ECE215 Final RC Part 3

Chapter 13 Magnetically Coupled Circuits

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Magnetically Coupled Circuits

Inductance comes from magnetic flux.

$$v = N\frac{d\phi}{dt} = N\frac{d\phi}{di}\frac{di}{dt} = L\frac{di}{dt}$$

$L = N\frac{d\phi}{DI}$ the self inductance of the coil

If two inductors are coupled, they will be affected by mutual inductance M.

The inducted voltage v=M$\frac{di}{dt}$

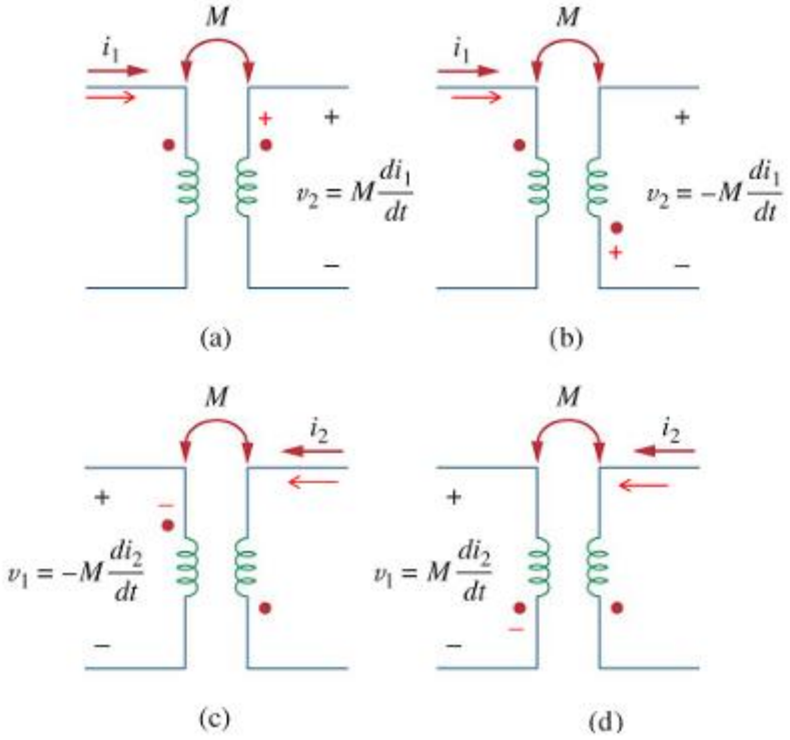
Remember the unit of M (H) henrys

Dot Convention

We use dot convention to reduce the need of using Lens' law.

The dot convention is stated as follows:

If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage of the second coil is positive at the dotted terminal of the second coil.

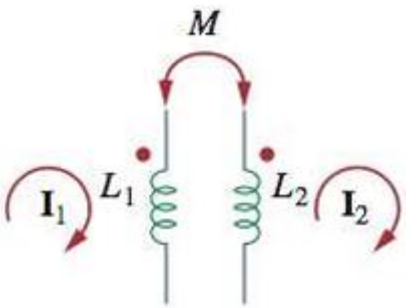


Understand or copy to your CTPP

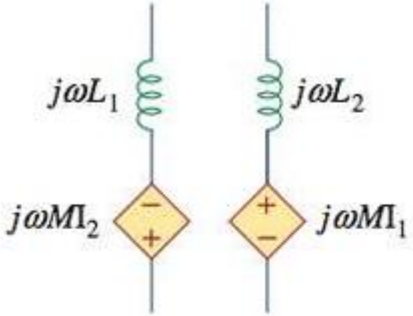
Dot Convention

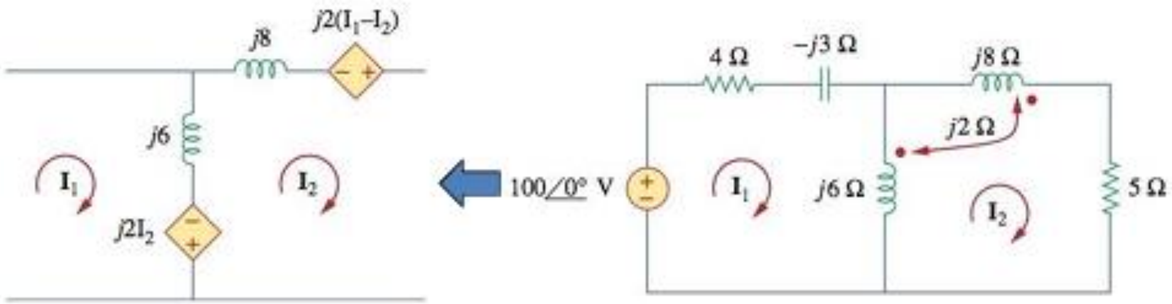
For simplicity on analyzing, we assign a dependent voltage source

Pay attention to the current value



Phasor representation





Energy in Magnetically Coupled Circuits

How to prove $M_{12} = M_{21} = M$ refers to slide #28

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

Positive if both entering the dot/ leaving the dot, otherwise negative.

$M \leq \sqrt{L_1 L_2}$ due to the power shouldn't be negative

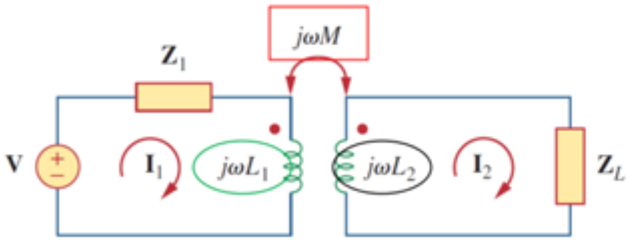
$$k = \frac{M}{\sqrt{L_1 L_2}}$$

defines how two coils are coupled.

k>0.5 tightly, k<0.5 loosely.

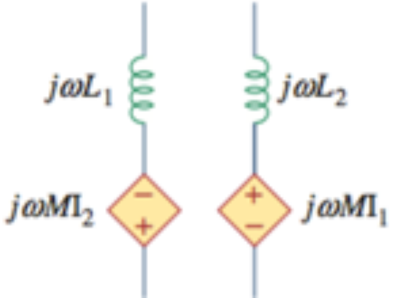
KVL analysis

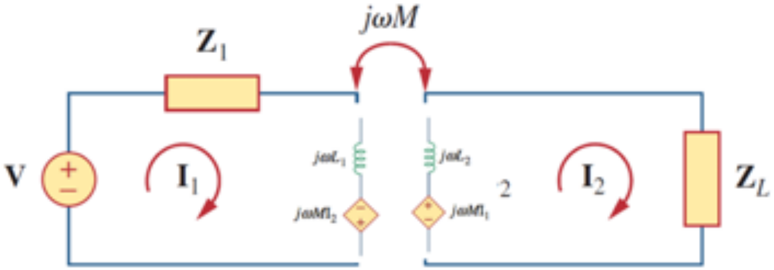
Pay attention to the dot convention



Loop 1 $\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - \boxed{j\omega M\mathbf{I}_2}$

Loop 2 $0 = \boxed{-j\omega M\mathbf{I}_1} + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2$





Loop 1

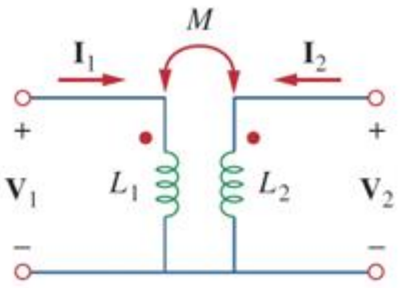
$$\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$

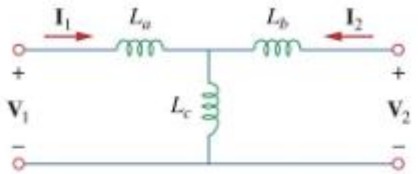
Loop 2

$$0 = -j\omega M\mathbf{I}_1 + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2$$

Equivalent circuit

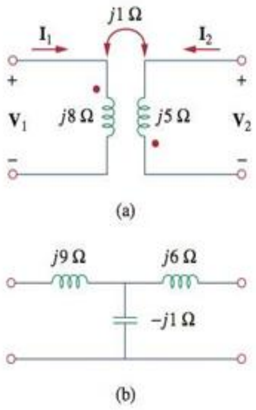
Equivalent T circuit





$$L_a = L_1 - M, L_b = L_2 - M, L_c = M$$

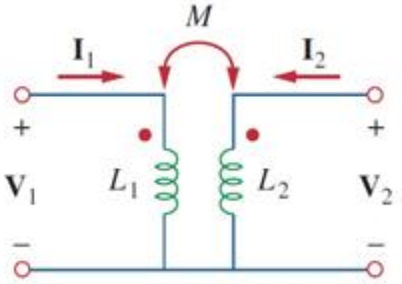
Dot conventions!
Should be -M if not both entering at dots

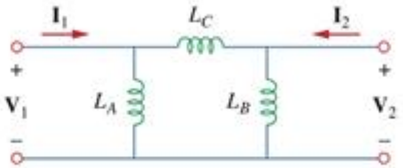


$$L_a = L_1 - (-M) = 8 + 1 = 9\text{ H}$$
$$L_b = L_2 - (-M) = 5 + 1 = 6\text{ H}, \qquad L_c = -M = -1\text{ H}$$

Dots on different sides → M changes to $-M$

Equivalent π circuit





$$L_A = \frac{L_1L_2 - M^2}{L_2 - M}, L_B = \frac{L_1L_2 - M^2}{L_1 - M}, L_C = \frac{L_1L_2 - M^2}{M}$$

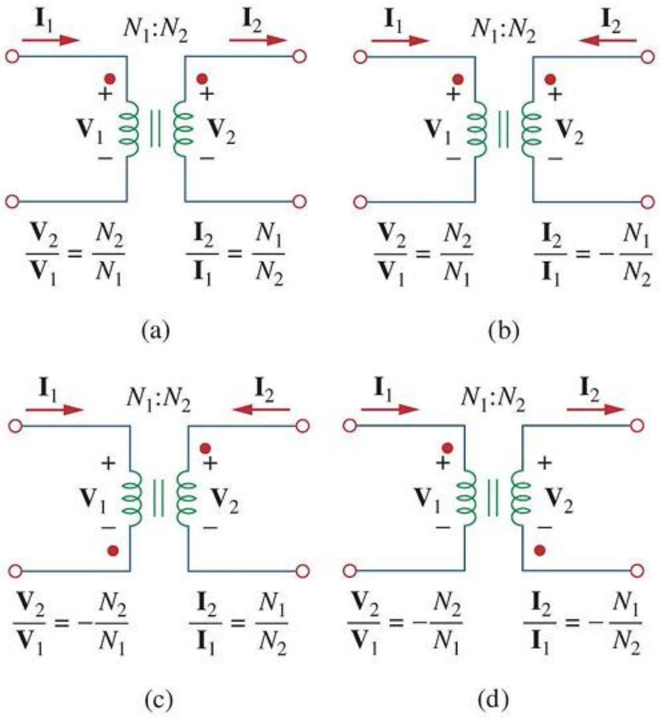
Ideal transformers

K=1, L=∞, indicated by vertical lines
Dot conventions!

Determine the sign of ration

(a) Voltages: both positive at the dotted terminals → +n
Currents: I_1 enters whereas I_2 leaves the dotted terminal → +n

(b) Voltages: both positive at the dotted terminals → +n
Currents: Both enter the dotted terminals → -n



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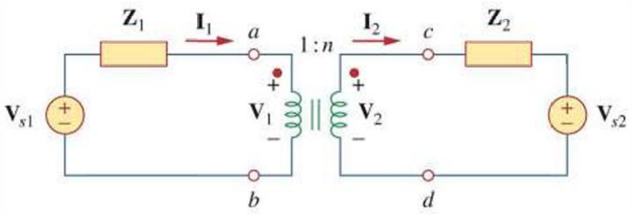
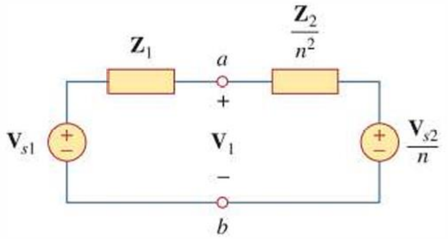
Thevenin equivalent circuit

Use to simplify the circuit

Only apply when there are no external connections





$I_1 = 0$

$I_2 = 0$

$V_{s2} = V_2$

$V_{TH} = \frac{V_{s2}}{n}$

Z_{TH}

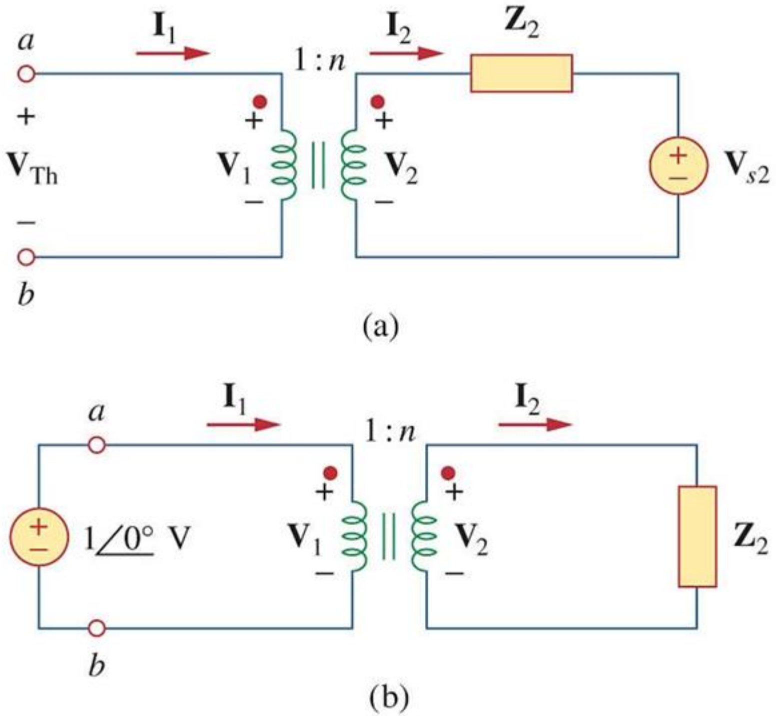


Figure 13.34 Obtaining the Thevenin equivalent for the circuit in Fig. 13.33.

$V_1 = 1$

$V_2 = n$

$I_2 = \frac{n}{Z_2}$

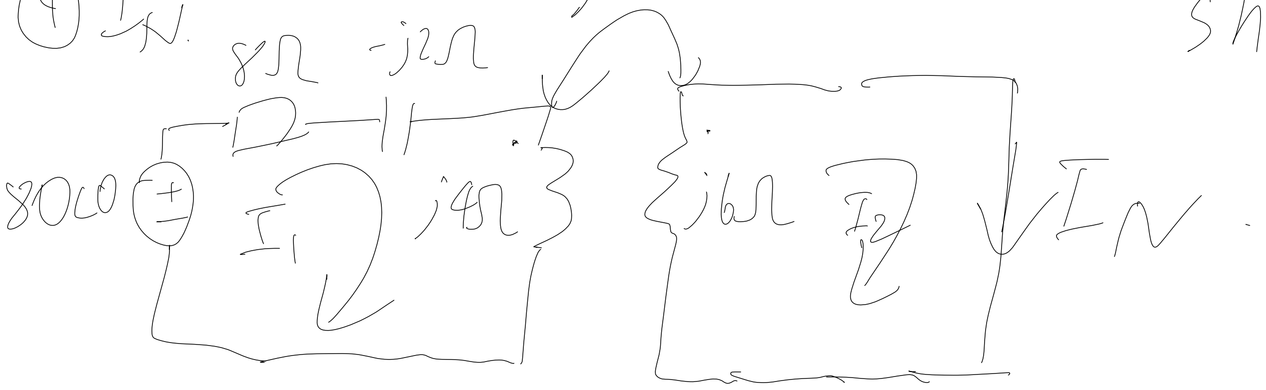
$I_1 = \frac{n^2}{Z_2}$

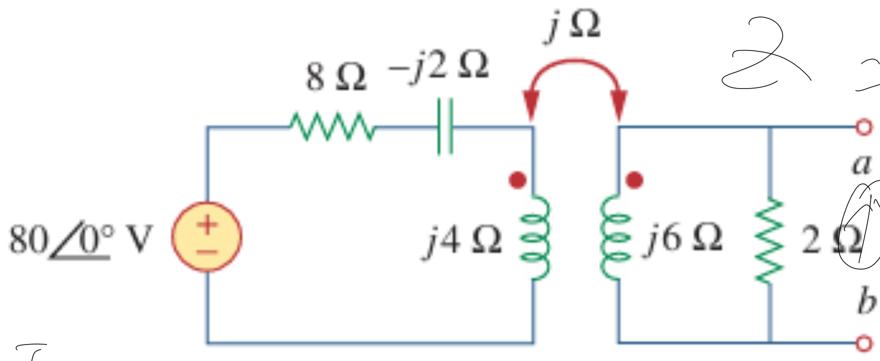
Exercise

Obtain the Norton equivalent at the terminals a-b

① I_N.

short circuit current



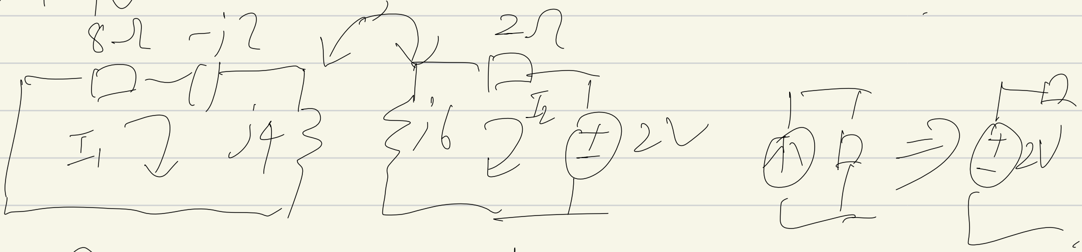


$Z = \frac{V}{I}$

① 1A

KVL
$$\begin{cases} -80+(8-j2)\cdot I_1 + j4 I_1 - jI_2 = 0. \\ -80+(8+j2)6I_2 - jI_2 = 0 \Rightarrow 80 = (48+15j)I_2 \\ j6I_2 - jI_1 = 0 \Rightarrow I_1 = 6I_2 \quad I_2 = \frac{80}{48+15j} = 1.58-j0.36 \end{cases}$$

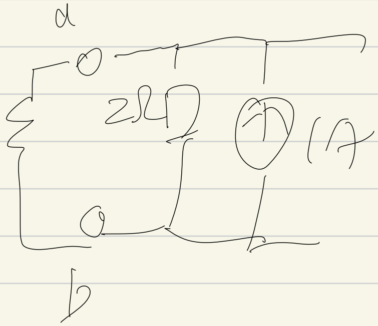
② R_N cut off ind → source



$$\begin{cases} (8-j2+j4)I_1 - jI_2 = 0 \Rightarrow I_1 = \frac{j}{8+j2} I_2 \\ (j6+2)I_2 + 2 - jI_1 = 0 \quad ② \end{cases}$$

$$I_2 = \frac{-2}{2+j6+\frac{1}{8+j2}} = -0.106+j0.298$$

$$V_{ab} = 2 + I_2 \cdot 2 = 1.788+j0.596$$

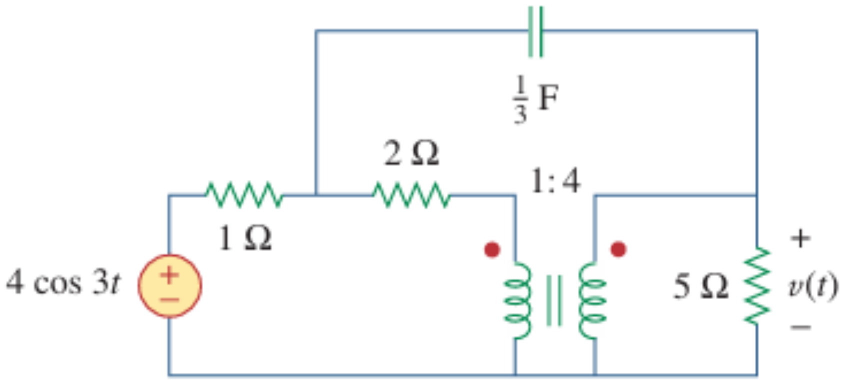


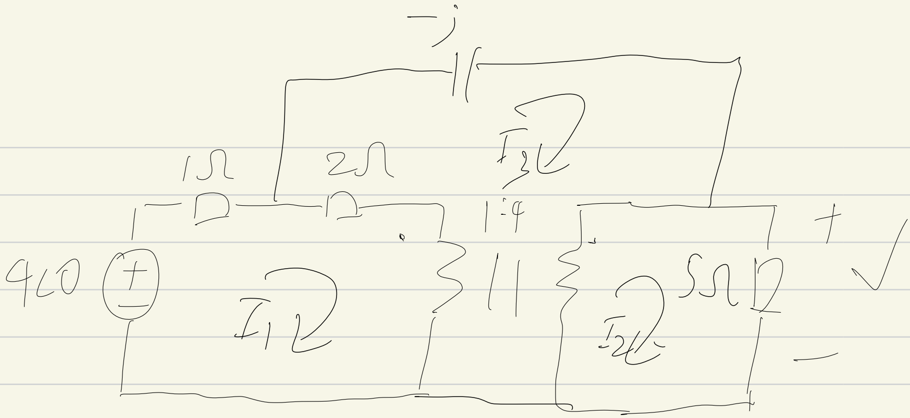
V_{ab}

$$Z_N = \frac{V_{ab}}{1} = 1.788+j0.596$$

Exercise

Find v(t) for the circuit





KVL
$$\begin{cases} I_1 + (I_1 - I_3)\cdot 2 - 4 + V_1 = 0 & (1) \\ 5I_2 - V_2 = 0 \Rightarrow V_2 = 5\cdot I_2 \\ V_2 - V_1 + 2(I_3 - I_1) - jI_3 = 0. \end{cases}$$

$$V_2 = 4V_1,\ (I_1 - I_3) = 4(I_2 - I_3) \quad (2).$$

$$V_2 = 5I_2, \quad V_1 = \frac{5}{4} I_2. \qquad I_1 = 4I_2 - 3I_3$$

$$(1) \Rightarrow 3I_1 - 2I_3 - 4 + \frac{5}{4} I_2 = 0$$

$$12 I_2 - 9 I_3 - 2 I_3 - 4 + \frac{5}{4} I_2 = 0$$

$$\Rightarrow I_3 = \frac{53}{44} I_2 - \frac{4}{11} \Rightarrow I_1 = \frac{17}{44} I_2 + \frac{12}{11}$$

$$\frac{15}{4} I_2 + (2 - j)\left(\frac{53}{44} I_2 - \frac{4}{11}\right) - 2\left(\frac{17}{44} I_2 + \frac{12}{11}\right) = 0.$$

$$\frac{237}{44} I_2 - j\frac{53}{44} I_2 = \frac{32}{11} - j\frac{4}{11} \Rightarrow I_2 = 0.529 + j0.051$$

$$V = 2.644 + j\,0.254$$

$V = 2.656 \angle 5.48^{\circ}$

$v(t) = 2.656 \cos(3t + 5.48)$

Tips

Review Homework and previous final exam

Understand the principle

If you can't, copy to CTPP

Good Luck!

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Reference

1. 2024 Fall VE215 slides, Sung-Liang Chen
2. 2023 Fall Final RC, Hengyi Cai
3. 2024 Fall RC 6, Yutin Cao

THANK YOU!

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