = _n;
        for (int i=0; i<n; i++) G[i].clear();
    }

    bool dfs(int u){
        for (int v:G[u]){
            if (vis[v]) continue;
            vis[v]=true;
            if (match[v]==-1 || dfs(match[v])){
                match[v] = u;
                match[u] = v;
            }
        }
    }
};
```

```

        return true;
    }
    }
    return false;
}

int solve(){
    int res = 0;
    memset(match, -1, sizeof(match));
    for (int i=0; i<n; i++){
        if (match[i]==-1){
            memset(vis, 0, sizeof(vis));
            if ( dfs(i) ) res++;
        }
    }
    return res;
}
} graph;

```

3.2 KM

```

const int MAXN = 400 + 10;
const long long INF64 = 0x3f3f3f3f3f3f3f3f;
int n1, nr;
int pre[MAXN];
long long slack[MAXN];
long long W[MAXN][MAXN];
long long lx[MAXN], ly[MAXN];
int mx[MAXN], my[MAXN];
bool vx[MAXN], vy[MAXN];
void augment(int u) {
    if(!u) return;
    augment(mx[pre[u]]);
    mx[pre[u]] = u;
    my[u] = pre[u];
}
void match(int x) {
    queue<int> que;
    que.push(x);
    while(1) {
        while(!que.empty()) {
            x = que.front(); que.pop();
            vx[x] = 1;
            for (int i=1; i<=nr; i++) {
                if(vy[i]) continue;
                long long t = lx[x] + ly[i] - W[x][i];
                if(t > 0) {
                    if(slack[i] >= t) slack[i] = t, pre[i] = x;
                    continue;
                }
                pre[i] = x;
                if(!my[i]) {
                    augment(i);
                    return;
                }
                vy[i] = 1;
                que.push(my[i]);
            }
        }
        long long t = INF64;
        for (int i=1; i<=nr; i++) if(!vy[i]) t = min(t, slack[i]);
        for (int i=1; i<=n1; i++) if(vx[i]) lx[i] -= t;
        for (int i=1; i<=nr; i++) {
            if(vy[i]) ly[i] += t;
            else slack[i] -= t;
        }
        for (int i=1; i<=nr; i++) {
            if(vy[i] || slack[i]) continue;
            if(!my[i]) {
                augment(i);
                return;
            }
        }
        vy[i] = 1;
        que.push(my[i]);
    }
}
}
int main() {

```

```

int m;
cin >> n1 >> nr >> m;
nr = max(n1, nr);
while(m--) {
    int u, v;
    long long w;
    cin >> u >> v >> w;
    W[u][v] = w;
    lx[u] = max(lx[u], w);
}
for (int i=1; i<=n1; i++) {
    for (int x=1; x<=n1; x++) vx[x] = 0;
    for (int y=1; y<=nr; y++) vy[y] = 0, slack[y] = INF64;
    match(i);
}
long long ans = 0;
for (int i=1; i<=n1; i++) ans += W[i][mx[i]];
cout << ans << '\n';
for (int i=1; i<=n1; i++) {
    if (i > 1) cout << ' ';
    cout << (W[i][mx[i]] ? mx[i] : 0);
}
cout << '\n';
}

```

3.3 Matching.txt

最大匹配 + 最小邊覆蓋 = V
 最大獨立集 + 最小點覆蓋 = V
 最大匹配 = 最小點覆蓋
 最小路徑覆蓋數 = V - 最大匹配數

3.4 Maximum General Matching

```

// Maximum Cardinality Matching
struct Graph {
    vector<int> G[MAXN];
    int pa[MAXN], match[MAXN], st[MAXN], S[MAXN], vis[
        MAXN];
    int t, n;

    void init(int _n) {
        n = _n;
        for ( int i = 1 ; i <= n ; i++ ) G[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].push_back(v);
        G[v].push_back(u);
    }
    int lca(int u, int v){
        for ( ++t ; ; swap(u, v) ) {
            if ( u == 0 ) continue;
            if ( vis[u] == t ) return u;
            vis[u] = t;
            u = st[ pa[ match[u] ] ];
        }
    }
    void flower(int u, int v, int l, queue<int> &q) {
        while ( st[u] != 1 ) {
            pa[u] = v;
            if ( S[ v = match[u] ] == 1 ) {
                q.push(v);
                S[v] = 0;
            }
            st[u] = st[v] = 1;
            u = pa[v];
        }
    }
    bool bfs(int u){
        for ( int i = 1 ; i <= n ; i++ ) st[i] = i;
        memset(S, -1, sizeof(S));
        queue<int> q;
        q.push(u);
        S[u] = 0;
        while ( !q.empty() ) {

```

```

    u = q.front(); q.pop();
    for ( int i = 0 ; i < (int)G[u].size(); i++) {
        int v = G[u][i];
        if ( S[v] == -1 ) {
            pa[v] = u;
            S[v] = 1;
            if ( !match[v] ) {
                for ( int lst ; u ; v = lst, u = pa[v] ) {
                    lst = match[u];
                    match[u] = v;
                    match[v] = u;
                }
                return 1;
            }
            q.push(match[v]);
            S[ match[v] ] = 0;
        } else if ( !S[v] && st[v] != st[u] ) {
            int l = lca(st[v], st[u]);
            flower(v, u, l, q);
            flower(u, v, l, q);
        }
    }
}
return 0;
}
int solve(){
    memset(pa, 0, sizeof(pa));
    memset(match, 0, sizeof(match));
    int ans = 0;
    for ( int i = 1 ; i <= n ; i++ )
        if ( !match[i] && bfs(i) ) ans++;
    return ans;
}
} graph;

```

3.5 Minimum General Weighted Matching

```

// Minimum Weight Perfect Matching (Perfect Match)

struct Graph {
    static const int MAXN = 105;
    int n, e[MAXN][MAXN];
    int match[MAXN], d[MAXN], onstk[MAXN];
    vector<int> stk;
    void init(int _n) {
        n = _n;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                e[i][j] = 0;
    }
    void add_edge(int u, int v, int w) {
        e[u][v] = e[v][u] = w;
    }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.push_back(u);
        onstk[u] = 1;
        for ( int v = 0 ; v < n ; v++ ) {
            if (u != v && match[u] != v && !onstk[v] )
            {
                int m = match[v];
                if ( d[m] > d[u] - e[v][m] + e[u][v] )
                {
                    d[m] = d[u] - e[v][m] + e[u][v];
                    onstk[v] = 1;
                    stk.push_back(v);
                    if (SPFA(m)) return true;
                    stk.pop_back();
                    onstk[v] = 0;
                }
            }
        }
        onstk[u] = 0;
        stk.pop_back();
        return false;
    }
    int solve() {
        for ( int i = 0 ; i < n ; i += 2 ) {
            match[i] = i+1;
            match[i+1] = i;
        }
    }
}

```

```

    }
    while (true){
        int found = 0;
        for ( int i = 0 ; i < n ; i++ )
            onstk[i] = d[i] = 0;
        for ( int i = 0 ; i < n ; i++ ) {
            stk.clear();
            if ( !onstk[i] && SPFA(i) ) {
                found = 1;
                while ( stk.size() >= 2 ) {
                    int u = stk.back(); stk.
                        pop_back();
                    int v = stk.back(); stk.
                        pop_back();
                    match[u] = v;
                    match[v] = u;
                }
            }
            if (!found) break;
        }
        int ret = 0;
        for ( int i = 0 ; i < n ; i++ )
            ret += e[i][match[i]];
        ret /= 2;
        return ret;
    }
} graph;

```

4 Graph

- Maximum Independent Set
 - General: [NPC] maximum clique of complement of G
 - Bipartite Graph: [P] Maximum Cardinality Bipartite Matching
 - Tree: [P] dp
- Minimum Dominating Set
 - General: [NPC]
 - Bipartite Graph: [NPC]
 - Tree: [P] DP
- Minimum Vertex Cover
 - General: [NPC] (?)maximum clique of complement of G
 - Bipartite Graph: [P] Maximum Cardinality Bipartite Matching
 - Tree: [P] Greedy, from leaf to root
- Minimum Edge Cover
 - General: [P] V - Maximum Matching
 - Bipartite Graph: [P] Greedy, strategy: cover small degree node first.
 - (Min/Max)Weighted: [P]: Minimum/Minimum Weight Matching

4.1 BCC edge

邊雙連通

任意兩點間至少有兩條不重疊的路徑連接，找法：

1. 標記出所有的橋
2. 對全圖進行 DFS，不走橋，每一次 DFS 就是一個新的邊雙連通

// from BCW

```

struct BccEdge {
    static const int MXN = 100005;
    struct Edge { int v, eid; };
    int n, m, step, par[MXN], dfn[MXN], low[MXN];
    vector<Edge> E[MXN];
    DisjointSet djs;
    void init(int _n) {
        n = _n; m = 0;
        for (int i=0; i<n; i++) E[i].clear();
        djs.init(n);
    }
    void add_edge(int u, int v) {
        E[u].PB({v, m});
    }
}

```

```

    E[v].PB({u, m});
    m++;
}
void DFS(int u, int f, int f_eid) {
    par[u] = f;
    dfn[u] = low[u] = step++;
    for (auto it:E[u]) {
        if (it.eid == f_eid) continue;
        int v = it.v;
        if (dfn[v] == -1) {
            DFS(v, u, it.eid);
            low[u] = min(low[u], low[v]);
        } else {
            low[u] = min(low[u], dfn[v]);
        }
    }
}
void solve() {
    step = 0;
    memset(dfn, -1, sizeof(int)*n);
    for (int i=0; i<n; i++) {
        if (dfn[i] == -1) DFS(i, i, -1);
    }
    djs.init(n);
    for (int i=0; i<n; i++) {
        if (low[i] < dfn[i]) djs.uni(i, par[i]);
    }
}
}graph;

```

4.2 Dijkstra

```

struct Edge{
    int v; long long len;
    bool operator < (const Edge &b) const { return len < b.len; }
};

const long long INF = 1LL<<60;

void Dijkstra(int n, vector<Edge> G[], long long d[],
    int s, int t=-1){
    static priority_queue<Edge> pq;
    while ( pq.size() )pq.pop();
    for (int i=1; i<=n; i++)d[i]=INF;
    d[s]=0; pq.push( {s,d[s]} );
    while ( pq.size() ){
        auto x = pq.top(); pq.pop();
        int u = x.v;
        if (d[u]<x.len)continue;
        if (u==t)return;
        for (auto &e:G[u]){
            if (d[e.v] > d[u]+e.len){
                d[e.v] = d[u]+e.len;
                pq.push( {e.v,d[e.v]} );
            }
        }
    }
}

```

4.3 Domination.txt

Maximum Independent Set
 General: [NPC] maximum clique of complement of G
 Tree: [P] Greedy
 Bipartite Graph: [P] Maximum Cardinality Bipartite Matching

 Minimum Dominating Set
 General: [NPC]
 Tree: [P] DP
 Bipartite Graph: [NPC]

 Minimum Vertex Cover
 General: [NPC] (?)maximum clique of complement of G
 Tree: [P] Greedy, from leaf to root
 Bipartite Graph: [P] Maximum Cardinality Bipartite Matching

 Minimum Edge Cover

General: [P] V - Maximum Matching

Bipartite Graph: [P] Greedy, strategy: cover small degree node first.

(Min/Max)Weighted: [P]: Minimum/Minimum Weight Matching

4.4 LCA

```

#define MAX 200000
vector<int> v[MAX];
int MAX_LOG, dep[MAX], par[(int)log2(MAX)+1][MAX], n,q;

void init() {
    for (int i = 1; i <= n; i++) v[i].clear();
    memset(dep, 0, sizeof(dep));
    memset(par, 0, sizeof(par));
    MAX_LOG = log2(n) + 1;
}

void dfs(int now, int fa, int d) {
    par[0][now] = fa, dep[now] = d;
    for (int i = 0; i < v[now].size(); i++)
        if (v[now][i] != fa) dfs(v[now][i], now, d + 1);
}

void build() {
    dfs(1, -1, 0);

    for (int i = 0; i < MAX_LOG; i++) {
        for (int j = 1; j <= n; j++) {
            if (par[i][j] <= 0) par[i + 1][j] = -1;
            else par[i + 1][j] = par[i][par[i][j]];
        }
    }

    int lca(int x, int y) {
        if (dep[x] > dep[y]) swap(x, y);
        for (int i = 0; i < MAX_LOG; i++)
            if ((dep[x] - dep[y]) >> i & 1) y = par[i][y];

        if (x == y) return x;

        for (int i = MAX_LOG - 1; i >= 0; i--)
            if (par[i][x] != par[i][y]) x = par[i][x], y = par[i][y];

        return par[0][x];
    }
}

```

4.5 max clique

```

const int MAXN = 105;
int best;
int n;
int num[MAXN];
int path[MAXN];
int G[MAXN][MAXN];

bool dfs( int *adj, int total, int cnt ){
    int t[MAXN];
    if (total == 0){
        if( best < cnt ){
            best = cnt;
            return true;
        }
        return false;
    }
    for(int i = 0; i < total; i++){
        if( cnt+(total-i) <= best ) return false;
        if( cnt+num[adj[i]] <= best ) return false;
        int k=0;
        for(int j=i+1; j<total; j++)
            if(G[ adj[i] ][ adj[j] ])
                t[k++] = adj[j];
        if (dfs(t, k, cnt+1)) return true;
    }
}

```

```

    }
    return false;
}
int MaximumClique(){
    int adj[MAXN];
    if (n <= 0) return 0;
    best = 0;
    for(int i = n-1; i >= 0; i--){
        int k=0;
        for(int j = i+1; j < n; j++){
            if (g[i][j]) adj[k++] = j;
            dfs( adj, k, 1 );
            num[i] = best;
        }
        return best;
    }
}

```

4.6 min mean cycle

```

// from BCW

/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
    int v,u;
    double c;
};
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
    for(int i=0; i<n; i++) d[0][i]=0;
    for(int i=0; i<n; i++) {
        fill(d[i+1], d[i+1]+n, inf);
        for(int j=0; j<m; j++) {
            int v = e[j].v, u = e[j].u;
            if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                d[i+1][u] = d[i][v]+e[j].c;
                prv[i+1][u] = v;
                prve[i+1][u] = j;
            }
        }
    }
}
double karp_mmc() {
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
        double avg=-inf;
        for(int k=0; k<n; k++) {
            if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
            else avg=max(avg,inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    for(int i=0; i<n; i++) vst[i] = 0;
    edgeID.clear(); cycle.clear(); rho.clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
}

```

4.7 SSSP related concepts

最短路問題分類：

三個工具 Bellman-Ford, Floyd, Dijkstra，

1. 可以把 Dijkstra Priority Queue 裡面存的東西想成「狀態」，他可以拿來統計甚至轉移。
2. 當遇到邊權會扣掉走的人的血量（或油量之類的），當不能有負值的時候，就要使用 Bellman-Ford 來做，一開始可以把起點設為最初的血量（油量），拿去做 Bellman-Ford，當做了 $n-1$ 次之後，還能轉移，那就是有負環或正環（端看如何轉移 Bellman-Ford，這部分的轉移式很自由可以依照題目敘述亂改。）
3. 特別注意如果要判到某一個點的長度是不是無限小，可在做了 $n-1$ 次之後，發現 $u \rightarrow v$ 可以更新，那我可以去看 v 是否可以到另一點 k ，如果是聯通的，代表 k 這個點的長度是無限小。

4.8 Tarjan.cpp

割點

點 u 為割點 **if and only if** 滿足 1. **or** 2.

1. u 為樹根，且 u 有多於一個子樹。
2. u 不為樹根，且滿足存在 (u,v) 為樹枝邊（或稱父子邊，即 u 為 v 在搜索樹中的父親），使得 $DFN(u) \leq Low(v)$ 。

橋

一條無向邊 (u,v) 是橋 **if and only if** (u,v) 為樹枝邊，且滿足 $DFN(u) < Low(v)$ 。

// 0 base

```

struct TarjanSCC{
    static const int MAXN = 1000006;
    int n, dfn[MAXN], low[MAXN], scc[MAXN], scn, count;
    vector<int> G[MAXN];
    stack<int> stk;
    bool ins[MAXN];

    void tarjan(int u){
        dfn[u] = low[u] = ++count;
        stk.push(u);
        ins[u] = true;

        for(auto v:G[u]){
            if(!dfn[v]){
                tarjan(v);
                low[u] = min(low[u], low[v]);
            }else if(ins[v]){
                low[u] = min(low[u], dfn[v]);
            }
        }

        if(dfn[u] == low[u]){
            int v;
            do {
                v = stk.top();
                stk.pop();
                scc[v] = scn;
                ins[v] = false;
            } while(v != u);
            scn++;
        }
    }

    void getSCC(){
        memset(dfn,0,sizeof(dfn));
        memset(low,0,sizeof(low));
        memset(ins,0,sizeof(ins));
        memset(scc,0,sizeof(scc));
        count = scn = 0;
        for(int i = 0 ; i < n ; i++) {
            if(!dfn[i]) tarjan(i);
        }
    }
}

```



```
}SCC;
```

4.9 2-SAT

```
const int MAXN = 2020;

struct TwoSAT{
    static const int MAXv = 2*MAXN;
    vector<int> GO[MAXv],BK[MAXv],stk;
    bool vis[MAXv];
    int SC[MAXv];

    void imply(int u,int v){ // u imply v
        GO[u].push_back(v);
        BK[v].push_back(u);
    }
    int dfs(int u,vector<int>*G,int sc){
        vis[u]=1, SC[u]=sc;
        for (int v:G[u])if (!vis[v])
            dfs(v,G,sc);
        if (G==GO)stk.push_back(u);
    }
    int scc(int n=MAXv){
        memset(vis,0,sizeof(vis));
        for (int i=0; i<n; i++)if (!vis[i])
            dfs(i,GO,-1);
        memset(vis,0,sizeof(vis));
        int sc=0;
        while (!stk.empty()){
            if (!vis[stk.back()])
                dfs(stk.back(),BK,sc++);
            stk.pop_back();
        }
    }
}SAT;

int main(){
    SAT.scc(2*n);
    bool ok=1;
    for (int i=0; i<n; i++){
        if (SAT.SC[2*i]==SAT.SC[2*i+1])ok=0;
    }
    if (ok){
        for (int i=0; i<n; i++){
            if (SAT.SC[2*i]>SAT.SC[2*i+1]){
                cout << i << endl;
            }
        }
    }
    else puts("NO");
}

void warshall(){
    bitset<2003> d[2003];
    for (int k=0; k<n; k++){
        for (int i=0; i<n; i++) if (d[i][k]) {
            d[i] |= d[k];
        }
    }
}

}
```

4.10 平面圖判定

```
//skydog
#include <bits/stdc++.h>
using namespace std;

typedef long long ll;
typedef pair<int, int> ii;
typedef pair<ll, ll> l4;

#define mp make_pair
#define pb push_back
#define debug(x) cerr << #x << " = " << x << " "

const int N=400+1;
```

```
struct Planar{
    int n,m,hash[N],fa[N],deep[N],low[N],ecp[N];
    vector<int> g[N],son[N];
    set< pair<int,int> > SDlist[N],proots[N];
    int nxt[N][2],back[N],rev[N];
    deque<int> q;
    void dfs(int u){
        hash[u]=1; q.pb(u);
        ecp[u]=low[u]=deep[u];
        int v;
        for (int i = 0; i < g[u].size(); ++i)
            if(!hash[v=g[u][i]]){
                fa[v]=u;
                deep[v]=deep[u]+1;
                dfs(v);
                low[u]=min(low[u],low[v]);
                SDlist[u].insert(mp(low[v],v));
            }
            else ecp[u]=min(ecp[u],deep[v]);
        low[u]=min(low[u],ecp[u]);
    }

    int visited[N];

    void addtree(int u,int t1,int v,int t2){
        nxt[u][t1]=v; nxt[v][t2]=u;
    }

    void findnxt(int u,int v,int& u1,int& v1){
        u1=nxt[u][v^1];
        if(nxt[u1][0]==u) v1=0;
        else v1=1;
    }

    void walkup(int u,int v){
        back[v]=u;
        int v1=v,v2=v,u1=1,u2=0,z;
        for (;;){
            if(hash[v1]==u || hash[v2]==u) break;
            hash[v1]=u;hash[v2]=u; z=max(v1,v2);
            if(z>n){
                int p=fa[z-n];
                if(p!=u){
                    proots[p].insert(mp(-low[z-n], z));
                    v1=p,v2=p,u1=0,u2=1;
                }
                else break;
            }
            else{
                findnxt(v1,u1,v1,u1);
                findnxt(v2,u2,v2,u2);
            }
        }
    }

    int topstack;
    pair<int,int> stack[N];

    int outer(int u,int v){
        return ecp[v]<deep[u] || (SDlist[v].size() &&
            SDlist[v].begin()->first<deep[u]);
    }

    int inside(int u,int v){
        return proots[v].size()>0 || back[v]==u;
    }

    int active(int u,int v){
        return inside(u,v) || outer(u,v);
    }

    void push(int a,int b){
        stack[++topstack]=mp(a,b);
    }

    void mergestack(){
        int v1,t1,v2,t2,s,s1;
        v1=stack[topstack].first;t1=stack[topstack].second;
        topstack--;
        v2=stack[topstack].first;t2=stack[topstack].second;
```



```

    topstack--;

    s=nxt[v1][t1^1];
    s1=(nxt[s][1]==v1);
    nxt[s][s1]=v2;
    nxt[v2][t2]=s;

    SDlist[v2].erase( make_pair(low[v1-n],v1-n) );
    proots[v2].erase( make_pair(-low[v1-n],v1) );
}

void findnxtActive(int u,int t,int& v,int& w1,int S
){
    findnxt(u,t,v,w1);
    while(u!=v && !active(S,v))
        findnxt(v,w1,v,w1);
}

void walkdown(int S,int u){
    topstack=0;
    int t1,v=S,w1,x2,y2,x1,y1,p;
    for(t1=0;t1<2;++t1){
        findnxt(S,t1^1,v,w1);
        while(v!=S){
            if(back[v]==u){
                while(topstack>0) mergestack();
                addtree(S,t1,v,w1); back[v]=0;
            }
            if(proots[v].size()){
                push(v,w1);
                p=proots[v].begin()->second;
                findnxtActive(p,1,x1,y1,u);
                findnxtActive(p,0,x2,y2,u);
                if(active(u,x1) && !outer(u,x1))
                    v=x1,w1=y1;
                else if(active(u,x2) && !outer(u,x2))
                    v=x2,w1=y2;
                else if(inside(u,x1) || back[x1]==u)
                    v=x1,w1=y1;
                else v=x2,w1=y2;
                push(p,v==x2);
            }
            else if(v>n || (ecp[v]>=deep[u] && !
                outer(u,v) ))
                findnxt(v,w1,v,w1);
            else if(v<=n && outer(u,v) && !topstack)
                addtree(S,t1,v,w1); break;
            else break;
        }
    }
}

int work(int u){
    int v;
    for (int i = 0; i < g[u].size(); ++i)
        if(fa[v=g[u][i]]==u){
            son[u].push_back(n+v);
            proots[n+v].clear();
            addtree(n+v,1,v,0);
            addtree(n+v,0,v,1);
        }
    for (int i = 0; i < g[u].size(); ++i)
        if(deep[v=g[u][i]]>deep[u]+1)
            walkup(u,v);
    topstack=0;
    for (int i = 0; i < son[u].size(); ++i)
        walkdown(son[u][i], u);
    for (int i = 0; i < g[u].size(); ++i)
        if(deep[v=g[u][i]]>deep[u]+1 && back[v])
            return 0;
    return 1;
}

void init(int _n){
    n = _n;
    m = 0;
    for(int i=1;i<=2*n;++i){
        g[i].clear();
        SDlist[i].clear();
        son[i].clear();
        proots[i].clear();
        nxt[i][0]=nxt[i][1]=0;
        fa[i]=0;
        hash[i]=0;low[i]=ecp[i]=deep[i]=back[i]=0;
        q.clear();
    }
}

void add(int u, int v){
    ++m;
    g[u].pb(v); g[v].pb(u);
}

bool check_planar(){
    if(m>3*n-5) return false;
    for(int i=1;i<=n;++i)
        if(!hash[i]){
            deep[i]=1;
            dfs(i);
        }
    memset(hash,0,sizeof(hash));
    // originally only looks at last n element
    assert(q.size() == n);
    while (!q.empty()){
        if (!work(q.back()))
            return false;
        q.pop_back();
    }
    return true;
}

} base, _new;
vector<ii> edges;
int n, m;
inline void build(int n, Planar &_new){
    _new.init(n);
    for (auto e : edges)
        _new.add(e.first, e.second);
}

void end(){
    puts("-1");
    exit(0);
}

bool vis[N];
const int maxp = 5;
int path[maxp], tp=0;
void dfs(int cur){
    vis[cur] = true;
    path[tp++] = cur;
    if (tp == maxp){
        auto it = lower_bound(base.g[cur].begin(), base.g[
            cur].end(), path[0]);
        if (it != base.g[cur].end() && *it == path[0])
        {
            //a cycle
            int x = n+1;
            for (int i = 0; i < 5; ++i) edges.pb(mp(x,
                path[i]));
            build(x, _new);
            if (_new.check_planar()){
                for (int i = 0; i < maxp; ++i) printf("
                    %d%c", path[i], i==maxp-1?'\\n':' ')
                    ;
                exit(0);
            }
            for (int i = 0; i < 5; ++i) edges.pop_back
                ();
        }
    }
    else{
        for (auto e : base.g[cur]) if (!vis[e]) dfs(e);
    }
    vis[cur] = false;
    --tp;
}

int main(){
    scanf("%d %d", &n, &m);
    if (n <= 4) {
        assert(false);
        puts("0");
        return 0;
    }
    for (int i = 0; i < m; ++i){

```

```

    int u, v; scanf("%d %d", &u, &v);
    edges.pb(mp(u, v));
}
build(n, base);
if (!base.check_planar()) end();
for (int i = 1; i <= n; ++i)
    sort(base.g[i].begin(), base.g[i].end());
for (int i = 1; i <= n; ++i)
    dfs(i);
end();
}

```

5 Math

- Stirling number of second kind
 $S(n, m)$: n 個相異球，放到 m 個相同的箱子，每個箱子至少 1
 $= m \times S(n-1, m) + S(n-1, m-1)$
 $= \frac{1}{m!} \sum_{j=0}^m \binom{m}{j} (m-j)^n (-1)^j$
- Stirling number of first kind
 $s(n, m)$: n 個相異球，分配到 m 個有向環，每個環至少 1
 $s(n+1, m) = n \times s(n, m) + s(n, m-1)$
 $s(n, m) \equiv \binom{[n/2]}{m-[n/2]} \pmod{2}$
- Pick's Theorem (Bangkok regional 2016 pD)
 多邊形頂點都在整數點上
 多邊形面積 = 內部整數點個數 + 邊上格子點個數/2 - 1
 $A = i + b/2 - 1$

5.1 $ax+by=\gcd(a,b)$

```

pair<int,int> extgcd(int a, int b){
    if (b==0) return {1,0};
    int k = a/b;
    pair<int,int> p = extgcd(b,a-k*b);
    return { p.second, p.first - k*p.second };
}

```

5.2 FFT

```

#define MAXN 262144
#define cplx complex<long double>
const long double PI = acos(-1);
const cplx I(0, 1);

cplx w[MAXN];

void pre_fft() {
    for (int i = 0; i < MAXN; i++)
        w[i] = exp(PI * i * 2 / MAXN * I);
}

int reverse_add(int x) {
    for (int l = (1 << 17); (x ^ l) < 1; l >>= 1);
    return x;
}

void bit_reverse(cplx a[], int n) {
    for (int i = 0, j = 0; i < n; i++) {
        if (i > j) swap(a[i], a[j]);
        j = reverse_add(j);
    }
}

void fft(cplx a[], int n) {
    bit_reverse(a, n);
    for (int i = 2; i <= n; i <= 1) {
        int m = i >> 1;
        for (int j = 0; j < n; j += i) {
            for (int k = 0; k < m; k++) {
                cplx z = w[n / i * k] * a[j + m + k];
                a[j + m + k] = a[j + k] - z;
                a[j + k] += z;
            }
        }
    }
}

void ifft(cplx a[], int n) {

```

```

    fft(a, n);
    vector<cplx> v(n);
    for (int i = 0; i < n; i++) v[i] = a[(n - i) % n] / (
        long double) n;
    for (int i = 0; i < n; i++) a[i] = v[i];
}

cplx g[MAXN], h[MAXN], x[MAXN];
int ans[MAXN], r;

void init() {
    memset(ans, 0, sizeof(ans));
    memset(g, 0, sizeof(g));
    memset(h, 0, sizeof(h));
    memset(x, 0, sizeof(x));
    r = 0;
}

int main() {
    cin.tie(0), cout.sync_with_stdio(false);
    pre_fft();
    string s1, s2;
    while (cin >> s1 >> s2) {
        init();
        for (int i = 0; i < s1.length(); i++)
            g[s1.length() - i - 1] = cplx(s1[i] - '0', 0);
        for (int i = 0; i < s2.length(); i++)
            h[s2.length() - i - 1] = cplx(s2[i] - '0', 0);

        fft(g, MAXN);
        fft(h, MAXN);

        for (int i = 0; i < MAXN; i++)
            x[i] = g[i] * h[i];

        ifft(x, MAXN);

        for (int i = 0; i < MAXN; i++)
            ans[i] = (int)(real(x[i]) + 0.5);

        for (int i = 0; i < MAXN; i++) {
            ans[i] = (ans[i] + r);
            r = ans[i] / 10;
            ans[i] %= 10;
        }

        int pos = MAXN - 1;
        while (pos > 0 && ans[pos] == 0) pos--;
        for (int i = pos; i >= 0; i--)
            cout << ans[i];
        cout << '\n';
    }
    return 0;
}

```

5.3 NTT

```

// Remember coefficient are mod P
// {n, 2^n, p, a, root} Note: p = a*2^n+1
// {16, 65536, 65537, 1, 3}
// {20, 1048576, 7340033, 7, 3}

template < LL P, LL root, int MAXN > // (must be 2^k)
struct NTT {
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
            if (b & 1) res = (res * bs) % P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if (a == 1) return 1;
        return (((LL)(a - inv(b % a, a)) * b + 1) / a)
            % b;
    }
    LL omega[MAXN + 1];
    NTT() {
        omega[0] = 1;
        LL r = bigmod(root, (P - 1) / MAXN);
        for (int i = 1; i <= MAXN; i++)

```

```

        omega[i] = (omega[i - 1] * r) % P;
    }
    // n must be 2^k
    void tran(int n, LL a[], bool inv_ntt = false) {
        int basic = MAXN / n, theta = basic;
        for (int m = n; m >= 2; m >= 1) {
            int mh = m >> 1;
            for (int i = 0; i < mh; i++) {
                LL w = omega[i * theta % MAXN];
                for (int j = i; j < n; j += m) {
                    int k = j + mh;
                    LL x = a[j] - a[k];
                    if (x < 0) x += P;
                    a[j] += a[k];
                    if (a[j] > P) a[j] -= P;
                    a[k] = (w * x) % P;
                }
            }
            theta = (theta * 2) % MAXN;
        }
        int i = 0;
        for (int j = 1; j < n - 1; j++) {
            for (int k = n >> 1; k > (i ^ k); k >= 1)
                ;
            if (j < i) swap(a[i], a[j]);
        }
        if (inv_ntt) {
            LL ni = inv(n, P);
            reverse(a + 1, a + n);
            for (i = 0; i < n; i++)
                a[i] = (a[i] * ni) % P;
        }
    }
};
const LL P=2013265921, root=31;
const int MAXN=4194304; // MAXN 的因數也可以跑
NTT<P, root, MAXN> ntt;

```

5.4 GaussElimination

```

// by bcw_codebook

const int MAXN = 300;
const double EPS = 1e-8;

int n;
double A[MAXN][MAXN];

void Gauss() {
    for(int i = 0; i < n; i++) {
        bool ok = 0;
        for(int j = i; j < n; j++) {
            if(fabs(A[j][i]) > EPS) {
                swap(A[j], A[i]);
                ok = 1;
                break;
            }
        }
        if(!ok) continue;

        double fs = A[i][i];
        for(int j = i+1; j < n; j++) {
            double r = A[j][i] / fs;
            for(int k = i; k < n; k++) {
                A[j][k] -= A[i][k] * r;
            }
        }
    }
}

```

5.5 inverse

```

const int MAXN = 1000006;
int inv[MAXN];
void invTable(int bound, int p){
    inv[1] = 1;
    for (int i=2; i<bound; i++){

```

```

        inv[i] = (long long)inv[p%i] * (p-p/i) %p;
    }
}

int inv(int b, int p){
    if (b==1) return 1;
    return (long long)inv(p%b,p) * (p-p/b) %p;
}

```

5.6 Miller-Rabin

```

typedef long long LL;

inline LL bin_mul(LL a, LL n, const LL& MOD){
    LL re=0;
    while (n>0){
        if (n&1) re += a;
        a += a; if (a>=MOD) a-=MOD;
        n>>=1;
    }
    return re%MOD;
}

inline LL bin_pow(LL a, LL n, const LL& MOD){
    LL re=1;
    while (n>0){
        if (n&1) re = bin_mul(re,a,MOD);
        a = bin_mul(a,a,MOD);
        n>>=1;
    }
    return re;
}

bool is_prime(LL n){
    //static LL sprp[3] = { 2LL, 7LL, 61LL};
    static LL sprp[7] = { 2LL, 325LL, 9375LL,
        28178LL, 450775LL, 9780504LL,
        1795265022LL };
    if (n==1 || (n&1)==0 ) return n==2;
    int u=n-1, t=0;
    while ( (u&1)==0 ) u>>=1, t++;
    for (int i=0; i<3; i++){
        LL x = bin_pow( sprp[i]%n, u, n);
        if (x==0 || x==1 || x==n-1)continue;

        for (int j=1; j<t; j++){
            x=x*x%n;
            if (x==1 || x==n-1)break;
        }
        if (x==n-1)continue;
        return 0;
    }
    return 1;
}

```

5.7 Mobius

```

void mobius() {
    fill(isPrime, isPrime + MAXN, 1);
    mu[1] = 1, num = 0;
    for (int i = 2; i < MAXN; ++i) {
        if (isPrime[i]) primes[num++] = i, mu[i] = -1;
        static int d;
        for (int j = 0; j < num && (d = i * primes[j])
            < MAXN; ++j) {
            isPrime[d] = false;
            if (i % primes[j] == 0) {
                mu[d] = 0; break;
            } else mu[d] = -mu[i];
        }
    }
}

```

5.8 pollardRho

```
// from PEC
// does not work when n is prime
Int f(Int x, Int mod){
    return add(mul(x, x, mod), 1, mod);
}
Int pollard_rho(Int n) {
    if ( !(n & 1) ) return 2;
    while (true) {
        Int y = 2, x = rand()%(n-1) + 1, res = 1;
        for ( int sz = 2 ; res == 1 ; sz *= 2 ) {
            for ( int i = 0 ; i < sz && res <= 1 ; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if ( res != 0 && res != n ) return res;
    }
}
```

5.9 SG

```
#include<bits/stdc++.h>
using namespace std;

int mex(set S) {
    // find the min number >= 0 that not in the S
    // e.g. S = {0, 1, 3, 4} mex(S) = 2
}

state = []
int SG(A) {
    if (A not in state) {
        S = sub_states(A)
        if( len(S) > 1 ) state[A] = reduce(operator.xor, [
            SG(B) for B in S])
        else state[A] = mex(set(SG(B) for B in next_states(
            A)))
    }
    return state[A]
}
```

5.10 theorem

```
/*
Lucas's Theorem
For non-negative integer n,m and prime P,
 $C(m,n) \bmod P = C(m/M,n/M) * C(m\%M,n\%M) \bmod P$ 
= mult_i ( C(m_i,n_i) )
where m_i is the i-th digit of m in base P.
-----
Kirchhoff's theorem
 $A_{ii} = \deg(i)$ ,  $A_{ij} = (i,j) \in E ? -1 : 0$ 
Deleting any one row, one column, and cal the det(A)
-----
Nth Catalan recursive function:
 $C_0 = 1$ ,  $C_{n+1} = C_n * 2(2n+1)/(n+2)$ 
-----
Mobius Formula
 $u(n) = 1$ , if  $n = 1$ 
 $(-1)^m$ , 若  $n$  無平方數因數, 且  $n = p_1 * p_2 * p_3 * \dots * p_k$ 
 $0$ , 若  $n$  有大於 1 的平方數因數
- Property
1. (積性函數)  $u(a)u(b) = u(ab)$ 
2.  $\sum_{d|n} u(d) = [n == 1]$ 
-----
Mobius Inversion Formula
if  $f(n) = \sum_{d|n} g(d)$ 
then  $g(n) = \sum_{d|n} u(n/d)f(d)$ 
 $= \sum_{d|n} u(d)f(n/d)$ 
- Application
the number/power of gcd(i, j) = k
- Trick
分塊,  $O(\sqrt{n})$ 
-----
Chinese Remainder Theorem ( $m_i$  兩兩互質)
```

```
x = a_1 (mod m_1)
x = a_2 (mod m_2)
....
x = a_i (mod m_i)
```

construct a solution:

```
Let  $M = m_1 * m_2 * m_3 * \dots * m_n$ 
Let  $M_i = M / m_i$ 
```

```
t_i = 1 / M_i
t_i * M_i = 1 (mod m_i)
```

```
solution  $x = a_1 * t_1 * M_1 + a_2 * t_2 * M_2 + \dots$ 
 $+ a_n * t_n * M_n + k * M$ 
 $= k * M + \sum a_i * t_i * M_i$ , k is positive integer.
```

```
under mod M, there is one solution  $x = \sum a_i * t_i * M_i$ 
```

Burnside's Lemma

$|G| * |X/G| = \sum (|X^g|)$ where g in G

總方法數：每一種旋轉下不動點的個數總和 除以 旋轉的方法數

*/

6 Geometry

6.1 2D point template

```
typedef double Double;
struct Point {
    Double x,y;

    bool operator < (const Point &b)const{
        //return tie(x,y) < tie(b.x,b.y);
        //return atan2(y,x) < atan2(b.y,b.x);
        assert(0 && "choose compare");
    }
    Point operator + (const Point &b)const{
        return {x+b.x,y+b.y};
    }
    Point operator - (const Point &b)const{
        return {x-b.x,y-b.y};
    }
    Point operator * (const Double &d)const{
        return {d*x,d*y};
    }
    Point operator / (const Double &d)const{
        return {x/d,y/d};
    }
    Double operator * (const Point &b)const{
        return x*b.x + y*b.y;
    }
    Double operator % (const Point &b)const{
        return x*b.y - y*b.x;
    }
    friend Double abs2(const Point &p){
        return p.x*p.x + p.y*p.y;
    }
    friend Double abs(const Point &p){
        return sqrt( abs2(p) );
    }
};
typedef Point Vector;

struct Line{
    Point P; Vector v;
    bool operator < (const Line &b)const{
        return atan2(v.y,v.x) < atan2(b.v.y,b.v.x);
    }
};
```

6.2 circumcentre

```
#include "2Dpoint.cpp"
```

```
Point circumcentre(Point &p0, Point &p1, Point &p2){
    Point a = p1-p0;
    Point b = p2-p0;
    Double c1 = abs2(a)*0.5;
    Double c2 = abs2(b)*0.5;
    Double d = a % b;
    Double x = p0.x + ( c1*b.y - c2*a.y ) / d;
    Double y = p0.y + ( c2*a.x - c1*b.x ) / d;
    return {x,y};
}
```

6.3 ConvexHull

```
#include "2Dpoint.cpp"

// return H, 第一個點會在 H 出現兩次
void ConvexHull(vector<Point> &P, vector<Point> &H){
    int n = P.size(), m=0;
    sort(P.begin(),P.end());
    H.clear();

    for (int i=0; i<n; i++){
        while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
            <0)H.pop_back(), m--;
        H.push_back(P[i]), m++;
    }

    for (int i=n-2; i>=0; i--){
        while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
            <0)H.pop_back(), m--;
        H.push_back(P[i]), m++;
    }
}
```

6.4 3D ConvexHull

```
// return the faces with pt indexes
int flag[MXN][MXN];
struct Point{
    ld x,y,z;
    Point operator - (const Point &b) const {
        return (Point){x-b.x,y-b.y,z-b.z};
    }
    Point operator * (const ld &b) const {
        return (Point){x*b,y*b,z*b};
    }
    ld len() const { return sqrt1(x*x+y*y+z*z); }
    ld dot(const Point &a) const {
        return x*a.x+y*a.y+z*a.z;
    }
    Point operator * (const Point &b) const {
        return (Point){y*b.z-b.y*z,z*b.x-b.z*x,x*b.y-b.x*y};
    }
};
Point ver(Point a, Point b, Point c) {
    return (b - a) * (c - a);
}
vector<Face> convex_hull_3D(const vector<Point> pt) {
    int n = SZ(pt);
    REP(i,n) REP(j,n)
        flag[i][j] = 0;

    vector<Face> now;
    now.push_back((Face){0,1,2});
    now.push_back((Face){2,1,0});
    int ftop = 0;
    for (int i=3; i<n; i++){
        ftop++;
        vector<Face> next;
        REP(j, SZ(now)) {
            Face& f=now[j];
            ld d=(pt[i]-pt[f.a]).dot(ver(pt[f.a], pt[f.b], pt
                [f.c]));
            if (d <= 0) next.push_back(f);
            int ff = 0;
            if (d > 0) ff=ftop;

```

```
        else if (d < 0) ff=-ftop;
        flag[f.a][f.b] = flag[f.b][f.c] = flag[f.c][f.a]
            = ff;
    }
    REP(j, SZ(now)) {
        Face& f=now[j];
        if (flag[f.a][f.b] > 0 and flag[f.a][f.b] != flag
            [f.b][f.a])
            next.push_back((Face){f.a,f.b,i});
        if (flag[f.b][f.c] > 0 and flag[f.b][f.c] != flag
            [f.c][f.b])
            next.push_back((Face){f.b,f.c,i});
        if (flag[f.c][f.a] > 0 and flag[f.c][f.a] != flag
            [f.a][f.c])
            next.push_back((Face){f.c,f.a,i});
    }
    now=next;
}
return now;
}
```

6.5 half plane intersection

```
bool OnLeft(const Line& L,const Point& p){
    return Cross(L.v,p-L.P)>0;
}
Point GetIntersection(Line a,Line b){
    Vector u = a.P-b.P;
    Double t = Cross(b.v,u)/Cross(a.v,b.v);
    return a.P + a.v*t;
}
int HalfplaneIntersection(Line* L,int n,Point* poly){
    sort(L,L+n);

    int first,last;
    Point *p = new Point[n];
    Line *q = new Line[n];
    q[first=last=0] = L[0];
    for(int i=1;i<n;i++){
        while(first < last && !OnLeft(L[i],p[last-1])) last
            --;
        while(first < last && !OnLeft(L[i],p[first])) first
            ++;
        q[++last]=L[i];
        if(fabs(Cross(q[last].v,q[last-1].v))<EPS){
            last--;
            if(OnLeft(q[last],L[i].P)) q[last]=L[i];
        }
        if(first < last) p[last-1]=GetIntersection(q[last-1],q[last]);
    }
    while(first<last && !OnLeft(q[first],p[last-1])) last
        --;
    if(last-first<=1) return 0;
    p[last]=GetIntersection(q[last],q[first]);

    int m=0;
    for(int i=first;i<=last;i++) poly[m++]=p[i];
    return m;
}
```

6.6 Intersection of two circle

```
vector<Point> interCircle(Point o1, Double r1, Point o2
    , Double r2) {
    Double d2 = abs2(o1 - o2);
    Double d = sqrt(d2);
    Point u = (o1+o2)*0.5 + (o1-o2)*(r2*r2-r1*r1)/(2.0*d2
        );
    if (abs((r1+r2)*(r1+r2) - d2) < 1e-6) return {u};
    if (d < fabs(r1-r2) || r1+r2 < d) return {};
    Double A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d) *
        (-r1+r2+d));
    Point v = Point{o1.y-o2.y, -o1.x+o2.x} * A / (2.0*d2)
        ;
    return {u+v, u-v};
}
```

6.7 Intersection of two lines

```
Point interPnt(Point p1, Point p2, Point q1, Point q2,
    bool &res){
    Double f1 = cross(p2, q1, p1);
    Double f2 = -cross(p2, q2, p1);
    Double f = (f1 + f2);

    if(fabs(f) < EPS) {
        res = false;
        return {};
    }

    res = true;
    return (f2 / f) * q1 + (f1 / f) * q2;
}
```

6.8 Smallest Circle

```
#include "circumcentre.cpp"
pair<Point,Double> SmallestCircle(int n, Point _p[]){
    Point *p = new Point[n];
    memcpy(p,_p,sizeof(Point)*n);
    random_shuffle(p,p+n);

    Double r2=0;
    Point cen;
    for (int i=0; i<n; i++){
        if ( abs2(cen-p[i]) <= r2)continue;
        cen = p[i], r2=0;
        for (int j=0; j<i; j++){
            if ( abs2(cen-p[j]) <= r2)continue;
            cen = (p[i]+p[j])*0.5;
            r2 = abs2(cen-p[i]);
            for (int k=0; k<j; k++){
                if ( abs2(cen-p[k]) <= r2)continue;
                cen = circumcentre(p[i],p[j],p[k]);
                r2 = abs2(cen-p[k]);
            }
        }
    }

    delete[] p;
    return {cen,r2};
}
// auto res = SmallestCircle(,);
```

7 String

7.1 AC automaton

```
// remember make_fail() !!!
// notice MLE

const int sigma = 62;
const int MAXC = 200005;

inline int idx(char c){
    if ('A'<= c && c <= 'Z')return c-'A';
    if ('a'<= c && c <= 'z')return c-'a' + 26;
    if ('0'<= c && c <= '9')return c-'0' + 52;
    assert(false);
}

struct ACautomaton{
    struct Node{
        Node *next[sigma], *fail;
        int cnt; // dp
        Node() : next{}, fail{}, cnt{}{}
    } buf[MAXC], *bufp, *ori, *root;

    void init(){
        bufp = buf;
        ori = new (bufp++) Node();
        root = new (bufp++) Node();
    }

    void insert(char *s){
```

```
Node *ptr = root;
for (int i=0; s[i]; i++){
    int c = idx(s[i]);
    if (!ptr->next[c])
        ptr->next[c] = new (bufp++) Node();
    ptr = ptr->next[c];
}
ptr->cnt=1;

Node* trans(Node *o, int c){
    while (!o->next[c]) o = o->fail;
    return o->next[c];
}

void make_fail(){
    static queue<Node*> que;

    for (int i=0; i<sigma; i++)
        ori->next[i] = root;
    root->fail = ori;

    que.push(root);
    while ( ! que.size() ){
        Node *u = que.front(); que.pop();
        for (int i=0; i<sigma; i++){
            if (!u->next[i])continue;
            u->next[i]->fail = trans(u->fail,i);
            que.push(u->next[i]);
        }
        u->cnt += u->fail->cnt;
    }
} ac;
```

7.2 KMP

```
template<typename T>
void build_KMP(int n, T *s, int *f){ // 1 base
    f[0]=-1, f[1]=0;
    for (int i=2; i<=n; i++){
        int w = f[i-1];
        while (w>=0 && s[w+1]!=s[i])w = f[w];
        f[i]=w+1;
    }
}

template<typename T>
int KMP(int n, T *a, int m, T *b){
    build_KMP(m,b,f);
    int ans=0;

    for (int i=1, w=0; i<=n; i++){
        while ( w>=0 && b[w+1]!=a[i] )w = f[w];
        w++;
        if (w==m){
            ans++;
            w=f[w];
        }
    }
    return ans;
}
```

7.3 palindromic tree

```
// remember init() !!!
// remember make_fail() !!!
// insert s need 1 base !!!
// notice MLE
const int sigma = 62;
const int MAXC = 1000006;
inline int idx(char c){
    if ('a'<= c && c <= 'z')return c-'a';
    if ('A'<= c && c <= 'Z')return c-'A'+26;
    if ('0'<= c && c <= '9')return c-'0'+52;
}

struct PalindromicTree{
    struct Node{
```

```

    Node *next[sigma], *fail;
    int len, cnt; // for dp
    Node(){
        memset(next,0,sizeof(next));
        fail=0;
        len = cnt = 0;
    }
} buf[MAXC], *bufp, *even, *odd;

void init(){
    bufp = buf;
    even = new (bufp++) Node();
    odd = new (bufp++) Node();
    even->fail = odd;
    odd->len = -1;
}

void insert(char *s){
    Node* ptr = even;
    for (int i=1; s[i]; i++){
        ptr = extend(ptr,s+i);
    }
}

Node* extend(Node *o, char *ptr){
    int c = idx(*ptr);
    while ( *ptr != *(ptr-1-o->len) )o=o->fail;
    Node *np = o->next[c];
    if (!np){
        np = new (bufp++) Node();
        np->len = o->len+2;
        Node *f = o->fail;
        if (f){
            while ( *ptr != *(ptr-1-f->len) )f=f->fail;
            np->fail = f->next[c];
        }
        else {
            np->fail = even;
        }
        np->cnt = np->fail->cnt;
    }
    np->cnt++;
    return np;
}
} PAM;

```

7.4 SAM

// par : fail link
 // val : a topological order (useful for DP)
 // go[x] : automata edge (x is integer in [0,26))

```

struct SAM{
    struct State{
        int par, go[26], val;
        State () : par(0), val(0){ FZ(go); }
        State (int _val) : par(0), val(_val){ FZ(go); }
    };
    vector<State> vec;
    int root, tail;

    void init(int arr[], int len){
        vec.resize(2);
        vec[0] = vec[1] = State(0);
        root = tail = 1;
        for (int i=0; i<len; i++)
            extend(arr[i]);
    }
    void extend(int w){
        int p = tail, np = vec.size();
        vec.pb(State(vec[p].val+1));
        for ( ; p && vec[p].go[w]==0; p=vec[p].par)
            vec[p].go[w] = np;
        if (p == 0){
            vec[np].par = root;
        } else {
            if (vec[vec[p].go[w]].val == vec[p].val+1){
                vec[np].par = vec[p].go[w];
            } else {

```

```

                int q = vec[p].go[w], r = vec.size();
                vec.pb(vec[q]);
                vec[r].val = vec[p].val+1;
                vec[q].par = vec[np].par = r;
                for ( ; p && vec[p].go[w] == q; p=vec[p].par)
                    vec[p].go[w] = r;
            }
        }
        tail = np;
    }
};

```

7.5 smallest rotation

```

string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

Contact GitHub API Training Shop Blog About

7.6 suffix array

*/*he[i]保存了在后缀数组中相邻两个后缀的最长公共前缀长度
 *sa[i]表示的是字典序排名为i的后缀是谁 (字典序越小的排名越靠前)
 *rk[i]表示的是后缀我所对应的排名是多少 */*

```

const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX];
int sa[MAX], tsa[MAX], tp[MAX][2];
void suffix_array(char *ip){
    int len = strlen(ip);
    int alp = 256;
    memset(ct, 0, sizeof(ct));
    for(int i=0;i<len;i++) ct[ip[i]+1]++;
    for(int i=1;i<alp;i++) ct[i]+=ct[i-1];
    for(int i=0;i<len;i++) rk[i]=ct[ip[i]];
    for(int i=1;i<len;i*=2){
        for(int j=0;j<len;j++){
            if(j+i>=len) tp[j][1]=0;
            else tp[j][1]=rk[j+i]+1;
            tp[j][0]=rk[j];
        }
        memset(ct, 0, sizeof(ct));
        for(int j=0;j<len;j++) ct[tp[j][1]+1]++;
        for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];
        for(int j=0;j<len;j++) tsa[ct[tp[j][1]]+1]=j;
        memset(ct, 0, sizeof(ct));
        for(int j=0;j<len;j++) ct[tp[j][0]+1]++;
        for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];
        for(int j=0;j<len;j++){
            sa[ct[tp[tsa[j]][0]]+1]=tsa[j];
            rk[sa[0]]=0;
            for(int j=1;j<len;j++){
                if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
                    tp[sa[j]][1] == tp[sa[j-1]][1] )
                    rk[sa[j]] = rk[sa[j-1]];
                else
                    rk[sa[j]] = j;
            }
        }
        for(int i=0,h=0;i<len;i++){
            if(rk[i]==0) h=0;
            else{
                int j=sa[rk[i]-1];
                h=max(0,h-1);

```



```

    for(;ip[i+h]==ip[j+h];h++);
  }
  he[rk[i]]=h;
}
}

```

7.7 Z value

```

z[0] = 0;
for ( int bst = 0, i = 1; i < len ; i++ ) {
    if ( z[bst] + bst <= i ) z[i] = 0;
    else z[i] = min(z[i - bst], z[bst] + bst - i);
    while ( str[i + z[i]] == str[z[i]] ) z[i]++;
    if ( i + z[i] > bst + z[bst] ) bst = i;
}

// 回文版

void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
    // centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        if (z[b]+b >= i) z[i] = min(z[2*b-i], b+z[b]-i);
        else z[i] = 0;
        while (i+z[i]+1 < len && i-z[i]-1 >= 0 &&
            s[i+z[i]+1] == s[i-z[i]-1]) z[i] ++;
        if (z[i]+i > z[b]+b) b = i;
    }
}

```

7.8 BWT (Burrows-Wheeler Transform)

```

string BWT(string); // by suffix array

string iBWT(string &s, int start=0){
    int n = (int) s.size();
    string ret(n, ' ');
    vector<int> next(n,0), box[256];

    for (int i=0; i<n; i++) // bucket sort
        box[ (int)s[i] ].push_back(i);

    for (int i=0, j=0; i<256; i++)
        for (int x:box[i])
            next[j++] = x;

    for (int i=0, p=start; i<n; i++)
        ret[i] = s[ p=next[p] ];

    return ret;
}

```

8 Data structure

8.1 2D range tree

```

// remember sort x !!!!!
typedef int T;
const int LGN = 20;
const int MAXN = 100005;

struct Point{
    T x, y;
    friend bool operator < (Point a, Point b){
        return tie(a.x,a.y) < tie(b.x,b.y);
    }
};

struct TREE{
    Point pt;
    int toleft;
}tree[LGN][MAXN];

struct SEG{

```

```

    T mx, Mx;
    int sz;
    TREE *st;
}seg[MAXN*4];

vector<Point> P;

void build(int l, int r, int o, int deep){
    seg[o].mx = P[l].x;
    seg[o].Mx = P[r].x;
    seg[o].sz = r-l+1;

    if(l == r){
        tree[deep][r].pt = P[r];
        tree[deep][r].toleft = 0;
        seg[o].st = &tree[deep][r];
        return;
    }
    int mid = (l+r)>>1;
    build(l,mid,o+o,deep+1);
    build(mid+1,r,o+o+1,deep+1);

    TREE *ptr = &tree[deep][l];
    TREE *pl = &tree[deep+1][l], *nl = &tree[deep+1][
        mid+1];
    TREE *pr = &tree[deep+1][mid+1], *nr = &tree[deep
        +1][r+1];

    int cnt = 0;
    while(pl != nl && pr != nr) {
        *(ptr) = pl->pt.y <= pr->pt.y ? cnt++, *(pl++):
            *(pr++);
        ptr -> toleft = cnt; ptr++;
    }
    while(pl != nl) *(ptr) = *(pl++), ptr -> toleft =
        ++cnt, ptr++;
    while(pr != nr) *(ptr) = *(pr++), ptr -> toleft =
        cnt, ptr++;
}

int main(){
    int n; cin >> n;
    for(int i = 0 ; i < n; i++){
        T x,y; cin >> x >> y;
        P.push_back((Point){x,y});
    }
    sort(P.begin(),P.end());
    build(0,n-1,1,0);
}

```

8.2 ext heap

```

#include <bits/extc++.h>
typedef __gnu_pbds::priority_queue<int> heap_t;
heap_t a,b;

int main() {
    a.clear();
    b.clear();
    a.push(1);
    a.push(3);
    b.push(2);
    b.push(4);
    assert(a.top() == 3);
    assert(b.top() == 4);
    // merge two heap
    a.join(b);
    assert(a.top() == 4);
    assert(b.empty());

    return 0;
}

```

8.3 KD tree

```

// from BCW

const int MXN = 100005;

```

```

struct KDTree {
    struct Node {
        int x,y,x1,y1,x2,y2;
        int id,f;
        Node *L, *R;
    }tree[MXN];
    int n;
    Node *root;

    long long dis2(int x1, int y1, int x2, int y2) {
        long long dx = x1-x2;
        long long dy = y1-y2;
        return dx*dx+dy*dy;
    }
    static bool cmpx(Node& a, Node& b){ return a.x<b.x; }
    static bool cmpy(Node& a, Node& b){ return a.y<b.y; }
    void init(vector<pair<int,int>> ip) {
        n = ip.size();
        for (int i=0; i<n; i++) {
            tree[i].id = i;
            tree[i].x = ip[i].first;
            tree[i].y = ip[i].second;
        }
        root = build_tree(0, n-1, 0);
    }
    Node* build_tree(int L, int R, int dep) {
        if (L>R) return nullptr;
        int M = (L+R)/2;
        tree[M].f = dep%2;
        nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
            cmpy : cmpx);
        tree[M].x1 = tree[M].x2 = tree[M].x;
        tree[M].y1 = tree[M].y2 = tree[M].y;

        tree[M].L = build_tree(L, M-1, dep+1);
        if (tree[M].L) {
            tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
        }

        tree[M].R = build_tree(M+1, R, dep+1);
        if (tree[M].R) {
            tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
        }

        return tree+M;
    }
    int touch(Node* r, int x, int y, long long d2){
        long long dis = sqrt(d2)+1;
        if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
            r->y2+dis)
            return 0;
        return 1;
    }
    void nearest(Node* r, int x, int y, int &mID, long
        long &md2) {
        if (!r || !touch(r, x, y, md2)) return;
        long long d2 = dis2(r->x, r->y, x, y);
        if (d2 < md2 || (d2 == md2 && mID < r->id)) {
            mID = r->id;
            md2 = d2;
        }
        // search order depends on split dim
        if ((r->f == 0 && x < r->x) ||
            (r->f == 1 && y < r->y)) {
            nearest(r->L, x, y, mID, md2);
            nearest(r->R, x, y, mID, md2);
        } else {
            nearest(r->R, x, y, mID, md2);
            nearest(r->L, x, y, mID, md2);
        }
    }
    int query(int x, int y) {
        int id = 1029384756;
        long long d2 = 102938475612345678LL;
        nearest(root, x, y, id, d2);

```

```

        return id;
    }
}tree;

```

8.4 Link-Cut tree

// from bcw codebook

```

const int MXN = 100005;
const int MEM = 100005;

struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay () : val(-1), rev(0), size(0) {
        f = ch[0] = ch[1] = &nil;
    }
    Splay (int _val) : val(_val), rev(0), size(1) {
        f = ch[0] = ch[1] = &nil;
    }
    bool isr() {
        return f->ch[0] != this && f->ch[1] != this;
    }
    int dir() {
        return f->ch[0] == this ? 0 : 1;
    }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push() {
        if (rev) {
            swap(ch[0], ch[1]);
            if (ch[0] != &nil) ch[0]->rev ^= 1;
            if (ch[1] != &nil) ch[1]->rev ^= 1;
            rev=0;
        }
    }
    void pull() {
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;

void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}

vector<Splay*> splayVec;
void splay(Splay *x) {
    splayVec.clear();
    for (Splay *q=x; q=q->f) {
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir()) rotate(x->f), rotate
            (x);
        else rotate(x), rotate(x);
    }
}

Splay* access(Splay *x) {
    Splay *q = nil;
    for (;x!=nil;x=x->f) {
        splay(x);

```

```

    x->setCh(q, 1);
    q = x;
}
return q;
}
void evert(Splay *x) {
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}
void link(Splay *x, Splay *y) {
    // evert(x);
    access(x);
    splay(x);
    evert(y);
    x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
    // evert(x);
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}

int N, Q;
Splay *vt[MXN];

int ask(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    int res = x->f->val;
    if (res == -1) res = x->val;
    return res;
}

int main(int argc, char** argv) {
    scanf("%d%d", &N, &Q);
    for (int i=1; i<=N; i++)
        vt[i] = new (Splay::pmem++) Splay(i);
    while (Q--) {
        char cmd[105];
        int u, v;
        scanf("%s", cmd);
        if (cmd[1] == 'i') {
            scanf("%d%d", &u, &v);
            link(vt[v], vt[u]);
        } else if (cmd[0] == 'c') {
            scanf("%d", &v);
            cut(vt[1], vt[v]);
        } else {
            scanf("%d%d", &u, &v);
            int res = ask(vt[u], vt[v]);
            printf("%d\n", res);
        }
    }
    return 0;
}

```

8.5 Treap

```

#define int long long
#define MAX 100010

struct Node {
    Node *ls, *rs;
    int val, sum, siz;

    Node (int val) {
        this->val = val;
        this->sum = val;
        siz = 1;
        ls = rs = NULL;
    }
}*rot;

int SZ(Node* now) {
    return now == NULL ? 0 : now->siz;
}

```

```

}

int getSum(Node* now) {
    return now == NULL ? 0 : now->sum;
}

void maintain(Node* now) {
    now->siz = 1 + SZ(now->ls) + SZ(now->rs);
    now->sum = now->val + getSum(now->ls) + getSum(now->rs);
}

void split(Node* now, int k, Node* &lef, Node* &rig) {
    if (now == NULL) {
        lef = rig = NULL;
        return;
    }
    if (k <= SZ(now->ls)) {
        rig = now;
        split(now->ls, k, lef, rig->ls);
    } else {
        lef = now;
        split(now->rs, k - SZ(now->ls) - 1, lef->rs, rig);
    }
    maintain(now);
}

Node* merge(Node* lef, Node* rig) {
    if (lef == NULL) return rig;
    if (rig == NULL) return lef;

    if (rand() % (SZ(lef) + SZ(rig)) < SZ(lef)) {
        lef->rs = merge(lef->rs, rig);
        maintain(lef);
        return lef;
    } else {
        rig->ls = merge(lef, rig->ls);
        maintain(rig);
        return rig;
    }
}

int32_t main() {
    cin.tie(0), cout.sync_with_stdio(false);

    int t; cin >> t;
    while (t--) {
        rot = NULL;

        int n, m; cin >> n >> m;
        for (int i = 1; i <= n; i++) {
            int x; cin >> x;
            rot = merge(rot, new Node(x));
        }

        Node *lef, *now, *rig, *tmp;
        while (m--) {
            int x, y; cin >> x >> y;

            split(rot, y, tmp, rig);
            split(tmp, x - 1, lef, now);

            int ans = getSum(now);
            cout << ans << '\n';

            rot = merge(merge(lef, new Node(ans)), rig);
        }
    }
    return 0;
}

```

8.6 Treap Lin

```

#include <bits/stdc++.h>
using namespace std;
const int INF = 1<<30;
int ran(){
    static unsigned x = 20190223;
    return x = 0xdefaced*x+1;
}

```

```

head = merge(ta, merge(tb, tc));
}else if(strcmp(ss, "REVERSE")==0){
    scanf("%d%d", &a, &b);
    split(head, tb, tc, b);
    split(tb, ta, tb, a-1);
    tb -> tag ^= 1;
    head = merge(ta, merge(tb, tc));
}else if(strcmp(ss, "REVOLVE")==0){
    scanf("%d%d%d", &a, &b, &c);
    split(head, tb, tc, b);
    split(tb, ta, tb, a-1);
    int szz = size(tb);
    c %= szz;
    split(tb, tb, td, szz-c);
    tb=merge(td, tb);
    head = merge(ta, merge(tb, tc));
}else if(strcmp(ss, "INSERT")==0){
    scanf("%d%d", &a, &b);
    split(head, ta, tc, a);
    tb = new Treap(b);
    head = merge(ta, merge(tb, tc));
}else if(strcmp(ss, "DELETE")==0){
    scanf("%d", &a);
    split(head, ta, tc, a-1);
    split(tc, tb, tc, 1);
    delete tb;
    head = merge(ta, tc);
}else if(strcmp(ss, "MIN")==0){
    scanf("%d%d", &a, &b);
    split(head, tb, tc, b);
    split(tb, ta, tb, a-1);
    printf("%d\n", min(tb));
    head = merge(ta, merge(tb, tc));
}
}
}

```

9.1 count spanning tree

```
{
    if(fabs(x) < eps) return 0;
```

```

    if(x < 0) return -1;
    else return 1;
}
double b[MAXN][MAXN];
double det(double a[][MAXN], int n)
{
    int i, j, k, sign = 0;
    double ret = 1;
    for(i = 0; i < n; i++)
    for(j = 0; j < n; j++) b[i][j] = a[i][j];
    for(i = 0; i < n; i++)
    {
        if(sgn(b[i][i]) == 0)
        {
            for(j = i + 1; j < n; j++)
            if(sgn(b[j][i]) != 0) break;
            if(j == n) return 0;
            for(k = i; k < n; k++) swap(b[i][k], b[j][k]);
            sign++;
        }
        ret *= b[i][i];
        for(k = i + 1; k < n; k++) b[i][k] /= b[i][i];
        for(j = i + 1; j < n; j++)
        for(k = i + 1; k < n; k++) b[j][k] -= b[j][i] * b[i][k];
    }
    if(sign & 1) ret = -ret;
    return ret;
}
double a[MAXN][MAXN];
int g[MAXN][MAXN];
int main()
{
    int T;
    int n, m;
    int u, v;
    scanf("%d", &T);
    while(T--)
    {
        scanf("%d%d", &n, &m);
        memset(g, 0, sizeof(g));
        while(m--)
        {
            scanf("%d%d", &u, &v);
            u--; v--;
            g[u][v] = g[v][u] = 1;
        }
        memset(a, 0, sizeof(a));
        for(int i = 0; i < n; i++)
        for(int j = 0; j < n; j++)
        if(i != j && g[i][j])
        {
            a[i][i]++;
            a[i][j] = -1;
        }
        double ans = det(a, n-1);
        printf("%.0Lf\n", ans);
    }
    return 0;
}

```

9.2 C++11 random

```

void init(){
    std::random_device rd;
    std::default_random_engine gen( rd() );
    std::uniform_int_distribution<unsigned long long>
        dis(0, ULLONG_MAX);

    for (int i=0; i<MAXN; i++){
        h[i] = dis(gen);
    }
}

```

9.3 Digit Counting

```

int dfs(int pos, int state1, int state2 ....., bool
    limit, bool zero) {

```

```

    if ( pos == -1 ) return 是否符合條件;
    int &ret = dp[pos][state1][state2][....];
    if ( ret != -1 && !limit ) return ret;
    int ans = 0;
    int upper = limit ? digit[pos] : 9;
    for ( int i = 0 ; i <= upper ; i++ ) {
        ans += dfs(pos - 1, new_state1, new_state2,
            limit & ( i == upper ), ( i == 0 ) && zero);
    }
    if ( !limit ) ret = ans;
    return ans;
}

int solve(int n) {
    int it = 0;
    for ( ; n ; n /= 10 ) digit[it++] = n % 10;
    return dfs(it - 1, 0, 0, 1, 1);
}

```

9.4 DP optimization

Monotonicity & 1D/1D DP & 2D/1D DP

Definition xD/yD
 1D/1D $DP[j] = \min(0 \leq i < j) \{ DP[i] + w(i, j) \}$; $DP[0] = k$
 2D/1D $DP[i][j] = \min(i < k \leq j) \{ DP[i][k-1] + DP[k][j] \}$
 $+ w(i, j)$; $DP[i][i] = 0$

Monotonicity

	c	d
a	$w(a, c)$	$w(a, d)$
b	$w(b, c)$	$w(b, d)$

Monge Condition

Concave (凹四邊形不等式): $w(a, c) + w(b, d) \geq w(a, d) + w(b, c)$

Convex (凸四邊形不等式): $w(a, c) + w(b, d) \leq w(a, d) + w(b, c)$

Totally Monotone

Concave (凹單調): $w(a, c) \leq w(b, d) \rightarrow w(a, d) \leq w(b, c)$

Convex (凸單調): $w(a, c) \geq w(b, d) \rightarrow w(a, d) \geq w(b, c)$

1D/1D DP $O(n^2) \rightarrow O(n \lg n)$

CONSIDER THE TRANSITION POINT

Solve 1D/1D Concave by Stack

Solve 1D/1D Convex by Deque

2D/1D Convex DP (Totally Monotone) $O(n^3) \rightarrow O(n^2)$

$h(i, j-1) \leq h(i, j) \leq h(i+1, j)$

9.5 DP 1D/1D

```
#include<bits/stdc++.h>
```

```

int t, n, L;
int p;
char s[MAXN][35];
ll sum[MAXN] = {0};
long double dp[MAXN] = {0};
int prevd[MAXN] = {0};

long double pw(long double a, int n) {
    if ( n == 1 ) return a;
    long double b = pw(a, n/2);
    if ( n & 1 ) return b*b*a;
    else return b*b;
}

long double f(int i, int j) {
    // cout << (sum[i] - sum[j]+i-j-1-L) << endl;
    return pw(abs(sum[i] - sum[j]+i-j-1-L), p) + dp[j];
}

struct INV {
    int L, R, pos;
}

```

```

};
INV stk[MAXN*10];
int top = 1, bot = 1;
void update(int i) {
    while ( top > bot && i < stk[top].L && f(stk[top].L
        , i) < f(stk[top].L, stk[top].pos) ) {
        stk[top - 1].R = stk[top].R;
        top--;
    }
    int lo = stk[top].L, hi = stk[top].R, mid, pos =
        stk[top].pos;
    //if ( i >= lo ) lo = i + 1;
    while ( lo != hi ) {
        mid = lo + (hi - lo) / 2;
        if ( f(mid, i) < f(mid, pos) ) hi = mid;
        else lo = mid + 1;
    }
    if ( hi < stk[top].R ) {
        stk[top + 1] = (INV) { hi, stk[top].R, i };
        stk[top++].R = hi;
    }
}

int main() {
    cin >> t;
    while ( t-- ) {
        cin >> n >> L >> p;
        dp[0] = sum[0] = 0;
        for ( int i = 1 ; i <= n ; i++ ) {
            cin >> s[i];
            sum[i] = sum[i-1] + strlen(s[i]);
            dp[i] = numeric_limits<long double>::max();
        }
        stk[top] = (INV) {1, n + 1, 0};
        for ( int i = 1 ; i <= n ; i++ ) {
            if ( i >= stk[bot].R ) bot++;
            dp[i] = f(i, stk[bot].pos);
            update(i);
        }
        // cout << (ll) f(i, stk[bot].pos) << endl;
        if ( dp[n] > 1e18 ) {
            cout << "Too hard to arrange" << endl;
        } else {
            vector<PI> as;
            cout << (ll)dp[n] << endl;
        }
    }
    return 0;
}

```

9.6 Manhattan MST.cpp

```

#include <bits/stdc++.h>
using namespace std;

const int MAXN = 100005;
const int OFFSET = 2000; // y-x may < 0, offset it, if
// y-x too large, please write a unique function
const int INF = 0xFFFFFFFF;
int n;
int x[MAXN], y[MAXN], p[MAXN];

typedef pair<int, int> pii;
pii bit[MAXN]; // [ val, pos ]

struct P {
    int x, y, id;
    bool operator<(const P&b) const {
        if ( x == b.x ) return y > b.y;
        else return x > b.x;
    }
};
vector<P> op;

struct E {
    int x, y, cost;
    bool operator<(const E&b) const {
        return cost < b.cost;
    }
};

```

```

vector<E> edges;

int find(int x) {
    return p[x] == x ? x : p[x] = find(p[x]);
}

void update(int i, int v, int p) {
    while ( i ) {
        if ( bit[i].first > v ) bit[i] = {v, p};
        i -= i & (-i);
    }
}

pii query(int i) {
    pii res = {INF, INF};
    while ( i < MAXN ) {
        if ( bit[i].first < res.first ) res = {bit[i].
            first, bit[i].second};
        i += i & (-i);
    }
    return res;
}

void input() {
    cin >> n;
    for ( int i = 0 ; i < n ; i++ ) cin >> x[i] >> y[i]
        ], op.push_back((P) {x[i], y[i], i});
}

void mst() {
    for ( int i = 0 ; i < MAXN ; i++ ) p[i] = i;
    int res = 0;
    sort(edges.begin(), edges.end());
    for ( auto e : edges ) {
        int x = find(e.x), y = find(e.y);
        if ( x != y ) {
            p[x] = y;
            res += e.cost;
        }
    }
    cout << res << endl;
}

void construct() {
    sort(op.begin(), op.end());
    for ( int i = 0 ; i < n ; i++ ) {
        pii q = query(op[i].y - op[i].x + OFFSET);
        update(op[i].y - op[i].x + OFFSET, op[i].x + op
            [i].y, op[i].id);
        if ( q.first == INF ) continue;
        edges.push_back((E) {op[i].id, q.second, abs(x[
            op[i].id]-x[q.second]) + abs(y[op[i].id]-y[
            q.second]) });
    }
}

void solve() {
    // [45 ~ 90 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,
        INF};
    construct();

    // [0 ~ 45 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,
        INF};
    for ( int i = 0 ; i < n ; i++ ) swap(op[i].x, op[i]
        ].y);
    construct();
    for ( int i = 0 ; i < n ; i++ ) swap(op[i].x, op[i]
        ].y);

    // [-90 ~ -45 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,
        INF};
    for ( int i = 0 ; i < n ; i++ ) op[i].y *= -1;
    construct();

    // [-45 ~ 0 deg]
    for ( int i = 0 ; i < MAXN ; i++ ) bit[i] = {INF,
        INF};
}

```

```

    for ( int i = 0 ; i < n ; i++ ) swap(op[i].x, op[i]
        ].y);
    construct();

    // mst
    mst();
}

int main () {
    input();
    solve();
    return 0;
}

```

9.7 stable marriage

```

// normal stable marriage problem
// input:
//3
//Albert Laura Nancy Marcy
//Brad Marcy Nancy Laura
//Chuck Laura Marcy Nancy
//Laura Chuck Albert Brad
//Marcy Albert Chuck Brad
//Nancy Brad Albert Chuck

#include<bits/stdc++.h>
using namespace std;
const int MAXN = 505;

int n;
int favor[MAXN][MAXN]; // favor[boy_id][rank] = girl_id
;
int order[MAXN][MAXN]; // order[girl_id][boy_id] = rank
;
int current[MAXN]; // current[boy_id] = rank; boy_id
will pursue current[boy_id] girl.
int girl_current[MAXN]; // girl[girl_id] = boy_id;

void initialize() {
    for ( int i = 0 ; i < n ; i++ ) {
        current[i] = 0;
        girl_current[i] = n;
        order[i][n] = n;
    }
}

map<string, int> male, female;
string bname[MAXN], gname[MAXN];
int fit = 0;

void stable_marriage() {
    queue<int> que;
    for ( int i = 0 ; i < n ; i++ ) que.push(i);
    while ( !que.empty() ) {
        int boy_id = que.front();
        que.pop();

        int girl_id = favor[boy_id][current[boy_id]];
        current[boy_id] ++;

        if ( order[girl_id][boy_id] < order[girl_id][
            girl_current[girl_id]] ) {
            girl_current[girl_id] ) {
                if ( girl_current[girl_id] < n ) que.push(
                    girl_current[girl_id]); // if not the first
                    time
            girl_current[girl_id] = boy_id;
        } else {
            que.push(boy_id);
        }
    }
}

int main() {
    cin >> n;

    for ( int i = 0 ; i < n ; i++ ) {

```

```

        string p, t;
        cin >> p;
        male[p] = i;
        bname[i] = p;
        for ( int j = 0 ; j < n ; j++ ) {
            cin >> t;
            if ( !female.count(t) ) {
                gname[fit] = t;
                female[t] = fit++;
            }
            favor[i][j] = female[t];
        }
    }

    for ( int i = 0 ; i < n ; i++ ) {
        string p, t;
        cin >> p;
        for ( int j = 0 ; j < n ; j++ ) {
            cin >> t;
            order[female[p]][male[t]] = j;
        }
    }

    initialize();
    stable_marriage();

    for ( int i = 0 ; i < n ; i++ ) {
        cout << bname[i] << " " << gname[favor[i][current[i]
            ] - 1]] << endl;
    }
}

```

9.8 Mo's algorithm

```

int l = 0, r = 0, nowAns = 0, BLOCK_SIZE, n, m;
int ans[];
struct QUE{
    int l, r, id;
    friend bool operator < (QUE a, QUE b){
        if(a.l / BLOCK_SIZE != b.l / BLOCK_SIZE)
            return a.l / BLOCK_SIZE < b.l / BLOCK_SIZE;
        return a.r < b.r;
    }
}qurys[];

inline void move(int pos, int sign) {
    // update nowAns
}

void solve() {
    BLOCK_SIZE = int(ceil(pow(n, 0.5)));
    sort(qurys, qurys + m);
    for (int i = 0; i < m; ++i) {
        const QUE &q = qurys[i];
        while (l > q.l) move(--l, 1);
        while (r < q.r) move(r++, 1);
        while (l < q.l) move(l++, -1);
        while (r > q.r) move(--r, -1);
        ans[q.id] = nowAns;
    }
}

```

9.9 Parser

```

using LL = long long;
const int MAXLEVEL = 2;
// binary operators
const vector<char> Ops[MAXLEVEL] = {
    {'+', '-'}, // Level 0
    {'*', '/'} // Level 1
};
// unary operators
const vector<pair<char, int>> Op1s = {
    {'-', 0} // operator negative works on Level 0
};
struct Node{

```



```

~Node(){ delete L; delete R; }
enum { op, op1, num } type;
LL val;
Node *L, *R;
} *root;
char getOp1(int LEVEL, istream& is){
    is >>ws;
    for (auto& x : Op1s){
        auto& op = x.first;
        auto& lev = x.second;
        if (LEVEL == lev && is.peek() == op)
            return is.get();
    }
    return 0;
}
template <int LEVEL> void parse(Node*& x, istream& is){
    char op1 = getOp1(LEVEL, is);
    parse<LEVEL+1>(x, is);
    if (op1) x = new Node{Node::op1, op1, x, nullptr};
    auto& ops = Ops[LEVEL];
    while (is>>ws && count(ops.begin(), ops.end(), is.
        peek())){
        x = new Node{Node::op, is.get(), x, nullptr};
        parse<LEVEL+1>(x->R, is);
    }
}
template <> void parse<MAXLEVEL>(Node*& x, istream& is)
{
    char op1 = getOp1(MAXLEVEL, is);
    is>>ws;
    if (is.peek()>='0' && is.peek()<='9'){
        LL t; is >>t;
        x = new Node{Node::num, t, nullptr, nullptr};
    } else if (is.peek() == '('){
        is.get();
        parse<0>(x, is);
        is>>ws;
        if (is.get()!=')') throw 0;
    } else throw 0;
    if (op1) x = new Node{Node::op1, op1, x, nullptr};
}
// throw when error occur !!!!!
void build(istream& is){
    parse<0>(root, is);
    if ((is>>ws).peek() != EOF) throw 0;
}

```

9.10 python cheat sheet

```

#!/usr/bin/env python3

# import
import math
from math import *
import math as M
from math import sqrt

# input
n = int( input() )
a = [ int(x) for x in input().split() ]

# EOF
while True:
    try:
        solve()
    except:
        break;

# output
print( x, sep=' ' )
print( ''.join( str(x)+' ' for x in a ) )
print( '{:5d}'.format(x) )

# sort
a.sort()
sorted(a)

# list
a = [ x for x in range(n) ]
a.append(x)

```

```

# Basic operator
a, b = 10, 20
a/b # 0.5
a//b # 0
a%b # 10
a**b # 10^20

# if, else if, else
if a==0:
    print('zero')
elif a>0:
    print('positive')
else:
    print('negative')

# loop
while a==b and b==c:
    for i in LIST:

# stack # C++
stack = [3,4,5]
stack.append(6) # push()
stack.pop() # pop()
stack[-1] # top()
len(stack) # size() 0(1)

# queue # C++
from collections import deque
queue = deque([3,4,5])
queue.append(6) # push()
queue.popleft() # pop()
queue[0] # front()
len(queue) # size() 0(1)

# random
from random import *
randrange(L,R,step) # [L,R) L+k*step
randint(L,R) # int from [L,R]
choice(list) # pick 1 item from list
choices(list,k) # pick k item
shuffle(list)
Uniform(L,R) # float from [L,R]

# Decimal
from fractions import Fraction
from decimal import Decimal, getcontext
getcontext().prec = 250 # set precision

```

```

itwo = Decimal(0.5)
two = Decimal(2)

```

```

N = 200
def angle(cost):
    """given cos(theta) in decimal return theta"""
    for i in range(N):
        cost = ((cost + 1) / two) ** itwo
        sinT = (1 - cost * cost) ** itwo
        return sinT * (2 ** N)
pi = angle(Decimal(-1))

```

```

# file IO
r = open("filename.in")
a = r.read() # read whole content into one string

```

```

w = open("filename.out", "w")
w.write('123\n')

```

```

# IO redirection
import sys
sys.stdin = open('filename.in')
sys.stdout = open('filename.out', 'w')

```

