Calculus $\boxed{1}$

Lecuture (1) :- function.

vids that the professor told us to watch before the lectures.

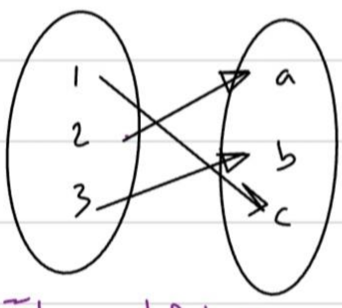
// الأقتران :- هو علاقة تربط كل عنصر بالمجال بعنصر واحد فقط في المدى

* Domian = مجال = input = x = D_f
* Range = مدى = output = y = R_f

Note:- اذا ارتبط عنصر واحد من المجال في عنصر واحد من المدى يسمى اقتران و اذا لم يتحقق هذا الشرط لا يسمى اقتران

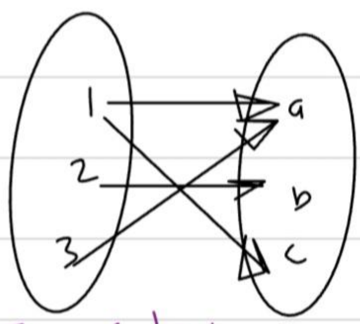
Example:-

(1)



يحقق الشرط = اقتران

(2)



لا يحقق الشرط ≠ اقتران

Note:- وتكتب قاعدة الأقتران بهذه الرموز $y = f(x)$

Example :- $f(x) = \sqrt{x+2}$

find :-

① $f(2)$:- $\sqrt{2+2} = \sqrt{4} = 2$

② $f(-4)$:- $\sqrt{-4+2} = \sqrt{-2}$ undefined غير معرف

③ $f(m+3)$:- $\sqrt{m+3+2} = \sqrt{m+5}$

④ Domain (f) :- $x \in [-2, \infty)$ // $x+2=0 \rightarrow x=-2$ //

⑤ Range (f) :- $x \geq 0$, $x \in [0, \infty)$

ايجاد مجال / مدى الأقتران من الرسم :-

Note :- - لايجاد المجال من الرسم نسقط رسمة الأقتران على محور السينات

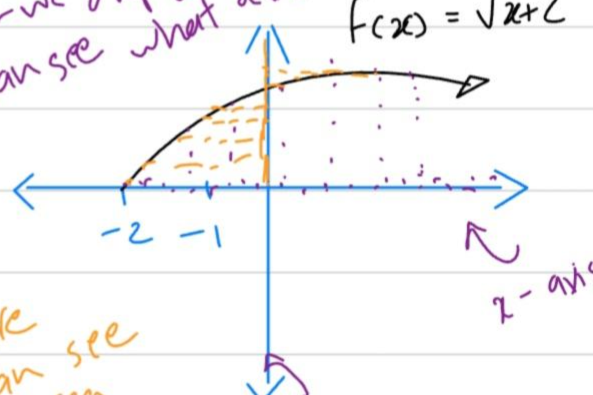
- لايجاد المدى من الرسم نسقط رسمة الأقتران على محور الصادات

$f(x) = \sqrt{x+2}$

مجال

مدى

when we drop like that we can see what area is it taking which is $x \in$

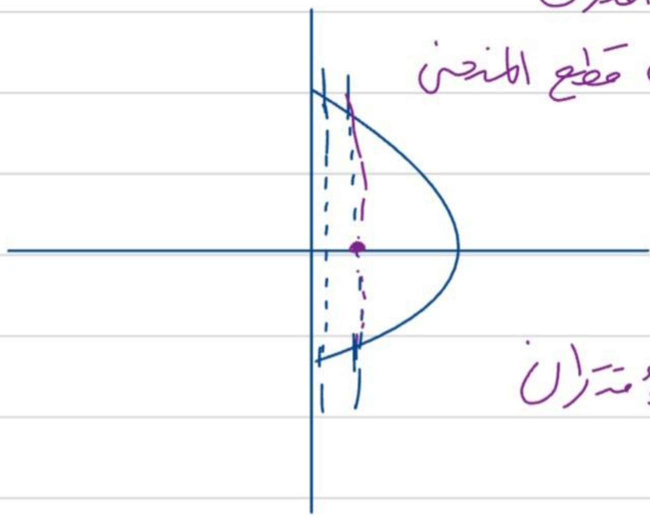


and here we can see the area its taking which is y +

Lecuture 2:-

The vertical Line test اختبار الخط العامودي

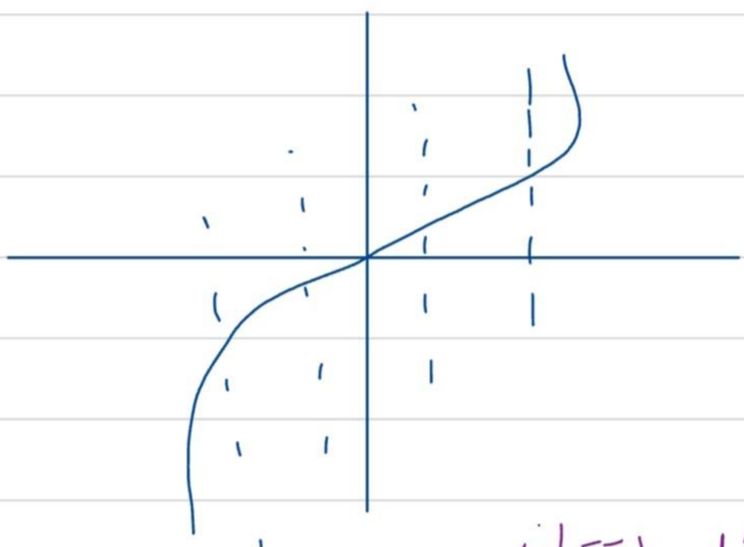
Example ①



• المنحنى لا يمثل اقتران
لان الخط العامودي قطع المنحنى
بنقطتين

⇐ هذه النقطة
لها صورتين اذا
لا تحقق شرط الاقتران

Example ②



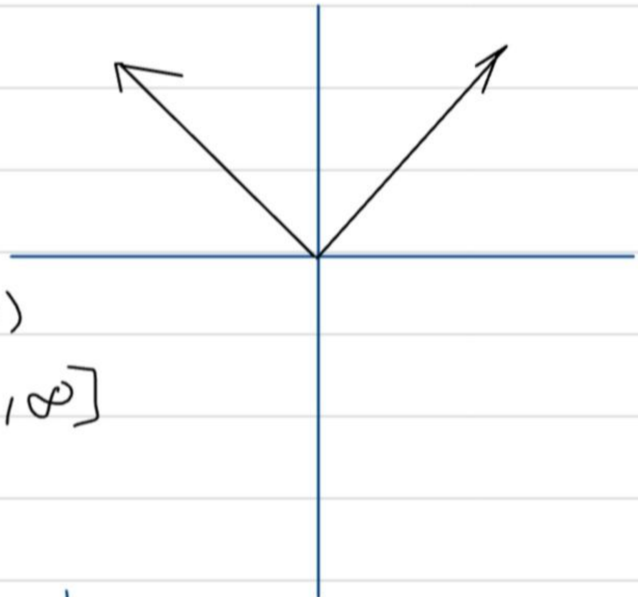
• المنحنى يمثل اقتران

The absolute value function

امتران القيمة المطلقة

// $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Find:- → $f(x) = |x|$



① $f(0)$:- $|0| = 0$

② $f(2)$:- $|2| = 2$

③ $f(-2)$:- $|-2| = 2$

④ Domain :- $\mathbb{R}$, $(-\infty, \infty)$

⑤ Range :- $x \geq 0$ $[0, \infty]$

① $|ab| = |a|\,|b|$

② $|a/b| = |a| / |b|$

③ $\sqrt{x^2} = |x|$

④ $(\sqrt{x})^2 = x$

⑤ $|x| = a \rightarrow x = -a, a$

⑥ $|x| \leq a \rightarrow -a \leq x \leq a$

⑦ $|x| \geq a \rightarrow x \geq a$ or $x \leq -a$

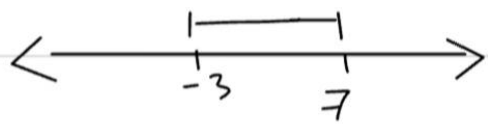
Exampl:- Find the value(s) of x

① $|x| = 3$ = $x = 3, x = -3$

② $|x-2| < 5$ = $-5 < x - 2 < 5$

$+2$

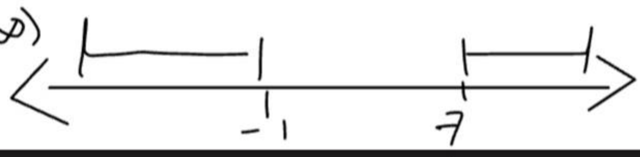
$-3 < x < 7$



③ $|x-3| \geq 4$ = $x - 3 \geq 4 = x \geq 7$

$x - 3 \leq 4 = x \leq -1$

$x \; (-\infty, -1] \cup [7, \infty)$

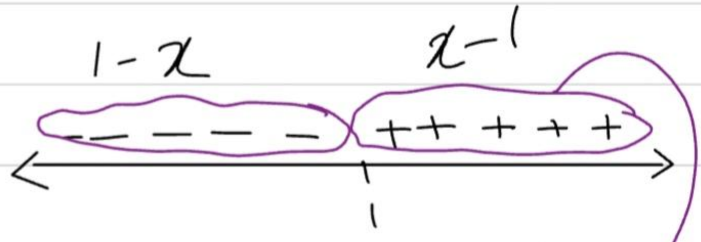


Rewrite :   اعادة تعريف

① $f(x) = |x-1|$

$x - 1 = 0$

$x = 1$



$$f(x) = \begin{cases} x-1 & , x > 1 \\ 1-x & , x \leq 1 \end{cases}$$

how did we know?

we can use 2 ways

① take a number after 1 for an example (2) and the find it from this

$(x-1) \Rightarrow 2-1 = +1$

the answer turn out positive so any number after (1) is gonna be positive and the same for number before (1) for example (-1)

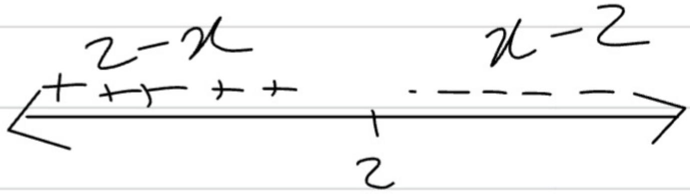
② or we can use the method اقتران خطي نفي / عكس

The home work

② $f(x) = |2-x|$

$2-x=0$

$x=2$



$$f(x) = \begin{cases} 2-x & , x < 2 \\ x-2 & , x \geq 2 \end{cases}$$

Lecuture 3: Polynomial

كثيرات الحدود

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} \ldots\ldots a_2 + a_1 + a_0$$

$$// \; a_n, a_{n-1}, a_{n-2} \cdots\cdots a_1, a_0 \in \mathbb{R} \;,\; a_n \neq 0$$

- The numbers $(a_n, a_{n-1} \cdots\cdots a_2, a_1, a_0)$ are Constants called the coefficionts of the polynomial

- The degree of the polynomial is (n)
- (n) is nonnegative integer number
- The domain of any polynomial is $(\mathbb{R})$

integer = صحيح
natural = طبيعي
coefficionts = معاملات

nonnegative
zero — Positive

nonpositive
zero — negative

Example:-

① $f(x) = 3x^{5} + 7x^{1} - 4$

المعاملات حقيقية // الدرجات اعداد صحيحة موجبة ⟸ تحقق الشروط

$\Rightarrow$ polynomial with degree 5, costant term $= -4$

② $f(x) = \sqrt{x} + 9x^4 + 1 = x^{\frac{1}{2}} + 9x^4 + 1$

The n here is not an integer number so its not a polynomial

Note:- even if there is just one number that does not follow the rules makes it a not polynomial function

③ $f(x) = 9x^{-5} - 4$ → the (n) is negative

not polynomial

④ $f(x) = 9 = 9x^0$

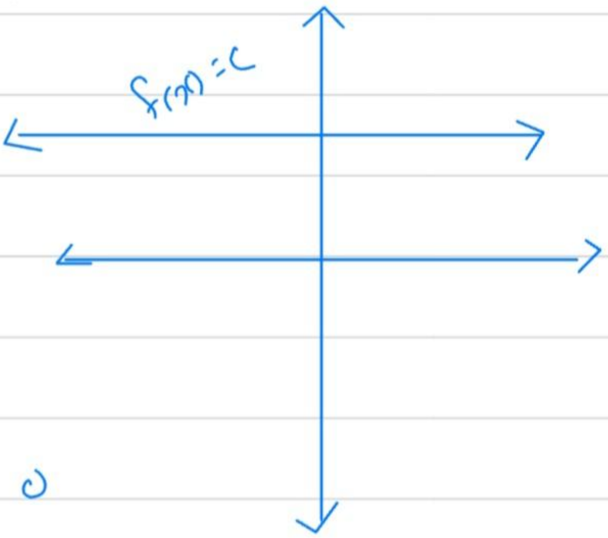
it is polynomial

Constant function :-

$f(x) = c$

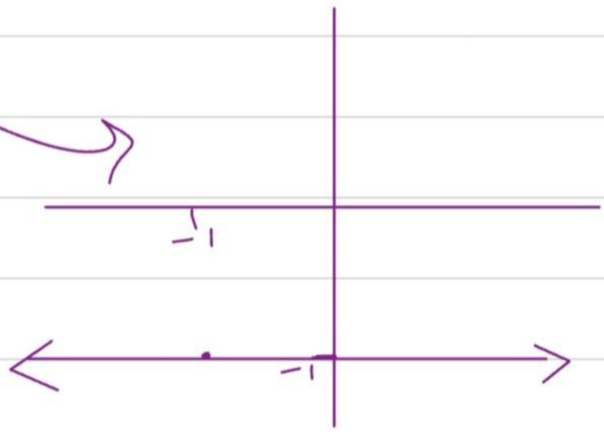
Domain $= \mathbb{R}$

Range $= \{c\}$



example :-

- $f(x) = 5$
- $f(x) = 0$
- $f(x) = -1$



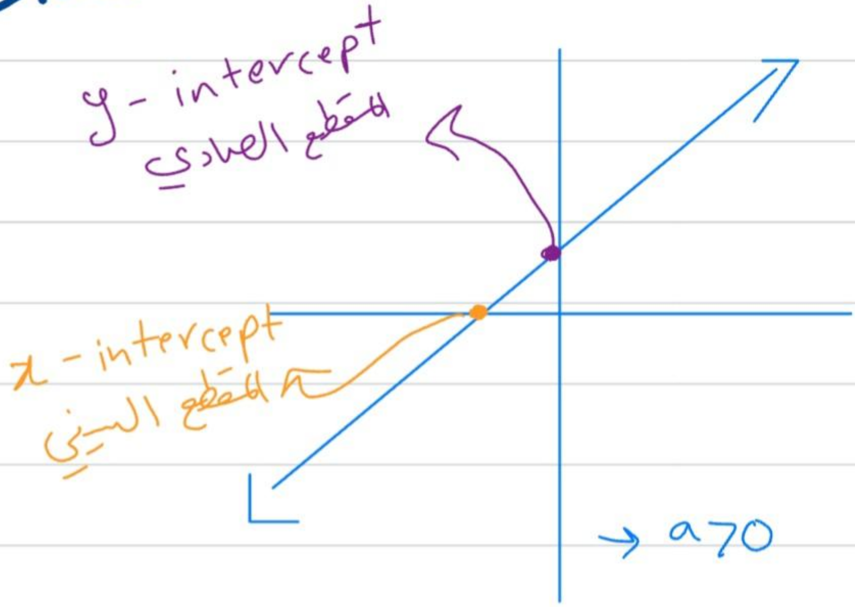
Lecuture 4 :-

Linear function :-

$f(x) = ax + b$

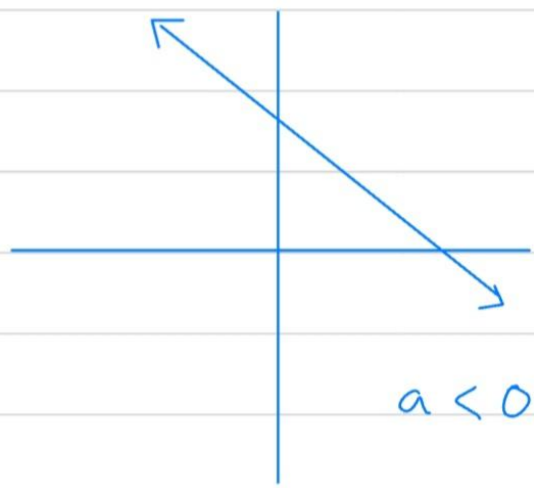
Domain $= \mathbb{R}$

Range $= \mathbb{R}$



// لايجاد المقطع الصادي نعوض الأكس $x = 0$

// لايجاد المقطع السيني نعوض الواي $y = 0$, $f(x) = 0$



Example:-

① $f(x) = 2x + 4$

x - intercept =

$2x + 4 = 0$

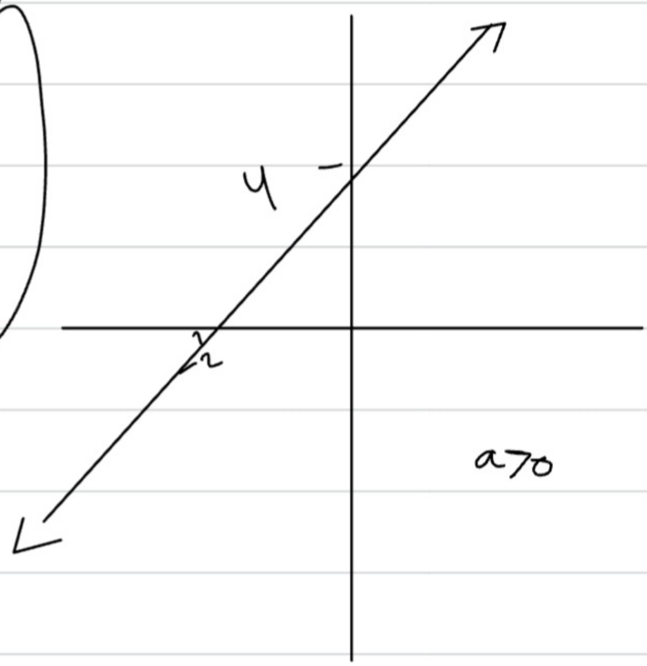
$\frac{\cancel{2}x}{\cancel{2}} = \frac{-4}{2} \rightarrow x = -2$

y - intercept =

$f(0) = 2(0) + 4$

$= 4$

$(-2, 4)$



Domain = Range = $\mathbb{R}$

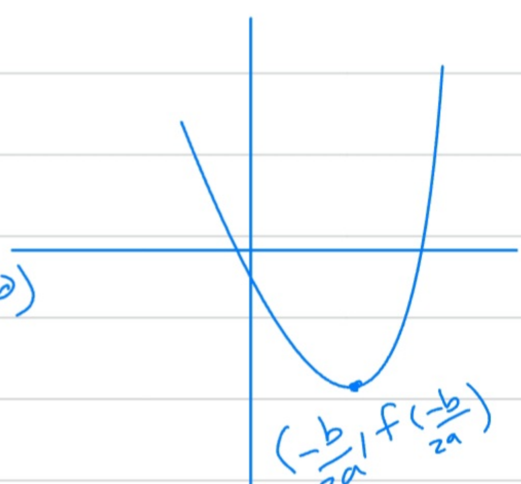
Quadratic function:-

$f(x) = ax^2 + bx + c$

Domain = $\mathbb{R}$

$\Rightarrow$ if $a > 0$ يكون مقعر للأعلى

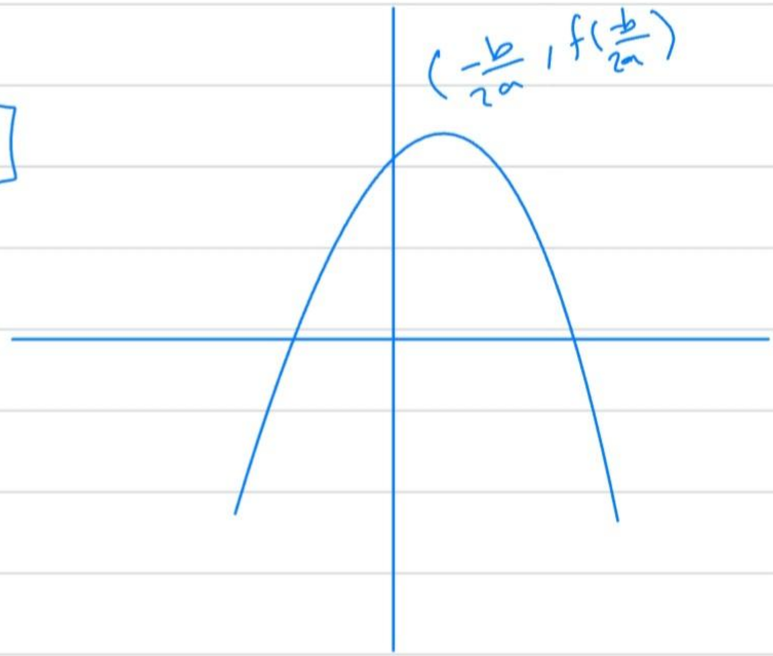
Domain = $\mathbb{R}$ Range:- $[f(\frac{-b}{2a}), \infty)$



$\Rightarrow$ if $a < 0$ يكون مقعر للأسفل

Domain $= \mathbb{R}$

range $= \left(-\infty, f\left(\frac{-b}{2a}\right)\right]$



Example:-

① $f(x) = x^2 - 6x + 7$

$a = 1, \quad b = -6, \quad c = 7$

$a > 0$ مفتوح للأعلى

Domain $= \mathbb{R}$

range $= [-2, \infty)$ $\rightarrow$ $\frac{-b}{2a} = -\frac{(-6)}{2(1)} = 3$

$f(3) = (3)^2 - 6(3) + 7$

$= 9 - 18 + 7$

-2

$\rightarrow x = 0 \rightarrow 7$

$y = 0 \rightarrow \quad 0 = x^2 - 6x + 7$

$(x - \quad)(x - \quad)$

لا يحل المميز = 8 ناخذ القانون العام

من اجل السرعة .....

② $f(x) = 3 + 2x - x^2$

$a = -1$, $b = 2$, $c = 3$

Domain $\mathbb{R}$

range $= \left(-\infty, f\left(\frac{-b}{2a}\right)\right]$

$f\left(\frac{-b}{2a}\right) = f\left(\frac{-2}{2(-1)}\right)$

$f(1) = 3 + 2(1) - (1)^2$

$= 4$

Lecture 5:-

Quadratic equation:-

Discriminant $= \Delta = b^2 - 4ac$

$\Delta \geq 0$

> 0 (حلين)

$= 0$ (حل)

$\Delta < 0$

لا يوجد حلول

$x = \dfrac{-b \mp \sqrt{\Delta}}{2a}$

Example: find the value(s) of x

① $x^2 + x + 12 = 0$ $a = 1$, $b = 1$, $c = 12$

$\Delta = b^2 - 4ac \rightarrow 1 - 4(12) \rightarrow 1 - 48 = -47$

$\therefore$ has no soulution

(2) $x^2 - 3x - 4 = 0$

$\Delta = b^2 - 4ac$

$(-3)^2 - 4(1)(-4) = 25$

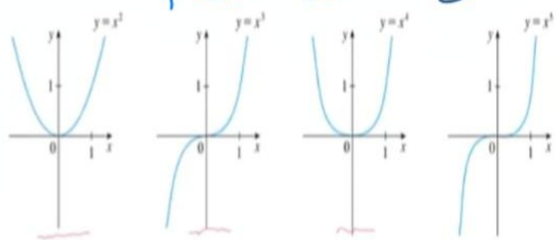
$x = \frac{-b \mp \sqrt{\Delta}}{2a} \rightarrow \frac{+3 \mp \sqrt{25}}{2}$

$= \frac{3-5}{2} \rightarrow -1$

$\rightarrow \frac{3+5}{2} = 4$

Power function

$f(x) = x^n$

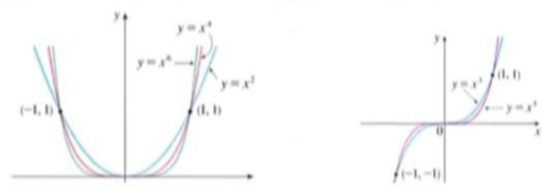


or we can do this

$x^2 - 3x - 4 = 0$

$(x-4)(x+1)$

$x = 4 \qquad x = -1$ #

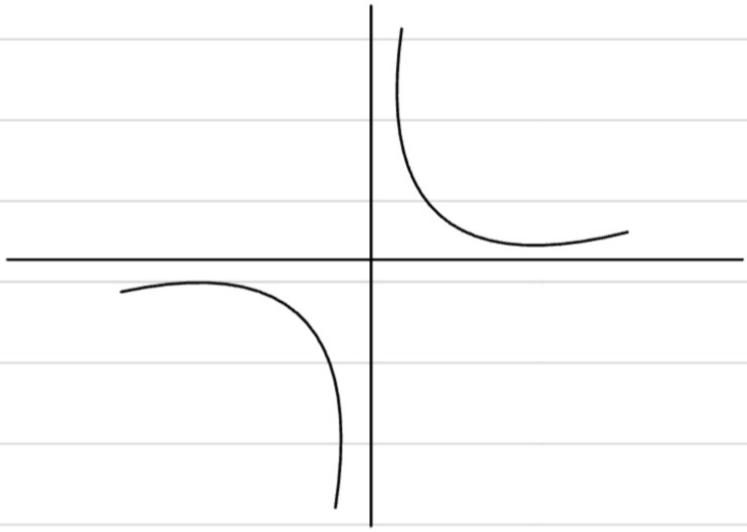


Rational function:-

$f(x) = \frac{Q(x)}{P(x)} = \frac{\text{كثر حدود}}{\text{كثير حدود}}$

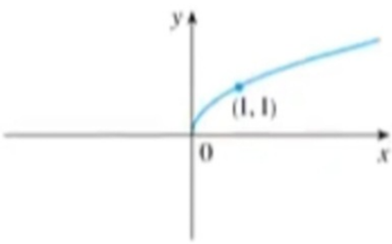
Example:- (1) $f(x) = \frac{x^2 + 5x}{x^7 - 6x + 1}$

(2) $f(x) = \frac{1}{x}$



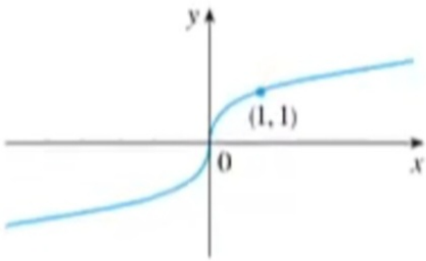
Domain = $\mathbb{R} - \{0\}$
range = $\mathbb{R} - \{0\}$

Root function :-



(a) $f(x) = \sqrt{x}$

Domain = $[0, \infty)$
Range = $[0, \infty)$



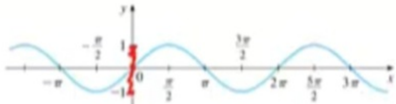
(b) $f(x) = \sqrt[3]{x}$

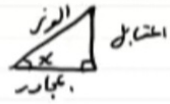
Domain = $\mathbb{R}$
Range = $\mathbb{R}$

Lecuture 6:-
trigonometric functions

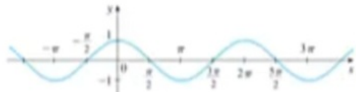
Trigonometric functions:

[1] $f(x) = \sin x$ (جاص) $= \frac{\text{المقابل}}{\text{الوتر}}$
Domain $f = \mathbb{R}$
Range $f = [-1, 1]$
$-1 \leq \sin x \leq 1$
$|\sin x| \leq 1$


(a) $f(x) = \sin x$



[2] $f(x) = \cos x$ (جتاص) $= \frac{\text{المجاور}}{\text{الوتر}}$
Domain $f = \mathbb{R}$
Range $f = [-1, 1]$


(b) $g(x) = \cos x$

[3] $f(x) = \tan x$ (ظاص) $= \frac{\sin x}{\cos x}$ (جاص / جتاص) $= \frac{\text{المقابل}}{\text{المجاور}}$
Domain $f = \mathbb{R} \setminus \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots\}$
Range $f = \mathbb{R}$

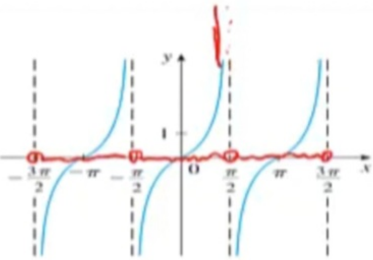


FIGURE 19
$y = \tan x$

[4] $f(x) = \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x} = \frac{\text{المجاور}}{\text{المقابل}}$

[5] $f(x) = \sec x = \frac{1}{\cos x} = \frac{\text{الوتر}}{\text{المجاور}}$

[6] $f(x) = \csc x = \frac{1}{\sin x} = \frac{\text{الوتر}}{\text{المقابل}}$

1) $\sin^2 x + \cos^2 x = 1$

2) $\sin 2x = 2\sin x \cos x$

3) $\cos 2x = \cos^2 x - \sin^2 x$

$1 - 2\sin^2 x$

$2\cos^2 x - 1$

4) $\sec^2 x = 1 + \tan^2 x$

5) $\csc^2 x = 1 + \cot^2 x$

6) $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

7) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

8) $\sin \alpha \cos \beta = \frac{1}{2}(\sin \alpha+\beta + \sin(\alpha-\beta))$

9) $\sin\alpha \sin\beta = -\frac{1}{2}(\cos \alpha+\beta) + \cos(\alpha-\beta))$

10) $\cos \alpha \cos\beta = \frac{1}{2}((\cos \alpha+\beta) + \cos(\alpha-\beta))$

11) $\sin(a+2\pi) = \sin a$

12) $\cos(a+2\pi) = \cos a$

cos, sec $\rightarrow$ even function

sin x, tan x, cot x, csc(x) $\rightarrow$ odd functions

(1) دائرة الوحدة

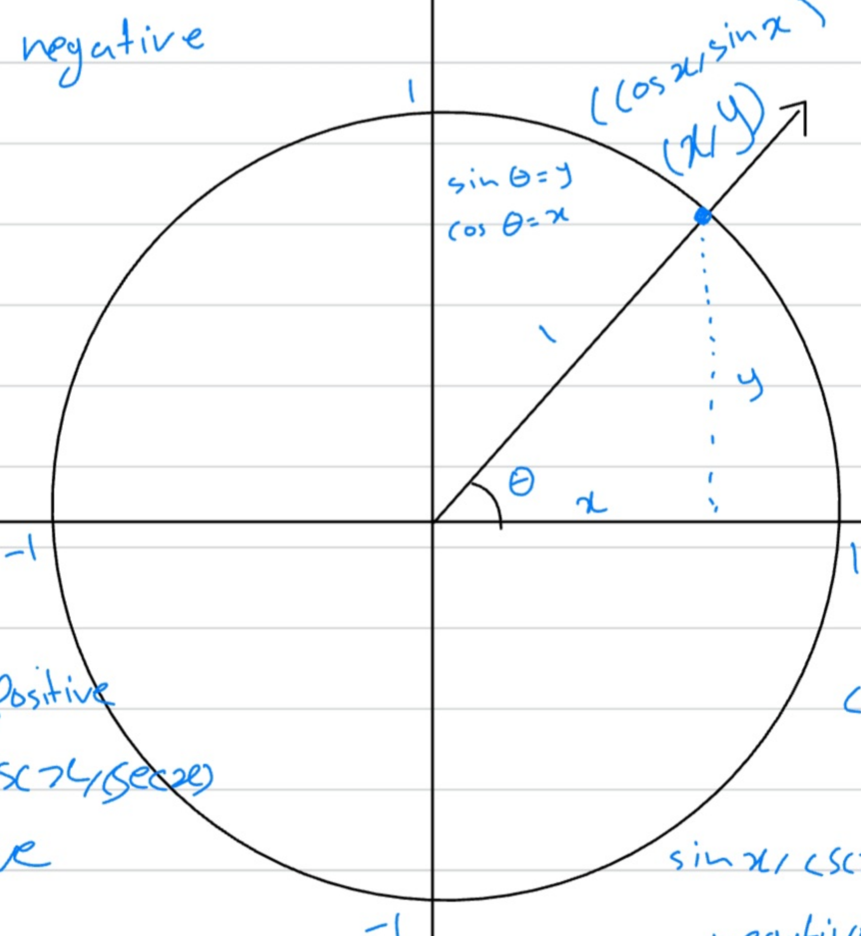
(2)

$\sin x, \csc x \to$ Positive

$\cos x, \tan x, \cot x, \sec x$

$\Rightarrow$ negative

All are positive



$\tan x, \cot x \to$ Positive

$\cos x, \sin x, \csc x, \sec x$

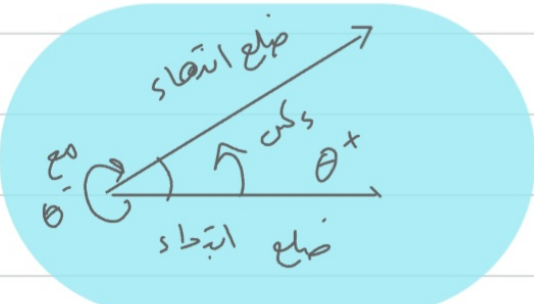
$\Rightarrow$ Negative

$\cos x, \sec x$

$=$ Positive

$\sin x, \csc x, \tan x, \cot x$

negative



(3)

(4)

Lecture 7:-

x	$\sin x$	$\cos x$	$\tan x$	$\cot x$	$\sec x$	$\csc x$
$0, 2\pi$ (360°)	0	1				
$90^\circ = \frac{\pi}{2}$	1	0				
$180^\circ = \pi$	0	-1				
$270^\circ = \frac{3\pi}{2}$	-1	0				
$30^\circ = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$				
$45^\circ = \frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$				
$60^\circ = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$				

* $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

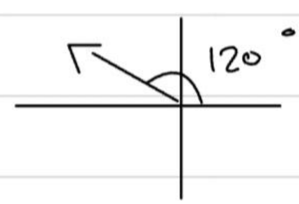
* $\csc\frac{\pi}{3} = \frac{2}{\sqrt{3}}$

* $\tan\frac{\pi}{4} = 1$

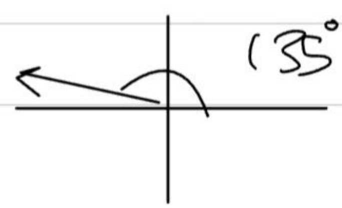
زاوية المرجع = هي زاوية حادة محصورة بين محور السينات وضلع انتهاء الزاوية

* درجات = (نحول) راديان = نضرب بـ $\frac{\pi}{180}$
* راديان = درجات = نضرب بـ $\frac{180}{\pi}$

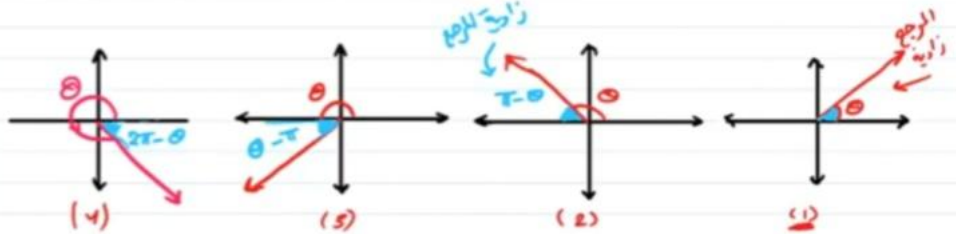
$120^\circ \rightarrow \cancel{120}^{2} \times \frac{\pi}{\cancel{180}_{3}} \rightarrow \frac{2\pi}{3}$



$\frac{3\pi}{4} \rightarrow \frac{3\cancel{\pi}}{4} \times \frac{\cancel{180}^{45}}{\cancel{\pi}} \rightarrow 135$



Example :-



① $\cos\left(\frac{3\pi}{4}\right) \rightarrow \frac{3\pi}{4} - \pi = \; = -\frac{\pi}{4} \rightarrow -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

② $\sin\left(\frac{11\pi}{6}\right) \rightarrow \frac{11\pi}{6} - 2\pi = -\frac{\pi}{6} \rightarrow -\sin\frac{\pi}{6} = -\frac{1}{2}$

③ $\tan\left(\frac{4\pi}{6}\right) \rightarrow \pi - \frac{4\pi}{6} = \frac{\pi}{3} \rightarrow \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$