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Scientific computing with Python

for beginners



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Introduction



1.1 History

- Project initiated by Guido Von Rossum in 1990
- Interpreted language written in C.
- Widely used in all domains (Web, Data Science, Scientific Computation).
- This is a high level language with a simple syntax.
- Python types are numerously and powerful.
- Bind Python with other languages is easy.
- You can perform a lot of operations with very few lines.
- Available on all platforms Unix, Windows, Mac OS X...
- Very few limits.
- Many libraries offer Python bindings.

1.2 Python 2 and 3 version

- Python 3.x isn't a simple improvement or extension of Python 2.x.
- All libraries exist in version 3 but both versions coexist.
- Every example are written in Python 3.x, it is the default version.
- Changes in official documentation: [https://docs.python.org/3/whatsnew/3.0.html]

- print is now a function print() with sep argument.
- Some function return "views" instead of "lists".
- In version 3 division operator isn't pure (7/2 = 3.5).
- range function doesn't return list anymore. Use list(range(n)).

1.3 Porting your code

- http://www.diveintopython3.net/porting-code-to-python-3-with-2to3.html
- Python-Future offers Python 2 compatibility.
- Migrating to Python 3 with pleasure

1.4 Python distributions

Python packages are available with all linux distributions but you can get standalone bundles:

- Anaconda
- Enthought Python Distribution
- Astropy
- SAGEMATH
- Pyzo

1.5 Performances

Python is not fast... but: - Sometimes it is. - Most of operations are optimized. - Package like numpy can reduce the CPU time. - With Python you can save time to achieve your project.

Some advices: - Write your program with Python language. - If it is fast enough, be happy. - After profiling, optimize costly parts of your code.

"Premature optimization is the root of all evil" (Donald Knuth 1974)

1.6 Jupyter - Start The Notebook

Open the notebook

cd python-notebooks
jupyter notebook

You should see the notebook open in your browser. If not, go to http://localhost:8888

The Jupyter Notebook is an interactive environment for writing and running code. The notebook is capable of running code in a wide range of languages. However, each notebook is associated with Python3 kernel.

1.7 Code cells allow you to enter and run code

Make a copy of this notebook by using the File menu.

Run a code cell using Shift-Enter or pressing the

button in the toolbar above:

There are two other keyboard shortcuts for running code:

- Alt-Enter runs the current cell and inserts a new one below.
- Ctrl-Enter run the current cell and enters command mode.

1.8 Managing the Kernel

Code is run in a separate process called the Kernel. The Kernel can be interrupted or restarted. Try running the following cell and then hit the

button in the toolbar above.

The "Cell" menu has a number of menu items for running code in different ways. These includes:

- Run and Select Below
- Run and Insert Below
- Run All
- Run All Above
- Run All Below

1.9 Restarting the kernels

The kernel maintains the state of a notebook's computations. You can reset this state by restarting the kernel. This is done by clicking on the

in the toolbar above.

Check the documentation.

1.10 First program

- Print out the string "Hello world!" and its type.
- Print out the value of a variable set to 6625 and its type.

1.11 Execute using python

```
In [2]: %%file hello.py
s = "Hello World!"
```

```
print(type(s),s)
a = 6625
print(type(a),a)

Writing hello.py

$ python3 hello.py

<class 'str'> Hello World!
<class 'int'> 6625
```

1.12 Execute with ipython

```
(my-env) $ ipython
Python 3.6.3 | packaged by conda-forge | (default, Nov 4 2017, 10:13:32)
Type 'copyright', 'credits' or 'license' for more information
IPython 6.2.1 -- An enhanced Interactive Python. Type '?' for help.

In [1]: run hello.py
<class 'str'> Hello World!
<class 'int'> 6625
In [3]: %run hello.py
<class 'str'> Hello World!
<class 'int'> 6625
```

1.13 Python Types

- Most of Python types are classes, typing is dynamic.
- ; symbol can be used to split two Python commands on the same line.

```
In [4]: s = int(2010); print(type(s))
    s = 3.14; print(type(s))
    s = True; print(type(s))
    s = None; print(type(s))
    s = 1.0j; print(type(s))
    s = type(type(s)); print(type(s))

<class 'int'>
<class 'float'>
<class 'bool'>
<class 'NoneType'>
<class 'complex'>
<class 'type'>
```

1.14 Calculate with Python

47 True

```
In [5]: x = 45  # This is a comment!
    x += 2  # equivalent to x = x + 2
    print(x, x > 45)
```

Multiple Assignment

- Variables can simultaneously get new values.
- Expressions on the right-hand side are all evaluated first before assignments take place.
- The right-hand side expressions are evaluated from the left to the right.
- Use it very carefully

input Function

```
In [14]: name = input("Please enter your name: ")
        {\tt StdinNotImplementedError}
                                                   Traceback (most recent call last)
        <ipython-input-14-11981014d572> in <module>
    ----> 1 name = input("Please enter your name: ")
          2 name
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/ipykernel/kernelbase.py in raw_in
        855
        856
                    if not self._allow_stdin:
    --> 857
                        raise StdinNotImplementedError(
                            "raw_input was called, but this frontend does not support input requests."
       858
        859
        StdinNotImplementedError: raw_input was called, but this frontend does not support input reques
In [15]: x = int(input("Please enter an integer: "))
       StdinNotImplementedError
                                                   Traceback (most recent call last)
       <ipython-input-15-a09915eb966c> in <module>
    ---> 1 x = int(input("Please enter an integer: "))
          2 x
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/ipykernel/kernelbase.py in raw_in
                    if not self._allow_stdin:
        856
```

```
--> 857
                       raise StdinNotImplementedError(
                            "raw_input was called, but this frontend does not support input requests."
       858
       859
       StdinNotImplementedError: raw_input was called, but this frontend does not support input reques
In [16]: 1 = list(input("Please enter 3 values "))
       StdinNotImplementedError
                                                Traceback (most recent call last)
       <ipython-input-16-31fbdd8846a0> in <module>
   ----> 1 l = list(input("Please enter 3 values "))
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/ipykernel/kernelbase.py in raw_in
       856
                   if not self._allow_stdin:
    --> 857
                       raise StdinNotImplementedError(
                           "raw_input was called, but this frontend does not support input requests."
       858
       859
                        )
```

StdinNotImplementedError: raw_input was called, but this frontend does not support input reques

Strings

```
In [1]: word = "bonjour"
In [2]: print(word, len(word))
bonjour 7
   Add a . to the variable and then press <TAB> to get all attached methods available.
In [3]: word.capitalize()
Out[3]: 'Bonjour'
   After choosing your method, press shift+<TAB> to get interface.
In [4]: word.upper()
Out[4]: 'BONJOUR'
In [5]: help(word.replace) # or word.replace?
Help on built-in function replace:
replace(old, new, count=-1, /) method of builtins.str instance
    Return a copy with all occurrences of substring old replaced by new.
      count
        Maximum number of occurrences to replace.
        -1 (the default value) means replace all occurrences.
    If the optional argument count is given, only the first count occurrences are
    replaced.
In [6]: word.replace('o','0',1)
Out[6]: 'bOnjour'
```

Strings and print Function

Strings can be enclosed in single quotes ('...') or double quotes ("...") with the same result. can be used to escape quotes:

```
In [7]: print('spam eggs')
                                   # single quotes
       print('doesn\'t')
                                   # use \' to escape the single quote...
       print("doesn't")
                                   # ...or use double quotes instead
       print('"Yes," he said.')
       print("\"Yes,\" he said.")
       print('"Isn\'t," she said.')
spam eggs
doesn't
doesn't
"Yes," he said.
"Yes," he said.
"Isn't," she said.
  print function translates C special characters
In [8]: s = '\tFirst line.\nSecond line.' # \n means newline \t inserts tab
        print(s) # with print(), \n produces a new line
       print(r'\tFirst line.\nSecond line.') # note the r before the quote
        First line.
Second line.
\tFirst line.\nSecond line.
```

String literals with multiple lines

```
In [9]: print("""\
       Usage: thingy [OPTIONS]
                                       Display this usage message
            -H hostname
                                       Hostname to connect to
Usage: thingy [OPTIONS]
                                  Display this usage message
     -H hostname
                                  Hostname to connect to
   character removes the initial newline.
   Strings can be concatenated (glued together) with the + operator, and repeated with *
In [10]: 3 * ("Re" + 2 * 'n' + 'es ')
Out[10]: 'Rennes Rennes '
   Two or more string literals next to each other are automatically concatenated.
In [11]: text = ('Put several strings within parentheses '
                  'to have them joined together.')
         text
Out[11]: 'Put several strings within parentheses to have them joined together.'
   Strings can be indexed, with the first character having index 0. There is no separate character type; a
character is simply a string of size one
In [12]: word = 'Python'
        print(word[0]) # character in position 0
        print(word[5]) # character in position 5
Ρ
n
   Indices may also be negative numbers, to start counting from the right
In [13]: print(word[-1]) # last character
         print(word[-2]) # second-last character
n
0
```

Slicing Strings

- Omitted first index defaults to zero,
- Omitted second index defaults to the size of the string being sliced.
- Step can be set with the third index

```
In [14]: print(word[:2]) # character from the beginning to position 2 (excluded)
        print(word[4:]) # characters from position 4 (included) to the end
        print(word[-2:]) # characters from the second-last (included) to the end
        print(word[::-1]) # This is the reversed string!
Ру
on
nohtyP
In [15]: word[::2]
Out[15]: 'Pto'
   Python strings cannot be changed — they are immutable. If you need a different string, you should
create a new or use Lists.
In [16]: word[0] = 'J'
                                                      Traceback (most recent call last)
        TypeError
        <ipython-input-16-91a956888ca7> in <module>
    ---> 1 \text{ word}[0] = 'J'
        TypeError: 'str' object does not support item assignment
In [17]: ## Some string methods
        print(word.startswith('P'))
True
In [18]: print(*(_ for _ in dir(word) if not _.startswith('_')) )
capitalize casefold center count encode endswith expandtabs find format format_map index isalnum isalph
```

7.0.1 Exercise

- Ask user to input a string.
- Print out the string length.
- Check if the last character is equal to the first character.
- Check if this string contains only letters.
- $\bullet\,$ Check if this string is lower case.
- Check if this string is a palindrome. A palindrome is a word, phrase, number, or other sequence of characters which reads the same backward as forward.

In [19]: # %load solutions/strings/demo.py

Python Lists and tuples

- List is the most versatile Python data type to group values with others
- Can be written as a list of comma-separated values (items) between square brackets.
- Tuples are written between parenthesis. They are read-only lists.
- Lists can contain items of different types.
- Like strings, lists can be indexed and sliced.
- Lists also support operations like concatenation.

8.1 Indexing

8.1.1 Unlike strings, which are immutable, lists are a mutable type.

```
In [6]: cubes = [1, 8, 27, 65, 125] # something's wrong here
    cubes[3] = 64 # replace the wrong value, the cube of 4 is 64, not 65!
    print(cubes)
```

8.2 Assignment

- You can change the size of the list or clear it entirely.
- The built-in function len() returns list size.
- It is possible to create lists containing other lists.

```
In [9]: letters = ['a', 'b', 'c', 'd', 'e', 'f', 'g']
        letters[2:5] = ['C', 'D', 'E'] # replace some values
       print(letters)
['a', 'b', 'C', 'D', 'E', 'f', 'g']
In [10]: letters[2:5] = [] # now remove them
        print(letters)
['a', 'b', 'f', 'g']
In [11]: a = ['a', 'b', 'c']
        n = [1, 2, 3]
        x = [a, n]
In [12]: x
Out[12]: [['a', 'b', 'c'], [1, 2, 3]]
In [13]: x[0]
Out[13]: ['a', 'b', 'c']
In [14]: x[0][1], len(x)
Out[14]: ('b', 2)
```

In [15]: a = [0, 1, 2, 3, 4] b = a

Assignment, Copy and Reference

9.1 Some useful List Methods

9.2 Dictionary

They are indexed by keys, which are often strings.

9.2.1 Exercise

- Split the string "python LILLE 2018" into the list ["python","LILLE", 2018]
- Insert "april" and value 10 before 2018 in the result list.
- Capitalize the first item to "Python"
- Create a dictionary with following keys (meeting, month, day, year)
- Print out the items.
- Append the key "place" to this dictionary and set the value to "LILLE". python ['python', 'LILLE', '2018'] ['python', 'LILLE', 'april', 10, '2018'] ['Python', 'LILLE', 'april', 10, '2018'] {'course': 'Python', 'month': 'april', 'day': 10, 'year': '2018', 'place': 'LILLE'}

Control Flow Tools

10.1 While loop

- Don't forget the ':' character.
- The body of the loop is indented

```
In [1]: # Fibonacci series:
    # the sum of two elements defines the next
    a, b = 0, 1
    while b < 1000:
        a, b = b, a+b
        print(round(b/a,3), end=",")

1.0,2.0,1.5,1.667,1.6,1.625,1.615,1.619,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,1.618,
```

10.2 if Statements

More

```
True, False, and, or, not, ==, is, !=, is not, >, >=, <, <=
In [2]: x = 42
    if x < 0:
        x = 0
        print('Negative changed to zero')
    elif x == 0:
        print('Zero')
    elif x == 1:
        print('Single')
    else:
        print('More')</pre>
```

switch or case statements don't exist in Python.

10.2.1 Exercise Collatz conjecture

Consider the following operation on an arbitrary positive integer: - If the number is even, divide it by two. - If the number is odd, triple it and add one.

The conjecture is that no matter what initial value of this integer, the sequence will always reach 1. - Test the Collatz conjecture for n = 100000. - How many steps do you need to reach 1?

Loop over an iterable object

We use for statement for looping over an iterable object. If we use it with a string, it loops over its characters.

11.0.1 Exercise: Anagram

An anagram is word or phrase formed by rearranging the letters of a different word or phrase, typically using all the original letters exactly once.

Write a code that print True if s1 is an anagram of s2. To do it, remove every character present in both strings. Check you obtain two empty strings.

Hint: s = s.replace(c,"",1) removes the character c in string s one time.

```
s1 = "pascal obispo"
s2 = "pablo picasso"
...
True
```

11.1 Loop with range function

- It generates arithmetic progressions
- It is possible to let the range start at another number, or to specify a different increment.

- Since Python 3, the object returned by range() doesn't return a list to save memory space. xrange no longer exists.
- Use function list() to creates it.

11.1.1 Exercise Exponential

• Write some code to compute the exponential mathematical constant $e \simeq 2.718281828459045$ using the taylor series developed at 0 and without any import of external modules:

$$e \simeq \sum_{n=0}^{50} \frac{1}{n!}$$

break Statement.

iter Function

13.1 Defining Function: def statement

- Body of the function start must be indented
- Functions without a return statement do return a value called None.

```
In [14]: def fib(n):
    """Print a Fibonacci series up to n."""
    a, b = 0, 1
    while a < n:
        print(a, end=' ') # the end optional argument is \n by default
        a, b = b, a+b
    print("\n") # new line

result = fib(2000)
    print(result) # is None</pre>
```

0 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597

None

Documentation string

• It's good practice to include docstrings in code that you write, so make a habit of it.

```
In [15]: def my_function( foo):
    """Do nothing, but document it.

    No, really, it doesn't do anything.
    """
    pass

    print(my_function.__doc__)

Do nothing, but document it.

    No, really, it doesn't do anything.

In [16]: help(my_function)

Help on function my_function in module __main__:
my_function(foo)
    Do nothing, but document it.

No, really, it doesn't do anything.
```

14.1 Default Argument Values

Important warning: The default value is evaluated only once.

14.2 Function Annotations

Completely optional metadata information about the types used by user-defined functions. These type annotations conforming to PEP 484 could be statically used by MyPy.

14.3 Arbitrary Argument Lists

Arguments can be wrapped up in a tuple or a list with form *args

- Normally, these variadic arguments will be last in the list of formal parameters.
- Any formal parameters which occur after the *args parameter are 'keyword-only' arguments.

Keyword Arguments Dictionary

A final formal parameter of the form **name receives a dictionary.

```
In [23]: def cheeseshop(kind, *arguments, **keywords):
            print("-- Do you have any", kind, "?")
            print("-- I'm sorry, we're all out of", kind)
            for arg in arguments:
                print(arg)
            print("-" * 40)
            for key, value in keywords.items():
                print(key, ":", value)
   *name must occur before **name
In [24]: cheeseshop("Limburger", "It's very runny, sir.",
                   "It's really very, VERY runny, sir.",
                   shopkeeper="Michael Palin",
                   client="John Cleese",
                   sketch="Cheese Shop Sketch")
-- Do you have any Limburger ?
-- I'm sorry, we're all out of Limburger
It's very runny, sir.
It's really very, VERY runny, sir.
shopkeeper : Michael Palin
client : John Cleese
sketch : Cheese Shop Sketch
```

15.1 Lambda Expressions

6

Lambda functions can be used wherever function objects are required.

lambda functions can reference variables from the containing scope:

15.2 Unpacking Argument Lists

Arguments are already in a list or tuple. They can be unpacked for a function call. For instance, the built-in range() function is called with the *-operator to unpack the arguments out of a list:

15.2.1 Exercise: Time converter

Write 3 functions to manipulate hours and minutes: - Function minutes return minutes from (hours, minutes). - Function hours the inverse function that return (hours, minutes) from minutes. - Function add_time to add (hh1,mm1) and (hh2, mm2) two couples (hours, minutes). It takes 2 tuples of length 2 as input arguments and return the tuple (hh,mm).

```
print(minutes(6,15)) # 375
print(minutes(7,46)) # 466
print(add_time((6,15),(7,46)) # (14,01)
```

15.3 Functions Scope

- All variable assignments in a function store the value in the local symbol table.
- Global variables cannot be directly assigned a value within a function (unless named in a global statement).
- The value of the function can be assigned to another name which can then also be used as a function.

```
In [30]: pi = 1.
         def deg2rad(theta):
             pi = 3.14
             return theta * pi / 180.
         print(deg2rad(45))
         print(pi)
0.785
1.0
In [31]: def rad2deg(theta):
             return theta*180./pi
         print(rad2deg(0.785))
         pi = 3.14
         print(rad2deg(0.785))
141.3
45.0
In [32]: def deg2rad(theta):
             global pi
             pi = 3.14
             return theta * pi / 180
         pi = 1
         print(deg2rad(45))
0.785
In [33]: print(pi)
3.14
```

15.4 enumerate Function

15.4.1 Exercise: Caesar cipher

In cryptography, a Caesar cipher, is one of the simplest and most widely known encryption techniques. It is a type of substitution cipher in which each letter in the plaintext is replaced by a letter some fixed number of positions down the alphabet. For example, with a left shift of 3, D would be replaced by A, E would become B, and so on.

- Create a function cipher that take the plain text and the key value as arguments and return the encrypted text.
- Create a funtion plain that take the crypted text and the key value as arguments that return the deciphered text.

15.5 zip Builtin Function

Loop over sequences simultaneously.

15.5.1 Exercise

Code a new version of your cypher function to crypt also upper case character. Use zip to loop over upper and lower case alphabets.

15.6 List comprehension

- Set or change values inside a list
- Create list from function

15.6.1 Exercise

Code a new version of cypher function using list comprehension.

Hints: -s = ''.join(L) convert the characters list L into a string s. -L.index(c) return the index position of c in list L - "c".islower() and "C".isupper() return True

map built-in function

```
Apply a function over a sequence.
```

Since Python 3.x, map process return an iterator. Save memory, and should make things go faster. Display result by using unpacking operator.

```
In [42]: print(*res)
Ox0 Ox1 Ox2 Ox3 Ox4 Ox5 Ox6 Ox7 Ox8 Ox9 Oxa Oxb Oxc Oxd Oxe Oxf
```

16.1 map with user-defined function

16.1.1 map is often faster than for loop

```
In [44]: M = range(10000)
    f = lambda x: x**2
        %timeit lmap = list(map(f,M))
3.51 ms ± 89.6 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
In [45]: M = range(10000)
    f = lambda x: x**2
        %timeit lfor = [f(m) for m in M]
3.86 ms ± 91.1 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

16.2 filter

creates a iterator of elements for which a function returns True.

16.2.1 As map, filter is often faster than for loop

16.2.2 Exercise with map:

Code a new version of your cypher function using map.

Hints: - Applied function must have only one argument, create a function called **shift** with the key value and use map.

16.2.3 Exercise with filter:

Create a function with a number n as single argument that returns True if n is a Kaprekar number. For example 45 is a Kaprekar number, because

$$45^2 = 2025$$

and

$$20 + 25 = 45$$

Use filter to give Kaprekar numbers list lower than 10000.

```
1, 9, 45, 55, 99, 297, 703, 999, 2223, 2728, 4879, 4950, 5050, 5292, 7272, 7777, 9999
```

Recursive Call

17.0.1 Exercise: factorial

• Write the function factorial with a recursive call

NB: Recursion is not recommended by Guido.

17.0.2 Exercise: Minimum number of rooms required for lectures.

Given an array of time intervals (start, end) for classroom lectures (possibly overlapping), find the minimum number of rooms required.

For example, given Input:

17.0.3 Exercise: non-palindromic skinny numbers

non-palindromic squares remaining square when written backwards

```
10^2 = 100 01^2 = 001

13^2 = 169 31^2 = 961

102^2 = 10404 201^2 = 40401
```

17.0.4 Exercise: Narcissistic number

A number is narcissistic if the sum of its own digits each raised to the power of the number of digits.

```
Example : 4150 = 4^5 + 1^5 + 5^5 + 0^5 or 153 = 1^3 + 5^3 + 3^3
```

Find narcissitic numbers with 3 digits

17.0.5 Exercise: Happy number

• Given a number $n = n_0$, define a sequence n_1, n_2, \ldots where n_{i+1} is the sum of the squares of the digits of n_i . Then n is happy if and only if there exists i such that $n_i = 1$.

For example, 19 is happy, as the associated sequence is:

- Write a function ishappy(n) that returns True if n is happy. - Write a function happy(n) that returns a list with all happy numbers < n.

17.0.6 Exercise: longuest increasing subsequence

Given N elements, write a program that prints the length of the longuest increasing subsequence whose adjacent element difference is one.

Examples:

```
a = [3, 10, 3, 11, 4, 5, 6, 7, 8, 12]
```

Output: 6

Explanation: 3, 4, 5, 6, 7, 8 is the longest increasing subsequence whose adjacent element differs by or

Input :
$$a = [6, 7, 8, 3, 4, 5, 9, 10]$$

Output: 5

Explanation: 6, 7, 8, 9, 10 is the longest increasing subsequence

17.0.7 Exercise: Polynomial derivative

- A Polynomial is represented by a Python list of its coefficients. $[1,5,-4] = > 1 + 5x 4x^2$
- Write the function diff(P,n) that return the nth derivative Q
- Don't use any external package

$$diff([3,2,1,5,7],2) = [2, 30, 84]$$

 $diff([-6,5,-3,-4,3,-4],3) = [-24, 72, -240]$

Modules

Writing fibo.py

If your Python program gets longer, you may want to split it into several files for easier maintenance. To support this, Python has a way to put definitions in a file and use them in a script or in an interactive instance of the interpreter. Such a file is called a module.

Run the cell below to create a file named fibo.py with several functions inside:

```
In [1]: %%file fibo.py
        """ Simple module with
            two functions to compute Fibonacci series """
        def fib1(n):
           """ write Fibonacci series up to n """
           a, b = 0, 1
           while b < n:
              print(b, end=', ')
              a, b = b, a+b
        def fib2(n):
            """ return Fibonacci series up to n """
           result = []
           a, b = 0, 1
            while b < n:
               result.append(b)
                a, b = b, a+b
            return result
        if __name__ == "__main__":
            import sys
            fib1(int(sys.argv[1]))
```

You can use the function fib by importing fibo which is the name of the file without .py extension.

```
In [3]: %run fibo.py 1000
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987,
In [4]: help(fibo)
Help on module fibo:
NAME
    fibo
DESCRIPTION
    Simple module with
    two functions to compute Fibonacci series
FUNCTIONS
    fib1(n)
        write Fibonacci series up to n
    fib2(n)
        return Fibonacci series up to n
FILE
    /home/runner/work/python-notebooks/python-notebooks/notebooks/fibo.py
```

18.1 Executing modules as scripts

When you run a Python module with

```
$ python fibo.py <arguments>
```

In [6]: fib1(1000)

the code in the module will be executed, just as if you imported it, but with the **name** set to "**main**". The following code will be executed only in this case and not when it is imported.

```
if __name__ == "__main__":
    import sys
    fib(int(sys.argv[1]))

In Jupyter notebook, you can run the fibo.py python script using magic command.
In [5]: %run fibo.py 1000

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987,
    The module is also imported.
```

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987,

18.2 Different ways to import a module

```
import fibo
import fibo as f
from fibo import fib1, fib2
from fibo import *
```

In [7]: from numpy import sqrt

sqrt(-1)

from scipy import sqrt

- Last command with '*' imports all names except those beginning with an underscore (_). In most cases, do not use this facility since it introduces an unknown set of names into the interpreter, possibly hiding some things you have already defined.
- If a function with same name is present in different modules imported. Last module function imported replace the previous one.

• For efficiency reasons, each module is only imported once per interpreter session. Therefore, if you change your modules, you must restart the interpreter – If you really want to test interactively after a long run, use:

```
import importlib
importlib.reload(modulename)
```

18.3 The Module Search Path

When a module is imported, the interpreter searches for a file named module.py in a list of directories given by the variable sys.path. - Python programs can modify sys.path - export the PYTHONPATH environment variable to change it on your system.

['/home/runner/work/python-notebooks/python-notebooks/notebooks', '/usr/share/miniconda3/envs/runenv/li

When you import a module foo, following files are searched in this order:

- foo.dll, foo.dylib or foo.so
- foo.py
- foo.pyc
- **foo/___init___.py**

18.4 Packages

- A package is a directory containing Python module files.
- This directory always contains a file name ___init___.py

18.5 Relative imports

These imports use leading dots to indicate the current and parent packages involved in the relative import. In the sugiton module, you can use:

```
from . import morgiou # import module in the same directory
from .. import cirm # import module in parent directory
from ..cirm import bastide # import module in another subdirectory of the parent directory
```

18.6 Reminder

```
Don't forget that importing \ast is not recommended
```

Out[14]: 10 In [15]: del sum # delete imported sum function from numpy help(sum) Help on built-in function sum in module builtins: sum(iterable, /, start=0) Return the sum of a 'start' value (default: 0) plus an iterable of numbers When the iterable is empty, return the start value. This function is intended specifically for use with numeric values and may reject non-numeric types. In [16]: import numpy as np help(np.sum) Help on function sum in module numpy: sum(a, axis=None, dtype=None, out=None, keepdims=<no value>, initial=<no value>, where=<no value>) Sum of array elements over a given axis. Parameters _____ a : array_like Elements to sum. axis : None or int or tuple of ints, optional Axis or axes along which a sum is performed. The default, axis=None, will sum all of the elements of the input array. If axis is negative it counts from the last to the first axis. .. versionadded:: 1.7.0 If axis is a tuple of ints, a sum is performed on all of the axes specified in the tuple instead of a single axis or all the axes as before. dtype : dtype, optional The type of the returned array and of the accumulator in which the elements are summed. The dtype of `a` is used by default unless `a` has an integer dtype of less precision than the default platform integer. In that case, if `a` is signed then the platform integer is used while if `a` is unsigned then an unsigned integer of the same precision as the platform integer is used. out : ndarray, optional Alternative output array in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary. keepdims : bool, optional If this is set to True, the axes which are reduced are left in the result as dimensions with size one. With this option, the result will broadcast correctly against the input array.

If the default value is passed, then `keepdims` will not be

passed through to the `sum` method of sub-classes of

`ndarray`, however any non-default value will be. If the sub-class' method does not implement `keepdims` any exceptions will be raised. initial : scalar, optional Starting value for the sum. See `~numpy.ufunc.reduce` for details. .. versionadded:: 1.15.0 where : array_like of bool, optional Elements to include in the sum. See `~numpy.ufunc.reduce` for details. .. versionadded:: 1.17.0 Returns sum_along_axis : ndarray An array with the same shape as `a`, with the specified axis removed. If `a` is a O-d array, or if `axis` is None, a scalar is returned. If an output array is specified, a reference to `out` is returned. See Also ndarray.sum : Equivalent method. add.reduce : Equivalent functionality of `add`. cumsum : Cumulative sum of array elements. trapz : Integration of array values using the composite trapezoidal rule. mean, average Notes Arithmetic is modular when using integer types, and no error is raised on overflow. The sum of an empty array is the neutral element 0: >>> np.sum([]) 0.0 For floating point numbers the numerical precision of sum (and ``np.add.reduce``) is in general limited by directly adding each number individually to the result causing rounding errors in every step. However, often numpy will use a numerically better approach (partial pairwise summation) leading to improved precision in many use-cases. This improved precision is always provided when no ``axis`` is given. When ``axis`` is given, it will depend on which axis is summed. Technically, to provide the best speed possible, the improved precision is only used when the summation is along the fast axis in memory.

Note that the exact precision may vary depending on other parameters. In contrast to NumPy, Python's ``math.fsum`` function uses a slower but

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more precise approach to summation.

Especially when summing a large number of lower precision floating point numbers, such as ``float32``, numerical errors can become significant. In such cases it can be advisable to use `dtype="float64"` to use a higher precision for the output.

```
Examples
>>> np.sum([0.5, 1.5])
>>> np.sum([0.5, 0.7, 0.2, 1.5], dtype=np.int32)
>>> np.sum([[0, 1], [0, 5]])
>>> np.sum([[0, 1], [0, 5]], axis=0)
array([0, 6])
>>> np.sum([[0, 1], [0, 5]], axis=1)
array([1, 5])
>>> np.sum([[0, 1], [np.nan, 5]], where=[False, True], axis=1)
array([1., 5.])
If the accumulator is too small, overflow occurs:
>>> np.ones(128, dtype=np.int8).sum(dtype=np.int8)
-128
You can also start the sum with a value other than zero:
>>> np.sum([10], initial=5)
15
```

Input and Output

- str() function return human-readable representations of values.
- repr() generate representations which can be read by the interpreter.
- For objects which don't have a particular representation for human consumption, str() will return the same value as repr().

```
In [1]: s = 'Hello, world.'
       str(s)
Out[1]: 'Hello, world.'
In [2]: 1 = list(range(4))
       str(1)
Out[2]: '[0, 1, 2, 3]'
In [3]: repr(s)
Out[3]: "'Hello, world.'"
In [4]: repr(1)
Out[4]: '[0, 1, 2, 3]'
In [5]: x = 10 * 3.25
       y = 200 * 200
       s = 'The value of x is ' + str(x) + ', and y is ' + repr(y) + '...'
The value of x is 32.5, and y is 40000...
   repr() of a string adds string quotes and backslashes:
In [6]: hello = 'hello, world\n'
       hellos = repr(hello)
       hellos
Out[6]: "'hello, world\\n'"
   The argument to repr() may be any Python object:
In [7]: repr((x, y, ('spam', 'eggs')))
Out[7]: "(32.5, 40000, ('spam', 'eggs'))"
```

```
In [8]: n = 7
      for x in range(1, n):
          for i in range(n):
             print(repr(x**i).rjust(i+2), end=' ') # rjust or center can be used
          print()
1
         1
               1
                     1
                             1
                                      1
1
              8
                     16
                            32
                                     64
              27
         9
                     81
                           243
                                    729
        16
                    256
                          1024
1
              64
                                   4096
   5
        25
             125
                    625
                          3125
                                  15625
                          7776
                                  46656
 1
        36
             216
                   1296
In [9]: for x in range(1, n):
          for i in range(n):
             print("%07d" % x**i, end=' ') # old C format
          print()
0000001 0000002 0000004 0000008 0000016 0000032 0000064
0000001 0000003 0000009 0000027 0000081 0000243 0000729
0000001 0000004 0000016 0000064 0000256 0001024 0004096
0000001 0000005 0000025 0000125 0000625 0003125 0015625
0000001 0000006 0000036 0000216 0001296 0007776 0046656
```

19.1 Usage of the str.format() method

19.2 Formatted string literals (Python 3.6)

```
In [14]: print(f'The value of PI is approximately {math.pi:.4f}.')
The value of PI is approximately 3.1416.
```

19.3. EXERCISE 63

```
In [15]: name = "Fred"
        print(f"He said his name is {name}.")
        print(f"He said his name is {name!r}.")
He said his name is Fred.
He said his name is 'Fred'.
In [16]: f"He said his name is {repr(name)}." # repr() is equivalent to !r
Out[16]: "He said his name is 'Fred'."
In [17]: width, precision = 10, 4
        value = 12.34567
        print(f"result: {value:{width}.{precision}f}") # nested fields
result:
           12.3457
In [18]: from datetime import *
        today = datetime(year=2017, month=1, day=27)
        print(f"{today: %B %d, %Y}") # using date format specifier
January 27, 2017
```

19.3 Exercise

Create a list containing the values of binomial coefficients and reproduce the Pascal's triangle

19.4 Reading and Writing Files

open() returns a file object, and is most commonly used with file name and accessing mode argument.

mode can be: - 'r' when the file will only be read, - 'w' for only writing (an existing file with the same name will be erased) - 'a' opens the file for appending; any data written to the file is automatically added to the end. - 'r+' opens the file for both reading and writing. - The mode argument is optional; 'r' will be assumed if it's omitted. - Normally, files are opened in text mode. - 'b' appended to the mode opens the file in binary mode.

```
In [22]: lines= []
    with open('workfile.txt') as f:
        lines.append(f.readline())
        lines.append(f.readline())
        lines.append(f.readline())

lines
Out[22]: ['1. This is a txt file.\n', '2. \\n is used to begin a new line', '']
```

- f.readline() returns an empty string when the end of the file has been reached.
- f.readlines() or list(f) read all the lines of a file in a list.

For reading lines from a file, you can loop over the file object. This is memory efficient, fast, and leads to simple code:

d['word2'] = 2

d

19.4.1 Exercise: Wordcount Example

WordCount is a simple application that counts the number of occurrences of each word in a given input set.

- Use lorem module to write a text in the file "sample.txt"
- Write a function words with file name as input that returns a sorted list of words present in the file.
- Write the function reduce to read the results of words and sum the occurrences of each word to a final count, and then output the results as a dictionary {word1:occurences1, word2:occurences2}.
- You can check the results using piped shell commands:

19.5 Saving structured data with json

- JSON (JavaScript Object Notation) is a popular data interchange format.
- JSON format is commonly used by modern applications to allow for data exchange.
- JSON can be used to communicate with applications written in other languages.

Errors and Exceptions

There are two distinguishable kinds of errors: *syntax errors* and *exceptions*. - Syntax errors, also known as parsing errors, are the most common. - Exceptions are errors caused by statement or expression syntactically corrects. - Exceptions are not unconditionally fatal.

Exceptions in Python documentation

```
In [1]: 10 * (1/0)
           ______
                                            Traceback (most recent call last)
       ZeroDivisionError
       <ipython-input-1-0b280f36835c> in <module>
   ----> 1 10 * (1/0)
      ZeroDivisionError: division by zero
In [2]: 4 + spam*3
      NameError
                                            Traceback (most recent call last)
       <ipython-input-2-c98bb92cdcac> in <module>
   ---> 1 4 + spam*3
      NameError: name 'spam' is not defined
In [3]: '2' + 2
                                            Traceback (most recent call last)
       TypeError
       <ipython-input-3-d2b23a1db757> in <module>
```

```
----> 1 '2' + 2
```

TypeError: can only concatenate str (not "int") to str

20.1 Handling Exceptions

- In example below, the user can interrupt the program with Control-C or the stop button in Jupyter Notebook.
- Note that a user-generated interruption is signalled by raising the **KeyboardInterrupt** exception.

```
In [4]: while True:
            x = int(input("Please enter a number: "))
            print(f' x = \{x\}')
          except ValueError:
            print("Oops! That was no valid number. Try again...")
        {\tt StdinNotImplementedError}
                                                    Traceback (most recent call last)
        <ipython-input-4-d9e83eb78fd3> in <module>
          1 while True:
          2
               try:
    ----> 3
                 x = int(input("Please enter a number: "))
          4
                 print(f' x = \{x\}')
                 break
          5
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/ipykernel/kernelbase.py in raw_in
        855
        856
                    if not self._allow_stdin:
    --> 857
                         raise StdinNotImplementedError(
        858
                             "raw_input was called, but this frontend does not support input requests."
        859
                         )
```

StdinNotImplementedError: raw_input was called, but this frontend does not support input reques

- A try statement may have more than one except clause
- The optional else clause must follow all except clauses.

```
In [5]: import sys

def process_file(file):
    try:
        i = int(open(file).readline().strip()) # Read the first line of f and convert to int
        print(i)
        assert i < 0 # check if i is negative
    except OSError as err:
        print(f"OS error: {err}")</pre>
```

```
except ValueError:
               print("Could not convert data to an integer.")
               print("Unexpected error:", sys.exc_info()[0])
        # Create the file workfile.txt
       with open('workfile.txt','w') as f:
           f.write("foo")
           f.write("bar")
In [6]: process_file('workfile.txt') # catch exception return by int() call
Could not convert data to an integer.
In [7]: # Change permission of the file, workfile.txt cannot be read
        !chmod u-r workfile.txt
In [8]: process_file('workfile.txt') # catch exception return by open() call
OS error: [Errno 13] Permission denied: 'workfile.txt'
In [9]: # Let's delete the file workfile.txt
       !rm -f workfile.txt
In [10]: process_file('workfile.txt') # catch another exception return by open() call
OS error: [Errno 2] No such file or directory: 'workfile.txt'
In [11]: # Insert the value 1 at the top of workfile.txt
        !echo "1" > workfile.txt
        %cat workfile.txt
1
In [12]: process_file('workfile.txt') # catch exception return by assert()
Unexpected error: <class 'AssertionError'>
```

20.2 Raising Exceptions

The raise statement allows the programmer to force a specified exception to occur.

20.3 Defining Clean-up Actions

- The try statement has an optional clause which is intended to define clean-up actions that must be executed under all circumstances.
- A finally clause is always executed before leaving the try statement

20.3.1 Exercise

- Write a function check_date that takes a string "DD/MM/YYYY" as argument and returns True if the date is valid.
- Use it with a try ... except statement to help the user to enter a valid date.
- raise ValueError "Not a valid date"
- Hints:
- Use string method split
- Year y is a leap year if y%400==0 or (y%4==0 and y%100!=0)

20.3.2 Wordcount Exercise

• Improve the function reduce to read the results of words by using the KeyError exception to fill in the dictionary.

Classes

- Classes provide a means of bundling data and functionality together.
- Creating a new class creates a **new type** of object.
- Assigned variables are new **instances** of that type.
- Each class instance can have attributes attached to it.
- Class instances can also have **methods** for modifying its state.
- Python classes provide the class **inheritance** mechanism.

21.1 Use class to store data

- A empty class can be used to bundle together a few named data items.
- You can easily save this class containing your data in JSON file.

21.2 namedtuple

```
In [6]: # Like tuples, namedtuples are immutable:
       dog.weight = 14.5
        AttributeError
                                                       Traceback (most recent call last)
        <ipython-input-6-44ca315d244f> in <module>
           1 # Like tuples, namedtuples are immutable:
    ---> 2 \text{ dog.weight} = 14.5
        AttributeError: can't set attribute
In [7]: class Animal:
            "A simple example class Animal with its name, weight and age"
           def __init__(self, name, weight, age): # constructor
               self.name = name
               self.weight = weight
               self.age = age
           def birthyear(self): # method
               import datetime
               now = datetime.datetime.now()
               return now.year - self.age
In [8]: dog = Animal('Dog', 18, 4) # Instance
       print(f' {dog.name}: {dog.weight} Kg, {dog.age} years')
       dog.birthyear()
Dog: 18 Kg, 4 years
Out[8]: 2016
In [9]: dog.age = 7
       dog.birthyear()
Out[9]: 2013
   • dog is an instance of Animal Class.
```

- dog.birthdate() is a *method* of Animal instance dog.
- name and weight are attributes of Animal instance dog.

21.3 Convert method to attribute

Use the property decorator

```
In [10]: class Animal:
             "A simple example class Animal with its name, weight and age"
             def __init__(self, name, weight, age): # constructor
                 self.name = name
```

```
self.weight = weight
self.age = age

@property
def birthyear(self): # method
    import datetime
    now = datetime.datetime.now()
    return now.year - self.age

In [11]: dog = Animal('Dog', 18, 4)
    dog.birthyear # birthyear can now be used as an attribute

Out[11]: 2016

In [12]: dog

Out[12]: <__main__.Animal at Ox7ff8886f98b0>
```

21.4 The new Python 3.7 DataClass

21.5 Method Overriding

• Every Python classes has a __repr__() method used when you call print() function.

```
In [15]: class Animal:
    """Simple example class with method overriding """

def __init__(self, name, weight, age):
    self.name = name
    self.weight = weight
    self.age = age

def __repr__(self):
    return f"{self.__class__.__name__}{({self.name}, {self.weight}, {self.age})"}

@property
def birthyear(self):
    import datetime
    now = datetime.datetime.now()
    return now.year - self.age
```

21.6 Inheritance

Out[17]: 2019

21.6.1 Exercise: Grocery list item

Let's create a class representing a grocery list. First we need a class to represent an item of this grocery list: - The GroceryItem class has seven attributes: - name (string) - price (double) - category (string) - vat_percentage (double) - quantity (integer) - ingredients (list of strings)

- The item class has two methods
 - get_total_vat returns the VAT value.
 - get_total_price returns the total price.

Implement the GroceryItem class and override the __repr__() method by returning the item name and its quantity.

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21.6.2 Exercice: Grocery list

Now implement the GroceryList containing GroceryItem defined above. In this class, add these functions:

- items_with_meat() return a list of items of 'Meat' category.
- prices_with_vat() return a dict with item names as keys and prices as values.
- ingredients_list() return a set of all ingredients contained in items.
- total invoice() return the total price of the shopping list.
- total_for(category) return the total price for a category
- price_by_category() return a dict with category as key and the price as value.
- total_vat() return the total VAT amount.
- top_ingredients(n) ranks the n most frequently founded ingredients

All articles names : ['Beef', 'Pork', 'Tomato Sauce', 'Beans', 'Tuna']

• all_item_names() return a list of item names

```
print(f"Articles with meat are : {shopping_list.items_with_meat()}")
print(f"Full prices are : {shopping_list.prices_with_vat()}")
print(f"Ingredients : {shopping_list.ingredients_list()}")
print(f"Total : {shopping_list.total_invoice()}")
print(f"Total for meat category : {shopping_list.total_for('Meat')}")
print(f"Prices by category : {shopping_list.price_by_category()}")
print(f"VAT amount : {shopping_list.total_vat()}")
print(f"First three ingedients : {shopping_list.top_ingredients(3)}")
print(f"All articles names : {shopping_list.all_item_names()}")
Articles with meat are : [Beef x 2, Pork x 1]
Full prices are : {'Beef': 27.06, 'Pork': 8.34, 'Tomato Sauce': 6.60, 'Beans': 17.32, 'Tuna': 7.19}
Ingredients: {'Tomato', 'Preservatives', 'Fish', 'Sugar', 'Water', 'Beef', 'Salt', 'Beans', 'Oil', 'Po
Total : 66.53
Total for meat category: 35.40
Prices by category : {'Meat': 35.40, 'Can': 31.125000000000004}
VAT amount: 6.59
First three ingedients : ['Water', 'Salt', 'Beef']
```

Private Variables and Methods

```
In [18]: class DemoClass:
            " Demo class for name mangling "
            def public_method(self):
                return 'public!'
            def __private_method(self): # Note the use of leading underscores
                return 'private!'
        object3 = DemoClass()
In [19]: object3.public_method()
Out[19]: 'public!'
In [20]: object3.__private_method()
        AttributeError
                                                     Traceback (most recent call last)
        <ipython-input-20-5a4a6ba27511> in <module>
    ----> 1 object3.__private_method()
        AttributeError: 'DemoClass' object has no attribute '__private_method'
In [21]: [ s for s in dir(object3) if "method" in s]
Out[21]: ['_DemoClass__private_method', 'public_method']
In [22]: object3._DemoClass__private_method()
Out[22]: 'private!'
In [23]: object3.public_method
Out[23]: <bound method DemoClass.public_method of <__main__.DemoClass object at 0x7ff888602e50>>
```

22.1 Use class as a Function.

```
In [24]: class Polynomial:
    " Class representing a polynom P(x) -> c_0+c_1*x+c_2*x^2+..."

    def __init__(self, coeffs):
        self.coeffs = coeffs

    def __call__(self, x):
        return sum([coef*x**exp for exp,coef in enumerate(self.coeffs)])

    p = Polynomial([2,4,-1])
    p(2)
Out [24]: 6
```

22.1.1 Exercise: Polynomial

- Improve the class above called Polynomial by creating a method diff(n) to compute the nth derivative.
- Override the __repr__() method to output a pretty printing.

Hint: f''(coeff:+d)'' forces to print sign before the value of an integer.

Operators Overriding

```
In [25]: class MyComplex:
            " Simple class representing a complex"
            width = 7
            precision = 3
            def __init__(self, real=0, imag=0):
                self.real = real
                self.imag = imag
            def __repr__(self):
                return (f"({self.real:{self.width}.{self.precision}f},"
                        f"{self.imag:+{self.width}.{self.precision}f}j)")
            def __eq__(self, other): # override '=='
                return (self.real == other.real) and (self.imag == other.imag)
            def __add__(self, other): # override '+'
                return MyComplex(self.real+other.real, self.imag+other.imag)
            def __sub__(self, other): # override '-'
                return MyComplex(self.real-other.real, self.imag-other.imag)
            def __mul__(self, other): # override '*'
                if isinstance(other, MyComplex):
                    return MyComplex(self.real * other.real - self.imag * other.imag,
                                     self.real * other.imag + self.imag * other.real)
                else:
                    return MyComplex(other*self.real, other*self.imag)
In [26]: u = MyComplex(0, 1)
        v = MyComplex(1, 0)
        print('u=', u, "; v=", v)
u = (0.000, +1.000j); v = (1.000, +0.000j)
In [27]: u+v, u-v, u*v, u==v
Out[27]: (( 1.000, +1.000j), ( -1.000, +1.000j), ( 0.000, +1.000j), False)
```

We can change the *class* attribute precision.

```
In [28]: MyComplex.precision=2
        print(u.precision)
        print(u)
    0.00, +1.00j
In [29]: v.precision
Out[29]: 2
  We can change the instance attribute precision.
In [30]: u.precision=1
        print(u)
     0.0, +1.0j
In [31]: print(v)
    1.00, +0.00j)
In [32]: MyComplex.precision=5
        u # set attribute keeps its value
Out[32]: (
            0.0,
                    +1.0j)
In [33]: v # unset attribute is set to the new value
Out[33]: (1.00000,+0.00000j)
        Rational example
23.1
In [34]: class Rational:
            " Class representing a rational number"
            def __init__(self, n, d):
                assert isinstance(n, int) and isinstance(d, int)
                def gcd(x, y):
                    if x == 0:
                        return y
                    elif x < 0:
                        return gcd(-x, y)
                    elif y < 0:
                        return -gcd(x, -y)
                    else:
                        return gcd(y % x, x)
                g = gcd(n, d)
                self.numer, self.denom = n//g, d//g
            def __add__(self, other):
                return Rational(self.numer * other.denom + other.numer * self.denom,
                                self.denom * other.denom)
```

23.1.1 Exercise

Improve the class Polynomial by implementing operations: - Overrides '+' operator (add) - Overrides '-' operator (neg) - Overrides '==' operator (eq) - Overrides '*' operator (mul)

Iterators

Most container objects can be looped over using a for statement:

```
In [1]: for element in [1, 2, 3]:
           print(element, end=' ')
1 2 3
In [2]: for element in (1, 2, 3):
           print(element, end=' ')
1 2 3
In [3]: for key in {'one': 1, 'two': 2}:
           print(key, end=' ')
one two
In [4]: for char in "123":
           print(char, end=' ')
1 2 3
In [5]: for line in open("environment.yml"):
           print(line, end= ' ')
        FileNotFoundError
                                                      Traceback (most recent call last)
        <ipython-input-5-e8ea7e33e965> in <module>
    ---> 1 for line in open("environment.yml"):
                 print(line, end= ' ')
        FileNotFoundError: [Errno 2] No such file or directory: 'environment.yml'
  • The for statement calls iter() on the container object.
  • The function returns an iterator object that defines the method __next__()
  • To add iterator behavior to your classes:
       - Define an __iter__() method which returns an object with a __next__().
```

```
- If the class defines __next__(), then __iter__() can just return self.
       - The StopIteration exception indicates the end of the loop.
In [6]: s = 'abc'
       it = iter(s)
        it
Out[6]: <str_iterator at 0x7fd7a0237820>
In [7]: next(it), next(it), next(it)
Out[7]: ('a', 'b', 'c')
In [8]: class Reverse:
            """Iterator for looping over a sequence backwards."""
            def __init__(self, data):
                self.data = data
                self.index = len(data)
            def __iter__(self):
                return self
            def __next__(self):
                if self.index == 0:
                    raise StopIteration
                self.index = self.index - 1
                return self.data[self.index]
In [9]: rev = Reverse('spam')
       for char in rev:
            print(char, end='')
maps
In [10]: def reverse(data): # Python 3.6
             yield from data[::-1]
         for char in reverse('bulgroz'):
             print(char, end='')
zorglub
```

24.1 Generators

- Generators are a simple and powerful tool for creating iterators.
- Write regular functions but use the yield statement when you want to return data.
- the __iter__() and __next__() methods are created automatically.

24.1.1 Exercise

Generates a list of IP addresses based on IP range.

```
ip_range =
for ip in ip_range("192.168.1.0", "192.168.1.10"):
    print(ip)

192.168.1.0
192.168.1.1
192.168.1.2
...
```

24.2 Generator Expressions

- Use a syntax similar to list comprehensions but with parentheses instead of brackets.
- Tend to be more memory friendly than equivalent list comprehensions.

```
In [13]: sum(i*i for i in range(10))
                                                    # sum of squares
Out[13]: 285
In [14]: %load_ext memory_profiler
In [15]: %memit doubles = [2 * n for n in range(10000)]
peak memory: 50.79 MiB, increment: 1.14 MiB
In [16]: %memit doubles = (2 * n for n in range(10000))
peak memory: 50.30 MiB, increment: -0.50 MiB
In [17]: # list comprehension
        doubles = [2 * n for n in range(10)]
        for x in doubles:
            print(x, end=' ')
0 2 4 6 8 10 12 14 16 18
In [18]: # generator expression
        doubles = (2 * n for n in range(10))
        for x in doubles:
            print(x, end=' ')
0 2 4 6 8 10 12 14 16 18
```

24.2.1 Exercise

The Chebyshev polynomials of the first kind are defined by the recurrence relation

$$T_o(x) = 1 (24.1)$$

$$T_1(x) = x \tag{24.2}$$

$$T_{n+1} = 2xT_n(x) - T_{n-1}(x) (24.3)$$

• Create a class Chebyshev that generates the sequence of Chebyshev polynomials

24.3 itertools

24.3.1 zip_longest

itertools.zip_longest() accepts any number of iterables as arguments and a fillvalue keyword argument that defaults to None.

```
In [19]: x = [1, 1, 1, 1, 1]
    y = [1, 2, 3, 4, 5, 6, 7]
    list(zip(x, y))
    from itertools import zip_longest
    list(map(sum,zip_longest(x, y, fillvalue=1)))
Out[19]: [2, 3, 4, 5, 6, 7, 8]
```

24.3.2 combinations

```
In [20]: loto_numbers = list(range(1,50))
```

A choice of 6 numbers from the sequence 1 to 49 is called a combination. The itertools.combinations() function takes two arguments—an iterable inputs and a positive integer n—and produces an iterator over tuples of all combinations of n elements in inputs.

24.3.3 permutations

24.3.4 count

k = 112

```
In [24]: from itertools import count
    n = 2024
    for k in count(): # replace    k = 0; while(True) : k += 1
        if n == 1:
            print(f"k = {k}")
            break
    elif n & 1:
            n = 3*n +1
    else:
            n = n // 2
```

24.4 cycle, islice, dropwhile, takewhile

In [25]: from itertools import cycle, islice, dropwhile, takewhile

```
L = list(range(10))
         cycled = cycle(L) # cycle through the list 'L'
         skipped = dropwhile(lambda x: x < 6 , cycled) # drop the values until x==4
         sliced = islice(skipped, None, 20) # take the first 20 values
         print(*sliced)
6\ 7\ 8\ 9\ 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 0\ 1\ 2\ 3\ 4\ 5
In [26]: result = takewhile(lambda x: x > 0, cycled) # cycled begins to 4
        print(*result)
6 7 8 9
24.4.1 product
In [27]: ranks = ['A', 'K', 'Q', 'J', '10', '9', '8', '7']
         suits = [ '\u2660', '\u2665', '\u2663', '\u2666']
         cards = [(rank, suit) for rank in ranks for suit in suits]
         len(cards)
         from itertools import product
         cards = product(ranks, suits)
         print(*cards)
```

('A', '') ('A', '') ('A', '') ('A', '') ('K', '') ('K', '') ('K', '') ('K', '') ('Q', '')

Multiprocessing

25.1 Map reduce example

```
In [2]: from time import sleep

def delayed_square(x):
    sleep(1)
    return x*x

data = list(range(8))
 data

Out[2]: [0, 1, 2, 3, 4, 5, 6, 7]

In [3]: %time sum(delayed_square(x) for x in data)

CPU times: user 0 ns, sys: 2.83 ms, total: 2.83 ms

Wall time: 8.01 s

Out[3]: 140

In [4]: %time sum(map(delayed_square,data))

CPU times: user 2.54 ms, sys: 605 µs, total: 3.15 ms

Wall time: 8.01 s

Out[4]: 140
```

We can process each delayed_square calls independently and in parallel. To accomplish this we'll apply that function across all list items in parallel using multiple processes.

25.2 Thread and Process: Differences

- A Process is an instance of a running program.
- Process may contain one or more threads, but a thread cannot contain a process.

- Process has a self-contained execution environment. It has its own memory space.
- Application running on your computer may be a set of cooperating processes.
- A Thread is made of and exist within a Process; every process has at least one.
- Multiple threads in a process share resources, which helps in efficient communication between threads.
- Threads can be concurrent on a multi-core system, with every core executing the separate threads simultaneously.

25.3 Multi-Processing vs Multi-Threading

25.3.1 Memory

- Each process has its own copy of the data segment of the parent process.
- Each thread has direct access to the data segment of its process.
- A process runs in separate memory spaces.
- A thread runs in shared memory spaces.

25.3.2 Communication

- Processes must use inter-process communication to communicate with sibling processes.
- Threads can directly communicate with other threads of its process.

25.3.3 Overheads

- Processes have considerable overhead.
- Threads have almost no overhead.

25.4 Multi-Processing vs Multi-Threading

25.4.1 Creation

- New processes require duplication of the parent process.
- New threads are easily created.

25.4.2 Control

- Processes can only exercise control over child processes.
- Threads can exercise considerable control over threads of the same process.

25.4.3 Changes

- Any change in the parent process does not affect child processes.
- Any change in the main thread may affect the behavior of the other threads of the process.

25.5 The Global Interpreter Lock (GIL)

- The Python interpreter is not thread safe.
- A few critical internal data structures may only be accessed by one thread at a time. Access to them
 is protected by the GIL.
- Attempts at removing the GIL from Python have failed until now. The main difficulty is maintaining the C API for extension modules.

- Multiprocessing avoids the GIL by having separate processes which each have an independent copy of the interpreter data structures.
- The price to pay: serialization of tasks, arguments, and results.

25.6 Multiprocessing (history)

- The multiprocessing allows the programmer to fully leverage multiple processors.
- The Pool object parallelizes the execution of a function across multiple input values.
- The if __name__ == '__main__' part is necessary.

 The next program does not work in a cell you need to save it and run with python in a terminal

```
python3 pool.py
In [5]: %%file pool.py
        from time import time, sleep
        from multiprocessing import Pool
        def delayed_square(x):
            sleep(1)
           return x*x
        if __name__ == '__main__': # Executed only on main process.
            start = time()
            data = list(range(8))
            with Pool() as p:
               result = sum(p.map(delayed_square, data))
            stop = time()
           print(f"result = {result} - Elapsed time {stop - start}")
Writing pool.py
In [6]: import sys
        !{sys.executable} pool.py
result = 140 - Elapsed time 4.025667428970337
```

25.7 Futures

The concurrent.futures module provides a high-level interface for asynchronously executing callables.

The asynchronous execution can be performed with threads, using ThreadPoolExecutor, or separate processes, using ProcessPoolExecutor. Both implement the same interface, which is defined by the abstract Executor class.

```
In [7]: %%file process_pool.py
    import os
    from time import time, sleep
    if os.name == "nt":
        from loky import ProcessPoolExecutor # for Windows users
    else:
        from concurrent.futures import ProcessPoolExecutor
        from time import time, sleep
```

```
def delayed_square(x):
           sleep(1)
           return x*x
       if __name__ == "__main__":
           start = time()
           data = list(range(8))
           with ProcessPoolExecutor() as pool:
               result = sum(pool.map(delayed_square, data))
           stop = time()
           print(f" result : {result} - elapsed time {stop - start}")
Writing process_pool.py
In [8]: !{sys.executable} process_pool.py
result: 140 - elapsed time 4.013125658035278
In [9]: %%time
       from concurrent.futures import ThreadPoolExecutor
       e = ThreadPoolExecutor()
       results = list(e.map(delayed_square, range(8)))
CPU times: user 1.65 ms, sys: 3.48 ms, total: 5.14 ms
Wall time: 2 s
```

25.8 Asynchronous Future

While many parallel applications can be described as maps, some can be more complex. In this section we look at the asynchronous Future interface, which provides a simple API for ad-hoc parallelism. This is useful for when your computations don't fit a regular pattern.

25.8.1 Executor.submit

The submit method starts a computation in a separate thread or process and immediately gives us a Future object that refers to the result. At first, the future is pending. Once the function completes the future is finished.

We collect the result of the task with the .result() method, which does not return until the results are available.

- Submit fires off a single function call in the background, returning a future.
- When you combine submit with a single for loop we recover the functionality of map.
- To collect your results, replace each of futures, f, with a call to f.result()
- Combine submit with multiple for loops and other general programming to get something more general than map.
- Sometimes, it did not speed up the code very much
- Threads and processes show some performance differences
- Use threads carefully, you can break your Python session.

Today most library designers are coordinating around the concurrent futures interface, so it's wise to move over.

- Profile your code
- Used concurrent.futures.ProcessPoolExecutor for simple parallelism
- Gained some speed boost (but not as much as expected)
- Lost ability to diagnose performance within parallel code
- Describing each task as a function call helps use tools like map for parallelism
- Making your tasks fast is often at least as important as parallelizing your tasks.

25.8.2 Exercise: Pi computation

Parallelize this computation with a ProcessPoolExecutor. ThreadPoolExecutor is not usable because of random function calls.

```
In [15]: import time
    import random

def compute_pi(n):
    count = 0
    for i in range(n):
        x = random.random()
        y = random.random()
        if x*x + y*y <= 1:
            count += 1
        return count</pre>
```

```
elapsed_time = time.time()
nb_simulations = 4
n = 10**7
result = [compute_pi(n) for i in range(nb_simulations)]
pi = 4 * sum(result) / (n*nb_simulations)
print(f"Estimated value of Pi : {pi:.8f} time : {time.time()-elapsed_time:.8f}")
```

Estimated value of Pi : 3.14118350 time : 13.81696153

25.8.3 Exercise

- $\bullet\,$ Do the same computation using a synchronous future
- Implement a joblib version (see example below)

Joblib (bonus)

Joblib provides a simple helper class to write parallel for loops using multiprocessing.

Standard Library

27.1 Operating System Interface

```
In [1]: import os
                        # Return the current working directory
       os.getcwd()
Out[1]: '/home/runner/work/python-notebooks/python-notebooks/notebooks'
In [2]: %env CC='/usr/local/bin/gcc-7'
        os.environ['CC']='/usr/local/bin/gcc-7' # Change the default C compiler to gcc-7
       os.system('mkdir today') # Run the command mkdir in the system shell
env: CC='/usr/local/bin/gcc-7'
Out[2]: 0
In [3]: os.chdir('today')
                          # Change current working directory
       os.system('touch data.db') # Create the empty file data.db
Out[3]: 0
In [4]: import shutil
       shutil.copyfile('data.db', 'archive.db')
       if os.path.exists('backup.db'): # If file backup.db exists
                                        # Remove it
            os.remove('backup.db')
       shutil.move('archive.db', 'backup.db',)
        shutil.os.chdir('...')
```

27.2 File Wildcards

The glob module provides a function for making file lists from directory wildcard searches:

```
replace "pattern" by "replace"
"""

for directory, subdirs, filenames in os.walk( root ):
    for filename in filenames:
        path = os.path.join( directory, filename )
        text = open( path ).read()
        if pattern in text:
        print('occurence in :' + filename)
        open(path,'w').write( text.replace( pattern, replace ) )
```

```
27.3
        Command Line Arguments
These arguments are stored in the sys module's argy attribute as a list.
  writefile magic_args="-a demo.py" slideshow={"slide_type": "fragment"} import sys
print(sys.argv)
In [7]: %run demo.py one two three
                                                  Traceback (most recent call last)
       OSError
       /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py
       702
                        fpath = arg_lst[0]
    --> 703
                        filename = file_finder(fpath)
       704
                    except IndexError:
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/utils/path.py in get_py_f
       108
    --> 109
                    raise IOError('File `%r` not found.' % name)
        110
       OSError: File `'demo.py'` not found.
   During handling of the above exception, another exception occurred:
       Exception
                                                  Traceback (most recent call last)
        <ipython-input-7-916df5e5b25f> in <module>
    ----> 1 get_ipython().run_line_magic('run', 'demo.py one two three')
       /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/interactiveshell.py
       2324
                            kwargs['local_ns'] = self.get_local_scope(stack_depth)
       2325
                        with self.builtin_trap:
    -> 2326
                            result = fn(*args, **kwargs)
       2327
                        return result
       2328
```

27.4. RANDOM 99

```
<decorator-gen-59> in run(self, parameter_s, runner, file_finder)
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magic.py in <lambda>
                # but it's overkill for just that one bit of state.
        186
                def magic_deco(arg):
    --> 187
                    call = lambda f, *a, **k: f(*a, **k)
        188
        189
                    if callable(arg):
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py
                        if os.name == 'nt' and re.match(r"^'.*',fpath):
       713
                            warn('For Windows, use double quotes to wrap a filename: %run "mypath\\myfi
    --> 714
                        raise Exception(msg)
        715
                    except TypeError:
        716
                        if fpath in sys.meta_path:
        Exception: File `'demo.py'` not found.
27.4 Random
In [8]: import random
       random.choice(['apple', 'pear', 'banana'])
Out[8]: 'banana'
In [9]: random.sample(range(100), 10) # sampling without replacement
Out[9]: [19, 31, 63, 15, 23, 61, 41, 68, 18, 92]
In [10]: random.random()
                        # random float
Out[10]: 0.12773468027453005
In [11]: random.randrange(6) # random integer chosen from range(6)
Out[11]: 4
27.5 Statistics
In [12]: import statistics
        data = [2.75, 1.75, 1.25, 0.25, 0.5, 1.25, 3.5]
        statistics.mean(data)
Out[12]: 1.6071428571428572
In [13]: statistics.median(data)
Out[13]: 1.25
In [14]: statistics.variance(data)
```

Out[14]: 1.3720238095238095

27.6 Performance Measurement

The profile and pstats modules provide tools for identifying time critical sections in larger blocks of code.

27.7 Quality Control

One approach for developing high quality software is to write tests for each function.

- The doctest module provides a tool for scanning a module and validating tests embedded in a program's
 docstrings.
- This improves the documentation by providing the user with an example and it allows the doctest module to make sure the code remains true to the documentation:

27.8 Python's standard library is very extensive

- Containers and iterators: collections, itertools
- Internet access: urllib, email, mailbox, cgi, ftplib
- Dates and Times: datetime, calendar,
- Data Compression: zlib, gzip, bz2, lzma, zipfile, tarfile
- File formats: csv, configparser, netrc, xdrlib, plistlib
- Cryptographic Services: hashlib, hmac, secrets
- Structure Markup Processing Tools: html, xml

Check the The Python Standard Library

Getting Started with matplotlib

- Python 2D plotting library which produces figures in many formats and interactive environments.
- Tries to make easy things easy and hard things possible.
- You can generate plots, histograms, power spectra, bar charts, errorcharts, scatterplots, etc., with just a few lines of code.
- Check the Matplotlib gallery.
- For simple plotting the pyplot module provides a MATLAB-like interface, particularly when combined with IPython.
- Matplotlib provides a set of functions familiar to MATLAB users.

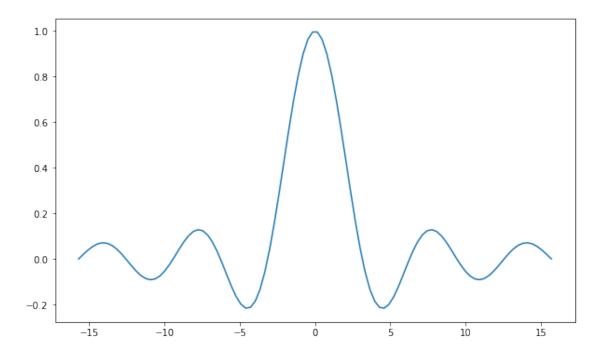
In this notebook we use some numpy command that will be explain more precisely later.

28.1 Line Plots

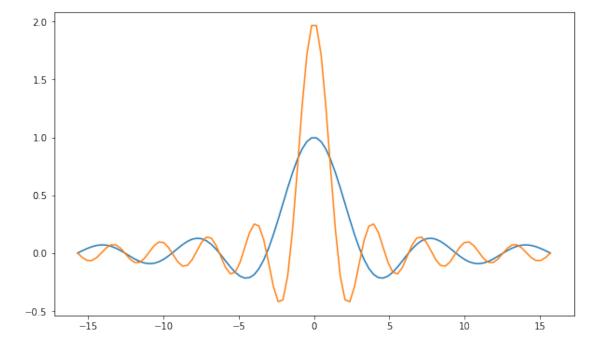
• np.linspace(0,1,10) return 10 evenly spaced values over [0,1].

```
In [1]: %matplotlib inline
    # inline can be replaced by notebook to get interactive plots
    import numpy as np
    import matplotlib.pyplot as plt

In [2]: plt.rcParams['figure.figsize'] = (10.0, 6.0) # set figures display bigger
    x = np.linspace(- 5*np.pi,5*np.pi,100)
    plt.plot(x,np.sin(x)/x);
```



In [3]: plt.plot(x,np.sin(x)/x,x,np.sin(2*x)/x);



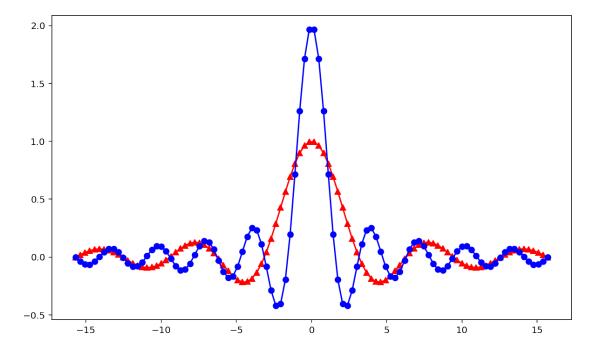
If you have a recent Macbook with a Retina screen, you can display high-resolution plot outputs. Running the next cell will give you double resolution plot output for Retina screens.

Note: the example below won't render on non-retina screens

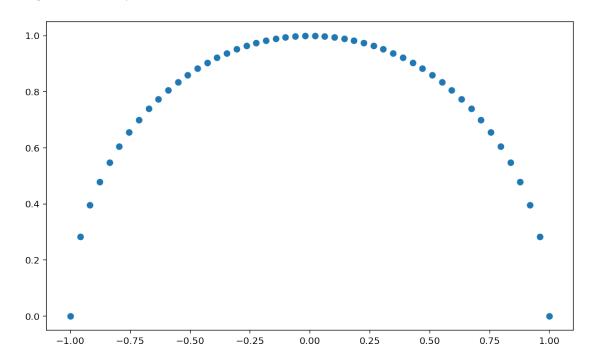
In [4]: %config InlineBackend.figure_format = 'retina'

28.1. LINE PLOTS 103

In [5]: # red, dot-dash, triangles and blue, dot-dash, bullet $plt.plot(x,np.sin(x)/x, 'r-^{-},x,np.sin(2*x)/x, 'b-o');$



Simple Scatter Plot



Colormapped Scatter Plot

-10

_5

-15

```
In [7]: theta = np.linspace(0,6*np.pi,50) # 50 steps from 0 to 6 PI
        size = 30*np.ones(50) # array with 50 values set to 30
        z = np.random.rand(50) # array with 50 random values in [0,1]
        x = theta*np.cos(theta)
        y = theta*np.sin(theta)
        plt.scatter(x,y,size,z)
        plt.colorbar();
       15
       10
                                                                                             0.8
         5
                                                                                            - 0.6
        0
                                                                                            0.4
       -5
      -10
                                                                                            - 0.2
      -15 -
```

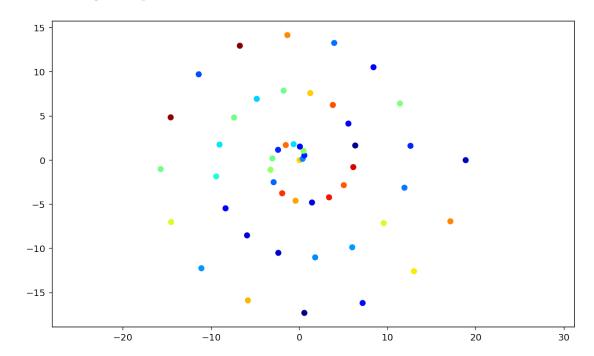
10

15

20

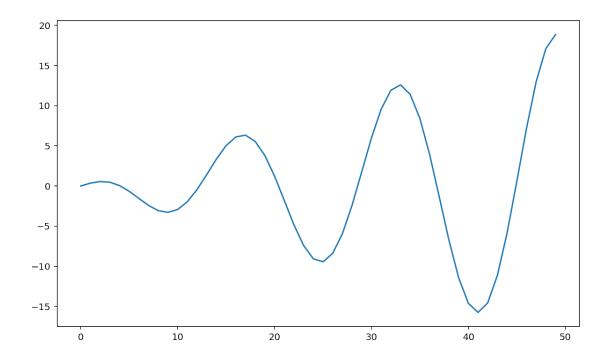
Change Colormap

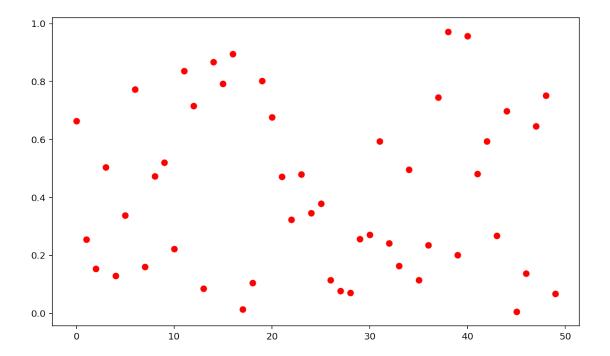
```
In [8]: fig = plt.figure() # create a figure
    ax = fig.add_subplot(1, 1, 1) # add a single plot
    ax.scatter(x,y,size,z,cmap='jet');
    ax.set_aspect('equal', 'datalim')
```



colormaps in matplotlib documentation.

Multiple Figures





10

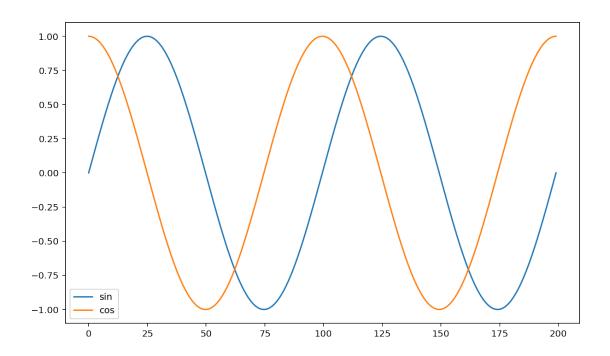
Multiple Plots Using subplot

```
In [10]: plt.subplot(1,2,1) # 1 row 1, 2 columns, active plot number 1
plt.plot(x,'b-*')
plt.subplot(1,2,2) # 1 row 1, 2 columns, active plot number 2
plt.plot(z,'ro');

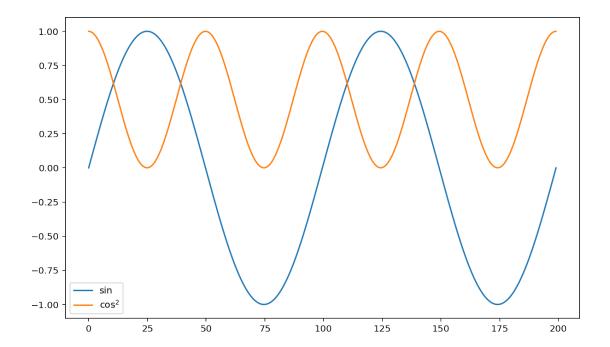
20
15
0-
0-
-5-
-10
-15-
0.0-
```

Legends

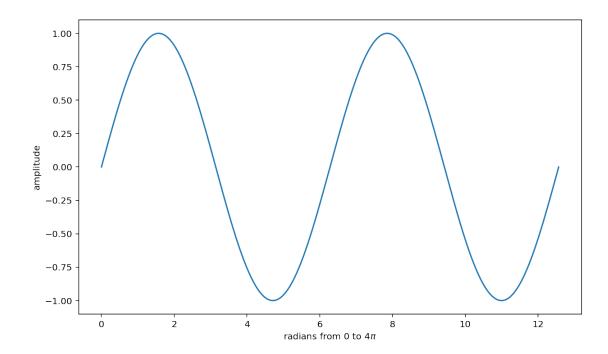
• Legends labels with plot



• Labelling with legend



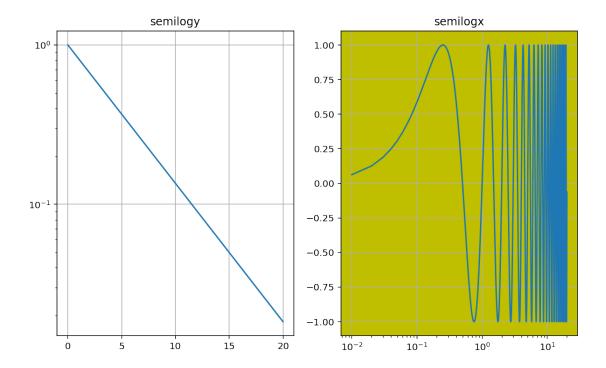
Titles and Axis Labels



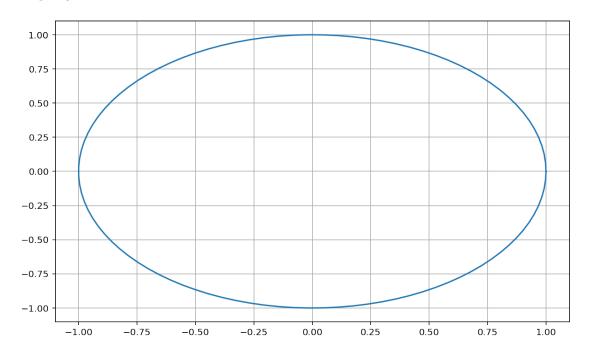
```
In [14]: t = np.arange(0.01, 20.0, 0.01)

    plt.subplot(121)
    plt.semilogy(t, np.exp(-t/5.0))
    plt.title('semilogy')
    plt.grid(True)

    plt.subplot(122,fc='y')
    plt.semilogx(t, np.sin(2*np.pi*t))
    plt.title('semilogx')
    plt.grid(True)
```



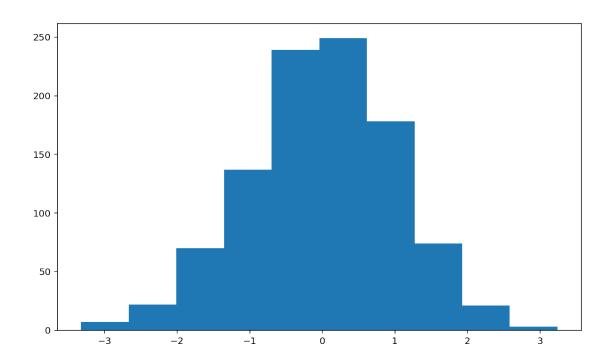
Plot Grid and Save to File



<Figure size 720x432 with 0 Axes>

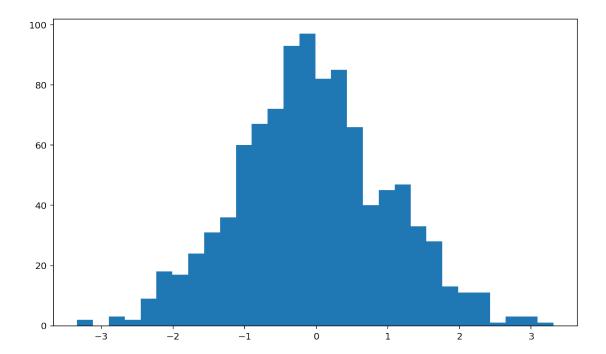
Histogram

```
In [17]: from numpy.random import randn
          plt.hist(randn(1000));
```



Change the number of bins and supress display of returned array with;

```
In [18]: plt.hist(randn(1000), 30);
```



Contour Plot

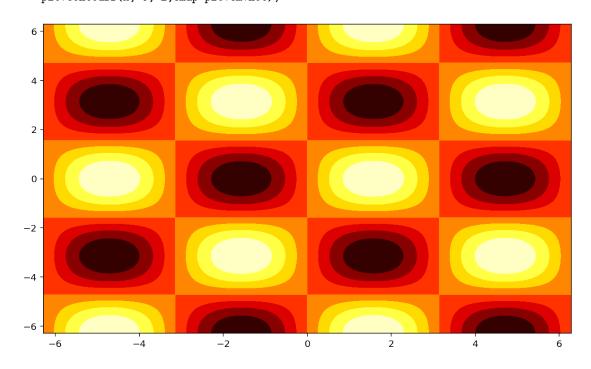


Image Display

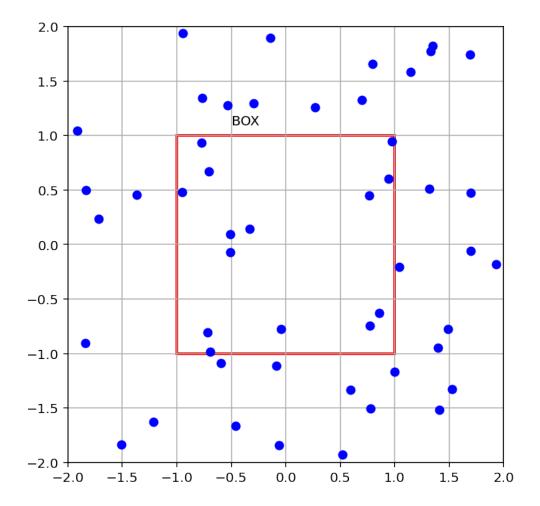
```
In [20]: img = plt.imread("https://hackage.haskell.org/package/JuicyPixels-extra-0.1.0/src/data-examples/lenna.png"
        plt.imshow(img)
       UnsupportedOperation
                                                   Traceback (most recent call last)
        <ipython-input-20-c27aa37b6b18> in <module>
    ----> 1 img = plt.imread("https://hackage.haskell.org/package/JuicyPixels-extra-0.1.0/src/data-exam
          2 plt.imshow(img)
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/matplotlib/pyplot.py in imread(fn
       2227 @_copy_docstring_and_deprecators(matplotlib.image.imread)
       2228 def imread(fname, format=None):
    -> 2229
                return matplotlib.image.imread(fname, format)
       2230
       2231
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/matplotlib/image.py in imread(fna
       1473
                        from urllib import request
       1474
                        with urllib.request.urlopen(fname) as response:
    -> 1475
                            return imread(response, format=ext)
                with img_open(fname) as image:
       1476
       1477
                    return (_pil_png_to_float_array(image)
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/matplotlib/image.py in imread(fna
       1474
                        with urllib.request.urlopen(fname) as response:
       1475
                            return imread(response, format=ext)
    -> 1476
                with img_open(fname) as image:
       1477
                    return (_pil_png_to_float_array(image)
       1478
                            if isinstance(image, PIL.PngImagePlugin.PngImageFile) else
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/PIL/ImageFile.py in __init__(self
        116
                        try:
```

```
--> 117
                        self._open()
    118
                   except (
   119
                        IndexError, # end of data
    /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/PIL/PngImagePlugin.py in _open(se
                    # get next chunk
   652
--> 653
                   cid, pos, length = self.png.read()
   654
    655
                    try:
   /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/PIL/PngImagePlugin.py in read(sel
   126
                    s = self.fp.read(8)
   127
                   cid = s[4:]
--> 128
                   pos = self.fp.tell()
   129
                    length = i32(s)
    130
```

UnsupportedOperation: seek

39.1 figure and axis

Best method to create a plot with many components



Exercise

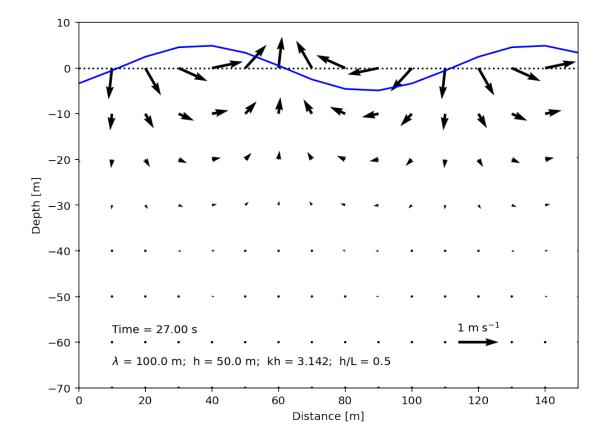
Recreate the image my_plots.png using the $delicate_arch.png$ file in images directory.

Alternatives

• bqplot : Jupyter Notebooks, Interactive.

```
• seaborn : Statistics.
       • toyplot : Nice graphes.
       • bokeh: Interactive and Server mode.
       • pygal : Charting
       • Altair : Data science
       • plot.ly: Data science and interactive
       • Mayavi: 3D
       • YT: Astrophysics (volume rendering, contours, particles).
       • VisIt: Powerful, easy to use but heavy.
       • Paraview: The most-used visualization application. Need high learning effort.
In [22]: #example from Filipe Fernandes
                       \verb| #http://nbviewer.jupyter.org/gist/ocefpaf/9730c697819e91b99f1d694983e39a8f| | for the first of the first
                      import numpy as np
                       import matplotlib.pyplot as plt
                      from matplotlib import animation
                      g = 9.81
                      denw = 1025.0  # Seawater density [kg/m**3].
                      sig = 7.3e-2 # Surface tension [N/m].
                      a = 1.0 # Wave amplitude [m].
                      L, h = 100.0, 50.0 # Wave height and water column depth.
                      k = 2 * np.pi / L
                      omega = np.sqrt((g * k + (sig / denw) * (k**3)) * np.tanh(k * h))
                      T = 2 * np.pi / omega
                      c = np.sqrt((g / k + (sig / denw) * k) * np.tanh(k * h))
                       # We'll solve the wave velocities in the `x` and `z` directions.
                      x, z = np.meshgrid(np.arange(0, 160, 10), np.arange(0, -80, -10),)
                      u, w = np.zeros_like(x), np.zeros_like(z)
                      def compute_vel(phase):
                                u = a * omega * (np.cosh(k * (z+h)) / np.sinh(k*h)) * np.cos(k * x - phase)
                                 w = a * omega * (np.sinh(k * (z+h)) / np.sinh(k*h)) * np.sin(k * x - phase)
                                 mask = -z > h
                                u[mask] = 0.0
                                 w[mask] = 0.0
                                 return u, w
```

```
def basic_animation(frames=91, interval=30, dt=0.3):
    fig = plt.figure(figsize=(8, 6))
    ax = plt.axes(xlim=(0, 150), ylim=(-70, 10))
    # Animated.
    quiver = ax.quiver(x, z, u, w, units='inches', scale=2)
    ax.quiverkey(quiver, 120, -60, 1,
                 label=r'1 m s\$^{-1}\$',
                 coordinates='data')
    line, = ax.plot([], [], 'b')
    \# Non-animated.
    ax.plot([0, 150], [0, 0], 'k:')
    ax.set_ylabel('Depth [m]')
    ax.set_xlabel('Distance [m]')
    text = (r'\$\absta = \%s m; h = \%s m; kh = \%2.3f; h/L = \%s' \%
            (L, h, k * h, h/L))
    ax.text(10, -65, text)
    time_step = ax.text(10, -58, '')
    line.set_data([], [])
    def init():
       return line, quiver, time_step
    def animate(i):
       time = i * dt
       phase = omega * time
       eta = a * np.cos(x[0] * k - phase)
       u, w = compute_vel(phase)
        quiver.set_UVC(u, w)
       line.set_data(x[0], 5 * eta)
        time_step.set_text('Time = {:.2f} s'.format(time))
        return line, quiver, time_step
    return animation.FuncAnimation(fig, animate, init_func=init,
                                   frames=frames, interval=interval)
```



References

- Simple examples with increasing difficulty https://matplotlib.org/examples/index.html
- Gallery https://matplotlib.org/gallery.html
- A matplotlib tutorial, part of the Lectures on Scientific Computing with Python by J.R. Johansson.
- Num Py Beginner | Sci Py 2016 Tutorial | Alexandre Chabot Le
Clerc
- matplotlib tutorial by Nicolas Rougier from LORIA.
- 10 Useful Python Data Visualization Libraries for Any Discipline

What provide Numpy to Python?

- ndarray multi-dimensional array object
- derived objects such as masked arrays and matrices
- ufunc fast array mathematical operations.
- Offers some Matlab-ish capabilities within Python
- Initially developed by Travis Oliphant.
- Numpy 1.0 released October, 2006.
- The SciPy.org website is very helpful.
- NumPy fully supports an object-oriented approach.

43.1 Routines for fast operations on arrays.

```
shape manipulation
sorting
I/O
FFT
basic linear algebra
basic statistical operations
random simulation
statistics
```

- and much more...

43.2 Getting Started with NumPy

• It is handy to import everything from NumPy into a Python console:

```
from numpy import *
```

• But it is easier to read and debug if you use explicit imports.

43.3 Why Arrays?

- Python lists are slow to process and use a lot of memory.
- For tables, matrices, or volumetric data, you need lists of lists of lists... which becomes messy to program.

43.4 Numpy Arrays: The ndarray class.

- There are important differences between NumPy arrays and Python lists:
 - NumPy arrays have a fixed size at creation.
 - NumPy arrays elements are all required to be of the same data type.
 - NumPy arrays operations are performed in compiled code for performance.
- Most of today's scientific/mathematical Python-based software use NumPy arrays.
- NumPy gives us the code simplicity of Python, but the operation is speedily executed by pre-compiled C code.

```
In [5]: a = np.array([0,1,2,3]) # list
    b = np.array((4,5,6,7)) # tuple
    c = np.matrix('8 9 0 1') # string (matlab syntax)
In [6]: print(a,b,c)
[0 1 2 3] [4 5 6 7] [[8 9 0 1]]
```

43.5 Element wise operations are the "default mode"

```
In [7]: a*b,a+b
Out[7]: (array([ 0,  5, 12, 21]), array([ 4,  6,  8, 10]))
In [8]: 5*a, 5+a
Out[8]: (array([ 0,  5, 10, 15]), array([5, 6, 7, 8]))
In [9]: a @ b, np.dot(a,b) # Matrix multiplication
Out[9]: (38, 38)
```

NumPy Arrays Properties

```
In [10]: a = np.array([1,2,3,4,5]) # Simple array creation
In [11]: type(a) # Checking the type
Out[11]: numpy.ndarray
In [12]: a.dtype # Print numeric type of elements
Out[12]: dtype('int64')
In [13]: a.itemsize # Print Bytes per element
Out[13]: 8
In [14]: a.shape # returns a tuple listing the length along each dimension
Out[14]: (5,)
In [15]: np.size(a), a.size # returns the entire number of elements.
Out[15]: (5, 5)
In [16]: a.ndim # Number of dimensions
Out[16]: 1
In [17]: a.nbytes # Memory used
Out[17]: 40
  • ** Always use shape or size for numpy arrays instead of len **
   • len gives same information only for 1d array.
```

44.1 Functions to allocate arrays

Setting Array Elements Values

Setting Array Elements Types

46.1 Slicing x[lower:upper:step]

- Extracts a portion of a sequence by specifying a lower and upper bound.
- The lower-bound element is included, but the upper-bound element is **not** included.
- The default step value is 1 and can be negative.

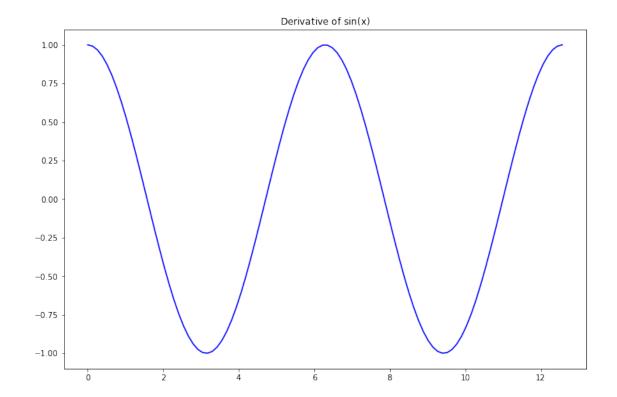
```
In [26]: a = np.array([10,11,12,13,14])
In [27]: a[:2], a[-5:-3], a[0:2], a[-2:] # negative indices work
Out[27]: (array([10, 11]), array([10, 11]), array([10, 11]), array([13, 14]))
In [28]: a[::2], a[::-1]
Out[28]: (array([10, 12, 14]), array([14, 13, 12, 11, 10]))
```

46.1.1 Exercise:

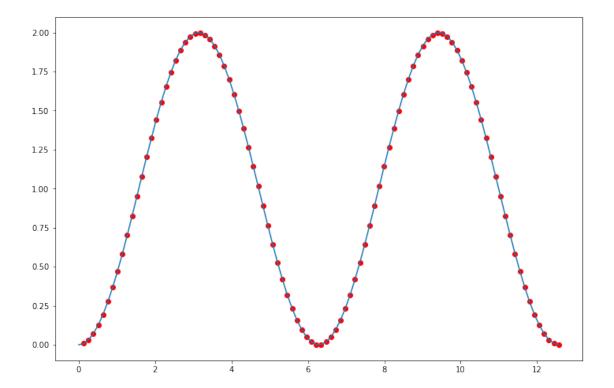
• Compute derivative of $f(x) = \sin(x)$ with finite difference method.

$$\frac{\partial f}{\partial x} \sim \frac{f(x+dx) - f(x)}{dx}$$

derivatives values are centered in-between sample points.



```
In [31]: # Compute integral of x numerically
    avg_height = 0.5*(y[1:]+y[:-1])
    int_sin = np.cumsum(dx*avg_height)
    plt.plot(x[1:], int_sin, 'ro', x, np.cos(0)-np.cos(x));
```



Multidimensional array

```
In [32]: a = np.arange(4*3).reshape(4,3) # NumPy array
        1 = [[0,1,2],[3,4,5],[6,7,8],[9,10,11]] # Python List
In [33]: print(a)
        print(1)
[[ 0 1 2]
[3 4 5]
[6 7 8]
[ 9 10 11]]
[[0, 1, 2], [3, 4, 5], [6, 7, 8], [9, 10, 11]]
In [34]: 1[-1][-1] # Access to last item
Out[34]: 11
In [35]: print(a[-1,-1]) # Indexing syntax is different with NumPy array
                         # returns the first item
        print(a[0,0])
        print(a[1,:])
                         # returns the second line
11
[3 \ 4 \ 5]
In [36]: print(a[1]) # second line with 2d array
        print(a[:,-1]) # last column
[3 \ 4 \ 5]
[2 5 8 11]
```

47.0.1 Exercise

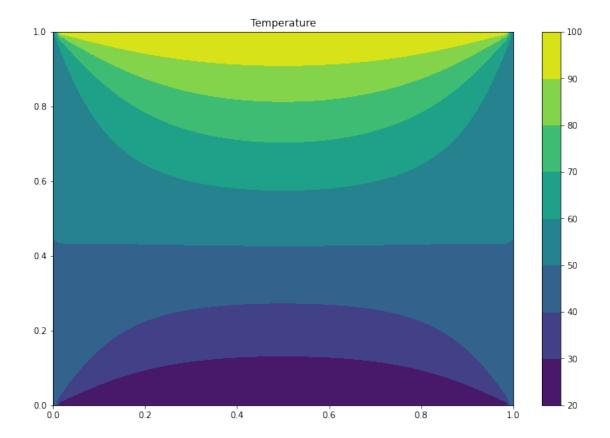
- We compute numerically the Laplace Equation Solution using Finite Difference Method
- Replace the computation of the discrete form of Laplace equation with numpy arrays

$$T_{i,j} = \frac{1}{4} (T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1})$$

• The function numpy.allclose can help you to compute the residual.

```
In [37]: %%time
        # Boundary conditions
        Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50
        # Set meshgrid
        n, 1 = 64, 1.0
        X, Y = np.meshgrid(np.linspace(0,1,n), np.linspace(0,1,n))
        T = np.zeros((n,n))
        # Set Boundary condition
        T[n-1:, :] = Tnorth
        T[:1, :] = Tsouth
        T[:, n-1:] = Teast
        T[:, :1] = Twest
        residual = 1.0
        istep = 0
        while residual > 1e-5:
            istep += 1
            print ((istep, residual), end="\r")
            residual = 0.0
            for i in range(1, n-1):
                for j in range(1, n-1):
                    T_old = T[i,j]
                    T[i, j] = 0.25 * (T[i+1,j] + T[i-1,j] + T[i,j+1] + T[i,j-1])
                        residual=max(residual,abs((T_old-T[i,j])/T[i,j]))
        print()
        print("iterations = ",istep)
        plt.title("Temperature")
        plt.contourf(X, Y, T)
        plt.colorbar()
```

```
(2457, 1.0022293826789268e-05)
iterations = 2457
CPU times: user 39.3 s, sys: 599 ms, total: 39.9 s
Wall time: 39.4 s
```



Arrays to ASCII files

Arrays from ASCII files

- save: Save an array to a binary file in NumPy .npy format
- savez : Save several arrays into an uncompressed .npz archive
- savez_compressed: Save several arrays into a compressed .npz archive
- load: Load arrays or pickled objects from .npy, .npz or pickled files.

49.1 H5py

Pythonic interface to the HDF5 binary data format. h5py user manual

<ipython-input-43-7b8ae0576ec5>:3: H5pyDeprecationWarning: dataset.value has been deprecated. Use datas
print(field+':',f[field].value)

Slices Are References

- Slices are references to memory in the original array.
- Changing values in a slice also changes the original array.

```
In [44]: a = np.arange(10)
    b = a[3:6]
    b # `b` is a view of array `a` and `a` is called base of `b`

Out[44]: array([3, 4, 5])
In [45]: b[0] = -1
    a # you change a view the base is changed.

Out[45]: array([0, 1, 2, -1, 4, 5, 6, 7, 8, 9])
    • Numpy does not copy if it is not necessary to save memory.
In [46]: c = a[7:8].copy() # Explicit copy of the array slice c[0] = -1
    a

Out[46]: array([0, 1, 2, -1, 4, 5, 6, 7, 8, 9])
```

Fancy Indexing

```
In [47]: a = np.fromfunction(lambda i, j: (i+1)*10+j, (4, 5), dtype=int)
Out[47]: array([[10, 11, 12, 13, 14],
                 [20, 21, 22, 23, 24],
                 [30, 31, 32, 33, 34],
                [40, 41, 42, 43, 44]])
In [48]: np.random.shuffle(a.flat) # shuffle modify only the first axis
Out[48]: array([[14, 11, 43, 34, 20],
                [44, 42, 32, 41, 30],
                [22, 21, 24, 31, 10],
                [23, 33, 40, 13, 12]])
In [49]: locations = a \% 3 == 0 # locations can be used as a mask
        a[locations] = 0 #set to 0 only the values that are divisible by 3
Out[49]: array([[14, 11, 43, 34, 20],
                 [44, 0, 32, 41, 0],
                 [22, 0, 0, 31, 10],
                [23, 0, 40, 13, 0]])
In [50]: a += a == 0
Out[50]: array([[14, 11, 43, 34, 20],
                 [44, 1, 32, 41, 1],
                 [22, 1, 1, 31, 10],
                [23, 1, 40, 13, 1]])
51.0.1 numpy.take
In [51]: a[1:3,2:5]
Out[51]: array([[32, 41, 1],
                [ 1, 31, 10]])
In [52]: np.take(a,[[6,7],[10,11]]) # Use flatten array indices
Out[52]: array([[ 1, 32],
                 [22, 1]])
```

Changing array shape

Sorting

Transpose-like operations

```
In [61]: a = np.array([5,3,6,1,6,7,9,0,8])
        b.shape = (3,3) # b is a reference so a will be changed
In [62]: a
Out[62]: array([[5, 3, 6],
                [1, 6, 7],
                [9, 0, 8]])
In [63]: c = a.T # Return a view so a is not changed
       np.may_share_memory(a,c)
Out[63]: True
In [64]: c[0,0] = -1 # c is stored in same memory so change c you change a
Out[64]: array([[-1, 3, 6],
                [1, 6, 7],
                [9, 0, 8]])
In [65]: c # is a transposed view of a
Out[65]: array([[-1, 1, 9],
                [3, 6, 0],
                [6, 7, 8]])
In [66]: b # b is a reference to a
Out[66]: array([[-1, 3, 6],
                [1, 6, 7],
                [9, 0, 8]])
In [67]: c.base # When the array is not a view `base` return None
Out[67]: array([[-1, 3, 6],
                [1, 6, 7],
                [9, 0, 8]])
```

Methods Attached to NumPy Arrays

```
In [68]: a = np.arange(20).reshape(4,5)
        np.random.shuffle(a.flat)
Out[68]: array([[12, 1, 9, 5, 15],
                [18, 3, 7, 6, 2],
                [4, 16, 10, 14, 11],
                [17, 8, 0, 13, 19]])
In [69]: a = (a - a.mean())/ a.std() # Standardize the matrix
        print(a)
[[ 0.43355498 -1.47408695 -0.086711
                                       -0.78039897 0.95382097]
 [ 1.47408695 -1.12724296 -0.43355498 -0.60697698 -1.30066495]
 [-0.95382097 1.12724296 0.086711
                                        0.78039897 0.26013299]
 [ 1.30066495 -0.26013299 -1.64750894   0.60697698   1.64750894]]
In [70]: np.set_printoptions(precision=4)
        print(a)
[[ 0.4336 -1.4741 -0.0867 -0.7804 0.9538]
 [ 1.4741 -1.1272 -0.4336 -0.607 -1.3007]
 [-0.9538 1.1272 0.0867 0.7804 0.2601]
 [ 1.3007 -0.2601 -1.6475 0.607
                                    1.6475]]
In [71]: a.argmax() # max position in the memory contiguous array
Out[71]: 19
In [72]: np.unravel_index(a.argmax(),a.shape) # get position in the matrix
Out[72]: (3, 4)
```

Array Operations over a given axis

```
In [73]: a = np.arange(20).reshape(5,4)
        np.random.shuffle(a.flat)
In [74]: a.sum(axis=0) # sum of each column
Out[74]: array([41, 56, 43, 50])
In [75]: np.apply_along_axis(sum, axis=0, arr=a)
Out[75]: array([41, 56, 43, 50])
In [76]: np.apply_along_axis(sorted, axis=0, arr=a)
Out[76]: array([[ 0, 1, 2, 7],
                 [3, 6, 4, 8],
                [5, 14, 9, 10],
                 [15, 16, 11, 12],
                 [18, 19, 17, 13]])
  You can replace the sorted builtin fonction by a user defined function.
In [77]: np.empty(10)
Out[77]: array([0., 0., 1., 0., 1., 1., 0., 1., 0., 0.])
In [78]: np.linspace(0,2*np.pi,10)
Out [78]: array([0. , 0.6981, 1.3963, 2.0944, 2.7925, 3.4907, 4.1888, 4.8869,
                5.5851, 6.2832])
In [79]: np.arange(0,2.+0.4,0.4)
Out[79]: array([0., 0.4, 0.8, 1.2, 1.6, 2.])
In [80]: np.eye(4)
Out[80]: array([[1., 0., 0., 0.],
                 [0., 1., 0., 0.],
                 [0., 0., 1., 0.],
                 [0., 0., 0., 1.]])
In [81]: a = np.diag(range(4))
```

```
Out[81]: array([[0, 0, 0, 0],
                [0, 1, 0, 0],
                [0, 0, 2, 0],
                [0, 0, 0, 3]])
In [82]: a[:,:,np.newaxis]
Out[82]: array([[[0],
                 [0],
                 [0]],
                [[0],
                 [1],
                 [0],
                 [0]],
                [[0],
                 [0],
                 [2],
                 [0]],
                [[0]]
                 [0],
                 [0],
                 [3]])
56.0.1 Create the following arrays
[100 101 102 103 104 105 106 107 108 109]
  Hint: numpy.arange
[-2. -1.8 -1.6 -1.4 -1.2 -1. -0.8 -0.6 -0.4 -0.2 0.
0.2 0.4 0.6 0.8 1. 1.2 1.4 1.6 1.8]
  Hint: numpy.linspace
           0.00129155 0.0016681 0.00215443 0.00278256
     0.003593810.00464159 0.00599484 0.00774264 0.01]
  Hint: numpy.logspace
[[0.0.-1.-1.-1.]
 [ 0. 0. 0. -1. -1.]
 [ 0. 0. 0. 0. -1. ]
 [ 0. 0. 0. 0. 0.]
 [ 0. 0. 0. 0. 0.]
 [ 0. 0. 0. 0. 0.]
 [ 0. 0. 0. 0. 0.]]
  Hint: numpy.tri, numpy.zeros, numpy.transpose
[[ 0. 1. 2. 3. 4.]
 [-1. 0. 1. 2. 3.]
 [-1. -1. 0. 1. 2.]
 [-1. -1. -1. 0. 1.]
 [-1. -1. -1. -1. 0.]
```

Hint: numpy.ones, numpy.diag

• Compute the integral numerically with Trapezoidal rule

$$I = \int_{-\infty}^{\infty} e^{-v^2} dv$$

with $v \in [-10; 10]$ and n=20.

None

Views and Memory Management

• If it exists one view of a NumPy array, it can be destroyed.

57.1 Change memory alignement

OWNDATA : True WRITEABLE : True ALIGNED : True

WRITEBACKIFCOPY : False UPDATEIFCOPY : False

```
In [87]: b.base is a
```

Out[87]: False

You can also create a fortran array with array function.

Broadcasting rules

Broadcasting rules allow you to make an outer product between two vectors: the first method involves array tiling, the second one involves broadcasting. The last method is significantly faster.

Numpy Matrix

Specialized 2-D array that retains its 2-D nature through operations. It has certain special operators, such as * (matrix multiplication) and ** (matrix power).

```
In [94]: m = np.matrix('1 2; 3 4') #Matlab syntax
Out[94]: matrix([[1, 2],
                  [3, 4]])
In [95]: a = np.matrix([[1, 2],[ 3, 4]]) #Python syntax
Out[95]: matrix([[1, 2],
                  [3, 4]])
In [96]: a = np.arange(1,4)
        b = np.mat(a) # 2D view, no copy!
        b, np.may_share_memory(a,b)
Out[96]: (matrix([[1, 2, 3]]), True)
In [97]: a = np.matrix([[1, 2, 3],[ 3, 4, 5]])
        a * b.T # Matrix vector product
Out[97]: matrix([[14],
                  [26]])
In [98]: m * a # Matrix multiplication
Out[98]: matrix([[ 7, 10, 13],
                  [15, 22, 29]])
```

59.1 StructuredArray using a compound data type specification

59.2 RecordArray

NumPy Array Programming

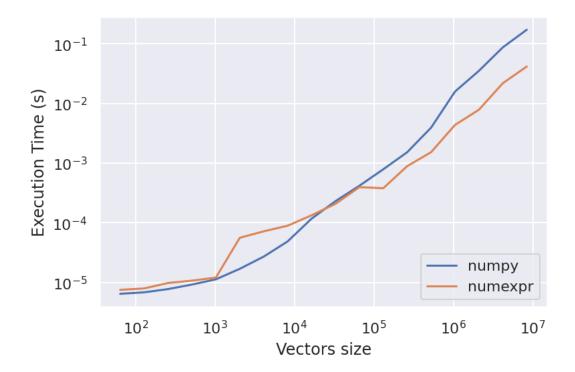
- Array operations are fast, Python loops are slow.
- Top priority: avoid loops
- It's better to do the work three times with array operations than once with a loop.
- This does require a change of habits.
- $\bullet\,$ This does require some experience.
- $\bullet\,$ NumPy's array operations are designed to make this possible.

Fast Evaluation Of Array Expressions

- The numexpr package supplies routines for the fast evaluation of array expressions elementwise by using a vector-based virtual machine.
- Expressions are cached, so reuse is fast.

Numexpr Users Guide

```
In [102]: import numexpr as ne
          import numpy as np
          nrange = (2 ** np.arange(6, 24)).astype(int)
          t_numpy = []
          t_numexpr = []
          for n in nrange:
              a = np.random.random(n)
              b = np.arange(n, dtype=np.double)
              c = np.random.random(n)
              c1 = ne.evaluate("a ** 2 + b ** 2 + 2 * a * b * c ", optimization='aggressive')
              t1 = \%timeit -oq -n 10 a ** 2 + b ** 2 + 2 * a * b * c
              t2 = %timeit -oq -n 10 ne.re_evaluate()
              t_numpy.append(t1.best)
              t_numexpr.append(t2.best)
          %matplotlib inline
          %config InlineBackend.figure_format = 'retina'
          import matplotlib.pyplot as plt
          import seaborn; seaborn.set()
          plt.loglog(nrange, t_numpy, label='numpy')
          plt.loglog(nrange, t_numexpr, label='numexpr')
          plt.legend(loc='lower right')
          plt.xlabel('Vectors size')
          plt.ylabel('Execution Time (s)');
```



References

- NumPy referenceGetting the Best Performance out of NumPy
- Numpy by Konrad Hinsen

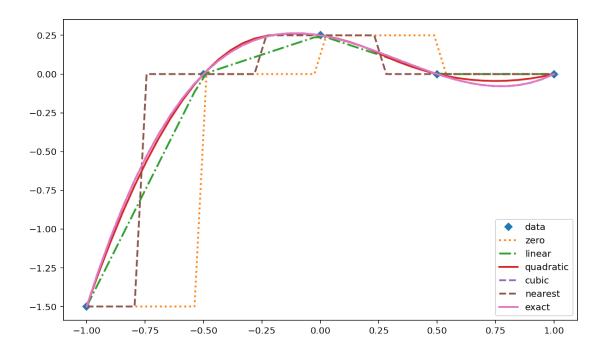
Scipy



Scipy is the scientific Python ecosystem : - fft, linear algebra, scientific computation,... - scipy contains numpy, it can be considered as an extension of numpy. - the add-on toolkits Scikits complements scipy.

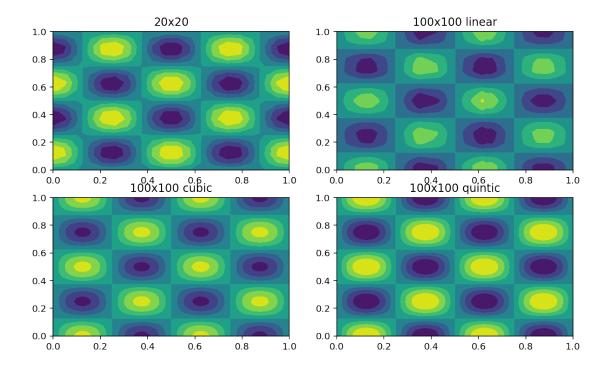
<ipython-input-3-52b38bd19582>:1: DeprecationWarning: scipy.log is deprecated and will be removed in Sc

```
sp.sqrt(-1.), sp.log(-2.)
Out [3]: (1j, (0.6931471805599453+3.141592653589793j))
In [4]: sp.exp(sp.log(-2.))
<ipython-input-4-021f9e257f98>:1: DeprecationWarning: scipy.log is deprecated and will be removed in Sc
  sp.exp(sp.log(-2.))
<ipython-input-4-021f9e257f98>:1: DeprecationWarning: scipy.exp is deprecated and will be removed in Sc
  sp.exp(sp.log(-2.))
Out[4]: (-2+2.4492935982947064e-16j)
        SciPy main packages
63.1
  • constants : Physical and mathematical constants
  • fftpack : Fast Fourier Transform routines
  • integrate: Integration and ordinary differential equation solvers
  • interpolate: Interpolation and smoothing splines
  • io: Input and Output
  • linalg: Linear algebra
  • signal : Signal processing
  • sparse : Sparse matrices and associated routines
In [5]: from scipy.interpolate import interp1d
       x = np.linspace(-1, 1, num=5) # 5 points evenly spaced in [-1,1].
       y = (x-1.)*(x-0.5)*(x+0.5)
                                    # x and y are numpy arrays
       f0 = interp1d(x,y, kind='zero')
       f1 = interp1d(x,y, kind='linear')
       f2 = interp1d(x,y, kind='quadratic')
       f3 = interp1d(x,y, kind='cubic')
       f4 = interp1d(x,y, kind='nearest')
In [6]: xnew = sp.linspace(-1, 1, num=40)
       ynew = (xnew-1.)*(xnew-0.5)*(xnew+0.5)
       plt.plot(x,y,'D',xnew,f0(xnew),':', xnew, f1(xnew),'-.',
                       xnew,f2(xnew),'-',xnew ,f3(xnew),'--',
                       xnew,f4(xnew),'--',xnew, ynew, linewidth=2)
       plt.legend(['data','zero','linear','quadratic','cubic','nearest','exact'],
                 loc='best');
<ipython-input-6-bb75b01c8201>:1: DeprecationWarning: scipy.linspace is deprecated and will be removed
  xnew = sp.linspace(-1, 1, num=40)
```



<ipython-input-7-3b5ca11518b4>:3: DeprecationWarning: scipy.cos is deprecated and will be removed in Sc
 z=sp.cos(4*sp.pi*x)*sp.sin(4*sp.pi*y) #initialize the field
<ipython-input-7-3b5ca11518b4>:3: DeprecationWarning: scipy.sin is deprecated and will be removed in Sc
 z=sp.cos(4*sp.pi*x)*sp.sin(4*sp.pi*y) #initialize the field

```
In [8]: X,Y=sp.mgrid[0:1:100j,0:1:100j] #create the interpolation grid 100x100
        # complex -> number of points, float -> step size
       plt.figure(1)
       plt.subplot(221) #Plot original data
       plt.contourf(x,y,z)
       plt.title('20x20')
       plt.subplot(222) #Plot linear interpolation
       plt.contourf(X,Y,T1(X[:,0],Y[0,:]))
       plt.title('100x100 linear')
       plt.subplot(223) #Plot cubic interpolation
       plt.contourf(X,Y,T2(X[:,0],Y[0,:]))
       plt.title('100x100 cubic')
       plt.subplot(224) #Plot quintic interpolation
       plt.contourf(X,Y,T3(X[:,0],Y[0,:]))
       plt.title('100x100 quintic')
Out[8]: Text(0.5, 1.0, '100x100 quintic')
```



63.2 FFT: scipy.fftpack

- FFT dimension 1, 2 and n : fft, ifft (inverse), rfft (real), irfft, fft2 (dimension 2), ifft2, fftn (dimension n), ifftn.
- Discrete cosinus transform : dct
- Convolution product : convolve

```
In [9]: from numpy.fft import fft, ifft
    x = np.random.random(1024)
    %timeit ifft(fft(x))
```

 $47.2 \text{ µs} \pm 1.02 \text{ µs}$ per loop (mean \pm std. dev. of 7 runs, 10000 loops each)

```
In [10]: from scipy.fftpack import fft, ifft
    x = np.random.random(1024)
    %timeit ifft(fft(x))
```

41.7 $\mu s \pm 638$ ns per loop (mean \pm std. dev. of 7 runs, 10000 loops each)

63.3 Linear algebra: scipy.linalg

- Sovers, decompositions, eigen values. (same as numpy).
- Matrix functions : expm, sinm, sinhm,...
- Block matrices diagonal, triangular, periodic,...

```
In [11]: import scipy.linalg as spl
                    b=sp.ones(5)
                    A=sp.array([[1.,3.,0., 0.,0.],
                                             [ 2.,1.,-4, 0.,0.],
                                             [6.,1.,2,-3.,0.],
                                             [0.,1.,4.,-2.,-3.],
                                             [0.,0.,6.,-3.,2.]
                    print("x=",spl.solve(A,b,sym_pos=False)) # LAPACK ( gesv ou posv )
                    AB=sp.array([[0.,3.,-4.,-3.,-3.],
                                                  [1.,1., 2.,-2., 2.],
                                                  [2.,1., 4.,-3., 0.],
                                                  [6.,1., 6., 0., 0.]])
                    print("x=",spl.solve_banded((2,1),AB,b)) # LAPACK ( gbsv )
x = \begin{bmatrix} -0.24074074 & 0.41358025 & -0.26697531 & -0.85493827 & 0.01851852 \end{bmatrix}
x = \begin{bmatrix} -0.24074074 & 0.41358025 & -0.26697531 & -0.85493827 & 0.01851852 \end{bmatrix}
<ipython-input-11-4af1e65b6e1b>:2: DeprecationWarning: scipy.ones is deprecated and will be removed in a contract of the 
     b=sp.ones(5)
<ipython-input-11-4af1e65b6e1b>:3: DeprecationWarning: scipy.array is deprecated and will be removed in
     A=sp.array([[1.,3.,0., 0.,0.],
<ipython-input-11-4af1e65b6e1b>:9: DeprecationWarning: scipy.array is deprecated and will be removed in
     AB=sp.array([[0.,3.,-4.,-3.,-3.],
In [12]: P,L,U = spl.lu(A) \# PA = LU
                    np.set_printoptions(precision=3)
                    for M in (P,L,U):
                             print(M, end="\n"+20*"-"+"\n")
[[0. 1. 0. 0. 0.]
  [0. 0. 0. 1. 0.]
  [1. 0. 0. 0. 0.]
  [0. \ 0. \ 0. \ 0. \ 1.]
  [0. 0. 1. 0. 0.]]
_____
[[ 1.
                        0.
                                          0.
                                                            0.
                                                                              0.
                                                                                          ]
  [ 0.167 1.
                                          0.
                                                                              0.
                                                                                          ]
                                                            0.
  [ 0. 0.
                                                                              0.
                                                                                          ٦
                                      1.
                                                            0.
  [ 0.333  0.235 -0.765  1.
                                                                                      ]
  [ 0. 0.353 0.686 0.083 1.
                                                                                          ]]
[[ 6.
                                                          -3.
                       1.
                                          2.
                                                                              0.
                                                                                          ]
  [ 0.
                    2.833 -0.333 0.5
                                                                              0.
                                                                                          ]
  [ 0.
                                                                                          ]
                   0.
                                                                              2.
                                          6. -3.
  [ 0.
                        0.
                                          0.
                                                         -1.412 1.529]
  [ 0.
                         0.
                                          0.
                                                            0. -4.5 ]]
```

63.4 CSC (Compressed Sparse Column)

- All operations are optimized
- Efficient "slicing" along axis=1.
- Fast Matrix-vector product.

• Conversion to other format could be costly.

```
In [13]: import scipy.sparse as spsp
        row = sp.array([0,2,2,0,1,2])
        col = sp.array([0,0,1,2,2,2])
        data = sp.array([1,2,3,4,5,6])
        Mcsc1 = spsp.csc_matrix((data,(row,col)),shape=(3,3))
        Mcsc1.todense()
<ipython-input-13-1852fd1225dd>:2: DeprecationWarning: scipy.array is deprecated and will be removed in
  row = sp.array([0,2,2,0,1,2])
<ipython-input-13-1852fd1225dd>:3: DeprecationWarning: scipy.array is deprecated and will be removed in
  col = sp.array([0,0,1,2,2,2])
<ipython-input-13-1852fd1225dd>:4: DeprecationWarning: scipy.array is deprecated and will be removed in
  data = sp.array([1,2,3,4,5,6])
Out[13]: matrix([[1, 0, 4],
                 [0, 0, 5],
                 [2, 3, 6]])
In [14]: indptr = sp.array([0,2,3,6])
        indices = sp.array([0,2,2,0,1,2])
              = sp.array([1,2,3,4,5,6])
        Mcsc2 = spsp.csc_matrix ((data,indices,indptr),shape=(3,3))
        Mcsc2.todense()
<ipython-input-14-31b5a43b6ca2>:1: DeprecationWarning: scipy.array is deprecated and will be removed in
  indptr = sp.array([0,2,3,6])
<ipython-input-14-31b5a43b6ca2>:2: DeprecationWarning: scipy.array is deprecated and will be removed in
  indices = sp.array([0,2,2,0,1,2])
<ipython-input-14-31b5a43b6ca2>:3: DeprecationWarning: scipy.array is deprecated and will be removed in
  data
          = sp.array([1,2,3,4,5,6])
Out[14]: matrix([[1, 0, 4],
                 [0, 0, 5],
                 [2, 3, 6]])
```

63.5 Dedicated format for assembling

- lil_matrix: Row-based linked list matrix. Easy format to build your matrix and convert to other format before solving.
- dok_matrix : A dictionary of keys based matrix. Ideal format for incremental matrix building. The conversion to csc/csr format is efficient.
- coo_matrix : coordinate list format. Fast conversion to formats CSC/CSR.

Lien vers la documentation scipy

63.6 Matrices creuses: scipy.sparse.linalg

- speigen, speigen_symmetric, lobpcg: (ARPACK).
- svd : (ARPACK).
- Direct methods (UMFPACK or SUPERLU) ou iteratives
- Minimization: lsqr and minres

For linear algebra: - Noobs: spsolve. - Intermmediate: dsolve.spsolve or isolve.spsolve - Advanced: splu, spilu (direct); cg, cgs, bicg, bicgstab, gmres, lgmres et qmr (iterative) - Boss: petsc4py et slepc4py.

63.7 Linear Operator

Out[19]: 1.5582722720639762e-15

The LinearOperator is used for matrix-free numerical methods.

```
In [15]: import scipy.sparse.linalg as spspl
        def mv(v):
          return sp.array([2*v[0],3*v[1]])
        A=spspl.LinearOperator((2 ,2),matvec=mv,dtype=float )
Out[15]: <2x2 _CustomLinearOperator with dtype=float64>
In [16]: A*sp.ones(2)
<ipython-input-16-8911e67c7fda>:1: DeprecationWarning: scipy.ones is deprecated and will be removed in a

  A*sp.ones(2)
<ipython-input-15-b76e52185657>:3: DeprecationWarning: scipy.array is deprecated and will be removed in
  return sp.array([2*v[0],3*v[1]])
Out[16]: array([2., 3.])
In [17]: A.matmat(sp.array([[1,-2],[3,6]]))
<ipython-input-17-80cc6cd3a514>:1: DeprecationWarning: scipy.array is deprecated and will be removed in
  A.matmat(sp.array([[1,-2],[3,6]]))
<ipython-input-15-b76e52185657>:3: DeprecationWarning: scipy.array is deprecated and will be removed in
  return sp.array([2*v[0],3*v[1]])
Out[17]: array([[ 2, -4],
                [ 9, 18]])
63.8 LU decomposition
In [18]: N = 50
```

```
In [18]: N = 50
    un = sp.ones(N)
    w = sp.rand(N+1)
    A = spsp.spdiags([w[1:],-2*un,w[:-1]],[-1,0,1],N,N) # tridiagonal matrix
    A = A.tocsc()
    b = un
    op = spspl.splu(A)
    op

<ipython-input-18-e97d13ba2dbc>:2: DeprecationWarning: scipy.ones is deprecated and will be removed in a company of the co
```

63.9 Conjugate Gradient

```
In [20]: global k
        k=0
        def f(xk): # function called at every iterations
            global k
            print ("iteration {0:2d} residu = {1:7.3g}".format(k,spl.norm(A*xk-b)))
            k += 1
        x,info=spspl.cg(A,b,x0=sp.zeros(N),tol=1.0e-12,maxiter=N,M=None,callback=f)
iteration 0 residu =
                         2.73
iteration 1 residu =
                         1.11
iteration 2 residu =
                        0.485
iteration 3 residu = 0.174
iteration 4 residu =
                       0.068
iteration 5 residu = 0.0281
iteration 6 residu = 0.0108
iteration 7 residu = 0.004
iteration 8 residu = 0.00148
iteration 9 residu = 0.000465
iteration 10 residu = 0.000194
iteration 11 residu = 7.56e-05
iteration 12 residu = 2.94e-05
iteration 13 residu = 1.01e-05
iteration 14 residu = 3.83e-06
iteration 15 residu = 1.12e-06
iteration 16 residu = 4.28e-07
iteration 17 residu = 1.98e-07
iteration 18 residu = 6.04e-08
iteration 19 residu = 2.26e-08
iteration 20 residu = 6.34e-09
iteration 21 residu = 1.63e-09
iteration 22 residu = 3.64e-10
iteration 23 residu = 1.1e-10
iteration 24 residu = 3.21e-11
iteration 25 residu = 7.73e-12
iteration 26 residu = 3.14e-12
```

<ipython-input-20-45fd1a29b4fe>:8: DeprecationWarning: scipy.zeros is deprecated and will be removed in x,info=spspl.cg(A,b,x0=sp.zeros(N),tol=1.0e-12,maxiter=N,M=None,callback=f)

63.10 Preconditioned conjugate gradient

```
iteration 0 residu = 0.196
iteration 1 residu = 0.00465
iteration 2 residu = 0.000175
iteration 3 residu = 5.61e-06
iteration 4 residu = 2.42e-07
iteration 5 residu = 3.57e-09
iteration 6 residu = 4.44e-11
iteration 7 residu = 2.33e-13
```

<ipython-input-22-dff3da9ff400>:5: DeprecationWarning: scipy.zeros is deprecated and will be removed in x,info=spspl.cg(A,b,x0=sp.zeros(N),tol=1.e-12,maxiter=N,M=lo,callback=f)

63.11 Numerical integration

• quad, dblquad, tplquad,... Fortran library QUADPACK.

63.12 Scipy ODE solver

It uses the Fortran ODEPACK library.

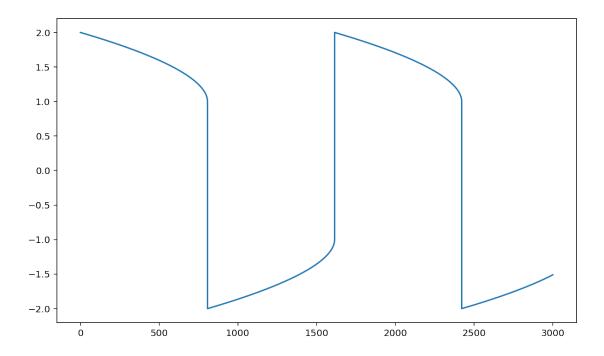
63.12.1 Van der Pol Oscillator

```
y_1'(t) = y_2(t), y_2'(t) = 1000(1 - y_1^2(t))y_2(t) - y_1(t) $$ with $y_1(0) = 2 $ and $ y_2(0) = 0. $.

In [25]: import numpy as np import scipy.integrate as spi
def \ vdp1000(y,t): \\ dy=np.zeros(2) \\ dy[0]=y[1] \\ dy[1]=1000.*(1.-y[0]**2)*y[1]-y[0]  return dy

In [26]: t0, tf =0, 3000 N = 3000000 t, dt = np.linspace(t0,tf,N, retstep=True)

In [27]: y=spi.odeint(vdp1000,[2.,0.],t) plt.plot(t,y[:,0]);
```



63.13 Exercise

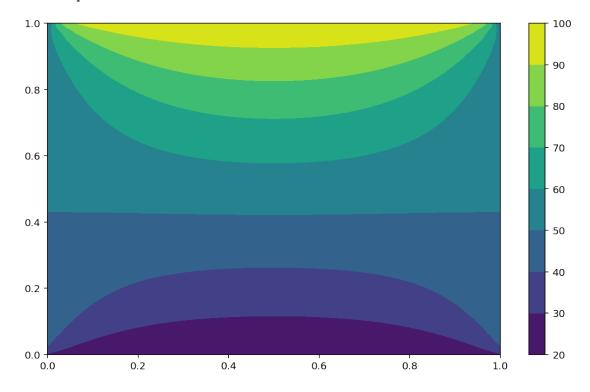
The following code solve the Laplace equation using a dense matrix. - Modified the code to use a sparse matrix

```
In [28]: %%time
         %matplotlib inline
         %config InlineBackend.figure_format = "retina"
         import numpy as np
         import matplotlib.pyplot as plt
         plt.rcParams['figure.figsize'] = (10,6)
         # Boundary conditions
        Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50
         # Set meshgrid
        n = 50
        1 = 1.0
        h = 1 / (n-1)
        X, Y = np.meshgrid(np.linspace(0,1,n), np.linspace(0,1,n))
        T = np.zeros((n,n),dtype='d')
         # Set Boundary condition
        T[n-1:, :] = Tnorth / h**2
        T[:1, :] = Tsouth / h**2
        T[:, n-1:] = Teast / h**2
        T[:, :1] = Twest / h**2
         A = np.zeros((n*n,n*n),dtype='d')
        nn = n*n
         ii = 0
```

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```
for j in range(n):
            for i in range(n):
              if j > 0:
                 jj = ii - n
                 A[ii,jj] = -1
              if j < n-1:
                 jj = ii + n
                 A[ii,jj] = -1
              if i > 0:
                 jj = ii - 1
                 A[ii,jj] = -1
              if i < n-1:
                 jj = ii + 1
                 A[ii,jj] = -1
              A[ii,ii] = 4
              ii = ii+1
        U = np.linalg.solve(A,np.ravel(h**2*T))
        T = U.reshape(n,n)
        plt.contourf(X,Y,T)
        plt.colorbar()
CPU times: user 536 ms, sys: 81.1 ms, total: 617 ms
Wall time: 339 ms
```

Out[28]: <matplotlib.colorbar.Colorbar at 0x7fa6b53096d0>

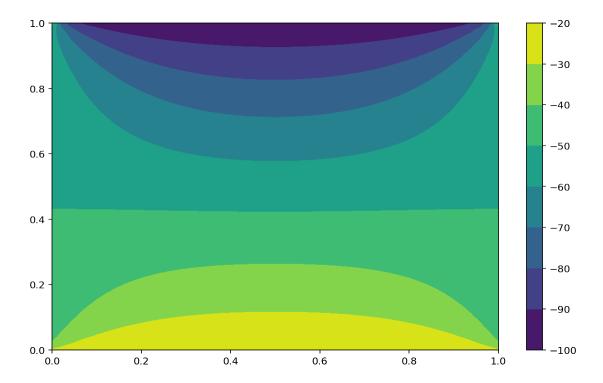


In [29]: %%time
 import scipy.sparse as spsp

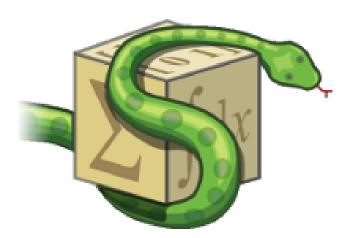
```
import scipy.sparse.linalg as spspl
# Boundary conditions
Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50
# Set meshgrid
n = 50
1 = 1.0
h = 1 / (n-1)
X, Y = np.meshgrid(np.linspace(0,1,n), np.linspace(0,1,n))
T = np.zeros((n,n),dtype='d')
# Set Boundary condition
T[n-1:, :] = Tnorth / h**2
T[ :1, :] = Tsouth / h**2
T[ :, n-1:] = Teast / h**2
T[ :, :1] = Twest / h**2
bdiag = -4 * np.eye(n)
bup = np.diag([1] * (n - 1), 1)
blow = np.diag([1] * (n - 1), -1)
block = bdiag + bup + blow
# Creat a list of n blocks
blist = [block] * n
S = spsp.block_diag(blist)
# Upper diagonal array offset by -n
upper = np.diag(np.ones(n * (n - 1)), n)
# Lower diagonal array offset by -n
lower = np.diag(np.ones(n * (n - 1)), -n)
S += upper + lower
T = sp.linalg.solve(S,np.ravel(h**2*T))
plt.contourf(X,Y,T.reshape(n,n))
plt.colorbar();
```

CPU times: user 616 ms, sys: 181 ms, total: 797 ms Wall time: 489 ms

63.13. EXERCISE 195



Sympy



The function init_printing() will enable LaTeX pretty printing in the notebook for SymPy expressions.

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Out [5]:
$$\frac{\sqrt{2}e^{-\frac{(-\mu+x)^2}{2\sigma^2}}}{2\sqrt{\pi}\sigma}$$

Why use sympy?

• Symbolic derivatives

x + 1

- Translate mathematics into low level code
- Deal with very large expressions
- Optimize code using mathematics

Dividing two integers in Python creates a float, like $1/2 \rightarrow 0.5$. If you want a rational number, use Rational(1, 2) or S(1)/2.

```
In [6]: x + sym.S(1)/2, sym.Rational(1,4)
   \left(x + \frac{1}{2}, \ \frac{1}{4}\right)
In [7]: y = Symbol('y')
        x \hat{y} # XOR operator (True only if <math>x != y)
Out[7]:
   x \vee y
In [8]: x**y
Out[8]:
   x^y
   SymPy expressions are immutable. Functions that operate on an expression return a new expression.
In [9]: expr = x + 1
         expr
Out [9]:
   x+1
In [10]: expr.subs(x, 2)
Out[10]:
   3
In [11]: expr
Out[11]:
```

65.0.1 Exercise: Lagrange polynomial

Given a set of k+1 data points $(x_0, y_0), \dots, (x_j, y_j), \dots, (x_k, y_k)$ the Lagrange interpolation polynomial is:

$$L(x) := \sum_{j=0}^{k} y_j \ell_j(x)$$

 ℓ_i are Lagrange basis polynomials:

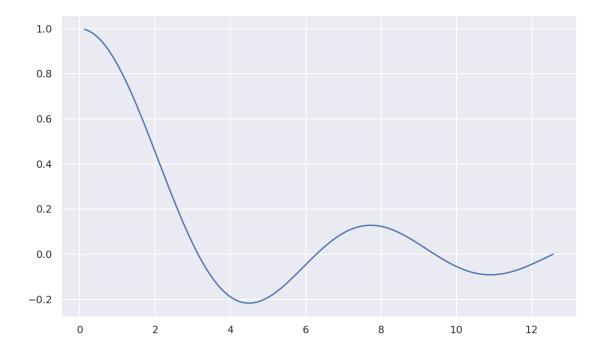
$$\ell_j(x) := \prod_{\substack{0 \le m \le k \\ m \ne j}} \frac{x - x_m}{x_j - x_m}$$

We can demonstrate that at each point x_i , $L(x_i) = y_i$ so L interpolates the function.

• Compute the Lagrange polynomial for points

$$(-2,21),(-1,1),(0,-1),(1,-3),(2,1)$$

65.1 Evaluate an expression



65.1.1 Exercise

• Plot the Lagrange polynomial computed above and interpolations points with matplotlib

65.2 Undefined functions and derivatives

Undefined functions are created with Function(). Undefined are useful to state that one variable depends on another (for the purposes of differentiation).

```
In [15]: from sympy import Function f = \text{Function}('f')
In [16]: f(x) + 1
Out[16]: f(x) + 1
In [17]: from sympy import diff, \sin, \cos \text{diff}(\sin(x+1)*\cos(y), x), \, \text{diff}(\sin(x+1)*\cos(y), x, y), \, \text{diff}(f(x), x)
Out[17]: \left(\cos(y)\cos(x+1), -\sin(y)\cos(x+1), \, \frac{d}{dx}f(x)\right)
In [18]: c, t = \text{sym.symbols}('t c') u = \text{sym.Function}('u') \text{sym.Eq}(\text{diff}(u(t,x),t,t), \, c**2*\text{diff}(u(t,x),x,2))
Out[18]: \frac{\partial^2}{\partial c^2}u(c,x) = t^2\frac{\partial^2}{\partial x^2}u(c,x)
```

Matrices

66.1 Matrix symbols

SymPy can also operate on matrices of symbolic dimension $(n \times m)$. MatrixSymbol("M", n, m) creates a matrix M of shape $n \times m$.

66.2 Solving systems of equations

solve solves equations symbolically (not numerically). The return value is a list of solutions. It automatically assumes that it is equal to 0.

```
In [23]: from sympy import Eq, solve
      solve(Eq(x**2, 4), x)
```

66.3 Solving differential equations

dsolve can (sometimes) produce an exact symbolic solution. Like solve, dsolve assumes that expressions are equal to 0.

```
In [26]: from sympy import Function, dsolve f = \text{Function('f')}  dsolve(f(x).diff(x, 2) + f(x)) Out[26]: f(x) = C_1 \sin(x) + C_2 \cos(x)
```

66.4 Code printers

The most basic form of code generation are the code printers. They convert SymPy expressions into over a dozen target languages.

```
In [27]: x = \text{symbols}(\cdot|\mathbf{x}') expr = abs(\sin(x**2)) expr

Out[27]: |\sin(x^2)|

In [28]: sym.ccode(expr)

Out[28]: 'fabs(\sin(pow(x, 2)))'

In [29]: sym.fcode(expr, standard=2003, source\_format='free')

Out[29]: 'abs(\sin(x**2))'

In [30]: from sympy.printing.cxxcode import cxxcode cxxcode(expr)

Out[30]: 'std::fabs(std::sin(std::pow(x, 2)))'

In [31]: sym.tanh(x).rewrite(sym.exp)

Out[31]: \frac{e^x - e^{-x}}{e^x + e^{-x}}
```

66.5 Creating a function from a symbolic expression

In SymPy there is a function to create a Python function which evaluates (usually numerically) an expression. SymPy allows the user to define the signature of this function (which is convenient when working with e.g. a numerical solver in scipy).

```
In [33]: from sympy import log
    x, y = symbols('x y')
        expr = 3*x**2 + log(x**2 + y**2 + 1)
        expr

Out[33]:
    3x² + log(x² + y² + 1)

In [34]: %timeit expr.subs({x: 17, y: 42}).evalf()

171    µs ± 1.88    µs    per loop (mean ± std. dev. of 7 runs, 10000 loops each)

In [35]: import math
        f = lambda x, y: 3*x**2 + math.log(x**2 + y**2 + 1)
        f(17, 42)

Out[35]:
    874.6275443904885

In [36]: %timeit f(17, 42)

1.17    µs ± 24.8    ns    per loop (mean ± std. dev. of 7 runs, 1000000 loops each)
```

Evaluate above expression numerically invoking the subs method followed by the evalf method can be quite slow and cannot be done repeatedly.

```
Out[39]:
   (5)
In [40]: z = z1, z2, z3 = symbols('z:3')
         expr2 = x*y*(z1 + z2 + z3)
         func2 = lambdify([x, y, z], expr2)
         func2(1, 2, (3, 4, 5))
Out [40]:
   24
```

Behind the scenes lambdify constructs a string representation of the Python code and uses Python's eval function to compile the function.

66.5.1 SIR model

$$\frac{dS(t)}{dt} = -\beta S(t)I(t) \tag{66.1}$$

$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$

$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t)$$
(66.1)

$$\frac{dR(t)}{dt} = \gamma I(t) \tag{66.3}$$

- S,I,R: ratio of suceptibles, infectious and recovered fraction of the population.
- t: time
- β : transmission coefficient.
- γ : healing rate.

We assume that total population is constant.

Solving the initial value problem numerically

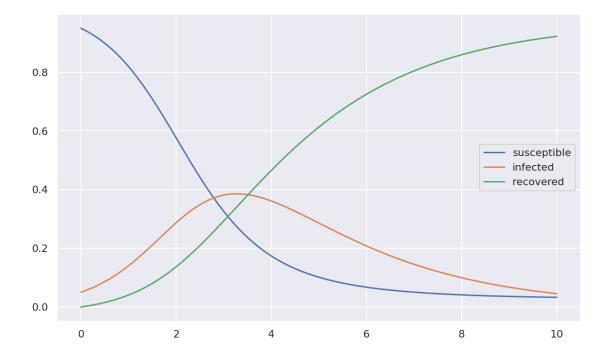
We will now integrate this system of ordinary differential equations numerically using the odeint solver provided by scipy:

By looking at the documentation of odeint we see that we need to provide a function which computes a vector of derivatives $(\dot{\mathbf{y}} = [\frac{dy_1}{dt}, \frac{dy_2}{dt}, \frac{dy_3}{dt}])$. The expected signature of this function is:

```
f(y: array[float64], t: float64, *args: arbitrary constants) -> dydt: array[float64]
```

in our case we can write it as:

```
In [41]: def rhs(y, t, beta, gamma):
             rb = beta * y[0]*y[1]
             rg = gamma * y[1]
             return [- rb , rb - rg, rg]
In [42]: import scipy.integrate as spi
         tout = np.linspace(0, 10, 100)
         k \text{ vals} = 1.66, 0.4545455
         y0 = [0.95, 0.05, 0]
         yout = spi.odeint(rhs, y0, tout, k_vals)
         plt.plot(tout, yout)
         plt.legend(['susceptible', 'infected', 'recovered']);
```



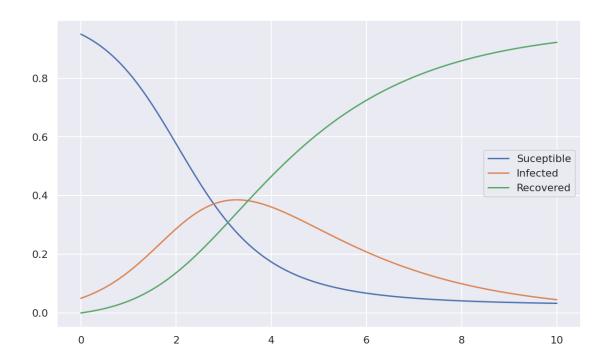
We will construct the system from a symbolic representation. But at the same time, we need the rhs function to be fast. Which means that we want to produce a fast function from our symbolic representation. Generating a function from our symbolic representation is achieved through *code generation*.

- 1. Construct a symbolic representation from some domain specific representation using SymPy.
- 2. Have SymPy generate a function with an appropriate signature (or multiple thereof), which we pass on to the solver.

We will achieve (1) by using SymPy symbols (and functions if needed). For (2) we will use a function in SymPy called lambdify it takes a symbolic expressions and returns a function. In a later notebook, we will look at (1), for now we will just use rhs which we've already written:

```
In [43]: y, k = sym.symbols('y:3'), sym.symbols('beta gamma') ydot = rhs(y, None, *k) y, ydot

Out[43]:  ((y_0, y_1, y_2), [-\beta y_0 y_1, \beta y_0 y_1 - \gamma y_1, \gamma y_1]) 
In [44]: f = sym.lambdify((y,t)+k, ydot) plt.plot(tout, spi.odeint(f, y0, tout, k_vals)) plt.legend(['Suceptible', 'Infected', 'Recovered']);
```



In this example the gains of using a symbolic representation are arguably limited. Let's take the same example with demography and n classes of subjects:

$$X_i = S_i, I_i, R_i \qquad i = 1 \dots n$$

$$\frac{dS_{i}}{dt} = \nu_{i} - \beta_{i}S_{i}I_{i} - \mu_{i}S_{i} + \sum_{j=1}^{n} m_{ji}S_{j} - \sum_{j=1}^{n} m_{ij}S_{i}\frac{dI_{i}}{dt} = \beta_{i}S_{i}I_{i} - (\gamma_{i} + \mu_{i})I_{i} + \sum_{j=1}^{n} m_{ij}I_{j} - \sum_{j=1}^{n} m_{ji}I_{i}\frac{dR_{i}}{dt} = -\frac{dS_{i}}{dt} - \frac{dI_{i}}{dt}$$

- β : transmission coefficient
- γ : healing rate
- μ : mortality rate
- ν : birth rate

66.5.3 Exercise

- Create the symbolic matrix m, symbols $\nu_i, \mu_i, \beta_i, \gamma_i$ for i = 0, 1, 2 and y_j for $j = 0, 1, 2, \dots, 8$
- Write the system $\dot{y} = f(t, y, m, \nu, \mu, \beta, \gamma)$
- lambdify the f function.
- Solve the system with:

$$m = \begin{bmatrix} 0 & 0.01 & 0.01 \\ 0.01 & 0 & 0.01 \\ 0.01 & 0.01 & 0 \end{bmatrix}$$

$$t = [0, 10] \text{ with } dt = 0.1$$

$$\nu_i = 0.0$$

$$\mu_i = 0.0$$

$$\beta_i = 1.66$$

$$\gamma_i = [0.4545, 0.3545, 0.2545]$$

$$S_i = 0.95$$

$$I_i = 0.05$$

$$R_i = 0.0$$

66.5.4 Exercise: Bezier curve

We want to compute and the draw the Bezier curve between the 3 points p_0 , p_1 , and p_2 , The middle point p_1 position is arbitrary.

$$p0 = (1,0);$$
 $p1 = (x,y);$ $p2 = (0,1)$

The n+1 Bernstein basis polynomials of degree n are defined as

$$b_{i,n}(x) = \binom{n}{i} x^i (1-x)^{n-i}, \quad i = 0, \dots, n.$$

where $\binom{n}{i}$ is the binomial coefficient.

The Bezier curve is defined by a linear combination of Bernstein basis polynomials:

$$B_n(x) = \sum_{i=0}^n \beta_i b_{i,n}(x)$$

- Withsympy.binomial, write a function bpoly(t,n,i) that returns the Bernstein basis polynomial $b_{i,n}(t)$.
- Compute the Berstein polynomial representing the Bezier curve between p_0, p_1, p_2 . $\beta_i = 1$.
- Plot the Bezier Curve for 3 positions of $p_1 = (0,0), (0.5,0.5), (1,1)$

66.6 Integrals quadrature

References

• SciPy 2017 tutorial

Call fortran from Python

```
In [1]: %matplotlib inline
       %config InlineBackend.figure_format = 'retina'
        import matplotlib.pyplot as plt
        import scipy.fftpack as sf
        import scipy.linalg as sl
        import numpy as np
In [2]: import sys
        %env FC=gfortran
        if sys.platform == "darwin":
            %env CC=gcc-10
        # change values for your configuration
```

env: FC=gfortran

68.1f2py

f2py is a part of Numpy and there are three ways to wrap Fortran with Python: - Write some fortran subroutines and just run f2py to create Python modules. - Insert special f2py directives inside Fortran source for complex wrapping. - Write a interface file (.pyf) to wrap Fortran files without changing them. f2py automatically generate the pyf template file that can be modified.

Simple Fortran subroutine to compute norm

69.0.1 Fortran 90/95 free format

```
In [3]: %%file euclidian_norm.f90
    subroutine euclidian_norm (a, b, c)
    real(8), intent(in) :: a, b
    real(8), intent(out) :: c
    c = sqrt (a*a+b*b)
    end subroutine euclidian_norm
```

Writing euclidian_norm.f90

69.0.2 Fortran 77 fixed format

Writing euclidian_norm.f

Build extension module with f2py program

```
In [5]: import sys
       !{sys.executable} -m numpy.f2py -c euclidian_norm.f90 -m vect --fcompiler=gnu95 --f90flags=-03
running build
running config_cc
unifing config_cc, config, build_clib, build_ext, build commands --compiler options
running config_fc
unifing config_fc, config, build_clib, build_ext, build commands --fcompiler options
running build_src
build_src
building extension "vect" sources
f2py options: []
f2py:> /tmp/tmp85hgxga_/src.linux-x86_64-3.8/vectmodule.c
creating /tmp/tmp85hgxga_/src.linux-x86_64-3.8
Reading fortran codes...
        Reading file 'euclidian_norm.f90' (format:free)
Post-processing...
        Block: vect
                        Block: euclidian norm
Post-processing (stage 2)...
Building modules...
        Building module "vect"...
                Constructing wrapper function "euclidian_norm"...
                  c = euclidian_norm(a,b)
        Wrote C/API module "vect" to file "/tmp/tmp85hgxga_/src.linux-x86_64-3.8/vectmodule.c"
  adding '/tmp/tmp85hgxga_/src.linux-x86_64-3.8/fortranobject.c' to sources.
  adding '/tmp/tmp85hgxga_/src.linux-x86_64-3.8' to include_dirs.
copying /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/f2py/src/fortranobject.c ->
copying /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/f2py/src/fortranobject.h ->
build_src: building npy-pkg config files
running build_ext
customize UnixCCompiler
customize UnixCCompiler using build_ext
customize Gnu95FCompiler
Found executable /usr/bin/gfortran
customize Gnu95FCompiler using build_ext
building 'vect' extension
```

```
compiling C sources
C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compiler_compat -W1,--sysroot=/ -Wsign-compiler_compat -W1,--sysroot=/ -Wsign-compiler_compat -W1,--sysroot=/ -Wsign-compiler_compat -W1,--sysroot=/ -Wsign-compiler_compat -W1,--sysroot=/ -Wsign-compiler_compiler_compat -W1,--sysroot=/ -Wsign-compiler_compiler_compat -W1,--sysroot=/ -Wsign-compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compile
creating /tmp/tmp85hgxga_/tmp
creating /tmp/tmp85hgxga_/tmp/tmp85hgxga_
creating /tmp/tmp85hgxga_/tmp/tmp85hgxga_/src.linux-x86_64-3.8
compile options: '-I/tmp/tmp85hgxga_/src.linux-x86_64-3.8 -I/usr/share/miniconda3/envs/runenv/lib/pythos
gcc: /tmp/tmp85hgxga_/src.linux-x86_64-3.8/fortranobject.c
gcc: /tmp/tmp85hgxga_/src.linux-x86_64-3.8/vectmodule.c
In file included from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/
                                from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/s
                                from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/
                                from /tmp/tmp85hgxga_/src.linux-x86_64-3.8/fortranobject.h:13,
                                from /tmp/tmp85hgxga_/src.linux-x86_64-3.8/fortranobject.c:2:
/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/numpy/npy_1_7_deprecat
  #warning "Using deprecated NumPy API, disable it with " \
In file included from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/
                                from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/s
                                from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/
                                from /tmp/tmp85hgxga_/src.linux-x86_64-3.8/fortranobject.h:13,
                                from /tmp/tmp85hgxga_/src.linux-x86_64-3.8/vectmodule.c:15:
/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/numpy/npy_1_7_deprecat
 #warning "Using deprecated NumPy API, disable it with " \
    ^~~~~~
compiling Fortran sources
Fortran f77 compiler: /usr/bin/gfortran -Wall -g -ffixed-form -fno-second-underscore -fPIC -03 -funroll
Fortran f90 compiler: /usr/bin/gfortran -03 -fPIC -03 -funroll-loops
Fortran fix compiler: /usr/bin/gfortran -Wall -g -ffixed-form -fno-second-underscore -03 -fPIC -03 -fun
compile options: '-I/tmp/tmp85hgxga_/src.linux-x86_64-3.8 -I/usr/share/miniconda3/envs/runenv/lib/pytho
gfortran:f90: euclidian_norm.f90
/usr/bin/gfortran -Wall -g -Wall -g -shared /tmp/tmp85hgxga_/tmp/tmp85hgxga_/src.linux-x86_64-3.8/vectm
Removing build directory /tmp/tmp85hgxga_
```

70.1 Use the extension module in Python

----c : float

Fortran magic

- Jupyter extension that help to use fortran code in an interactive session.
- It adds a %% fortran cell magic that compile and import the Fortran code in the cell, using F2py.
- The contents of the cell are written to a .f90 file in the directory IPYTHONDIR/fortran using a filename with the hash of the code. This file is then compiled. The resulting module is imported and all of its symbols are injected into the user's namespace.

Documentation

In [8]: %load_ext fortranmagic

F2py directives

- F2PY introduces also some extensions to Fortran 90/95 language specification that help designing Fortran to Python interface, make it more "Pythonic".
- If editing Fortran codes is acceptable, these specific attributes can be inserted directly to Fortran source codes. Special comment lines are ignored by Fortran compilers but F2PY interprets them as normal lines.

```
In [9]: %%fortran
       subroutine euclidian_norm(a,c,n)
         integer :: n
         real(8),dimension(n),intent(in) :: a
          !f2py optional , depend(a) :: n=len(a)
         real(8),intent(out) :: c
         real(8) :: sommec
          integer :: i
          sommec = 0
          do i=1,n
           sommec=sommec+a( i )*a( i )
         end do
         c = sqrt (sommec)
       end subroutine euclidian_norm
In [10]: a=[2,3,4] # Python list
        type(a)
Out[10]: list
In [11]: euclidian_norm(a)
Out[11]: 5.385164807134504
In [12]: a=np.arange(2,5) # numpy array
        type(a)
Out[12]: numpy.ndarray
In [13]: euclidian_norm(a)
Out[13]: 5.385164807134504
In [14]: print(euclidian_norm.__doc__) # Documentation
c = euclidian_norm(a,[n])
```

Wrapper for ``euclidian_norm``.

Parameters

a : input rank-1 array('d') with bounds (n)

Other Parameters

n : input int, optional
 Default: len(a)

Returns

c : float

F2py directives

- optional: The corresponding argument is moved to the end.
- required: This is default. Use it to disable automatic optional setting.
- intent(in | inout | out | hide), intent(in) is the default.
- intent(out) is implicitly translated to intent(out, hide).
- intent(copy) and intent(overwrite) control changes for input arguments.
- check performs some assertions, it is often automatically generated.
- depend: f2py detects cyclic dependencies.
- allocatable, parameter
- intent(callback), external: for function as arguments.
- intent(c) C-type argument, array or function.
- C expressions: rank, shape, len, size, slen.

Callback

You can call a python function inside your fortran code

```
In [15]: %%fortran
         subroutine sum_f (f ,n, s)
           !Compute sum(f(i), i=1,n)
          external f
          integer, intent(in) :: n
          real, intent(out) :: s
          s = 0.0
          do i=1,n
            s=s+f(i)
         end subroutine sum_f
In [16]: def fonction(i) : # python function
            return i*i
         sum_f(fonction,3)
Out[16]: 14.0
In [17]: sum_f(lambda x :x**2,3) # lambda function
Out[17]: 14.0
```

Fortran arrays and Numpy arrays

Let's see how to pass numpy arrays to fortran subroutine.

```
In [18]: %%fortran --extra "-DF2PY_REPORT_ON_ARRAY_COPY=1"
        subroutine push( positions, velocities, dt, n)
           integer, intent(in) :: n
          real(8), intent(in) :: dt
          real(8), dimension(n,3), intent(in) :: velocities
          real(8), dimension(n,3) :: positions
          do i = 1, n
            positions(i,:) = positions(i,:) + dt*velocities(i,:)
           end do
        end subroutine push
In [19]: positions = [[0, 0, 0], [0, 0, 0], [0, 0, 0]]
        velocities = [[0, 1, 2], [0, 3, 2], [0, 1, 3]]
In [20]: import sys
        push(positions, velocities, 0.1)
        positions # memory is not updated because we used C memory storage
Out[20]: [[0, 0, 0], [0, 0, 0], [0, 0, 0]]
```

During execution, the message "created an array from object" is displayed, because a copy of is made when passing multidimensional array to fortran subroutine.

Signature file

This file contains descriptions of wrappers to Fortran or C functions, also called as signatures of the functions. F2PY can create initial signature file by scanning Fortran source codes and catching all relevant information needed to create wrapper functions.

```
f2py vector.f90 -h vector.pyf

• vector.pyf

!     -*- f90 -*-
! Note: the context of this file is case sensitive.

subroutine euclidian_norm(a,c,n) ! in vector.f90
     real(kind=8) dimension(n),intent(in) :: a
     real(kind=8) intent(out) :: c
     integer optional,check(len(a)>=n),depend(a) :: n=len(a)
end subroutine euclidian_norm

! This file was auto-generated with f2py (version:2).
! See http://cens.ioc.ee/projects/f2py2e/
```

Wrap lapack function dgemm with f2py

• Generate the signature file In [22]: %rm -f dgemm.f dgemm.pyf !wget http://ftp.mcs.anl.gov/pub/MINPACK-2/blas/dgemm.f --2020-08-09 16:33:55-- http://ftp.mcs.anl.gov/pub/MINPACK-2/blas/dgemm.f Resolving ftp.mcs.anl.gov (ftp.mcs.anl.gov)... 140.221.6.23 Connecting to ftp.mcs.anl.gov (ftp.mcs.anl.gov)|140.221.6.23|:80... connected. HTTP request sent, awaiting response... 301 Moved Permanently Location: https://ftp.mcs.anl.gov/pub/MINPACK-2/blas/dgemm.f [following] --2020-08-09 16:33:56-- https://ftp.mcs.anl.gov/pub/MINPACK-2/blas/dgemm.f Connecting to ftp.mcs.anl.gov (ftp.mcs.anl.gov)|140.221.6.23|:443... connected. HTTP request sent, awaiting response... 200 OK Length: 9851 (9.6K) [text/plain] Saving to: 'dgemm.f' 100%[======>] dgemm.f 9.62K --.-KB/sin Os 2020-08-09 16:33:56 (228 MB/s) - 'dgemm.f' saved [9851/9851] In [23]: # %load dgemm.f In [24]: !{sys.executable} -m numpy.f2py -m mylapack --overwrite-signature -h dgemm.pyf dgemm.f Reading fortran codes... Reading file 'dgemm.f' (format:fix,strict) rmbadname1: Replacing "max" with "max_bn". Post-processing... Block: mylapack Block: dgemm Post-processing (stage 2)... Saving signatures to file "./dgemm.pyf" -*- f90 -*-! Note: the context of this file is case sensitive.

```
python module mylapack ! in
    interface ! in :mylapack
        subroutine dgemm(transa,transb,m,n,k,alpha,a,lda,b,ldb,beta,c,ldc) ! in :mylapack:dgemm.f
            character*1 :: transa
            character*1 :: transb
            integer :: m
            integer :: n
            integer :: k
            double precision :: alpha
            double precision dimension(lda,*) :: a
            integer, optional, check(shape(a,0)==lda), depend(a) :: lda=shape(a,0)
            double precision dimension(ldb,*) :: b
            integer, optional,check(shape(b,0)==ldb),depend(b) :: ldb=shape(b,0)
            double precision :: beta
            double precision dimension(ldc,*) :: c
            integer, optional,check(shape(c,0)==ldc),depend(c) :: ldc=shape(c,0)
        end subroutine dgemm
   end interface
end python module mylapack
! This file was auto-generated with f2py (version:2).
! See http://cens.ioc.ee/projects/f2py2e/
In [25]: !{sys.executable} -m numpy.f2py -c dgemm.pyf -llapack
running build
running config_cc
unifing config_cc, config, build_clib, build_ext, build commands --compiler options
running config_fc
unifing config_fc, config, build_clib, build_ext, build commands --fcompiler options
running build_src
build_src
building extension "mylapack" sources
creating /tmp/tmp44thhw01/src.linux-x86_64-3.8
f2py options: []
f2py: dgemm.pyf
Reading fortran codes...
       Reading file 'dgemm.pyf' (format:free)
Post-processing...
        Block: mylapack
                        Block: dgemm
Post-processing (stage 2)...
Building modules...
        Building module "mylapack"...
                Constructing wrapper function "dgemm"...
getarrdims:warning: assumed shape array, using 0 instead of '*'
getarrdims:warning: assumed shape array, using 0 instead of '*'
getarrdims:warning: assumed shape array, using 0 instead of '*'
                  dgemm(transa,transb,m,n,k,alpha,a,b,beta,c,[lda,ldb,ldc])
        Wrote C/API module "mylapack" to file "/tmp/tmp44thhw01/src.linux-x86_64-3.8/mylapackmodule.c"
  adding '/tmp/tmp44thhw01/src.linux-x86_64-3.8/fortranobject.c' to sources.
  adding '/tmp/tmp44thhw01/src.linux-x86_64-3.8' to include_dirs.
copying /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/f2py/src/fortranobject.c ->
copying /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/f2py/src/fortranobject.h ->
```

```
build_src: building npy-pkg config files
running build_ext
customize UnixCCompiler
customize UnixCCompiler using build_ext
building 'mylapack' extension
compiling C sources
C compiler: gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compiler_compat -W1,--sysroot=/ -Wsign-compiler_compat -W1,--sysroot=/ -Wsign-compiler_compat -W1,--sysroot=/ -Wsign-compiler_compat -W1,--sysroot=/ -Wsign-compiler_compat -W1,--sysroot=/ -Wsign-compiler_compiler_compat -W1,--sysroot=/ -Wsign-compiler_compiler_compat -W1,--sysroot=/ -Wsign-compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compile
creating /tmp/tmp44thhw01/tmp
creating /tmp/tmp44thhw01/tmp/tmp44thhw01
creating /tmp/tmp44thhw01/tmp/tmp44thhw01/src.linux-x86_64-3.8
compile options: '-I/tmp/tmp44thhw01/src.linux-x86_64-3.8 -I/usr/share/miniconda3/envs/runenv/lib/pytho.
gcc: /tmp/tmp44thhw01/src.linux-x86_64-3.8/fortranobject.c
gcc: /tmp/tmp44thhw01/src.linux-x86_64-3.8/mylapackmodule.c
In file included from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/
                               from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/
                              from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/s
                               from /tmp/tmp44thhw01/src.linux-x86_64-3.8/fortranobject.h:13,
                               from /tmp/tmp44thhw01/src.linux-x86_64-3.8/fortranobject.c:2:
/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/numpy/npy_1_7_deprecat
 #warning "Using deprecated NumPy API, disable it with " \
In file included from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/
                              from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/s
                               from /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/
                               from /tmp/tmp44thhw01/src.linux-x86_64-3.8/fortranobject.h:13,
                               from /tmp/tmp44thhw01/src.linux-x86_64-3.8/mylapackmodule.c:16:
/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/core/include/numpy/npy_1_7_deprecat
 #warning "Using deprecated NumPy API, disable it with " \
/tmp/tmp44thhw01/src.linux-x86_64-3.8/mylapackmodule.c:142:12: warning: 'f2py_size' defined but not use
  static int f2py_size(PyArrayObject* var, ...)
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
Removing build directory /tmp/tmp44thhw01
In [26]: import numpy as np
              import mylapack
              a = np.array([[7,8],[3,4],[1,2]])
              b = np.array([[1,2,3],[4,5,6]])
              print("a=",a)
              print("b=",b)
              assert a.shape[1] == b.shape[0]
              c = np.zeros((a.shape[0],b.shape[1]),'d',order='F')
              mylapack.dgemm('N','N',a.shape[0],b.shape[1],a.shape[1],1.0,a,b,1.0,c)
              print(c)
              np.all(c == a @ b) # check with numpy matrix multiplication
a= [[7 8]
 [3 4]
  [1 2]]
b = [[1 \ 2 \ 3]]
 [4 5 6]]
[[39. 54. 69.]
  [19. 26. 33.]
```

[9. 12. 15.]]

Out[26]: True

77.0.1 Exercise

• Modify the file dgemm.pyf to set all arguments top optional and keep only the two matrices as input.

In [27]: # $\label{local_solutions_fortran_dgemm2.pyf}$

Build the pythoni module

```
In [28]: !{sys.executable} -m numpy.f2py -c dgemm2.pyf -llapack --f90flags=-03
Traceback (most recent call last):
  File "/usr/share/miniconda3/envs/runenv/lib/python3.8/runpy.py", line 194, in _run_module_as_main
   return _run_code(code, main_globals, None,
  File "/usr/share/miniconda3/envs/runenv/lib/python3.8/runpy.py", line 87, in _run_code
    exec(code, run_globals)
  File "/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/f2py/__main__.py", line 4,
  File "/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/f2py/f2py2e.py", line 692,
   run_compile()
  File "/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/f2py/f2py2e.py", line 603,
    modulename = get_f2py_modulename(f)
  File "/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numpy/distutils/command/build_src
    with open(source) as f:
FileNotFoundError: [Errno 2] No such file or directory: 'dgemm2.pyf'
In [29]: import mylapack2
        a = np.array([[7,8],[3,4],[1,2]])
        b = np.array([[1,2,3],[4,5,6]])
        c = mylapack2.dgemm(a,b)
        np.all(c == a @ b)
                                                   Traceback (most recent call last)
        ModuleNotFoundError
        <ipython-input-29-bc49cfff5b3c> in <module>
    ----> 1 import mylapack2
          2 a = np.array([[7,8],[3,4],[1,2]])
          3 b = np.array([[1,2,3],[4,5,6]])
          4 c = mylapack2.dgemm(a,b)
          5 np.all( c == a @ b)
        ModuleNotFoundError: No module named 'mylapack2'
```

78.1 Check performance between numpy and mylapack

```
In [30]: a = np.random.random((512,128))
        b = np.random.random((128,512))
In [31]: %timeit c = mylapack2.dgemm(a,b)
       NameError
                                                   Traceback (most recent call last)
        <ipython-input-31-0ea4b0142b0d> in <module>
    ----> 1 get_ipython().run_line_magic('timeit', 'c = mylapack2.dgemm(a,b)')
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/interactiveshell.py
                            kwargs['local_ns'] = self.get_local_scope(stack_depth)
       2325
                        with self.builtin_trap:
    -> 2326
                            result = fn(*args, **kwargs)
       2327
                        return result
       2328
        <decorator-gen-60> in timeit(self, line, cell, local_ns)
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magic.py in <lambda>
        185
                # but it's overkill for just that one bit of state.
        186
                def magic_deco(arg):
    --> 187
                    call = lambda f, *a, **k: f(*a, **k)
        188
        189
                    if callable(arg):
       /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py
       1167
                        for index in range(0, 10):
       1168
                            number = 10 ** index
    -> 1169
                            time_number = timer.timeit(number)
       1170
                            if time_number >= 0.2:
       1171
                                break
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py
                    gc.disable()
        167
        168
    --> 169
                        timing = self.inner(it, self.timer)
        170
                    finally:
        171
                        if gcold:
        <magic-timeit> in inner(_it, _timer)
        NameError: name 'mylapack2' is not defined
```

```
In [32]: %timeit c = a @ b
1.39 ms \pm 9.24 \mus per loop (mean \pm std. dev. of 7 runs, 1000 loops each)
   Fortran arrays allocated in a subroutine share same memory in Python
In [33]: %%fortran
        module f90module
           implicit none
           real(8), dimension(:), allocatable :: farray
        contains
            subroutine init( n ) !Allocation du tableau farray
            integer, intent(in) :: n
            allocate(farray(n))
            end subroutine init
        end module f90module
In [34]: f90module.init(10)
        len(f90module.farray)
Out[34]: 10
   Numpy arrays allocated in Python passed to Fortran are already allocated
In [35]: %%fortran
        module f90module
           implicit none
           real(8), dimension(:), allocatable :: farray
            subroutine test_array( allocated_flag, array_size )
           logical, intent(out) :: allocated_flag
            integer, intent(out) :: array_size
           allocated_flag = allocated(farray)
           array_size = size(farray)
            end subroutine test_array
        end module f90module
In [36]: f90module.farray = np.random.rand(10).astype(np.float64)
        f90module.test_array()
```

Out[36]: (1, 10)

f2py + OpenMP

```
In [37]: %env OMP_NUM_THREADS=4
env: OMP_NUM_THREADS=4
In [38]: %%fortran
        subroutine hello()
          integer :: i
          do i = 1, 4
            call sleep(1)
          end do
        end subroutine
In [39]: %%time
        hello()
CPU times: user 21.8 ms, sys: 4.54 ms, total: 26.3 ms
Wall time: 4 s
In [40]: %%fortran --f90flags "-fopenmp" --extra "-L/usr/local/lib -lgomp"
        subroutine hello_omp( )
          integer :: i
          !$OMP PARALLEL PRIVATE(I)
          !$OMP DO
          do i = 1, 4
            call sleep(1)
          end do
          !$OMP END DO
          !$OMP END PARALLEL
        end subroutine
In [41]: %%time
        hello_omp()
CPU times: user 6.61 ms, sys: 158 µs, total: 6.77 ms
Wall time: 1 s
```

Conclusions

- Easy to use, it works with modern fortran, legacy fortran and also C.
- Works with common and modules and arrays dynamically allocated.
- Python function callback can be very useful combined with Sympy
- Documentation is automatically generated
- All fortran compilers are supported: GNU, Portland, Sun, Intel,...
- F2py is integrated in numpy library.

80.1 cons

- Derived types and fortran pointers are not well supported.
- Absolutely not compatible with fortran 2003-2008 new features (classes)
- f2py is maintained but not really improved. Development is stopped.

distutils

81.1 setup.py

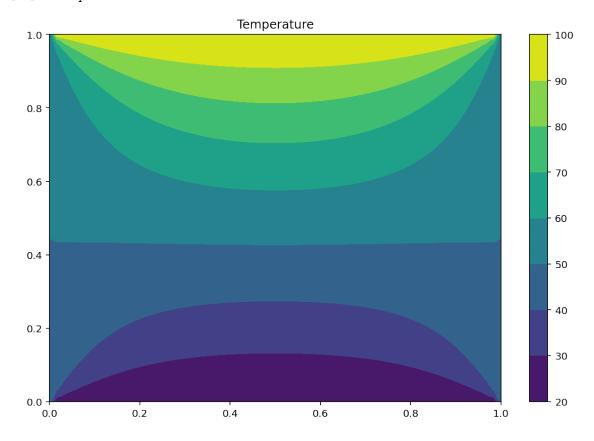
81.1.1 Exercice: Laplace problem

• Replace the laplace function by a fortran subroutine

```
In [42]: %%time
         %matplotlib inline
         %config InlineBackend.figure_format = 'retina'
         import numpy as np
         import matplotlib.pyplot as plt
         import itertools
         # Boundary conditions
        Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50
         # Set meshgrid
        n, 1 = 64, 1.0
        X, Y = np.meshgrid(np.linspace(0,1,n), np.linspace(0,1,n))
        T = np.zeros((n,n))
         # Set Boundary condition
        T[n-1:, :] = Tnorth
        T[:1, :] = Tsouth
        T[:, n-1:] = Teast
        T[:, :1] = Twest
         def laplace(T, n):
            residual = 0.0
             for i in range(1, n-1):
                for j in range(1, n-1):
```

```
T_old = T[i,j]
                    T[i, j] = 0.25 * (T[i+1,j] + T[i-1,j] + T[i,j+1] + T[i,j-1])
                    if T[i,j]>0:
                        residual=max(residual,abs((T_old-T[i,j])/T[i,j]))
            return residual
        residual = 1.0
        istep = 0
        while residual > 1e-5:
            istep += 1
            residual = laplace(T, n)
            print ((istep, residual), end="\r")
        print("iterations = ",istep)
        plt.rcParams['figure.figsize'] = (10,6.67)
        plt.title("Temperature")
        plt.contourf(X, Y, T)
        plt.colorbar()
iterations = 2457
CPU times: user 32.2 s, sys: 382 ms, total: 32.6 s
Wall time: 32.2 s
```

Out[42]: <matplotlib.colorbar.Colorbar at 0x7ff92b009820>



81.1. SETUP.PY 247

```
real(8), intent(inout) :: T(0:n-1,0:n-1) ! Python indexing
          integer, intent(in)
                                :: n
          real(8), intent(out) :: residual
          real(8) :: T_old
          residual = 0.0
          do i = 1, n-2
            do j = 1, n-2
                T_old = T(i,j)
                T(i, j) = 0.25 * (T(i+1,j) + T(i-1,j) + T(i,j+1) + T(i,j-1))
                    if (T(i,j) > 0) then
                        residual=max(residual,abs((T_old-T(i,j))/T(i,j)))
                    end if
             end do
          end do
         end subroutine laplace_fortran
          File "<ipython-input-43-666eb7786496>", line 2
        subroutine laplace_fortran( T, n, residual )
    SyntaxError: invalid syntax
In [44]: %%time
        %matplotlib inline
        %config InlineBackend.figure_format = 'retina'
        import numpy as np
        import matplotlib.pyplot as plt
         import itertools
         # Boundary conditions
        Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50
        # Set meshgrid
        n, 1 = 64, 1.0
        X, Y = np.meshgrid(np.linspace(0,1,n), np.linspace(0,1,n))
        T = np.zeros((n,n), order='F') ## We need to declare a new order in memory
         # Set Boundary condition
        T[n-1:, :] = Tnorth
        T[:1, :] = Tsouth
        T[:, n-1:] = Teast
        T[:, :1] = Twest
        residual = 1.0
        istep = 0
        while residual > 1e-5:
            istep += 1
            residual = laplace_fortran(T, n)
            print ((istep, residual), end="\r")
        print()
        print("iterations = ",istep)
        plt.rcParams['figure.figsize'] = (10,6.67)
        plt.title("Temperature")
        plt.contourf(X, Y, T)
```

nl+	colorbar	٠/١
рт с.	COTOLDAL	()

NameError

Traceback (most recent call last)

<timed exec> in <module>

 ${\tt NameError:\ name\ 'laplace_fortran'\ is\ not\ defined}$

References

- Talk by E. Sonnendrücker
- SciPy
- Sagemath Documentation
- $\bullet\,$ Hans Petter Langtangen. Python Scripting for Computational Science. Springer 2004

Cython

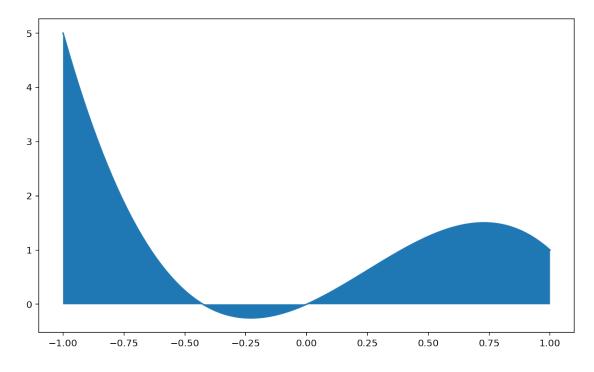
```
In [1]: %matplotlib inline
    import matplotlib.pyplot as plt
    plt.rcParams['figure.figsize'] = (10,6)
    %config InlineBackend.figure_format = 'retina'
    import numpy as np
```



- Cython provides extra syntax allowing for static type declarations (remember: Python is generally dynamically typed)
- Python code gets translated into optimised C/C++ code and compiled as Python extension modules
- Cython allows you to write fast C code in a Python-like syntax.
- Furthermore, linking to existing C libraries is simplified.

Pure Python Function

```
\begin{split} f(x) &= -2x^3 + 5x^2 + x, \\ \text{In [2]: def } f(x): \\ & \text{return } -4*x**3 + 3*x**2 + 2*x \\ \\ & x = \text{np.linspace}(-1,1,100) \\ & \text{ax} = \text{plt.subplot}(1,1,1) \\ & \text{ax.plot}(x, f(x)) \\ & \text{ax.fill_between}(x, 0, f(x)); \end{split}
```



we compute integral $\int_a^b f(x)dx$ numerically with N points.

```
In [3]: def integrate_f_py(a,b,N): s = 0 dx = (b - a) / (N-1) for i in range(N-1): # we intentionally use the bad way to do this with a loop
```

```
x = a + i*dx
              s += (f(x)+f(x+dx))/2
          return s*dx
In [4]: %timeit integrate_f_py(-1,1,10**3)
       print(integrate_f_py(-1,1,1000))
961 µs ± 13 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
2.0000040080120174
In [5]: %load_ext heat
        ModuleNotFoundError
                                                  Traceback (most recent call last)
        <ipython-input-5-7b0de4361b3c> in <module>
    ---> 1 get_ipython().run_line_magic('load_ext', 'heat')
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/interactiveshell.py
                            kwargs['local_ns'] = self.get_local_scope(stack_depth)
       2324
       2325
                        with self.builtin_trap:
    -> 2326
                            result = fn(*args, **kwargs)
       2327
                        return result
       2328
        <decorator-gen-64> in load_ext(self, module_str)
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magic.py in <lambda>
        185
                # but it's overkill for just that one bit of state.
        186
                def magic_deco(arg):
    --> 187
                   call = lambda f, *a, **k: f(*a, **k)
        188
        189
                    if callable(arg):
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/extension.py
        31
                    if not module_str:
         32
                        raise UsageError('Missing module name.')
    ---> 33
                    res = self.shell.extension_manager.load_extension(module_str)
         34
         35
                    if res == 'already loaded':
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/extensions.py in loa
         78
                        if module_str not in sys.modules:
         79
                            with prepended_to_syspath(self.ipython_extension_dir):
    ---> 80
                                mod = import_module(module_str)
                                if mod.__file__.startswith(self.ipython_extension_dir):
         81
         82
                                    print(("Loading extensions from {dir} is deprecated. "
```

```
/usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/__init__.py in import_module(name, pa
        125
                            break
        126
                        level += 1
    --> 127
                return _bootstrap._gcd_import(name[level:], package, level)
        128
        129
        /usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/_bootstrap.py in _gcd_import(name, pa
        /usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/_bootstrap.py in _find_and_load(name,
        /usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/_bootstrap.py in _find_and_load_unloc
        ModuleNotFoundError: No module named 'heat'
In [6]: %%heat
       def f(x):
          return -4*x**3 +3*x**2 +2*x
       def integrate_f(a, b, N):
          s = 0
          dx = (b - a) / (N-1)
          for i in range(N-1):
              x = a + i*dx
              s += (f(x)+f(x+dx))/2
```

UsageError: Cell magic `%%heat` not found.

return s*dx

integrate_f(0, 10, 1000)

Pure C function

```
In [7]: %%file integral_f_c.c
        #include <stdlib.h>
        #include <stdio.h>
        #include <time.h>
        \#define\ NB\_RUNS\ 1000
        double f(double x) {
           return -4*x*x*x +3*x*x +2*x;
        double integrate_f_c(double a, double b, int N) {
            double s = 0;
            double dx = (b - a) / (N-1);
            for(int i=0; i<N-1; ++i){
                double x = a + i*dx;
                s += (f(x)+f(x+dx))/2.0;
           return s*dx;
        }
        int main(int argc, char **argv)
          double a = atof(argv[1]);
          double b = atof(argv[2]);
          int N = atoi(argv[3]);
          double res = 0;
          clock_t begin = clock();
          for (int i=0; i<NB_RUNS; ++i)</pre>
              res += integrate_f_c( a, b, N );
          clock_t end = clock();
          fprintf(stdout, "integral_f(%3.1f, %3.1f, %d) = %f \n", a, b, N, res / NB_RUNS);
          fprintf( stdout, "time = \%e ms \n", (double)(end - begin) / CLOCKS_PER_SEC );
          return 0;
        }
```

```
Writing integral_f_c.c
```

```
In [8]: !gcc -03 integral_f_c.c; ./a.out -1 1 1000
integral_f(-1.0, 1.0, 1000) = 2.000004
time = 8.663000e-03 ms
```

Cython compilation: Generating C code

Load Cython in jupyter notebook.

86.1 C Variable and Type definitions

In general, use cdef to declare C variables. The command :

```
$ cython -a mycode.pyx
```

outputs an html file. It shows what parts of your code are C, which parts are Python, and where C-Python conversion occurs.

Out[11]: <IPython.core.display.HTML object>

86.1.1 Another Python vs. Cython coloring guide

```
In [12]: %%cython -a
        cdef int m, n
        cdef double cy_total = 0.0
        for m in range(10):
            n = 2*m
            cy_total += n
        a, b = 0, 0
        py_total = 0.0
        for a in range(10):
            b = 2*a
            py_total += b
        print(cy_total, py_total)
90.0 90.0
Out[12]: <IPython.core.display.HTML object>
In [13]: %%cython -a
        cdef struct Grail:
            int age
            float volume
        cdef union Food:
            char *spam
            float *eggs
        cdef enum CheeseType:
            cheddar, edam,
            camembert
        cdef enum CheeseState:
            hard = 1
            soft = 2
            runny = 3
        cdef Grail holy
        holy.age
                  = 500
        holy.volume = 10.0
        print (holy.age, holy.volume)
500 10.0
Out[13]: <IPython.core.display.HTML object>
```

Cython Functions

Use **cdef** to define a Cython function.

- Cython function can accept either (inclusive) Python and C values as well as return either Python or C values, - *Within a Cython module* Python and Cython functions can call each other freely. However, only **Python** functions can be called from outside the module by Python code. (i.e. importing/exporting a Cython module into some Python code)

cpdef define a Cython function with a simple Python wrapper. However, when called from Cython the Cython / C code is called directly, bypassing the Python wrapper.

Writing pure code in Cython gives a small speed boost. Note that none of the code below is Cython-specific. Just add .pyx instead of .py extension.

```
In [14]: %%file cython_f_example.pyx
    def f(x):
        return -4*x**3 +3*x**2 +2*x
    def integrate_f(a, b, N):
        s = 0
        dx = (b - a) / (N-1)
        for i in range(N-1):
            x = a + i*dx
            s += (f(x)+f(x+dx))/2
        return s*dx
```

Writing cython_f_example.pyx

Cython Compilation

- The .pyx source file is compiled by Cython to a .c file.
- The .c source file contains the code of a Python extension module.
- The .c file is compiled by a C compiler to a .so (shared object library) file which can be imported directly into a Python session.

88.1 Build with CMake

```
project(cython_f_example CXX)
include(UseCython) # Load Cython functions
# Set C++ output
set_source_file_properties(cython_f_example.pyx PROPERTIES CYTHON_IS_CXX TRUE )
# Build the extension module
cython_add_module( modname cython_f_example.pyx cython_f_example.cpp )
```

88.2 C/C++ generation with cython application

```
cython -3 cython_f_example.pyx # create the C file for Python 3
cython -3 --cplus cython_f_example.pyx # create the C++ file for Python 3
```

88.3 build with a C/C++ compiler

To build use the Makefile:

```
CC=gcc
CFLAGS=`python-config --cflags`
LDFLAGS=`python-config --ldflags`
cython_f_example:
    ${CC} -c $0.c ${CFLAGS}
    ${CC} $0.o -o $0.so -shared ${LDFLAGS}

Import the module in Python session
import cython_f_example
```

pyximport

Building a Cython module using distutils

```
Create the setup.py script:
In [16]: %%file setup.py
        from distutils.core import setup
        from Cython.Build import cythonize
        setup(
          name = 'Cython Example Integrate f Function',
          ext_modules = cythonize("cython_f_example.pyx"),
Writing setup.py
In [17]: %run setup.py build_ext --inplace
Compiling cython_f_example.pyx because it changed.
[1/1] Cythonizing cython_f_example.pyx
running build_ext
building 'cython_f_example' extension
creating build
creating build/temp.linux-x86_64-3.8
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
<Figure size 720x432 with 0 Axes>
In [18]: from cython_f_example import integrate_f
        %timeit integrate_f(-1,1,10**3)
        integrate_f(-1,1,10**3)
757 µs ± 4.88 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
Out[18]: 2.0000040080120174
```

Why is it faster with Cython?

- Python code is interpreted at every execution to machine code.
- Compiled C code is already in machine code.
- C is a statically-typed language. It gives to the compiler more information which allows it to optimize both computations and memory access.
- To add two variables, Python checks the type before calling the right **add** function and store it to a value that can be new.
- C just add the variables and return the result.

Add Cython types

We coerce Python types to C types when calling the function. Still a "Python function" so callable from the global namespace.

```
In [19]: %%cython
    def f(x):
        return -4*x**3 +3*x**2 +2*x
    def cy_integrate_f(double a, double b, int N):
        cdef int i
        cdef double s, x, dx
        s = 0
        dx = (b - a) / (N-1)
        for i in range(N-1):
            x = a + i*dx
            s += (f(x)+f(x+dx))/2
        return s*dx
```

building '_cython_magic_8c0baa0b730cb71c660b134eef8e4b03' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/

- typing the iterator variable i with C semantics, tells Cython to compile the for-loop to pure C code.
- typing a, s and dx is important as they are involved in arithmetic within the for-loop
- Cython type declarations can make the source code less readable
- Do not use them without good reason, i.e. only in performance critical sections.

Finally, we integrate a Cython function instead of a Python function. This eliminates the Python-C conversion at the function call as seen above thus giving a pure Cython/C algorithm.

The primary downside is not being allowed to call the function cy_f, from Python unless cpdef is used.

```
def cycy_integrate_f(double a, double b, int N):
            cdef int i
            cdef double s, x, dx
            s = 0
            dx = (b - a) / (N-1)
            for i in range(N-1):
                x = a + i*dx
                s += (cy_f(x)+cy_f(x+dx))/2
            return s*dx
building '_cython_magic_c253bb1961eda445c34ddc2d7b4ab8bc' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
In [22]: %timeit cycy_integrate_f(-1,1,10**3)
        print(cycy_integrate_f(-1,1,1000))
156 \mu s \pm 1.95 \mu s per loop (mean \pm std. dev. of 7 runs, 10000 loops each)
2.0000040080120174
```

Exercise: Cythonize the trivial exponential function.

```
In [23]: %%cython -a
        def exp_python(x,terms=50):
            sum = 0.
            power = 1.
           fact = 1.
            for i in range(terms):
               sum += power/fact
               power *= x
               fact *= i+1
            return sum
building '_cython_magic_a96a095f6c3175b142027432bac51f88' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
Out[23]: <IPython.core.display.HTML object>
In [24]: %timeit exp_python(1.,50)
4.39~\mu s~\pm~149~ns per loop (mean \pm~std.~dev.~of~7~runs,~100000~loops~each)
In [25]: %%cython
        # %load solutions/cython/exponential.pyx
building '_cython_magic_eb57b1dfc3f16e554f6b2bfcf272fbe2' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
In [26]: %timeit exp_cython(1.,50)
        NameError
                                                    Traceback (most recent call last)
        <ipython-input-26-e456486158ce> in <module>
    ----> 1 get_ipython().run_line_magic('timeit', 'exp_cython(1.,50)')
```

```
/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/interactiveshell.py
   2324
                        kwargs['local_ns'] = self.get_local_scope(stack_depth)
   2325
                   with self.builtin_trap:
-> 2326
                        result = fn(*args, **kwargs)
  2327
                   return result
   2328
    <decorator-gen-60> in timeit(self, line, cell, local_ns)
    /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magic.py in <lambda>
            # but it's overkill for just that one bit of state.
    185
    186
            def magic_deco(arg):
--> 187
                call = lambda f, *a, **k: f(*a, **k)
    188
    189
                if callable(arg):
   /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py
   1167
                   for index in range(0, 10):
   1168
                        number = 10 ** index
-> 1169
                        time_number = timer.timeit(number)
  1170
                        if time_number >= 0.2:
   1171
                            break
    /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py
    167
                gc.disable()
    168
                try:
--> 169
                    timing = self.inner(it, self.timer)
    170
                finally:
    171
                    if gcold:
    <magic-timeit> in inner(_it, _timer)
   NameError: name 'exp_cython' is not defined
```

Cython and Numpy

The Numpy library contains many fast numerics routines. Their speed comes from manipulating the low-level C-arrays that the numpy array object wraps rather than computing over slow Python lists. Using Cython one can access those low-level arrays and implement their own fast algorithms while allowing the easy interaction afforded by Python + Numpy.

The examples below are various implementations of the naive matrix multiplication algorithm. We will start with a pure Python implementation and then incrementally add structures that allow Cython to exploit the low-level speed of the numpy array object.

94.0.1 Pure Python implementation compiled in Cython without specific optimizations.

```
In [27]: %%cython
    def matmul1(A, B, out=None):
        assert A.shape[1] == B.shape[0]
    for i in range(A.shape[0]):
        for j in range(B.shape[1]):
        s = 0
        for k in range(A.shape[1]):
        s += A[i,k] * B[k,j]
        out[i,j] = s
    return out
```

building '_cython_magic_899b76c7c47e2932ff2e677e49192f5a' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/run

Import numpy as a Cython module

We now take advantage of the ability to access the underlying C arrays in the numpy.array object from Cython, thanks to a special numpy.pxd file included with Cython. (The Cython developers worked closely with Numpy developers to make this optimal.)

To begin with, we have to cimport numpy: that is, import numpy as a **Cython** module rather than a **Python** module. To do so, simply type:

```
cimport numpy as np
```

In [28]: %%cython

In [29]: import numpy as np

In [30]: %timeit matmul1(A,B,C)

from timeit import timeit

C = np.zeros((64,64))

A = np.random.random_sample((64,64)) B = np.random.random_sample((64,64))

Another important thing to note is the type of Numpy indexers. There is a special Numpy variable type used for numpy.array indices called Py_ssize_t. To take full advantage of the speedups that Cython can provide we should make sure to type the variables used for indexing as such.

```
import numpy as np
        cimport numpy as np
        ctypedef np.float64_t dtype_t
                                          # shorthand type. easy to change
        def matmul2(np.ndarray[dtype_t, ndim=2] A,
                    np.ndarray[dtype_t, ndim=2] B,
                    np.ndarray[dtype_t, ndim=2] out=None):
            cdef Py_ssize_t i, j, k
            cdef dtype_t s
            assert A.shape[1] == B.shape[0]
            for i in range(A.shape[0]):
                for j in range(B.shape[1]):
                    for k in range(A.shape[1]):
                        s += A[i,k] * B[k,j]
                    out[i,j] = s
            return out
building '_cython_magic_44357700cb4ec61887ddbee898be64d7' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
```

```
155 ms \pm 659 \mus per loop (mean \pm std. dev. of 7 runs, 10 loops each)

In [31]: %timeit matmul2(A,B,C)

338 \mus \pm 4.85 \mus per loop (mean \pm std. dev. of 7 runs, 1000 loops each)
```

95.0.1 Tuning indexing

The array lookups are still slowed down by two factors: * Bounds checking is performed. * Negative indices are checked for and handled correctly.

The code doesn't use negative indices, and always access to arrays within bounds. We can add a decorator to disable bounds checking:

```
In [32]: %%cython
         cimport cython
                                                               # cython tools
         import numpy as np
         cimport numpy as np
         ctypedef np.float64_t dtype_t
         @cython.boundscheck(False) # turn off bounds-checking for entire function
         @cython.wraparound(False) # turn off negative index wrapping for entire function
         def matmul3(np.ndarray[dtype_t, ndim=2] A,
                     np.ndarray[dtype_t, ndim=2] B,
                     np.ndarray[dtype_t, ndim=2] out=None):
             cdef Py_ssize_t i, j, k
             cdef dtype_t s
             assert A.shape[1] == B.shape[0]
             for i in range(A.shape[0]):
                 for j in range(B.shape[1]):
                     s = 0
                     for k in range(A.shape[1]):
                        s += A[i,k] * B[k,j]
                     out[i,j] = s
             return out
```

building '_cython_magic_75874d7a10a3fb62ace5ee487efe1f5a' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/

```
In [33]: %timeit matmul3(A,B,C)

292 µs ± 9.72 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

Cython Build Options

- boundcheck(True,False): array bounds checking
- wraparound(True,False) : negative indexing.
- initialized check(True,False): checks that a memory view is initialized
- nonecheck(True,False): Check if one argument is None
- overflowcheck(True,False) : Check if int are too big
- cdivision(True,False): If False, adjust the remainder and quotient operators C types to match those of Python ints. Could be very effective when it is set to True.
- profile (True / False): Write hooks for Python profilers into the compiled C code. Default is False.

Cython Compiler directives

Numpy objects with external C program.

Note that this can actually be slower because the C function is not the best implementation of matrix multiplication. Call cblas with same technique is an interesting exercise.

Writing mydgemm.c

- The np.ndarray[double, ndim=2, mode="c"] assures that you get a C-contiguous numpy array of doubles
- The &input[0,0] passed in the address of the beginning of the data array.

```
cdef int m = A.shape[0]
            cdef int n = A.shape[1]
            cdef int k = B.shape[1]
           cdef dtype_t s
           my_dgemm(m, n, k, &A[0,0], &B[0,0], &C[0,0])
            return C
building '_cython_magic_ac507ac6e349df671064279adff4ef16' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
In [36]: %timeit matmul4(A,B,C)
        NameError
                                                   Traceback (most recent call last)
        <ipython-input-36-edb42162d6ab> in <module>
    ----> 1 get_ipython().run_line_magic('timeit', 'matmul4(A,B,C)')
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/interactiveshell.py
       2324
                            kwargs['local_ns'] = self.get_local_scope(stack_depth)
       2325
                        with self.builtin_trap:
    -> 2326
                            result = fn(*args, **kwargs)
       2327
                        return result
       2328
        <decorator-gen-60> in timeit(self, line, cell, local_ns)
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magic.py in <lambda>
        185
                # but it's overkill for just that one bit of state.
        186
                def magic deco(arg):
                    call = lambda f, *a, **k: f(*a, **k)
    --> 187
        188
        189
                    if callable(arg):
       /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py
                        for index in range(0, 10):
       1167
       1168
                            number = 10 ** index
    -> 1169
                            time_number = timer.timeit(number)
       1170
                            if time_number >= 0.2:
       1171
                                break
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py
        167
                    gc.disable()
        168
    --> 169
                        timing = self.inner(it, self.timer)
        170
                    finally:
```

```
171
                      if gcold:
       <magic-timeit> in inner(_it, _timer)
       NameError: name 'matmul4' is not defined
97.0.1 Exercise: Find prime numbers < 10000
In [37]: # %load solutions/cython/is_primeO.py
In [38]: [ p for p in range(20) if is_primeO(p)]
       NameError
                                               Traceback (most recent call last)
       <ipython-input-38-3839f7983258> in <module>
   ---> 1 [ p for p in range(20) if is_primeO(p)]
       <ipython-input-38-3839f7983258> in <listcomp>(.0)
   ---> 1 [ p for p in range(20) if is_primeO(p)]
       NameError: name 'is_prime0' is not defined
In [39]: L = list(range(10000))
       %timeit [ p for p in L if is_primeO(p)]
       ______
       NameError
                                               Traceback (most recent call last)
       <ipython-input-39-227d4c8590e5> in <module>
         1 L = list(range(10000))
   ----> 2 get_ipython().run_line_magic('timeit', '[ p for p in L if is_primeO(p)]')
       /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/interactiveshell.py
      2324
                          kwargs['local_ns'] = self.get_local_scope(stack_depth)
      2325
                      with self.builtin_trap:
   -> 2326
                          result = fn(*args, **kwargs)
      2327
                     return result
      2328
       <decorator-gen-60> in timeit(self, line, cell, local_ns)
       /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magic.py in <lambda>
```

```
185
                 # but it's overkill for just that one bit of state.
        186
                 def magic_deco(arg):
    --> 187
                     call = lambda f, *a, **k: f(*a, **k)
        188
        189
                     if callable(arg):
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py
       1167
                         for index in range(0, 10):
       1168
                              number = 10 ** index
    -> 1169
                              time_number = timer.timeit(number)
       1170
                              if time_number >= 0.2:
       1171
                                  break
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/execution.py
        167
                     gc.disable()
        168
                     try:
    --> 169
                          timing = self.inner(it, self.timer)
        170
                     finally:
        171
                          if gcold:
        <magic-timeit> in inner(_it, _timer)
        <magic-timeit> in <listcomp>(.0)
        NameError: name 'is_prime0' is not defined
In [40]: %%cython
        def is_prime1(n):
            if n < 4: return True
            if n % 2 == 0 : return False
            k = 3
            while k*k <= n:
                if n % k == 0: return False
                k += 2
            return True
building '_cython_magic_f81b23461181c5d78ab3700de598d3b5' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
In [41]: [ p for p in range(20) if is_prime1(p)]
Out [41]: [0, 1, 2, 3, 5, 7, 11, 13, 17, 19]
In [42]: %timeit [p for p in L if is_prime1(p)]
4.7 \text{ ms} \pm 103 \text{ } \mu \text{s} \text{ per loop (mean} \pm \text{ std. dev. of } 7 \text{ runs, } 100 \text{ loops each)}
```

97.0.2 Add Cython types without modifying the Python Code

```
In [43]: %%cython
        import cython
        @cython.locals(n=int, k=int)
        def is_prime2(n):
            if n < 4: return True
            if n \% 2 == 0 : return False
            k = 3
            while k*k \le n:
                if n % k == 0: return False
                k += 2
            return True
building '_cython_magic_745b858a4456e55af5f92e69b9968cd3' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
In [44]: [ p for p in range(20) if is_prime2(p)]
Out[44]: [0, 1, 2, 3, 5, 7, 11, 13, 17, 19]
In [45]: %timeit [p for p in L if is_prime2(p) ]
680 \mu s \pm 8.78 \mu s per loop (mean \pm std. dev. of 7 runs, 1000 loops each)
97.0.3 Cython function
In [46]: %%cython
        import cython
        cdef bint is_prime3(int n):
            if n < 4: return True
            if n % 2 == 0: return False
            cdef int k = 3
            while k*k <= n:
                if n % k == 0: return False
                k += 2
            return True
        def prime_list(L):
            return [p for p in L if is_prime3(p)]
building '_cython_magic_666b225df8666611ff2af72a85a03e9d' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
In [47]: prime_list(list(range(20)))
Out [47]: [0, 1, 2, 3, 5, 7, 11, 13, 17, 19]
In [48]: %timeit prime_list(L)
369 \mu s \pm 4.2 \ \mu s per loop (mean \pm std. dev. of 7 runs, 1000 loops each)
In [49]: %%cython
        import cython
        from numpy cimport ndarray
```

```
import numpy
        cdef bint is_prime3(int n):
            if n < 4: return True
            if n % 2 == 0: return False
            cdef int k = 3
            while k*k <= n:
               if n % k == 0: return False
               k += 2
            return True
        def prime_array(ndarray[int, ndim=1] L):
            cdef ndarray[int, ndim=1] res = ndarray(shape=(L.shape[0]),dtype=numpy.int32)
            cdef int i
            for i in range(L.shape[0]):
               res[i] = is_prime3(L[i])
            return L[res==1]
building '_cython_magic_2a5b09b5e9c574f8e7fa68917eb3edc0' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/
In [50]: import numpy as np
        prime_array(np.arange(20,dtype=np.int32))
Out[50]: array([ 0, 1, 2, 3, 5, 7, 11, 13, 17, 19], dtype=int32)
In [51]: npL = numpy.array(L,dtype=np.int32)
        %timeit prime_array(npL)
```

376 $\mu s \pm 2.83 \mu s$ per loop (mean \pm std. dev. of 7 runs, 1000 loops each)

Using Parallelism

- Cython supports native parallelism via OpenMP
- by default, Python's Global Interpreter Lock (GIL) prevents that several threads use the Python interpreter simultaneously
- to use this kind of parallelism, the GIL must be released

If you have a default compiler with openmp support you can use this magic command in your notebook.

```
%%cython --compile-args=-fopenmp --link-args=-fopenmp
In [52]: %%file cython_omp.pyx
        import cython
        from cython.parallel cimport parallel, prange # import parallel functions
        import numpy as np
        from numpy cimport ndarray
         cdef bint is_prime4(int n) nogil:
                                              #release the gil
            if n < 4: return True
            if n % 2 == 0: return False
             cdef int k = 3
             while k*k <= n:
                if n % k == 0: return False
                k += 2
             return True
         @cython.boundscheck(False)
        def prime_array_omp(ndarray[int, ndim=1] L):
            cdef ndarray[int, ndim=1] res = ndarray(shape=(L.shape[0]),dtype=np.int32)
            cdef Py_ssize_t i
             with nogil, parallel(num_threads=4):
                 for i in prange(L.shape[0]):
                                                 #Parallel loop
                    res[i] = is_prime4(L[i])
             return L[res==1]
```

To use the OpenMP support, you need to enable OpenMP. For gcc this can be done as follows in a setup.py:

Writing cython_omp.pyx

```
import os, sys
                     import numpy
                     if sys.platform == "darwin":
                               os.environ["CC"] = "gcc-8" # Change the compiler to gcc on Mac
                               os.environ["CXX"] = "g++-8"
                     ext modules = [
                               Extension(
                                         "cython_omp",
                                         ["cython_omp.pyx"],
                                         extra_compile_args=['-fopenmp'],
                                         extra_link_args=['-fopenmp'],
                                        include_dirs=[numpy.get_include()]
                               )
                     ]
                     setup(
                               name='Cython OpenMP Example',
                               ext_modules=cythonize(ext_modules),
                      # python setup.py build_ext --inplace
Overwriting setup.py
In [54]: %run setup.py build_ext --inplace
Compiling cython_omp.pyx because it changed.
[1/1] Cythonizing cython_omp.pyx
/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/Cython/Compiler/Main.py:369: FutureWarnin
     tree = Parsing.p_module(s, pxd, full_module_name)
running build_ext
building 'cython_omp' extension
gcc -pthread -B /usr/share/miniconda3/envs/runenv/compiler_compat -W1,--sysroot=/ -Wsign-compare -DNDEB
gcc -pthread -shared -B /usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/runenv/compiler_compat -L/usr/share/miniconda3/envs/runenv/compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_compiler_c
In [55]: from cython_omp import prime_array_omp
In [56]: prime_array_omp(np.arange(20,dtype=np.int32))
Out[56]: array([ 0, 1, 2, 3, 5, 7, 11, 13, 17, 19], dtype=int32)
In [57]: %timeit prime_array_omp(npL)
311 \mu s \pm 9.38 \ \mu s per loop (mean \pm std. dev. of 7 runs, 1000 loops each)
```

98.1 References

- Cython documentation
- An Interactive Introduction to Cython by Chris Swierczewski
- Introduction To Python by Michael Kraus

98.1. REFERENCES 289

- Cython by Xavier Juvigny
- Cython: C-Extensions for Python, Wiki
- Kurt W. Smith
 - Cython A Guide for Python Programmers
 - Cython: Blend the Best of Python and C++ | SciPy 2015 Tutorial | Kurt Smith
 - Cython: Speed up Python and NumPy, Pythonize C, C++, and Fortran, SciPy2013 Kurt W. Smith
 - SciPy 2017 Cython by
 - Cython Book examples
- Parallel computing in Cython/threads Neal Hughes

Numba

```
In [1]: import numpy as np
```

- Numba is a compiler for Python array and numerical functions.
- Numba generates optimized machine code from pure Python code with a few simple annotations
- Python code is just-in-time optimized to performance similar as C, C++ and Fortran, without having to switch languages or Python interpreters.
- The code is generated on-the-fly for CPU (default) or GPU hardware.

99.1 Python decorator

A decorator is used to modify a function or a class. A reference to a function "func" or a class "C" is passed to a decorator and the decorator returns a modified function or class. The modified functions or classes usually contain calls to the original function "func" or class "C".

99.2 First example

```
In [3]: from numba import jit
    @jit
    def sum(a, b):
        return a + b
```

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```
• Compilation will be deferred until the first function execution.
```

• Numba will infer the argument types at call time.

99.3 Performance

```
In [6]: x = np.random.rand(10000000)
In [7]: %timeit x.sum() # Numpy
9.22 ms ± 141 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
In [8]: @jit
    def numba_sum(x):
        res= 0
        for i in range(x.size):
            res += x[i]
        return res
In [9]: %timeit numba_sum(x)
13.4 ms ± 92.5 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

99.4 Numba methods

```
# --- LINE 3 ---
   # label 0
     a = arg(0, name=a) :: int64
     b = arg(1, name=b) :: int64
      $6binary_add.2 = a + b :: int64
   #
      del b
      $8return_value.3 = cast(value=$6binary_add.2) :: int64
      del $6binary_add.2
      return $8return_value.3
   return a + b
In [13]: jit_sum(1., 2.) # call it once with doubles
      jit_sum.inspect_types()
jit_sum (int64, int64)
______
# File: <ipython-input-10-ebda2b2f7dda>
# --- LINE 1 ---
@jit
# --- LINE 2 ---
def jit_sum(a, b):
   # --- LINE 3 ---
   # label 0
   \# a = arg(0, name=a) :: int64
   # b = arg(1, name=b) :: int64
      $6binary_add.2 = a + b :: int64
      del b
   #
      del a
      $8return_value.3 = cast(value=$6binary_add.2) :: int64
      del $6binary_add.2
      return $8return_value.3
   return a + b
______
jit_sum (float64, float64)
# File: <ipython-input-10-ebda2b2f7dda>
# --- LINE 1 ---
@jit
# --- LINE 2 ---
```

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```
def jit_sum(a, b):
    # --- LINE 3 ---
    # label 0
    # a = arg(0, name=a) :: float64
    # b = arg(1, name=b) :: float64
    # $6binary_add.2 = a + b :: float64
    # del b
    # del a
    # $8return_value.3 = cast(value=$6binary_add.2) :: float64
    # del $6binary_add.2
    # return $8return_value.3
```

• jit_sum.inspect_llvm() returns a dict with llvm representation.

LLVM is a library that is used to construct, optimize and produce intermediate and/or binary machine code.

• jit_sum.inspect_asm() returns a dict with assembler information.

```
In [14]: jit_sum.py_func(1, 2) # call origin python function without numba process
Out[14]: 3
```

99.5 Types coercion

Tell Numba the function signature you are expecting.

TypeError: No matching definition for argument type(s) array(float64, 1d, C), array(float64, 1d, C)

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99.6 Numba types

```
void,
intp, uintp,
intc, uintc,
int8, uint8, int16, uint16, int32, uint32, int64, uint64,
float32, float64,
complex64, complex128.

99.6.1 Arrays
float32[:]
float64[:, :]
```

99.7 Numba compilation options

- ** nopython ** : Compilation fails if you use pure Python objects.
- ** nogil **: release Python's global interpreter lock (GIL).
- ** cache **: Do not recompile the function each time you invoke a Python program.
- ** parallel ** : experimental feature that automatically parallelizes must be used in conjunction with nopython= True :

Inlining

Numba-compiled functions can call other compiled functions. The function calls may even be inlined in the native code, depending on optimizer heuristics.

100.1 @vectorize decorator

- Numba's vectorize allows Python functions taking scalar input arguments to be used as NumPy ufuncs.
- Write your function as operating over input scalars, rather than arrays. Numba will generate the surrounding loop (or kernel) allowing efficient iteration over the actual inputs.

100.1.1 Two modes of operation:

- 1. Eager mode: If you pass one or more type signatures to the decorator, you will be building a Numpy universal function (ufunc).
- 2. Call-time mode: When not given any signatures, the decorator will give you a Numba dynamic universal function (DUFunc) that dynamically compiles a new kernel when called with a previously unsupported input type.

If you pass several signatures, beware that you have to pass most specific signatures before least specific ones (e.g., single-precision floats before double-precision floats)

```
In [22]: @vectorize([int32(int32, int32),
                    int64(int64, int64),
                    float32(float32, float32),
                    float64(float64, float64)])
        def f(x, y):
            return x + y
In [23]: a = np.arange(6)
        f(a, a)
Out[23]: array([ 0, 2, 4, 6, 8, 10])
In [24]: a = np.linspace(0, 1, 6)
        f(a, a)
Out[24]: array([0., 0.4, 0.8, 1.2, 1.6, 2.])
In [25]: a = np.linspace(0, 1+1j, 6)
        f(a, a)
        TypeError
                                                     Traceback (most recent call last)
        <ipython-input-25-b196490ab338> in <module>
          1 a = np.linspace(0, 1+1j, 6)
    ---> 2 f(a, a)
```

TypeError: ufunc 'f' not supported for the input types, and the inputs could not be safely coer

100.1.2 Why not using a simple iteration loop using the @jit decorator?

The answer is that NumPy ufuncs automatically get other features such as reduction, accumulation or broadcasting.

```
In [26]: a = np.arange(12).reshape(3, 4)
Out[26]: array([[ 0, 1, 2, 3],
                [4, 5, 6, 7],
                [8, 9, 10, 11]])
In [27]: f.reduce(a, axis=0)
Out[27]: array([12, 15, 18, 21])
In [28]: f.reduce(a, axis=1)
Out[28]: array([ 6, 22, 38])
In [29]: f.accumulate(a)
Out[29]: array([[ 0, 1, 2, 3],
                [4, 6, 8, 10],
                [12, 15, 18, 21]])
In [30]: f.accumulate(a, axis=1)
Out[30]: array([[ 0, 1, 3, 6],
                [4, 9, 15, 22],
                [8, 17, 27, 38]])
```

100.2 The vectorize() decorator supports multiple ufunc targets:

- cpu Single-threaded CPU: small data sizes (approx. less than 1KB), no overhead.
- parallel Multi-core CPU: medium data sizes (approx. less than 1MB), small overhead.
- cuda CUDA GPU big data sizes (approx. greater than 1MB), significant overhead.

100.3 The @guvectorize decorator

• It allows you to write ufuncs that will work on an arbitrary number of elements of input arrays, and take and return arrays of differing dimensions.

```
In [31]: from numba import guvectorize
          @guvectorize([(int64[:], int64[:], int64[:])], '(n),()->(n)')
           def g(x, y, res):
                for i in range(x.shape[0]):
                res[i] = x[i] + y[0] # adds the scalar y to all elements of x
```

This decorator has two arguments: - the declaration (n),()->(n) tells NumPy that the function takes a n-element one-dimension array, a scalar (symbolically denoted by the empty tuple ()) and returns a n-element one-dimension array; - the list of supported concrete signatures as in @vectorize; here we only support int64 arrays.

100.4 Automatic parallelization with @jit

- Setting the parallel option for jit() enables this experimental Numba feature.
- Array Expressions like element-wise or point-wise array operations are supported.

```
unary operators: + - ~
binary operators: + - * / /? % | >> ^ << & ** //</li>
comparison operators: ==!= < <= >>=
```

- Numpy usuncs that are supported in nopython mode.
- Numpy reduction functions sum and prod.
- Numpy array creation functions zeros, ones, and several random functions (rand, randn, ranf, random_sample, sample, random, standard_normal, chisquare, weibull, power, geometric, exponential, poisson, rayleigh, normal, uniform, beta, binomial, f, gamma, lognormal, laplace, randint, triangular).

Numpy dot function between a matrix and a vector, or two vectors. In all other cases, Numba's default implementation is used.

Multi-dimensional arrays are also supported for the above operations when operands have matching dimension and size. The full semantics of Numpy broadcast between arrays with mixed dimensionality or size is not supported, nor is the reduction across a selected dimension.

http://numba.pydata.org/numba-doc/latest/user/parallel.html

100.5 Explicit Parallel Loops

Another experimental feature of this module is support for explicit parallel loops. One can use Numba's prange instead of range to specify that a loop can be parallelized. The user is required to make sure that the loop does not have cross iteration dependencies except the supported reductions. Currently, reductions on scalar values are supported and are inferred from in-place operations. The example below demonstrates a parallel loop with a reduction (A is a one-dimensional Numpy array):

```
In [32]: from numba import njit, prange
    @njit(parallel=True)
    def prange_test(A):
```

```
s = 0
for i in prange(A.shape[0]):
    s += A[i]
return s
```

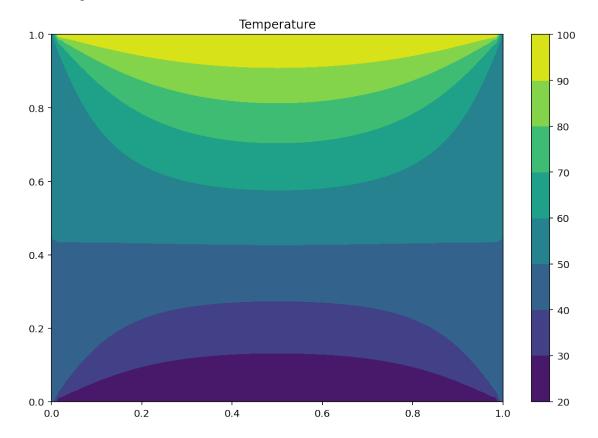
100.6 Exercise

- Optimize the Laplace equation solver with numba.
 - 1. Use only @jit
 - 2. Try to use @jit(nopython=True) option
 - 3. Optimize the laplace function with the right signature.
 - 4. Try to parallelize.

```
In [33]: %%time
         %matplotlib inline
         %config InlineBackend.figure_format = 'retina'
         import numpy as np
         import matplotlib.pyplot as plt
         import itertools
         from numba import jit, float64
         # Boundary conditions
         Tnorth, Tsouth, Twest, Teast = 100, 20, 50, 50
         # Set meshgrid
        n, 1 = 64, 1.0
        X, Y = np.meshgrid(np.linspace(0,1,n), np.linspace(0,1,n))
        T = np.zeros((n,n))
         # Set Boundary condition
         T[n-1:, :] = Tnorth
         T[:1, :] = Tsouth
        T[:, n-1:] = Teast
        T[:, :1] = Twest
         def laplace(T, n):
            residual = 0.0
             for i in range(1, n-1):
                 for j in range(1, n-1):
                     T_old = T[i,j]
                     T[i, j] = 0.25 * (T[i+1,j] + T[i-1,j] + T[i,j+1] + T[i,j-1])
                     if T[i,j]>0:
                         residual=max(residual,abs((T_old-T[i,j])/T[i,j]))
             return residual
         residual = 1.0
         istep = 0
         while residual > 1e-5:
            istep += 1
             residual = laplace(T, n)
            print ((istep, residual), end="\r")
         print("\n iterations = ",istep)
        plt.rcParams['figure.figsize'] = (10,6.67)
        plt.title("Temperature")
         plt.contourf(X, Y, T)
        plt.colorbar()
```

```
(2457, 9.997295133247811e-06)
iterations = 2457
CPU times: user 33.4 s, sys: 402 ms, total: 33.8 s
Wall time: 33.3 s
```

Out[33]: <matplotlib.colorbar.Colorbar at 0x7f3a5d3f77c0>



100.7 Vectorize performance

```
In [34]: import socket
    import numpy as np
    from numba import vectorize

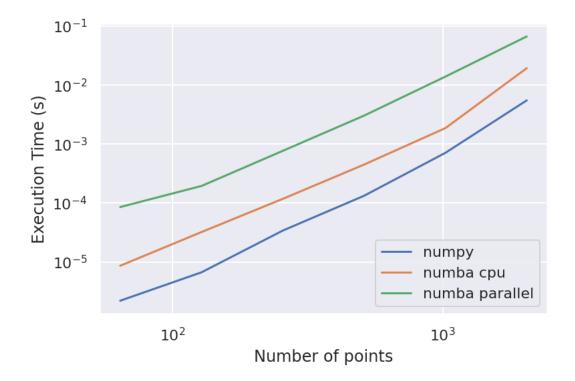
    @vectorize(['float64(float64, float64)'], target="cpu", cache=True, nopython=True)
    def cpu_add(a, b):
        return a + b

    @vectorize(['float64(float64, float64)'], target="parallel", cache=True, nopython=True)
    def parallel_add(a, b):
        return a + b

    if socket.gethostname() == "gpu-irmar.insa-rennes.fr":
        @vectorize(['float64(float64, float64)'], target="cuda", cache=True, nopython=True)
        def parallel_add(a, b):
            return a + b
```

/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/numba/np/ufunc/parallel.py:355: NumbaWarn warnings.warn(problem)

```
In [35]: %matplotlib inline
         import matplotlib.pyplot as plt
         import seaborn; seaborn.set()
         import progressbar
         Nrange = (2 ** np.arange(6, 12)).astype(int)
         t_numpy = []
         t_numba_cpu = []
         t_numba_parallel = []
        bar = progressbar.ProgressBar()
         for N in bar(Nrange):
             # Initialize arrays
             A = np.ones(N*N, dtype=np.float32).reshape(N,N)
             B = np.ones(A.shape, dtype=A.dtype)
             C = np.empty_like(A, dtype=A.dtype)
            t1 = \%timeit -oq C = A + B
             t2 = %timeit -oq C = cpu_add(A, B)
             t3 = %timeit -oq C = parallel_add(A, B)
             t_numpy.append(t1.best)
             t_numba_cpu.append(t2.best)
             t_numba_parallel.append(t3.best)
         plt.loglog(Nrange, t_numpy, label='numpy')
         plt.loglog(Nrange, t_numba_cpu, label='numba cpu')
         plt.loglog(Nrange, t_numba_parallel, label='numba parallel')
         plt.legend(loc='lower right')
        plt.xlabel('Number of points')
        plt.ylabel('Execution Time (s)');
```



References

- Numba by Loic Gouarin
- Numba Documentation
- Numbapro
- $\bullet~$ Numba examples

Semi-Lagrangian method

Let us consider an abstract scalar advection equation of the form

$$\frac{\partial f}{\partial t} + a(x,t) \cdot \nabla f = 0.$$

The characteristic curves associated to this equation are the solutions of the ordinary differential equations

$$\frac{dX}{dt} = a(X(t), t)$$

We shall denote by X(t, x, s) the unique solution of this equation associated to the initial condition X(s) = x. The classical semi-Lagrangian method is based on a backtracking of characteristics. Two steps are needed to update the distribution function f^{n+1} at t^{n+1} from its value f^n at time $t^n: 1$. For each grid point x_i compute $X(t^n; x_i, t^{n+1})$ the value of the characteristic at t^n which takes the value x_i at t^{n+1} . 2. As the distribution solution of first equation verifies

$$f^{n+1}(x_i) = f^n(X(t^n; x_i, t^{n+1})),$$

we obtain the desired value of $f^{n+1}(x_i)$ by computing $f^n(X(t^n; x_i, t^{n+1}))$ by interpolation as $X(t^n; x_i, t^{n+1})$ is in general not a grid point.

Eric Sonnendrücker - Numerical methods for the Vlasov equations

```
In [2]: # Disable the pager for lprun
    from IPython.core import page
    page.page = print
```

102.1 Bspline interpolator

• De Boor's Algorithm - Wikipedia

102.1.1 Numpy

```
In [3]: import numpy as np
    from scipy.fftpack import fft, ifft

def bspline_python(p, j, x):
    """Return the value at x in [0,1[ of the B-spline with
```

```
integer nodes of degree p with support starting at j.
        Implemented recursively using the de Boor's recursion formula"""
        assert (x >= 0.0) & (x <= 1.0)
        assert (type(p) == int) & (type(j) == int)
        if p == 0:
           if j == 0:
               return 1.0
            else:
               return 0.0
        else:
           w = (x - j) / p
           w1 = (x - j - 1) / p
        return w * bspline_python(p - 1, j, x) + (1 - w1) * bspline_python(p - 1, j + 1, x)
class BSplineNumpy:
    """ Class to compute BSL advection of 1d function """
    def __init__(self, p, xmin, xmax, ncells):
       assert p & 1 == 1 # check that p is odd
       self.p = p
        self.ncells = ncells
        # compute eigenvalues of degree p b-spline matrix
        self.modes = 2 * np.pi * np.arange(ncells) / ncells
        self.deltax = (xmax - xmin) / ncells
        self.eig_bspl = bspline_python(p, -(p + 1) // 2, 0.0)
        for j in range(1, (p + 1) // 2):
            self.eig\_bspl += bspline\_python(p, j - (p + 1) // 2, 0.0) * 2 * np.cos(j * self.modes)
        self.eigalpha = np.zeros(ncells, dtype=complex)
    def interpolate_disp(self, f, alpha):
        """compute the interpolating spline of degree p of odd degree
        of a function f on a periodic uniform mesh, at
        all points xi-alpha"""
        p = self.p
        assert (np.size(f) == self.ncells)
        # compute eigenvalues of cubic splines evaluated at displaced points
        ishift = np.floor(-alpha / self.deltax)
        beta = -ishift - alpha / self.deltax
        self.eigalpha.fill(0.)
        for j in range(-(p-1)//2, (p+1)//2 + 1):
            self.eigalpha += bspline_python(p, j-(p+1)//2, beta) * np.exp((ishift+j)*1j*self.modes)
        # compute interpolating spline using fft and properties of circulant matrices
        return np.real(ifft(fft(f) * self.eigalpha / self.eig_bspl))
```

102.1.2 Interpolation test

sin function after a displacement of alpha

```
In [4]: def interpolation_test(BSplineClass):
    """ Test to check interpolation"""
    n = 64
    cs = BSplineClass(3,0,1,n)
    x = np.linspace(0,1,n, endpoint=False)
    f = np.sin(x*4*np.pi)
    alpha = 0.2
```

return np.allclose(np.sin((x-alpha)*4*np.pi), cs.interpolate_disp(f, alpha))

interpolation_test(BSplineNumpy)

Out[4]: True

102.2 Profiling the code

Total time: 0.000902 s

File: <ipython-input-3-e4558e235572>
Function: interpolate_disp at line 38

Line #	Hits	Time	Per Hit	% Time	Line Contents
38			=======		def interpolate_disp(self, f, alpha):
39					"""compute the interpolating spline of degree
40					of a function f on a periodic uniform mesh, at
41					all points xi-alpha"""
42	1	2.0	2.0	0.2	-
43	1	10.0	10.0	1.1	assert (np.size(f) == self.ncells)
44					<pre># compute eigenvalues of cubic splines evaluate</pre>
45	1	9.0	9.0	1.0	ishift = np.floor(-alpha / self.deltax)
46	1	3.0	3.0	0.3	beta = -ishift - alpha / self.deltax
47	1	6.0	6.0	0.7	self.eigalpha.fill(0.)
48	5	6.0	1.2	0.7	for j in range(-(p-1)//2, (p+1)//2 + 1):
49	4	654.0	163.5	72.5	self.eigalpha += bspline_python(p, j-(p+1),
50					
51					# compute interpolating spline using fft and p
52	1	212.0	212.0	23.5	return np.real(ifft(fft(f) * self.eigalpha / se

*** Profile printout saved to text file 'lp_results.txt'.

102.3 Fortran

Replace the bspline computation by a fortran function, call it **bspline_fortran**.

```
real(8) :: res
            if (p == 0) then
                if (j == 0) then
                   res = 1.0
                    return
                else
                    res = 0.0
                    return
                end if
            else
                w = (x - j) / p
                w1 = (x - j - 1) / p
            end if
            res = w * bspline_fortran(p-1,j,x) &
            +(1-w1)*bspline_fortran(p-1,j+1,x)
        end function bspline_fortran
In [9]: import numpy as np
        from scipy.fftpack import fft, ifft
        class BSplineFortran:
            def __init__(self, p, xmin, xmax, ncells):
                assert p \& 1 == 1 # check that p is odd
                self.p = p
                self.ncells = ncells
                # compute eigenvalues of degree p b-spline matrix
                self.modes = 2 * np.pi * np.arange(ncells) / ncells
                self.deltax = (xmax - xmin) / ncells
                self.eig_bspl = bspline_fortran(p, -(p+1)//2, 0.0)
                for j in range(1, (p+1)//2):
                    self.eig_bspl += bspline_fortran(p, j-(p+1)//2,0.0)*2*np.cos(j*self.modes)
                self.eigalpha = np.zeros(ncells, dtype=complex)
            def interpolate_disp(self, f, alpha):
                """compute the interpolating spline of degree p of odd degree
                of a function f on a periodic uniform mesh, at
                all points xi-alpha"""
                p = self.p
                assert (np.size(f) == self.ncells)
                # compute eigenvalues of cubic splines evaluated at displaced points
                ishift = np.floor(-alpha / self.deltax)
                beta = -ishift - alpha / self.deltax
                self.eigalpha.fill(0.)
                for j in range(-(p-1)//2, (p+1)//2 + 1):
                    self.eigalpha += bspline_fortran(p, j-(p+1)//2, beta) * np.exp((ishift+j)*1j*self.modes)
                # compute interpolating spline using fft and properties of circulant matrices
                return np.real(ifft(fft(f) * self.eigalpha / self.eig_bspl))
In [10]: interpolation_test(BSplineFortran)
Out[10]: True
```

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102.4 Numba

Create a optimized function of bspline python function with Numba. Call it bspline numba.

```
In [11]: # %load solutions/landau_damping/bspline_numba.py
         from numba import jit, int32, float64
         from scipy.fftpack import fft, ifft
         @jit("float64(int32,int32,float64)",nopython=True)
         def bspline_numba(p, j, x):
                 """Return the value at x in [0,1[ of the B-spline with
                 integer nodes of degree p with support starting at j.
                 Implemented recursively using the de Boor's recursion formula"""
                 assert ((x >= 0.0) & (x <= 1.0))
                 if p == 0:
                     if j == 0:
                         return 1.0
                     else:
                         return 0.0
                 else:
                     w = (x-j)/p
                     w1 = (x-j-1)/p
                 return w * bspline_numba(p-1,j,x)+(1-w1)*bspline_numba(p-1,j+1,x)
In [12]: class BSplineNumba:
             def __init__(self, p, xmin, xmax, ncells):
                 assert p & 1 == 1 # check that p is odd
                 self.p = p
                 self.ncells = ncells
                 \# compute eigenvalues of degree p b-spline matrix
                 self.modes = 2 * np.pi * np.arange(ncells) / ncells
                 self.deltax = (xmax - xmin) / ncells
                 self.eig_bspl = bspline_numba(p, -(p+1)//2, 0.0)
                 for j in range(1, (p + 1) // 2):
                     \texttt{self.eig\_bspl} \; +\!= \; \texttt{bspline\_numba(p,j-(p+1)//2,0.0)*2*np.cos(j*self.modes)}
                 self.eigalpha = np.zeros(ncells, dtype=complex)
             def interpolate_disp(self, f, alpha):
                 """compute the interpolating spline of degree p of odd degree
                 of a function f on a periodic uniform mesh, at
                 all points xi-alpha"""
                 p = self.p
                 assert (np.size(f) == self.ncells)
                 # compute eigenvalues of cubic splines evaluated at displaced points
                 ishift = np.floor(-alpha / self.deltax)
                 beta = -ishift - alpha / self.deltax
                 self.eigalpha.fill(0.)
                 for j in range(-(p-1)//2, (p+1)//2+1):
                     self.eigalpha += bspline_numba(p, j-(p+1)//2, beta)*np.exp((ishift+j)*1j*self.modes)
                 # compute interpolating spline using fft and properties of circulant matrices
                 return np.real(ifft(fft(f) * self.eigalpha / self.eig_bspl))
```

In [13]: interpolation_test(BSplineNumba)

Out[13]: True

Pythran

```
In [14]: import pythran
        ModuleNotFoundError
                                                  Traceback (most recent call last)
        <ipython-input-14-b13d9faa9001> in <module>
    ----> 1 import pythran
        ModuleNotFoundError: No module named 'pythran'
In [15]: %load_ext pythran.magic
        {\tt ModuleNotFoundError}
                                                  Traceback (most recent call last)
        <ipython-input-15-60954a8043dc> in <module>
   ---> 1 get_ipython().run_line_magic('load_ext', 'pythran.magic')
       /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/interactiveshell.py
       2324
                            kwargs['local_ns'] = self.get_local_scope(stack_depth)
       2325
                        with self.builtin_trap:
    -> 2326
                            result = fn(*args, **kwargs)
       2327
                        return result
       2328
        <decorator-gen-64> in load_ext(self, module_str)
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magic.py in <lambda>
                # but it's overkill for just that one bit of state.
        185
        186
                def magic deco(arg):
                    call = lambda f, *a, **k: f(*a, **k)
    --> 187
```

```
188
        189
                    if callable(arg):
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/magics/extension.py
                    if not module str:
                        raise UsageError('Missing module name.')
         32
    ---> 33
                    res = self.shell.extension_manager.load_extension(module_str)
         34
         35
                    if res == 'already loaded':
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/extensions.py in loa
         78
                        if module_str not in sys.modules:
         79
                            with prepended_to_syspath(self.ipython_extension_dir):
    ---> 80
                                mod = import_module(module_str)
         81
                                if mod.__file__.startswith(self.ipython_extension_dir):
         82
                                    print(("Loading extensions from {dir} is deprecated. "
        /usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/__init__.py in import_module(name, pa
                        level += 1
        126
    --> 127
                return _bootstrap._gcd_import(name[level:], package, level)
        128
        129
        /usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/_bootstrap.py in _gcd_import(name, pa
        /usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/_bootstrap.py in _find_and_load(name,
        /usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/_bootstrap.py in _find_and_load_unloc
        /usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/_bootstrap.py in _call_with_frames_real_
        /usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/_bootstrap.py in _gcd_import(name, pa
        /usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/_bootstrap.py in _find_and_load(name,
        /usr/share/miniconda3/envs/runenv/lib/python3.8/importlib/_bootstrap.py in _find_and_load_unloc
        ModuleNotFoundError: No module named 'pythran'
In [16]: # %load solutions/landau_damping/bspline_pythran.py
```

#pythran export bspline_pythran(int,int,float64)

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```
def bspline_pythran(p, j, x):
            if p == 0:
                if j == 0:
                    return 1.0
                else:
                    return 0.0
             else:
                w = (x-j)/p
                w1 = (x-j-1)/p
             return w * bspline_pythran(p-1,j,x)+(1-w1)*bspline_pythran(p-1,j+1,x)
In [17]: class BSplinePythran:
             def __init__(self, p, xmin, xmax, ncells):
                assert p & 1 == 1 # check that p is odd
                self.p = p
                self.ncells = ncells
                 # compute eigenvalues of degree p b-spline matrix
                self.modes = 2 * np.pi * np.arange(ncells) / ncells
                self.deltax = (xmax - xmin) / ncells
                self.eig_bspl = bspline_pythran(p, -(p+1)//2, 0.0)
                for j in range(1, (p + 1) // 2):
                    self.eig_bspl += bspline_pythran(p,j-(p+1)//2,0.0)*2*np.cos(j*self.modes)
                self.eigalpha = np.zeros(ncells, dtype=complex)
             def interpolate_disp(self, f, alpha):
                 """compute the interpolating spline of degree p of odd degree
                 of a function f on a periodic uniform mesh, at
                all points xi-alpha"""
                p = self.p
                assert (f.size == self.ncells)
                 # compute eigenvalues of cubic splines evaluated at displaced points
                ishift = np.floor(-alpha / self.deltax)
                beta = -ishift - alpha / self.deltax
                self.eigalpha.fill(0.)
                for j in range(-(p-1)//2, (p+1)//2+1):
                     self.eigalpha += bspline_pythran(p, j-(p+1)//2, beta)*np.exp((ishift+j)*1j*self.modes)
                 # compute interpolating spline using fft and properties of circulant matrices
                return np.real(ifft(fft(f) * self.eigalpha / self.eig_bspl))
In [18]: interpolation_test(BSplinePythran)
Out[18]: True
103.1 Cython
   • Create bspline_cython function.
In [19]: %load_ext cython
In [20]: %%cython -a
        def bspline_cython(p, j, x):
                 """Return the value at x in [0,1[ of the B-spline with
                 integer nodes of degree p with support starting at j.
```

Implemented recursively using the de Boor's recursion formula"""

```
assert (x >= 0.0) & (x <= 1.0)
                 assert (type(p) == int) & (type(j) == int)
                 if p == 0:
                     if j == 0:
                         return 1.0
                     else:
                         return 0.0
                 else:
                     w = (x - j) / p
                     w1 = (x - j - 1) / p
                 return w * bspline_cython(p - 1, j, x) + (1 - w1) * bspline_cython(p - 1, j + 1, x)
Out[20]: <IPython.core.display.HTML object>
In [21]: %%cython
         import cython
         import numpy as np
         cimport numpy as np
         from scipy.fftpack import fft, ifft
         @cython.cdivision(True)
         cdef double bspline_cython(int p, int j, double x):
                 """Return the value at x in [0,1[ of the B-spline with
                 integer nodes of degree p with support starting at j.
                 Implemented recursively using the de Boor's recursion formula"""
                 cdef double w, w1
                 if p == 0:
                     if j == 0:
                        return 1.0
                     else:
                         return 0.0
                 else:
                     w = (x - j) / p
                     w1 = (x - j - 1) / p
                 return w * bspline_cython(p-1,j,x)+(1-w1)*bspline_cython(p-1,j+1,x)
         class BSplineCython:
             def __init__(self, p, xmin, xmax, ncells):
                self.p = p
                 self.ncells = ncells
                 # compute eigenvalues of degree p b-spline matrix
                 self.modes = 2 * np.pi * np.arange(ncells) / ncells
                 self.deltax = (xmax - xmin) / ncells
                 self.eig_bspl = bspline_cython(p,-(p+1)//2, 0.0)
                 for j in range(1, (p + 1) // 2):
                     self.eig_bspl += bspline_cython(p, j-(p+1)//2, 0.0)*2*np.cos(j*self.modes)
                 self.eigalpha = np.zeros(ncells, dtype=complex)
             @cython.boundscheck(False)
             @cython.wraparound(False)
             def interpolate_disp(self, f, alpha):
                 """compute the interpolating spline of degree p of odd degree
                 of a function f on a periodic uniform mesh, at
                 all points xi-alpha"""
                 cdef Py_ssize_t j
                 cdef int p = self.p
```

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```
# compute eigenvalues of cubic splines evaluated at displaced points
                                         cdef int ishift = np.floor(-alpha / self.deltax)
                                         cdef double beta = -ishift - alpha / self.deltax
                                         self.eigalpha.fill(0)
                                         for j in range(-(p-1)//2, (p+1)//2+1):
                                                   self.eigalpha += bspline_cython(p,j-(p+1)//2,beta)*np.exp((ishift+j)*1j*self.modes)
                                         # compute interpolating spline using fft and properties of circulant matrices
                                         return np.real(ifft(fft(f) * self.eigalpha / self.eig_bspl))
In [22]: interpolation_test(BSplineCython)
Out [22]: True
       import seaborn; seaborn.set() import progressbar Mrange = (2 ** np.arange(5, 10)).astype(int)
       t \text{ numpy} = [t \text{ fortran} = [t \text{ numba} = t \text{ pythran} = t \text{ cython} = t]
       bar = progressbar.ProgressBar()
       for M in bar(Mrange): x = np.linspace(0,1,M, endpoint=False) f = np.sin(x \not =
= BSplineNumpy(5,0,1,M) cs2 = BSplineFortran(5,0,1,M) cs3 = BSplineNumba(5,0,1,M) cs4 =
BSplinePythran(5,0,1,M) cs5 = BSplineCython(5,0,1,M)
alpha = 0.1
t1 = %timeit -oq cs1.interpolate_disp(f, alpha)
t2 = %timeit -oq cs2.interpolate_disp(f, alpha)
t3 = %timeit -oq cs3.interpolate_disp(f, alpha)
t4 = %timeit -oq cs4.interpolate_disp(f, alpha)
t5 = %timeit -oq cs5.interpolate_disp(f, alpha)
t_numpy.append(t1.best)
t fortran.append(t2.best)
t_numba.append(t3.best)
t pythran.append(t4.best)
t_cython.append(t5.best)
       plt.loglog(Mrange,
                                                                                     label='numpy') plt.loglog(Mrange,
                                                                                                                                                                                t fortran,
                                                                                                                                                                                                              label='fortran')
                                                       t_numpy,
                                                                               label='numba') plt.loglog(Mrange, t_pythran,
plt.loglog(Mrange, t_numba,
                                                                                                                                                                                                            label='pythran')
plt.loglog(Mrange, t_cython, label='cython') plt.legend(loc='lower right') plt.xlabel('Number of points')
plt.ylabel('Execution Time (s)');
```

Vlasov-Poisson equation

We consider the dimensionless Vlasov-Poisson equation for one species with a neutralizing background.

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f + E(t, x) \cdot \nabla_v f = 0, \quad -\Delta \phi = 1 - \rho, E = -\nabla \phi \rho(t, x) = \int f(t, x, v) dv.$$

• Vlasov Equation - Wikipedia

```
In [23]: import progressbar
         BSpline = dict(numpy=BSplineNumpy,
                        fortran=BSplineFortran,
                        cython=BSplineCython,
                        numba=BSplineNumba,
                        pythran=BSplinePythran)
         class VlasovPoisson:
             def __init__(self, xmin, xmax, nx, vmin, vmax, nv, opt='numpy'):
                 # Grid
                 self.nx = nx
                 self.x, self.dx = np.linspace(xmin, xmax, nx, endpoint=False, retstep=True)
                 self.nv = nv
                 self.v, self.dv = np.linspace(vmin, vmax, nv, endpoint=False, retstep=True)
                 # Distribution function
                 self.f = np.zeros((nx,nv))
                 # Interpolators for advection
                 BSplineClass = BSpline[opt]
                 self.cs_x = BSplineClass(3, xmin, xmax, nx)
                 self.cs_v = BSplineClass(3, vmin, vmax, nv)
                 # Modes for Poisson equation
                 self.modes = np.zeros(nx)
                 k = 2* np.pi / (xmax - xmin)
                 self.modes[:nx//2] = k * np.arange(nx//2)
                 self.modes[nx//2:] = -k * np.arange(nx//2,0,-1)
                 self.modes += self.modes == 0 # avoid division by zero
             def advection_x(self, dt):
                 for j in range(self.nv):
                     alpha = dt * self.v[j]
```

```
self.f[j,:] = self.cs_x.interpolate_disp(self.f[j,:], alpha)
def advection_v(self, e, dt):
   for i in range(self.nx):
        alpha = dt * e[i]
        self.f[:,i] = self.cs_v.interpolate_disp(self.f[:,i], alpha)
def compute_rho(self):
   rho = self.dv * np.sum(self.f, axis=0)
   return rho - rho.mean()
def compute_e(self, rho):
    # compute Ex using that ik*Ex = rho
   rhok = fft(rho)/self.modes
   return np.real(ifft(-1j*rhok))
def run(self, f, nstep, dt):
   self.f = f
   nrj = []
   bar = progressbar.ProgressBar()
   self.advection_x(0.5*dt)
   for istep in bar(range(nstep)):
       rho = self.compute_rho()
       e = self.compute_e(rho)
       self.advection_v(e, dt)
        self.advection_x(dt)
        nrj.append( 0.5*np.log(np.sum(e*e)*self.dx))
    return nrj
```

Landau Damping

```
Landau damping - Wikipedia
In [24]: from time import time
        elapsed_time = {}
        fig, axes = plt.subplots()
        for opt in ['numpy', 'fortran', 'numba', 'cython', 'pythran']:
             # Set grid
            nx, nv = 32, 64
            xmin, xmax = 0.0, 4*np.pi
            vmin, vmax = -6., 6.
             # Create Vlasov-Poisson simulation
            sim = VlasovPoisson(xmin, xmax, nx, vmin, vmax, nv, opt=opt)
             # Initialize distribution function
            X, V = np.meshgrid(sim.x, sim.v)
            eps, kx = 0.001, 0.5
            f = (1.0 + eps*np.cos(kx*X))/np.sqrt(2.0*np.pi)*np.exp(-0.5*V*V)
             # Set time domain
             t, dt = np.linspace(0.0, 60.0, nstep, retstep=True)
             # Run simulation
             etime = time()
            nrj = sim.run(f, nstep, dt)
            print(" {0:12s} : {1:.4f} ".format(opt, time()-etime))
             # Plot energy
             axes.plot(t, nrj, label=opt)
        axes.plot(t, -0.1533*t-5.50)
        plt.legend();
100% (600 of 600) | ############################ Elapsed Time: 0:00:12 Time: 0:00:12
  2% (16 of 600) |
                                             | Elapsed Time: 0:00:00 ETA:
                                                                                0:00:05
               : 12.9626
numpy
```

fortran : 5.9712

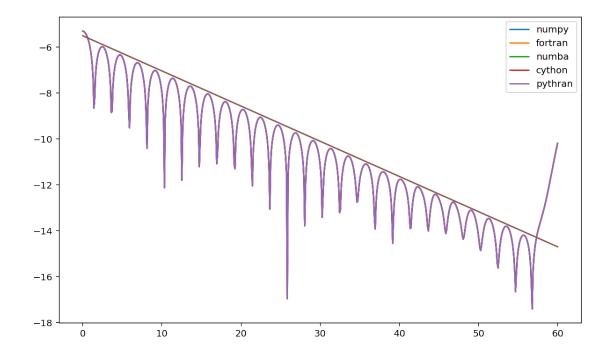
100% (600 of 600) | ################ | Elapsed Time: 0:00:06 Time: 0:00:06 3% (23 of 600) | | Elapsed Time: 0:00:00 ETA: 0:00:04

numba : 6.1876

cython : 4.7824

100% (600 of 600) | ################ | Elapsed Time: 0:00:11 Time: 0:00:11

pythran : 11.5004



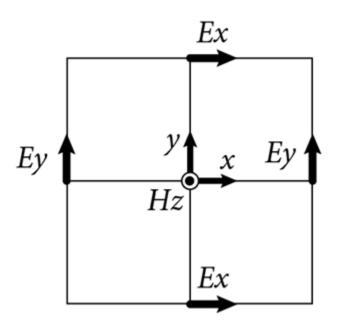
References

Chapter 107

Maxwell solver in two dimensions with FDTD scheme

$$\frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}; \qquad \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y}; \qquad \frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x}$$

Description of the scheme



$$\begin{split} H_z\big|_{i+1/2,j+1/2}^{n+1/2} &= H_z\big|_{i+1/2,j+1/2}^{n-1/2} + \frac{dt}{dy} \big(E_x\big|_{i+1/2,j+1}^n - E_x\big|_{i+1/2,j}^n \big) - \frac{dt}{dx} \big(E_y\big|_{i+1,j+1/2}^n - E_y\big|_{i,j+1/2}^n \big) \\ & E_x\big|_{i+1/2,j}^{n+1} &= E_x\big|_{i+1/2,j}^n + \frac{dt}{dy} \big(H_z\big|_{i+1/2,j+1/2}^{n+1/2} - H_z\big|_{i-1/2,j-1/2}^{n+1/2} \big) \\ & E_y\big|_{i,j+1/2}^{n+1} &= E_y\big|_{i,j+1/2}^n - \frac{dt}{dx} \big(H_z\big|_{i+1/2,j+1/2}^{n+1/2} - H_z\big|_{i-1/2,j+1/2}^{n+1/2} \big) \end{split}$$

```
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import axes3d
import matplotlib.animation as animation
from IPython.display import HTML

plt.rcParams['figure.figsize'] = (10,6)
```

```
In [2]: # Mesh parameters
    nx, ny = 101, 101
    vx, dx = np.linspace(0, 1, nx, endpoint=True, retstep=True)
    vy, dy = np.linspace(0, 1, ny, endpoint=True, retstep=True)

#Initialize Ex, Ey when time = 0
    ex = np.zeros((nx-1, ny), dtype=np.double)
    ey = np.zeros((nx, ny-1), dtype=np.double)
    nbiter = 500  # time loop size
    dt = 0.001  # time step
    m, n = 2, 2
    omega = np.sqrt((m*np.pi)**2+(n*np.pi)**2)
    # Create the staggered grid for Bz
    x, y = np.meshgrid(0.5*(vx[:-1]+vx[1:]), 0.5*(vy[:-1]+vy[1:]))
```

```
In [3]: fig = plt.figure()
    ax = axes3d.Axes3D(fig)

#Initialize Bz when time = - dt / 2
    hz = - np.cos(m*np.pi*y) * np.cos(n*np.pi*x) * np.cos(omega*(-0.5*dt))
    wframe = ax.plot_wireframe(x, y, hz, rstride=2, cstride=2)
    ax.set_zlim(-1,1);
```

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107.1 numpy

```
return ex, ey
In [5]: def update(i, ax, fig):
                            ax.cla()
                            global ex, ey, hz
                            hz = faraday( ex, ey, hz)
                             ex, ey = ampere_maxwell( hz, ex, ey)
                            wframe = ax.plot_wireframe(x, y, hz, rstride=2, cstride=2)
                            ax.set_zlim(-1, 1)
                            return wframe,
In [6]: ani = animation.FuncAnimation(fig, update,
                                                                                           frames=range(200),
                                                                                           fargs=(ax, fig), interval=100)
In [7]: %%time
                  HTML(ani.to_html5_video())
                    RuntimeError
                                                                                                                                      Traceback (most recent call last)
                     <timed eval> in <module>
                    /usr/share/miniconda 3/envs/runenv/lib/python 3.8/site-packages/matplotlib/animation.py in to {\tt htm} in the context of the 
                  1306
                                                                           # We create a writer manually so that we can get the
                  1307
                                                                           # appropriate size for the tag
                                                                           Writer = writers[mpl.rcParams['animation.writer']]
          -> 1308
                  1309
                                                                           writer = Writer(codec='h264',
                  1310
                                                                                                                      bitrate=mpl.rcParams['animation.bitrate'],
                     /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/matplotlib/animation.py in __geti
                     164
                                                     if self.is_available(name):
                     165
                                                                return self._registered[name]
          --> 166
                                                     raise RuntimeError(f"Requested MovieWriter ({name}) not available")
                     167
                     168
                     RuntimeError: Requested MovieWriter (ffmpeg) not available
In [8]: %%time
                   from tqdm import tqdm_notebook as tqdm
                  nx, ny = 512, 512
                  vx, dx = np.linspace(0, 1, nx, endpoint=True, retstep=True)
                   vy, dy = np.linspace(0, 1, ny, endpoint=True, retstep=True)
```

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```
ex = np.zeros((nx-1, ny), dtype=np.double)
       ey = np.zeros((nx, ny-1), dtype=np.double)
       dt = 0.001
                      # time step
       m, n = 2, 2
       omega = np.sqrt((m*np.pi)**2+(n*np.pi)**2)
       x, y = np.meshgrid(0.5*(vx[:-1]+vx[1:]), 0.5*(vy[:-1]+vy[1:]))
       hz = - np.cos(m*np.pi*y) * np.cos(n*np.pi*x) * np.cos(omega*(-0.5*dt))
       for t in tqdm(range(1000)):
           hz = faraday( ex, ey, hz)
           ex, ey = ampere_maxwell( hz, ex, ey)
<timed exec>:16: TqdmDeprecationWarning: This function will be removed in tqdm==5.0.0
Please use `tqdm.notebook.tqdm` instead of `tqdm.tqdm_notebook`
HBox(children=(FloatProgress(value=0.0, max=1000.0), HTML(value='')))
CPU times: user 5.32 s, sys: 49.7 ms, total: 5.37 s
Wall time: 5.31 s
In [9]: %load_ext fortranmagic
107.2 fortran
In [10]: %%fortran
        subroutine faraday_fortran( ex, ey, bz, dx, dy, dt, nx, ny)
        implicit none
        real(8), intent(in) :: ex(nx-1,ny)
        real(8), intent(in) :: ey(nx,ny-1)
        real(8), intent(inout) :: bz(nx-1,ny-1)
        integer, intent(in) :: nx, ny
        real(8), intent(in) :: dx, dy, dt
        integer :: i, j
        real(8) :: dex_dx, dey_dy
        real(8) :: dex_dy, dey_dx
        do j=1,ny-1
        do i=1,nx-1
           dex_dy = (ex(i,j+1)-ex(i,j)) / dy
           dey_dx = (ey(i+1,j)-ey(i,j)) / dx
           bz(i,j) = bz(i,j) + dt * (dex_dy - dey_dx)
        end do
        end do
        end subroutine faraday_fortran
In [11]: %%fortran
```

```
subroutine amperemaxwell_fortran(ex, ey, bz, dx, dy, dt, nx, ny)
        implicit none
        integer, intent(in):: nx, ny
        real(8), intent(in):: dx, dy, dt
        real(8), dimension(nx-1, ny-1), intent(inout) :: bz
        real(8), dimension(nx-1, ny), intent(inout) :: ex
        real(8), dimension(nx, ny-1), intent(inout) :: ey
        integer:: i, j
        real(8):: dbz_dx, dbz_dy
        real(8), parameter:: csq = 1d0
        do i = 1, nx-1
           dbz_dy = (bz(i, 1)-bz(i, ny-1)) / dy ! periodic BC
           ex(i, 1) = ex(i, 1) + dt*csq*dbz_dy
            ex(i, ny) = ex(i, 1)
         end do
        do j = 1, ny-1
           dbz_dx = (bz(1,j)-bz(nx-1,j)) / dx ! periodic BC
            ey(1,j) = ey(1,j) - dt*csq*dbz_dx
            ey(nx,j) = ey(1,j)
        end do
         do j=2,ny-1
            do i=1,nx-1
               dbz_dy = (bz(i,j)-bz(i,j-1)) / dy
                ex(i,j) = ex(i,j) + dt*csq*dbz_dy
             end do
        end do
        do j=1,ny-1
             do i=2,nx-1
               dbz_dx = (bz(i,j)-bz(i-1,j)) / dx
               ey(i,j) = ey(i,j) - dt*csq*dbz_dx
            end do
         end do
         end subroutine amperemaxwell_fortran
In [12]: %%time
        from tqdm import tqdm_notebook as tqdm
        ex.fill(0.0)
        ey.fill(0.0)
        hz = -np.cos(m*np.pi*y) * np.cos(n*np.pi*x) * np.cos(omega*(-0.5*dt))
        ex = np.asfortranarray(ex)
        ey = np.asfortranarray(ey)
        hz = np.asfortranarray(hz)
        for t in tqdm(range(1000)):
             faraday_fortran( ex, ey, hz, dx, dy, dt, nx, ny)
             amperemaxwell_fortran(ex, ey, hz, dx, dy, dt, nx, ny)
<timed exec>:10: TqdmDeprecationWarning: This function will be removed in tqdm==5.0.0
Please use `tqdm.notebook.tqdm` instead of `tqdm.tqdm_notebook`
```

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```
HBox(children=(FloatProgress(value=0.0, max=1000.0), HTML(value='')))
CPU times: user 1.87 s, sys: 7.45 ms, total: 1.88 s
Wall time: 1.86 s
In []:
```

Chapter 108

Gray-Scott Model

```
In [1]: import numpy as np
In [2]: %config InlineBackend.figure_format = 'retina'
```

The reaction-diffusion system described here involves two generic chemical species U and V, whose concentration at a given point in space is referred to by variables u and v. As the term implies, they react with each other, and they diffuse through the medium. Therefore the concentration of U and V at any given location changes with time and can differ from that at other locations.

The overall behavior of the system is described by the following formula, two equations which describe three sources of increase and decrease for each of the two chemicals:

$$\frac{\partial u}{\partial t} = D_u \Delta u - uv^2 + F(1 - u)$$
$$\frac{\partial v}{\partial t} = D_v \Delta v + uv^2 - (F + k)v$$

The laplacian is computed with the following numerical scheme

$$\Delta u_{i,j} \approx u_{i,j-1} + u_{i-1,j} - 4u_{i,j} + u_{i+1,j} + u_{i,j+1}$$

The classic Euler scheme is used to integrate the time derivative.

108.1 Initialization

u is 1 everywhere et v is 0 in the domain except in a square zone where v = 0.25 and u = 0.5. This square located in the center of the domain is $[0,1] \times [0,1]$ with a size of 0.2.

```
In [3]: def init(n):
    u = np.ones((n+2,n+2))
    v = np.zeros((n+2,n+2))

x, y = np.meshgrid(np.linspace(0, 1, n+2), np.linspace(0, 1, n+2))

mask = (0.4<x) & (x<0.6) & (0.4<y) & (y<0.6)

u[mask] = 0.50
 v[mask] = 0.25

return u, v</pre>
```

108.2 Boundary conditions

We assume that the domain is periodic.

108.3 Laplacian

108.4 Gray-Scott model

108.5 Visualization

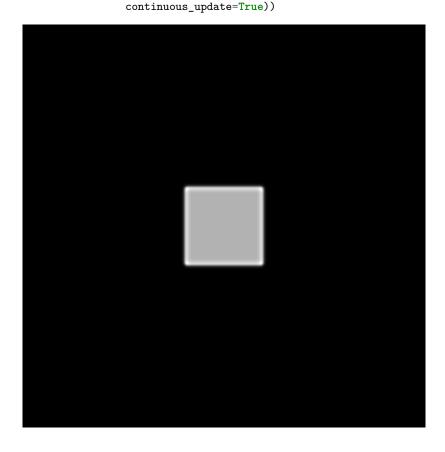
Nous utiliserons les données suivantes.

```
In [7]: Du, Dv = .1, .05
    F, k = 0.0545, 0.062
In [8]: %%time
    from tqdm import tqdm_notebook as tqdm
    from PIL import Image
    U, V = init(300)

def create_image():
        global U, V
        for t in range(40):
            grayscott(U, V, Du, Dv, F, k)
        V_scaled = np.uint8(255*(V-V.min()) / (V.max()-V.min()))
        return V_scaled

def create_frames(n):
```

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108.6 References

• Reaction-Diffusion by the Gray-Scott Model: Pearson's Parametrization

108.7 Cython

```
Program by Hyry
In [12]: %load_ext cython
         %env CC='gcc-8'
env: CC='gcc-8'
In [13]: %%cython
         #cython: boundscheck=False
         #cython: wraparound=False
         cimport cython
         import numpy as np
         cimport numpy as np
         cpdef cython_grayscott(int counts, double Du, double Dv, double F, double k):
             cdef int n = 300
             cdef np.ndarray U = np.zeros((n+2,n+2), dtype=np.float_)
             cdef np.ndarray V = np.zeros((n+2,n+2), dtype=np.float_)
             cdef np.ndarray u = U[1:-1,1:-1]
             cdef np.ndarray v = V[1:-1,1:-1]
             cdef int r = 20
             u[:] = 1.0
             U[n/2-r:n/2+r,n/2-r:n/2+r] = 0.50
             V[n/2-r:n/2+r,n/2-r:n/2+r] = 0.25
             u += 0.15*np.random.random((n,n))
             v += 0.15*np.random.random((n,n))
             cdef np.ndarray Lu = np.zeros_like(u)
             cdef np.ndarray Lv = np.zeros_like(v)
             cdef int i, c, r1, c1, r2, c2
             cdef double uvv
             cdef double[:, ::1] bU = U
             cdef double[:, ::1] bV = V
             cdef double[:, ::1] bLu = Lu
             cdef double[:, ::1] bLv = Lv
             for i in range(counts):
```

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for r in range(n):
   r1 = r + 1
   r2 = r + 2
   for c in range(n):
       c1 = c + 1
       c2 = c + 2
       bLu[r,c] = bU[r1,c2] + bU[r1,c] + bU[r2,c1] + bU[r,c1] - 4*bU[r1,c1]
       bLv[r,c] = bV[r1,c2] + bV[r1,c] + bV[r2,c1] + bV[r,c1] - 4*bV[r1,c1]
for r in range(n):
   r1 = r + 1
   for c in range(n):
       c1 = c + 1
       uvv = bU[r1,c1]*bV[r1,c1]*bV[r1,c1]
       bU[r1,c1] += Du*bLu[r,c] - uvv + F*(1 - bU[r1,c1])
       bV[r1,c1] += Dv*bLv[r,c] + uvv - (F + k)*bV[r1,c1]
```

return V

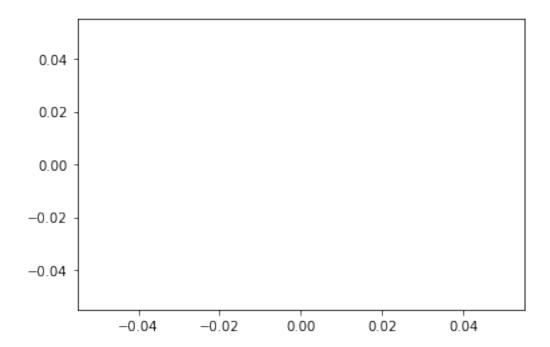
In []:

Chapter 109

Animation with matplotlib

```
In [1]: %matplotlib inline
In [2]: import numpy as np
        import matplotlib.pyplot as plt
        from matplotlib import animation, rc
        from IPython.display import HTML
        from matplotlib.animation import FuncAnimation
In [3]: fig, ax = plt.subplots()
        ax.set_xlim(( 0, 2))
        ax.set_ylim((-2, 2))
        line, = ax.plot([], [], lw=2)
           2.0
           1.5
           1.0
           0.5
           0.0
         -0.5
         -1.0
         -1.5
         -2.0
                       0.25
                                0.50
                                         0.75
                                                  1.00
                                                           1.25
                                                                    1.50
              0.00
                                                                            1.75
                                                                                      2.00
```

```
In [4]: def init():
           line.set_data([], [])
           return (line,)
In [5]: def animate(i):
           x = np.linspace(0, 2, 1000)
           y = np.sin(2 * np.pi * (x - 0.01 * i))
           line.set_data(x, y)
           return (line,)
In [6]: anim = animation.FuncAnimation(fig, animate, init_func=init,
                                    frames=100, interval=20,
                                    blit=True)
In [7]: HTML(anim.to_html5_video())
        RuntimeError
                                                    Traceback (most recent call last)
        <ipython-input-7-f885f4acf935> in <module>
    ----> 1 HTML(anim.to_html5_video())
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/matplotlib/animation.py in to_htm
       1306
                             # We create a writer manually so that we can get the
       1307
                             # appropriate size for the tag
    -> 1308
                             Writer = writers[mpl.rcParams['animation.writer']]
                             writer = Writer(codec='h264',
       1309
       1310
                                              bitrate=mpl.rcParams['animation.bitrate'],
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/matplotlib/animation.py in __geti
                    if self.is_available(name):
        164
        165
                         return self._registered[name]
    --> 166
                    raise RuntimeError(f"Requested MovieWriter ({name}) not available")
        167
        168
        RuntimeError: Requested MovieWriter (ffmpeg) not available
In [8]: HTML(anim.to_jshtml())
Out[8]: <IPython.core.display.HTML object>
In [9]: fig = plt.figure()
       ax1 = fig.add_subplot(1,1,1)
       xdata, ydata = [], []
       line1, = plt.plot([], [], 'r-', animated=True)
```



```
In [10]: def init():
            ax1.set_xlim((0,1))
            ax1.set_ylim((-1,1))
            return line1,
        def update(frame):
            xdata.append(frame)
            ydata.append(np.sin(8*np.pi*frame))
            line1.set_data(xdata, ydata)
            return line1,
        ani = FuncAnimation(fig, update, frames=np.linspace(0, 1.0, 100),
                            init_func=init, blit=True)
        plt.rc('animation', html='html5')
        ani
        RuntimeError
                                                      Traceback (most recent call last)
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/IPython/core/formatters.py in __c
        343
                         method = get_real_method(obj, self.print_method)
                          if method is not None:
        344
    --> 345
                              return method()
        346
                          return None
```

347

else:

```
/usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/matplotlib/animation.py in _repr_
                    fmt = mpl.rcParams['animation.html']
       1369
       1370
                    if fmt == 'html5':
    -> 1371
                        return self.to_html5_video()
                    elif fmt == 'jshtml':
       1372
       1373
                        return self.to_jshtml()
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/matplotlib/animation.py in to_htm
                            # We create a writer manually so that we can get the
       1306
      1307
                            # appropriate size for the tag
    -> 1308
                            Writer = writers[mpl.rcParams['animation.writer']]
       1309
                            writer = Writer(codec='h264',
       1310
                                            bitrate=mpl.rcParams['animation.bitrate'],
        /usr/share/miniconda3/envs/runenv/lib/python3.8/site-packages/matplotlib/animation.py in __geti
                    if self.is_available(name):
        165
                        return self._registered[name]
                    raise RuntimeError(f"Requested MovieWriter ({name}) not available")
    --> 166
        167
        168
       RuntimeError: Requested MovieWriter (ffmpeg) not available
Out[10]: <matplotlib.animation.FuncAnimation at 0x7f1d32c22d00>
In []:
```