上 海 交 通 大 学 试 卷

（ 2023~ 2024~1Academic Year/Fall Semester ）

Class No.____________________ Name in English or Pinyin: ____________________

Student ID No._______________ Name in Hanzi(if applicable): ____________________

ECE2150J/VE215 Introduction to Circuits

Final Exam

2023/December/12th 10:00 – 11:40 am

The exam paper has 15 pages in total.

You are to abide by the University of Michigan-Shanghai Jiao Tong University Joint Institute (UM-SJTU JI) honor code. Please sign below to signify that you have kept the honor code pledge.

THE UM-SJTU JI HONOR CODE

I accept the letter and spirit of the honor code:

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code by myself or others.

Signature:____________

Please enter grades here:

Exercises No. **题号**	**Points** **得分**	**Grader's Signature** **流水批阅人签名**
1		
2		
3		
4		
5		
Total 总分		

Q1. Discrete small questions. (20 points)

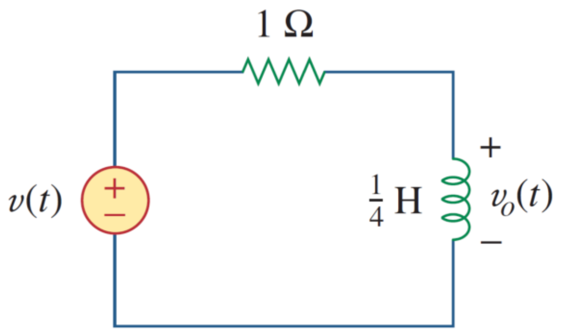
Please note that letters in **bold** indicate a **phasor form** of elements/parameters.

Q1.1 If $v_1 = 30\sin(\omega t + 10°)$ and $v_2 = 20\sin(\omega t + 50°)$, which of these statements are true? (3 Points)

(a) v_1 leads v_2 (b) v_2 leads v_1 (c) v_2 lags v_1

(d) v_1 lags v_2 (e) v_1 and v_2 are in phase

Q1.2 At what frequency will the output voltage *$v_o(t)$* in the figure below be equal to the input voltage *$v(t)$*? (3 points)



(a) 0 rad/s (b) 1 rad/s (c) 4 rad/s

(d) ∞ rad/s (e) None of the above

Q1.3 A source is connected to three loads $\mathbf{Z_1}$, $\mathbf{Z_2}$, and $\mathbf{Z_3}$ in parallel. Which of these is not true? (4 points)

(a) $P = P_1 + P_2 + P_3$ (b) $Q = Q_1 + Q_2 + Q_3$ (c) $S = S_1 + S_2 + S_3$

(d) $\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} + \mathbf{S_3}$ (e) None of the above

Q1.4 Which of these is not a required condition for a balanced system? (4 points)

(a) $|\boldsymbol{V_{an}}| = |\boldsymbol{V_{bn}}| = |\boldsymbol{V_{cn}}|$

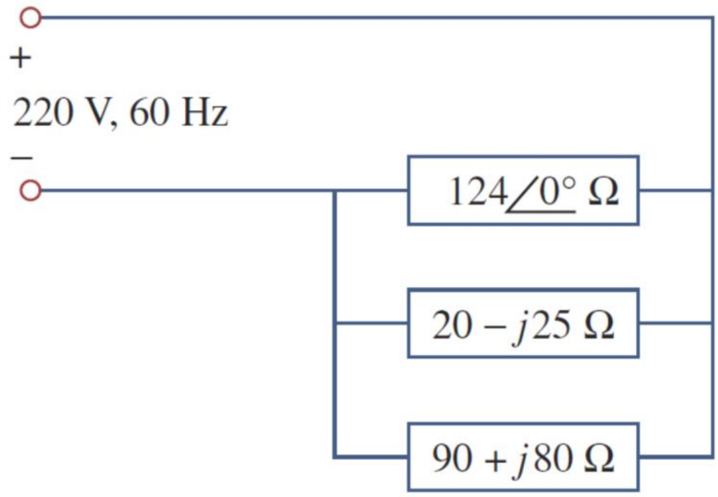
(b) $\mathbf{I_a} + \mathbf{I_b} + \mathbf{I_c} = 0$

(c) $V_{an} + V_{bn} + V_{cn} = 0$

(d) Source voltages are 120° out of phase with each other.

(e) Load impedances for the three phases are equal.

Q1.5 For the power system below, find the average power, the reactive power, and the power factor. Note that 220 V is the **rms value**. (6 points)

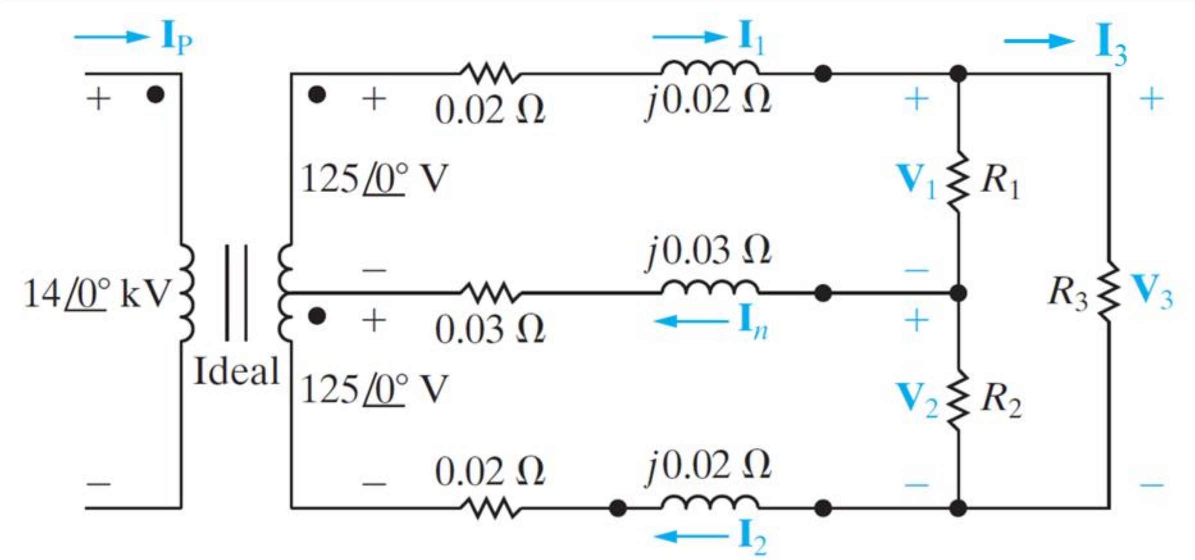


Q2. A residential wiring circuit is shown below. In this model, the resistor R_3 is used to model a 250 V appliance, and the resistors R_1 and R_2 are used to model 125 V appliances. The branches carrying I_1 and I_2 are modeling what electricians refer to as the hot conductors in the circuit, and the branch carrying I_3 is modeling the neutral conductor. (18 points)

(a) Show that $\mathbf{I_n}$ is zero if $R_1 = R_2$. (6 points)

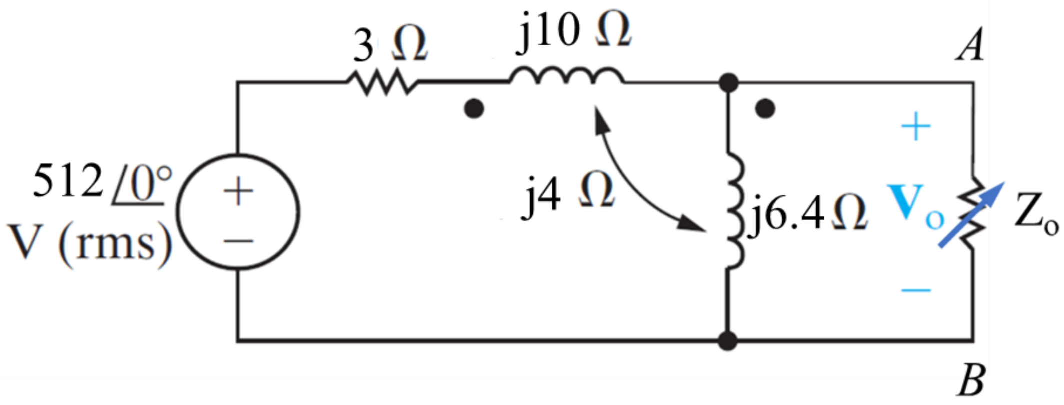
(b) Show that $\mathbf{V_1} = \mathbf{V_2}$ if $R_1 = R_2$. (3 points)

(c) If $R_1 = 40\ \Omega$, $R_2 = 400\ \Omega$, and $R_3 = 8\ \Omega$, please compare $\mathbf{V_1}$ and $\mathbf{V_2}$ with and without the neutral line. And please explain the need of the neutral line. *Hint*: The circuit with the neutral line has $\mathbf{I_1} = 34.2 - j0.18$ A and $\mathbf{I_2} = 31.4 - j0.16$ A. (9 points)



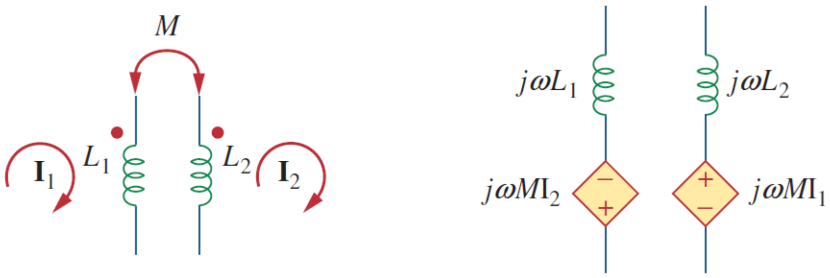
(c) An open neutral can result in severely unbalanced voltages across the 125 V loads at R_1 and R_2. (+2)

Q3. Assume a variable impedance $\mathbf{Z_o}$ is adjusted for maximum average power transfer to $\mathbf{Z_o}$. Please answer the following questions. (18 points)



(a) Please draw the equivalent circuit using the dependent voltage sources as the example below shows. (4 points)

Example: Magnetically coupled circuit and its equivalent circuit with dependent voltage sources.

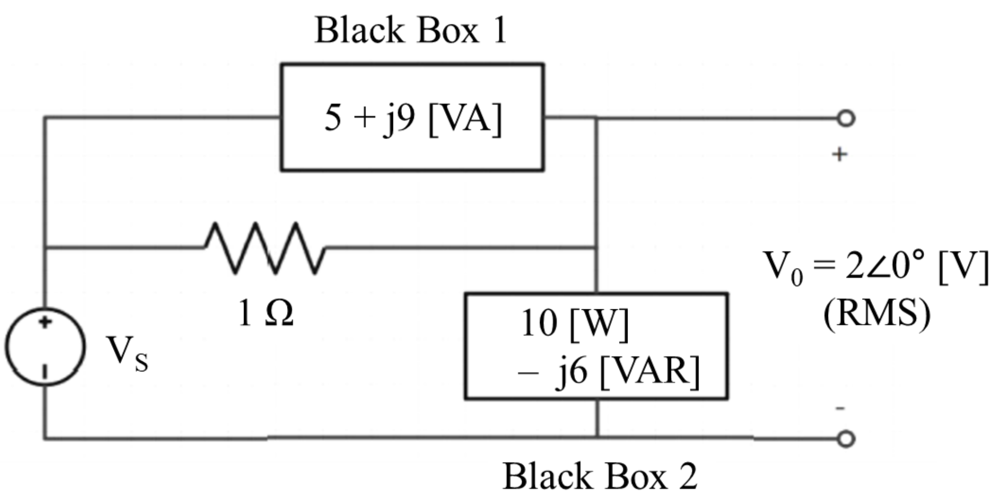


(b) Find open circuit voltage $\mathbf{V_{OC}} = \mathbf{V_{TH}}$, and short circuit current $\mathbf{I_{SC}} = \mathbf{I_N}$ at the terminal between *A* and *B*. (8 points)

(c) Find the equivalent impedance at the terminal between A and B. (2 points)

(d) What is the maximum average power that can be delivered to $\mathbf{Z_o}$? (4 points)

Q4. In the circuit below, the two black boxes are composed of linear, passive circuit elements (resistors, capacitors, and inductors) without a power source. (24 points)

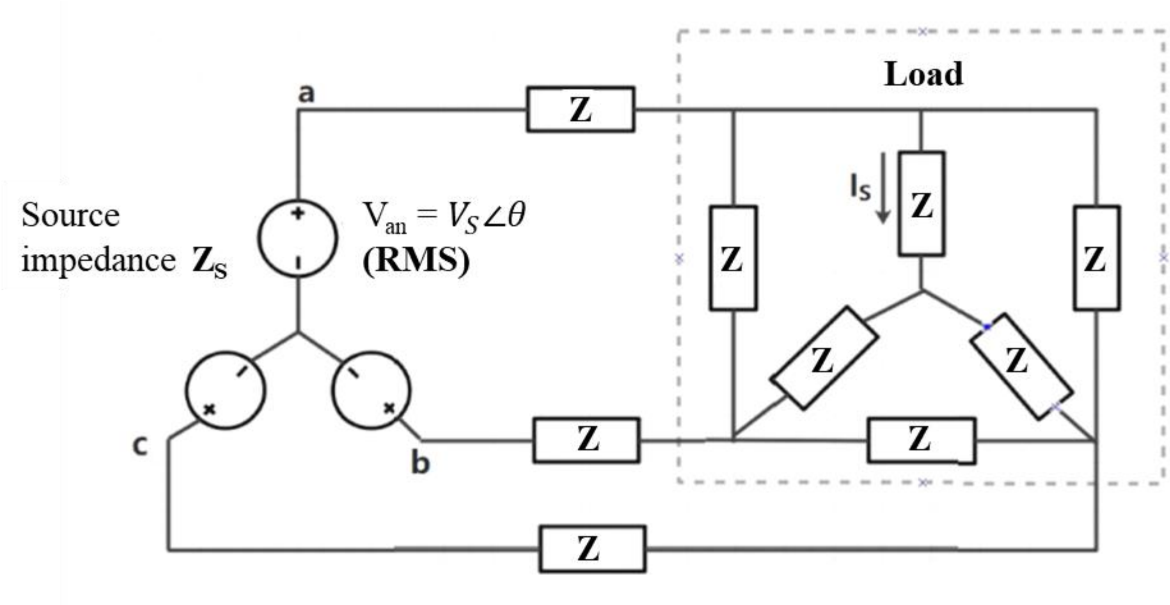


(a) Please draw the power triangles of the two black boxes and then determine its type, i.e. leading or lagging. (4 points)

(b) Please find all possible values of source voltage $\mathbf{V_S}$. Please use a phasor form. Hint: Current through resistor is a complex number. (12 points)

(c) Suppose the angular frequency of the source is 100 rad/s and you want to correct the overall power factor by connecting pure reactive loads in parallel with the source. Please calculate all possible values of such additional capacitance (or inductance) that will change the overall power factor to 0.95 for all possible $\mathbf{V_S}$. (8 points)

Q5. The circuit below shows a balanced source, a-b-c sequence three phase system. Line impedance ($\mathbf{Z}_l$) is $\mathbf{Z}$, load impedance ($\mathbf{Z}_L$) Δ and Y is $\mathbf{Z}$, respectively, and source impedance ($\mathbf{Z}_S$) is $\mathbf{Z}_S$. Suppose $\mathbf{V}_{an}$ is $V_S\angle\theta$. Both Δ and Y load impedances are present. (20 points)



(a) Find an equivalent circuit in the Y-Y form. Please mark values of $\mathbf{Z}_S$, $\mathbf{Z}_l$, and $\mathbf{Z}_L$ clearly. (4 points)

(b) Derive line currents and the voltage drops caused by the line ($\mathbf{Z}_l$) and load impedance ($\mathbf{Z}_L$) of three lines. (6 points)

(c) Please derive the total complex power absorbed by the loads ($\mathbf{Z}_L$). (6 points)

(d) Please derive $\mathbf{I}_S$. (4 points)

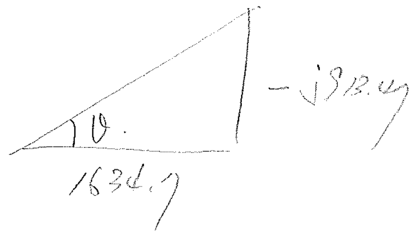
Q1.5

220 Vrms, 60Hz

$$S_1 = \frac{220^2}{124\angle 0^\circ} = 390.22$$

$$S_2 = \frac{220^2}{20+j25} = 944.4 - j1180.5$$

$$S_3 = \frac{220^2}{90-j80} = 300 + j267.03$$



$P = 390.22 + 944.4 + 300 = 1634.7$ [W] (+2)

$Q = -j1180.5 + j267.03 = -j913.47$ or 913.47 leading [VAR]
(+2) (+1)

$pf = 0.8732$ (+1)

finding total complex power is okay
but needs to specify leading or lagging
otherwise −1

Q2.

(a) I_1 loop.

$$(0.02+j0.02)I_1 + R(I_1-\cancel{I_3}) + (0.03+j0.03)(I_1-I_2) = 125 \quad ① \quad (+1)$$

I_2 loop.

$$(0.02+j0.02)I_2 + (0.03+j0.03)(I_2-I_1) + R(I_2-\cancel{I_3}) = 125 \quad ② \quad (+1)$$

$$① - ② \Rightarrow (0.08+j0.08+R)I_1 - (0.08+j0.08+R)I_2 = 0.$$

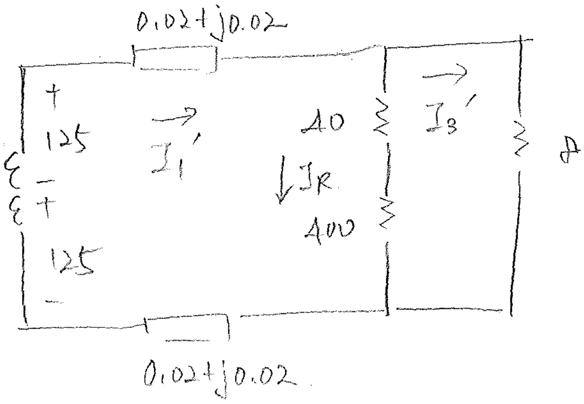
or $(0.08+j0.08+R)I_1 = (0.08+j0.08+R)I_2$ (+2)

$\underline{I_1=I_2}$ thus, $\underline{I_n=0}$ proof +2.

(b) $V_1 = R_1(I_1-I_3)$ $V_2 = R_2(I_2-I_3)$ (+1)

if $R_1=R_2 \Rightarrow I_1=I_2$ (+1) thus, $\underline{V_1=V_2}$ (+1)

(c) (i) without neutral line.



for (a) and (b)
Other methods are okay as long as they are right with clear processes or logic.

① $(0.04+j0.04)I_1' + 440(I_1'-I_3') = 250.$

② $440(I_3'-I_1') + 8I_3' = 0.$

Solve the above. $I_1' = 31.66 - j0.16$ (+1)

$I_3' = 31.09 - j0.16.$ (+1)

$I_R = I_1' - I_3' = \underline{0.57}.$

$V_1 = 40 \times 0.57 = \underline{22.8}$ (+1)

$V_2 = 400 \times 0.57 = \underline{228}$ (+1)

(ii) with neutral line.

we get ① $(40.05+j0.05)I_1 - (0.03+j0.03)I_2 - 40I_3 = 125$

② $-(0.03+j0.03)I_1 + (400.05+j0.05)I_2 - 400I_3 = 125$

③ $-40I_1 - 400I_2 + 448I_3 = 0$

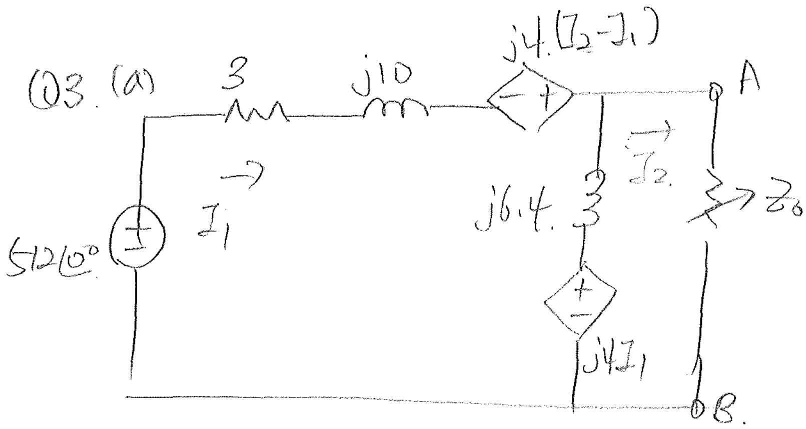
Since $I_1 = 34.19 - j0.18$ and $I_2 = 31.4 - j0.16.$

$\underline{I_3 = 31.09 - j0.16.}$ (+1)

$V_1 = 40(I_1-I_3) = 124.2 \angle -0.35°$ (+1)

$V_2 = 400(I_2-I_3) = 124.4 \angle -0.18°$ (+1)

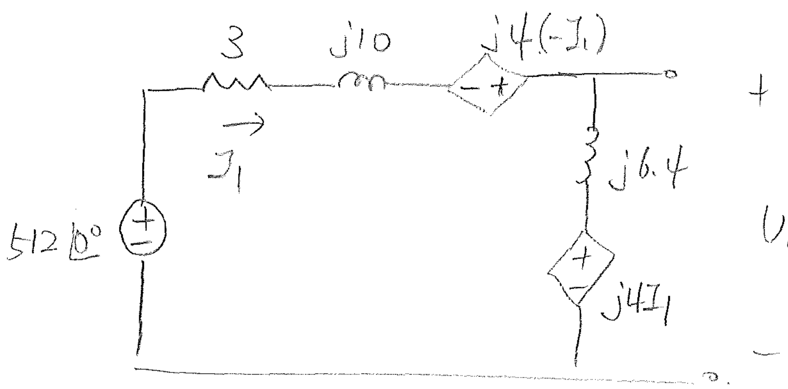
Q3. (a)



by setting the current I_1 and I_2 direction as above

mutual voltage < 0

(b) $V_{oc} \Rightarrow I_2 = 0$.



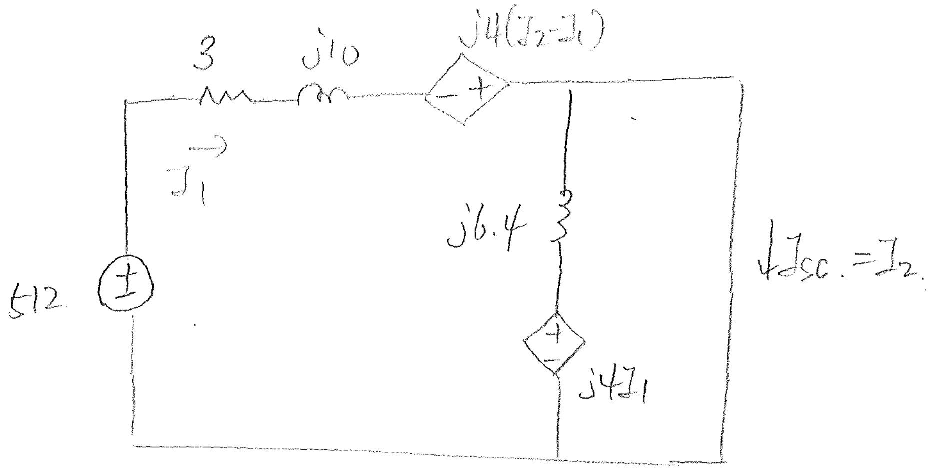
$$-512 + (3+j10)I_1 + j4I_1 + j6.4I_1 + j4I_1 = 0 \quad (+1)$$

$$I_1 = \frac{512}{3+j24.4} = 2.57 - j20.66 \quad (+1)$$

$$= 20.83\angle -82.90^\circ$$

$$V_{TH} = j6.4 \times I_1 + j4 \times I_1 = 216.53\angle 7.1^\circ$$

(+1) (+1)



① $-512 + (3+j10)I_1 - j4(I_2 - I_1) + j6.4(I_1 - I_2) + j4I_1 = 0$ (+1)

② $-j4I_1 + j6.4(I_2 - I_1) = 0$ (+1)

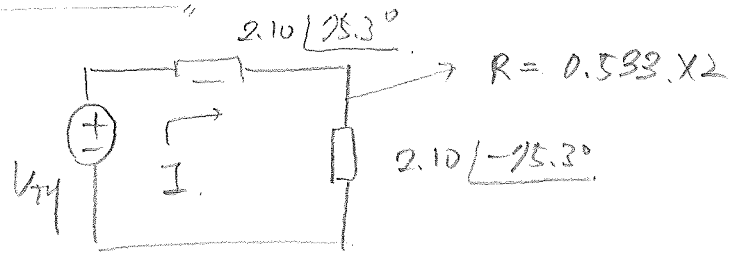
Solving ① and ② $I_{sc} = I_2 = 38.25 - j95.63$ (+2)

$= 102.99\angle -68.20°$

(c) $Z_{TH} = \dfrac{V_{oc}}{I_{sc}} = 2.10\angle 75.3°$ (+2)

(d) $Z_0 = Z_{TH}^* = 2.10\angle -75.3$ (+2)

Max power

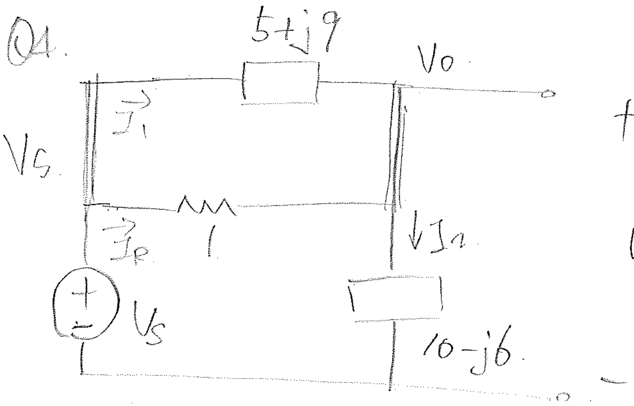


$I = \dfrac{216.53\angle 2.1°}{0.533 \times 2} = 203.12\angle 2.1°$ (+1)

$P_{max} = 203.12^2 \times 0.533 = 21.99\,KW$ (+2)

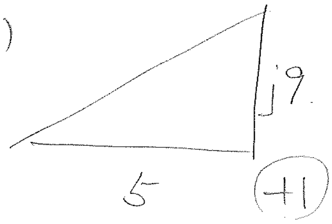
$V_{TH}^2 / 4R_{Th}$ is based on Peak Value not RMS

Q4.



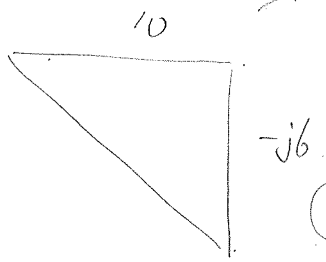
$V_o = 2\angle 0°$

(a)



(+1)

with or without j either is okay.



(+1)

$pf = \cos \tan^{-1}\frac{9}{5} = 0.49$
Lagging. (+1)

$pf = \cos \tan^{-1}\frac{-6}{10} = 0.86$
leading. (+1)

(b) Black box 2

$S_2 = V_o \times I_2^* = 10 - j6 \Rightarrow I_2^* = \dfrac{10-j6}{2\angle 0°} = 5 - j3$

thus $I_2 = 5 + j3$ (+2)

Voltage at R, $V_R = V_S - V_o = I_R \times 1$,

$I_R = V_S - 2$ or $a + jb$. (+1)

by KCL $I_2 = I_1 + I_R$, $I_1 = (5-a) + j(3-b)$ (+1)

Black box 1

$S_1 = (V_S - 2) I_1^* = (a+jb) \times (5 - a + j(3-b)) = 5 + j9$

$P = 5a - a^2 + 3b - b^2 = 5$ (+1)

$Q = -3a + 5b = 9$ (+1)

Solve P and Q we get $a = 0.8$ and $b = 2.28$ (+1)

or $a = 2.61$ and $b = 3.37$ (+1)

therefore, by $V_s - 2 = a + jb$.

$$V_s = 2.8 + j2.28 \text{ or } 3.61 \angle 39.16^\circ \quad (+2)$$

$$\text{or } V_s = 4.61 + j3.37 \text{ or } 5.71 \angle 36.17^\circ \quad (+2)$$

(C) $\omega = 100$ rad/s

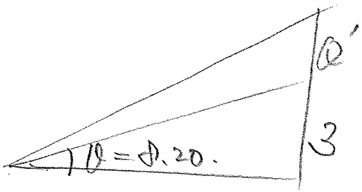
(1) for $V_s = 3.61 \angle 39.16^\circ$ or $2.8 + j2.28$

at R, $S_R = (2.8 + j2.28 - 2)(0.8 - j2.28)$

$$= 0.8^2 + 2.28^2 = 5.84$$

total $S = 5 + j9 + 10 - j6 + 5.84 = 20.84 + j3$

$pf = 0.9898 \quad \theta = 8.20^\circ$ (+1)



pf to be 0.85 $\theta = 31.79^\circ$

inductor needed (+1)

$$\tan\theta' = \frac{Q' + 3}{20.84} \qquad Q' = 3.85 \text{ VAR}$$

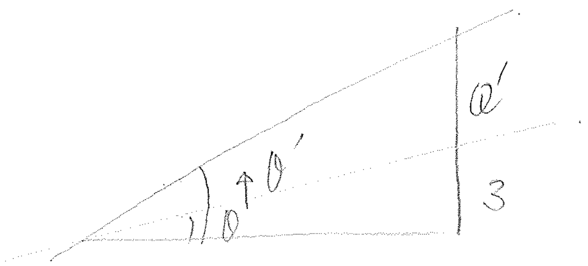
$$3.85 = \frac{3.61^2}{100 \times L} \qquad L = 0.034 \text{ H} \quad (+2)$$

② for $V_S = 5.71\angle 36.17°$ or $4.61 + j3.37$

$S_R = (4.61 + j3.37 - 2)(2.61 - j3.37) = 2.61^2 + 3.37^2 = 18.17$

total $S = 5 + j9 + 10 - j6 + 18.17 = \underline{33.17 + j3} \Rightarrow \underline{pf = 0.9959}$ (+1)

$\underline{\theta = 5.71°}$



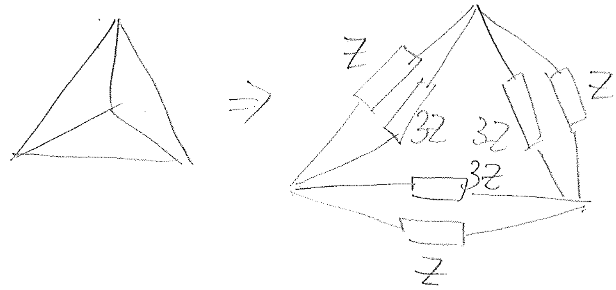
pf to be 0.95 $\theta' = 18.19$ $\underline{\text{inductor is needed.}}$ (+1)

$\tan\theta' = \dfrac{Q' + 3}{33.17}$ $Q' = 7.90$ VAR required.

$7.90 = \dfrac{5.71^2}{100 \times L}$ $\underline{L = 0.041\ H}$ (+2)

Q5. (a) Change Y load to Δ.

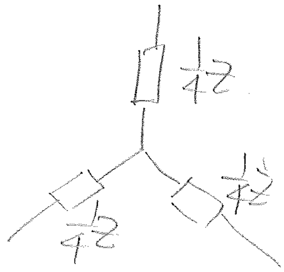
$Z_\Delta = 3Z_Y$.



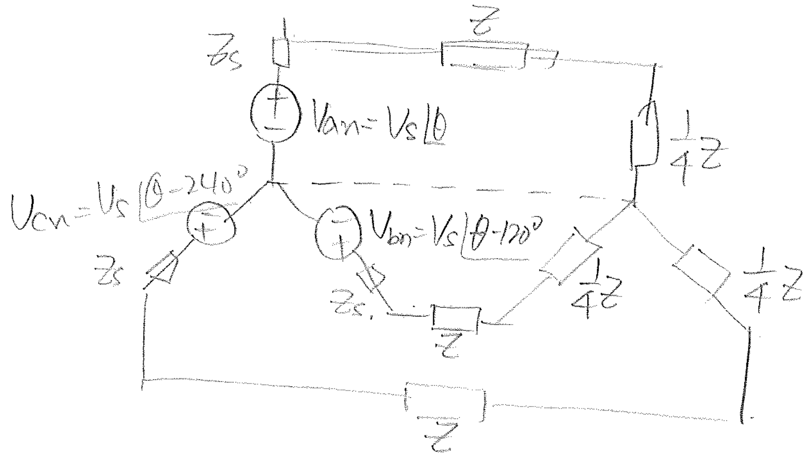
now we have two parallel Δ with the values of Z and 3Z.

$$Z_{\Delta\text{-}eq} = \frac{3Z^2}{3Z+Z} = \frac{3}{4}Z \quad (+1)$$

Change Δ back to Y, $Z_Y = \frac{1}{3}Z_\Delta$



(+1)



(+2)

(b) Y-Y circuit. Line current = phase current.

$$I_a = \frac{V_s\angle\theta}{Z_s + Z + \frac{1}{4}Z} = \frac{V_s\angle\theta}{Z_s + \frac{5}{4}Z}$$

$$I_b = \frac{V_s\angle\theta-120°}{Z_s + \frac{5}{4}Z}, \quad I_c = \frac{V_s\angle\theta-240°}{Z_s + \frac{5}{4}Z} \quad \text{each } (+1)$$

define as V_L

Line voltage drop by Z_ℓ and Z_L.

$$V_{L-a} = \frac{V_s\angle\theta}{Z_s+\frac{5}{4}Z}\times\left(Z+\frac{1}{4}Z\right) = \frac{\frac{5}{4}Z\times V_s\angle\theta}{Z_s+\frac{5}{4}Z}$$ each (+1)

$$V_{L-b} = \frac{\frac{5}{4}Z\times V_s\angle\theta-120^\circ}{Z_s+\frac{5}{4}Z}, \quad V_{L-c} = \frac{\frac{5}{4}Z\times V_s\angle\theta-240^\circ}{Z_s+\frac{5}{4}Z}$$

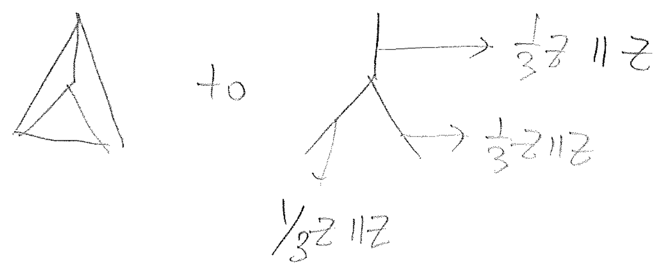
typo it shows (b)

(C) $S_a = I_a^2 Z_Y = \frac{1}{4}Z\times\frac{V_s^2}{(Z_s+5/4Z)^2}$ (+3)

thus total complex power $S = 3\times S_a = \frac{3}{4}Z\times\frac{V_s^2}{(Z_s+5/4Z)^2}$ (+3)

typo it shows (c).

(d) in Y-Y, we have $I_a = \dfrac{V_s\angle\theta}{Z_s+5/4Z}$

Change 

(+2)

by current division, $$I_s = \frac{\frac{1}{3}Z}{\frac{1}{3}Z+Z}\,\frac{V_s\angle\theta}{Z_s+5/4Z}$$

$$= \frac{1}{4}\,\frac{V_s\angle\theta}{Z_s+5/4Z}$$ (+2)