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Scientific computing with
Python
for beginners



Publisher

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Chapter 1

Introduction



1.1 History

- Project initiated by Guido Von Rossum in 1990
- Interpreted language written in C.
- Widely used in all domains (Web, Data Science, Scientific Computation).
- This is a high level language with a simple syntax.
- Python types are numerous and powerful.
- Bind Python with other languages is easy.
- You can perform a lot of operations with very few lines.
- Available on all platforms Unix, Windows, Mac OS X...
- Many libraries offer Python bindings.
- Python 2 is retired use only Python 3

1.2 Python distributions

Python packages are available with all linux distributions but you can get standalone bundles:

- [Anaconda](#)
- [Enthought Tools Suite](#)

- [Astropy](#)
- [SAGEMATH](#)
- [Pyzo](#)

1.3 Performances

Python is not fast... but: - Sometimes it is. - Most of operations are optimized. - Package like numpy can reduce the CPU time. - With Python you can save time to achieve your project.

Some advices: - Write your program with Python language. - If it is fast enough, be happy. - After profiling, optimize costly parts of your code.

”Premature optimization is the root of all evil” (Donald Knuth 1974)

1.4 Jupyter - Start The Notebook

Open the notebook

```
git clone https://github.com/pnavaro/python-notebooks
cd python-notebooks/notebooks
jupyter notebook
```

You should see the notebook open in your browser. If not, go to <http://localhost:8888>

The Jupyter Notebook is an interactive environment for writing and running code. The notebook is capable of running code in a wide range of languages. However, each notebook is associated with Python3 kernel.

1.5 Code cells allow you to enter and run code

Make a copy of this notebook by using the File menu.

Run a code cell using **Shift-Enter** or pressing the button in the toolbar above:

There are two other keyboard shortcuts for running code:

- **Alt-Enter** runs the current cell and inserts a new one below.
- **Ctrl-Enter** run the current cell and enters command mode.

1.6 Managing the Kernel

Code is run in a separate process called the Kernel. The Kernel can be interrupted or restarted. Try running the following cell and then hit the

button in the toolbar above.

The ”Cell” menu has a number of menu items for running code in different ways. These includes:

- Run and Select Below
- Run and Insert Below
- Run All
- Run All Above
- Run All Below

1.7 Restarting the kernels

The kernel maintains the state of a notebook’s computations. You can reset this state by restarting the kernel. This is done by clicking on the

in the toolbar above.

Check the [documentation](#).

1.8 First program

- Print out the string "Hello world!" and its type.
- Print out the value of a variable set to 6625 and its type.

```
In [1]: s = "Hello World!"
        print(type(s),s)
```

```
<class 'str'> Hello World!
```

```
In [2]: a = 6625
        print(type(a),a)
```

```
<class 'int'> 6625
```

```
In [3]: # a+s
```

1.9 Execute using python

```
In [4]: %%file hello.py
```

```
s = "Hello World!"
print(type(s),s)
a = 6625
print(type(a),a)
```

Writing hello.py

```
$ python3 hello.py
<class 'str'> Hello World!
<class 'int'> 6625
```

1.10 Execute with ipython

```
(my-env) $ ipython
Python 3.6.3 | packaged by conda-forge | (default, Nov  4 2017, 10:13:32)
Type 'copyright', 'credits' or 'license' for more information
IPython 6.2.1 -- An enhanced Interactive Python. Type '?' for help.
```

```
In [1]: run hello.py
<class 'str'> Hello World!
<class 'int'> 6625
```

```
In [5]: %run hello.py
```

```
<class 'str'> Hello World!
<class 'int'> 6625
```

1.11 Python Types

- Most of Python types are classes, typing is dynamic.
- ; symbol can be used to split two Python commands on the same line.

```
In [6]: s = int(2010); print(type(s))
        s = 3.14; print(type(s))
        s = True; print(type(s))
        s = None; print(type(s))
        s = 1.0j; print(type(s))
        s = type(type(s)); print(type(s))
```

```
<class 'int'>
<class 'float'>
<class 'bool'>
<class 'NoneType'>
<class 'complex'>
<class 'type'>
```

1.12 Calculate with Python

```
In [7]: x = 45          # This is a comment!
        x += 2          # equivalent to x = x + 2
        print(x, x > 45)
```

```
47 True
```

```
In [8]: y = 2.5
        print("x+y=",x+y, type(x+y)) # Add float to integer, result will be a float
```

```
x+y= 49.5 <class 'float'>
```

```
In [9]: print(x*10/y)    # true division returns a float
        print(x*10//3)   # floor division discards the fractional part
```

```
188.0
156
```

```
In [10]: print( x % 8) # the % operator returns the remainder of the division
```

```
7
```

```
In [11]: print( f" x = {x:05d} ") # You can use C format rules to improve print output

x = 00047
```

1.13 Multiple Assignment

- Variables can simultaneously get new values.
- Expressions on the right-hand side are all evaluated first before assignments take place.
- The right-hand side expressions are evaluated from the left to the right.

- Use it very carefully

```
In [12]: a = b = c = 1  
        print(a, b, c)
```

```
1 1 1
```

```
In [13]: a, b, c = 1, 2, 3  
        print (a, b, c)
```

```
1 2 3
```

```
In [14]: a, c = c, a      # Nice way to permute values  
        print (a, b, c)
```

```
3 2 1
```

```
In [15]: a < b < c, a > b > c
```

```
Out[15]: (False, True)
```

1.14 input Function

- Value returned by input is a string.
- You must cast input call to get the type you want.

```
name = input("Please enter your name: ")  
x = int(input("Please enter an integer: "))  
L = list(input("Please enter 3 integers "))
```

Copy-pase code above in three different cells and print returned values.

Chapter 2

Strings

```
In [1]: word = "bonjour"
```

```
In [2]: print(word, len(word))
```

```
bonjour 7
```

Add a `.` to the variable and then press `<TAB>` to get all attached methods available.

```
In [3]: word.capitalize()
```

```
Out[3]: 'Bonjour'
```

After choosing your method, press `shift+<TAB>` to get interface.

```
In [4]: word.upper()
```

```
Out[4]: 'BONJOUR'
```

```
In [5]: help(word.replace) # or word.replace?
```

Help on built-in function replace:

replace(old, new, count=-1, /) method of builtins.str instance

Return a copy with all occurrences of substring old replaced by new.

count

Maximum number of occurrences to replace.

-1 (the default value) means replace all occurrences.

If the optional argument count is given, only the first count occurrences are replaced.

```
In [6]: word.replace('o','0',1)
```

```
Out[6]: 'b0njour'
```

2.1 Strings and print Function

Strings can be enclosed in single quotes ('...') or double quotes ("...") with the same result. `\` can be used to escape quotes:

```
In [7]: print('spam eggs')           # single quotes
        print('doesn\'t')           # use \' to escape the single quote...
        print("doesn't")           # ...or use double quotes instead
        print('"Yes," he said.')    #
        print("\\"Yes,\" he said.")
        print('"Isn\'t," she said.')
```

```
spam eggs
doesn't
doesn't
"Yes," he said.
"Yes," he said.
"Isn't," she said.
```

`print` function translates C special characters

```
In [8]: s = '\tFirst line.\nSecond line.' # \n means newline \t inserts tab
        print(s) # with print(), \n produces a new line
        print(r'\tFirst line.\nSecond line.') # note the r before the quote
```

```
First line.
Second line.
\tFirst line.\nSecond line.
```

2.2 String literals with multiple lines

```
In [9]: print("""\
Usage: thingy [OPTIONS]
    -h                Display this usage message
    -H hostname       Hostname to connect to
""")
```

```
Usage: thingy [OPTIONS]
    -h                Display this usage message
    -H hostname       Hostname to connect to
```

`r` character removes the initial newline.

Strings can be concatenated (glued together) with the `+` operator, and repeated with `*`

```
In [10]: 3 * ("Re" + 2 * 'n' + 'es ')
```

```
Out[10]: 'Rennes Rennes Rennes '
```

Two or more string literals next to each other are automatically concatenated.

```
In [11]: text = ('Put several strings within parentheses '
                 'to have them joined together.')
        text
```

```
Out[11]: 'Put several strings within parentheses to have them joined together.'
```

Strings can be indexed, with the first character having index 0. There is no separate character type; a character is simply a string of size one

```
In [12]: word = 'Python @ ENSAI'
         print(word[0]) # character in position 0
         print(word[5]) # character in position 5
```

```
P
n
```

Indices may also be negative numbers, to start counting from the right

```
In [13]: print(word[-1]) # last character
         print(word[-2]) # second-last character
```

```
I
A
```

2.3 Slicing Strings

- Omitted first index defaults to zero,
- Omitted second index defaults to the size of the string being sliced.
- Step can be set with the third index

```
In [14]: print(word[:2]) # character from the beginning to position 2 (excluded)
         print(word[4:]) # characters from position 4 (included) to the end
         print(word[-2:]) # characters from the second-last (included) to the end
         print(word[::-1]) # This is the reversed string!
```

```
Py
on @ ENSAI
AI
IASNE @ nohtyP
```

```
In [15]: word[:2]
```

```
Out[15]: 'Pto NA'
```

Python strings cannot be changed — they are immutable. If you need a different string, you should create a new one or use Lists.

```
In [16]: word[0] = 'J'
```

```
-----
TypeError                                Traceback (most recent call last)

<ipython-input-16-91a956888ca7> in <module>
----> 1 word[0] = 'J'

TypeError: 'str' object does not support item assignment
```

```
In [17]: ## Some string methods
         print(word.startswith('P'))
```

True

```
In [18]: print(*("\n"+w for w in dir(word) if not w.startswith('_')) )
```

capitalize
casefold
center
count
encode
endswith
expandtabs
find
format
format_map
index
isalnum
isalpha
isascii
isdecimal
isdigit
isidentifier
islower
isnumeric
isprintable
isspace
istitle
isupper
join
ljust
lower
lstrip
maketrans
partition
replace
rfind
rindex
rjust
rpartition
rsplit
rstrip
split
splitlines
startswith
strip
swapcase
title
translate
upper
zfill

2.4 Exercise

- Ask user to input a string.
- Print out the string length.
- Check if the last character is equal to the first character.
- Check if this string contains only letters.
- Check if this string is lower case.
- Check if this string is a palindrome. A palindrome is a word, phrase, number, or other sequence of characters which reads the same backward as forward.

In [19]: `# %load solutions/strings/demo.py`

Chapter 3

Python lists and tuples

- List is the most versatile Python data type to group values with others
- Can be written as a list of comma-separated values (items) between square brackets.
- Tuples are written between parenthesis. They are read-only lists.
- Lists can contain items of different types.
- Like strings, lists can be indexed and sliced.
- Lists also support operations like concatenation.

3.1 Indexing

```
In [1]: squares = [1, 4, 9, 16, 25]
        print(squares)
```

```
[1, 4, 9, 16, 25]
```

```
In [2]: print(squares[0])  # indexing returns the item
```

```
1
```

```
In [3]: squares[-1]
```

```
Out[3]: 25
```

```
In [4]: squares[-3:]  # slicing returns a new list
```

```
Out[4]: [9, 16, 25]
```

```
In [5]: squares += [36, 49, 64, 81, 100]
        print(squares)
```

```
[1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
```

- Unlike strings, which are immutable, lists are a mutable type.

```
In [6]: cubes = [1, 8, 27, 65, 125]  # something's wrong here
        cubes[3] = 64  # replace the wrong value, the cube of 4 is 64, not 65!
        print(cubes)
```

```
[1, 8, 27, 64, 125]
```

```
In [7]: cubes.append(216)  # add the cube of 6
        print(cubes)
```

```
[1, 8, 27, 64, 125, 216]
```

```
In [8]: cubes.remove(1)
        print(cubes)
```

```
[8, 27, 64, 125, 216]
```

3.2 Assignment

- You can change the size of the list or clear it entirely.
- The built-in function `len()` returns list size.
- It is possible to create lists containing other lists.

```
In [9]: letters = ['a', 'b', 'c', 'd', 'e', 'f', 'g']
        letters[2:5] = ['C', 'D', 'E'] # replace some values
        print(letters)
```

```
['a', 'b', 'C', 'D', 'E', 'f', 'g']
```

```
In [10]: letters[2:5] = [] # now remove them
         print(letters)
```

```
['a', 'b', 'f', 'g']
```

```
In [11]: a = ['a', 'b', 'c']
         n = [1, 2, 3]
         x = [a, n]
```

```
In [12]: x
```

```
Out[12]: [['a', 'b', 'c'], [1, 2, 3]]
```

```
In [13]: x[0]
```

```
Out[13]: ['a', 'b', 'c']
```

```
In [14]: x[0][1], len(x)
```

```
Out[14]: ('b', 2)
```

3.3 Assignment, Copy and Reference

```
In [15]: a = [0, 1, 2, 3, 4]
         b = a
         print("b = ", b)
```

```
b = [0, 1, 2, 3, 4]
```

```
In [16]: b[1] = 20          # Change one value in b
         print("a = ", a)  # Y
```



```
a = [0, 20, 2, 3, 4]
```

b is a reference to a, they occupy same space memory

```
In [17]: b = a[:] # assign a slice of a and you create a new list
         b[2] = 10
         print("b = ",b)
         print("a = ",a)

b = [0, 20, 10, 3, 4]
a = [0, 20, 2, 3, 4]
```

3.4 Some useful List Methods

```
In [18]: a = list("Python-2020")
         a

Out[18]: ['P', 'y', 't', 'h', 'o', 'n', '-', '2', '0', '2', '0']

In [19]: a.sort()
         a

Out[19]: ['-', '0', '0', '2', '2', 'P', 'h', 'n', 'o', 't', 'y']

In [20]: a.reverse()
         a

Out[20]: ['y', 't', 'o', 'n', 'h', 'P', '2', '2', '0', '0', '-']

In [21]: a.pop() #pop the last item and remove it from the list
         a

Out[21]: ['y', 't', 'o', 'n', 'h', 'P', '2', '2', '0', '0']
```

3.5 Dictionary

They are indexed by keys, which are often strings.

```
In [22]: person = dict(firstname="John", lastname="Smith", email="john.doe@domain.fr")
         person['size'] = 1.80
         person['weight'] = 70

In [23]: person

Out[23]: {'firstname': 'John',
         'lastname': 'Smith',
         'email': 'john.doe@domain.fr',
         'size': 1.8,
         'weight': 70}

In [24]: print(person.keys())

dict_keys(['firstname', 'lastname', 'email', 'size', 'weight'])

In [25]: print(person.items())

dict_items([('firstname', 'John'), ('lastname', 'Smith'), ('email', 'john.doe@domain.fr'), ('size', 1.8), ('weight', 70)])
```

3.6 Exercises

- Split the string "python ENSAI 2020" into the list ["python","ENSAI", 2020]
 - Insert "september" and value 7 before 2020 in the result list.
 - Capitalize the first item to "Python"
 - Create a dictionary with following keys (meeting, month, day, year)
 - Print out the items.
 - Append the key "place" to this dictionary and set the value to "ENSAI".
- ```
python
['python', 'ENSAI', '2020']
['python', 'ENSAI', 'september', 7, '2020']
['Python', 'ENSAI', 'september', 7, '2020']
{'course': 'Python', 'september': 'september', 'day': 7, 'year': '2020', 'place': 'ENSAI'}
```

# Chapter 4

## Control Flow Tools

### 4.1 While loop

- Don't forget the ':' character.
- The body of the loop is indented

```
In [1]: # Fibonacci series:
 # the sum of two elements defines the next
 a, b = 0, 1
 while b < 500:
 a, b = b, a+b
 print(round(b/a,5), end=",")
```

1.0,2.0,1.5,1.66667,1.6,1.625,1.61538,1.61905,1.61765,1.61818,1.61798,1.61806,1.61803,1.61804,

### 4.2 if Statements

True, False, and, or, not, ==, is, !=, is not, >, >=, <, <=

```
In [2]: x = 42
 if x < 0:
 x = 0
 print('Negative changed to zero')
 elif x == 0:
 print('Zero')
 elif x == 1:
 print('Single')
 else:
 print('More')
```

More

switch or case statements don't exist in Python.

#### 4.2.1 Exercise Collatz conjecture

Consider the following operation on an arbitrary positive integer: - If the number is even, divide it by two.  
- If the number is odd, triple it and add one.

The conjecture is that no matter what initial value of this integer, the sequence will always reach 1. -  
Test the Collatz conjecture for  $n = 100000$ . - How many steps do you need to reach 1 ?

### 4.3 Loop over an iterable object

We use `for` statement for looping over an iterable object. If we use it with a string, it loops over its characters.

```
In [3]: for c in "python":
 print(c)
```

```
p
y
t
h
o
n
```

```
In [4]: for word in "Python ENSAI september 7th 2020".split(" "):
 print(word, len(word))
```

```
Python 6
ENSAI 5
september 9
7th 3
2020 4
```

#### 4.3.1 Exercise: Anagram

An anagram is word or phrase formed by rearranging the letters of a different word or phrase, typically using all the original letters exactly once.

Write a code that print `True` if `s1` is an anagram of `s2`. To do it, remove every character present in both strings. Check you obtain two empty strings.

Hint: `s = s.replace(c, "", 1)` removes the character `c` in string `s` one time.

```
s1 = "pascal obispo"
s2 = "pablo picasso"
..
True
```

### 4.4 Loop with range function

- It generates arithmetic progressions
- It is possible to let the range start at another number, or to specify a different increment.
- Since Python 3, the object returned by `range()` doesn't return a list to save memory space. `xrange` no longer exists.
- Use function `list()` to creates it.

```
In [5]: list(range(5))
```

```
Out[5]: [0, 1, 2, 3, 4]
```

```
In [6]: list(range(2, 5))
```

```
Out[6]: [2, 3, 4]
```

```
In [7]: list(range(-1, -5, -1))
```

```
Out[7]: [-1, -2, -3, -4]
```

```
In [8]: for i in range(5):
 print(i, end=' ')
```

```
0 1 2 3 4
```

### 4.4.1 Exercise Exponential

- Write some code to compute the exponential mathematical constant  $e \simeq 2.718281828459045$  using the Taylor series developed at 0 and without any import of external modules:

$$e \simeq \sum_{n=0}^{50} \frac{1}{n!}$$

## 4.5 break Statement.

```
In [9]: for n in range(2, 10): # n = 2,3,4,5,6,7,8,9
 for x in range(2, n): # x = 2, ..., n-1
 if n % x == 0: # Return the division remain (mod)
 print(n, " = ", x, "*", n/x)
 break
 else:
 print("%d is a prime number" % n)
 break
```

```
3 is a prime number
4 = 2 * 2
5 is a prime number
6 = 2 * 3
7 is a prime number
8 = 2 * 4
9 is a prime number
```

### 4.5.1 iter Function

```
In [10]: course = """ Python september 7, 14 2020 ENSAI Rennes """.split()
 print(course)
```

```
['Python', 'september', '7,', '14', '2020', 'ENSAI', 'Rennes']
```

```
In [11]: iterator = iter(course)
 print(iterator.__next__())
```

```
Python
```

```
In [12]: print(iterator.__next__())
```

```
september
```

## 4.6 Defining Function: def statement

```
In [13]: def is_palindromic(s):
 "Return True if the input sequence is a palindrome"
 return s == s[::-1]
```

```
is_palindromic("kayak")
```

Out[13]: True

- Body of the function start must be indented
- Functions without a return statement do return a value called `None`.

```
In [14]: def fib(n):
 """Print a Fibonacci series up to n."""
 a, b = 0, 1
 while a < n:
 print(a, end=' ') # the end optional argument is \n by default
 a, b = b, a+b
 print("\n") # new line

 result = fib(2000)
 print(result) # is None

0 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597
```

None

## 4.7 Documentation string

- It's good practice to include docstrings in code that you write, so make a habit of it.

```
In [15]: def my_function(foo):
 """Do nothing, but document it.
 No, really, it doesn't do anything.
 """
 pass

 print(my_function.__doc__)
```

Do nothing, but document it.

No, really, it doesn't do anything.

```
In [16]: help(my_function)

Help on function my_function in module __main__:

my_function(foo)
 Do nothing, but document it.

 No, really, it doesn't do anything.
```

## 4.8 Default Argument Values

```
In [17]: def f(a,b=5):
 return a+b

 print(f(1))
 print(f(b="a",a="bc"))
```

```
6
bca
```

**Important warning:** The default value is evaluated only once.

```
In [18]: def f(a, L=[]):
 L.append(a)
 return L

 print(f(1))
```

```
[1]
```

```
In [19]: print(f(2)) # L = [1]
```

```
[1, 2]
```

```
In [20]: print(f(3)) # L = [1,2]
```

```
[1, 2, 3]
```

## 4.9 Function Annotations

Completely optional metadata information about the types used by user-defined functions. These type annotations conforming to [PEP 484](#) could be statically used by [MyPy](#).

```
In [21]: def f(ham: str, eggs: str = 'eggs') -> str:
 print("Annotations:", f.__annotations__)
 print("Arguments:", ham, eggs)
 return ham + ' and ' + eggs

 f('spam')
 help(f)
 print(f.__doc__)
```

```
Annotations: {'ham': <class 'str'>, 'eggs': <class 'str'>, 'return': <class 'str'>}
```

```
Arguments: spam eggs
```

```
Help on function f in module __main__:
```

```
f(ham: str, eggs: str = 'eggs') -> str
```

```
None
```

## 4.10 Arbitrary Argument Lists

Arguments can be wrapped up in a tuple or a list with form `*args`

```
In [22]: def f(*args, sep=" "):
 print(args)
 return sep.join(args)

 print(f("big", "data"))
```

```
('big', 'data')
big data
```

- Normally, these variadic arguments will be last in the list of formal parameters.
- Any formal parameters which occur after the `*args` parameter are ‘keyword-only’ arguments.

## 4.11 Keyword Arguments Dictionary

A final formal parameter of the form `**name` receives a dictionary.

```
In [23]: def add_contact(kind, *args, **kwargs):
 print(args)
 print("-" * 40)
 for key, value in kwargs.items():
 print(key, ":", value)
```

`*name` must occur before `**name`

```
In [24]: add_contact("John", "Smith",
 phone="555 8765",
 email="john.smith@python.org")
```

```
('Smith',)
```

```

```

```
NameError
```

```
Traceback (most recent call last)
```

```
<ipython-input-24-9581700007e6> in <module>
----> 1 add_contact("John", "Smith",
 2 phone="555 8765",
 3 email="john.smith@python.org")
```

```
<ipython-input-23-5a2bcf0288b1> in add_contact(kind, *args, **kwargs)
 2 print(args)
 3 print("-" * 40)
----> 4 for key, value in kwargs.items():
 5 print(key, ":", value)
```

```
NameError: name 'kwargs' is not defined
```

## 4.12 Lambda Expressions

Lambda functions can be used wherever function objects are required.

```
In [25]: f = lambda x : 2 * x + 2
 f(3)
```

```
Out[25]: 8
```



```
In [26]: taxicab_distance = lambda x_a,y_a,x_b,y_b: abs(x_b-x_a)+abs(y_b-y_a)
 print(taxicab_distance(3,4,7,2))
```

```
6
```

lambda functions can reference variables from the containing scope:

```
In [27]: def make_incrementor(n):
 return lambda x: x + n

 f = make_incrementor(42)
 f(0),f(1)
```

```
Out[27]: (42, 43)
```

## 4.13 Unpacking Argument Lists

Arguments are already in a list or tuple. They can be unpacked for a function call. For instance, the built-in `range()` function is called with the `*`-operator to unpack the arguments out of a list:

```
In [28]: def chessboard_distance(x_a, y_a, x_b, y_b):
 """
 Compute the rectilinear distance between
 point (x_a,y_a) and (x_b, y_b)
 """
 return max(abs(x_b-x_a),abs(y_b-y_a))

 coordinates = [3,4,7,2]
 chessboard_distance(*coordinates)
```

```
Out[28]: 4
```

In the same fashion, dictionaries can deliver keyword arguments with the `**`-operator:

```
In [29]: def parrot(voltage, state='a stiff', action='vroom'):
 print("-- This parrot wouldn't", action, end=' ')
 print("if you put", voltage, "volts through it.", end=' ')
 print("E's", state, "!")

 d = {"voltage": "four million", "state": "bleedin' demised", "action": "VOOM"}
 parrot(**d)
```

```
-- This parrot wouldn't VOOM if you put four million volts through it. E's bleedin' demised !
```

## 4.14 Exercise: Time converter

Write 3 functions to manipulate hours and minutes : - Function `minutes` return minutes from (hours, minutes). - Function `hours` the inverse function that return (hours, minutes) from minutes. - Function `add_time` to add (hh1,mm1) and (hh2, mm2) two couples (hours, minutes). It takes 2 tuples of length 2 as input arguments and return the tuple (hh,mm).

```
print(minutes(6,15)) # 375
print(minutes(7,46)) # 466
print(add_time((6,15),(7,46)) # (14,01)
```

## 4.15 Functions Scope

- All variable assignments in a function store the value in the local symbol table.
- Global variables cannot be directly assigned a value within a function (unless named in a global statement).
- The value of the function can be assigned to another name which can then also be used as a function.

```
In [30]: pi = 1.
 def deg2rad(theta):
 pi = 3.14
 return theta * pi / 180.

 print(deg2rad(45))
 print(pi)
```

0.785

1.0

```
In [31]: def rad2deg(theta):
 return theta*180./pi

 print(rad2deg(0.785))
 pi = 3.14
 print(rad2deg(0.785))
```

141.3

45.0

```
In [32]: def deg2rad(theta):
 global pi
 pi = 3.14
 return theta * pi / 180

 pi = 1
 print(deg2rad(45))
```

0.785

```
In [33]: print(pi)
```

3.14

## 4.16 enumerate Function

```
In [34]: primes = [1,2,3,5,7,11,13]
 for idx, ele in enumerate (primes):
 print(idx, " --- ", ele)
```

```
0 --- 1
1 --- 2
2 --- 3
3 --- 5
4 --- 7
5 --- 11
6 --- 13
```

### 4.16.1 Exercise: Caesar cipher

In cryptography, a Caesar cipher, is one of the simplest and most widely known encryption techniques. It is a type of substitution cipher in which each letter in the plaintext is replaced by a letter some fixed number of positions down the alphabet. For example, with a left shift of 3, D would be replaced by A, E would become B, and so on.

- Create a function `cipher` that take the plain text and the key value as arguments and return the encrypted text.
- Create a function `plain` that take the crypted text and the key value as arguments that return the deciphered text.

## 4.17 zip Builtin Function

Loop over sequences simultaneously.

```
In [35]: L1 = [1, 2, 3]
 L2 = [4, 5, 6]

 for (x, y) in zip(L1, L2):
 print (x, y, '--', x + y)

1 4 -- 5
2 5 -- 7
3 6 -- 9
```

## 4.18 List comprehension

- Set or change values inside a list
- Create list from function

```
In [36]: lsingle = [1, 3, 9, 4]
 ldouble = []
 for k in lsingle:
 ldouble.append(2*k)
 ldouble

Out[36]: [2, 6, 18, 8]

In [37]: ldouble = [k*2 for k in lsingle]

In [38]: [n*n for n in range(1,10)]

Out[38]: [1, 4, 9, 16, 25, 36, 49, 64, 81]

In [39]: [n*n for n in range(1,10) if n%1]

Out[39]: [1, 9, 25, 49, 81]

In [40]: [n+1 if n%1 else n//2 for n in range(1,10)]

Out[40]: [2, 1, 4, 2, 6, 3, 8, 4, 10]
```

### 4.18.1 Exercise

Code a new version of cypher function using list comprehension.

Hints: - `s = ''.join(L)` convert the characters list `L` into a string `s`. - `L.index(c)` return the index position of `c` in list `L` - `"c".islower()` and `"C".isupper()` return `True`

## 4.19 map built-in function

Apply a function over a sequence.

```
In [41]: res = map(hex,range(16))
 print(res)
```

```
<map object at 0x7fa1b896e130>
```

Since Python 3.x, `map` process return an iterator. Save memory, and should make things go faster. Display result by using unpacking operator.

```
In [42]: print(*res)
```

```
0x0 0x1 0x2 0x3 0x4 0x5 0x6 0x7 0x8 0x9 0xa 0xb 0xc 0xd 0xe 0xf
```

## 4.20 map with user-defined function

```
In [43]: def add(x,y):
 return x+y

 L1 = [1, 2, 3]
 L2 = [4, 5, 6]
 print(*map(add,L1,L2))
```

```
5 7 9
```

- `map` is often faster than `for` loop

```
In [44]: M = range(10000)
 f = lambda x: x**2
 %timeit lmap = list(map(f,M))
```

```
3.21 ms ± 39.1 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

```
In [45]: M = range(10000)
 f = lambda x: x**2
 %timeit lfor = [f(m) for m in M]
```

```
3.58 ms ± 30.9 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

## 4.21 filter

creates a iterator of elements for which a function returns `True`.

```
In [46]: number_list = range(-5, 5)
 odd_numbers = filter(lambda x: x & 1 , number_list)
 print(*odd_numbers)
```

```
-5 -3 -1 1 3
```

- As `map`, `filter` is often faster than `for` loop

```
In [47]: M = range(1000)
 f = lambda x: x % 3 == 0
 %timeit lmap = filter(f,M)
```

161 ns ± 0.494 ns per loop (mean ± std. dev. of 7 runs, 10000000 loops each)

```
In [48]: M = range(1000)
 %timeit lfor = (m for m in M if m % 3 == 0)
```

320 ns ± 3.49 ns per loop (mean ± std. dev. of 7 runs, 1000000 loops each)

## 4.22 Exercise with map:

Code a new version of your cypher function using map.

Hints: - Applied function must have only one argument, create a function called `shift` with the key value and use map.

## 4.23 Exercise with filter:

Create a function with a number `n` as single argument that returns True if `n` is a [Kaprekar number](#). For example 45 is a Kaprekar number, because

$$45^2 = 2025$$

and

$$20 + 25 = 45$$

Use `filter` to give Kaprekar numbers list lower than 10000.

1, 9, 45, 55, 99, 297, 703, 999, 2223, 2728, 4879, 4950, 5050, 5292, 7272, 7777, 9999

## 4.24 Recursive Call

```
“python slideshow={”slide_type”: ”fragment”} def gcd(x, y): """ returns the greatest common divisor."""
if x == 0: return y else : return gcd(y % x, x)
gcd(12,16) ““
```

## 4.25 Exercises

### 4.25.1 Factorial

- Write the function `factorial` with a recursive call

NB: Recursion is not recommended by [Guido](#).

### 4.25.2 Minimum number of rooms required for lectures.

Given an array of time intervals (start, end) for classroom lectures (possibly overlapping), find the minimum number of rooms required.

For example, given Input:

```
lectures = ["9:00-10:30", "9:30-11:30", "11:00-12:00", "14:00-18:00", "15:00-16:00", "15:30-17:30", "16:00-17:30"]
```

should output 3.

### 4.25.3 Non-palindromic skinny numbers

non-palindromic squares remaining square when written backwards

$$\begin{array}{rclclcl} 10^2 & = & 100 & 01^2 & = & 001 \\ 13^2 & = & 169 & 31^2 & = & 961 \\ 102^2 & = & 10404 & 201^2 & = & 40401 \end{array}$$

### 4.25.4 Narcissistic number

A number is narcissistic if the sum of its own digits each raised to the power of the number of digits.

Example :  $4150 = 4^5 + 1^5 + 5^5 + 0^5$  or  $153 = 1^3 + 5^3 + 3^3$

Find narcissistic numbers with 3 digits

### 4.25.5 Happy number

- Given a number  $n = n_0$ , define a sequence  $n_1, n_2, \dots$  where  $n_{i+1}$  is the sum of the squares of the digits of  $n_i$ . Then  $n$  is happy if and only if there exists  $i$  such that  $n_i = 1$ .

For example, 19 is happy, as the associated sequence is:

$$\begin{array}{rclcl} 1^2 & + & 9^2 & = & 82 \\ 8^2 & + & 2^2 & = & 68 \\ 6^2 & + & 8^2 & = & 100 \\ 1^2 & + & 0^2 & + & 0^2 = 1 \end{array}$$

- Write a function `ishappy(n)` that returns True if `n` is happy. - Write a function `happy(n)` that returns a list with all happy numbers  $< n$ .

`happy(100) = [1, 7, 10, 13, 19, 23, 28, 31, 32, 44, 49, 68, 70, 79, 82, 86, 91, 94, 97]`

### 4.25.6 Longest increasing subsequence

Given  $N$  elements, write a program that prints the length of the longest increasing subsequence whose adjacent element difference is one.

Examples:

`a = [3, 10, 3, 11, 4, 5, 6, 7, 8, 12]`

Output : 6

Explanation: 3, 4, 5, 6, 7, 8 is the longest increasing subsequence whose adjacent element differs by one

Input : `a = [6, 7, 8, 3, 4, 5, 9, 10]`

Output : 5

Explanation: 6, 7, 8, 9, 10 is the longest increasing subsequence

### 4.25.7 Polynomial derivative

- A Polynomial is represented by a Python list of its coefficients.  $[1, 5, -4] \Rightarrow 1 + 5x - 4x^2$
- Write the function `diff(P,n)` that return the  $n$ th derivative  $Q$
- Don't use any external package

`diff([3,2,1,5,7],2) = [2, 30, 84]`

`diff([-6,5,-3,-4,3,-4],3) = [-24, 72, -240]`

## Chapter 5

# Modules

If your Python program gets longer, you may want to split it into several files for easier maintenance. To support this, Python has a way to put definitions in a file and use them in a script or in an interactive instance of the interpreter. Such a file is called a module.

Run the cell below to create a file named `fibonacci.py` with several functions inside:

```
In [1]: %%file fibonacci.py
 """ Simple module with
 two functions to compute Fibonacci series """

 def fib1(n):
 """ write Fibonacci series up to n """
 a, b = 0, 1
 while b < n:
 print(b, end=' ', ' ')
 a, b = b, a+b

 def fib2(n):
 """ return Fibonacci series up to n """
 result = []
 a, b = 0, 1
 while b < n:
 result.append(b)
 a, b = b, a+b
 return result

 if __name__ == "__main__":
 import sys
 fib1(int(sys.argv[1]))
```

Writing `fibonacci.py`

You can use the function `fib` by importing `fibonacci` which is the name of the file without `.py` extension.

```
In [2]: import fibonacci
 print(fibonacci.__name__)
 print(fibonacci.__file__)
 fibonacci.fib1(1000)
```

`fibonacci`

```
/home/runner/work/python-notebooks/python-notebooks/notebooks/fibonacci.py
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987,
```

```
In [3]: %run fibo.py 1000

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987,

In [4]: help(fibo)

Help on module fibo:

NAME
 fibo

DESCRIPTION
 Simple module with
 two functions to compute Fibonacci series

FUNCTIONS
 fib1(n)
 write Fibonacci series up to n

 fib2(n)
 return Fibonacci series up to n

FILE
 /home/runner/work/python-notebooks/python-notebooks/notebooks/fibo.py
```

## 5.1 Executing modules as scripts

When you run a Python module with

```
$ python fibo.py <arguments>
```

the code in the module will be executed, just as if you imported it, but with the **name** set to **"main"**. The following code will be executed only in this case and not when it is imported.

```
if __name__ == "__main__":
 import sys
 fib(int(sys.argv[1]))
```

In Jupyter notebook, you can run the fibo.py python script using magic command.

```
In [5]: %run fibo.py 1000

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987,
```

The module is also imported.

```
In [6]: fib1(1000)

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987,
```



## 5.2 Different ways to import a module

```
import fibo
import fibo as f
from fibo import fib1, fib2
from fibo import *
```

- Last command with '\*' imports all names except those beginning with an underscore (\_). In most cases, do not use this facility since it introduces an unknown set of names into the interpreter, possibly hiding some things you have already defined.
- If a function with same name is present in different modules imported. Last module function imported replace the previous one.

```
In [7]: from numpy import sqrt
 from scipy import sqrt
 sqrt(-1)
```

```
<ipython-input-7-f3f47bc91153>:3: DeprecationWarning: scipy.sqrt is deprecated and will be removed in S
sqrt(-1)
```

```
Out[7]: 1j
```

```
In [8]: from scipy import sqrt
 from numpy import sqrt
 sqrt(-1)
```

```
<ipython-input-8-8a25f477b688>:3: RuntimeWarning: invalid value encountered in sqrt
sqrt(-1)
```

```
Out[8]: nan
```

```
In [9]: import numpy as np
 import scipy as sp

 print(np.sqrt(-1+0j), sp.sqrt(-1))
```

```
1j 1j
```

```
<ipython-input-9-235de4d5ffbb>:4: DeprecationWarning: scipy.sqrt is deprecated and will be removed in S
print(np.sqrt(-1+0j), sp.sqrt(-1))
```

- For efficiency reasons, each module is only imported once per interpreter session. Therefore, if you change your modules, you must restart the interpreter – If you really want to test interactively after a long run, use :

```
import importlib
importlib.reload(modulename)
```

## 5.3 The Module Search Path

When a module is imported, the interpreter searches for a file named module.py in a list of directories given by the variable sys.path. - Python programs can modify sys.path - export the PYTHONPATH environment variable to change it on your system.

```
In [10]: import sys
 sys.path
```

```
Out[10]: ['/home/runner/work/python-notebooks/python-notebooks/notebooks',
 '/usr/share/miniconda3/envs/runenv/lib/python38.zip',
 '/usr/share/miniconda3/envs/runenv/lib/python3.8',
 '/usr/share/miniconda3/envs/runenv/lib/python3.8/lib-dynload',
 '',
 '/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages',
 '/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/extensions',
 '/home/runner/.ipython']
```

```
In [11]: import collections
 collections.__path__
```

```
Out[11]: ['/usr/share/miniconda3/envs/runenv/lib/python3.8/collections']
```

`sys.path` is a list and you can append some directories:

```
In [12]: sys.path.append("/Users/navaro/python-notebooks/")
 print(sys.path)
```

```
['/home/runner/work/python-notebooks/python-notebooks/notebooks', '/usr/share/miniconda3/envs/runenv/lib/python3.8/collections', '/Users/navaro/python-notebooks/']
```

When you import a module `foo`, following files are searched in this order:

- `foo.dll`, `foo.dylib` or `foo.so`
- `foo.py`
- `foo.pyc`
- `**foo/___init___.py**`

## 5.4 Packages

- A package is a directory containing Python module files.
- This directory always contains a file name `___init___.py`

```
cluster __init__.py
```

## 5.5 Relative imports

These imports use leading dots to indicate the current and parent packages involved in the relative import.

In the `sugiton` module, you can use:

```
from . import cluster # import module in the same directory
from .. import base # import module in parent directory
from ..ensemble import _forest # import module in another subdirectory of the parent directory
```

## 5.6 Reminder

Don't forget that importing `*` is not recommended

```
In [13]: sum(range(5), -1)
```

```
Out[13]: 9
```

```
In [14]: from numpy import *
 sum(range(5),-1)
```

```
Out[14]: 10
```

```
In [15]: del sum # delete imported sum function from numpy
 help(sum)
```

Help on built-in function sum in module builtins:

```
sum(iterable, /, start=0)
 Return the sum of a 'start' value (default: 0) plus an iterable of numbers

 When the iterable is empty, return the start value.
 This function is intended specifically for use with numeric values and may
 reject non-numeric types.
```

```
In [16]: import numpy as np
 help(np.sum)
```

Help on function sum in module numpy:

```
sum(a, axis=None, dtype=None, out=None, keepdims=<no value>, initial=<no value>, where=<no value>)
 Sum of array elements over a given axis.
```

Parameters

-----

a : array\_like

Elements to sum.

axis : None or int or tuple of ints, optional

Axis or axes along which a sum is performed. The default, axis=None, will sum all of the elements of the input array. If axis is negative it counts from the last to the first axis.

.. versionadded:: 1.7.0

If axis is a tuple of ints, a sum is performed on all of the axes specified in the tuple instead of a single axis or all the axes as before.

dtype : dtype, optional

The type of the returned array and of the accumulator in which the elements are summed. The dtype of `a` is used by default unless `a` has an integer dtype of less precision than the default platform integer. In that case, if `a` is signed then the platform integer is used while if `a` is unsigned then an unsigned integer of the same precision as the platform integer is used.

out : ndarray, optional

Alternative output array in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

keepdims : bool, optional

If this is set to True, the axes which are reduced are left in the result as dimensions with size one. With this option, the result will broadcast correctly against the input array.

If the default value is passed, then ``keepdims`` will not be passed through to the ``sum`` method of sub-classes of ``ndarray``, however any non-default value will be. If the sub-class' method does not implement ``keepdims`` any exceptions will be raised.

`initial` : scalar, optional  
Starting value for the sum. See ``~numpy.ufunc.reduce`` for details.

`.. versionadded:: 1.15.0`

`where` : array\_like of bool, optional  
Elements to include in the sum. See ``~numpy.ufunc.reduce`` for details.

`.. versionadded:: 1.17.0`

Returns  
-----

`sum_along_axis` : ndarray  
An array with the same shape as ``a``, with the specified axis removed. If ``a`` is a 0-d array, or if ``axis`` is None, a scalar is returned. If an output array is specified, a reference to ``out`` is returned.

See Also  
-----

`ndarray.sum` : Equivalent method.

`add.reduce` : Equivalent functionality of ``add``.

`cumsum` : Cumulative sum of array elements.

`trapez` : Integration of array values using the composite trapezoidal rule.

`mean`, `average`

Notes  
-----

Arithmetic is modular when using integer types, and no error is raised on overflow.

The sum of an empty array is the neutral element 0:

```
>>> np.sum([])
0.0
```

For floating point numbers the numerical precision of sum (and ``np.add.reduce``) is in general limited by directly adding each number individually to the result causing rounding errors in every step. However, often numpy will use a numerically better approach (partial pairwise summation) leading to improved precision in many use-cases. This improved precision is always provided when no ``axis`` is given. When ``axis`` is given, it will depend on which axis is summed. Technically, to provide the best speed possible, the improved precision

is only used when the summation is along the fast axis in memory. Note that the exact precision may vary depending on other parameters. In contrast to NumPy, Python's `math.fsum` function uses a slower but more precise approach to summation. Especially when summing a large number of lower precision floating point numbers, such as `float32`, numerical errors can become significant. In such cases it can be advisable to use `dtype="float64"` to use a higher precision for the output.

#### Examples

```

>>> np.sum([0.5, 1.5])
2.0
>>> np.sum([0.5, 0.7, 0.2, 1.5], dtype=np.int32)
1
>>> np.sum([[0, 1], [0, 5]])
6
>>> np.sum([[0, 1], [0, 5]], axis=0)
array([0, 6])
>>> np.sum([[0, 1], [0, 5]], axis=1)
array([1, 5])
>>> np.sum([[0, 1], [np.nan, 5]], where=[False, True], axis=1)
array([1., 5.])

```

If the accumulator is too small, overflow occurs:

```

>>> np.ones(128, dtype=np.int8).sum(dtype=np.int8)
-128

```

You can also start the sum with a value other than zero:

```

>>> np.sum([10], initial=5)
15

```



## Chapter 6

# Input and Output

- `str()` function return human-readable representations of values.
- `repr()` generate representations which can be read by the interpreter.
- For objects which don't have a particular representation for human consumption, `str()` will return the same value as `repr()`.

```
In [1]: s = 'Hello, world.'
 str(s)
```

```
Out[1]: 'Hello, world.'
```

```
In [2]: l = list(range(4))
 str(l)
```

```
Out[2]: '[0, 1, 2, 3]'
```

```
In [3]: repr(s)
```

```
Out[3]: "'Hello, world.'"
```

```
In [4]: repr(l)
```

```
Out[4]: '[0, 1, 2, 3]'
```

```
In [5]: x = 10 * 3.25
 y = 200 * 200
 s = 'The value of x is ' + str(x) + ', and y is ' + repr(y) + '...'
 print(s)
```

The value of x is 32.5, and y is 40000...

`repr()` of a string adds string quotes and backslashes:

```
In [6]: hello = 'hello, world\n'
 hellos = repr(hello)
 hellos
```

```
Out[6]: "'hello, world\\n'"
```

The argument to `repr()` may be any Python object:

```
In [7]: repr((x, y, ('spam', 'eggs')))
```

```
Out[7]: "(32.5, 40000, ('spam', 'eggs'))"
```

```
In [8]: n = 7
 for x in range(1, n):
 for i in range(n):
 print(repr(x**i).rjust(i+2), end=' ') # rjust or center can be used
 print()
```

```
1 1 1 1 1 1 1
1 2 4 8 16 32 64
1 3 9 27 81 243 729
1 4 16 64 256 1024 4096
1 5 25 125 625 3125 15625
1 6 36 216 1296 7776 46656
```

```
In [9]: for x in range(1, n):
 for i in range(n):
 print("%07d" % x**i, end=' ') # old C format
 print()
```

```
0000001 0000001 0000001 0000001 0000001 0000001 0000001
0000001 0000002 0000004 0000008 0000016 0000032 0000064
0000001 0000003 0000009 0000027 0000081 0000243 0000729
0000001 0000004 0000016 0000064 0000256 0001024 0004096
0000001 0000005 0000025 0000125 0000625 0003125 0015625
0000001 0000006 0000036 0000216 0001296 0007776 0046656
```

## 6.1 Usage of the `str.format()` method

```
In [10]: print('We are at the {} in {}'.format('ENSAI', 'Rennes'))
```

We are at the ENSAI in Rennes!

```
In [11]: print('From {0} to {1}'.format('September 7', 'September 14'))
```

From September 7 to September 14

```
In [12]: print('It takes place at {place}'.format(place='Milon room'))
```

It takes place at Milon room

```
In [13]: import math
 print('The value of PI is approximately {:.7g}'.format(math.pi))
```

The value of PI is approximately 3.141593.

## 6.2 Formatted string literals (Python 3.6)

```
In [14]: print(f'The value of PI is approximately {math.pi:.4f}.')
```

The value of PI is approximately 3.1416.



```
In [15]: name = "Fred"
 print(f"He said his name is {name}.")
 print(f"He said his name is {name!r}.")

He said his name is Fred.
He said his name is 'Fred'.
```

```
In [16]: f"He said his name is {repr(name)}." # repr() is equivalent to !r

Out[16]: "He said his name is 'Fred'."
```

```
In [17]: width, precision = 10, 4
 value = 12.34567
 print(f"result: {value:{width}.{precision}f}") # nested fields

result: 12.3457
```

```
In [18]: from datetime import *
 today = datetime(year=2017, month=1, day=27)
 print(f"{today:%B %d, %Y}") # using date format specifier

January 27, 2017
```

## 6.3 Reading and Writing Files

`open()` returns a file object, and is most commonly used with file name and accessing mode argument.

```
In [19]: f = open('workfile.txt', 'w')
 f.write("1. This is a txt file.\n")
 f.write("2. \n is used to begin a new line")
 f.close()
 !cat workfile.txt
```

```
1. This is a txt file.
2. \n is used to begin a new line
```

mode can be : - 'r' when the file will only be read, - 'w' for only writing (an existing file with the same name will be erased) - 'a' opens the file for appending; any data written to the file is automatically added to the end. - 'r+' opens the file for both reading and writing. - The mode argument is optional; 'r' will be assumed if it's omitted. - Normally, files are opened in text mode. - 'b' appended to the mode opens the file in binary mode.

```
In [20]: with open('workfile.txt') as f:
 read_text = f.read()
 f.closed

Out[20]: True
```

```
In [21]: read_text

Out[21]: '1. This is a txt file.\n2. \n is used to begin a new line'
```

```
In [22]: lines= []
 with open('workfile.txt') as f:
 lines.append(f.readline())
 lines.append(f.readline())
 lines.append(f.readline())

 lines
```

```
Out[22]: ['1. This is a txt file.\n', '2. \n is used to begin a new line', '']
```

- `f.readline()` returns an empty string when the end of the file has been reached.
- `f.readlines()` or `list(f)` read all the lines of a file in a list.

For reading lines from a file, you can loop over the file object. This is memory efficient, fast, and leads to simple code:

```
In [23]: with open('workfile.txt') as f:
 for line in f:
 print(line, end='')

1. This is a txt file.
2. \n is used to begin a new line
```

### 6.3.1 Exercise: Wordcount Example

**WordCount** is a simple application that counts the number of occurrences of each word in a given input set.

- Use `lorem` module to write a text in the file "sample.txt"
- Write a function `words` with file name as input that returns a sorted list of words present in the file.
- Write the function `reduce` to read the results of words and sum the occurrences of each word to a final count, and then output the results as a dictionary `{word1:occurences1, word2:occurences2}`.
- You can check the results using piped shell commands:

```
cat sample.txt | fmt -1 | tr [:upper:] [:lower:] | tr -d '.' | sort | uniq -c
```

```
In [24]: from lorem import text
```

```
 text()
```

```
Out[24]: 'Quaerat consectetur quiquia aliquam quaerat. Neque dolorem quiquia etincidunt eius amet conse
```

```
In [25]: def words(file):
 """ Parse a file and returns a sorted list of words """
 pass

 words('sample.txt')
 # [('adipisci', 1),
 # ('adipisci', 1),
 # ('adipisci', 1),
 # ('aliquam', 1),
 # ('aliquam', 1),
```

```
In [26]: d = {}
 d['word1'] = 3
 d['word2'] = 2
 d
```

```
Out[26]: {'word1': 3, 'word2': 2}
```

```
In [27]: def reduce (words):
 """ Count the number of occurrences of a word in list
 and return a dictionary """
 pass

 reduce(words('sample.txt'))
```

```
#{'neque': 80),
'ut': 80,
'est': 76,
'amet': 74,
'magnam': 74,
'adipisci': 73,
```

## 6.4 Saving structured data with json

- JSON (JavaScript Object Notation) is a popular data interchange format.
- JSON format is commonly used by modern applications to allow for data exchange.
- JSON can be used to communicate with applications written in other languages.

```
In [28]: import json
 json.dumps([1, 'simple', 'list'])
```

```
Out[28]: '[1, "simple", "list"]'
```

```
In [29]: x = dict(name="Pierre Navaro", organization="CNRS", position="IR")
 with open('workfile.json','w') as f:
 json.dump(x, f)
```

```
In [30]: with open('workfile.json','r') as f:
 x = json.load(f)
 x
```

```
Out[30]: {'name': 'Pierre Navaro', 'organization': 'CNRS', 'position': 'IR'}
```

```
In [31]: %cat workfile.json
```

```
{"name": "Pierre Navaro", "organization": "CNRS", "position": "IR"}
```

Use `ujson` for big data structures <https://pypi.python.org/pypi/ujson>

For common file formats used in data science (CSV, xls, feather, parquet, ORC, HDF, avro, ...) use packages like `pandas` or better `pyarrow`. It depends of what you want to do with your data but `Dask` and `pyspark` offer features to read and write (big) data files.



## Chapter 7

# Errors and Exceptions

There are two distinguishable kinds of errors: *syntax errors* and *exceptions*. - Syntax errors, also known as parsing errors, are the most common. - Exceptions are errors caused by statement or expression syntactically corrects. - Exceptions are not unconditionally fatal.

[Exceptions in Python documentation](#)

```
In [1]: 10 * (1/0)
```

```

ZeroDivisionError Traceback (most recent call last)

 <ipython-input-1-0b280f36835c> in <module>
----> 1 10 * (1/0)

ZeroDivisionError: division by zero
```

```
In [2]: 4 + spam*3
```

```

NameError Traceback (most recent call last)

 <ipython-input-2-c98bb92cdcac> in <module>
----> 1 4 + spam*3

NameError: name 'spam' is not defined
```

```
In [3]: '2' + 2
```

```

TypeError Traceback (most recent call last)

 <ipython-input-3-d2b23a1db757> in <module>
```

```
----> 1 '2' + 2
```

```
TypeError: can only concatenate str (not "int") to str
```

## 7.1 Handling Exceptions

- In example below, the user can interrupt the program with Control-C or the `stop` button in Jupyter Notebook.
- Note that a user-generated interruption is signalled by raising the **KeyboardInterrupt** exception.

```
In [4]: while True:
```

```
 try:
 x = int(input("Please enter a number: "))
 print(f' x = {x}')
 break
 except ValueError:
 print("Oops! That was no valid number. Try again...")
```

```

StdinNotImplementedError Traceback (most recent call last)
```

```
<ipython-input-4-d9e83eb78fd3> in <module>
 1 while True:
 2 try:
----> 3 x = int(input("Please enter a number: "))
 4 print(f' x = {x}')
 5 break
```

```
/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/ipykernel/kernelbase.py in raw_input
855 """
856 if not self._allow_stdin:
--> 857 raise StdinNotImplementedError(
858 "raw_input was called, but this frontend does not support input requests."
859)
```

```
StdinNotImplementedError: raw_input was called, but this frontend does not support input requests
```

- A try statement may have more than one except clause
- The optional else clause must follow all except clauses.

```
In [5]: import sys
```

```
def process_file(file):
 " Read the first line of f and convert to int and check if this integer is positive"
 try:
 i = int(open(file).readline().strip())
 print(i)
 assert i > 0
 except OSError as err:
```

```

 print(f"OS error: {err}")
 except ValueError:
 print("Could not convert data to an integer.")
 except:
 print("Unexpected error:", sys.exc_info()[0])

 # Create the file workfile.txt
 with open('workfile.txt', 'w') as f:
 f.write("foo")
 f.write("bar")

In [6]: process_file('workfile.txt') # catch exception return by int() call
Could not convert data to an integer.

In [7]: # Change permission of the file, workfile.txt cannot be read
 !chmod u-r workfile.txt

In [8]: process_file('workfile.txt') # catch exception return by open() call
OS error: [Errno 13] Permission denied: 'workfile.txt'

In [9]: # Let's delete the file workfile.txt
 !rm -f workfile.txt

In [10]: process_file('workfile.txt') # catch another exception return by open() call
OS error: [Errno 2] No such file or directory: 'workfile.txt'

In [11]: # Insert the value -1 at the top of workfile.txt
 !echo "-1" > workfile.txt
 %cat workfile.txt

-1

In [12]: process_file('workfile.txt') # catch exception return by assert()

-1
Unexpected error: <class 'AssertionError'>

```

## 7.2 Raising Exceptions

The raise statement allows the programmer to force a specified exception to occur.

```
In [13]: raise NameError('HiThere')
```

```

NameError Traceback (most recent call last)

<ipython-input-13-72c183edb298> in <module>
----> 1 raise NameError('HiThere')

NameError: HiThere

```

## 7.3 Defining Clean-up Actions

- The try statement has an optional clause which is intended to define clean-up actions that must be executed under all circumstances.
- A finally clause is always executed before leaving the try statement

```
In [14]: try:
 raise KeyboardInterrupt
 finally:
 print('Goodbye, world!')
```

Goodbye, world!

```

KeyboardInterrupt Traceback (most recent call last)

<ipython-input-14-7786db0b9fd1> in <module>
 1 try:
----> 2 raise KeyboardInterrupt
 3 finally:
 4 print('Goodbye, world!')

KeyboardInterrupt:
```

### 7.3.1 Wordcount Exercise

- Improve the function `reduce` to read the results of `words` by using the `KeyError` exception to fill in the dictionary.



# Chapter 8

## Classes

- Classes provide a means of bundling data and functionality together.
- Creating a new class creates a **new type** of object.
- Assigned variables are new **instances** of that type.
- Each class instance can have **attributes** attached to it.
- Class instances can also have **methods** for modifying its state.
- Python classes provide the class **inheritance** mechanism.

### 8.1 Use class to store data

- A empty class can be used to bundle together a few named data items.
- You can easily save this class containing your data in JSON file.

```
In [1]: class Car:
 pass

 mycar = Car() # Create an empty car record

 # Fill the fields of the record
 mycar.brand = 'Peugeot'
 mycar.model = 308
 mycar.year = 2015

In [2]: mycar.__dict__

Out[2]: {'brand': 'Peugeot', 'model': 308, 'year': 2015}
```

### 8.2 namedtuple

```
In [3]: from collections import namedtuple

 Car = namedtuple('Car', 'brand, model, year')

In [4]: mycar = Car('Peugeot', 308, 2015)
 mycar

Out[4]: Car(brand='Peugeot', model=308, year=2015)

In [5]: mycar.year

Out[5]: 2015
```

```
In [6]: # Like tuples, namedtuples are immutable:
 mycar.model = 3008
```

```

AttributeError Traceback (most recent call last)

<ipython-input-6-09089c3eb392> in <module>
 1 # Like tuples, namedtuples are immutable:
----> 2 mycar.model = 3008

AttributeError: can't set attribute
```

```
In [7]: class Car:
```

```
 "A simple example class Animal with its name, weight and age"
```

```
 def __init__(self, brand, model, year): # constructor
 self.brand = brand
 self.model = model
 self.year = year
```

```
 def age(self): # method
 import datetime
 now = datetime.datetime.now()
 return now.year - self.year
```

```
In [8]: mycar = Car('Peugeot', 308, 2015) # Instance
 print(f' {mycar.brand} {mycar.model} {mycar.year} ')
 print(f' {mycar.age()} years old')
```

```
Peugeot 308 2015
5 years old
```

```
In [9]: mycar.year = 2017
 mycar.age()
```

```
Out[9]: 3
```

- mycar is an *instance* of Car Class.
- mycar.age() is a *method* of Car instance mycar.
- brand and model are attributes of Car instance mycar.

## 8.3 Convert method to attribute

Use the property decorator

```
In [10]: class Car:
```

```
 "A simple example class Car with its model, brand and year"
```

```
 def __init__(self, brand, model, year): # constructor
 self.model = model
```

```

 self.brand = brand
 self.year = year

 @property
 def age(self): # method
 import datetime
 now = datetime.datetime.now()
 return now.year - self.year

```

```

In [11]: mycar = Car('Peugeot', 308, 2015)
 mycar.age # age can now be used as an attribute

```

```

Out[11]: 5

```

```

In [12]: mycar.age = 3 # a protected attribute

```

```

AttributeError Traceback (most recent call last)

<ipython-input-12-d6fe717cdbfa> in <module>
----> 1 mycar.age = 3 # a protected attribute

AttributeError: can't set attribute

```

## 8.4 The new Python 3.7 DataClass

```

In [13]: from dataclasses import dataclass

```

```

@dataclass
class Car:

 brand: str
 model: int
 year: int

 @property
 def age(self) -> int:
 import datetime
 now = datetime.datetime.now()
 return now.year - self.year

```

```

In [14]: mycar = Car('Peugeot', 308, 2015)
 mycar

```

```

Out[14]: Car(brand='Peugeot', model=308, year=2015)

```

```

In [15]: myothercar = Car('BMW', "1 Series", 2009)
 myothercar

```

```

Out[15]: Car(brand='BMW', model='1 Series', year=2009)

```

## 8.5 Method Overriding

- Every Python classes has a `__repr__()` method used when you call `print()` function.

```
In [16]: class Car:
 """Simple example class with method overriding """

 def __init__(self, brand, model, year):
 self.brand = brand
 self.model = model
 self.year = year

 def __repr__(self):
 return f"{self.year} {self.brand} {self.model} {self.__class__.__name__}"

In [17]: mycar = Car('Peugeot', 308, 2015)
 print(mycar)
```

2015 Peugeot 308 Car

## 8.6 Inheritance

```
In [18]: class Rectangle(): # Parent class is defined here

 def __init__(self, width, height):
 self.width, self.height = width, height
 @property
 def area(self):
 return self.width * self.height

 class Square(Rectangle):

 def __init__(self, edge):
 super().__init__(edge, edge) # Call method in the parent class

 r = Rectangle(2, 3)
 print(f"Rectangle area \t = {r.area:7.3f}")
 s = Square(4)
 print(f"Square area \t = {s.area:7.3f}")
```

```
Rectangle area = 6.000
Square area = 16.000
```

## 8.7 Private Variables and Methods

```
In [19]: class DemoClass:
 " Demo class for name mangling "

 def public_method(self):
 return 'public!'

 def __private_method(self): # Note the use of leading underscores
 return 'private!'
```

```

object3 = DemoClass()

In [20]: object3.public_method()

Out[20]: 'public!'

In [21]: object3.__private_method()

AttributeError Traceback (most recent call last)

<ipython-input-21-5a4a6ba27511> in <module>
----> 1 object3.__private_method()

AttributeError: 'DemoClass' object has no attribute '__private_method'

In [22]: [s for s in dir(object3) if "method" in s]

Out[22]: ['_DemoClass__private_method', 'public_method']

In [23]: object3._DemoClass__private_method()

Out[23]: 'private!'

In [24]: object3.public_method

Out[24]: <bound method DemoClass.public_method of <__main__.DemoClass object at 0x7ffb801d9b50>>

```

## 8.8 Use class as a Function.

```

In [25]: class Polynomial:

 " Class representing a polynom P(x) -> c_0+c_1*x+c_2*x^2+..."

 def __init__(self, coeffs):
 self.coeffs = coeffs

 def __call__(self, x):
 return sum(coef*x**exp for exp,coef in enumerate(self.coeffs))

p = Polynomial([2,4,-1])
p(2)

Out[25]: 6

```

### 8.8.1 Exercise: Polynomial

- Improve the class above called Polynomial by creating a method `diff(n)` to compute the  $n$ th derivative.
- Override the `__repr__()` method to output a pretty printing.

Hint: `f"{coeff:+d}"` forces to print sign before the value of an integer.

## 8.9 Operators Overriding

```
In [26]: class MyComplex:
 " Simple class representing a complex"
 width = 7
 precision = 3

 def __init__(self, real=0, imag=0):
 self.real = real
 self.imag = imag

 def __repr__(self):
 return (f"({self.real:{self.width}.{self.precision}f},"
 f"{self.imag:+{self.width}.{self.precision}f}j)")

 def __eq__(self, other): # override '=='
 return (self.real == other.real) and (self.imag == other.imag)

 def __add__(self, other): # override '+'
 return MyComplex(self.real+other.real, self.imag+other.imag)

 def __sub__(self, other): # override '-'
 return MyComplex(self.real-other.real, self.imag-other.imag)

 def __mul__(self, other): # override '*'
 if isinstance(other, MyComplex):
 return MyComplex(self.real * other.real - self.imag * other.imag,
 self.real * other.imag + self.imag * other.real)
 else:
 return MyComplex(other*self.real, other*self.imag)
```

```
In [27]: u = MyComplex(0, 1)
 v = MyComplex(1, 0)
 print('u=', u, "; v=", v)
```

```
u= (0.000, +1.000j) ; v= (1.000, +0.000j)
```

```
In [28]: u+v, u-v, u*v, u==v
```

```
Out[28]: ((1.000, +1.000j), (-1.000, +1.000j), (0.000, +1.000j), False)
```

We can change the *class* attribute precision.

```
In [29]: MyComplex.precision=2
 print(u.precision)
 print(u)
```

```
2
(0.00, +1.00j)
```

```
In [30]: v.precision
```

```
Out[30]: 2
```

We can change the *instance* attribute precision.

```
In [31]: u.precision=1
 print(u)
```

```
(0.0, +1.0j)

In [32]: print(v)

(1.00, +0.00j)

In [33]: MyComplex.precision=5
 u # set attribute keeps its value

Out[33]: (0.0, +1.0j)

In [34]: v # unset attribute is set to the new value

Out[34]: (1.00000,+0.00000j)
```

## 8.10 Rational example

```
In [35]: class Rational:
 " Class representing a rational number"

 def __init__(self, n, d):
 assert isinstance(n, int) and isinstance(d, int)

 def gcd(x, y):
 if x == 0:
 return y
 elif x < 0:
 return gcd(-x, y)
 elif y < 0:
 return -gcd(x, -y)
 else:
 return gcd(y % x, x)

 g = gcd(n, d)
 self.numer, self.denom = n//g, d//g

 def __add__(self, other):
 return Rational(self.numer * other.denom + other.numer * self.denom,
 self.denom * other.denom)

 def __sub__(self, other):
 return Rational(self.numer * other.denom - other.numer * self.denom,
 self.denom * other.denom)

 def __mul__(self, other):
 return Rational(self.numer * other.numer, self.denom * other.denom)

 def __truediv__(self, other):
 return Rational(self.numer * other.denom, self.denom * other.numer)

 def __repr__(self):
 return f"{self.numer:d}/{self.denom:d}"

In [36]: r1 = Rational(2,3)
 r2 = Rational(3,4)
 r1+r2, r1-r2, r1*r2, r1/r2

Out[36]: (17/12, -1/12, 1/2, 8/9)
```

### 8.10.1 Exercise

Improve the class Polynomial by implementing operations: - Overrides '+' operator (**add**) - Overrides '-' operator (**neg**) - Overrides '==' operator (**eq**) - Overrides '\*' operator (**mul**)



## Chapter 9

# Iterators

Most container objects can be looped over using a for statement:

```
In [1]: for element in [1, 2, 3]:
 print(element, end=' ')
```

1 2 3

```
In [2]: for element in (1, 2, 3):
 print(element, end=' ')
```

1 2 3

```
In [3]: for key in {'one': 1, 'two': 2}:
 print(key, end=' ')
```

one two

```
In [4]: for char in "123":
 print(char, end=' ')
```

1 2 3

```
In [5]: for line in open("environment.yml"):
 print(line, end= ' ')
```

```

FileNotFoundError Traceback (most recent call last)

<ipython-input-5-e8ea7e33e965> in <module>
----> 1 for line in open("environment.yml"):
 2 print(line, end= ' ')
```

FileNotFoundError: [Errno 2] No such file or directory: 'environment.yml'

- The for statement calls `iter()` on the container object.
- The function returns an iterator object that defines the method `__next__()`
- To add iterator behavior to your classes:
  - Define an `__iter__()` method which returns an object with a `__next__()`.

- If the class defines `__next__()`, then `__iter__()` can just return self.
- The **StopIteration** exception indicates the end of the loop.

```
In [6]: s = 'abc'
 it = iter(s)
 it
```

```
Out[6]: <str_iterator at 0x7f9d702660d0>
```

```
In [7]: next(it), next(it), next(it)
```

```
Out[7]: ('a', 'b', 'c')
```

```
In [8]: class Reverse:
 """Iterator for looping over a sequence backwards."""

 def __init__(self, data):
 self.data = data
 self.index = len(data)

 def __iter__(self):
 return self

 def __next__(self):
 if self.index == 0:
 raise StopIteration
 self.index = self.index - 1
 return self.data[self.index]
```

```
In [9]: rev = Reverse('spam')
 for char in rev:
 print(char, end='')
```

maps

```
In [10]: def reverse(data): # Python 3.6
 yield from data[::-1]

 for char in reverse('bulgroz'):
 print(char, end='')
```

zorglub

## 9.1 Generators

- Generators are a simple and powerful tool for creating iterators.
- Write regular functions but use the `yield` statement when you want to return data.
- the `__iter__()` and `__next__()` methods are created automatically.

```
In [11]: def reverse(data):
 for index in range(len(data)-1, -1, -1):
 yield data[index]
```

```
In [12]: for char in reverse('bulgroz'):
 print(char, end='')
```

zorglub

### 9.1.1 Exercise

Generates a list of IP addresses based on IP range.

```
ip_range =
for ip in ip_range("192.168.1.0", "192.168.1.10"):
 print(ip)

192.168.1.0
192.168.1.1
192.168.1.2
...
```

## 9.2 Generator Expressions

- Use a syntax similar to list comprehensions but with parentheses instead of brackets.
- Tend to be more memory friendly than equivalent list comprehensions.

```
In [13]: sum(i*i for i in range(10)) # sum of squares
```

```
Out[13]: 285
```

```
In [14]: %load_ext memory_profiler
```

```
In [15]: %memit doubles = [2 * n for n in range(10000)]
```

```
peak memory: 50.82 MiB, increment: 1.13 MiB
```

```
In [16]: %memit doubles = (2 * n for n in range(10000))
```

```
peak memory: 50.82 MiB, increment: 0.00 MiB
```

```
In [17]: # list comprehension
doubles = [2 * n for n in range(10)]
for x in doubles:
 print(x, end=' ')
```

```
0 2 4 6 8 10 12 14 16 18
```

```
In [18]: # generator expression
doubles = (2 * n for n in range(10))
for x in doubles:
 print(x, end=' ')
```

```
0 2 4 6 8 10 12 14 16 18
```

### 9.2.1 Exercise

The [Chebyshev polynomials](#) of the first kind are defined by the recurrence relation

$$T_0(x) = 1 \tag{9.1}$$

$$T_1(x) = x \tag{9.2}$$

$$T_{n+1} = 2xT_n(x) - T_{n-1}(x) \tag{9.3}$$

- Create a class `Chebyshev` that generates the sequence of Chebyshev polynomials

## 9.3 itertools

### 9.3.1 zip\_longest

`itertools.zip_longest()` accepts any number of iterables as arguments and a `fillvalue` keyword argument that defaults to `None`.

```
In [19]: x = [1, 1, 1, 1, 1]
 y = [1, 2, 3, 4, 5, 6, 7]
 list(zip(x, y))
 from itertools import zip_longest
 list(map(sum, zip_longest(x, y, fillvalue=1)))
```

```
Out[19]: [2, 3, 4, 5, 6, 7, 8]
```

### 9.3.2 combinations

```
In [20]: loto_numbers = list(range(1,50))
```

A choice of 6 numbers from the sequence 1 to 49 is called a combination. The `itertools.combinations()` function takes two arguments—an iterable inputs and a positive integer `n`—and produces an iterator over tuples of all combinations of `n` elements in inputs.

```
In [21]: from itertools import combinations
 len(list(combinations(loto_numbers, 6)))
```

```
Out[21]: 13983816
```

```
In [22]: from math import factorial
 factorial(49)/ factorial(6) / factorial(49-6)
```

```
Out[22]: 13983816.0
```

### 9.3.3 permutations

```
In [23]: from itertools import permutations
 for s in permutations('dsi'):
 print("".join(s), end=" ",)
```

```
dsi, dis, sdi, sid, ids, isd,
```

### 9.3.4 count

```
In [24]: from itertools import count
 n = 2024
 for k in count(): # replace k = 0; while(True) : k += 1
 if n == 1:
 print(f"k = {k}")
 break
 elif n & 1:
 n = 3*n +1
 else:
 n = n // 2
```

```
k = 112
```

### 9.3.5 cycle, islice, dropwhile, takewhile

```
In [25]: from itertools import cycle, islice, dropwhile, takewhile
L = list(range(10))
cycled = cycle(L) # cycle through the list 'L'
skipped = dropwhile(lambda x: x < 6 , cycled) # drop the values until x==4
sliced = islice(skipped, None, 20) # take the first 20 values
print(*sliced)
```

6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

```
In [26]: result = takewhile(lambda x: x > 0, cycled) # cycled begins to 4
 print(*result)
```

6 7 8 9

### 9.3.6 product

```
In [27]: ranks = ['A', 'K', 'Q', 'J', '10', '9', '8', '7']
 suits = ['\u2660', '\u2665', '\u2663', '\u2666']
 cards = [(rank, suit) for rank in ranks for suit in suits]
 len(cards)
 from itertools import product
 cards = product(ranks, suits)
 print(*cards)
```

( 'A', ' ' ) ( 'A', ' ' ) ( 'A', ' ' ) ( 'A', ' ' ) ( 'K', ' ' ) ( 'K', ' ' ) ( 'K', ' ' ) ( 'K', ' ' ) ( 'Q', ' ' ) ( 'Q', ' ' )



# Chapter 10

## Multiprocessing

```
In [1]: from multiprocessing import cpu_count

 cpu_count()
```

```
Out[1]: 2
```

### 10.1 Map reduce example

```
In [2]: from time import sleep
 def delayed_square(x):
 sleep(1)
 return x*x
 data = list(range(8))
 data
```

```
Out[2]: [0, 1, 2, 3, 4, 5, 6, 7]
```

```
In [3]: %time sum(delayed_square(x) for x in data)
```

```
CPU times: user 0 ns, sys: 3.24 ms, total: 3.24 ms
Wall time: 8.01 s
```

```
Out[3]: 140
```

```
In [4]: %time sum(map(delayed_square,data))
```

```
CPU times: user 3.02 ms, sys: 336 µs, total: 3.36 ms
Wall time: 8.01 s
```

```
Out[4]: 140
```

We can process each `delayed_square` calls independently and in parallel. To accomplish this we'll apply that function across all list items in parallel using multiple processes.

### 10.2 Thread and Process: Differences

- A Process is an instance of a running program.
- Process may contain one or more threads, but a thread cannot contain a process.

- Process has a self-contained execution environment. It has its own memory space.
- Application running on your computer may be a set of cooperating processes.
- A Thread is made of and exist within a Process; every process has at least one.
- Multiple threads in a process share resources, which helps in efficient communication between threads.
- Threads can be concurrent on a multi-core system, with every core executing the separate threads simultaneously.

## 10.3 Multi-Processing vs Multi-Threading

### 10.3.1 Memory

- Each process has its own copy of the data segment of the parent process.
- Each thread has direct access to the data segment of its process.
- A process runs in separate memory spaces.
- A thread runs in shared memory spaces.

### 10.3.2 Communication

- Processes must use inter-process communication to communicate with sibling processes.
- Threads can directly communicate with other threads of its process.

### 10.3.3 Overheads

- Processes have considerable overhead.
- Threads have almost no overhead.

## 10.4 Multi-Processing vs Multi-Threading

### 10.4.1 Creation

- New processes require duplication of the parent process.
- New threads are easily created.

### 10.4.2 Control

- Processes can only exercise control over child processes.
- Threads can exercise considerable control over threads of the same process.

### 10.4.3 Changes

- Any change in the parent process does not affect child processes.
- Any change in the main thread may affect the behavior of the other threads of the process.

## 10.5 The Global Interpreter Lock (GIL)

- The Python interpreter is not thread safe.
- A few critical internal data structures may only be accessed by one thread at a time. Access to them is protected by the GIL.
- Attempts at removing the GIL from Python have failed until now. The main difficulty is maintaining the C API for extension modules.



- Multiprocessing avoids the GIL by having separate processes which each have an independent copy of the interpreter data structures.
- The price to pay: serialization of tasks, arguments, and results.

## 10.6 Multiprocessing (history)

- The multiprocessing allows the programmer to fully leverage multiple processors.
- The Pool object parallelizes the execution of a function across multiple input values.
- The if `__name__ == '__main__'` part is necessary.

The next program does not work in a cell you need to save it and run with python in a terminal

python3 pool.py

In [5]: %%file pool.py

```
from time import time, sleep

from multiprocessing import Pool

def delayed_square(x):
 sleep(1)
 return x*x

if __name__ == '__main__': # Executed only on main process.
 start = time()
 data = list(range(8))
 with Pool() as p:
 result = sum(p.map(delayed_square, data))
 stop = time()
 print(f"result = {result} - Elapsed time {stop - start}")
```

Writing pool.py

In [6]: import sys  
!{sys.executable} pool.py

result = 140 - Elapsed time 4.024413585662842

## 10.7 Futures

The `concurrent.futures` module provides a high-level interface for asynchronously executing callables.

The asynchronous execution can be performed with threads, using `ThreadPoolExecutor`, or separate processes, using `ProcessPoolExecutor`. Both implement the same interface, which is defined by the abstract `Executor` class.

In [7]: %%file process\_pool.py

```
import os
from time import time, sleep
if os.name == "nt":
 from loky import ProcessPoolExecutor # for Windows users
else:
 from concurrent.futures import ProcessPoolExecutor

from time import time, sleep
```

```
def delayed_square(x):
 sleep(1)
 return x*x

if __name__ == "__main__":
 start = time()
 data = list(range(8))
 with ProcessPoolExecutor() as pool:
 result = sum(pool.map(delayed_square, data))
 stop = time()
 print(f" result : {result} - elapsed time {stop - start}")
```

Writing process\_pool.py

```
In [8]: !{sys.executable} process_pool.py
```

```
result : 140 - elapsed time 4.013685464859009
```

```
In [9]: %%time
 from concurrent.futures import ThreadPoolExecutor

 e = ThreadPoolExecutor()

 results = list(e.map(delayed_square, range(8)))
```

```
CPU times: user 4.11 ms, sys: 572 µs, total: 4.68 ms
Wall time: 2 s
```

## 10.8 Asynchronous Future

While many parallel applications can be described as maps, some can be more complex. In this section we look at the asynchronous Future interface, which provides a simple API for ad-hoc parallelism. This is useful for when your computations don't fit a regular pattern.

### 10.8.1 Executor.submit

The `submit` method starts a computation in a separate thread or process and immediately gives us a `Future` object that refers to the result. At first, the future is pending. Once the function completes the future is finished.

We collect the result of the task with the `.result()` method, which does not return until the results are available.

```
In [10]: from time import sleep

 def slowadd(a, b, delay=1):
 sleep(delay)
 return a + b

In [11]: from concurrent.futures import ThreadPoolExecutor
 e = ThreadPoolExecutor(4)
 future = e.submit(slowadd, 1, 2)
 future
```

```
Out[11]: <Future at 0x7f9e18da5cd0 state=running>
```

```
In [12]: future.result()
```

Out[12]: 3

Submit many tasks all at once and they be will executed in parallel.

```
In [13]: %%time
 results = [slowadd(i, i, delay=1) for i in range(8)]
```

```
CPU times: user 0 ns, sys: 3.48 ms, total: 3.48 ms
```

Wall time: 8.01 s

```
In [14]: %%time
 futures = [e.submit(slowadd, 1, 1, delay=1) for i in range(8)]
 results = [f.result() for f in futures]
```

```
CPU times: user 0 ns, sys: 3.02 ms, total: 3.02 ms
```

Wall time: 2 s

- Submit fires off a single function call in the background, returning a future.
- When you combine submit with a single for loop we recover the functionality of map.
- To collect your results, replace each of futures, `f`, with a call to `f.result()`
- Combine submit with multiple for loops and other general programming to get something more general than map.
- Sometimes, it did not speed up the code very much
- Threads and processes show some performance differences
- Use threads carefully, you can break your Python session.

Today most library designers are coordinating around the `concurrent.futures` interface, so it's wise to move over.

- Profile your code
- Used `concurrent.futures.ProcessPoolExecutor` for simple parallelism
- Gained some speed boost (but not as much as expected)
- Lost ability to diagnose performance within parallel code
- Describing each task as a function call helps use tools like `map` for parallelism
- Making your tasks fast is often at least as important as parallelizing your tasks.

### 10.8.2 Exercise: Pi computation

Parallelize this computation with a `ProcessPoolExecutor`. `ThreadPoolExecutor` is not usable because of `random` function calls.

```
In [15]: import time
import random

def compute_pi(n):
 count = 0
 for i in range(n):
 x = random.random()
 y = random.random()
 if x*x + y*y <= 1:
 count += 1
 return count
```

```
elapsed_time = time.time()
nb_simulations = 4
n = 10**7
result = [compute_pi(n) for i in range(nb_simulations)]
pi = 4 * sum(result) / (n*nb_simulations)
print(f"Estimated value of Pi : {pi:.8f} time : {time.time()-elapsed_time:.8f}")
```

Estimated value of Pi : 3.14231230 time : 13.55466604

### 10.8.3 Exercise

- Do the same computation using asynchronous future
- Implement a joblib version (see example below)

## 10.9 Parallel tools for Python

The parallel tools from standard library are very limited. You will have more powerful features with:

- [Joblib](#) provides a simple helper class to write parallel for loops using multiprocessing.
- [Dask](#)
- [PySpark](#)
- [mpi4py](#)

# Chapter 11

## Standard Library

### 11.1 Operating System Interface

```
In [1]: import os
 os.getcwd() # Return the current working directory

Out[1]: '/home/runner/work/python-notebooks/python-notebooks/notebooks'

In [2]: %env CC='/usr/local/bin/gcc-7'
 os.environ['CC']='/usr/local/bin/gcc-7' # Change the default C compiler to gcc-7
 os.system('mkdir today') # Run the command mkdir in the system shell

env: CC='/usr/local/bin/gcc-7'

Out[2]: 0

In [3]: os.chdir('today') # Change current working directory
 os.system('touch data.db') # Create the empty file data.db

Out[3]: 0

In [4]: import shutil
 shutil.copyfile('data.db', 'archive.db')
 if os.path.exists('backup.db'): # If file backup.db exists
 os.remove('backup.db') # Remove it
 shutil.move('archive.db', 'backup.db',)
 shutil.os.chdir('..')
```

### 11.2 File Wildcards

The glob module provides a function for making file lists from directory wildcard searches:

```
In [5]: import glob
 glob.glob('*.py')

Out[5]: ['process_pool.py', 'pool.py']

In [6]: def recursive_replace(root, pattern, replace) :
 """
 Function to replace a string inside a directory
 root : directory
 pattern : searched string
```

```

replace "pattern" by "replace"
"""
for directory, subdirs, filenames in os.walk(root):
 for filename in filenames:
 path = os.path.join(directory, filename)
 text = open(path).read()
 if pattern in text:
 print('occurence in ' + filename)
 open(path,'w').write(text.replace(pattern, replace))

```

## 11.3 Command Line Arguments

These arguments are stored in the sys module's argv attribute as a list.

```

writefile magic_args="-a demo.py" slideshow={"slide_type": "fragment"} import sys
print(sys.argv)

```

In [7]: %run demo.py one two three

```

OSError Traceback (most recent call last)

/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py :
702 fpath = arg_lst[0]
--> 703 filename = file_finder(fpath)
704 except IndexError:

/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/utils/path.py in get_py_f
108 else:
--> 109 raise IOError('File `%r` not found.' % name)
110

OSError: File `demo.py` not found.

```

During handling of the above exception, another exception occurred:

```

Exception Traceback (most recent call last)

<ipython-input-7-916df5e5b25f> in <module>
----> 1 get_ipython().run_line_magic('run', 'demo.py one two three')

/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/interactiveshell.py :
2324 kwargs['local_ns'] = self.get_local_scope(stack_depth)
2325 with self.builtin_trap:
-> 2326 result = fn(*args, **kwargs)
2327 return result
2328

```

```

<decorator-gen-59> in run(self, parameter_s, runner, file_finder)

/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magic.py in <lambda>
185 # but it's overkill for just that one bit of state.
186 def magic_deco(arg):
--> 187 call = lambda f, *a, **k: f(*a, **k)
188
189 if callable(arg):

/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py :
712 if os.name == 'nt' and re.match(r"^\.*$", fpath):
713 warn('For Windows, use double quotes to wrap a filename: %run "mypath\\myfi
--> 714 raise Exception(msg)
715 except TypeError:
716 if fpath in sys.meta_path:

Exception: File `demo.py` not found.

```

## 11.4 Random

```

In [8]: import random
 random.choice(['apple', 'pear', 'banana'])

Out[8]: 'pear'

In [9]: random.sample(range(100), 10) # sampling without replacement

Out[9]: [24, 35, 16, 17, 14, 23, 55, 90, 31, 34]

In [10]: random.random() # random float

Out[10]: 0.28056595262227313

In [11]: random.randrange(6) # random integer chosen from range(6)

Out[11]: 0

```

## 11.5 Statistics

```

In [12]: import statistics
 data = [2.75, 1.75, 1.25, 0.25, 0.5, 1.25, 3.5]
 statistics.mean(data)

Out[12]: 1.6071428571428572

In [13]: statistics.median(data)

Out[13]: 1.25

In [14]: statistics.variance(data)

Out[14]: 1.3720238095238095

```

## 11.6 Performance Measurement

```
In [15]: from timeit import Timer
 Timer('t=a; a=b; b=t', 'a=1; b=2').timeit()
```

```
Out[15]: 0.02690286400002151
```

```
In [16]: Timer('a,b = b,a', 'a=1; b=2').timeit()
```

```
Out[16]: 0.023747071000002506
```

```
In [17]: %%timeit a=1; b=2
 a,b = b,a
```

```
23.6 ns ± 0.271 ns per loop (mean ± std. dev. of 7 runs, 10000000 loops each)
```

The [profile](#) and [pstats](#) modules provide tools for identifying time critical sections in larger blocks of code.

## 11.7 Quality Control

One approach for developing high quality software is to write tests for each function.

- The doctest module provides a tool for scanning a module and validating tests embedded in a program's docstrings.
- This improves the documentation by providing the user with an example and it allows the doctest module to make sure the code remains true to the documentation:

```
In [18]: def average(values):
 """Computes the arithmetic mean of a list of numbers."""

 >>> print(average([20, 30, 70]))
 40.0
 """
 return sum(values) / len(values)

 import doctest
 doctest.testmod() # automatically validate the embedded tests
```

```
Out[18]: TestResults(failed=0, attempted=1)
```

## 11.8 Python's standard library is very extensive

- Containers and iterators: `collections`, `itertools`
- Internet access: `urllib`, `email`, `mailbox`, `cgi`, `ftplib`
- Dates and Times: `datetime`, `calendar`,
- Data Compression: `zlib`, `gzip`, `bz2`, `lzma`, `zipfile`, `tarfile`
- File formats: `csv`, `configparser`, `netrc`, `xdrllib`, `plistlib`
- Cryptographic Services: `hashlib`, `hmac`, `secrets`
- Structure Markup Processing Tools: `html`, `xml`

Check the [The Python Standard Library](#)



# Chapter 12

## Matplotlib

- Python 2D plotting library which produces figures in many formats and interactive environments.
- Tries to make easy things easy and hard things possible.
- You can generate plots, histograms, power spectra, bar charts, errorcharts, scatterplots, etc., with just a few lines of code.
- Check the [Matplotlib gallery](#).
- For simple plotting the pyplot module provides a MATLAB-like interface, particularly when combined with IPython.
- Matplotlib provides a set of functions familiar to MATLAB users.

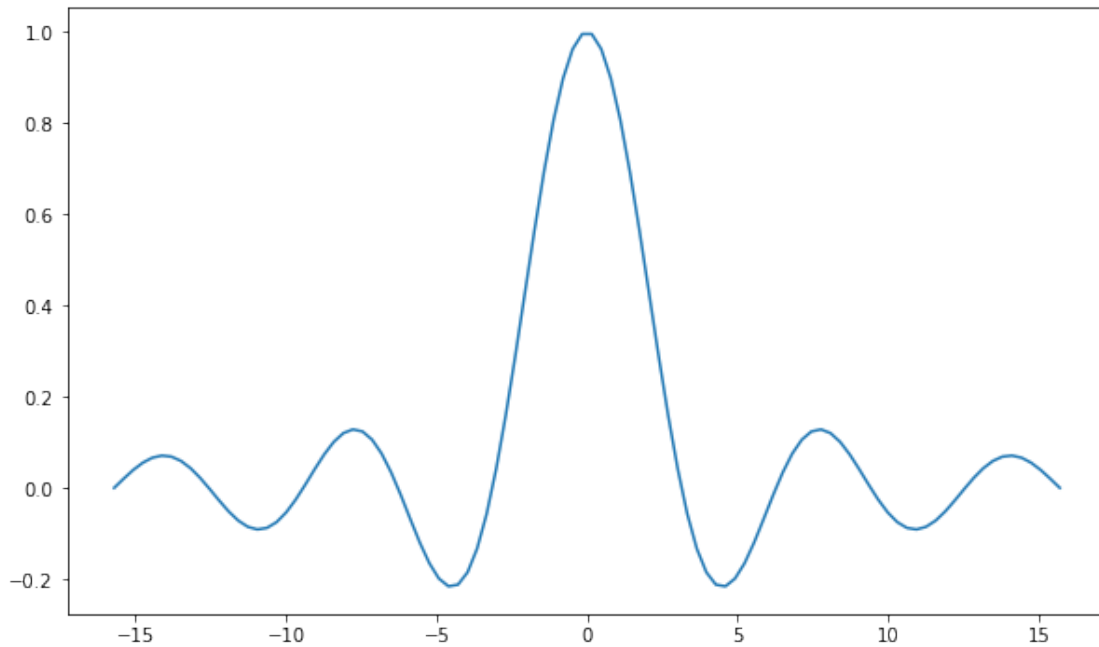
*In this notebook we use some numpy command that will be explain more precisely later.*

### 12.1 Line Plots

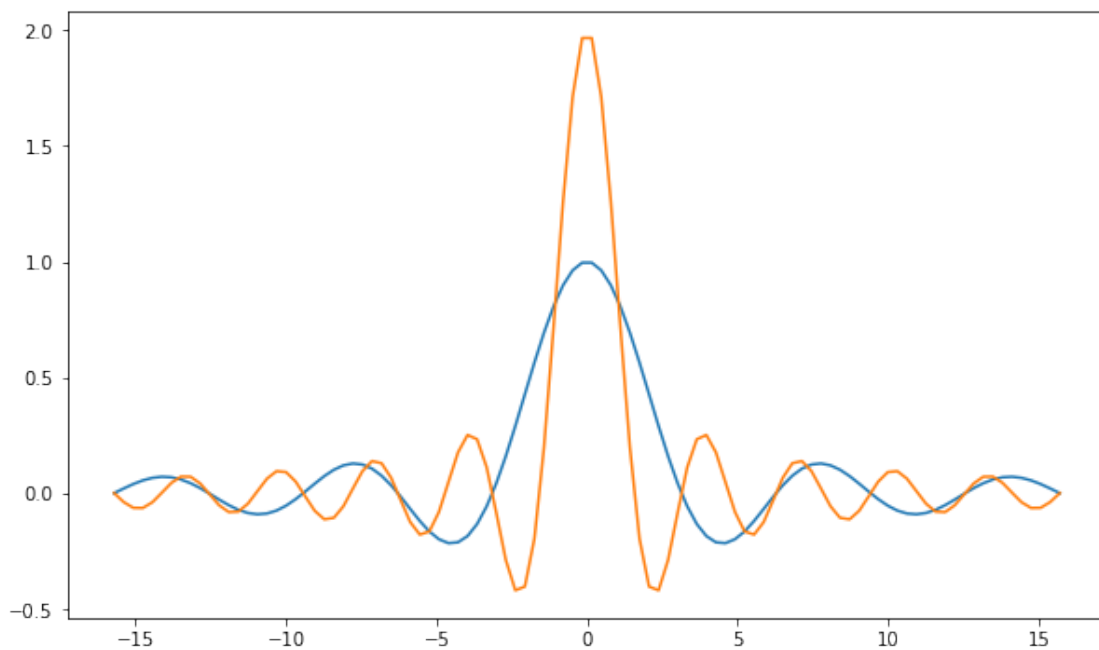
- `np.linspace(0,1,10)` return 10 evenly spaced values over  $[0, 1]$ .

```
In [1]: %matplotlib inline
 # inline can be replaced by notebook to get interactive plots
 import numpy as np
 import matplotlib.pyplot as plt
```

```
In [2]: plt.rcParams['figure.figsize'] = (10.0, 6.0) # set figures display bigger
 x = np.linspace(- 5*np.pi,5*np.pi,100)
 plt.plot(x,np.sin(x)/x);
```



```
In [3]: plt.plot(x,np.sin(x)/x,x,np.sin(2*x)/x);
```

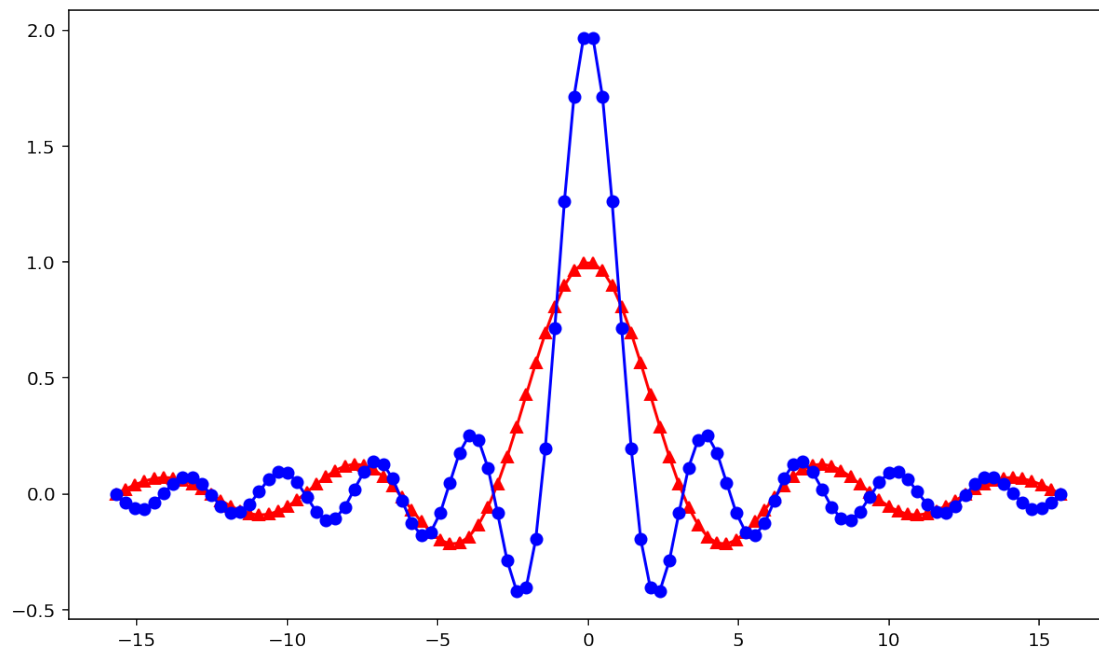


If you have a recent Macbook with a Retina screen, you can display high-resolution plot outputs. Running the next cell will give you double resolution plot output for Retina screens.

*Note: the example below won't render on non-retina screens*

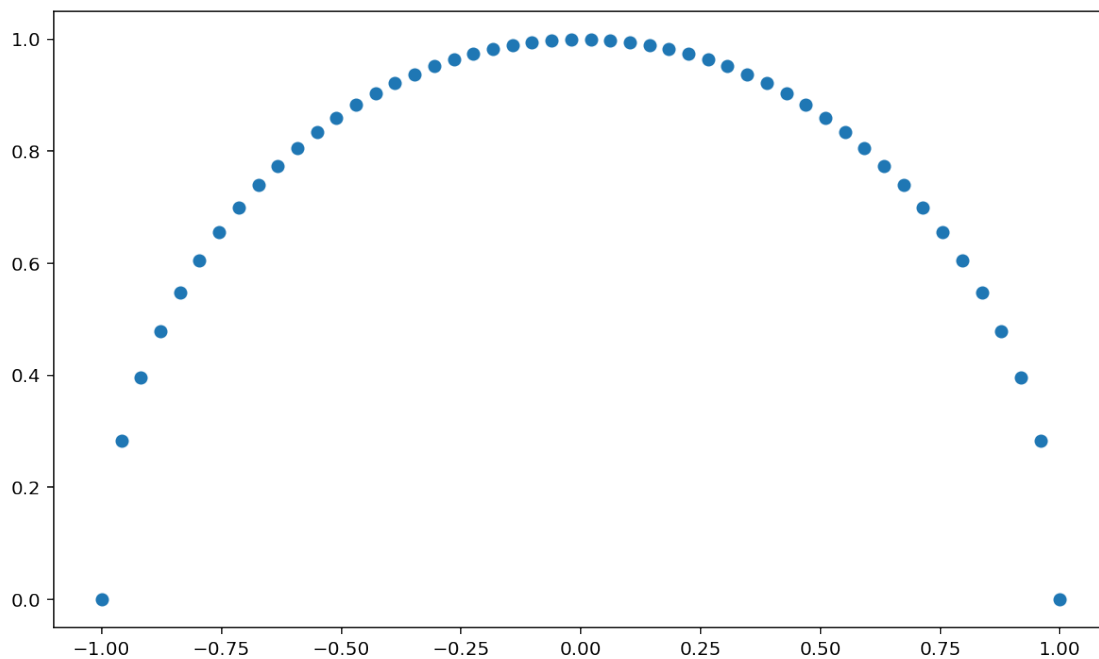
```
In [4]: %config InlineBackend.figure_format = 'retina'
```

```
In [5]: # red, dot-dash, triangles and blue, dot-dash, bullet
plt.plot(x,np.sin(x)/x, 'r-^',x,np.sin(2*x)/x, 'b-o');
```



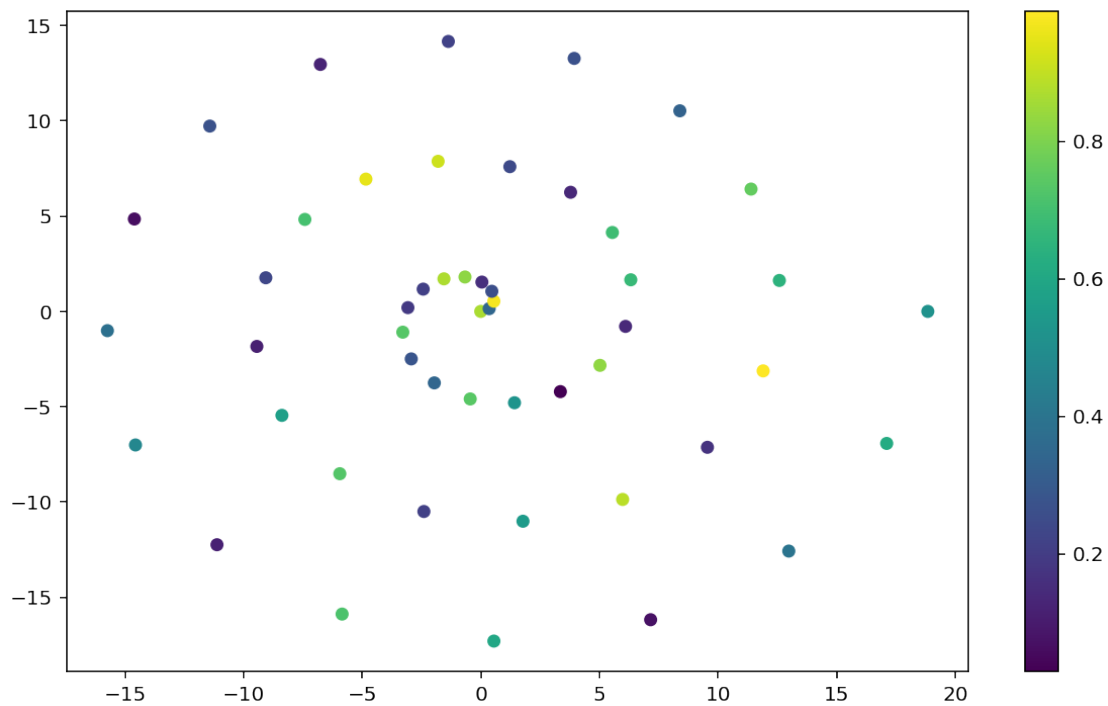
## 12.2 Simple Scatter Plot

```
In [6]: x = np.linspace(-1,1,50)
y = np.sqrt(1-x**2)
plt.scatter(x,y);
```



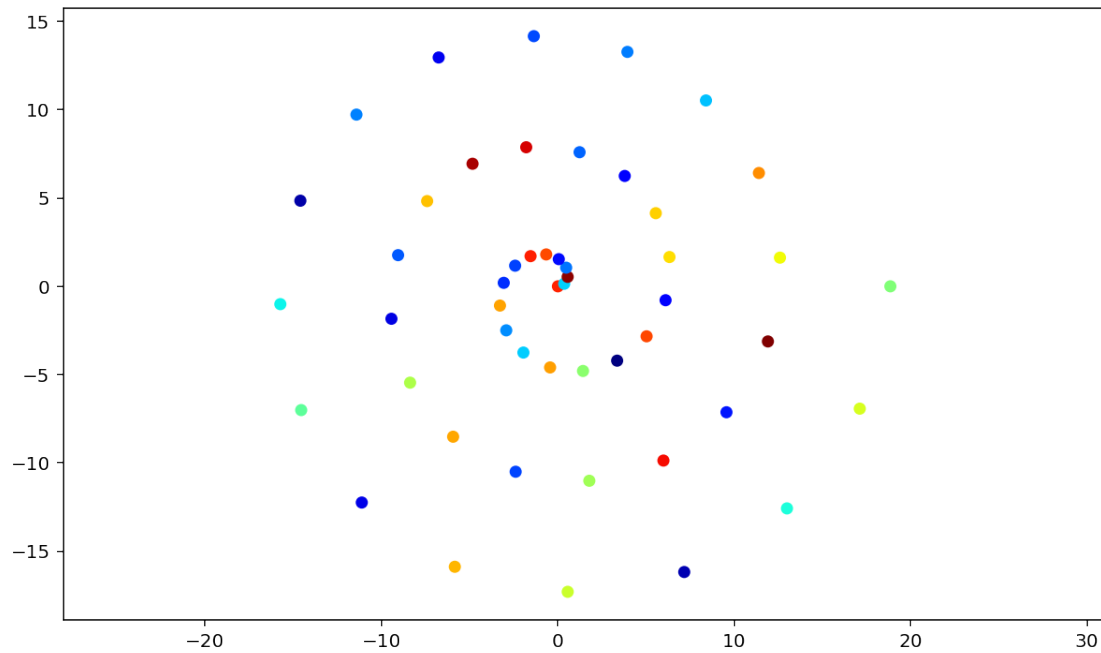
## 12.3 Colormapped Scatter Plot

```
In [7]: theta = np.linspace(0,6*np.pi,50) # 50 steps from 0 to 6 PI
 size = 30*np.ones(50) # array with 50 values set to 30
 z = np.random.rand(50) # array with 50 random values in [0,1]
 x = theta*np.cos(theta)
 y = theta*np.sin(theta)
 plt.scatter(x,y,size,z)
 plt.colorbar();
```



## 12.4 Change Colormap

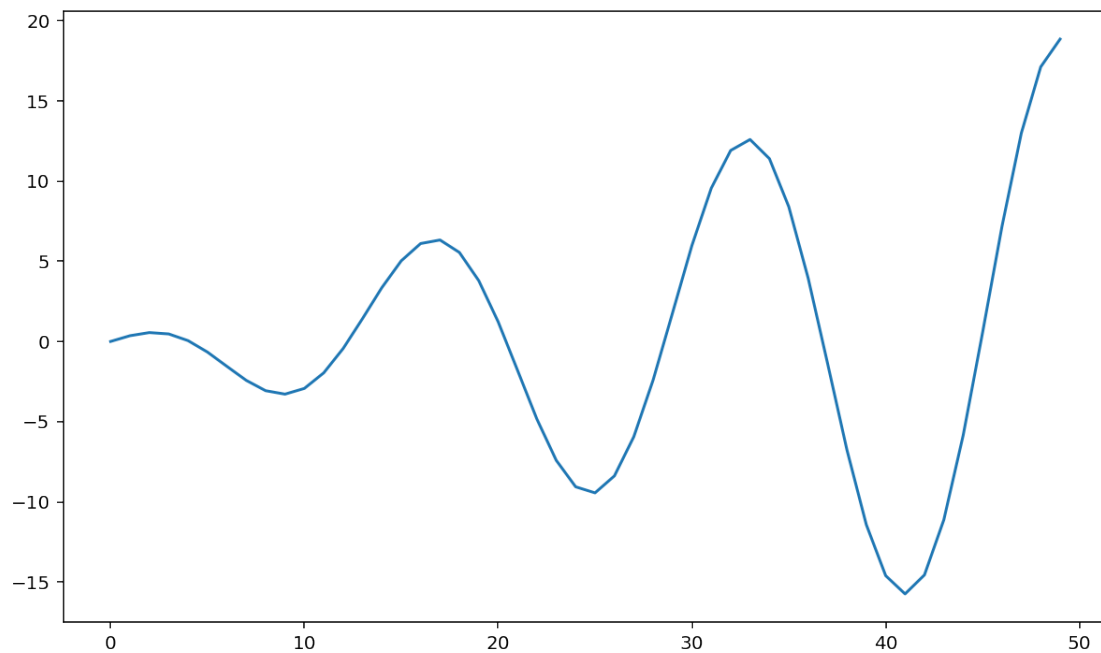
```
In [8]: fig = plt.figure() # create a figure
 ax = fig.add_subplot(1, 1, 1) # add a single plot
 ax.scatter(x,y,size,z,cmap='jet');
 ax.set_aspect('equal', 'datalim')
```

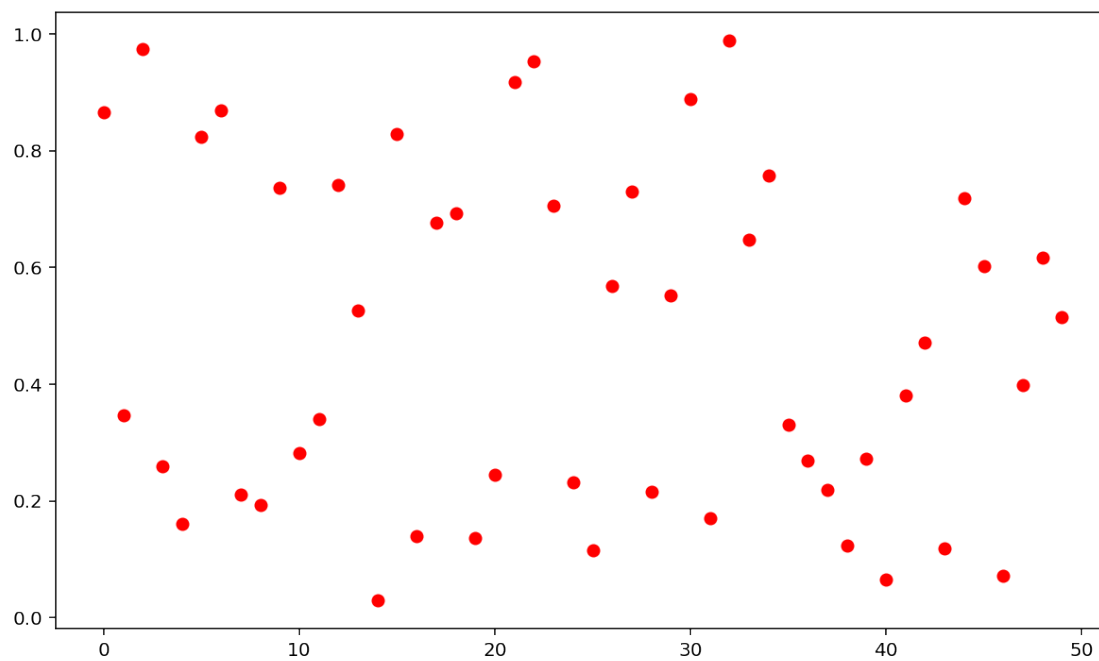


[colormaps](#) in matplotlib documentation.

## 12.5 Multiple Figures

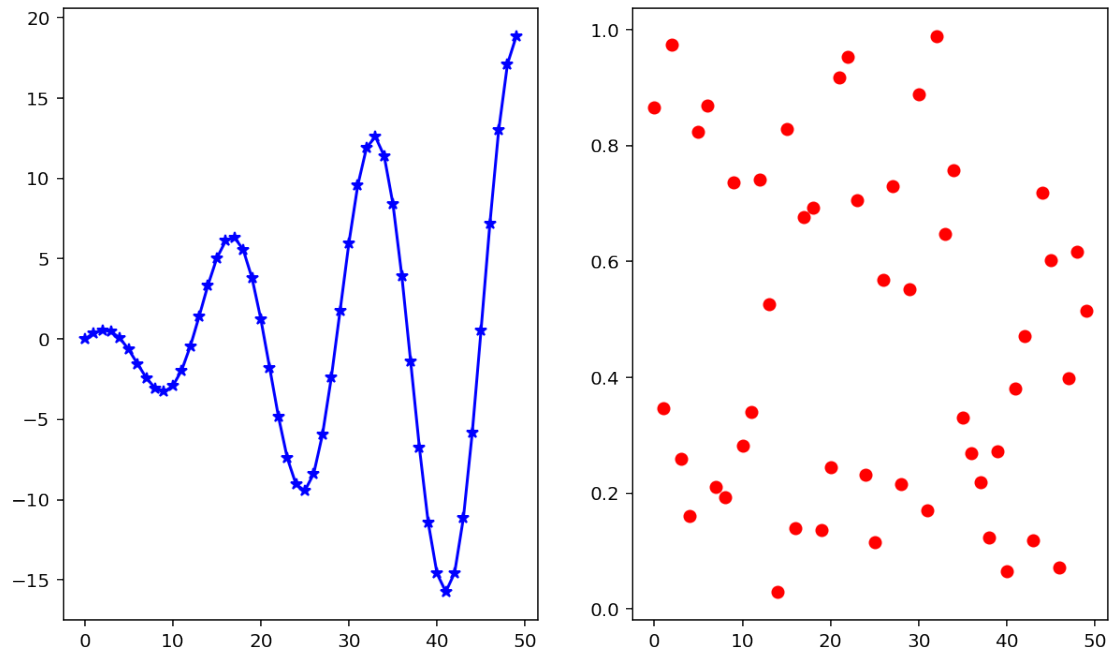
```
In [9]: plt.figure()
plt.plot(x)
plt.figure()
plt.plot(z, 'ro');
```





## 12.6 Multiple Plots Using subplot

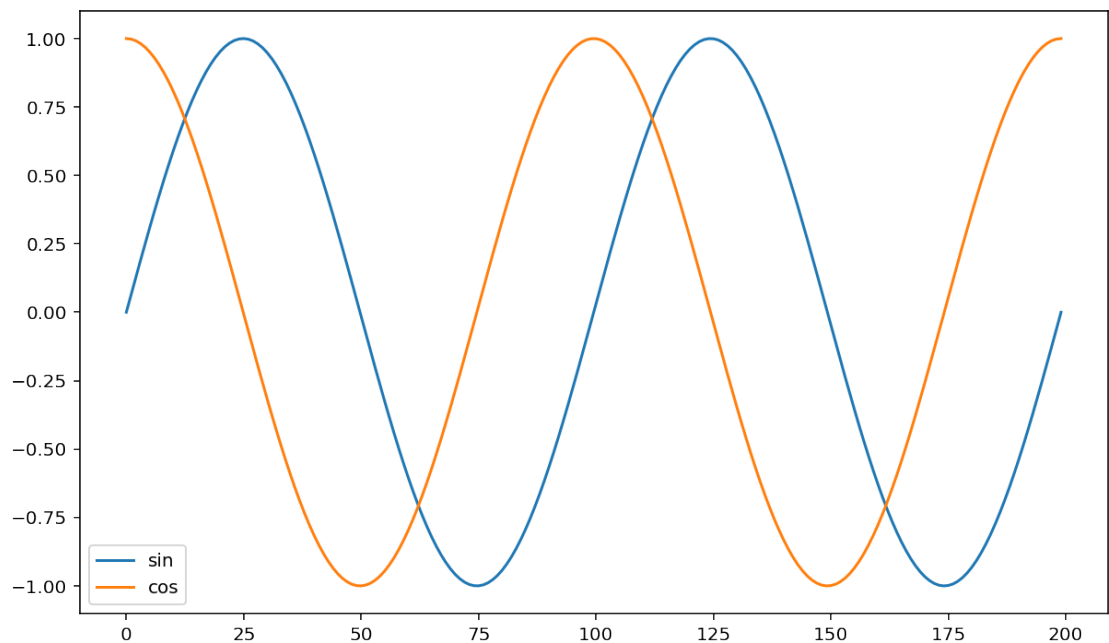
```
In [10]: plt.subplot(1,2,1) # 1 row 1, 2 columns, active plot number 1
plt.plot(x, 'b-*)
plt.subplot(1,2,2) # 1 row 1, 2 columns, active plot number 2
plt.plot(z, 'ro');
```



## 12.7 Legends

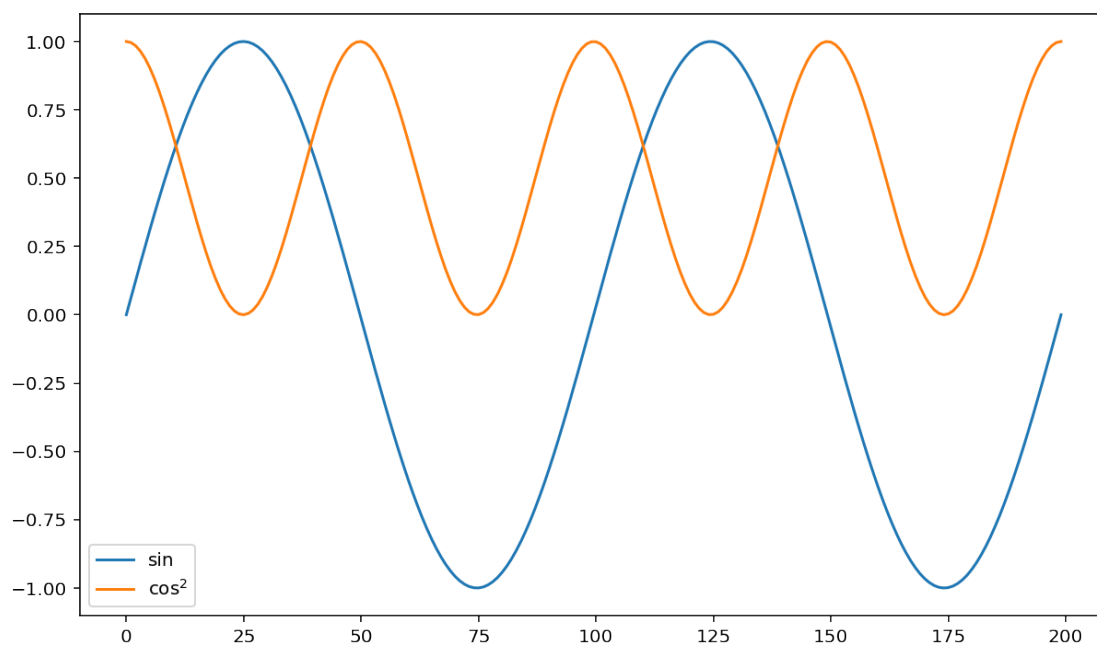
- Legends labels with plot

```
In [11]: theta = np.linspace(0, 4*np.pi, 200)
plt.plot(np.sin(theta), label='sin')
plt.plot(np.cos(theta), label='cos')
plt.legend();
```



- Labelling with legend

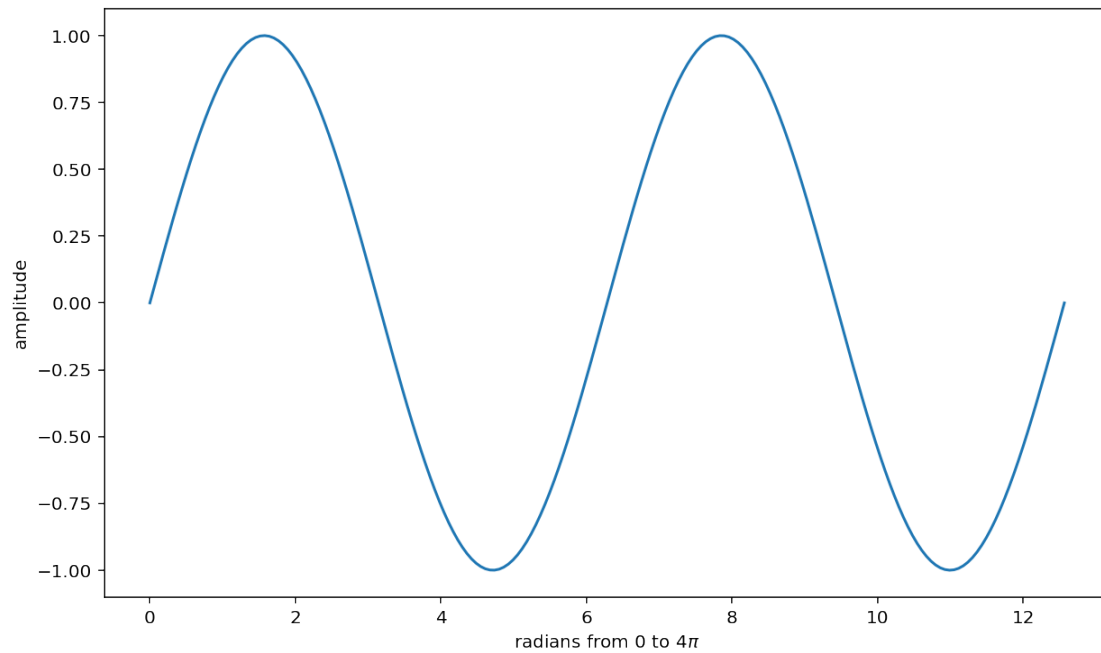
```
In [12]: plt.plot(np.sin(theta))
 plt.plot(np.cos(theta)**2)
 plt.legend(['sin', '\cos^2']);
```



## 12.8 Titles and Axis Labels

```
In [13]: plt.plot(theta, np.sin(theta))
 plt.xlabel('radians from 0 to 4π')
 plt.ylabel('amplitude');
```

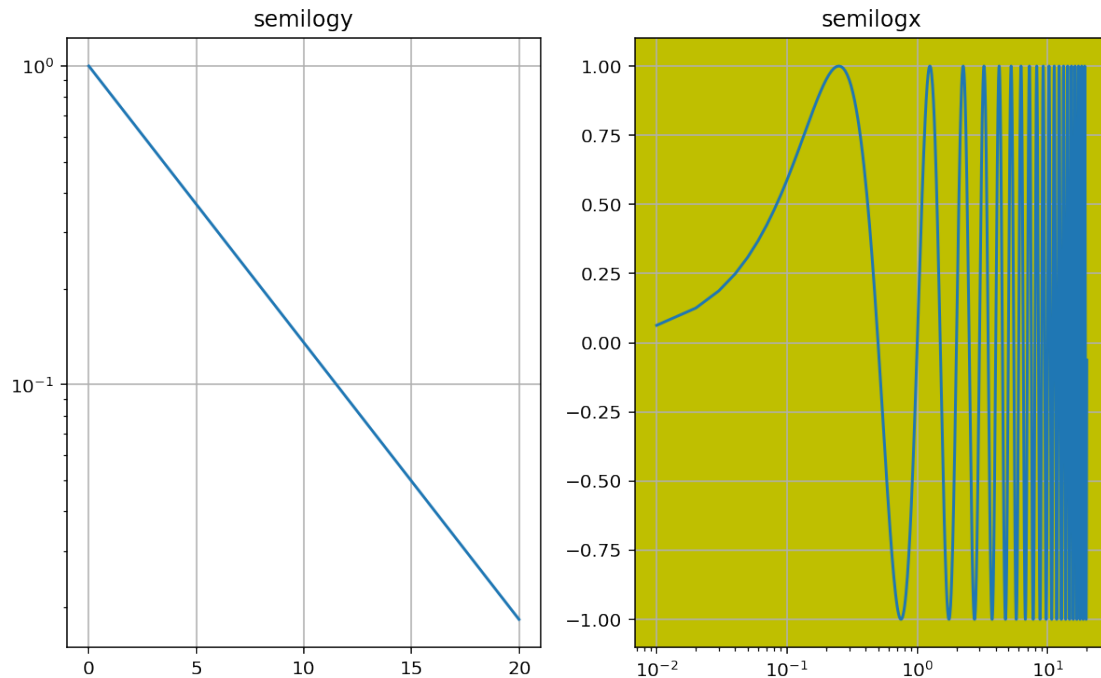




```
In [14]: t = np.arange(0.01, 20.0, 0.01)

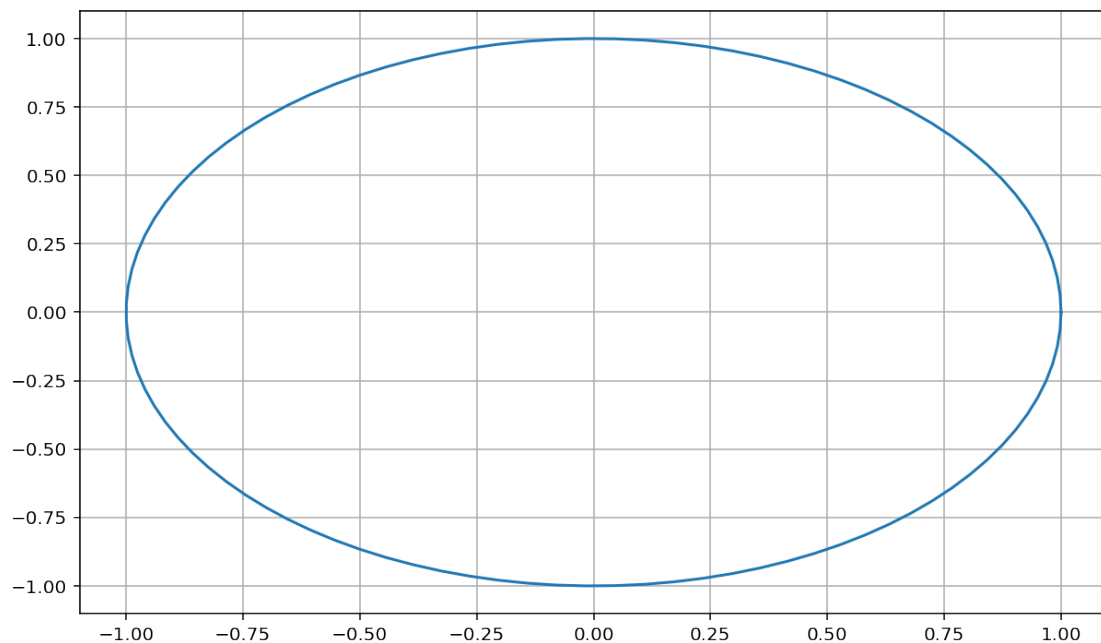
plt.subplot(121)
plt.semilogy(t, np.exp(-t/5.0))
plt.title('semilogy')
plt.grid(True)

plt.subplot(122,fc='y')
plt.semilogx(t, np.sin(2*np.pi*t))
plt.title('semilogx')
plt.grid(True)
```



## 12.9 Plot Grid and Save to File

```
In [15]: theta = np.linspace(0,2*np.pi,100)
plt.plot(np.cos(theta),np.sin(theta))
plt.grid();
```



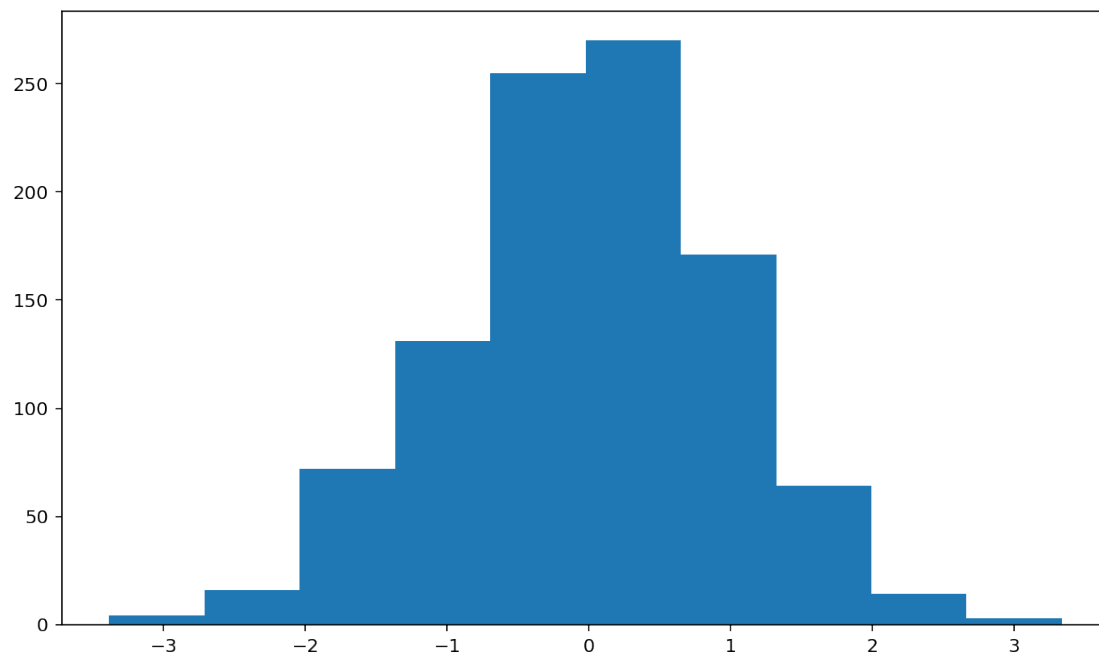
```
In [16]: plt.savefig('circle.png');
 %ls *.png
```

circle.png

<Figure size 720x432 with 0 Axes>

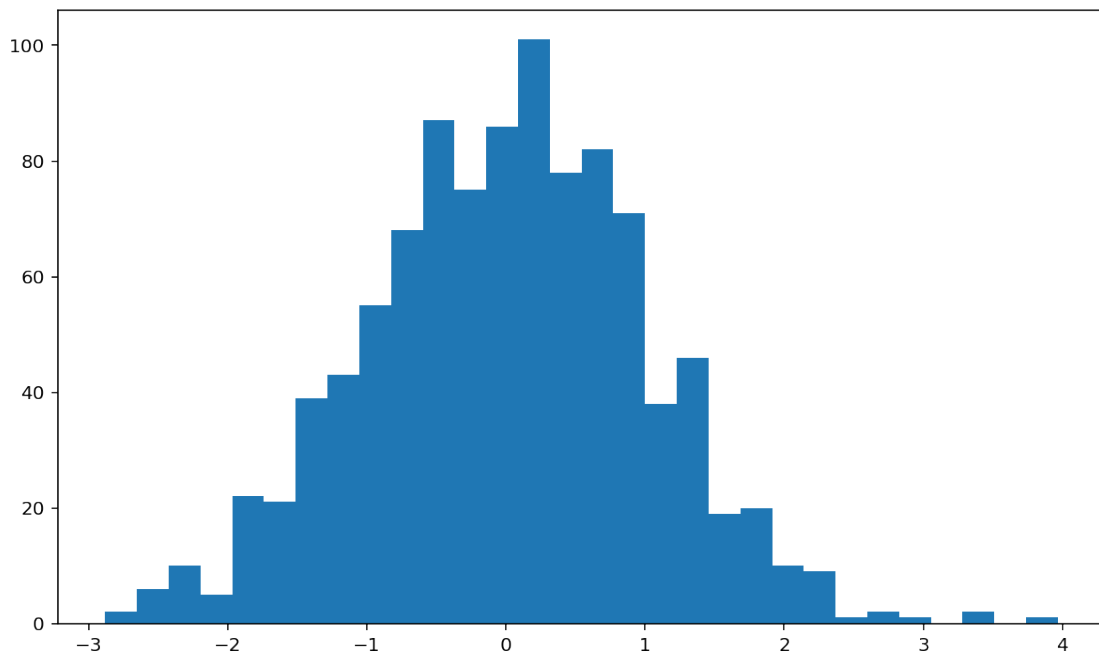
## 12.10 Histogram

```
In [17]: from numpy.random import randn
 plt.hist(randn(1000));
```



Change the number of bins and suppress display of returned array with ;

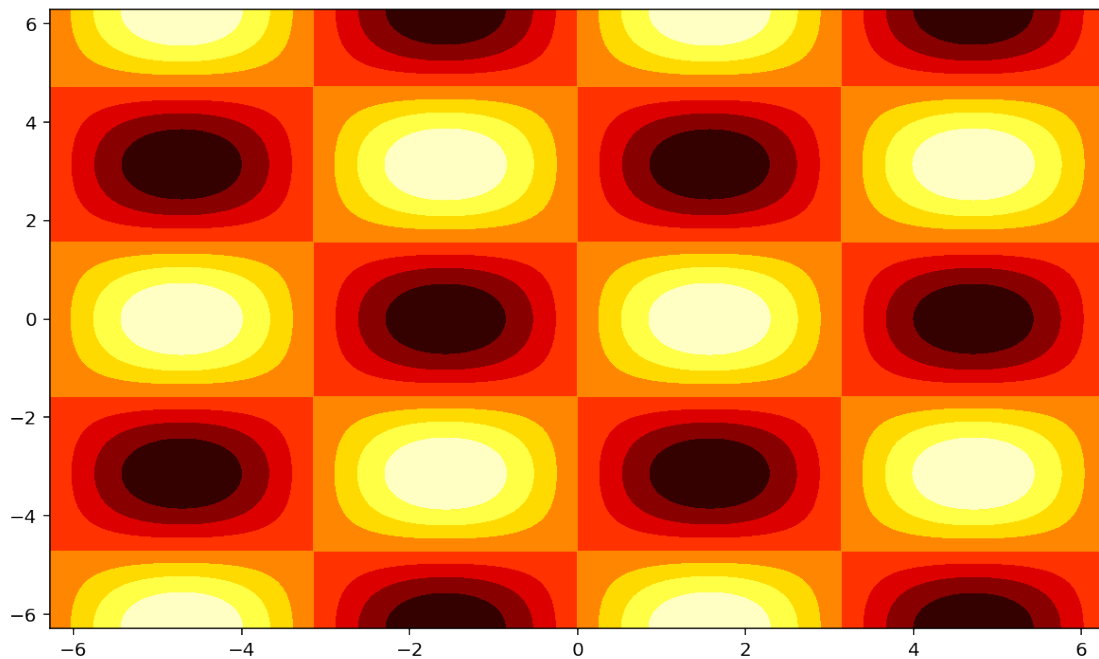
```
In [18]: plt.hist(randn(1000), 30);
```



## 12.11 Contour Plot

```
In [19]: x = y = np.arange(-2.0*np.pi, 2.0*np.pi+0.01, 0.01)
 X, Y = np.meshgrid(x, y)
 Z = np.sin(X)*np.cos(Y)

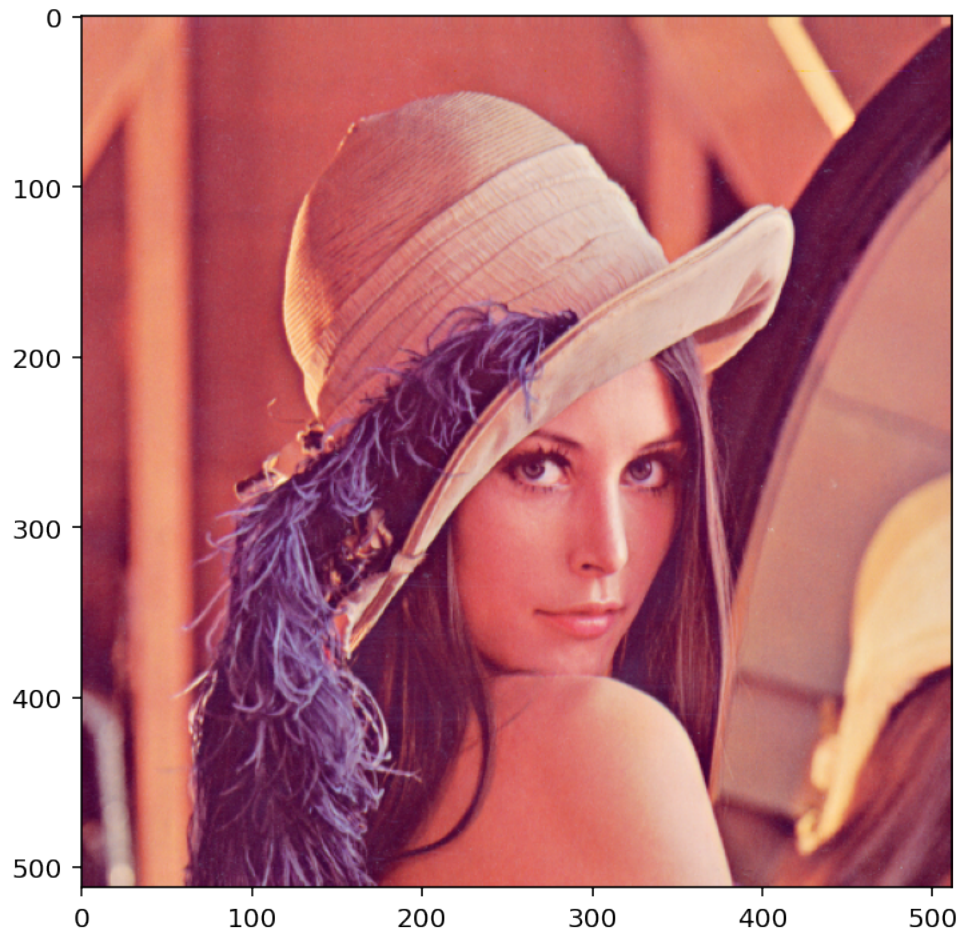
 plt.contourf(X, Y, Z, cmap=plt.cm.hot);
```



## 12.12 Image Display

```
In [20]: img = plt.imread("https://hackage.haskell.org/package/JuicyPixels-extra-0.1.0/src/data-examples/lenna.png")
plt.imshow(img)
```

```
Out[20]: <matplotlib.image.AxesImage at 0x7fa1b68f7d00>
```



## 12.13 figure and axis

Best method to create a plot with many components

```
In [21]: fig = plt.figure()
axis = fig.add_subplot(111, aspect='equal',
 xlim=(-2, 2), ylim=(-2, 2))

state = -0.5 + np.random.random((50, 4))
state[:, :2] *= 3.9
```

```

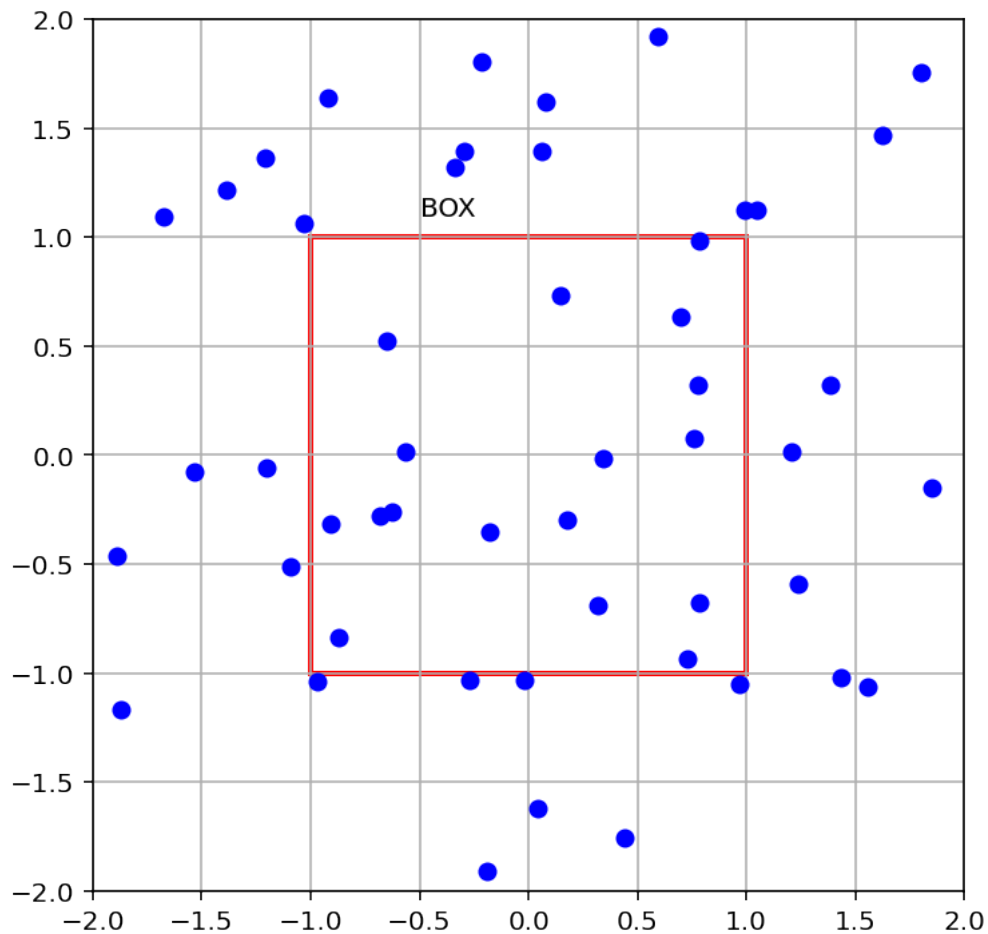
bounds = [-1, 1, -1, 1]

particles = axis.plot(state[:,0], state[:,1], 'bo', ms=6)
rect = plt.Rectangle(bounds[:2],
 bounds[1] - bounds[0],
 bounds[3] - bounds[2],
 ec='r', lw=2, fc='none')

axis.grid()
axis.add_patch(rect)
axis.text(-0.5, 1.1, "BOX")

```

Out[21]: Text(-0.5, 1.1, 'BOX')



## 12.14 Exercises

Recreate the image `my_plots.png` using the `delicate_arch.png` file in `images` directory.

## 12.15 Alternatives

- [bqplot](#) : Jupyter Notebooks, Interactive.

- [seaborn](#) : Statistics.
- [toyplot](#) : Nice graphes.
- [bokeh](#) : Interactive and Server mode.
- [pygal](#) : Charting
- [Altair](#) : Data science
- [plot.ly](#) : Data science and interactive
- [Mayavi](#): 3D
- [YT](#): Astrophysics (volume rendering, contours, particles).
- [VisIt](#): Powerful, easy to use but heavy.
- [Paraview](#): The most-used visualization application. Need high learning effort.
- [PyVista](#): 3D plotting and mesh analysis through a streamlined interface for the Visualization Toolkit (VTK)
- [Yellowbrick](#) : Yellowbrick: Machine Learning Visualization

```
In [22]: #example from Filipe Fernandes
#http://nbviewer.jupyter.org/gist/ocefpaf/9730c697819e91b99f1d694983e39a8f
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import animation

g = 9.81
denw = 1025.0 # Seawater density [kg/m**3].
sig = 7.3e-2 # Surface tension [N/m].
a = 1.0 # Wave amplitude [m].

L, h = 100.0, 50.0 # Wave height and water column depth.
k = 2 * np.pi / L
omega = np.sqrt((g * k + (sig / denw) * (k**3)) * np.tanh(k * h))
T = 2 * np.pi / omega
c = np.sqrt((g / k + (sig / denw) * k) * np.tanh(k * h))

We'll solve the wave velocities in the `x` and `z` directions.
x, z = np.meshgrid(np.arange(0, 160, 10), np.arange(0, -80, -10),)
u, w = np.zeros_like(x), np.zeros_like(z)

def compute_vel(phase):
 u = a * omega * (np.cosh(k * (z+h)) / np.sinh(k*h)) * np.cos(k * x - phase)
 w = a * omega * (np.sinh(k * (z+h)) / np.sinh(k*h)) * np.sin(k * x - phase)
 mask = -z > h
 u[mask] = 0.0
 w[mask] = 0.0
 return u, w

def basic_animation(frames=91, interval=30, dt=0.3):
 fig = plt.figure(figsize=(8, 6))
 ax = plt.axes(xlim=(0, 150), ylim=(-70, 10))

 # Animated.
 quiver = ax.quiver(x, z, u, w, units='inches', scale=2)
 ax.quiverkey(quiver, 120, -60, 1,
 label=r'1 m s$^{-1}$',
 coordinates='data')
 line, = ax.plot([], [], 'b')

 # Non-animated.
 ax.plot([0, 150], [0, 0], 'k:')
```

```

ax.set_ylabel('Depth [m]')
ax.set_xlabel('Distance [m]')
text = (r'λ = %s m; h = %s m; kh = %2.3f; h/L = %s' %
 (L, h, k * h, h/L))
ax.text(10, -65, text)
time_step = ax.text(10, -58, '')
line.set_data([], [])

def init():
 return line, quiver, time_step

def animate(i):
 time = i * dt
 phase = omega * time
 eta = a * np.cos(x[0] * k - phase)
 u, w = compute_vel(phase)
 quiver.set_UVC(u, w)
 line.set_data(x[0], 5 * eta)
 time_step.set_text('Time = {:.2f} s'.format(time))
 return line, quiver, time_step

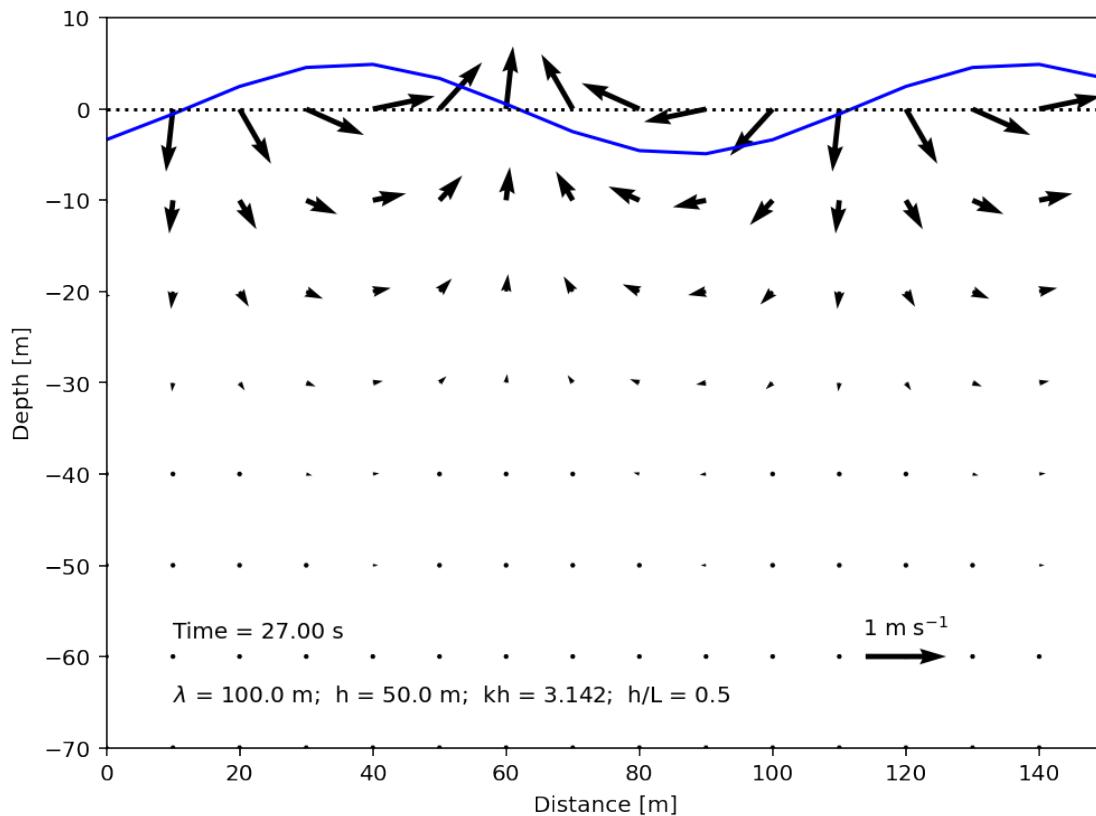
return animation.FuncAnimation(fig, animate, init_func=init,
 frames=frames, interval=interval)

```

In [23]: `from IPython.display import HTML`

`HTML(basic_animation(dt=0.3).to_jshtml())`

Out[23]: <IPython.core.display.HTML object>





## 12.16 References

- Simple examples with increasing difficulty <https://matplotlib.org/examples/index.html>
- Gallery <https://matplotlib.org/gallery.html>
- A [matplotlib tutorial](#), part of the [Lectures on Scientific Computing with Python](#) by J.R. Johansson.
- [NumPy Beginner | SciPy 2016 Tutorial | Alexandre Chabot LeClerc](#)
- [matplotlib tutorial](#) by Nicolas Rougier from LORIA.
- [10 Useful Python Data Visualization Libraries for Any Discipline](#)



## Chapter 13

# What provide Numpy to Python ?

- `ndarray` multi-dimensional array object
- derived objects such as masked arrays and matrices
- `ufunc` fast array mathematical operations.
- Offers some Matlab-ish capabilities within Python
- Initially developed by [Travis Oliphant](#).
- Numpy 1.0 released October, 2006.
- The [SciPy.org website](#) is very helpful.
- NumPy fully supports an object-oriented approach.

### 13.1 Routines for fast operations on arrays.

- shape manipulation
- sorting
- I/O
- FFT
- basic linear algebra
- basic statistical operations
- random simulation
- statistics
- and much more...

### 13.2 Getting Started with NumPy

- It is handy to import everything from NumPy into a Python console:

```
from numpy import *
```

- But it is easier to read and debug if you use explicit imports.

```
import numpy as np
import scipy as sp
import matplotlib.pyplot as plt
```

```
In [1]: import numpy as np
 print(np.__version__)
```

1.19.1

### 13.3 Why Arrays ?

- Python lists are slow to process and use a lot of memory.
- For tables, matrices, or volumetric data, you need lists of lists of lists... which becomes messy to program.

```
In [2]: from random import random
 from operator import truediv
```

```
In [3]: l1 = [random() for i in range(1000)]
 l2 = [random() for i in range(1000)]
 %timeit s = sum(map(truediv,l1,l2))
```

36.3  $\mu$ s  $\pm$  68.1 ns per loop (mean  $\pm$  std. dev. of 7 runs, 10000 loops each)

```
In [4]: a1 = np.array(l1)
 a2 = np.array(l2)
 %timeit s = np.sum(a1/a2)
```

6.73  $\mu$ s  $\pm$  28.8 ns per loop (mean  $\pm$  std. dev. of 7 runs, 100000 loops each)

### 13.4 Numpy Arrays: The ndarray class.

- There are important differences between NumPy arrays and Python lists:
  - NumPy arrays have a fixed size at creation.
  - NumPy arrays elements are all required to be of the same data type.
  - NumPy arrays operations are performed in compiled code for performance.
- Most of today’s scientific/mathematical Python-based software use NumPy arrays.
- NumPy gives us the code simplicity of Python, but the operation is speedily executed by pre-compiled C code.

```
In [5]: a = np.array([0,1,2,3]) # list
 b = np.array((4,5,6,7)) # tuple
 c = np.matrix('8 9 0 1') # string (matlab syntax)
```

```
In [6]: print(a,b,c)
```

```
[0 1 2 3] [4 5 6 7] [[8 9 0 1]]
```

### 13.5 Element wise operations are the “default mode”

```
In [7]: a*b,a+b
```

```
Out[7]: (array([0, 5, 12, 21]), array([4, 6, 8, 10]))
```

```
In [8]: 5*a, 5+a
```

```
Out[8]: (array([0, 5, 10, 15]), array([5, 6, 7, 8]))
```

```
In [9]: a @ b, np.dot(a,b) # Matrix multiplication
```

```
Out[9]: (38, 38)
```

## 13.6 NumPy Arrays Properties

```
In [10]: a = np.array([1,2,3,4,5]) # Simple array creation
In [11]: type(a) # Checking the type
Out[11]: numpy.ndarray
In [12]: a.dtype # Print numeric type of elements
Out[12]: dtype('int64')
In [13]: a.itemsize # Print Bytes per element
Out[13]: 8
In [14]: a.shape # returns a tuple listing the length along each dimension
Out[14]: (5,)
In [15]: np.size(a), a.size # returns the entire number of elements.
Out[15]: (5, 5)
In [16]: a.ndim # Number of dimensions
Out[16]: 1
In [17]: a.nbytes # Memory used
Out[17]: 40
```

- **\*\* Always use shape or size for numpy arrays instead of len \*\***
- len gives same information only for 1d array.

## 13.7 Functions to allocate arrays

```
In [18]: x = np.zeros((2,),dtype=('i4,f4,a10'))
 x
Out[18]: array([(0, 0., b''), (0, 0., b'')],
 dtype=[('f0', '<i4'), ('f1', '<f4'), ('f2', 'S10')])
empty, empty_like, ones, ones_like, zeros, zeros_like, full, full_like
```

## 13.8 Setting Array Elements Values

```
In [19]: a = np.array([1,2,3,4,5])
 print(a.dtype)
int64
In [20]: a[0] = 10 # Change first item value
 a, a.dtype
Out[20]: (array([10, 2, 3, 4, 5]), dtype('int64'))
In [21]: a.fill(0) # slightly faster than a[:] = 0
 a
Out[21]: array([0, 0, 0, 0, 0])
```

## 13.9 Setting Array Elements Types

```
In [22]: b = np.array([1,2,3,4,5.0]) # Last item is a float
 b, b.dtype
```

```
Out[22]: (array([1., 2., 3., 4., 5.]), dtype('float64'))
```

```
In [23]: a.fill(3.0) # assigning a float into a int array
 a[1] = 1.5 # truncates the decimal part
 print(a.dtype, a)
```

```
int64 [3 1 3 3 3]
```

```
In [24]: a.astype('float64') # returns a new array containing doubles
```

```
Out[24]: array([3., 1., 3., 3., 3.])
```

```
In [25]: np.asfarray([1,2,3,4]) # Return an array converted to a float type
```

```
Out[25]: array([1., 2., 3., 4.])
```

## 13.10 Slicing x[lower:upper:step]

- Extracts a portion of a sequence by specifying a lower and upper bound.
- The lower-bound element is included, but the upper-bound element is **not** included.
- The default step value is 1 and can be negative.

```
In [26]: a = np.array([10,11,12,13,14])
```

```
In [27]: a[:2], a[-5:-3], a[0:2], a[-2:] # negative indices work
```

```
Out[27]: (array([10, 11]), array([10, 11]), array([10, 11]), array([13, 14]))
```

```
In [28]: a[::-2], a[::-1]
```

```
Out[28]: (array([10, 12, 14]), array([14, 13, 12, 11, 10]))
```

### 13.10.1 Exercise:

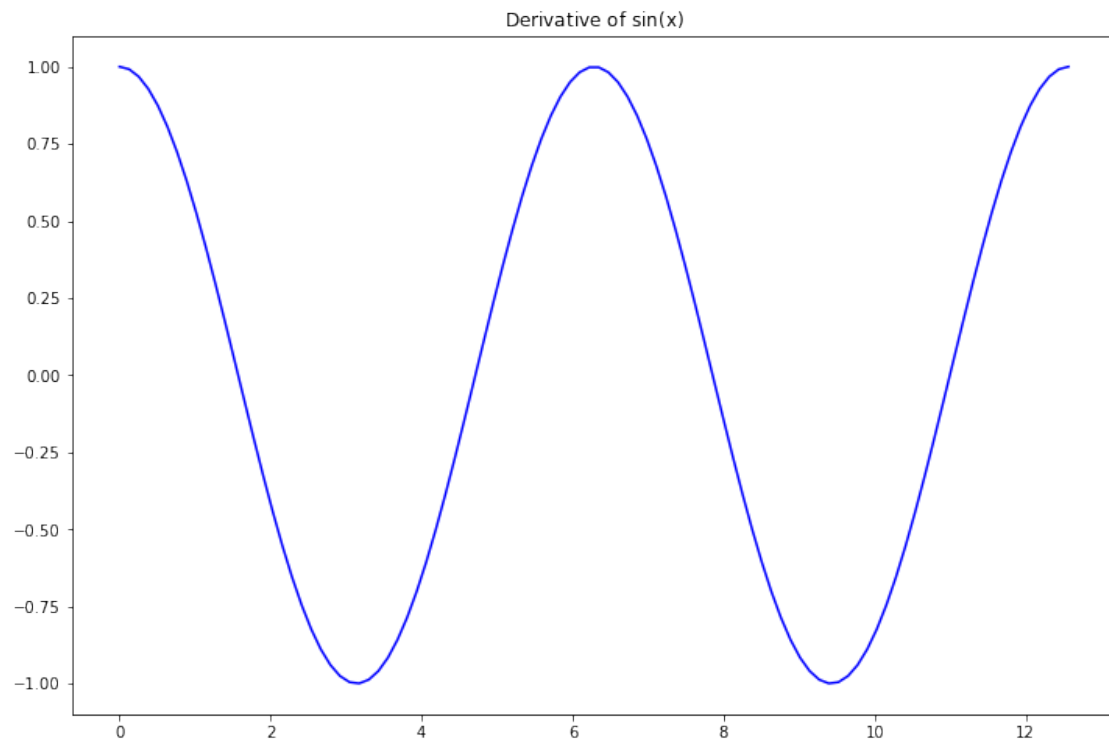
- Compute derivative of  $f(x) = \sin(x)$  with finite difference method.

$$\frac{\partial f}{\partial x} \sim \frac{f(x+dx) - f(x)}{dx}$$

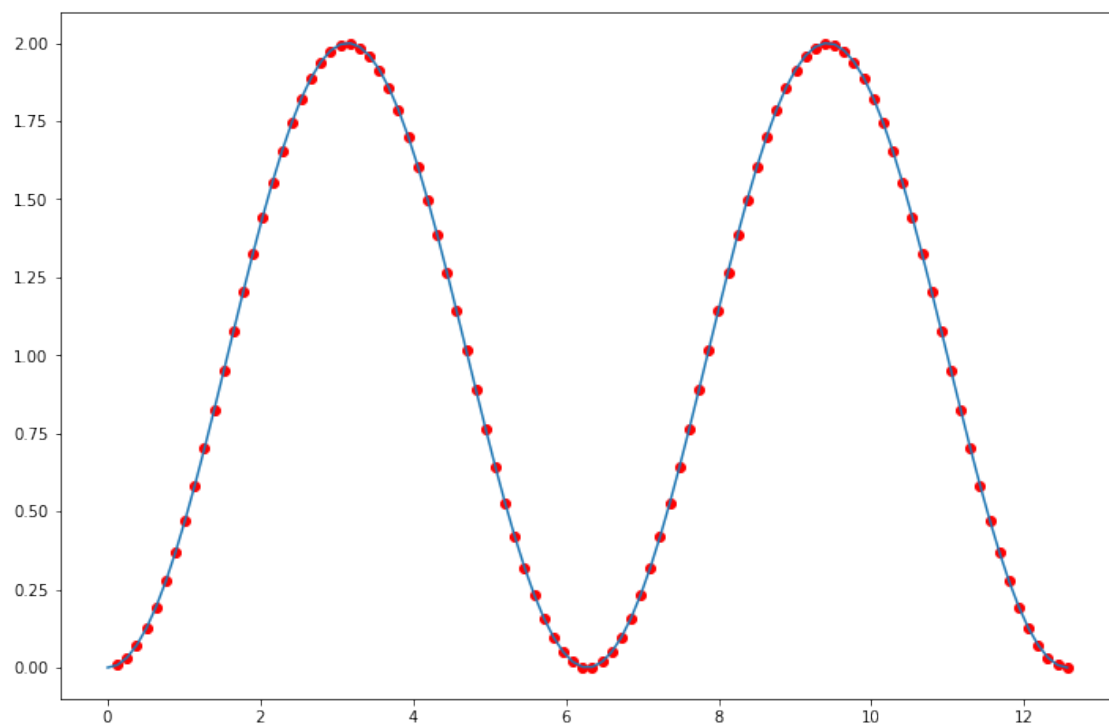
derivatives values are centered in-between sample points.

```
In [29]: x, dx = np.linspace(0,4*np.pi,100, retstep=True)
 y = np.sin(x)
```

```
In [30]: %matplotlib inline
 import matplotlib.pyplot as plt
 plt.rcParams['figure.figsize'] = [12.,8.] # Increase plot size
 plt.plot(x, np.cos(x), 'b')
 plt.title(r"$\rm{Derivative\ of}\ \sin(x)$");
```



```
In [31]: # Compute integral of x numerically
avg_height = 0.5*(y[1:]+y[:-1])
int_sin = np.cumsum(dx*avg_height)
plt.plot(x[1:], int_sin, 'ro', x, np.cos(0)-np.cos(x));
```



## 13.11 Multidimensional array

```
In [32]: a = np.arange(4*3).reshape(4,3) # NumPy array
 l = [[0,1,2],[3,4,5],[6,7,8],[9,10,11]] # Python List
```

```
In [33]: print(a)
 print(l)
```

```
[[0 1 2]
 [3 4 5]
 [6 7 8]
 [9 10 11]]
[[0, 1, 2], [3, 4, 5], [6, 7, 8], [9, 10, 11]]
```

```
In [34]: l[-1][-1] # Access to last item
```

```
Out[34]: 11
```

```
In [35]: print(a[-1,-1]) # Indexing syntax is different with NumPy array
 print(a[0,0]) # returns the first item
 print(a[1,:]) # returns the second line
```

```
11
0
[3 4 5]
```

```
In [36]: print(a[1]) # second line with 2d array
 print(a[:,-1]) # last column
```

```
[3 4 5]
[2 5 8 11]
```

### 13.11.1 Exercise

- We compute numerically the Laplace Equation Solution using Finite Difference Method
- Replace the computation of the discrete form of Laplace equation with numpy arrays

$$T_{i,j} = \frac{1}{4}(T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1})$$

- The function `numpy.allclose` can help you to compute the residual.

```
In [37]: %%time
 # Boundary conditions
 Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50

 # Set meshgrid
 n, l = 64, 1.0
 X, Y = np.meshgrid(np.linspace(0,1,n), np.linspace(0,1,n))
 T = np.zeros((n,n))

 # Set Boundary condition
```



```

T[n-1:, :] = Tnorth
T[:, 1] = Tsouth
T[:, n-1:] = Teast
T[:, :1] = Twest

residual = 1.0
istep = 0
while residual > 1e-5 :
 istep += 1
 print ((istep, residual), end="\r")
 residual = 0.0
 for i in range(1, n-1):
 for j in range(1, n-1):
 T_old = T[i,j]
 T[i, j] = 0.25 * (T[i+1,j] + T[i-1,j] + T[i,j+1] + T[i,j-1])
 if T[i,j]>0:
 residual=max(residual,abs((T_old-T[i,j])/T[i,j]))

print()
print("iterations = ",istep)
plt.title("Temperature")
plt.contourf(X, Y, T)
plt.colorbar()

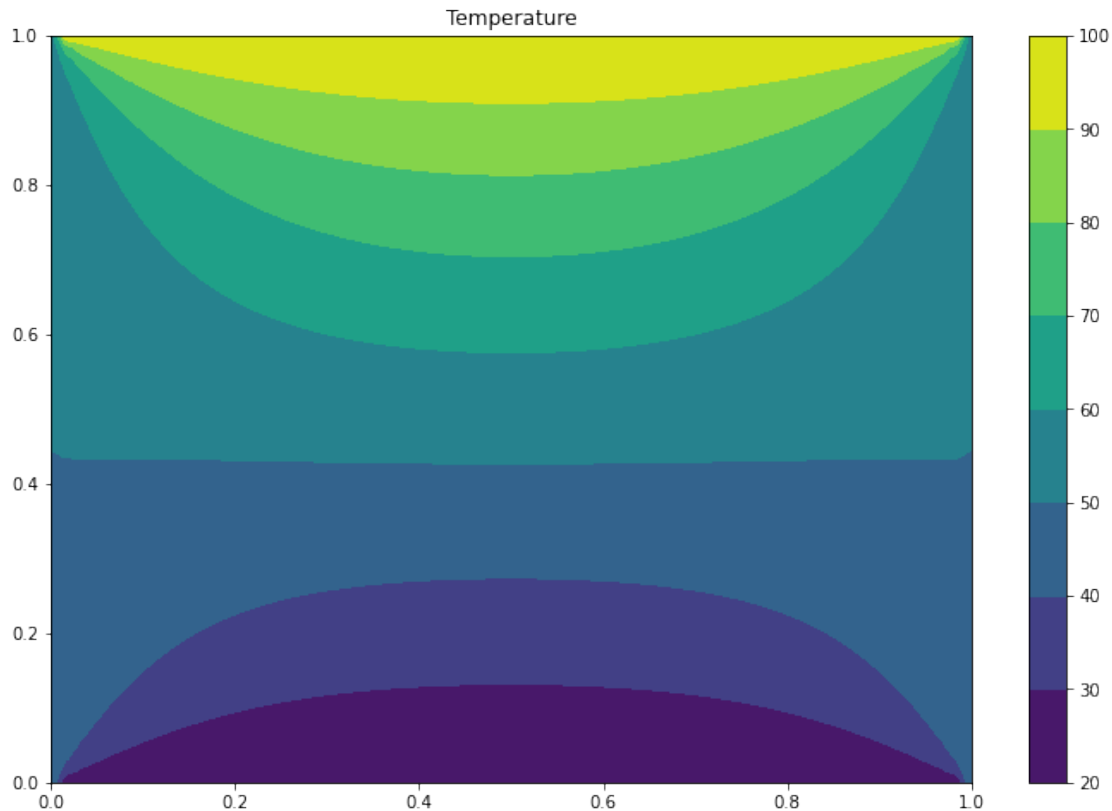
```

```

(2457, 1.0022293826789268e-05)
iterations = 2457
CPU times: user 37 s, sys: 316 ms, total: 37.3 s
Wall time: 36.7 s

```

```
Out[37]: <matplotlib.colorbar.Colorbar at 0x7f4c24b97520>
```



## 13.12 Arrays to ASCII files

In [38]: `x = y = z = np.arange(0.0,5.0,1.0)`

In [39]: `np.savetxt('test.out', (x,y,z), delimiter=',') # X is an array`  
`%cat test.out`

```
0.0000000000000000e+00,1.0000000000000000e+00,2.0000000000000000e+00,3.0000000000000000e+00,4.0000000000000000e+00,
0.0000000000000000e+00,1.0000000000000000e+00,2.0000000000000000e+00,3.0000000000000000e+00,4.0000000000000000e+00,
0.0000000000000000e+00,1.0000000000000000e+00,2.0000000000000000e+00,3.0000000000000000e+00,4.0000000000000000e+00,
```

In [40]: `np.savetxt('test.out', (x,y,z), fmt='%1.4e') # use exponential notation`  
`%cat test.out`

```
0.0000e+00 1.0000e+00 2.0000e+00 3.0000e+00 4.0000e+00
0.0000e+00 1.0000e+00 2.0000e+00 3.0000e+00 4.0000e+00
0.0000e+00 1.0000e+00 2.0000e+00 3.0000e+00 4.0000e+00
```

## 13.13 Arrays from ASCII files

In [41]: `np.loadtxt('test.out')`

```
Out[41]: array([[0., 1., 2., 3., 4.],
 [0., 1., 2., 3., 4.],
 [0., 1., 2., 3., 4.]])
```

- `save`: Save an array to a binary file in NumPy .npy format
- `savez` : Save several arrays into an uncompressed .npz archive
- `savez_compressed`: Save several arrays into a compressed .npz archive
- `load`: Load arrays or pickled objects from .npy, .npz or pickled files.

## 13.14 H5py

Pythonic interface to the HDF5 binary data format. [h5py user manual](#)

```
In [42]: import h5py as h5
```

```
with h5.File('test.h5', 'w') as f:
 f['x'] = x
 f['y'] = y
 f['z'] = z
```

```
In [43]: with h5.File('test.h5', 'r') as f:
 for field in f.keys():
 print(field+':', f[field].value)
```

```
x: [0. 1. 2. 3. 4.]
```

```
y: [0. 1. 2. 3. 4.]
```

```
z: [0. 1. 2. 3. 4.]
```

```
<ipython-input-43-7b8ae0576ec5>:3: H5pyDeprecationWarning: dataset.value has been deprecated. Use dataset.get()
print(field+':', f[field].value)
```

## 13.15 Slices Are References

- Slices are references to memory in the original array.
- Changing values in a slice also changes the original array.

```
In [44]: a = np.arange(10)
 b = a[3:6]
 b # `b` is a view of array `a` and `a` is called base of `b`
```

```
Out[44]: array([3, 4, 5])
```

```
In [45]: b[0] = -1
 a # you change a view the base is changed.
```

```
Out[45]: array([0, 1, 2, -1, 4, 5, 6, 7, 8, 9])
```

- Numpy does not copy if it is not necessary to save memory.

```
In [46]: c = a[7:8].copy() # Explicit copy of the array slice
 c[0] = -1
 a
```

```
Out[46]: array([0, 1, 2, -1, 4, 5, 6, 7, 8, 9])
```

## 13.16 Fancy Indexing

```
In [47]: a = np.fromfunction(lambda i, j: (i+1)*10+j, (4, 5), dtype=int)
a
```

```
Out[47]: array([[10, 11, 12, 13, 14],
 [20, 21, 22, 23, 24],
 [30, 31, 32, 33, 34],
 [40, 41, 42, 43, 44]])
```

```
In [48]: np.random.shuffle(a.flat) # shuffle modify only the first axis
a
```

```
Out[48]: array([[23, 34, 44, 12, 10],
 [30, 22, 24, 41, 31],
 [33, 43, 20, 14, 13],
 [42, 21, 11, 40, 32]])
```

```
In [49]: locations = a % 3 == 0 # locations can be used as a mask
a[locations] = 0 #set to 0 only the values that are divisible by 3
a
```

```
Out[49]: array([[23, 34, 44, 0, 10],
 [0, 22, 0, 41, 31],
 [0, 43, 20, 14, 13],
 [0, 0, 11, 40, 32]])
```

```
In [50]: a += a == 0
a
```

```
Out[50]: array([[23, 34, 44, 1, 10],
 [1, 22, 1, 41, 31],
 [1, 43, 20, 14, 13],
 [1, 1, 11, 40, 32]])
```

### 13.16.1 numpy.take

```
In [51]: a[1:3,2:5]
```

```
Out[51]: array([[1, 41, 31],
 [20, 14, 13]])
```

```
In [52]: np.take(a, [[6,7],[10,11]]) # Use flatten array indices
```

```
Out[52]: array([[22, 1],
 [1, 43]])
```

## 13.17 Changing array shape

```
In [53]: grid = np.indices((2,3)) # Return an array representing the indices of a grid.
grid[0]
```

```
Out[53]: array([[0, 0, 0],
 [1, 1, 1]])
```

```
In [54]: grid[1]
```

```
Out[54]: array([[0, 1, 2],
 [0, 1, 2]])
```

```
In [55]: grid.flat[:] # Return a view of grid array
Out[55]: array([0, 0, 0, 1, 1, 1, 0, 1, 2, 0, 1, 2])
In [56]: grid.flatten() # Return a copy
Out[56]: array([0, 0, 0, 1, 1, 1, 0, 1, 2, 0, 1, 2])
In [57]: np.ravel(grid, order='C') # A copy is made only if needed.
Out[57]: array([0, 0, 0, 1, 1, 1, 0, 1, 2, 0, 1, 2])
```

## 13.18 Sorting

```
In [58]: a=np.array([5,3,6,1,6,7,9,0,8])
 np.sort(a) #. Return a view
Out[58]: array([0, 1, 3, 5, 6, 6, 7, 8, 9])
In [59]: a
Out[59]: array([5, 3, 6, 1, 6, 7, 9, 0, 8])
In [60]: a.sort() # Change the array inplace
 a
Out[60]: array([0, 1, 3, 5, 6, 6, 7, 8, 9])
```

## 13.19 Transpose-like operations

```
In [61]: a = np.array([5,3,6,1,6,7,9,0,8])
 b = a
 b.shape = (3,3) # b is a reference so a will be changed
In [62]: a
Out[62]: array([[5, 3, 6],
 [1, 6, 7],
 [9, 0, 8]])
In [63]: c = a.T # Return a view so a is not changed
 np.may_share_memory(a,c)
Out[63]: True
In [64]: c[0,0] = -1 # c is stored in same memory so change c you change a
 a
Out[64]: array([[-1, 3, 6],
 [1, 6, 7],
 [9, 0, 8]])
In [65]: c # is a transposed view of a
Out[65]: array([[-1, 1, 9],
 [3, 6, 0],
 [6, 7, 8]])
In [66]: b # b is a reference to a
```

```
Out[66]: array([[-1, 3, 6],
 [1, 6, 7],
 [9, 0, 8]])
```

```
In [67]: c.base # When the array is not a view `base` return None
```

```
Out[67]: array([[-1, 3, 6],
 [1, 6, 7],
 [9, 0, 8]])
```

## 13.20 Methods Attached to NumPy Arrays

```
In [68]: a = np.arange(20).reshape(4,5)
 np.random.shuffle(a.flat)
 a
```

```
Out[68]: array([[17, 4, 6, 0, 16],
 [10, 14, 3, 18, 8],
 [13, 12, 19, 5, 1],
 [15, 2, 11, 7, 9]])
```

```
In [69]: a = (a - a.mean())/ a.std() # Standardize the matrix
 print(a)
```

```
[[1.30066495 -0.95382097 -0.60697698 -1.64750894 1.12724296]
 [0.086711 0.78039897 -1.12724296 1.47408695 -0.26013299]
 [0.60697698 0.43355498 1.64750894 -0.78039897 -1.47408695]
 [0.95382097 -1.30066495 0.26013299 -0.43355498 -0.086711]]
```

```
In [70]: np.set_printoptions(precision=4)
 print(a)
```

```
[[1.3007 -0.9538 -0.607 -1.6475 1.1272]
 [0.0867 0.7804 -1.1272 1.4741 -0.2601]
 [0.607 0.4336 1.6475 -0.7804 -1.4741]
 [0.9538 -1.3007 0.2601 -0.4336 -0.0867]]
```

```
In [71]: a.argmax() # max position in the memory contiguous array
```

```
Out[71]: 12
```

```
In [72]: np.unravel_index(a.argmax(),a.shape) # get position in the matrix
```

```
Out[72]: (2, 2)
```

## 13.21 Array Operations over a given axis

```
In [73]: a = np.arange(20).reshape(5,4)
 np.random.shuffle(a.flat)
```

```
In [74]: a.sum(axis=0) # sum of each column
```

```
Out[74]: array([20, 54, 49, 67])
```

```
In [75]: np.apply_along_axis(sum, axis=0, arr=a)
```

```
Out[75]: array([20, 54, 49, 67])
```

```
In [76]: np.apply_along_axis(sorted, axis=0, arr=a)
```

```
Out[76]: array([[0, 4, 2, 7],
 [1, 5, 8, 11],
 [3, 14, 9, 13],
 [6, 15, 12, 17],
 [10, 16, 18, 19]])
```

You can replace the `sorted` builtin function by a user defined function.

```
In [77]: np.empty(10)
```

```
Out[77]: array([0., 0., 1., 0., 1., 1., 0., 1., 0., 0.])
```

```
In [78]: np.linspace(0,2*np.pi,10)
```

```
Out[78]: array([0. , 0.6981, 1.3963, 2.0944, 2.7925, 3.4907, 4.1888, 4.8869,
 5.5851, 6.2832])
```

```
In [79]: np.arange(0,2.+0.4,0.4)
```

```
Out[79]: array([0. , 0.4, 0.8, 1.2, 1.6, 2.])
```

```
In [80]: np.eye(4)
```

```
Out[80]: array([[1., 0., 0., 0.],
 [0., 1., 0., 0.],
 [0., 0., 1., 0.],
 [0., 0., 0., 1.]])
```

```
In [81]: a = np.diag(range(4))
 a
```

```
Out[81]: array([[0, 0, 0, 0],
 [0, 1, 0, 0],
 [0, 0, 2, 0],
 [0, 0, 0, 3]])
```

```
In [82]: a[:, :, np.newaxis]
```

```
Out[82]: array([[0],
 [0],
 [0],
 [0]],

 [[0],
 [1],
 [0],
 [0]],

 [[0],
 [0],
 [2],
 [0]],

 [[0],
 [0],
 [0],
 [3]])
```

### 13.21.1 Create the following arrays

```
[100 101 102 103 104 105 106 107 108 109]
```

Hint: `numpy.arange`

```
[-2. -1.8 -1.6 -1.4 -1.2 -1. -0.8 -0.6 -0.4 -0.2 0.
 0.2 0.4 0.6 0.8 1. 1.2 1.4 1.6 1.8]
```

Hint: `numpy.linspace`

```
[[0.001 0.00129155 0.0016681 0.00215443 0.00278256
 0.00359381 0.00464159 0.00599484 0.00774264 0.01]]
```

Hint: `numpy.logspace`

```
[[0. 0. -1. -1. -1.]
 [0. 0. 0. -1. -1.]
 [0. 0. 0. 0. -1.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
```

Hint: `numpy.tri`, `numpy.zeros`, `numpy.transpose`

```
[[0. 1. 2. 3. 4.]
 [-1. 0. 1. 2. 3.]
 [-1. -1. 0. 1. 2.]
 [-1. -1. -1. 0. 1.]
 [-1. -1. -1. -1. 0.]]
```

Hint: `numpy.ones`, `numpy.diag`

- Compute the integral numerically with Trapezoidal rule

$$I = \int_{-\infty}^{\infty} e^{-v^2} dv$$

with  $v \in [-10; 10]$  and  $n=20$ .

## 13.22 Views and Memory Management

- If it exists one view of a NumPy array, it can be destroyed.

```
In [83]: big = np.arange(1000000)
 small = big[:5]
 del big
 small.base
```

```
Out[83]: array([0, 1, 2, ..., 999997, 999998, 999999])
```

- Array called `big` is still allocated.
- Sometimes it is better to create a copy.

```
In [84]: big = np.arange(1000000)
 small = big[:5].copy()
 del big
 print(small.base)
```

None



### 13.22.1 Change memory alignment

```
In [85]: del(a)
 a = np.arange(20).reshape(5,4)
 print(a.flags)
```

```
C_CONTIGUOUS : True
F_CONTIGUOUS : False
OWNDATA : False
WRITEABLE : True
ALIGNED : True
WRITEBACKIFCOPY : False
UPDATEIFCOPY : False
```

```
In [86]: b = np.asfortranarray(a) # makes a copy
 b.flags
```

```
Out[86]: C_CONTIGUOUS : False
 F_CONTIGUOUS : True
 OWNDATA : True
 WRITEABLE : True
 ALIGNED : True
 WRITEBACKIFCOPY : False
 UPDATEIFCOPY : False
```

```
In [87]: b.base is a
```

```
Out[87]: False
```

You can also create a fortran array with array function.

```
In [88]: c = np.array([[1,2,3],[4,5,6]])
 f = np.asfortranarray(c)
```

```
In [89]: print(f.ravel(order='K')) # Return a 1D array using memory order
 print(c.ravel(order='K')) # Copy is made only if necessary
```

```
[1 4 2 5 3 6]
[1 2 3 4 5 6]
```

## 13.23 Broadcasting rules

Broadcasting rules allow you to make an outer product between two vectors: the first method involves array tiling, the second one involves broadcasting. The last method is significantly faster.

```
In [90]: n = 1000
 a = np.arange(n)
 ac = a[:, np.newaxis] # column matrix
 ar = a[np.newaxis, :] # row matrix
```

```
In [91]: %timeit np.tile(a, (n,1)).T * np.tile(a, (n,1))
```

11.3 ms  $\pm$  352  $\mu$ s per loop (mean  $\pm$  std. dev. of 7 runs, 100 loops each)

```
In [92]: %timeit ac * ar
```

```
1.3 ms ± 10.7 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

```
In [93]: np.all(np.tile(a, (n,1)).T * np.tile(a, (n,1)) == ac * ar)
```

```
Out[93]: True
```

## 13.24 Numpy Matrix

Specialized 2-D array that retains its 2-D nature through operations. It has certain special operators, such as `*` (matrix multiplication) and `**` (matrix power).

```
In [94]: m = np.matrix('1 2; 3 4') #Matlab syntax
 m
```

```
Out[94]: matrix([[1, 2],
 [3, 4]])
```

```
In [95]: a = np.matrix([[1, 2],[3, 4]]) #Python syntax
 a
```

```
Out[95]: matrix([[1, 2],
 [3, 4]])
```

```
In [96]: a = np.arange(1,4)
 b = np.mat(a) # 2D view, no copy!
 b, np.may_share_memory(a,b)
```

```
Out[96]: (matrix([[1, 2, 3]]), True)
```

```
In [97]: a = np.matrix([[1, 2, 3],[3, 4, 5]])
 a * b.T # Matrix vector product
```

```
Out[97]: matrix([[14],
 [26]])
```

```
In [98]: m * a # Matrix multiplication
```

```
Out[98]: matrix([[7, 10, 13],
 [15, 22, 29]])
```

## 13.25 StructuredArray using a compound data type specification

```
In [99]: data = np.zeros(4, dtype={'names':('name', 'age', 'weight'),
 'formats':('U10', 'i4', 'f8')})
 print(data.dtype)
```

```
[('name', '<U10'), ('age', '<i4'), ('weight', '<f8')]
```

```
In [100]: data['name'] = ['Pierre', 'Paul', 'Jacques', 'Francois']
 data['age'] = [45, 10, 71, 39]
 data['weight'] = [95.0, 75.0, 88.0, 71.0]
 print(data)
```

```
[('Pierre', 45, 95.) ('Paul', 10, 75.) ('Jacques', 71, 88.)
 ('Francois', 39, 71.)]
```

## 13.26 RecordArray

```
In [101]: data_rec = data.view(np.recarray)
 data_rec.age
```

```
Out[101]: array([45, 10, 71, 39], dtype=int32)
```

## 13.27 NumPy Array Programming

- Array operations are fast, Python loops are slow.
- Top priority: **avoid loops**
- It's better to do the work three times with array operations than once with a loop.
- This does require a change of habits.
- This does require some experience.
- NumPy's array operations are designed to make this possible.

## 13.28 Fast Evaluation Of Array Expressions

- The `numexpr` package supplies routines for the fast evaluation of array expressions elementwise by using a vector-based virtual machine.
- Expressions are cached, so reuse is fast.

### Numexpr Users Guide

```
In [102]: import numexpr as ne
 import numpy as np
 nrange = (2 ** np.arange(6, 24)).astype(int)

 t_numpy = []
 t_numexpr = []

 for n in nrange:
 a = np.random.random(n)
 b = np.arange(n, dtype=np.double)
 c = np.random.random(n)

 c1 = ne.evaluate("a ** 2 + b ** 2 + 2 * a * b * c ", optimization='aggressive')

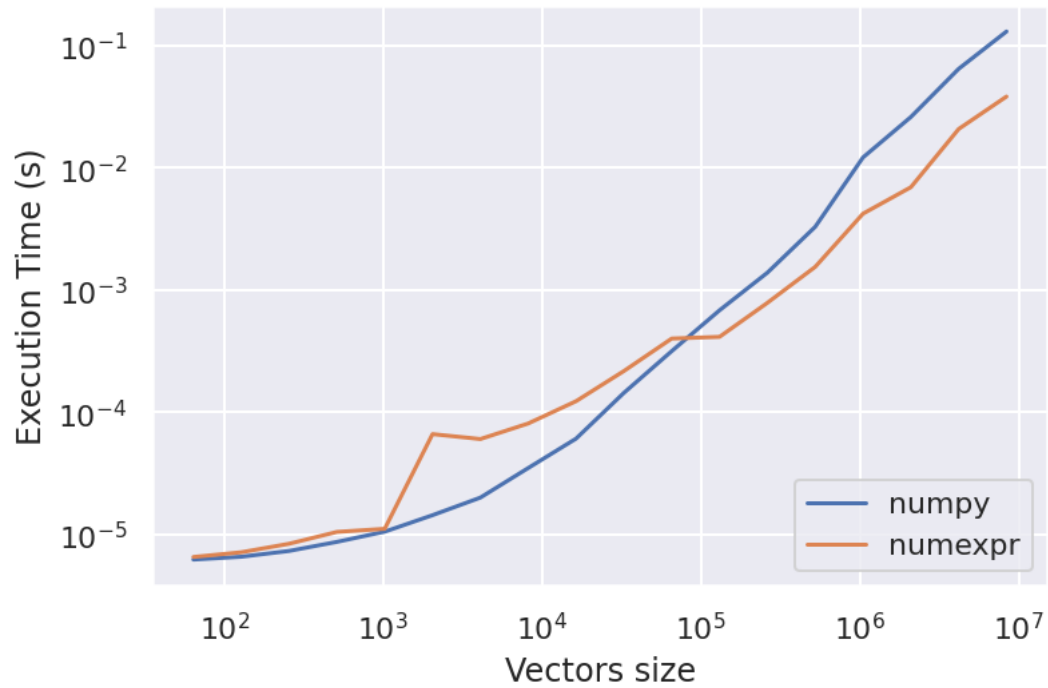
 t1 = %timeit -oq -n 10 a ** 2 + b ** 2 + 2 * a * b * c
 t2 = %timeit -oq -n 10 ne.re_evaluate()

 t_numpy.append(t1.best)
 t_numexpr.append(t2.best)

 %matplotlib inline
 %config InlineBackend.figure_format = 'retina'
 import matplotlib.pyplot as plt
 import seaborn; seaborn.set()

 plt.loglog(nrange, t_numpy, label='numpy')
 plt.loglog(nrange, t_numexpr, label='numexpr')

 plt.legend(loc='lower right')
 plt.xlabel('Vectors size')
 plt.ylabel('Execution Time (s)');
```



## 13.29 References

- [NumPy reference](#)
- [Getting the Best Performance out of NumPy](#)
- [Numpy by Konrad Hinsén](#)

## Chapter 14

# Scipy



Scipy is the scientific Python ecosystem : - fft, linear algebra, scientific computation,... - scipy contains numpy, it can be considered as an extension of numpy. - the add-on toolkits [Scikits](#) complements scipy.

```
In [1]: %matplotlib inline
 %config InlineBackend.figure_format = 'retina'
 import matplotlib.pyplot as plt
 plt.rcParams['figure.figsize'] = (10,6)
```

```
In [2]: import numpy as np
 import scipy as sp
 np.sqrt(-1.), np.log(-2.)
```

```
<ipython-input-2-5c51392f47b6>:3: RuntimeWarning: invalid value encountered in sqrt
 np.sqrt(-1.), np.log(-2.)
```

```
<ipython-input-2-5c51392f47b6>:3: RuntimeWarning: invalid value encountered in log
 np.sqrt(-1.), np.log(-2.)
```

```
Out[2]: (nan, nan)
```

```
In [3]: sp.sqrt(-1.), sp.log(-2.)
```

```
<ipython-input-3-52b38bd19582>:1: DeprecationWarning: scipy.sqrt is deprecated and will be removed in S
 sp.sqrt(-1.), sp.log(-2.)
```

```
<ipython-input-3-52b38bd19582>:1: DeprecationWarning: scipy.log is deprecated and will be removed in SciPy 1.0.0
 sp.sqrt(-1.), sp.log(-2.)
```

```
Out[3]: (1j, (0.6931471805599453+3.141592653589793j))
```

```
In [4]: sp.exp(sp.log(-2.))
```

```
<ipython-input-4-021f9e257f98>:1: DeprecationWarning: scipy.log is deprecated and will be removed in SciPy 1.0.0
 sp.exp(sp.log(-2.))
```

```
<ipython-input-4-021f9e257f98>:1: DeprecationWarning: scipy.exp is deprecated and will be removed in SciPy 1.0.0
 sp.exp(sp.log(-2.))
```

```
Out[4]: (-2+2.4492935982947064e-16j)
```

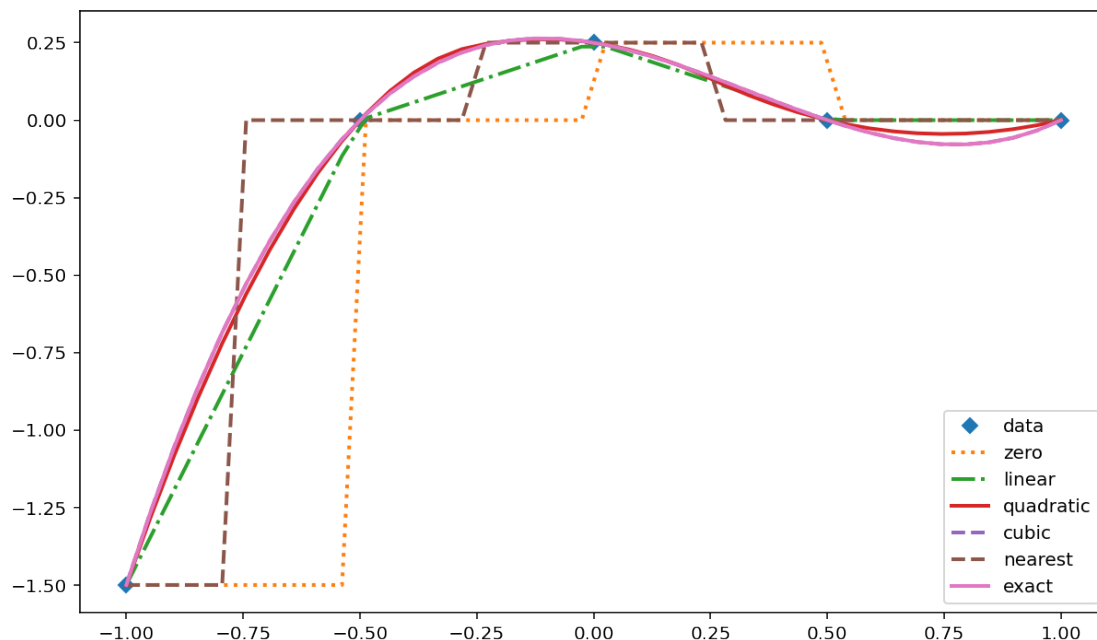
## 14.1 SciPy main packages

- `constants` : Physical and mathematical constants
- `fftpack` : Fast Fourier Transform routines
- `integrate` : Integration and ordinary differential equation solvers
- `interpolate` : Interpolation and smoothing splines
- `io` : Input and Output
- `linalg` : Linear algebra
- `signal` : Signal processing
- `sparse` : Sparse matrices and associated routines

```
In [5]: from scipy.interpolate import interp1d
 x = np.linspace(-1, 1, num=5) # 5 points evenly spaced in [-1,1].
 y = (x-1.)*(x-0.5)*(x+0.5) # x and y are numpy arrays
 f0 = interp1d(x,y, kind='zero')
 f1 = interp1d(x,y, kind='linear')
 f2 = interp1d(x,y, kind='quadratic')
 f3 = interp1d(x,y, kind='cubic')
 f4 = interp1d(x,y, kind='nearest')
```

```
In [6]: xnew = sp.linspace(-1, 1, num=40)
 ynew = (xnew-1.)*(xnew-0.5)*(xnew+0.5)
 plt.plot(x,y, 'D', xnew, f0(xnew), ':', xnew, f1(xnew), '-.',
 xnew, f2(xnew), '-', xnew, f3(xnew), '--',
 xnew, f4(xnew), '--', xnew, ynew, linewidth=2)
 plt.legend(['data', 'zero', 'linear', 'quadratic', 'cubic', 'nearest', 'exact'],
 loc='best');
```

```
<ipython-input-6-bb75b01c8201>:1: DeprecationWarning: scipy.linspace is deprecated and will be removed in SciPy 1.0.0
 xnew = sp.linspace(-1, 1, num=40)
```

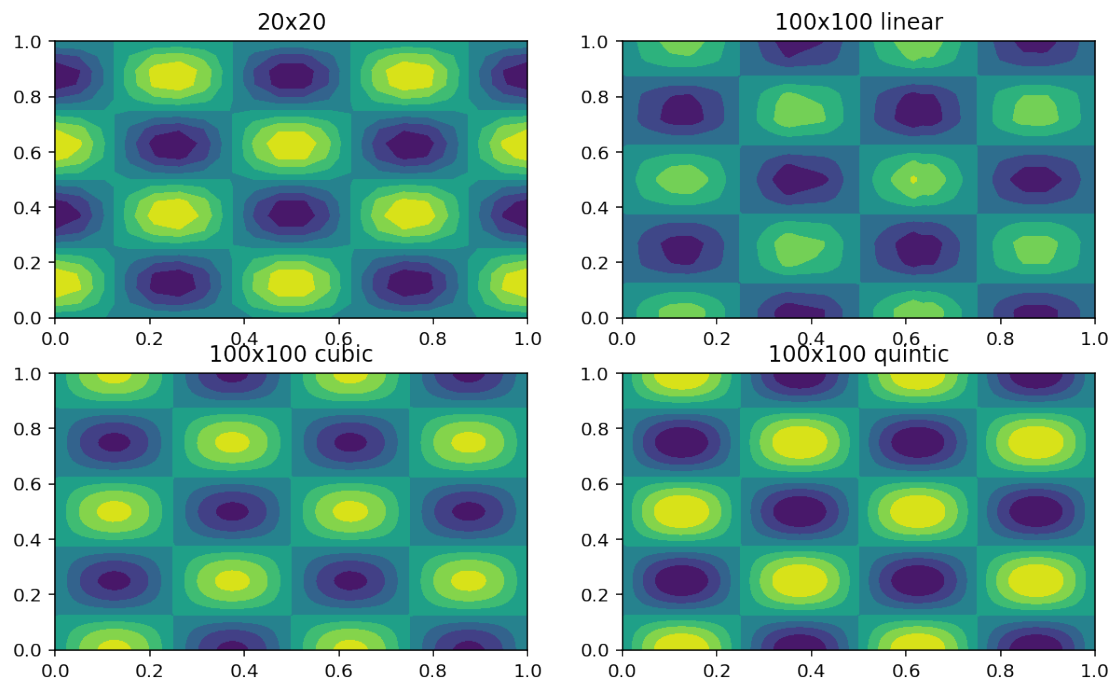


```
In [7]: from scipy.interpolate import interp2d
x,y=sp.mgrid[0:1:20j,0:1:20j] #create the grid 20x20
z=sp.cos(4*sp.pi*x)*sp.sin(4*sp.pi*y) #initialize the field
T1=interp2d(x,y,z,kind='linear')
T2=interp2d(x,y,z,kind='cubic')
T3=interp2d(x,y,z,kind='quintic')
```

```
<ipython-input-7-3b5ca11518b4>:3: DeprecationWarning: scipy.cos is deprecated and will be removed in SciPy 1.0.0
z=sp.cos(4*sp.pi*x)*sp.sin(4*sp.pi*y) #initialize the field
<ipython-input-7-3b5ca11518b4>:3: DeprecationWarning: scipy.sin is deprecated and will be removed in SciPy 1.0.0
z=sp.cos(4*sp.pi*x)*sp.sin(4*sp.pi*y) #initialize the field
```

```
In [8]: X,Y=sp.mgrid[0:1:100j,0:1:100j] #create the interpolation grid 100x100
complex -> number of points, float -> step size
plt.figure(1)
plt.subplot(221) #Plot original data
plt.contourf(x,y,z)
plt.title('20x20')
plt.subplot(222) #Plot linear interpolation
plt.contourf(X,Y,T1(X[:,0],Y[0,:]))
plt.title('100x100 linear')
plt.subplot(223) #Plot cubic interpolation
plt.contourf(X,Y,T2(X[:,0],Y[0,:]))
plt.title('100x100 cubic')
plt.subplot(224) #Plot quintic interpolation
plt.contourf(X,Y,T3(X[:,0],Y[0,:]))
plt.title('100x100 quintic')
```

```
Out[8]: Text(0.5, 1.0, '100x100 quintic')
```



## 14.2 FFT : `scipy.fftpack`

- FFT dimension 1, 2 and n : `fft`, `ifft` (inverse), `rfft` (real), `irfft`, `fft2` (dimension 2), `ifft2`, `fftn` (dimension n), `ifftn`.
- Discrete cosine transform : `dct`
- Convolution product : `convolve`

```
In [9]: from numpy.fft import fft, ifft
 x = np.random.random(1024)
 %timeit ifft(fft(x))
```

37.7  $\mu$ s  $\pm$  837 ns per loop (mean  $\pm$  std. dev. of 7 runs, 10000 loops each)

```
In [10]: from scipy.fftpack import fft, ifft
 x = np.random.random(1024)
 %timeit ifft(fft(x))
```

33.3  $\mu$ s  $\pm$  1.45  $\mu$ s per loop (mean  $\pm$  std. dev. of 7 runs, 10000 loops each)

## 14.3 Linear algebra : `scipy.linalg`

- Solvers, decompositions, eigen values. (same as `numpy`).
- Matrix functions : `expm`, `sinm`, `sinhm`,...
- Block matrices diagonal, triangular, periodic,...



```
In [11]: import scipy.linalg as spl
 b=sp.ones(5)
 A=sp.array([[1.,3.,0., 0.,0.],
 [2.,1.,-4, 0.,0.],
 [6.,1., 2,-3.,0.],
 [0.,1., 4.,-2.,-3.],
 [0.,0., 6.,-3., 2.]])
 print("x=",spl.solve(A,b,sym_pos=False)) # LAPACK (gesv ou posv)
 AB=sp.array([[0.,3.,-4.,-3.,-3.],
 [1.,1., 2.,-2., 2.],
 [2.,1., 4.,-3., 0.],
 [6.,1., 6., 0., 0.]])
 print("x=",spl.solve_banded((2,1),AB,b)) # LAPACK (gbsv)
```

```
x= [-0.24074074 0.41358025 -0.26697531 -0.85493827 0.01851852]
x= [-0.24074074 0.41358025 -0.26697531 -0.85493827 0.01851852]
```

```
<ipython-input-11-4af1e65b6e1b>:2: DeprecationWarning: scipy.ones is deprecated and will be removed in 1.10.0
 b=sp.ones(5)
<ipython-input-11-4af1e65b6e1b>:3: DeprecationWarning: scipy.array is deprecated and will be removed in 1.10.0
 A=sp.array([[1.,3.,0., 0.,0.],
<ipython-input-11-4af1e65b6e1b>:9: DeprecationWarning: scipy.array is deprecated and will be removed in 1.10.0
 AB=sp.array([[0.,3.,-4.,-3.,-3.],
```

```
In [12]: P,L,U = spl.lu(A) # P A = L U
 np.set_printoptions(precision=3)
 for M in (P,L,U):
 print(M, end="\n"+20*"-"+"\n")
```

```
[[0. 1. 0. 0. 0.]
 [0. 0. 0. 1. 0.]
 [1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 1.]
 [0. 0. 1. 0. 0.]]

[[1. 0. 0. 0. 0.]
 [0.167 1. 0. 0. 0.]
 [0. 0. 1. 0. 0.]
 [0.333 0.235 -0.765 1. 0.]
 [0. 0.353 0.686 0.083 1.]]

[[6. 1. 2. -3. 0.]
 [0. 2.833 -0.333 0.5 0.]
 [0. 0. 6. -3. 2.]
 [0. 0. 0. -1.412 1.529]
 [0. 0. 0. 0. -4.5]]
```

## 14.4 CSC (Compressed Sparse Column)

- All operations are optimized
- Efficient "slicing" along axis=1.
- Fast Matrix-vector product.

- Conversion to other format could be costly.

```
In [13]: import scipy.sparse as spsp
 row = sp.array([0,2,2,0,1,2])
 col = sp.array([0,0,1,2,2,2])
 data = sp.array([1,2,3,4,5,6])
 Mcsc1 = spsp.csc_matrix((data,(row,col)),shape=(3,3))
 Mcsc1.todense()
```

```
<ipython-input-13-1852fd1225dd>:2: DeprecationWarning: scipy.array is deprecated and will be removed in
row = sp.array([0,2,2,0,1,2])
<ipython-input-13-1852fd1225dd>:3: DeprecationWarning: scipy.array is deprecated and will be removed in
col = sp.array([0,0,1,2,2,2])
<ipython-input-13-1852fd1225dd>:4: DeprecationWarning: scipy.array is deprecated and will be removed in
data = sp.array([1,2,3,4,5,6])
```

```
Out[13]: matrix([[1, 0, 4],
 [0, 0, 5],
 [2, 3, 6]])
```

```
In [14]: indptr = sp.array([0,2,3,6])
 indices = sp.array([0,2,2,0,1,2])
 data = sp.array([1,2,3,4,5,6])
 Mcsc2 = spsp.csc_matrix ((data,indices,indptr),shape=(3,3))
 Mcsc2.todense()
```

```
<ipython-input-14-31b5a43b6ca2>:1: DeprecationWarning: scipy.array is deprecated and will be removed in
indptr = sp.array([0,2,3,6])
<ipython-input-14-31b5a43b6ca2>:2: DeprecationWarning: scipy.array is deprecated and will be removed in
indices = sp.array([0,2,2,0,1,2])
<ipython-input-14-31b5a43b6ca2>:3: DeprecationWarning: scipy.array is deprecated and will be removed in
data = sp.array([1,2,3,4,5,6])
```

```
Out[14]: matrix([[1, 0, 4],
 [0, 0, 5],
 [2, 3, 6]])
```

## 14.5 Dedicated format for assembling

- `lil_matrix` : Row-based linked list matrix. Easy format to build your matrix and convert to other format before solving.
- `dok_matrix` : A dictionary of keys based matrix. Ideal format for incremental matrix building. The conversion to csc/csr format is efficient.
- `coo_matrix` : coordinate list format. Fast conversion to formats CSC/CSR.

[Lien vers la documentation scipy](#)

## 14.6 Sparse matrices : `scipy.sparse.linalg`

- `speigen`, `speigen_symmetric`, `lobpcg` : (ARPACK).
- `svd` : (ARPACK).
- Direct methods (UMFPACK or SUPERLU) or Krylov based methods
- Minimization : `lsqr` and `minres`

For linear algebra: - Noobs: `spsolve`. - Intermmediate: `dsolve.spsolve` or `isolve.spsolve` - Advanced: `splu`, `spilu` (direct); `cg`, `cgs`, `bicg`, `bicgstab`, `gmres`, `lgmres` et `qmr` (iterative) - Boss: `petsc4py` et `slepc4py`.

## 14.7 LinearOperator

The LinearOperator is used for matrix-free numerical methods.

```
In [15]: import scipy.sparse.linalg as spspl
 def mv(v):
 return sp.array([2*v[0],3*v[1]])

 A=spspl.LinearOperator((2,2),matvec=mv,dtype=float)
 A
```

```
Out[15]: <2x2 _CustomLinearOperator with dtype=float64>
```

```
In [16]: A*sp.ones(2)
```

```
<ipython-input-16-8911e67c7fda>:1: DeprecationWarning: scipy.ones is deprecated and will be removed in 1.10
 A*sp.ones(2)
<ipython-input-15-b76e52185657>:3: DeprecationWarning: scipy.array is deprecated and will be removed in 1.10
 return sp.array([2*v[0],3*v[1]])
```

```
Out[16]: array([2., 3.])
```

```
In [17]: A.matmat(sp.array([[1,-2],[3,6]]))
```

```
<ipython-input-17-80cc6cd3a514>:1: DeprecationWarning: scipy.array is deprecated and will be removed in 1.10
 A.matmat(sp.array([[1,-2],[3,6]]))
<ipython-input-15-b76e52185657>:3: DeprecationWarning: scipy.array is deprecated and will be removed in 1.10
 return sp.array([2*v[0],3*v[1]])
```

```
Out[17]: array([[2, -4],
 [9, 18]])
```

## 14.8 LU decomposition

```
In [18]: N = 50
 un = sp.ones(N)
 w = sp.rand(N+1)
 A = spsp.spdiags([w[1:],-2*un,w[:-1]],[-1,0,1],N,N) # tridiagonal matrix
 A = A.tocsc()
 b = un
 op = spspl.splu(A)
 op
```

```
<ipython-input-18-e97d13ba2dbc>:2: DeprecationWarning: scipy.ones is deprecated and will be removed in 1.10
 un = sp.ones(N)
<ipython-input-18-e97d13ba2dbc>:3: DeprecationWarning: scipy.rand is deprecated and will be removed in 1.10
 w = sp.rand(N+1)
```

```
Out[18]: <SuperLU at 0x7fecf9024ae0>
```

```
In [19]: x=op.solve(b)
 spl.norm(A*x-b)
```

```
Out[19]: 1.4130832128153975e-15
```

## 14.9 Conjugate Gradient

```
In [20]: global k
 k=0
 def f(xk): # function called at every iterations
 global k
 print ("iteration {0:2d} residu = {1:7.3g}".format(k,spl.norm(A*xk-b)))
 k += 1

 x,info=spspl.cg(A,b,x0=sp.zeros(N),tol=1.0e-12,maxiter=N,M=None,callback=f)
```

```
iteration 0 residu = 2.62
iteration 1 residu = 1.21
iteration 2 residu = 0.688
iteration 3 residu = 0.286
iteration 4 residu = 0.168
iteration 5 residu = 0.0493
iteration 6 residu = 0.0128
iteration 7 residu = 0.0055
iteration 8 residu = 0.00178
iteration 9 residu = 0.000819
iteration 10 residu = 0.000288
iteration 11 residu = 7.9e-05
iteration 12 residu = 2.35e-05
iteration 13 residu = 4.91e-06
iteration 14 residu = 1.05e-06
iteration 15 residu = 3.9e-07
iteration 16 residu = 1.72e-07
iteration 17 residu = 4.75e-08
iteration 18 residu = 1.69e-08
iteration 19 residu = 4.39e-09
iteration 20 residu = 1.16e-09
iteration 21 residu = 2.79e-10
iteration 22 residu = 8.82e-11
iteration 23 residu = 1.51e-11
iteration 24 residu = 4.15e-12
```

```
<ipython-input-20-45fd1a29b4fe>:8: DeprecationWarning: scipy.zeros is deprecated and will be removed in
 x,info=spspl.cg(A,b,x0=sp.zeros(N),tol=1.0e-12,maxiter=N,M=None,callback=f)
```

## 14.10 Preconditioned conjugate gradient

```
In [21]: pc=spspl.spilu(A,drop_tol=0.1) # pc is an ILU decomposition
 xp=pc.solve(b)
 spl.norm(A*xp-b)
```

```
Out[21]: 0.3975967040362023
```

```
In [22]: def mv(v):
 return pc.solve(v)
 lo = spspl.LinearOperator((N,N),matvec=mv)
 k = 0
 x,info=spspl.cg(A,b,x0=sp.zeros(N),tol=1.e-12,maxiter=N,M=lo,callback=f)
```

```

iteration 0 residu = 0.362
iteration 1 residu = 0.0222
iteration 2 residu = 0.00144
iteration 3 residu = 7.73e-05
iteration 4 residu = 4.74e-06
iteration 5 residu = 2.72e-07
iteration 6 residu = 2.03e-08
iteration 7 residu = 8.38e-10
iteration 8 residu = 6.72e-11
iteration 9 residu = 4.38e-12

```

```

<ipython-input-22-dff3da9ff400>:5: DeprecationWarning: scipy.zeros is deprecated and will be removed in
x,info=spspl.cg(A,b,x0=sp.zeros(N),tol=1.e-12,maxiter=N,M=lo,callback=f)

```

## 14.11 Numerical integration

- quad, dblquad, tplquad,... Fortran library QUADPACK.

```
In [23]: import scipy.integrate as spi
```

```

x2=lambda x: x**2
4.**3/3 # int(x2) in [0,4]

```

```
Out[23]: 21.333333333333332
```

```
In [24]: spi.quad(x2,0.,4.)
```

```
Out[24]: (21.333333333333336, 2.368475785867001e-13)
```

## 14.12 Scipy ODE solver

It uses the Fortran ODEPACK library.

### 14.12.1 Van der Pol Oscillator

$$\begin{aligned}
 y_1'(t) &= y_2(t), \\
 y_2'(t) &= 1000(1 - y_1^2(t))y_2(t) - y_1(t)
 \end{aligned}$$

\$\$ with  $y_1(0) = 2$  and  $y_2(0) = 0$ .

```

In [25]: import numpy as np
import scipy.integrate as spi

def vdp1000(y,t):
 dy=np.zeros(2)
 dy[0]=y[1]
 dy[1]=1000.*(1.-y[0]**2)*y[1]-y[0]
 return dy

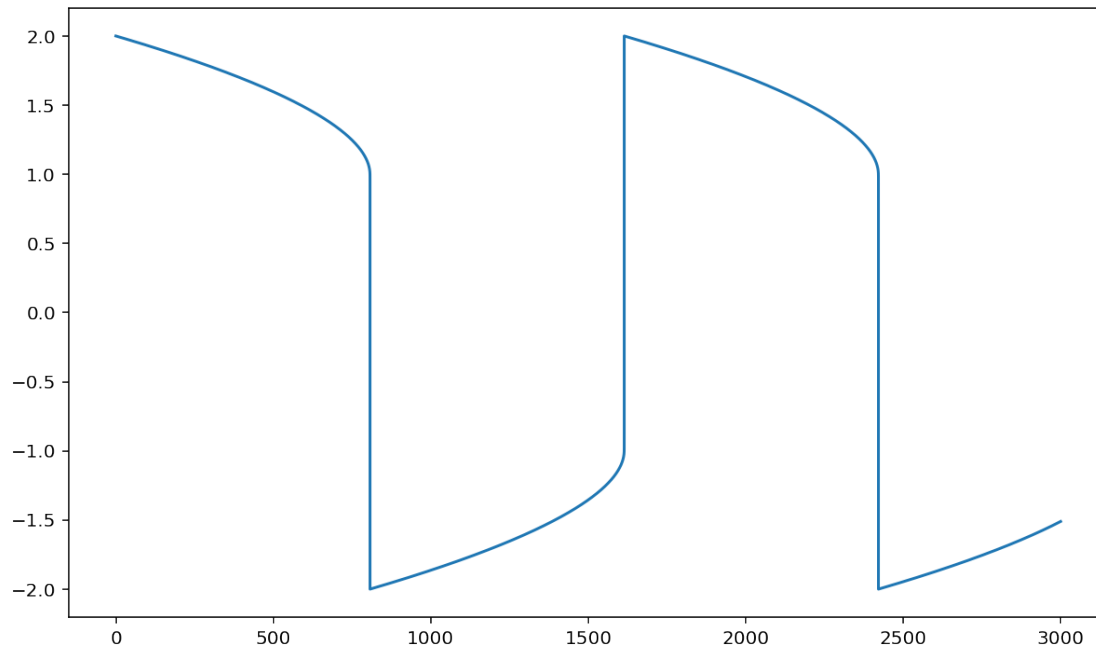
```

```

In [26]: t0, tf =0, 3000
N = 300000
t, dt = np.linspace(t0,tf,N, retstep=True)

```

```
In [27]: y=spi.odeint(vdp1000,[2.,0.],t)
 plt.plot(t,y[:,0]);
```



## 14.13 Exercise

The following code solve the Laplace equation using a dense matrix. - Modified the code to use a sparse matrix

```
In [28]: %%time
 %matplotlib inline
 %config InlineBackend.figure_format = "retina"
 import numpy as np
 import matplotlib.pyplot as plt
 plt.rcParams['figure.figsize'] = (10,6)

 # Boundary conditions
 Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50

 # Set meshgrid
 n = 50
 l = 1.0
 h = l / (n-1)
 X, Y = np.meshgrid(np.linspace(0,l,n), np.linspace(0,l,n))
 T = np.zeros((n,n),dtype='d')

 # Set Boundary condition
 T[n-1:, :] = Tnorth / h**2
 T[:1, :] = Tsouth / h**2
 T[:, n-1:] = Teast / h**2
 T[:, :1] = Twest / h**2
```

```

A = np.zeros((n*n,n*n),dtype='d')
nn = n*n
ii = 0
for j in range(n):
 for i in range(n):
 if j > 0:
 jj = ii - n
 A[ii,jj] = -1
 if j < n-1:
 jj = ii + n
 A[ii,jj] = -1
 if i > 0:
 jj = ii - 1
 A[ii,jj] = -1
 if i < n-1:
 jj = ii + 1
 A[ii,jj] = -1
 A[ii,ii] = 4
 ii = ii+1

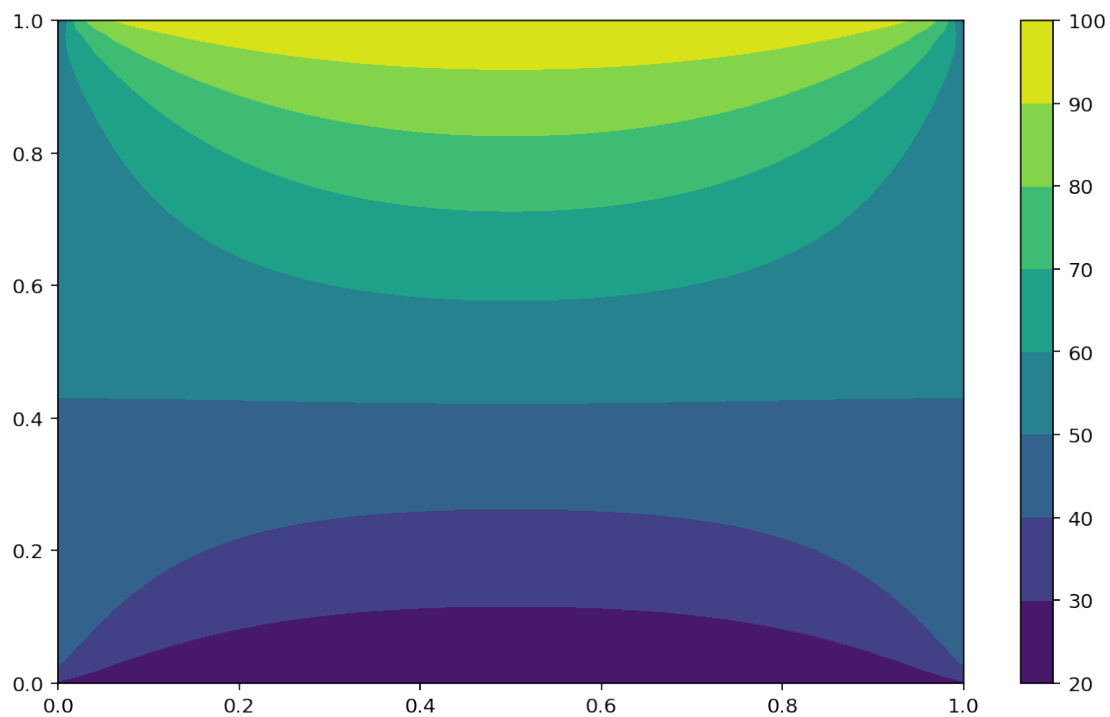
U = np.linalg.solve(A,np.ravel(h**2*T))
T = U.reshape(n,n)
plt.contourf(X,Y,T)
plt.colorbar()

```

CPU times: user 366 ms, sys: 68.1 ms, total: 434 ms

Wall time: 244 ms

Out[28]: <matplotlib.colorbar.Colorbar at 0x7fece75480a0>



```

In [29]: %%time
import scipy.sparse as spsp
import scipy.sparse.linalg as spspl

Boundary conditions
Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50

Set meshgrid
n = 50
l = 1.0
h = l / (n-1)
X, Y = np.meshgrid(np.linspace(0,l,n), np.linspace(0,l,n))
T = np.zeros((n,n),dtype='d')

Set Boundary condition
T[n-1:, :] = Tnorth / h**2
T[:, 1, :] = Tsouth / h**2
T[:, :, n-1:] = Teast / h**2
T[:, :, 1] = Twest / h**2

bdiag = -4 * np.eye(n)
bup = np.diag([1] * (n - 1), 1)
blow = np.diag([1] * (n - 1), -1)
block = bdiag + bup + blow
Creat a list of n blocks
blist = [block] * n
S = spsp.block_diag(blist)
Upper diagonal array offset by -n
upper = np.diag(np.ones(n * (n - 1)), n)
Lower diagonal array offset by -n
lower = np.diag(np.ones(n * (n - 1)), -n)
S += upper + lower

T = sp.linalg.solve(S,np.ravel(h**2*T))
plt.contourf(X,Y,T.reshape(n,n))
plt.colorbar();

```

```

CPU times: user 425 ms, sys: 176 ms, total: 600 ms
Wall time: 382 ms

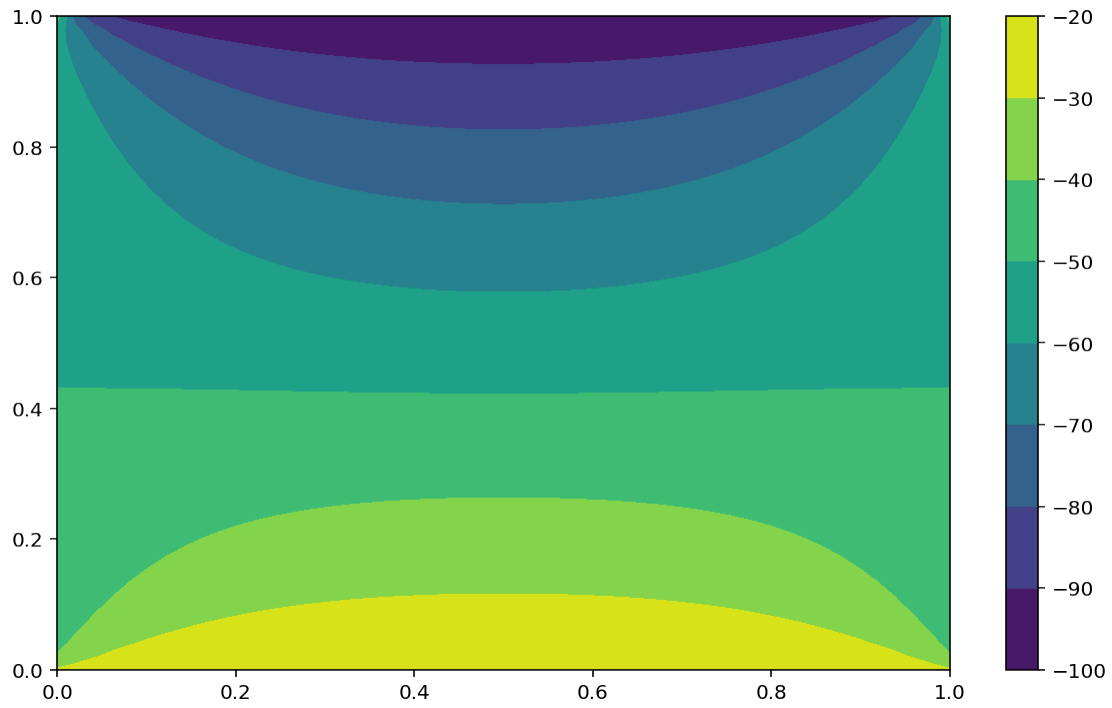
```

```

Out[29]: <matplotlib.colorbar.Colorbar at 0x7fece7479610>

```







## Chapter 15

# Sympy

```
In [1]: %matplotlib inline
 %config InlineBackend.figure_format = "retina"
 import matplotlib.pyplot as plt
 import numpy as np
 import seaborn as sns
 sns.set()
 plt.rcParams['figure.figsize'] = (10.0, 6.0)
```



The function `init_printing()` will enable LaTeX pretty printing in the notebook for SymPy expressions.

```
In [2]: import sympy as sym
 from sympy import symbols, Symbol
 sym.init_printing()
```

```
In [3]: x= Symbol('x')
 (sym.pi + x)**2
```

```
Out[3]:

$$(x + \pi)^2$$

```

```
In [4]: alpha1, omega_2 = symbols('alpha1 omega_2')
 alpha1, omega_2
```

```
Out[4]:

$$(\alpha_1, \omega_2)$$

```

```
In [5]: mu, sigma = sym.symbols('mu sigma', positive = True)
 1/sym.sqrt(2*sym.pi*sigma**2)* sym.exp(-(x-mu)**2/(2*sigma**2))
```

Out [5] :

$$\frac{\sqrt{2}e^{-\frac{(-\mu+x)^2}{2\sigma^2}}}{2\sqrt{\pi}\sigma}$$

## 15.1 Why use sympy?

- Symbolic derivatives
- Translate mathematics into low level code
- Deal with very large expressions
- Optimize code using mathematics

Dividing two integers in Python creates a float, like  $1/2 \rightarrow 0.5$ . If you want a rational number, use `Rational(1, 2)` or `S(1)/2`.

In [6]: `x + sym.S(1)/2 , sym.Rational(1,4)`

Out [6] :

$$\left(x + \frac{1}{2}, \frac{1}{4}\right)$$

In [7]: `y = Symbol('y')`  
`x ^ y # XOR operator (True only if x != y)`

Out [7] :

$$x \vee y$$

In [8]: `x**y`

Out [8] :

$$x^y$$

SymPy expressions are immutable. Functions that operate on an expression return a new expression.

In [9]: `expr = x + 1`  
`expr`

Out [9] :

$$x + 1$$

In [10]: `expr.subs(x, 2)`

Out [10] :

$$3$$

In [11]: `expr`

Out [11] :

$$x + 1$$

### 15.1.1 Exercise: Lagrange polynomial

Given a set of  $k + 1$  data points  $:(x_0, y_0), \dots, (x_j, y_j), \dots, (x_k, y_k)$  the Lagrange interpolation polynomial is:

$$L(x) := \sum_{j=0}^k y_j \ell_j(x)$$

$\ell_j$  are Lagrange basis polynomials:

$$\ell_j(x) := \prod_{\substack{0 \leq m \leq k \\ m \neq j}} \frac{x - x_m}{x_j - x_m}$$

We can demonstrate that at each point  $x_i$ ,  $L(x_i) = y_i$  so  $L$  interpolates the function.

- Compute the Lagrange polynomial for points

$$(-2, 21), (-1, 1), (0, -1), (1, -3), (2, 1)$$

## 15.2 Evaluate an expression

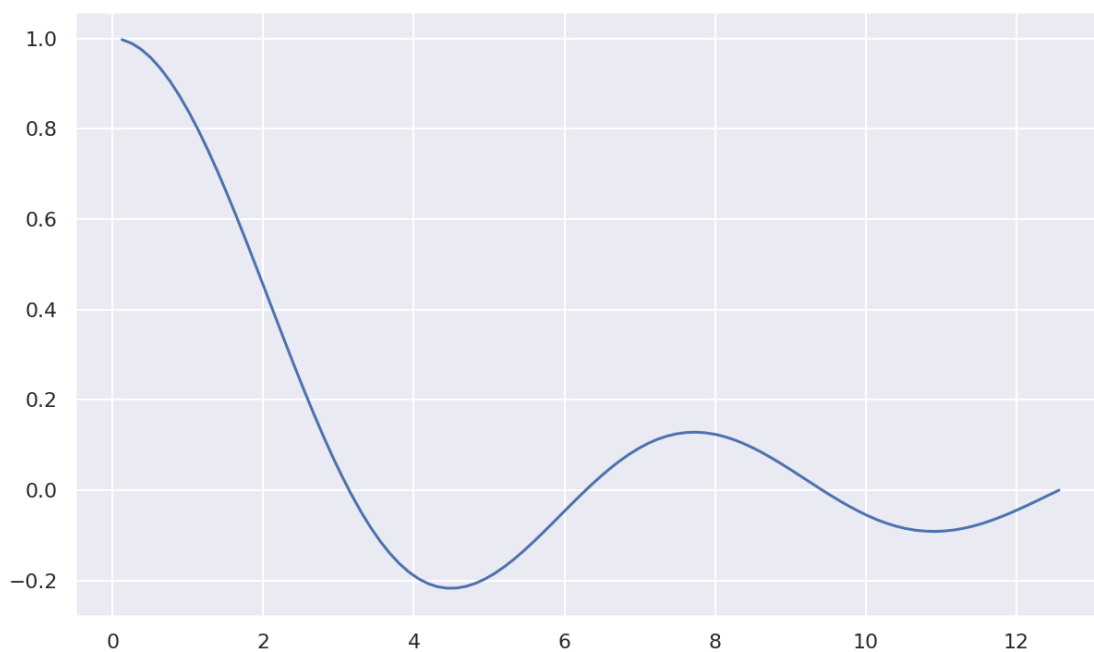
In [12]: `sym.sqrt(2), sym.sqrt(2).evalf(7) # set the precision to 7 digits`

Out[12]:  
 $(\sqrt{2}, 1.414214)$

In [13]: `from sympy import sin  
x = Symbol('x')  
expr = sin(x)/x  
expr.evalf(subs={x: 3.14}) # substitute the symbol x by Pi value`

Out[13]:  
0.00050721430461364

In [14]: `from sympy.utilities.autowrap import ufuncify  
f = ufuncify([x], expr, backend='f2py')  
  
t = np.linspace(0, 4*np.pi, 100)  
plt.plot(t, f(t));`



### 15.2.1 Exercise

- Plot the Lagrange polynomial computed above and interpolations points with matplotlib

## 15.3 Undefined functions and derivatives

Undefined functions are created with `Function()`. Undefined are useful to state that one variable depends on another (for the purposes of differentiation).

```
In [15]: from sympy import Function
 f = Function('f')
```

```
In [16]: f(x) + 1
```

```
Out[16]:
 f(x) + 1
```

```
In [17]: from sympy import diff, sin, cos
 diff(sin(x + 1)*cos(y), x), diff(sin(x + 1)*cos(y), x, y), diff(f(x), x)
```

```
Out[17]:
 (cos(y) cos(x + 1), -sin(y) cos(x + 1), $\frac{d}{dx}f(x)$)
```

```
In [18]: c, t = sym.symbols('t c')
 u = sym.Function('u')
 sym.Eq(diff(u(t,x),t,t), c**2*diff(u(t,x),x,2))
```

```
Out[18]:
 $\frac{\partial^2}{\partial c^2}u(c, x) = t^2 \frac{\partial^2}{\partial x^2}u(c, x)$
```

## 15.4 Matrices

```
In [19]: from sympy import Matrix
 Matrix([[1, 2], [3, 4]])*Matrix([x, y])
```

```
Out[19]:
 $\begin{bmatrix} x + 2y \\ 3x + 4y \end{bmatrix}$
```

```
In [20]: x, y, z = sym.symbols('x y z')
 Matrix([sin(x) + y, cos(y) + x, z]).jacobian([x, y, z])
```

```
Out[20]:
 $\begin{bmatrix} \cos(x) & 1 & 0 \\ 1 & -\sin(y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$
```

## 15.5 Matrix symbols

SymPy can also operate on matrices of symbolic dimension ( $n \times m$ ). `MatrixSymbol("M", n, m)` creates a matrix  $M$  of shape  $n \times m$ .

```
In [21]: from sympy import MatrixSymbol, Transpose
```

```
 n, m = sym.symbols('n m', integer=True)
 M = MatrixSymbol("M", n, m)
 b = MatrixSymbol("b", m, 1)
 Transpose(M*b)
```

```
Out[21]:
 $(Mb)^T$
```

```
In [22]: Transpose(M*b).doit()
```

```
Out[22]:
 $b^T M^T$
```

## 15.6 Solving systems of equations

`solve` solves equations symbolically (not numerically). The return value is a list of solutions. It automatically assumes that it is equal to 0.

```
In [23]: from sympy import Eq, solve
 solve(Eq(x**2, 4), x)
```

```
Out[23]:
[-2, 2]
```

```
In [24]: solve(x**2 + 3*x - 3, x)
```

```
Out[24]:

$$\left[-\frac{3}{2} + \frac{\sqrt{21}}{2}, -\frac{\sqrt{21}}{2} - \frac{3}{2} \right]$$

```

```
In [25]: eq1 = x**2 + y**2 - 4 # circle of radius 2
 eq2 = 2*x + y - 1 # straight line: y(x) = -2*x + 1
 solve([eq1, eq2], [x, y])
```

```
Out[25]:

$$\left[\left(\frac{2}{5} - \frac{\sqrt{19}}{5}, \frac{1}{5} + \frac{2\sqrt{19}}{5} \right), \left(\frac{2}{5} + \frac{\sqrt{19}}{5}, \frac{1}{5} - \frac{2\sqrt{19}}{5} \right) \right]$$

```

## 15.7 Solving differential equations

`dsolve` can (sometimes) produce an exact symbolic solution. Like `solve`, `dsolve` assumes that expressions are equal to 0.

```
In [26]: from sympy import Function, dsolve
 f = Function('f')
 dsolve(f(x).diff(x, 2) + f(x))
```

```
Out[26]:

$$f(x) = C_1 \sin(x) + C_2 \cos(x)$$

```

## 15.8 Code printers

The most basic form of code generation are the code printers. They convert SymPy expressions into over a dozen target languages.

```
In [27]: x = symbols('x')
 expr = abs(sin(x**2))
 expr
```

```
Out[27]:

$$|\sin(x^2)|$$

```

```
In [28]: sym.ccode(expr)
```

```
Out[28]: 'fabs(sin(pow(x, 2)))'
```

```
In [29]: sym.fcode(expr, standard=2003, source_format='free')
```

```
Out[29]: 'abs(sin(x**2))'
```

```
In [30]: from sympy.printing.cxxcode import cxxcode
 cxxcode(expr)
```

```
Out[30]: 'std::fabs(std::sin(std::pow(x, 2)))'
```

```
In [31]: sym.tanh(x).rewrite(sym.exp)
```

```
Out[31]:

$$\frac{e^x - e^{-x}}{e^x + e^{-x}}$$

```

```
In [32]: from sympy import sqrt, exp, pi
 expr = 1/sqrt(2*pi*sigma**2)* exp(-(x-mu)**2/(2*sigma**2))
 print(sym.fcode(expr, standard=2003, source_format='free'))
```

```
parameter (pi = 3.1415926535897932d0)
(1.0d0/2.0d0)*sqrt(2.0d0)*exp(-0.5d0*(-mu + x)**2/sigma**2)/(sqrt(pi)* &
sigma)
```

## 15.9 Creating a function from a symbolic expression

In SymPy there is a function to create a Python function which evaluates (usually numerically) an expression. SymPy allows the user to define the signature of this function (which is convenient when working with e.g. a numerical solver in `scipy`).

```
In [33]: from sympy import log
 x, y = symbols('x y')
 expr = 3*x**2 + log(x**2 + y**2 + 1)
 expr
```

```
Out[33]:

$$3x^2 + \log(x^2 + y^2 + 1)$$

```

```
In [34]: %timeit expr.subs({x: 17, y: 42}).evalf()
```

```
141 µs ± 1.02 µs per loop (mean ± std. dev. of 7 runs, 10000 loops each)
```

```
In [35]: import math
 f = lambda x, y: 3*x**2 + math.log(x**2 + y**2 + 1)
 f(17, 42)
```

```
Out[35]:
874.6275443904885
```

```
In [36]: %timeit f(17, 42)
```

```
1.15 µs ± 6.8 ns per loop (mean ± std. dev. of 7 runs, 1000000 loops each)
```

Evaluate above expression numerically invoking the `subs` method followed by the `evalf` method can be quite slow and cannot be done repeatedly.

```
In [37]: from sympy import lambdify
 g = lambdify([x, y], expr, modules=['math'])
 g(17, 42)
```



Out[37]:

874.6275443904885

In [38]: %timeit g(17, 42)

1.08 µs ± 0.795 ns per loop (mean ± std. dev. of 7 runs, 1000000 loops each)

```
In [39]: xarr = np.linspace(17, 18, 5)
 h = lambdify([x, y], expr) # lambdify return a python function
 out = h(xarr, 42)
 out.shape
```

Out[39]:

(5)

```
In [40]: z = z1, z2, z3 = symbols('z:3')
 expr2 = x*y*(z1 + z2 + z3)
 func2 = lambdify([x, y, z], expr2)
 func2(1, 2, (3, 4, 5))
```

Out[40]:

24

Behind the scenes `lambdify` constructs a string representation of the Python code and uses Python's `eval` function to compile the function.

## 15.10 SIR model

$$\frac{dS(t)}{dt} = -\beta S(t)I(t) \quad (15.1)$$

$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t) \quad (15.2)$$

$$\frac{dR(t)}{dt} = \gamma I(t) \quad (15.3)$$

- S,I,R: ratio of suceptibles, infectious and recovered fraction of the population.
- t: time
- $\beta$  : transmission coefficient.
- $\gamma$  : healing rate.

We assume that total population is constant.

### 15.10.1 Solving the initial value problem numerically

We will now integrate this system of ordinary differential equations numerically using the `odeint` solver provided by `scipy`:

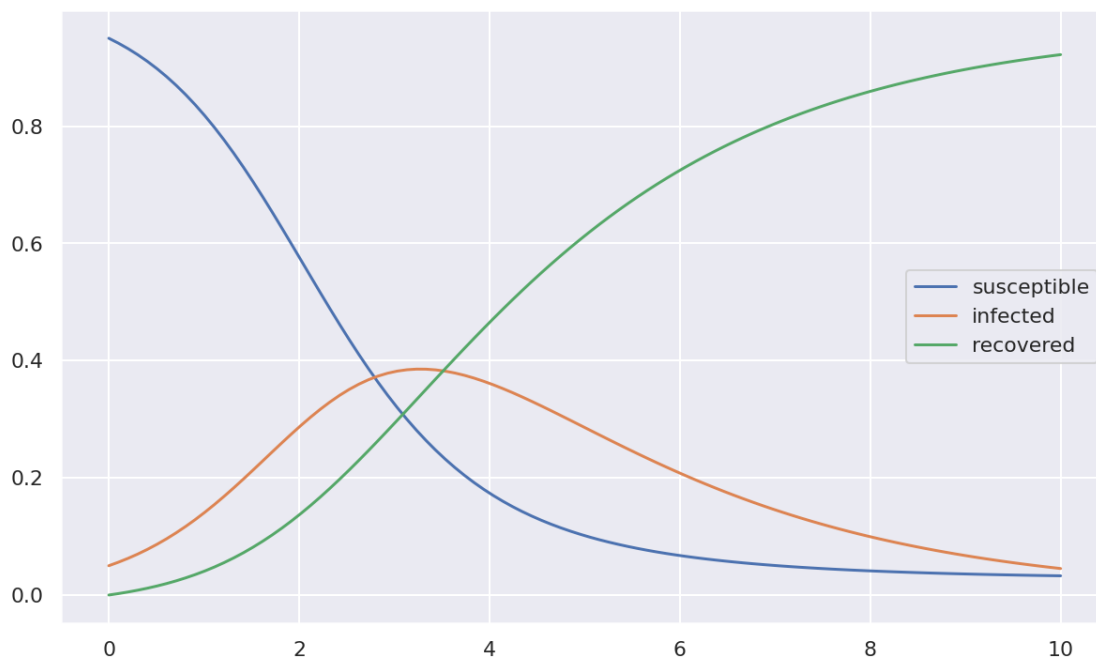
By looking at the [documentation](#) of `odeint` we see that we need to provide a function which computes a vector of derivatives ( $\dot{\mathbf{y}} = [\frac{dy_1}{dt}, \frac{dy_2}{dt}, \frac{dy_3}{dt}]$ ). The expected signature of this function is:

`f(y: array[float64], t: float64, *args: arbitrary constants) -> dydt: array[float64]`

in our case we can write it as:

```
In [41]: def rhs(y, t, beta, gamma):
 rb = beta * y[0]*y[1]
 rg = gamma * y[1]
 return [- rb , rb - rg, rg]
```

```
In [42]: import scipy.integrate as spi
 tout = np.linspace(0, 10, 100)
 k_vals = 1.66, 0.4545455
 y0 = [0.95, 0.05, 0]
 yout = spi.odeint(rhs, y0, tout, k_vals)
 plt.plot(tout, yout)
 plt.legend(['susceptible', 'infected', 'recovered']);
```



We will construct the system from a symbolic representation. But at the same time, we need the `rhs` function to be fast. Which means that we want to produce a fast function from our symbolic representation. Generating a function from our symbolic representation is achieved through *code generation*.

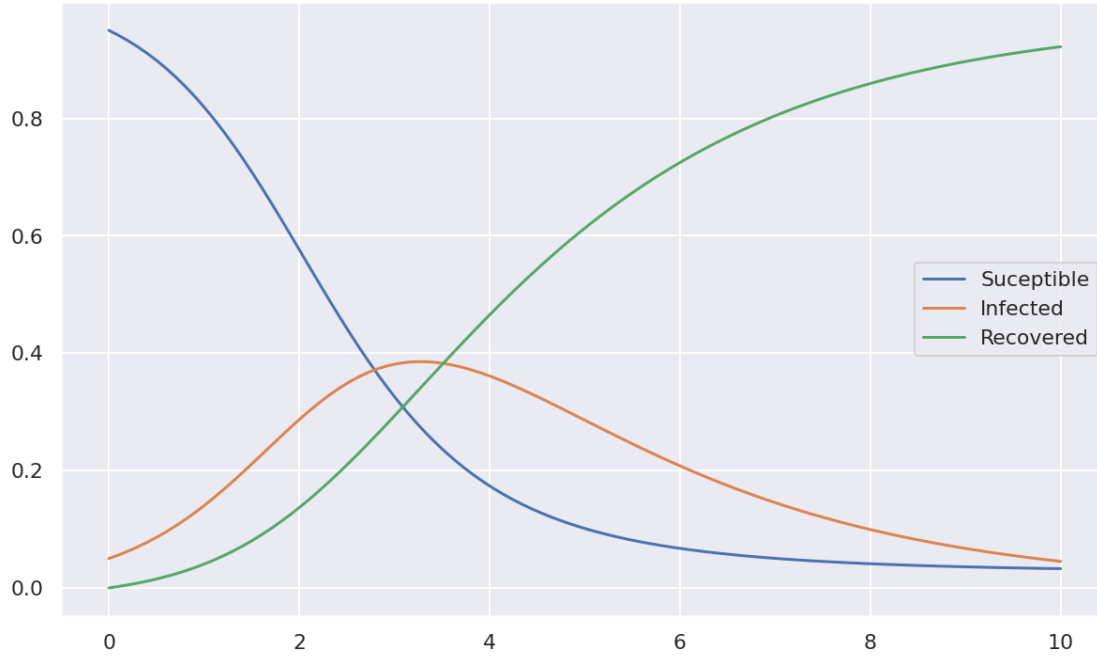
1. Construct a symbolic representation from some domain specific representation using SymPy.
2. Have SymPy generate a function with an appropriate signature (or multiple thereof), which we pass on to the solver.

We will achieve (1) by using SymPy symbols (and functions if needed). For (2) we will use a function in SymPy called `lambdify` it takes a symbolic expressions and returns a function. In a later notebook, we will look at (1), for now we will just use `rhs` which we've already written:

```
In [43]: y, k = sym.symbols('y:3'), sym.symbols('beta gamma')
 ydot = rhs(y, None, *k)
 y, ydot
```

```
Out[43]: ((y0, y1, y2), [-beta*y0*y1, beta*y0*y1 - gamma*y1, gamma*y1])
```

```
In [44]: f = sym.lambdify((y,t)+k, ydot)
 plt.plot(tout, spi.odeint(f, y0, tout, k_vals))
 plt.legend(['Suceptible', 'Infected', 'Recovered']);
```



In this example the gains of using a symbolic representation are arguably limited.

Let's take the same example with demography and  $n$  classes of subjects:

$$X_i = S_i, I_i, R_i \quad i = 1 \dots n$$

$$\frac{dS_i}{dt} = \nu_i - \beta_i S_i I_i - \mu_i S_i + \sum_{j=1}^n m_{ji} S_j - \sum_{j=1}^n m_{ij} S_i \frac{dI_i}{dt} = \beta_i S_i I_i - (\gamma_i + \mu_i) I_i + \sum_{j=1}^n m_{ij} I_j - \sum_{j=1}^n m_{ji} I_i \frac{dR_i}{dt} = -\frac{dS_i}{dt} - \frac{dI_i}{dt}$$

- $\beta$  : transmission coefficient
- $\gamma$  : healing rate
- $\mu$  : mortality rate
- $\nu$  : birth rate

### 15.10.2 Exercise

- Create the symbolic matrix  $m$ , symbols  $\nu_i, \mu_i, \beta_i, \gamma_i$  for  $i = 0, 1, 2$  and  $y_j$  for  $j = 0, 1, 2, \dots, 8$
- Write the system  $\dot{y} = f(t, y, m, \nu, \mu, \beta, \gamma)$
- `lambdify` the  $f$  function.
- Solve the system with:

$$m = \begin{bmatrix} 0 & 0.01 & 0.01 \\ 0.01 & 0 & 0.01 \\ 0.01 & 0.01 & 0 \end{bmatrix}$$

$$\begin{aligned}
t &= [0, 10] \text{ with } dt = 0.1 \\
\nu_i &= 0.0 \\
\mu_i &= 0.0 \\
\beta_i &= 1.66 \\
\gamma_i &= [0.4545, 0.3545, 0.2545] \\
S_i &= 0.95 \\
I_i &= 0.05 \\
R_i &= 0.0
\end{aligned}$$

### 15.10.3 Exercise : Bezier curve

We want to compute and the draw the Bezier curve between the 3 points  $p_0$ ,  $p_1$ , and  $p_2$ , The middle point  $p_1$  position is arbitrary.

$$p_0 = (1, 0); \quad p_1 = (x, y); \quad p_2 = (0, 1)$$

The  $n + 1$  Bernstein basis polynomials of degree  $n$  are defined as

$$b_{i,n}(x) = \binom{n}{i} x^i (1-x)^{n-i}, \quad i = 0, \dots, n.$$

where  $\binom{n}{i}$  is the binomial coefficient.

The Bezier curve is defined by a linear combination of Bernstein basis polynomials:

$$B_n(x) = \sum_{i=0}^n \beta_i b_{i,n}(x)$$

- With `sympy.binomial`, write a function `bpoly(t,n,i)` that returns the Bernstein basis polynomial  $b_{i,n}(t)$ .
- Compute the Bernstein polynomial representing the Bezier curve between  $p_0, p_1, p_2$ .  $\beta_i = 1$ .
- Plot the Bezier Curve for 3 positions of  $p_1 = (0, 0), (0.5, 0.5), (1, 1)$

## 15.11 Integrals quadrature

```
In [45]: from sympy.integrals.quadrature import *
 x, w = gauss_legendre(3, 5)
 x, w
```

```
Out[45]: ([-0.7746, 0, 0.7746], [0.55556, 0.88889, 0.55556])
```

```
In [46]: x, w = gauss_lobatto(3,12)
 x, w
```

```
Out[46]: ([-1, 0, 1], [0.333333333333, 1.33333333333, 0.33333333333])
```

## 15.12 References

- [SciPy 2017 tutorial](#)

## Chapter 16

# Call fortran from Python

```
In [1]: %matplotlib inline
 %config InlineBackend.figure_format = 'retina'
 import matplotlib.pyplot as plt
 import scipy.fftpack as sf
 import scipy.linalg as sl
 import numpy as np
 import warnings
 warnings.filterwarnings('ignore')
```

### 16.1 f2py

f2py is a part of Numpy and there are three ways to wrap Fortran with Python : - Write some fortran subroutines and just run f2py to create Python modules. - Insert special f2py directives inside Fortran source for complex wrapping. - Write a interface file (.pyf) to wrap Fortran files without changing them. f2py automatically generate the pyf template file that can be modified.

### 16.2 Simple Fortran subroutine to compute norm

#### 16.2.1 Fortran 90/95 free format

```
In [2]: %%file euclidian_norm.f90
 subroutine euclidian_norm (a, b, c)
 real(8), intent(in) :: a, b
 real(8), intent(out) :: c
 c = sqrt (a*a+b*b)
 end subroutine euclidian_norm
```

Writing euclidian\_norm.f90

#### 16.2.2 Fortran 77 fixed format

```
In [3]: %%file euclidian_norm.f
 subroutine euclidian_norm (a, b, c)
 real*8 a,b,c
 Cf2py intent(out) c
 c = sqrt (a*a+b*b)
 end
```

Writing euclidian\_norm.f

## 16.3 Build extension module with f2py program

```
In [4]: import sys
 ![sys.executable} -m numpy.f2py --quiet -c euclidian_norm.f90 -m vect --fcompiler=g95 --f90flags=-O3
```

### 16.3.1 Use the extension module in Python

```
In [5]: import vect
 c = vect.euclidian_norm(3,4)
 c
```

```
Out[5]: 5.0
```

```
In [6]: print(vect.euclidian_norm.__doc__) # Docstring is automatically generate
```

```
c = euclidian_norm(a,b)
```

```
Wrapper for ``euclidian_norm``.
```

```
Parameters
```

```

a : input float
b : input float
```

```
Returns
```

```

c : float
```

## 16.4 Fortran magic

- Jupyter extension that help to use fortran code in an interactive session.
- It adds a %%fortran cell magic that compile and import the Fortran code in the cell, using F2py.
- The contents of the cell are written to a .f90 file in the directory IPYTHONDIR/fortran using a filename with the hash of the code. This file is then compiled. The resulting module is imported and all of its symbols are injected into the user's namespace.

[Documentation](#)

```
In [7]: %load_ext fortranmagic
```

## 16.5 F2py directives

- F2PY introduces also some extensions to Fortran 90/95 language specification that help designing Fortran to Python interface, make it more “Pythonic”.
- If editing Fortran codes is acceptable, these specific attributes can be inserted directly to Fortran source codes. Special comment lines are ignored by Fortran compilers but F2PY interprets them as normal lines.

```
In [8]: %%fortran
 subroutine euclidian_norm(a,c,n)
 integer :: n
 real(8),dimension(n),intent(in) :: a
 !f2py optional , depend(a) :: n=len(a)
 real(8),intent(out) :: c
```

```

real(8) :: sommec
integer :: i
sommec = 0
do i=1,n
 sommec=sommec+a(i)*a(i)
end do
c = sqrt (sommec)
end subroutine euclidian_norm

```

```

In [9]: a=[2,3,4] # Python list
 type(a)

```

```
Out[9]: list
```

```
In [10]: euclidian_norm(a)
```

```
Out[10]: 5.385164807134504
```

```

In [11]: a=np.arange(2,5) # numpy array
 type(a)

```

```
Out[11]: numpy.ndarray
```

```
In [12]: euclidian_norm(a)
```

```
Out[12]: 5.385164807134504
```

```
In [13]: print(euclidian_norm.__doc__) # Documentation
```

```
c = euclidian_norm(a,[n])
```

```
Wrapper for ``euclidian_norm``.
```

```
Parameters
```

```

```

```
a : input rank-1 array('d') with bounds (n)
```

```
Other Parameters
```

```

```

```
n : input int, optional
 Default: len(a)
```

```
Returns
```

```

```

```
c : float
```

## 16.6 F2py directives

- optional: The corresponding argument is moved to the end.
- required: This is default. Use it to disable automatic optional setting.
- intent(in | inout | out | hide) , intent(in) is the default.
- intent(out) is implicitly translated to intent(out,hide).
- intent(copy) and intent(overwrite) control changes for input arguments.
- check performs some assertions, it is often automatically generated.
- depend: f2py detects cyclic dependencies.

- allocatable, parameter
- intent(callback), external: for function as arguments.
- intent(c) C-type argument , array or function.
- C expressions: rank, shape, len, size, slen.

## 16.7 Callback

You can call a python function inside your fortran code

```
In [14]: %%fortran
 subroutine sum_f (f ,n, s)
 !Compute sum(f(i), i=1,n)
 external f
 integer, intent(in) :: n
 real, intent(out) :: s
 s = 0.0
 do i=1,n
 s=s+f(i)
 end do
 end subroutine sum_f

In [15]: def fonction(i) : # python function
 return i*i

 sum_f(fonction,3)

Out[15]: 14.0

In [16]: sum_f(lambda x :x**2,3) # lambda function

Out[16]: 14.0
```

## 16.8 Fortran arrays and Numpy arrays

Let's see how to pass numpy arrays to fortran subroutine.

```
In [17]: %%fortran --extra "-DF2PY_REPORT_ON_ARRAY_COPY=1"
 subroutine push(positions, velocities, dt, n)
 integer, intent(in) :: n
 real(8), intent(in) :: dt
 real(8), dimension(n,3), intent(in) :: velocities
 real(8), dimension(n,3) :: positions
 do i = 1, n
 positions(i,:) = positions(i,:) + dt*velocities(i,:)
 end do
 end subroutine push

In [18]: positions = [[0, 0, 0], [0, 0, 0], [0, 0, 0]]
 velocities = [[0, 1, 2], [0, 3, 2], [0, 1, 3]]

In [19]: import sys
 push(positions, velocities, 0.1)
 positions # memory is not updated because we used C memory storage

Out[19]: [[0, 0, 0], [0, 0, 0], [0, 0, 0]]
```

During execution, the message "created an array from object" is displayed, because a copy of is made when passing multidimensional array to fortran subroutine.



```
In [20]: positions = np.array(positions, dtype='f8', order='F')
 push(positions, velocities, 0.1)
 positions # the memory is updated

Out[20]: array([[0. , 0.1, 0.2],
 [0. , 0.3, 0.2],
 [0. , 0.1, 0.3]])
```

## 16.9 Signature file

This file contains descriptions of wrappers to Fortran or C functions, also called as signatures of the functions. F2PY can create initial signature file by scanning Fortran source codes and catching all relevant information needed to create wrapper functions.

```
f2py vector.f90 -h vector.pyf
```

- vector.pyf

```
! -*- f90 -*-
! Note: the context of this file is case sensitive.

subroutine euclidian_norm(a,c,n) ! in vector.f90
 real(kind=8) dimension(n),intent(in) :: a
 real(kind=8) intent(out) :: c
 integer optional,check(len(a)>=n),depend(a) :: n=len(a)
end subroutine euclidian_norm

! This file was auto-generated with f2py (version:2).
! See http://cens.ioc.ee/projects/f2py2e/
```

## 16.10 Wrap lapack function dgemm with f2py

- Generate the signature file

```
In [21]: %rm -f dgemm.f dgemm.pyf
 !wget -q http://ftp.mcs.anl.gov/pub/MINPACK-2/blas/dgemm.f

In [22]: # %load dgemm.f

In [23]: !{sys.executable} -m numpy.f2py -m mylapack --overwrite-signature -h dgemm.pyf dgemm.f

Reading fortran codes...
 Reading file 'dgemm.f' (format:fix,strict)
rmbadname1: Replacing "max" with "max_bn".
Post-processing...
 Block: mylapack
 Block: dgemm
Post-processing (stage 2)...
Saving signatures to file "./dgemm.pyf"

! -*- f90 -*-
! Note: the context of this file is case sensitive.

python module mylapack ! in
```

```

interface ! in :mylapack
 subroutine dgemm(transa,transb,m,n,k,alpha,a,lda,b,ldb,beta,c,ldc) ! in :mylapack:dgemm.f
 character*1 :: transa
 character*1 :: transb
 integer :: m
 integer :: n
 integer :: k
 double precision :: alpha
 double precision dimension(lda,*) :: a
 integer, optional, check(shape(a,0)==lda), depend(a) :: lda=shape(a,0)
 double precision dimension(ldb,*) :: b
 integer, optional, check(shape(b,0)==ldb), depend(b) :: ldb=shape(b,0)
 double precision :: beta
 double precision dimension(ldc,*) :: c
 integer, optional, check(shape(c,0)==ldc), depend(c) :: ldc=shape(c,0)
 end subroutine dgemm
end interface
end python module mylapack

```

*! This file was auto-generated with f2py (version:2).*  
*! See <http://cens.ioc.ee/projects/f2py2e/>*

In [24]: `!{sys.executable} -m numpy.f2py --quiet -c dgemm.pyf -llapack`

```

In [25]: import numpy as np
import mylapack
a = np.array([[7,8],[3,4],[1,2]])
b = np.array([[1,2,3],[4,5,6]])
print("a=",a)
print("b=",b)
assert a.shape[1] == b.shape[0]
c = np.zeros((a.shape[0],b.shape[1]),'d',order='F')
mylapack.dgemm('N','N',a.shape[0],b.shape[1],a.shape[1],1.0,a,b,1.0,c)
print(c)
np.all(c == a @ b) # check with numpy matrix multiplication

```

```

a= [[7 8]
 [3 4]
 [1 2]]
b= [[1 2 3]
 [4 5 6]]
[[39. 54. 69.]
 [19. 26. 33.]
 [9. 12. 15.]]

```

Out[25]: True

### 16.10.1 Exercise

- Modify the file dgemm.pyf to set all arguments top optional and keep only the two matrices as input.

In [26]: `# %load solutions/fortran/dgemm2.pyf`

- Build the python module

In [27]: `!{sys.executable} -m numpy.f2py --quiet -c solutions/fortran/dgemm2.pyf -llapack --f90flags=-O3`

```
In [28]: import mylapack2
 a = np.array([[7,8],[3,4],[1,2]])
 b = np.array([[1,2,3],[4,5,6]])
 c = mylapack2.dgemm(a,b)
 np.all(c == a @ b)
```

```
Out[28]: True
```

## 16.11 Check performance between numpy and mylapack

```
In [29]: a = np.random.random((512,128))
 b = np.random.random((128,512))
```

```
In [30]: %timeit c = mylapack2.dgemm(a,b)
```

```
975 µs ± 14 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

```
In [31]: %timeit c = a @ b
```

```
693 µs ± 4.56 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

Fortran arrays allocated in a subroutine share same memory in Python

```
In [32]: %%fortran
 module f90module
 implicit none
 real(8), dimension(:), allocatable :: farray
 contains
 subroutine init(n) !Allocation du tableau farray
 integer, intent(in) :: n
 allocate(farray(n))
 end subroutine init
 end module f90module
```

```
In [33]: f90module.init(10)
 len(f90module.farray)
```

```
Out[33]: 10
```

Numpy arrays allocated in Python passed to Fortran are already allocated

```
In [34]: %%fortran
 module f90module
 implicit none
 real(8), dimension(:), allocatable :: farray
 contains
 subroutine test_array(allocated_flag, array_size)
 logical, intent(out) :: allocated_flag
 integer, intent(out) :: array_size
 allocated_flag = allocated(farray)
 array_size = size(farray)
 end subroutine test_array
 end module f90module
```

```
In [35]: f90module.farray = np.random.rand(10).astype(np.float64)
 f90module.test_array()
```

```
Out[35]: (1, 10)
```

## 16.12 f2py + OpenMP

In [36]: `%env OMP_NUM_THREADS=4`

env: OMP\_NUM\_THREADS=4

In [37]: `%%fortran`

```
subroutine hello()
 integer :: i
 do i = 1, 4
 call sleep(1)
 end do
end subroutine
```

In [38]: `%%time`

```
hello()
```

CPU times: user 29.3 ms, sys: 9.85 ms, total: 39.1 ms

Wall time: 4 s

In [39]: `%%fortran --f90flags "-fopenmp" --extra "-L/usr/local/lib -lgomp"`

```
subroutine hello_omp()
 integer :: i
 !$OMP PARALLEL PRIVATE(I)
 !$OMP DO
 do i = 1, 4
 call sleep(1)
 end do
 !$OMP END DO
 !$OMP END PARALLEL

end subroutine
```

In [40]: `%%time`

```
hello_omp()
```

CPU times: user 0 ns, sys: 10.6 ms, total: 10.6 ms

Wall time: 1 s

## 16.13 Conclusions

### 16.13.1 pros

- Easy to use, it works with modern fortran, legacy fortran and also C.
- Works with common and modules and arrays dynamically allocated.
- Python function callback can be very useful combined with Sympy
- Documentation is automatically generated
- All fortran compilers are supported: GNU, Portland, Sun, Intel,...
- F2py is integrated in numpy library.

### 16.13.2 cons

- Derived types and fortran pointers are not well supported.
- Absolutely not compatible with fortran 2003-2008 new features (classes)
- f2py is maintained but not really improved. Development is stopped.

## 16.14 distutils

### 16.14.1 setup.py

```
from numpy.distutils.core import Extension, setup
ext1 = Extension(name = 'scalar',
 sources = ['scalar.f'])
ext2 = Extension(name = 'fib2',
 sources = ['fib2.pyf', 'fib1.f'])

setup(name = 'f2py_example', ext_modules = [ext1, ext2])
```

Compilation

```
python3 setup.py build_ext --inplace
```

## 16.15 Exercice: Laplace problem

- Replace the laplace function by a fortran subroutine

```
In [41]: %%time
 %matplotlib inline
 %config InlineBackend.figure_format = 'retina'
 import numpy as np
 import matplotlib.pyplot as plt
 import itertools
 # Boundary conditions
 Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50

 # Set meshgrid
 n, l = 64, 1.0
 X, Y = np.meshgrid(np.linspace(0,1,n), np.linspace(0,1,n))
 T = np.zeros((n,n))

 # Set Boundary condition
 T[n-1:, :] = Tnorth
 T[:, 0] = Tsouth
 T[:, n-1:] = Teast
 T[:, :1] = Twest

 def laplace(T, n):
 residual = 0.0
 for i in range(1, n-1):
 for j in range(1, n-1):
 T_old = T[i,j]
 T[i, j] = 0.25 * (T[i+1,j] + T[i-1,j] + T[i,j+1] + T[i,j-1])
 if T[i,j]>0:
 residual=max(residual,abs((T_old-T[i,j])/T[i,j]))
 return residual

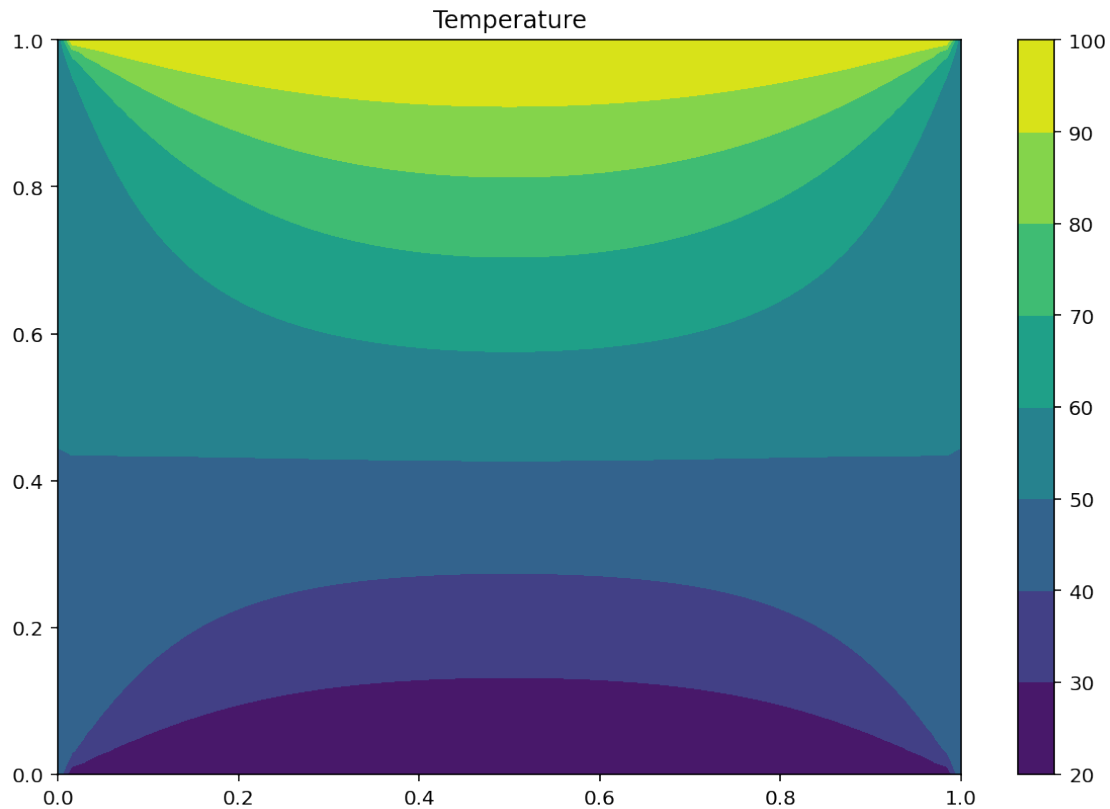
 residual = 1.0
 istep = 0
 while residual > 1e-5 :
 istep += 1
 residual = laplace(T, n)
 print ((istep, residual), end="\r")

 print("iterations = ",istep)
```

```
plt.rcParams['figure.figsize'] = (10,6.67)
plt.title("Temperature")
plt.contourf(X, Y, T)
plt.colorbar()
```

```
iterations = 2457
CPU times: user 31.1 s, sys: 676 ms, total: 31.8 s
Wall time: 31.3 s
```

Out[41]: <matplotlib.colorbar.Colorbar at 0x7fd7f99ae160>



```
In [42]: %%fortran
%load solutions/fortran/laplace_fortran.F90
subroutine laplace_fortran(T, n, residual)

 real(8), intent(inout) :: T(0:n-1,0:n-1) ! Python indexing
 integer, intent(in) :: n
 real(8), intent(out) :: residual
 real(8) :: T_old

 residual = 0.0
 do i = 1, n-2
 do j = 1, n-2
 T_old = T(i,j)
 T(i, j) = 0.25 * (T(i+1,j) + T(i-1,j) + T(i,j+1) + T(i,j-1))
 if (T(i,j) > 0) then
```

```

 residual=max(residual,abs((T_old-T(i,j))/T(i,j)))
 end if
end do
end do

end subroutine laplace_fortran

```

```

In [43]: %%time
%matplotlib inline
%config InlineBackend.figure_format = 'retina'
import numpy as np
import matplotlib.pyplot as plt
import itertools
Boundary conditions
Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50

Set meshgrid
n, l = 64, 1.0
X, Y = np.meshgrid(np.linspace(0,l,n), np.linspace(0,l,n))
T = np.zeros((n,n), order='F') ## We need to declare a new order in memory

Set Boundary condition
T[n-1:, :] = Tnorth
T[:, 0] = Tsouth
T[:, n-1:] = Teast
T[:, 1] = Twest

residual = 1.0
istep = 0
while residual > 1e-5 :
 istep += 1
 residual = laplace_fortran(T, n)
 print ((istep, residual), end="\r")

print()
print("iterations = ",istep)
plt.rcParams['figure.figsize'] = (10,6.67)
plt.title("Temperature")
plt.contourf(X, Y, T)
plt.colorbar()

```

```

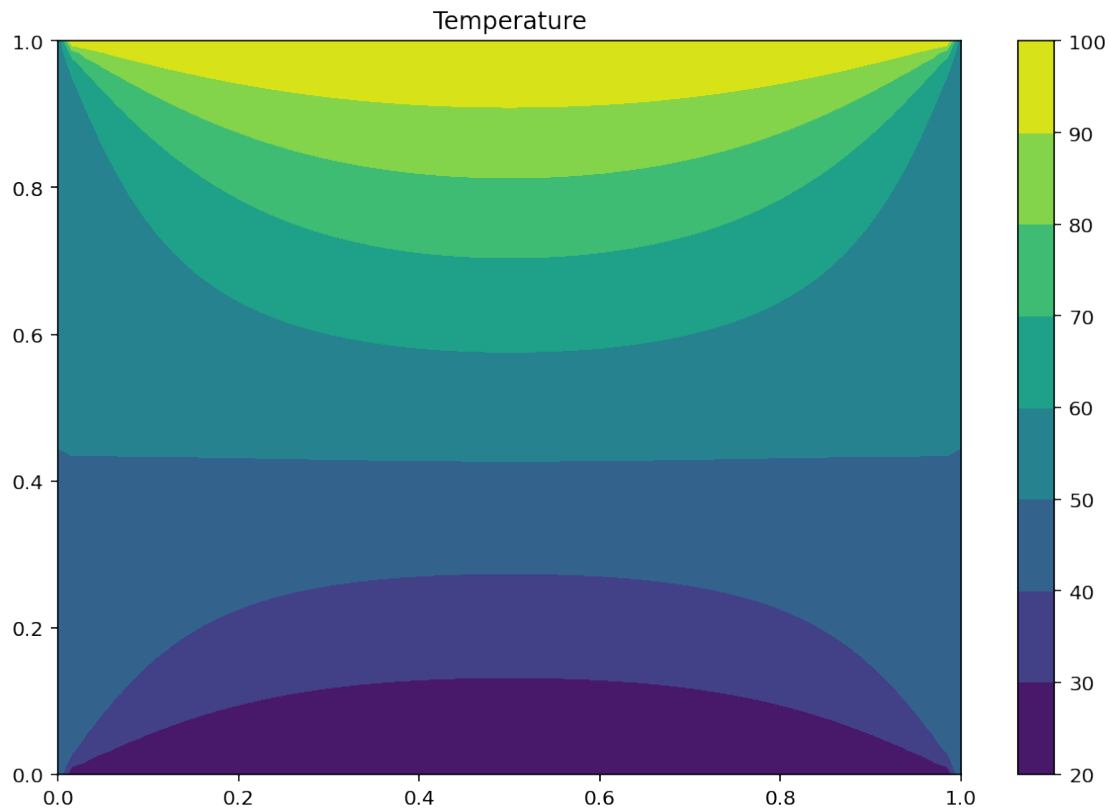
(2457, 9.997295133247811e-06)
iterations = 2457
CPU times: user 364 ms, sys: 110 ms, total: 474 ms
Wall time: 326 ms

```

```

Out[43]: <matplotlib.colorbar.Colorbar at 0x7fd7efe34130>

```



## 16.16 References

- [Talk by E. Sonnendrücker](#)
- [SciPy](#)
- [Sagemath Documentation](#)
- Hans Petter Langtangen. *Python Scripting for Computational Science*. Springer 2004



## Chapter 17

# Cython

```
In [1]: %matplotlib inline
import matplotlib.pyplot as plt
plt.rcParams['figure.figsize'] = (10,6)
%config InlineBackend.figure_format = 'retina'
import numpy as np

In [2]: import warnings
warnings.filterwarnings("ignore")
```



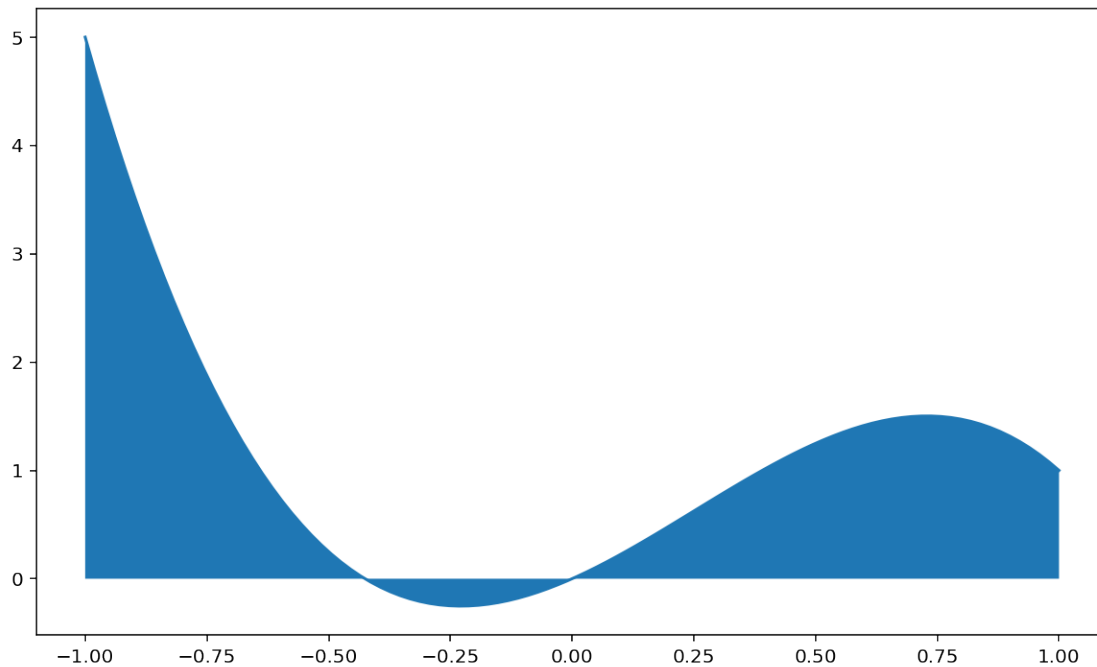
- Cython provides extra syntax allowing for static type declarations (remember: Python is generally dynamically typed)
- Python code gets translated into optimised C/C++ code and compiled as Python extension modules
- Cython allows you to write fast C code in a Python-like syntax.
- Furthermore, linking to existing C libraries is simplified.
- Pure Python Function

$$f(x) = -2x^3 + 5x^2 + x,$$

```
In [3]: def f(x):
 return -4*x**3 + 3*x**2 + 2*x

x = np.linspace(-1,1,100)
```

```
ax = plt.subplot(1,1,1)
ax.plot(x, f(x))
ax.fill_between(x, 0, f(x));
```



we compute integral  $\int_a^b f(x)dx$  numerically with  $N$  points.

```
In [4]: def integrate_f_py(a,b,N):
 s = 0
 dx = (b - a) / (N-1)
 for i in range(N-1): # we intentionally use the bad way to do this with a loop
 x = a + i*dx
 s += (f(x)+f(x+dx))/2
 return s*dx
```

```
In [5]: %timeit integrate_f_py(-1,1,10**3)
 print(integrate_f_py(-1,1,1000))
```

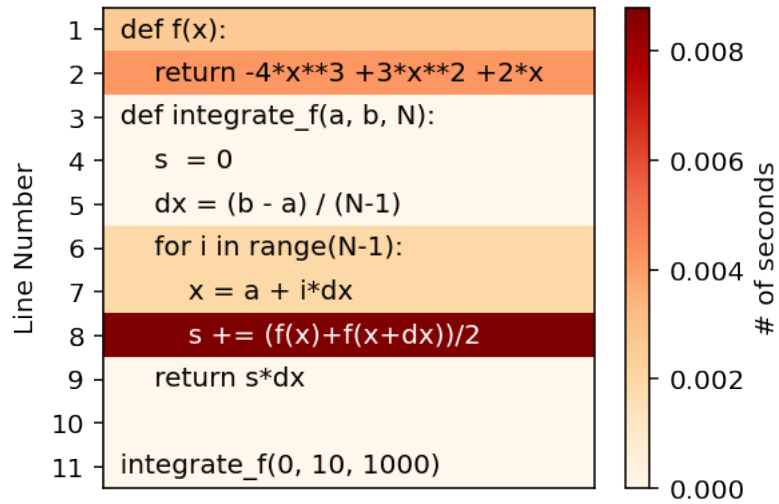
959  $\mu$ s  $\pm$  7.17  $\mu$ s per loop (mean  $\pm$  std. dev. of 7 runs, 1000 loops each)

2.0000040080120174

```
In [6]: %load_ext heat
```

```
In [7]: %%heat
 def f(x):
 return -4*x**3 +3*x**2 +2*x
 def integrate_f(a, b, N):
 s = 0
 dx = (b - a) / (N-1)
 for i in range(N-1):
 x = a + i*dx
 s += (f(x)+f(x+dx))/2
 return s*dx

 integrate_f(0, 10, 1000)
```



- Pure C function

In [8]: %%file integral\_f.c.c

```
#include <stdlib.h>
#include <stdio.h>
#include <time.h>

#define NB_RUNS 1000

double f(double x) {
 return -4*x*x*x + 3*x*x + 2*x;
}

double integrate_f_c(double a, double b, int N) {
 double s = 0;
 double dx = (b - a) / (N-1);
 for(int i=0; i<N-1; ++i){
 double x = a + i*dx;
 s += (f(x)+f(x+dx))/2.0;
 }
 return s*dx;
}

int main(int argc, char **argv)
{
 double a = atof(argv[1]);
 double b = atof(argv[2]);
 int N = atoi(argv[3]);
 double res = 0;

 clock_t begin = clock();

 for (int i=0; i<NB_RUNS; ++i)
 res += integrate_f_c(a, b, N);
```

```

 clock_t end = clock();

 fprintf(stdout, "integral_f(%3.1f, %3.1f, %d) = %f \n", a, b, N, res / NB_RUNS);
 fprintf(stdout, "time = %e ms \n", (double)(end - begin) / CLOCKS_PER_SEC);

 return 0;
}

```

Writing `integral_f.c`

```
In [9]: !gcc -O3 integral_f.c; ./a.out -1 1 1000
```

```

integral_f(-1.0, 1.0, 1000) = 2.000004
time = 5.529000e-03 ms

```

## 17.1 Cython compilation: Generating C code

Load Cython in jupyter notebook.

```
In [10]: %load_ext Cython
```

### 17.1.1 C Variable and Type definitions

In general, use `cdef` to declare C variables. The command :

```
$ cython -a mycode.pyx
```

outputs an html file. It shows what parts of your code are C, which parts are Python, and where C-Python conversion occurs.

```

In [11]: %%cython -a
 cdef int i, j = 2, k = 3 # assigning values at declaration
 i = 1 # assigning values afterwards
 # avoid Python-C conversion! It's expensive:
 a = 5
 i = a
 # same with C-Python conversion:
 b = j
 print("a = %d" % a)
 print("i = %d" % i)
 print("b = %d" % b)

```

```

a = 5
i = 5
b = 2

```

```
Out[11]: <IPython.core.display.HTML object>
```

### 17.1.2 Another Python vs. Cython coloring guide

```

In [12]: %%cython -a
 cdef int m, n
 cdef double cy_total = 0.0
 for m in range(10):
 n = 2*m

```

```

 cy_total += n
 a, b = 0, 0
 py_total = 0.0
 for a in range(10):
 b = 2*a
 py_total += b
 print(cy_total, py_total)

```

90.0 90.0

Out[12]: <IPython.core.display.HTML object>

```

In [13]: %%cython -a
 cdef struct Grail:
 int age
 float volume
 cdef union Food:
 char *spam
 float *eggs
 cdef enum CheeseType:
 cheddar, edam,
 camembert
 cdef enum CheeseState:
 hard = 1
 soft = 2
 runny = 3
 cdef Grail holy
 holy.age = 500
 holy.volume = 10.0
 print (holy.age, holy.volume)

```

500 10.0

Out[13]: <IPython.core.display.HTML object>

## 17.2 Cython Functions

Use **cdef** to define a Cython function.

- Cython function can accept either (inclusive) Python and C values as well as return either Python or C values, - *Within a Cython module* Python and Cython functions can call each other freely. However, only **Python** functions can be called from outside the module by Python code. (i.e. importing/exporting a Cython module into some Python code)

**cpdef** define a Cython function with a simple Python wrapper. However, when called from Cython the Cython / C code is called directly, bypassing the Python wrapper.

Writing pure code in Cython gives a small speed boost. Note that none of the code below is Cython-specific. Just add `.pyx` instead of `.py` extension.

```

In [14]: %%file cython_f_example.pyx
 def f(x):
 return -4*x**3 + 3*x**2 + 2*x
 def integrate_f(a, b, N):
 s = 0
 dx = (b - a) / (N-1)
 for i in range(N-1):
 x = a + i*dx
 s += (f(x)+f(x+dx))/2
 return s*dx

```

Writing `cython_f_example.pyx`

## 17.3 Cython Compilation

- The `.pyx` source file is compiled by Cython to a `.c` file.
- The `.c` source file contains the code of a Python extension module.
- The `.c` file is compiled by a C compiler to a `.so` (shared object library) file which can be imported directly into a Python session.

### 17.3.1 Build with CMake

```
project(cython_f_example CXX)
include(UseCython) # Load Cython functions
Set C++ output
set_source_file_properties(cython_f_example.pyx PROPERTIES CYTHON_IS_CXX TRUE)
Build the extension module
cython_add_module(modname cython_f_example.pyx cython_f_example.cpp)
```

### 17.3.2 C/C++ generation with cython application

```
cython -3 cython_f_example.pyx # create the C file for Python 3
cython -3 --cplus cython_f_example.pyx # create the C++ file for Python 3
```

### 17.3.3 build with a C/C++ compiler

To build use the Makefile:

```
CC=gcc
CFLAGS=`python-config --cflags`
LDFLAGS=`python-config --ldflags`
cython_f_example:
 ${CC} -c $@.c ${CFLAGS}
 ${CC} $@.o -o $@.so -shared ${LDFLAGS}
```

Import the module in Python session

```
import cython_f_example
```

## 17.4 pyximport

import Cython `.pyx` files as if they were `.py` files:

```
In [15]: import pyximport
 pyximport.install()
 import cython_f_example
 %timeit cython_f_example.integrate_f(-1,1,10**3)
 print(cython_f_example.integrate_f(-1,1,1000))
```

```
769 µs ± 6.02 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
2.0000040080120174
```

## 17.5 Building a Cython module using distutils

Create the setup.py script:

```
In [16]: %%file setup.py
 from distutils.core import setup
 from Cython.Build import cythonize

 setup(
 name = 'Cython Example Integrate f Function',
 ext_modules = cythonize("cython_f_example.pyx"),
)
```

Writing setup.py

```
In [17]: %run setup.py build_ext --inplace --quiet
```

Compiling cython\_f\_example.pyx because it changed.

[1/1] Cythonizing cython\_f\_example.pyx

running build\_ext

building 'cython\_f\_example' extension

C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler\_compat -Wl,--sysroot=/ -Wsign-compare

creating build

creating build/temp.linux-x86\_64-3.8

compile options: '-I/usr/share/miniconda3/envs/runenv/include/python3.8 -c'

gcc: cython\_f\_example.c

gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler\_compat -L/usr/share/miniconda3/envs/runenv/lib -Wl,-rpath=/usr/share/miniconda3/envs/runenv/lib

<Figure size 720x432 with 0 Axes>

```
In [18]: from cython_f_example import integrate_f
 %timeit integrate_f(-1,1,10**3)
 integrate_f(-1,1,10**3)
```

778 µs ± 8.33 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)

```
Out[18]: 2.0000040080120174
```

## 17.6 Why is it faster with Cython ?

- Python code is interpreted at every execution to machine code.
- Compiled C code is already in machine code.
- C is a statically-typed language. It gives to the compiler more information which allows it to optimize both computations and memory access.
- To add two variables, Python checks the type before calling the right **add** function and store it to a value that can be new.
- C just add the variables and return the result.

## 17.7 Add Cython types

We coerce Python types to C types when calling the function. Still a "Python function" so callable from the global namespace.

```
In [19]: %%cython
def f(x):
 return -4*x**3 + 3*x**2 + 2*x
def cy_integrate_f(double a, double b, int N):
 cdef int i
 cdef double s, x, dx
 s = 0
 dx = (b - a) / (N-1)
 for i in range(N-1):
 x = a + i*dx
 s += (f(x)+f(x+dx))/2
 return s*dx

building '_cython_magic_8c0baa0b730cb71c660b134eef8e4b03' extension
C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -Wl,--sysroot=/ -Wsign-con
compile options: '-I/usr/share/miniconda3/envs/runenv/include/python3.8 -c'
gcc: /home/runner/.cache/ipython/cython/_cython_magic_8c0baa0b730cb71c660b134eef8e4b03.c
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
```

- typing the iterator variable `i` with C semantics, tells Cython to compile the for-loop to pure C code.
- typing `a`, `s` and `dx` is important as they are involved in arithmetic within the for-loop
- Cython type declarations can make the source code less readable
- Do not use them without good reason, i.e. only in performance critical sections.

```
In [20]: %timeit cy_integrate_f(-1,1,10**3)
print(cy_integrate_f(-1,1,1000))
```

```
687 µs ± 3.44 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
2.0000040080120174
```

Finally, we integrate a Cython function instead of a Python function. This eliminates the Python-C conversion at the function call as seen above thus giving a pure Cython/C algorithm.

The primary downside is not being allowed to call the function `cy_f`, from Python unless `cpdef` is used.

```
In [21]: %%cython
cdef double cy_f(double x):
 return -4*x**3 + 3*x**2 + 2*x
def ccy_integrate_f(double a, double b, int N):
 cdef int i
 cdef double s, x, dx
 s = 0
 dx = (b - a) / (N-1)
 for i in range(N-1):
 x = a + i*dx
 s += (cy_f(x)+cy_f(x+dx))/2
 return s*dx

building '_cython_magic_c253bb1961eda445c34ddc2d7b4ab8bc' extension
C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -Wl,--sysroot=/ -Wsign-con
compile options: '-I/usr/share/miniconda3/envs/runenv/include/python3.8 -c'
gcc: /home/runner/.cache/ipython/cython/_cython_magic_c253bb1961eda445c34ddc2d7b4ab8bc.c
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
```



```
In [22]: %timeit cicy_integrate_f(-1,1,10**3)
 print(cicy_integrate_f(-1,1,1000))
```

172  $\mu$ s  $\pm$  704 ns per loop (mean  $\pm$  std. dev. of 7 runs, 10000 loops each)  
2.0000040080120174

## 17.8 Exercise : Cythonize the trivial exponential function.

```
In [23]: %%cython -a
 def exp_python(x,terms=50):
 sum = 0.
 power = 1.
 fact = 1.
 for i in range(terms):
 sum += power/fact
 power *= x
 fact *= i+1
 return sum
```

building '\_cython\_magic\_a96a095f6c3175b142027432bac51f88' extension

C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler\_compat -Wl,--sysroot=/ -Wsign-compare

compile options: '-I/usr/share/miniconda3/envs/runenv/include/python3.8 -c'

gcc: /home/runner/.cache/ipython/cython/\_cython\_magic\_a96a095f6c3175b142027432bac51f88.c

gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler\_compat -L/usr/share/miniconda3/envs/runenv/lib -Wl,-rpath=/usr/share/miniconda3/envs/runenv/lib

Out[23]: <IPython.core.display.HTML object>

```
In [24]: %timeit exp_python(1.,50)
```

4.45  $\mu$ s  $\pm$  71 ns per loop (mean  $\pm$  std. dev. of 7 runs, 100000 loops each)

```
In [25]: %%cython
```

```
%load solutions/cython/exponential.pyx
#cython: profile=False
#cython: cdivision=True
def exp_cython(double x, int terms = 50):
 cdef double sum
 cdef double power
 cdef double fact
 cdef int i
 sum = 0.
 power = 1.
 fact = 1.
 for i in range(terms):
 sum += power/fact
 power *= x
 fact *= i+1
 return sum
```

building '\_cython\_magic\_f09447b346fe20b620ea715f5b8935ac' extension

C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler\_compat -Wl,--sysroot=/ -Wsign-compare

compile options: '-I/usr/share/miniconda3/envs/runenv/include/python3.8 -c'

```
gcc: /home/runner/.cache/ipython/cython/_cython_magic_f09447b346fe20b620ea715f5b8935ac.c
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
```

```
In [26]: %timeit exp_cython(1.,50)
```

```
165 ns ± 1.68 ns per loop (mean ± std. dev. of 7 runs, 10000000 loops each)
```

## 17.9 Cython and Numpy

The Numpy library contains many fast numerics routines. Their speed comes from manipulating the low-level C-arrays that the `numpy.array` object wraps rather than computing over slow Python lists. Using Cython one can access those low-level arrays and implement their own fast algorithms while allowing the easy interaction afforded by Python + Numpy.

The examples below are various implementations of the naive matrix multiplication algorithm. We will start with a pure Python implementation and then incrementally add structures that allow Cython to exploit the low-level speed of the `numpy.array` object.

### 17.9.1 Pure Python implementation compiled in Cython without specific optimizations.

```
In [27]: %%cython
```

```
def matmul1(A, B, out=None):
 assert A.shape[1] == B.shape[0]
 for i in range(A.shape[0]):
 for j in range(B.shape[1]):
 s = 0
 for k in range(A.shape[1]):
 s += A[i,k] * B[k,j]
 out[i,j] = s
 return out
```

```
building '_cython_magic_899b76c7c47e2932ff2e677e49192f5a' extension
```

```
C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -Wl,--sysroot=/ -Wsign-con
```

```
compile options: '-I/usr/share/miniconda3/envs/runenv/include/python3.8 -c'
```

```
gcc: /home/runner/.cache/ipython/cython/_cython_magic_899b76c7c47e2932ff2e677e49192f5a.c
```

```
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
```

### 17.9.2 Import numpy as a Cython module

We now take advantage of the ability to access the underlying C arrays in the `numpy.array` object from Cython, thanks to a special `numpy.pxd` file included with Cython. (The Cython developers worked closely with Numpy developers to make this optimal.)

To begin with, we have to `cimport` numpy: that is, import numpy as a **Cython** module rather than a **Python** module. To do so, simply type:

```
cimport numpy as np
```

Another important thing to note is the type of Numpy indexers. There is a special Numpy variable type used for `numpy.array` indices called `Py_ssize_t`. To take full advantage of the speedups that Cython can provide we should make sure to type the variables used for indexing as such.

```
In [28]: %%cython
import numpy as np
cimport numpy as np
ctypedef np.float64_t dtype_t # shorthand type. easy to change
def matmul2(np.ndarray[dtype_t, ndim=2] A,
 np.ndarray[dtype_t, ndim=2] B,
 np.ndarray[dtype_t, ndim=2] out=None):
 cdef Py_ssize_t i, j, k
 cdef dtype_t s
 assert A.shape[1] == B.shape[0]
 for i in range(A.shape[0]):
 for j in range(B.shape[1]):
 s = 0
 for k in range(A.shape[1]):
 s += A[i,k] * B[k,j]
 out[i,j] = s
 return out
```

building '\_cython\_magic\_44357700cb4ec61887ddbee898be64d7' extension

C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler\_compat -Wl,--sysroot=/ -Wsign-compare

compile options: '-I/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include -I

gcc: /home/runner/.cache/ipython/cython/\_cython\_magic\_44357700cb4ec61887ddbee898be64d7.c

gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler\_compat -L/usr/share/miniconda3/envs/

```
In [29]: import numpy as np
from timeit import timeit
A = np.random.random_sample((64,64))
B = np.random.random_sample((64,64))
C = np.zeros((64,64))
```

```
In [30]: %timeit matmul1(A,B,C)
```

149 ms ± 1.79 ms per loop (mean ± std. dev. of 7 runs, 10 loops each)

```
In [31]: %timeit matmul2(A,B,C)
```

361 µs ± 3.67 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)

### 17.9.3 Tuning indexing

The array lookups are still slowed down by two factors: \* Bounds checking is performed. \* Negative indices are checked for and handled correctly.

The code doesn't use negative indices, and always access to arrays within bounds. We can add a decorator to disable bounds checking:

```
In [32]: %%cython
import cython # cython tools
import numpy as np
cimport numpy as np
ctypedef np.float64_t dtype_t
@cython.boundscheck(False) # turn off bounds-checking for entire function
@cython.wraparound(False) # turn off negative index wrapping for entire function
def matmul3(np.ndarray[dtype_t, ndim=2] A,
 np.ndarray[dtype_t, ndim=2] B,
```

```

 np.ndarray[dtype_t, ndim=2] out=None):
cdef Py_ssize_t i, j, k
cdef dtype_t s
assert A.shape[1] == B.shape[0]
for i in range(A.shape[0]):
 for j in range(B.shape[1]):
 s = 0
 for k in range(A.shape[1]):
 s += A[i,k] * B[k,j]
 out[i,j] = s
return out

```

building '\_cython\_magic\_75874d7a10a3fb62ace5ee487efe1f5a' extension

C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler\_compat -Wl,--sysroot=/ -Wsign-compare

compile options: '-I/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include -I

gcc: /home/runner/.cache/ipython/cython/\_cython\_magic\_75874d7a10a3fb62ace5ee487efe1f5a.c

gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler\_compat -L/usr/share/miniconda3/envs/

In [33]: %timeit matmul3(A,B,C)

324 µs ± 8.76 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)

## 17.10 Cython Build Options

- `boundscheck(True,False)` : array bounds checking
- `wraparound(True,False)` : negative indexing.
- `initializedcheck(True,False)`: checks that a memoryview is initialized
- `nonecheck(True,False)` : Check if one argument is None
- `overflowcheck(True,False)` : Check if int are too big
- `cdivision(True,False)` : If False, adjust the remainder and quotient operators C types to match those of Python ints. Could be very effective when it is set to True.
- `profile (True / False)` : Write hooks for Python profilers into the compiled C code. Default is False.

[Cython Compiler directives](#)

## 17.11 Numpy objects with external C program.

Note that this can actually be slower because the C function is not the best implementation of matrix multiplication. Call `cblas` with same technique is an interesting exercise.

In [34]: %%file mydgemm.c

```

void my_dgemm(int m, int n, int k,
 double a[m][n], double b[n][k], float c[m][k])
{
 double ab = 0;
 for(int j = 0 ; j < m ; j++) {
 for(int i = 0 ; i < k ; i++) {
 for(int l = 0 ; l < n ; l++){
 ab += a[j][l] * b[l][i];
 }
 c[j][i] = ab ;
 ab = 0;
 }
 }
}

```

```
 }
}
```

Writing mydgemm.c

- The `np.ndarray[double, ndim=2, mode="c"]` assures that you get a C-contiguous numpy array of doubles
- The `&input[0,0]` passed in the address of the beginning of the data array.

```
In [35]: from pyximport import install
import os
here = os.getcwd()
```

```
In [36]: %%cython -I {here}
do not forget to change the file path
cdef extern from "mydgemm.c":
 void my_dgemm (int m, int n, int k,
 double *A, double *B, double *C)

cimport cython
import numpy as np
cimport numpy as np
ctypedef np.float64_t dtype_t
@cython.boundscheck(False)
@cython.wraparound(False)
def matmul4(np.ndarray[dtype_t, ndim=2, mode="c"] A,
 np.ndarray[dtype_t, ndim=2, mode="c"] B,
 np.ndarray[dtype_t, ndim=2, mode="c"] C=None):
 cdef int m = A.shape[0]
 cdef int n = A.shape[1]
 cdef int k = B.shape[1]
 cdef dtype_t s

 my_dgemm(m, n, k, &A[0,0], &B[0,0], &C[0,0])

 return C
```

building '\_cython\_magic\_3182af7b83c2474db8f1f7bb74b9f4d7' extension

C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler\_compat -Wl,--sysroot=/ -Wsign-compare

compile options: '-I/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include -I

gcc: /home/runner/.cache/ipython/cython/\_cython\_magic\_3182af7b83c2474db8f1f7bb74b9f4d7.c

gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler\_compat -L/usr/share/miniconda3/envs/

```
In [37]: %timeit matmul4(A,B,C)
```

311  $\mu$ s  $\pm$  3.29  $\mu$ s per loop (mean  $\pm$  std. dev. of 7 runs, 1000 loops each)

### 17.11.1 Exercise : Find prime numbers $< 10000$

```
In [38]: # %load solutions/cython/is_prime0.py
```

```
def is_prime0(n):
 if n < 4: return True
 if n % 2 == 0 : return False
 k = 3
 while k*k <= n:
```

```

 if n % k == 0: return False
 k += 2
 return True

```

In [39]: [ p for p in range(20) if is\_prime0(p)]

Out[39]: [0, 1, 2, 3, 5, 7, 11, 13, 17, 19]

In [40]: L = list(range(10000))  
 %timeit [ p for p in L if is\_prime0(p)]

8.55 ms  $\pm$  72.7  $\mu$ s per loop (mean  $\pm$  std. dev. of 7 runs, 100 loops each)

```

In [41]: %%cython
def is_prime1(n):
 if n < 4: return True
 if n % 2 == 0 : return False
 k = 3
 while k*k <= n:
 if n % k == 0: return False
 k += 2
 return True

```

building '\_cython\_magic\_f81b23461181c5d78ab3700de598d3b5' extension

C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler\_compat -Wl,--sysroot=/ -Wsign-con

compile options: '-I/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include -I

gcc: /home/runner/.cache/ipython/cython/\_cython\_magic\_f81b23461181c5d78ab3700de598d3b5.c

gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler\_compat -L/usr/share/miniconda3/envs/

In [42]: [ p for p in range(20) if is\_prime1(p)]

Out[42]: [0, 1, 2, 3, 5, 7, 11, 13, 17, 19]

In [43]: %timeit [p for p in L if is\_prime1(p)]

4.44 ms  $\pm$  77.4  $\mu$ s per loop (mean  $\pm$  std. dev. of 7 runs, 100 loops each)

### 17.11.2 Add Cython types without modifying the Python Code

```

In [44]: %%cython
import cython
@cython.locals(n=int, k=int)
def is_prime2(n):
 if n < 4: return True
 if n % 2 == 0 : return False
 k = 3
 while k*k <= n:
 if n % k == 0: return False
 k += 2
 return True

```

building '\_cython\_magic\_745b858a4456e55af5f92e69b9968cd3' extension

C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler\_compat -Wl,--sysroot=/ -Wsign-con

compile options: '-I/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include -I

gcc: /home/runner/.cache/ipython/cython/\_cython\_magic\_745b858a4456e55af5f92e69b9968cd3.c

gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler\_compat -L/usr/share/miniconda3/envs/

```
In [45]: [p for p in range(20) if is_prime2(p)]
```

```
Out[45]: [0, 1, 2, 3, 5, 7, 11, 13, 17, 19]
```

```
In [46]: %timeit [p for p in L if is_prime2(p)]
```

672  $\mu$ s  $\pm$  1.16  $\mu$ s per loop (mean  $\pm$  std. dev. of 7 runs, 1000 loops each)

### 17.11.3 Cython function

```
In [47]: %%cython
```

```
import cython
cdef bint is_prime3(int n):
 if n < 4: return True
 if n % 2 == 0: return False
 cdef int k = 3
 while k*k <= n:
 if n % k == 0: return False
 k += 2
 return True
def prime_list(L):
 return [p for p in L if is_prime3(p)]
```

building '\_cython\_magic\_666b225df8666611ff2af72a85a03e9d' extension

C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler\_compat -Wl,--sysroot=/ -Wsign-compare

compile options: '-I/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include -I

gcc: /home/runner/.cache/ipython/cython/\_cython\_magic\_666b225df8666611ff2af72a85a03e9d.c

gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler\_compat -L/usr/share/miniconda3/envs/

```
In [48]: prime_list(list(range(20)))
```

```
Out[48]: [0, 1, 2, 3, 5, 7, 11, 13, 17, 19]
```

```
In [49]: %timeit prime_list(L)
```

350  $\mu$ s  $\pm$  3.36  $\mu$ s per loop (mean  $\pm$  std. dev. of 7 runs, 1000 loops each)

```
In [50]: %%cython
```

```
import cython
from numpy cimport ndarray
import numpy

cdef bint is_prime3(int n):
 if n < 4: return True
 if n % 2 == 0: return False
 cdef int k = 3
 while k*k <= n:
 if n % k == 0: return False
 k += 2
 return True

def prime_array(ndarray[int, ndim=1] L):
 cdef ndarray[int, ndim=1] res = ndarray(shape=(L.shape[0]), dtype=numpy.int32)
 cdef int i
 for i in range(L.shape[0]):
 res[i] = is_prime3(L[i])
 return L[res==1]
```

```
building '_cython_magic_2a5b09b5e9c574f8e7fa68917eb3edc0' extension
C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -Wl,--sysroot=/ -Wsign-con
```

```
compile options: '-I/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include -I.
gcc: /home/runner/.cache/ipython/cython/_cython_magic_2a5b09b5e9c574f8e7fa68917eb3edc0.c
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
```

```
In [51]: import numpy as np
 prime_array(np.arange(20, dtype=np.int32))
```

```
Out[51]: array([0, 1, 2, 3, 5, 7, 11, 13, 17, 19], dtype=int32)
```

```
In [52]: nPL = numpy.array(L, dtype=np.int32)
 %timeit prime_array(nPL)
```

```
358 µs ± 4.06 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

## 17.12 Using Parallelism

- Cython supports native parallelism via OpenMP
- by default, Python's Global Interpreter Lock (GIL) prevents that several threads use the Python interpreter simultaneously
- to use this kind of parallelism, the GIL must be released

If you have a default compiler with openmp support you can use this magic command in your notebook.

```
%%cython --compile-args=-fopenmp --link-args=-fopenmp
```

```
In [53]: %%file cython_omp.pyx
 import cython
 from cython.parallel cimport parallel, prange # import parallel functions
 import numpy as np
 from numpy cimport ndarray

 cdef bint is_prime4(int n) nogil: #release the gil
 if n < 4: return True
 if n % 2 == 0: return False
 cdef int k = 3
 while k*k <= n:
 if n % k == 0: return False
 k += 2
 return True

 @cython.boundscheck(False)
 def prime_array_omp(ndarray[int, ndim=1] L):
 cdef ndarray[int, ndim=1] res = ndarray(shape=(L.shape[0]), dtype=np.int32)
 cdef Py_ssize_t i
 with nogil, parallel(num_threads=4):
 for i in prange(L.shape[0]): #Parallel loop
 res[i] = is_prime4(L[i])
 return L[res==1]
```

Writing cython\_omp.pyx

To use the OpenMP support, you need to enable OpenMP. For gcc this can be done as follows in a setup.py:



```
In [54]: %%file setup.py
from distutils.core import setup
from distutils.extension import Extension
from Cython.Build import cythonize
import os, sys
import numpy

if sys.platform == "darwin": # for omp, use gcc installed with brew
 os.environ["CC"]="gcc-10"
 os.environ["CXX"]="g++-10"

ext_modules = [
 Extension(
 "cython_omp",
 ["cython_omp.pyx"],
 extra_compile_args=['-fopenmp'],
 extra_link_args=['-fopenmp'],
 include_dirs=[numpy.get_include()]
)
]

setup(
 name='Cython OpenMP Example',
 ext_modules=cythonize(ext_modules),
)
python setup.py build_ext --inplace
```

Overwriting setup.py

```
In [55]: %run setup.py build_ext --inplace --quiet
```

Compiling cython\_omp.pyx because it changed.

[1/1] Cythonizing cython\_omp.pyx

running build\_ext

building 'cython\_omp' extension

C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler\_compat -Wl,--sysroot=/ -Wsign-con

compile options: '-I/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include -I

extra options: '-fopenmp'

gcc: cython\_omp.c

gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler\_compat -L/usr/share/miniconda3/envs/

```
In [56]: from cython_omp import prime_array_omp
```

```
In [57]: prime_array_omp(np.arange(20, dtype=np.int32))
```

```
Out[57]: array([0, 1, 2, 3, 5, 7, 11, 13, 17, 19], dtype=int32)
```

```
In [58]: %timeit prime_array_omp(npL)
```

279  $\mu$ s  $\pm$  8.53  $\mu$ s per loop (mean  $\pm$  std. dev. of 7 runs, 1000 loops each)

## 17.13 References

- [Cython documentation](#)

- [An Interactive Introduction to Cython by Chris Swierczewski](#)
- [Introduction To Python by Michael Kraus](#)
- [Cython by Xavier Juvigny](#)
- [Cython: C-Extensions for Python, Wiki](#)
- [Kurt W. Smith](#)
  - [Cython A Guide for Python Programmers](#)
  - [Cython: Blend the Best of Python and C++ | SciPy 2015 Tutorial | Kurt Smith](#)
  - [Cython: Speed up Python and NumPy, Pythonize C, C++, and Fortran, SciPy2013 Kurt W. Smith](#)
  - [SciPy 2017 - Cython by](#)
  - [Cython Book examples](#)
- [Parallel computing in Cython/threads - Neal Hughes](#)

# Chapter 18

## Numba

```
In [1]: import numpy as np
```

- Numba is a compiler for Python array and numerical functions.
- Numba generates optimized machine code from pure Python code with a few simple annotations
- Python code is just-in-time optimized to performance similar as C, C++ and Fortran, without having to switch languages or Python interpreters.
- The code is generated on-the-fly for CPU (default) or GPU hardware.

### 18.1 Python decorator

A decorator is used to modify a function or a class. A reference to a function "func" or a class "C" is passed to a decorator and the decorator returns a modified function or class. The modified functions or classes usually contain calls to the original function "func" or class "C".

```
In [2]: def timeit(function):
 def wrapper(*args, **kwargs):
 import time
 t1 = time.time()
 result = function(*args, **kwargs)
 t2 = time.time()
 print("execution time", t2-t1)
 return result
 return wrapper

 @timeit
 def f(a, b):
 return a + b

 print(f(1, 2))
```

```
execution time 9.5367431640625e-07
```

```
3
```

### 18.2 First example

```
In [3]: from numba import jit
 @jit
 def sum(a, b):
 return a + b
```

- Compilation will be deferred until the first function execution.
- Numba will infer the argument types at call time.

```
In [4]: sum(1, 2), sum(1j, 2)
```

```
Out[4]: (3, (2+1j))
```

```
In [5]: x = np.random.rand(10)
 y = np.random.rand(10)
 sum(x, y)
```

```
Out[5]: array([0.53752801, 1.04247199, 1.36680709, 1.5649025 , 1.36627693,
 1.08140921, 0.62730358, 0.99310516, 1.34317215, 0.71797285])
```

## 18.3 Performance

```
In [6]: x = np.random.rand(10000000)
```

```
In [7]: %timeit x.sum() # Numpy
```

```
6.67 ms ± 27.6 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

```
In [8]: @jit
 def numba_sum(x):
 res= 0
 for i in range(x.size):
 res += x[i]
 return res
```

```
In [9]: %timeit numba_sum(x)
```

```
17 ms ± 113 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

## 18.4 Numba methods

```
In [10]: @jit
 def jit_sum(a, b):
 return a + b
```

```
In [11]: jit_sum.inspect_types() # jit_sum has not been compiled
```

```
In [12]: jit_sum(1, 2) # call it once with ints
 jit_sum.inspect_types()
```

```
jit_sum (int64, int64)
```

```

File: <ipython-input-10-ebda2b2f7dda>
--- LINE 1 ---
```

```
@jit
```

```
--- LINE 2 ---
```

```
def jit_sum(a, b):
```

```

--- LINE 3 ---
label 0
a = arg(0, name=a) :: int64
b = arg(1, name=b) :: int64
$6binary_add.2 = a + b :: int64
del b
del a
$8return_value.3 = cast(value=$6binary_add.2) :: int64
del $6binary_add.2
return $8return_value.3

return a + b

```

```

=====

In [13]: jit_sum(1., 2.) # call it once with doubles
 jit_sum.inspect_types()

```

```

jit_sum (int64, int64)

```

```

File: <ipython-input-10-ebda2b2f7dda>
--- LINE 1 ---

```

```

@jit

```

```

--- LINE 2 ---

```

```

def jit_sum(a, b):

```

```

--- LINE 3 ---
label 0
a = arg(0, name=a) :: int64
b = arg(1, name=b) :: int64
$6binary_add.2 = a + b :: int64
del b
del a
$8return_value.3 = cast(value=$6binary_add.2) :: int64
del $6binary_add.2
return $8return_value.3

return a + b

```

```

=====

jit_sum (float64, float64)

```

```

File: <ipython-input-10-ebda2b2f7dda>
--- LINE 1 ---

```

```

@jit

```

```

--- LINE 2 ---

```

```
def jit_sum(a, b):

 # --- LINE 3 ---
 # label 0
 # a = arg(0, name=a) :: float64
 # b = arg(1, name=b) :: float64
 # $6binary_add.2 = a + b :: float64
 # del b
 # del a
 # $8return_value.3 = cast(value=$6binary_add.2) :: float64
 # del $6binary_add.2
 # return $8return_value.3

 return a + b
```

```
=====
```

- `jit_sum.inspect_llvm()` returns a dict with llvm representation.

LLVM is a library that is used to construct, optimize and produce intermediate and/or binary machine code.

- `jit_sum.inspect_asm()` returns a dict with assembler information.

```
In [14]: jit_sum.py_func(1, 2) # call origin python function without numba process
```

```
Out[14]: 3
```

## 18.5 Types coercion

Tell Numba the function signature you are expecting.

```
In [15]: @jit(['int32[:](int32[:], int32[:])', 'int32(int32, int32)'])
 def product(a, b):
 return a*b
```

```
In [16]: product(2, 3), product(2.2, 3.2)
```

```
Out[16]: (6, 6)
```

```
In [17]: a = np.arange(10, dtype=np.int32)
 b = np.arange(10, dtype=np.int32)
 product(a, b)
```

```
Out[17]: array([0, 1, 4, 9, 16, 25, 36, 49, 64, 81], dtype=int32)
```

```
In [18]: a = np.random.random(10) # Numpy arrays contain double by default
 b = np.random.random(10)
 try:
 product(a, b)
 except TypeError as e:
 print("TypeError:", e)
```

```
TypeError: No matching definition for argument type(s) array(float64, 1d, C), array(float64, 1d, C)
```

## 18.6 Numba types

```
void,
intp, uintp,
intc, uintc,
int8, uint8, int16, uint16, int32, uint32, int64, uint64,
float32, float64,
complex64, complex128.
```

### 18.6.1 Arrays

```
float32[:]
float64[:, :]
```

## 18.7 Numba compilation options

- `** nopython **` : Compilation fails if you use pure Python objects.
- `** nogil **` : release Python's global interpreter lock (GIL).
- `** cache **` : Do not recompile the function each time you invoke a Python program.
- `** parallel **` : experimental feature that automatically parallelizes must be used in conjunction with `nopython=True`:

## 18.8 Inlining

Numba-compiled functions can call other compiled functions. The function calls may even be inlined in the native code, depending on optimizer heuristics.

```
In [19]: import math
 from numba import njit

 @njit
 def square(x):
 return x ** 2

 @njit
 def hypot(x, y):
 return math.sqrt(square(x) + square(y)) # square function is inlined
```

```
In [20]: hypot(2., 3.)
```

```
Out[20]: 3.605551275463989
```

## 18.9 @vectorize decorator

- Numba's `vectorize` allows Python functions taking scalar input arguments to be used as NumPy ufuncs.
- Write your function as operating over input scalars, rather than arrays. Numba will generate the surrounding loop (or kernel) allowing efficient iteration over the actual inputs.

### 18.9.1 Two modes of operation:

1. Eager mode: If you pass one or more type signatures to the decorator, you will be building a NumPy universal function (ufunc).
2. Call-time mode: When not given any signatures, the decorator will give you a Numba dynamic universal function (DUFunc) that dynamically compiles a new kernel when called with a previously unsupported input type.

```
In [21]: from numba import vectorize, float64, float32, int32, int64
```

```
 @vectorize([float64(float64, float64)])
 def f(x, y):
 return x + y
```

If you pass several signatures, beware that you have to pass most specific signatures before least specific ones (e.g., single-precision floats before double-precision floats)

```
In [22]: @vectorize([int32(int32, int32),
 int64(int64, int64),
 float32(float32, float32),
 float64(float64, float64)])
 def f(x, y):
 return x + y
```

```
In [23]: a = np.arange(6)
 f(a, a)
```

```
Out[23]: array([0, 2, 4, 6, 8, 10])
```

```
In [24]: a = np.linspace(0, 1, 6)
 f(a, a)
```

```
Out[24]: array([0. , 0.4, 0.8, 1.2, 1.6, 2.])
```

```
In [25]: a = np.linspace(0, 1+1j, 6)
 f(a, a)
```

```

TypeError Traceback (most recent call last)

<ipython-input-25-b196490ab338> in <module>
 1 a = np.linspace(0, 1+1j, 6)
----> 2 f(a, a)
```

```
TypeError: ufunc 'f' not supported for the input types, and the inputs could not be safely coerced
```

### 18.9.2 Why not using a simple iteration loop using the @jit decorator?

The answer is that NumPy ufuncs automatically get other features such as reduction, accumulation or broadcasting.

```
In [26]: a = np.arange(12).reshape(3, 4)
 a
```

```
Out[26]: array([[0, 1, 2, 3],
 [4, 5, 6, 7],
 [8, 9, 10, 11]])
```

```
In [27]: f.reduce(a, axis=0)
```

```
Out[27]: array([12, 15, 18, 21])
```

```
In [28]: f.reduce(a, axis=1)
```



```

Out[28]: array([6, 22, 38])

In [29]: f.accumulate(a)

Out[29]: array([[0, 1, 2, 3],
 [4, 6, 8, 10],
 [12, 15, 18, 21]])

In [30]: f.accumulate(a, axis=1)

Out[30]: array([[0, 1, 3, 6],
 [4, 9, 15, 22],
 [8, 17, 27, 38]])

```

## 18.10 The vectorize() decorator supports multiple ufunc targets:

- **cpu** *Single-threaded CPU* : small data sizes (approx. less than 1KB), no overhead.
- **parallel** *Multi-core CPU* : medium data sizes (approx. less than 1MB), small overhead.
- **cuda** *CUDA GPU* big data sizes (approx. greater than 1MB), significant overhead.

## 18.11 The @guvectorize decorator

- It allows you to write ufuncs that will work on an arbitrary number of elements of input arrays, and take and return arrays of differing dimensions.

```

In [31]: from numba import guvectorize
 @guvectorize([(int64[:], int64[:], int64[:])], '(n),()->(n)')
 def g(x, y, res):
 for i in range(x.shape[0]):
 res[i] = x[i] + y[0] # adds the scalar y to all elements of x

```

This decorator has two arguments: - the declaration `(n),()->(n)` tells NumPy that the function takes a `n`-element one-dimension array, a scalar (symbolically denoted by the empty tuple `()`) and returns a `n`-element one-dimension array; - the list of supported concrete signatures as in `@vectorize`; here we only support `int64` arrays.

## 18.12 Automatic parallelization with @jit

- Setting the `parallel` option for `jit()` enables this experimental Numba feature.
- **Array Expressions like element-wise or point-wise array operations are supported.**
  - unary operators: `+` `-` `~`
  - binary operators: `+` `-` `*` `/` `/?` `%` `|` `>>` `^` `<<` `&` `**` `//`
  - comparison operators: `==` `!=` `<` `<=` `>` `>=`
  - Numpy ufuncs that are supported in nopython mode.
  - Numpy reduction functions `sum` and `prod`.
- Numpy array creation functions `zeros`, `ones`, and several random functions (`rand`, `randn`, `ranf`, `random_sample`, `sample`, `random`, `standard_normal`, `chisquare`, `weibull`, `power`, `geometric`, `exponential`, `poisson`, `rayleigh`, `normal`, `uniform`, `beta`, `binomial`, `f`, `gamma`, `lognormal`, `laplace`, `randint`, `triangular`).

Numpy dot function between a matrix and a vector, or two vectors. In all other cases, Numba's default implementation is used.

Multi-dimensional arrays are also supported for the above operations when operands have matching dimension and size. The full semantics of Numpy broadcast between arrays with mixed dimensionality or size is not supported, nor is the reduction across a selected dimension.

<http://numba.pydata.org/numba-doc/latest/user/parallel.html>

## 18.13 Explicit Parallel Loops

Another experimental feature of this module is support for explicit parallel loops. One can use Numba's `prange` instead of `range` to specify that a loop can be parallelized. The user is required to make sure that the loop does not have cross iteration dependencies except the supported reductions. Currently, reductions on scalar values are supported and are inferred from in-place operations. The example below demonstrates a parallel loop with a reduction (A is a one-dimensional Numpy array):

```
In [32]: from numba import njit, prange
 @njit(parallel=True)
 def prange_test(A):
 s = 0
 for i in prange(A.shape[0]):
 s += A[i]
 return s
```

## 18.14 Exercise

- Optimize the Laplace equation solver with numba.
  1. Use only `@jit`
  2. Try to use `@jit(nopython=True)` option
  3. Optimize the laplace function with the right signature.
  4. Try to parallelize.

```
In [33]: %%time
 %matplotlib inline
 %config InlineBackend.figure_format = 'retina'
 import numpy as np
 import matplotlib.pyplot as plt
 import itertools
 from numba import jit, float64
 # Boundary conditions
 Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50

 # Set meshgrid
 n, l = 64, 1.0
 X, Y = np.meshgrid(np.linspace(0,1,n), np.linspace(0,1,n))
 T = np.zeros((n,n))

 # Set Boundary condition
 T[n-1:, :] = Tnorth
 T[:, 0] = Tsouth
 T[:, n-1:] = Teast
 T[:, :1] = Twest

 def laplace(T, n):
 residual = 0.0
 for i in range(1, n-1):
 for j in range(1, n-1):
 T_old = T[i,j]
 T[i, j] = 0.25 * (T[i+1,j] + T[i-1,j] + T[i,j+1] + T[i,j-1])
 if T[i,j]>0:
 residual=max(residual,abs((T_old-T[i,j])/T[i,j]))
 return residual

 residual = 1.0
 istep = 0
```

```

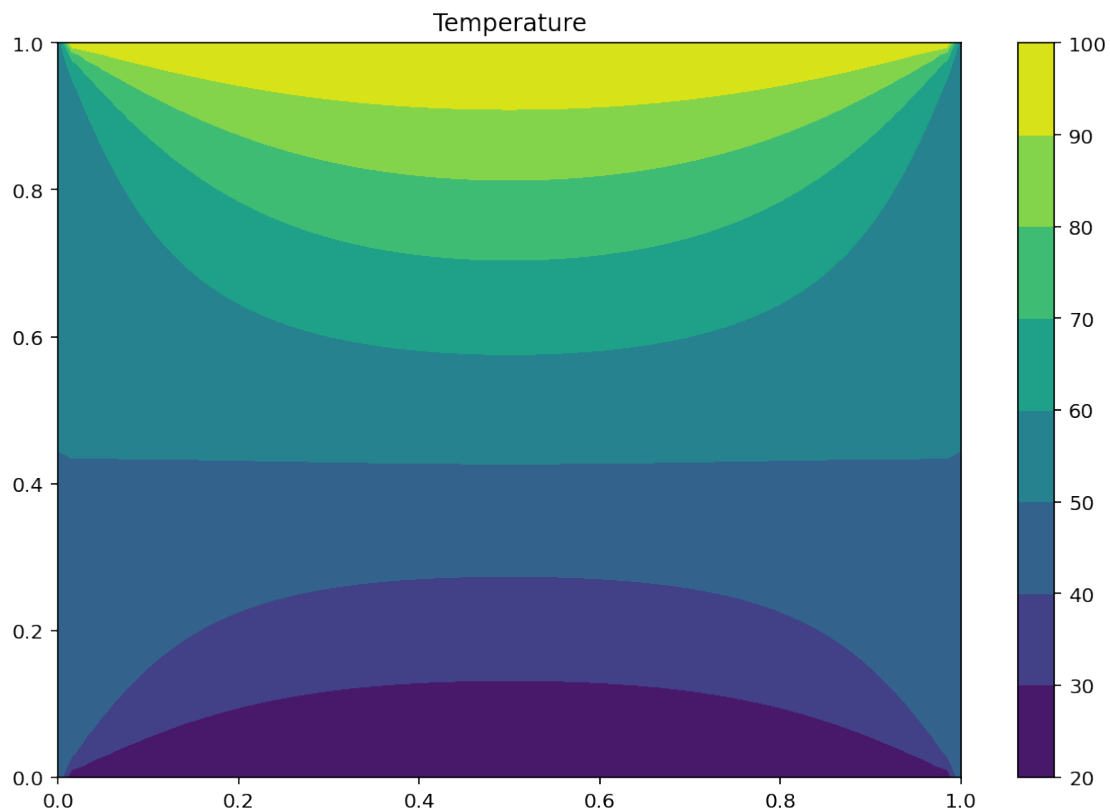
while residual > 1e-5 :
 istep += 1
 residual = laplace(T, n)
 print ((istep, residual), end="\r")

print("\n iterations = ",istep)
plt.rcParams['figure.figsize'] = (10,6.67)
plt.title("Temperature")
plt.contourf(X, Y, T)
plt.colorbar()

(2457, 9.997295133247811e-06)
iterations = 2457
CPU times: user 30.6 s, sys: 296 ms, total: 30.9 s
Wall time: 30.4 s

```

Out[33]: <matplotlib.colorbar.Colorbar at 0x7f09419be190>



## 18.15 Vectorize performance

```

In [34]: import socket
import numpy as np
from numba import vectorize

```

```

@vectorize(['float64(float64, float64)'], target="cpu", cache=True, nopython=True)
def cpu_add(a, b):
 return a + b

@vectorize(['float64(float64, float64)'], target="parallel", cache=True, nopython=True)
def parallel_add(a, b):
 return a + b

if socket.gethostname() == "gpu-irmar.insa-rennes.fr":
 @vectorize(['float64(float64, float64)'], target="cuda", cache=True, nopython=True)
 def parallel_add(a, b):
 return a + b

```

```

/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numba/np/ufunc/parallel.py:363: NumbaWarning
warnings.warn(problem)

```

```

In [35]: %matplotlib inline
import matplotlib.pyplot as plt
import seaborn; seaborn.set()
import progressbar
Nrange = (2 ** np.arange(6, 12)).astype(int)

t_numpy = []
t_numba_cpu = []
t_numba_parallel = []

bar = progressbar.ProgressBar()

for N in bar(Nrange):
 # Initialize arrays

 A = np.ones(N*N, dtype=np.float32).reshape(N,N)
 B = np.ones(A.shape, dtype=A.dtype)
 C = np.empty_like(A, dtype=A.dtype)

 t1 = %timeit -oq C = A + B
 t2 = %timeit -oq C = cpu_add(A, B)
 t3 = %timeit -oq C = parallel_add(A, B)

 t_numpy.append(t1.best)
 t_numba_cpu.append(t2.best)
 t_numba_parallel.append(t3.best)

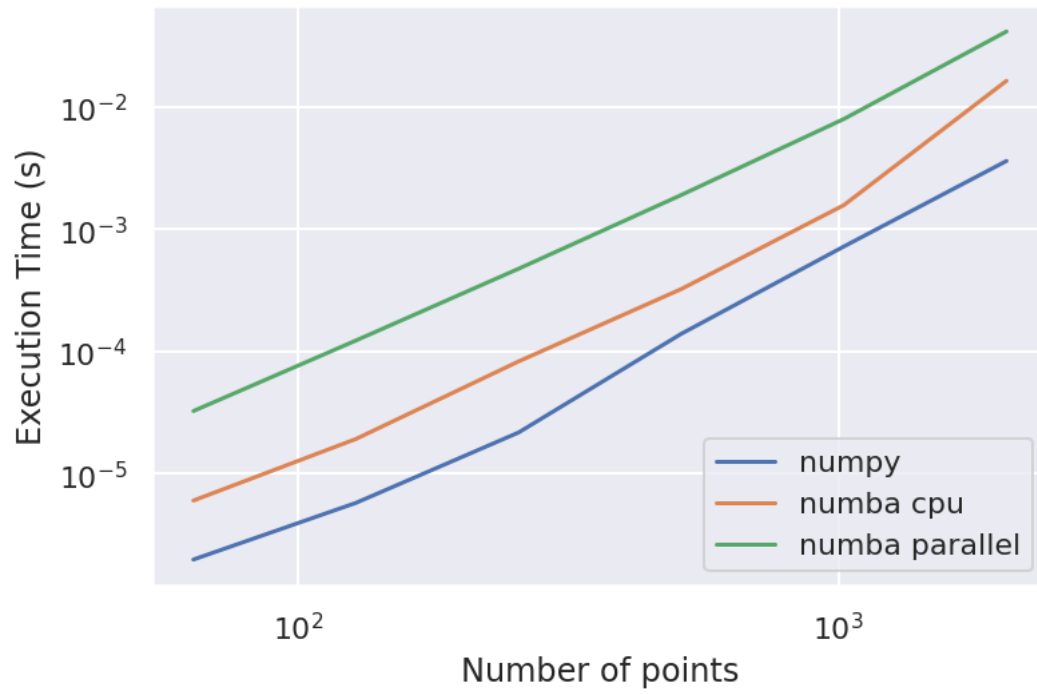
plt.loglog(Nrange, t_numpy, label='numpy')
plt.loglog(Nrange, t_numba_cpu, label='numba cpu')
plt.loglog(Nrange, t_numba_parallel, label='numba parallel')
plt.legend(loc='lower right')
plt.xlabel('Number of points')
plt.ylabel('Execution Time (s)');

```

```

100% (6 of 6) |#####| Elapsed Time: 0:02:07 Time: 0:02:07

```



## 18.16 References

- [Numba by Loic Gouarin](#)
- [Numba Documentation](#)
- [Numbapro](#)
- [Numba examples](#)



## Chapter 19

# Semi-Lagrangian method

Let us consider an abstract scalar advection equation of the form

$$\frac{\partial f}{\partial t} + a(x, t) \cdot \nabla f = 0.$$

The characteristic curves associated to this equation are the solutions of the ordinary differential equations

$$\frac{dX}{dt} = a(X(t), t)$$

We shall denote by  $X(t, x, s)$  the unique solution of this equation associated to the initial condition  $X(s) = x$ .

The classical semi-Lagrangian method is based on a backtracking of characteristics. Two steps are needed to update the distribution function  $f^{n+1}$  at  $t^{n+1}$  from its value  $f^n$  at time  $t^n$  : 1. For each grid point  $x_i$  compute  $X(t^n; x_i, t^{n+1})$  the value of the characteristic at  $t^n$  which takes the value  $x_i$  at  $t^{n+1}$ . 2. As the distribution solution of first equation verifies

$$f^{n+1}(x_i) = f^n(X(t^n; x_i, t^{n+1})),$$

we obtain the desired value of  $f^{n+1}(x_i)$  by computing  $f^n(X(t^n; x_i, t^{n+1}))$  by interpolation as  $X(t^n; x_i, t^{n+1})$  is in general not a grid point.

*Eric Sonnendrücker - Numerical methods for the Vlasov equations*

```
In [1]: %matplotlib inline
 %config InlineBackend.figure_format = 'retina'
 import matplotlib.pyplot as plt
 plt.rcParams['figure.figsize'] = (10.0, 6.0)
```

```
In [2]: # Disable the pager for lprun
 from IPython.core import page
 page.page = print
```

## 19.1 Bspline interpolator

- [De Boor's Algorithm - Wikipedia](#)

### 19.1.1 Numpy

```
In [3]: def bspline_python(p, j, x):
 """Return the value at x in [0,1[of the B-spline with
 integer nodes of degree p with support starting at j.
 Implemented recursively using the de Boor's recursion formula"""
 assert (x >= 0.0) & (x <= 1.0)
```

```

assert (type(p) == int) & (type(j) == int)
if p == 0:
 if j == 0:
 return 1.0
 else:
 return 0.0
else:
 w = (x - j) / p
 w1 = (x - j - 1) / p
 return w * bspline_python(p - 1, j, x) + (1 - w1) * bspline_python(p - 1, j + 1, x)

```

```

In [4]: import numpy as np
 from scipy.fftpack import fft, ifft

 class BSplineNumpy:

 """ Class to compute BSL advection of 1d function """

 def __init__(self, p, xmin, xmax, ncells):
 assert p & 1 == 1 # check that p is odd
 self.p = p
 self.ncells = ncells
 # compute eigenvalues of degree p b-spline matrix
 self.modes = 2 * np.pi * np.arange(ncells) / ncells
 self.deltax = (xmax - xmin) / ncells

 self.eig_bspl = bspline_python(p, -(p + 1) // 2, 0.0)
 for j in range(1, (p + 1) // 2):
 self.eig_bspl += bspline_python(p, j - (p + 1) // 2, 0.0) * 2 * np.cos(j * self.modes)

 self.eigalpha = np.zeros(ncells, dtype=complex)

 def interpolate_disp(self, f, alpha):
 """compute the interpolating spline of degree p of odd degree
 of a function f on a periodic uniform mesh, at
 all points xi-alpha"""
 p = self.p
 assert (np.size(f) == self.ncells)
 # compute eigenvalues of cubic splines evaluated at displaced points
 ishift = np.floor(-alpha / self.deltax)
 beta = -ishift - alpha / self.deltax
 self.eigalpha.fill(0.)
 for j in range(-(p-1)//2, (p+1)//2 + 1):
 self.eigalpha += bspline_python(p, j-(p+1)//2, beta) * np.exp((ishift+j)*1j*self.modes)

 # compute interpolating spline using fft and properties of circulant matrices
 return np.real(ifft(fft(f) * self.eigalpha / self.eig_bspl))

```

### 19.1.2 Interpolation test

sin function after a displacement of alpha

```

In [5]: def interpolation_test(BSplineClass):
 """ Test to check interpolation """
 n = 64
 cs = BSplineClass(3,0,1,n)
 x = np.linspace(0,1,n, endpoint=False)
 f = np.sin(x*4*np.pi)
 alpha = 0.2

```



```
return np.allclose(np.sin((x-alpha)*4*np.pi), cs.interpolate_disp(f, alpha))
```

```
interpolation_test(BSplineNumpy)
```

```
Out[5]: True
```

### 19.1.3 Profiling the code

```
In [6]: %load_ext line_profiler
```

```
In [7]: n = 1024
 cs = BSplineNumpy(3,0,1,n)
 x = np.linspace(0,1,n, endpoint=False)
 f = np.sin(x*4*np.pi)
 alpha = 0.2;
```

```
In [8]: %lprun -s -f cs.interpolate_disp -T lp_results.txt cs.interpolate_disp(f, alpha);
```

```
Timer unit: 1e-06 s
```

```
Total time: 0.001091 s
```

```
File: <ipython-input-4-c5af2f880bfd>
```

```
Function: interpolate_disp at line 22
```

| Line # | Hits | Time  | Per Hit | % Time | Line Contents                                    |
|--------|------|-------|---------|--------|--------------------------------------------------|
| 22     |      |       |         |        | def interpolate_disp(self, f, alpha):            |
| 23     |      |       |         |        | """compute the interpolating spline of degree p  |
| 24     |      |       |         |        | of a function f on a periodic uniform mesh, at   |
| 25     |      |       |         |        | all points xi-alpha"""                           |
| 26     | 1    | 3.0   | 3.0     | 0.3    | p = self.p                                       |
| 27     | 1    | 12.0  | 12.0    | 1.1    | assert (np.size(f) == self.ncells)               |
| 28     |      |       |         |        | # compute eigenvalues of cubic splines evaluated |
| 29     | 1    | 15.0  | 15.0    | 1.4    | ishift = np.floor(-alpha / self.deltax)          |
| 30     | 1    | 3.0   | 3.0     | 0.3    | beta = -ishift - alpha / self.deltax             |
| 31     | 1    | 21.0  | 21.0    | 1.9    | self.eigalpha.fill(0.)                           |
| 32     | 5    | 6.0   | 1.2     | 0.5    | for j in range(-(p-1)//2, (p+1)//2 + 1):         |
| 33     | 4    | 856.0 | 214.0   | 78.5   | self.eigalpha += bspline_python(p, j-(p+1)/2)    |
| 34     |      |       |         |        |                                                  |
| 35     |      |       |         |        | # compute interpolating spline using fft and p   |
| 36     | 1    | 175.0 | 175.0   | 16.0   | return np.real(ifft(fft(f) * self.eigalpha / s   |

```
*** Profile printout saved to text file 'lp_results.txt'.
```

### 19.1.4 Fortran

Replace the bspline computation by a fortran function, call it `bspline_fortran`.

```
In [9]: %load_ext fortranmagic
```

```
In [10]: %%fortran
 recursive function bspline_fortran(p, j, x) result(res)
 integer :: p, j
 real(8) :: x, w, w1
 real(8) :: res
```

```

 if (p == 0) then
 if (j == 0) then
 res = 1.0
 return
 else
 res = 0.0
 return
 end if
 else
 w = (x - j) / p
 w1 = (x - j - 1) / p
 end if

 res = w * bspline_fortran(p-1,j,x) &
 +(1-w1)*bspline_fortran(p-1,j+1,x)

end function bspline_fortran

```

```

In [11]: import numpy as np
 from scipy.fftpack import fft, ifft

 class BSplineFortran:

 def __init__(self, p, xmin, xmax, ncells):
 assert p & 1 == 1 # check that p is odd
 self.p = p
 self.ncells = ncells
 # compute eigenvalues of degree p b-spline matrix
 self.modes = 2 * np.pi * np.arange(ncells) / ncells
 self.deltax = (xmax - xmin) / ncells

 self.eig_bspl = bspline_fortran(p, -(p+1)//2, 0.0)
 for j in range(1, (p+1)//2):
 self.eig_bspl += bspline_fortran(p, j-(p+1)//2, 0.0)*2*np.cos(j*self.modes)

 self.eigalpha = np.zeros(ncells, dtype=complex)

 def interpolate_disp(self, f, alpha):
 """compute the interpolating spline of degree p of odd degree
 of a function f on a periodic uniform mesh, at
 all points xi-alpha"""
 p = self.p
 assert (np.size(f) == self.ncells)
 # compute eigenvalues of cubic splines evaluated at displaced points
 ishift = np.floor(-alpha / self.deltax)
 beta = -ishift - alpha / self.deltax
 self.eigalpha.fill(0.)
 for j in range(-(p-1)//2, (p+1)//2 + 1):
 self.eigalpha += bspline_fortran(p, j-(p+1)//2, beta) * np.exp((ishift+j)*1j*self.modes)

 # compute interpolating spline using fft and properties of circulant matrices
 return np.real(ifft(fft(f) * self.eigalpha / self.eig_bspl))

```

```

In [12]: interpolation_test(BSplineFortran)

```

```

Out[12]: True

```

### 19.1.5 Numba

Create a optimized function of bspline python function with Numba. Call it bspline\_numba.

```
In [13]: # %load solutions/landau_damping/bspline_numba.py
from numba import jit, int32, float64
from scipy.fftpack import fft, ifft

@jit("float64(int32,int32,float64)", nopython=True)
def bspline_numba(p, j, x):

 """Return the value at x in [0,1[of the B-spline with
 integer nodes of degree p with support starting at j.
 Implemented recursively using the de Boor's recursion formula"""

 assert ((x >= 0.0) & (x <= 1.0))
 if p == 0:
 if j == 0:
 return 1.0
 else:
 return 0.0
 else:
 w = (x-j)/p
 w1 = (x-j-1)/p
 return w * bspline_numba(p-1,j,x)+(1-w1)*bspline_numba(p-1,j+1,x)

In [14]: class BSplineNumba:

 def __init__(self, p, xmin, xmax, ncells):
 assert p & 1 == 1 # check that p is odd
 self.p = p
 self.ncells = ncells
 # compute eigenvalues of degree p b-spline matrix
 self.modes = 2 * np.pi * np.arange(ncells) / ncells
 self.deltax = (xmax - xmin) / ncells

 self.eig_bspl = bspline_numba(p, -(p+1)//2, 0.0)
 for j in range(1, (p + 1) // 2):
 self.eig_bspl += bspline_numba(p, j-(p+1)//2, 0.0)*2*np.cos(j*self.modes)

 self.eigalpha = np.zeros(ncells, dtype=complex)

 def interpolate_disp(self, f, alpha):
 """compute the interpolating spline of degree p of odd degree
 of a function f on a periodic uniform mesh, at
 all points xi-alpha"""

 p = self.p
 assert (np.size(f) == self.ncells)
 # compute eigenvalues of cubic splines evaluated at displaced points
 ishift = np.floor(-alpha / self.deltax)
 beta = -ishift - alpha / self.deltax
 self.eigalpha.fill(0.)
 for j in range(-(p-1)//2, (p+1)//2+1):
 self.eigalpha += bspline_numba(p, j-(p+1)//2, beta)*np.exp((ishift+j)*1j*self.modes)

 # compute interpolating spline using fft and properties of circulant matrices
 return np.real(ifft(fft(f) * self.eigalpha / self.eig_bspl))

In [15]: interpolation_test(BSplineNumba)
```

Out[15]: True

### 19.1.6 Pythran

In [16]: `import pythran`

In [17]: `%load_ext pythran.magic`

In [18]: `# %load solutions/landau_damping/bspline_pythran.py`

```
#pythran export bspline_pythran(int,int,float64)
def bspline_pythran(p, j, x):
 if p == 0:
 if j == 0:
 return 1.0
 else:
 return 0.0
 else:
 w = (x-j)/p
 w1 = (x-j-1)/p
 return w * bspline_pythran(p-1,j,x)+(1-w1)*bspline_pythran(p-1,j+1,x)
```

In [19]: `class BSplinePythran:`

```
def __init__(self, p, xmin, xmax, ncells):
 assert p & 1 == 1 # check that p is odd
 self.p = p
 self.ncells = ncells
 # compute eigenvalues of degree p b-spline matrix
 self.modes = 2 * np.pi * np.arange(ncells) / ncells
 self.deltax = (xmax - xmin) / ncells

 self.eig_bspl = bspline_pythran(p, -(p+1)//2, 0.0)
 for j in range(1, (p + 1) // 2):
 self.eig_bspl += bspline_pythran(p,j-(p+1)//2,0.0)*2*np.cos(j*self.modes)

 self.eigalpha = np.zeros(ncells, dtype=complex)

def interpolate_disp(self, f, alpha):
 """compute the interpolating spline of degree p of odd degree
 of a function f on a periodic uniform mesh, at
 all points xi-alpha"""

 p = self.p
 assert (f.size == self.ncells)
 # compute eigenvalues of cubic splines evaluated at displaced points
 ishift = np.floor(-alpha / self.deltax)
 beta = -ishift - alpha / self.deltax
 self.eigalpha.fill(0.)
 for j in range(-(p-1)//2, (p+1)//2+1):
 self.eigalpha += bspline_pythran(p, j-(p+1)//2, beta)*np.exp((ishift+j)*1j*self.modes)

 # compute interpolating spline using fft and properties of circulant matrices
 return np.real(iff(fft(f) * self.eigalpha / self.eig_bspl))
```

In [20]: `interpolation_test(BSplinePythran)`

Out[20]: True

### 19.1.7 Cython

- Create `bspline_cython` function.

In [21]: `%load_ext cython`

```
In [22]: %%cython -a
def bspline_cython(p, j, x):
 """Return the value at x in [0,1[of the B-spline with
 integer nodes of degree p with support starting at j.
 Implemented recursively using the de Boor's recursion formula"""
 assert (x >= 0.0) & (x <= 1.0)
 assert (type(p) == int) & (type(j) == int)
 if p == 0:
 if j == 0:
 return 1.0
 else:
 return 0.0
 else:
 w = (x - j) / p
 w1 = (x - j - 1) / p
 return w * bspline_cython(p - 1, j, x) + (1 - w1) * bspline_cython(p - 1, j + 1, x)
```

Out[22]: <IPython.core.display.HTML object>

```
In [23]: %%cython
import cython
import numpy as np
cimport numpy as np
from scipy.fftpack import fft, ifft

@cython.cdivision(True)
cdef double bspline_cython(int p, int j, double x):
 """Return the value at x in [0,1[of the B-spline with
 integer nodes of degree p with support starting at j.
 Implemented recursively using the de Boor's recursion formula"""
 cdef double w, w1
 if p == 0:
 if j == 0:
 return 1.0
 else:
 return 0.0
 else:
 w = (x - j) / p
 w1 = (x - j - 1) / p
 return w * bspline_cython(p-1,j,x)+(1-w1)*bspline_cython(p-1,j+1,x)

class BSplineCython:

 def __init__(self, p, xmin, xmax, ncells):
 self.p = p
 self.ncells = ncells
 # compute eigenvalues of degree p b-spline matrix
 self.modes = 2 * np.pi * np.arange(ncells) / ncells
 self.deltax = (xmax - xmin) / ncells

 self.eig_bspl = bspline_cython(p,-(p+1)//2, 0.0)
 for j in range(1, (p + 1) // 2):
 self.eig_bspl += bspline_cython(p,j-(p+1)//2,0.0)*2*np.cos(j*self.modes)
```

```

self.eigalpha = np.zeros(ncells, dtype=complex)

@cython.boundscheck(False)
@cython.wraparound(False)
def interpolate_disp(self, f, alpha):
 """compute the interpolating spline of degree p of odd degree
 of a function f on a periodic uniform mesh, at
 all points xi-alpha"""
 cdef Py_ssize_t j
 cdef int p = self.p
 # compute eigenvalues of cubic splines evaluated at displaced points
 cdef int ishift = np.floor(-alpha / self.deltax)
 cdef double beta = -ishift - alpha / self.deltax
 self.eigalpha.fill(0)
 for j in range(-(p-1)//2, (p+1)//2+1):
 self.eigalpha += bspline_cython(p, j-(p+1)//2, beta)*np.exp((ishift+j)*1j*self.modes)

 # compute interpolating spline using fft and properties of circulant matrices
 return np.real(iff(fft(f) * self.eigalpha / self.eig_bspl))

```

In [24]: interpolation\_test(BSplineCython)

Out[24]: True

```

In [25]: import seaborn; seaborn.set()
from tqdm.notebook import tqdm
Mrange = (2 ** np.arange(5, 10)).astype(int)

t_numpy = []
t_fortran = []
t_numba = []
t_pythran = []
t_cython = []

for M in tqdm(Mrange):
 x = np.linspace(0,1,M, endpoint=False)
 f = np.sin(x*4*np.pi)
 cs1 = BSplineNumpy(5,0,1,M)
 cs2 = BSplineFortran(5,0,1,M)
 cs3 = BSplineNumba(5,0,1,M)
 cs4 = BSplinePythran(5,0,1,M)
 cs5 = BSplineCython(5,0,1,M)

 alpha = 0.1
 t1 = %timeit -oq cs1.interpolate_disp(f, alpha)
 t2 = %timeit -oq cs2.interpolate_disp(f, alpha)
 t3 = %timeit -oq cs3.interpolate_disp(f, alpha)
 t4 = %timeit -oq cs4.interpolate_disp(f, alpha)
 t5 = %timeit -oq cs5.interpolate_disp(f, alpha)

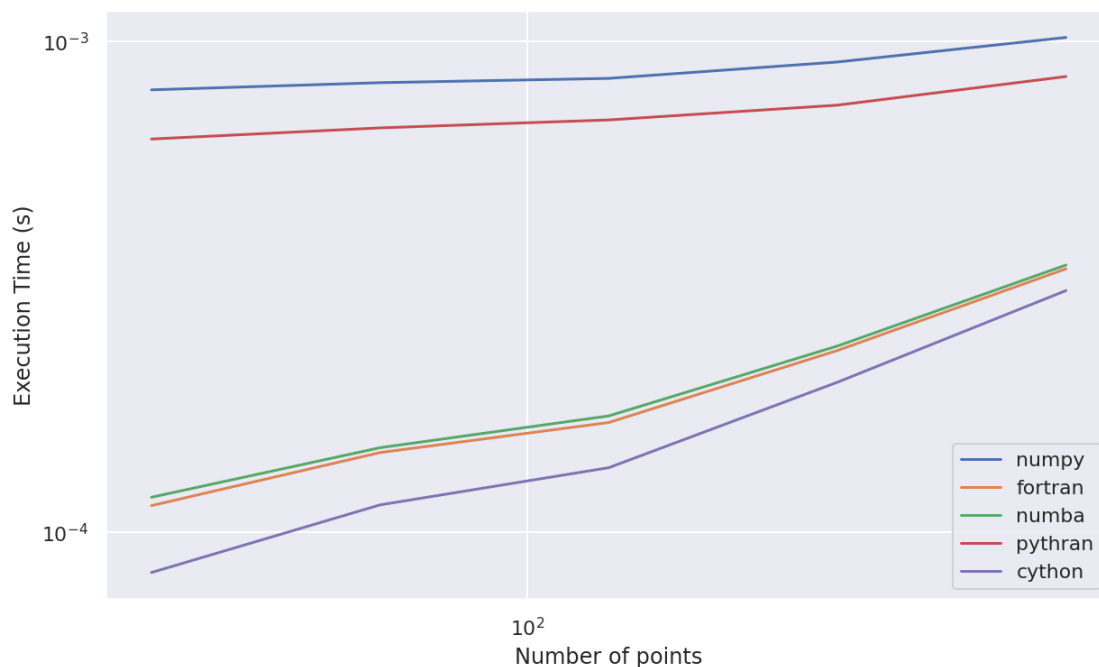
 t_numpy.append(t1.best)
 t_fortran.append(t2.best)
 t_numba.append(t3.best)
 t_pythran.append(t4.best)
 t_cython.append(t5.best)

plt.loglog(Mrange, t_numpy, label='numpy')
plt.loglog(Mrange, t_fortran, label='fortran')
plt.loglog(Mrange, t_numba, label='numba')
plt.loglog(Mrange, t_pythran, label='pythran')

```

```
plt.loglog(Mrange, t_cython, label='cython')
plt.legend(loc='lower right')
plt.xlabel('Number of points')
plt.ylabel('Execution Time (s)');

HBox(children=(FloatProgress(value=0.0, max=5.0), HTML(value='')))
```



## 19.2 Vlasov-Poisson equation

We consider the dimensionless Vlasov-Poisson equation for one species with a neutralizing background.

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f + E(t, x) \cdot \nabla_v f = 0, \quad -\Delta \phi = 1 - \rho, \quad E = -\nabla \phi \rho(t, x) = \int f(t, x, v) dv.$$

- [Vlasov Equation - Wikipedia](#)

```
In [26]: BSpline = dict(numpy=BSplineNumpy,
 fortran=BSplineFortran,
 cython=BSplineCython,
 numba=BSplineNumba,
 pythran=BSplinePythran)

class VlasovPoisson:

 def __init__(self, xmin, xmax, nx, vmin, vmax, nv, opt='numpy'):

 # Grid
```

```

self.nx = nx
self.x, self.dx = np.linspace(xmin, xmax, nx, endpoint=False, retstep=True)
self.nv = nv
self.v, self.dv = np.linspace(vmin, vmax, nv, endpoint=False, retstep=True)

Distribution function
self.f = np.zeros((nx,nv))

Interpolators for advection
BSplineClass = BSpline[opt]
self.cs_x = BSplineClass(3, xmin, xmax, nx)
self.cs_v = BSplineClass(3, vmin, vmax, nv)

Modes for Poisson equation
self.modes = np.zeros(nx)
k = 2* np.pi / (xmax - xmin)
self.modes[:nx//2] = k * np.arange(nx//2)
self.modes[nx//2:] = - k * np.arange(nx//2,0,-1)
self.modes += self.modes == 0 # avoid division by zero

def advection_x(self, dt):
 for j in range(self.nv):
 alpha = dt * self.v[j]
 self.f[j,:] = self.cs_x.interpolate_disp(self.f[j,:], alpha)

def advection_v(self, e, dt):
 for i in range(self.nx):
 alpha = dt * e[i]
 self.f[:,i] = self.cs_v.interpolate_disp(self.f[:,i], alpha)

def compute_rho(self):
 rho = self.dv * np.sum(self.f, axis=0)
 return rho - rho.mean()

def compute_e(self, rho):
 # compute Ex using that ik*Ex = rho
 rhok = fft(rho)/self.modes
 return np.real(iff(-1j*rhok))

def run(self, f, nstep, dt):
 self.f = f
 nrj = []
 self.advection_x(0.5*dt)
 for istep in tqdm(range(nstep)):
 rho = self.compute_rho()
 e = self.compute_e(rho)
 self.advection_v(e, dt)
 self.advection_x(dt)
 nrj.append(0.5*np.log(np.sum(e*e)*self.dx))

 return nrj

```

## 19.3 Landau Damping

[Landau damping - Wikipedia](#)

In [27]: `from time import time`



```

elapsed_time = {}
fig, axes = plt.subplots()
for opt in ('numpy', 'fortran', 'numba', 'cython', 'pythran'):

 # Set grid
 nx, nv = 32, 64
 xmin, xmax = 0.0, 4*np.pi
 vmin, vmax = -6., 6.

 # Create Vlasov-Poisson simulation
 sim = VlasovPoisson(xmin, xmax, nx, vmin, vmax, nv, opt=opt)

 # Initialize distribution function
 X, V = np.meshgrid(sim.x, sim.v)
 eps, kx = 0.001, 0.5
 f = (1.0+eps*np.cos(kx*X))/np.sqrt(2.0*np.pi)* np.exp(-0.5*V*V)

 # Set time domain
 nstep = 600
 t, dt = np.linspace(0.0, 60.0, nstep, retstep=True)

 # Run simulation
 etime = time()
 nrj = sim.run(f, nstep, dt)
 print(" {0:12s} : {1:.4f} ".format(opt, time()-etime))

 # Plot energy
 axes.plot(t, nrj, label=opt)

 axes.plot(t, -0.1533*t-5.50)
 plt.legend();

HBox(children=(FloatProgress(value=0.0, max=600.0), HTML(value='')))

numpy : 11.6214

HBox(children=(FloatProgress(value=0.0, max=600.0), HTML(value='')))

fortran : 4.9955

HBox(children=(FloatProgress(value=0.0, max=600.0), HTML(value='')))

numba : 5.1997

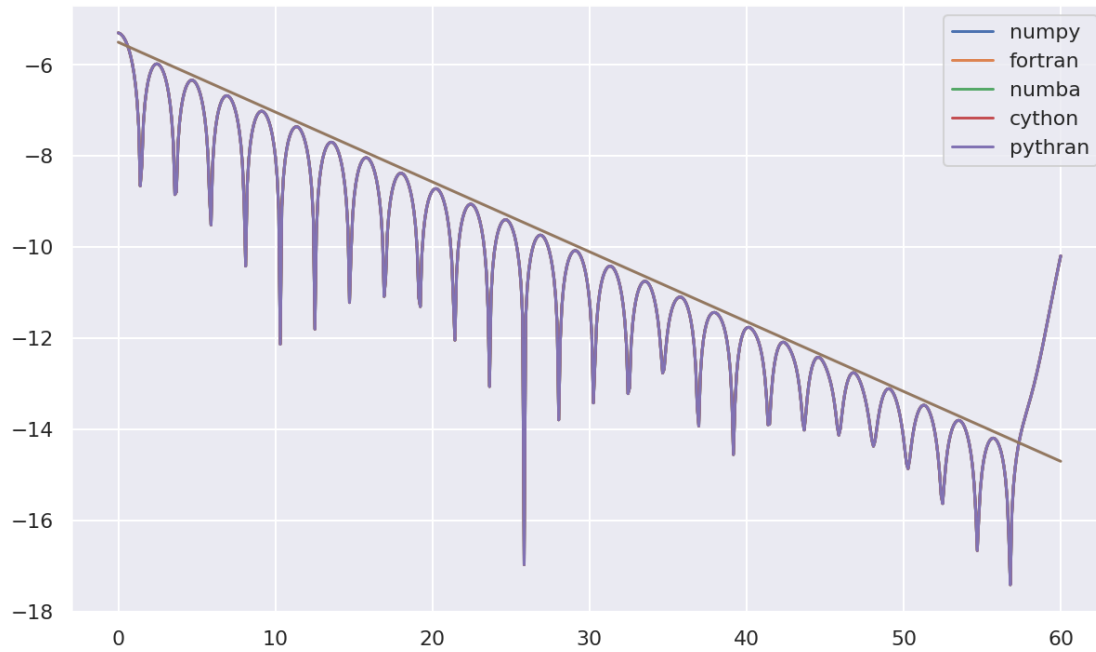
HBox(children=(FloatProgress(value=0.0, max=600.0), HTML(value='')))

cython : 3.7105

```

```
HBox(children=(FloatProgress(value=0.0, max=600.0), HTML(value='')))
```

pythran : 9.7878



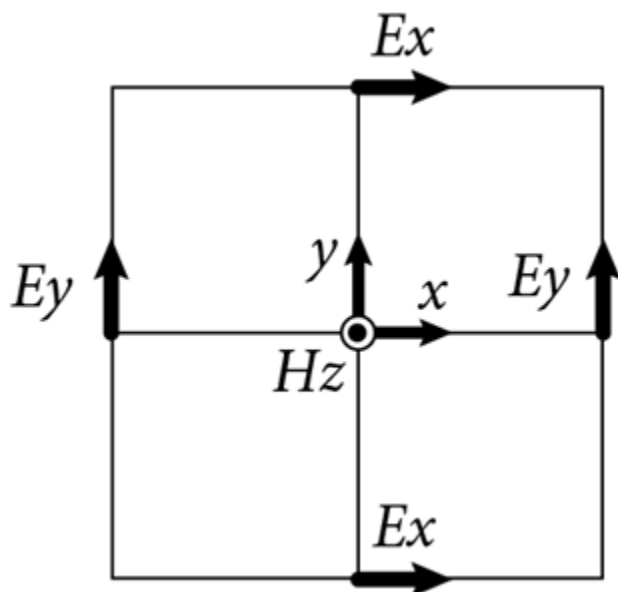
## 19.4 References

- [Optimizing Python with NumPy and Numba](#)

## Chapter 20

# Maxwell solver in two dimensions with FDTD scheme

$$\frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}; \quad \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y}; \quad \frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x}$$



$$H_z|_{i+1/2,j+1/2}^{n+1/2} = H_z|_{i+1/2,j+1/2}^{n-1/2} + \frac{dt}{dy}(E_x|_{i+1/2,j+1}^n - E_x|_{i+1/2,j}^n) - \frac{dt}{dx}(E_y|_{i+1,j+1/2}^n - E_y|_{i,j+1/2}^n)$$

$$E_x|_{i+1/2,j}^{n+1} = E_x|_{i+1/2,j}^n + \frac{dt}{dy}(H_z|_{i+1/2,j+1/2}^{n+1/2} - H_z|_{i-1/2,j-1/2}^{n+1/2})$$

$$E_y|_{i,j+1/2}^{n+1} = E_y|_{i,j+1/2}^n - \frac{dt}{dx}(H_z|_{i+1/2,j+1/2}^{n+1/2} - H_z|_{i-1/2,j+1/2}^{n+1/2})$$

Description of the scheme

```
In [1]: %matplotlib inline
 %config InlineBackend.figure_format = 'retina'

 import matplotlib.pyplot as plt
 import numpy as np
 from mpl_toolkits.mplot3d import axes3d
```

```

import matplotlib.animation as animation
from IPython.display import HTML

plt.rcParams['figure.figsize'] = (10,6)

```

```

In [2]: # Mesh parameters
nx, ny = 101, 101
vx, dx = np.linspace(0, 1, nx, endpoint=True, retstep=True)
vy, dy = np.linspace(0, 1, ny, endpoint=True, retstep=True)

#Initialize Ex, Ey when time = 0
ex = np.zeros((nx-1, ny), dtype=np.double)
ey = np.zeros((nx, ny-1), dtype=np.double)
nbiter = 500 # time loop size
dt = 0.001 # time step
m, n = 2, 2
omega = np.sqrt((m*np.pi)**2+(n*np.pi)**2)
Create the staggered grid for Bz
x, y = np.meshgrid(0.5*(vx[:-1]+vx[1:]), 0.5*(vy[:-1]+vy[1:]))

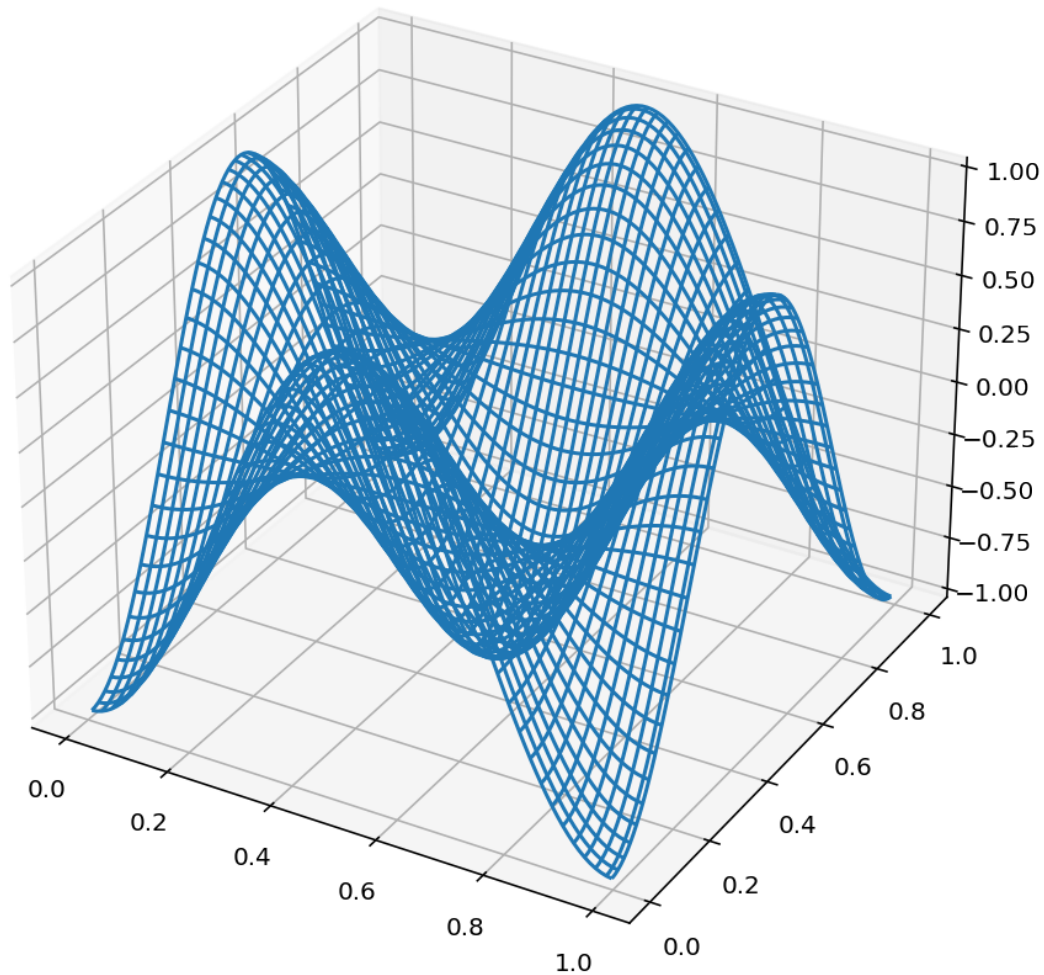
```

```

In [3]: fig = plt.figure()
ax = axes3d.Axes3D(fig)

#Initialize Bz when time = - dt / 2
hz = - np.cos(m*np.pi*y) * np.cos(n*np.pi*x) * np.cos(omega*(-0.5*dt))
wframe = ax.plot_wireframe(x, y, hz, rstride=2, cstride=2)
ax.set_zlim(-1,1);

```



## 20.1 numpy

```
In [4]: def faraday(ex, ey, hz) :
 "faraday equation Bz(t+dt/2) -> Bz(t-dt/2) + dt f(E(t))"
 return hz + dt * ((ex[:, 1:]-ex[:, :-1]) / dy - (ey[1:, :]-ey[:-1, :]) / dx)

def ampere_maxwell(hz, ex, ey):
 " Ampere-Maxwell equation E(t+dt) -> E(t) + dt g(Bz(t+dt/2)) "
 ex[:, 1:-1] += dt*(hz[:, 1:]-hz[:, :-1]) / dy
 ey[1:-1, :] += - dt*(hz[1:, :]-hz[:-1, :]) / dx

 # periodic boundary conditions
 ex[:, 0] += dt*(hz[:, 0]-hz[:, -1]) / dy
 ex[:, -1] = ex[:, 0]
 ey[0, :] += - dt*(hz[0, :]-hz[-1, :]) / dx
 ey[-1, :] = ey[0, :]
```

```

 return ex, ey

In [5]: def update(i, ax, fig):
 ax.cla()

 global ex, ey, hz

 hz = faraday(ex, ey, hz)
 ex, ey = ampere_maxwell(hz, ex, ey)

 wframe = ax.plot_wireframe(x, y, hz, rstride=2, cstride=2)
 ax.set_zlim(-1, 1)
 return wframe,

```

```

In [6]: ani = animation.FuncAnimation(fig, update,
 frames=range(200),
 fargs=(ax, fig), interval=100)

```

```

In [7]: %%time
 HTML(ani.to_html5_video())

```

CPU times: user 27 s, sys: 559 ms, total: 27.6 s  
 Wall time: 28.1 s

Out[7]: <IPython.core.display.HTML object>

```

In [8]: %%time

 from tqdm.notebook import tqdm

 nx, ny = 512, 512
 vx, dx = np.linspace(0, 1, nx, endpoint=True, retstep=True)
 vy, dy = np.linspace(0, 1, ny, endpoint=True, retstep=True)

 ex = np.zeros((nx-1, ny), dtype=np.double)
 ey = np.zeros((nx, ny-1), dtype=np.double)
 dt = 0.001 # time step
 m, n = 2, 2
 omega = np.sqrt((m*np.pi)**2+(n*np.pi)**2)
 x, y = np.meshgrid(0.5*(vx[:-1]+vx[1:]), 0.5*(vy[:-1]+vy[1:]))

 hz = - np.cos(m*np.pi*y) * np.cos(n*np.pi*x) * np.cos(omega*(-0.5*dt))

 for t in tqdm(range(1000)):

 hz = faraday(ex, ey, hz)
 ex, ey = ampere_maxwell(hz, ex, ey)

```

```

HBox(children=(FloatProgress(value=0.0, max=1000.0), HTML(value='')))

```

CPU times: user 3.65 s, sys: 12.1 ms, total: 3.66 s  
 Wall time: 3.64 s

```

In [9]: %load_ext fortranmagic

```

## 20.2 fortran

In [10]: %%fortran

```

subroutine faraday_fortran(ex, ey, bz, dx, dy, dt, nx, ny)
implicit none

real(8), intent(in) :: ex(nx-1,ny)
real(8), intent(in) :: ey(nx,ny-1)
real(8), intent(inout) :: bz(nx-1,ny-1)
integer, intent(in) :: nx, ny
real(8), intent(in) :: dx, dy, dt

integer :: i, j
real(8) :: dex_dx, dey_dy
real(8) :: dex_dy, dey_dx

do j=1,ny-1
do i=1,nx-1
 dex_dy = (ex(i,j+1)-ex(i,j)) / dy
 dey_dx = (ey(i+1,j)-ey(i,j)) / dx
 bz(i,j) = bz(i,j) + dt * (dex_dy - dey_dx)
end do
end do

end subroutine faraday_fortran

```

In [11]: %%fortran

```

subroutine amperemaxwell_fortran(ex, ey, bz, dx, dy, dt, nx, ny)

implicit none
integer, intent(in):: nx, ny
real(8), intent(in):: dx, dy, dt
real(8), dimension(nx-1, ny-1), intent(inout) :: bz
real(8), dimension(nx-1, ny), intent(inout) :: ex
real(8), dimension(nx, ny-1), intent(inout) :: ey
integer:: i, j
real(8):: dbz_dx, dbz_dy
real(8), parameter:: csq = 1d0

do i = 1, nx-1
 dbz_dy = (bz(i, 1)-bz(i, ny-1)) / dy ! periodic BC
 ex(i, 1) = ex(i, 1) + dt*csq*dbz_dy
 ex(i, ny) = ex(i, 1)
end do

do j = 1, ny-1
 dbz_dx = (bz(1,j)-bz(nx-1,j)) / dx ! periodic BC
 ey(1,j) = ey(1,j) - dt*csq*dbz_dx
 ey(nx,j) = ey(1,j)
end do

do j=2,ny-1
 do i=1,nx-1
 dbz_dy = (bz(i,j)-bz(i,j-1)) / dy
 ex(i,j) = ex(i,j) + dt*csq*dbz_dy
 end do
end do

```

```

do j=1,ny-1
 do i=2,nx-1
 dbz_dx = (bz(i,j)-bz(i-1,j)) / dx
 ey(i,j) = ey(i,j) - dt*csq*dbz_dx
 end do
end do

end subroutine amperemaxwell_fortran

```

In [12]: %%time

```

from tqdm.notebook import tqdm

ex.fill(0.0)
ey.fill(0.0)
hz = - np.cos(m*np.pi*y) * np.cos(n*np.pi*x) * np.cos(omega*(-0.5*dt))
ex = np.asfortranarray(ex)
ey = np.asfortranarray(ey)
hz = np.asfortranarray(hz)

for t in tqdm(range(1000)):

 faraday_fortran(ex, ey, hz, dx, dy, dt, nx, ny)
 amperemaxwell_fortran(ex, ey, hz, dx, dy, dt, nx, ny)

```

```
HBox(children=(FloatProgress(value=0.0, max=1000.0), HTML(value='')))
```

```

CPU times: user 1e+03 ms, sys: 16.2 ms, total: 1.02 s
Wall time: 1 s

```



## Chapter 21

# Gray-Scott Model

```
In [1]: import numpy as np
```

```
In [2]: %config InlineBackend.figure_format = 'retina'
```

The reaction-diffusion system described here involves two generic chemical species U and V, whose concentration at a given point in space is referred to by variables u and v. As the term implies, they react with each other, and they diffuse through the medium. Therefore the concentration of U and V at any given location changes with time and can differ from that at other locations.

The overall behavior of the system is described by the following formula, two equations which describe three sources of increase and decrease for each of the two chemicals:

$$\begin{aligned}\frac{\partial u}{\partial t} &= D_u \Delta u - uv^2 + F(1 - u) \\ \frac{\partial v}{\partial t} &= D_v \Delta v + uv^2 - (F + k)v\end{aligned}$$

The laplacian is computed with the following numerical scheme

$$\Delta u_{i,j} \approx u_{i,j-1} + u_{i-1,j} - 4u_{i,j} + u_{i+1,j} + u_{i,j+1}$$

The classic Euler scheme is used to integrate the time derivative.

### 21.1 Initialization

$u$  is 1 everywhere et  $v$  is 0 in the domain except in a square zone where  $v = 0.25$  and  $u = 0.5$ . This square located in the center of the domain is  $[0, 1] \times [0, 1]$  with a size of 0.2.

```
In [3]: def init(n):

 u = np.ones((n+2,n+2))
 v = np.zeros((n+2,n+2))

 x, y = np.meshgrid(np.linspace(0, 1, n+2), np.linspace(0, 1, n+2))

 mask = (0.4<x) & (x<0.6) & (0.4<y) & (y<0.6)

 u[mask] = 0.50
 v[mask] = 0.25

 return u, v
```

## 21.2 Boundary conditions

We assume that the domain is periodic.

```
In [4]: def periodic_bc(u):
 u[0, :] = u[-2, :]
 u[-1, :] = u[1, :]
 u[:, 0] = u[:, -2]
 u[:, -1] = u[:, 1]
```

## 21.3 Laplacian

```
In [5]: def laplacian(u):
 """
 second order finite differences
 """
 return (
 u[:, :-2, 1:-1] +
 u[1:-1, :-2] - 4*u[1:-1, 1:-1] + u[1:-1, 2:] +
 u[2:, 1:-1])
```

## 21.4 Gray-Scott model

```
In [6]: def grayscott(U, V, Du, Dv, F, k):

 u, v = U[1:-1, 1:-1], V[1:-1, 1:-1]

 Lu = laplacian(U)
 Lv = laplacian(V)

 uvv = u*v*v
 u += Du*Lu - uvv + F*(1 - u)
 v += Dv*Lv + uvv - (F + k)*v

 periodic_bc(U)
 periodic_bc(V)
```

## 21.5 Visualization

Nous utiliserons les données suivantes.

```
In [7]: Du, Dv = .1, .05
 F, k = 0.0545, 0.062

In [8]: %%time
 from tqdm.notebook import tqdm
 from PIL import Image
 U, V = init(300)

 def create_image():
 global U, V
 for t in range(40):
 grayscott(U, V, Du, Dv, F, k)
 V_scaled = np.uint8(255*(V-V.min()) / (V.max()-V.min()))
 return V_scaled

 def create_frames(n):
```

```

 return [create_image() for i in tqdm(range(n))]

frames = create_frames(500)

HBox(children=(FloatProgress(value=0.0, max=500.0), HTML(value='')))

```

CPU times: user 57.3 s, sys: 29.1 s, total: 1min 26s  
 Wall time: 1min 26s

```

In [9]: from ipywidgets import interact, IntSlider

def display_sequence(iframe):

 return Image.fromarray(frames[iframe])

interact(display_sequence,
 iframe=IntSlider(min=0,
 max=len(frames)-1,
 step=1,
 value=0,
 continuous_update=True))

```



Out[9]: <function \_\_main\_\_.display\_sequence(iframe)>

```
In [10]: import imageio
 frames_scaled = [np.uint8(255 * frame) for frame in frames]
 imageio.mimsave('images/movie.gif', frames_scaled, format='gif', fps=60)

In [11]: from IPython.display import HTML
 HTML('')

Out[11]: <IPython.core.display.HTML object>
```

## 21.6 References

- [Reaction-Diffusion by the Gray-Scott Model: Pearson's Parametrization](#)

## Chapter 22

# Animation with matplotlib

```
In [1]: %matplotlib inline
```

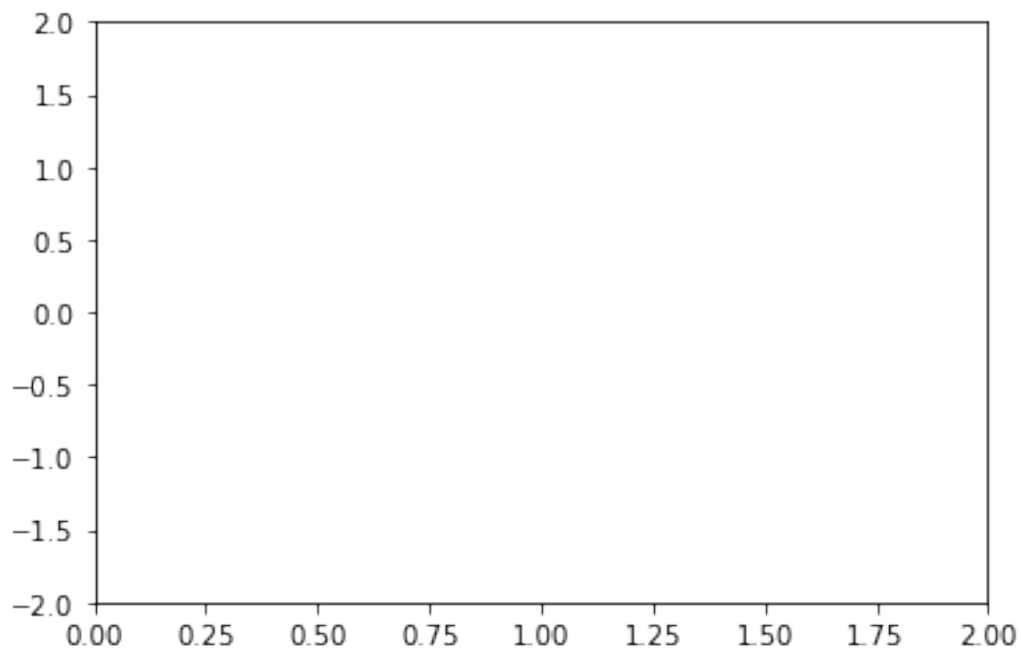
```
In [2]: import numpy as np
import matplotlib.pyplot as plt

from matplotlib import animation, rc
from IPython.display import HTML
from matplotlib.animation import FuncAnimation
```

```
In [3]: fig, ax = plt.subplots()

ax.set_xlim((0, 2))
ax.set_ylim((-2, 2))

line, = ax.plot([], [], lw=2)
```



```

In [4]: def init():
 line.set_data([], [])
 return (line,)

In [5]: def animate(i):
 x = np.linspace(0, 2, 1000)
 y = np.sin(2 * np.pi * (x - 0.01 * i))
 line.set_data(x, y)
 return (line,)

In [6]: anim = animation.FuncAnimation(fig, animate, init_func=init,
 frames=100, interval=20,
 blit=True)

In [7]: HTML(anim.to_html5_video())

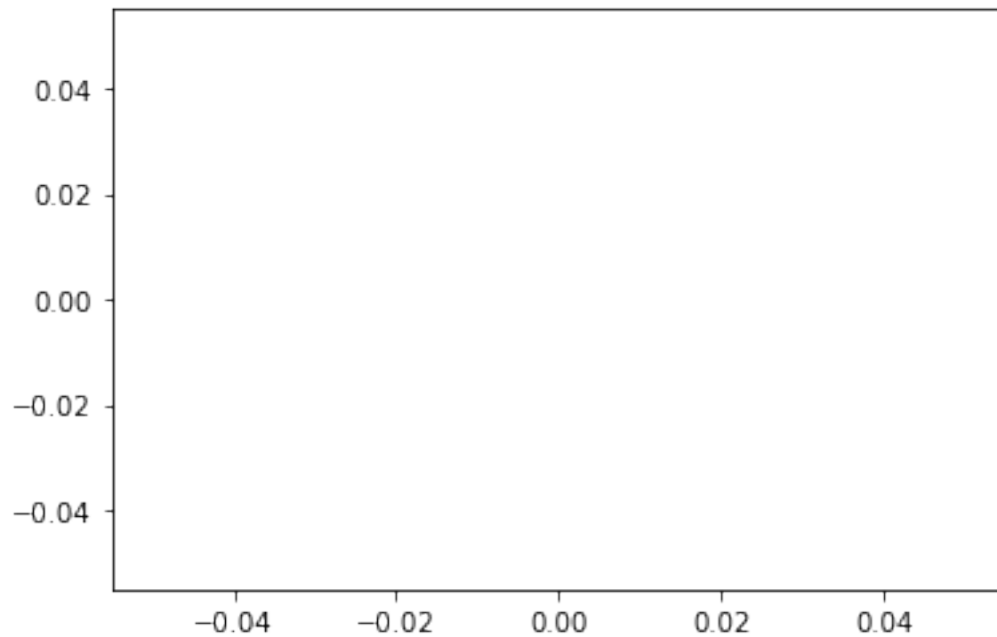
Out[7]: <IPython.core.display.HTML object>

In [8]: HTML(anim.to_jshtml())

Out[8]: <IPython.core.display.HTML object>

In [9]: fig = plt.figure()
 ax1 = fig.add_subplot(1,1,1)
 xdata, ydata = [], []
 line1, = plt.plot([], [], 'r-', animated=True)

```



```

In [10]: def init():
 ax1.set_xlim((0,1))
 ax1.set_ylim((-1,1))
 return line1,

 def update(frame):

```

```
xdata.append(frame)
ydata.append(np.sin(8*np.pi*frame))
line1.set_data(xdata, ydata)

return line1,

ani = FuncAnimation(fig, update, frames=np.linspace(0, 1.0, 100),
 init_func=init, blit=True)

plt.rc('animation', html='html5')
ani

Out[10]: <matplotlib.animation.FuncAnimation at 0x7fc40a9250d0>

In []:
```