<u>Definition of System</u>:

A **system** is a quantitative description of a physical process which transforms signals (at its “input") to signals (at its “output"). More precisely, a system is a “black box" (viewed as a mathematical abstraction) that deterministically transforms input signals into output signals.

A **system** is a set of interacting or interdependent component parts forming a complex/intricate whole. Every system is delineated by its spatial and temporal boundaries, surrounded and influenced by its environment, described by its structure and purpose and expressed in its functioning.

A **physical system** is a collection of physical objects connected together to serve an objective. Examples of physical systems may be cited from laboratory, industrial plant, utility services – ranging from a simple electronic amplifier to the satellite orbiting the earth falls under the category of physical systems.

In a more general perspective the term **system** is used to describe a combination of components which may not all be physical, e.g., economical, social, management systems.

<u>Classification of Systems:</u>

<u>*Continuous-time System* vs. *Discrete-time System*</u>

This may be the simplest classification to understand as the idea of discrete-time and continuous-time is one of the most fundamental properties to all of signals and system. A system where the input and output signals are continuous is a continuous system, and one where the input and output signals are discrete is a discrete-time system.

<u>*Linear System* vs. *Nonlinear System:*</u>

A linear system is any system that obeys the properties of scaling (homogeneity) and superposition (additivity), while a nonlinear system is any system that does not obey at least one of these.

To show that a system H obeys the scaling property is to show that $H(kf(t)) = k\,H(f(t))$.

To demonstrate that a system H obeys the superposition property of linearity is to show that $H(f_1(t) + f_2(t)) = H(f_1(t)) + H(f_2(t))$.

It is possible to check a system for linearity in a single (though larger) step by simply combining the first two steps to get $H(k_1 f_1(t) + k_2 f_2(t)) = k_1 H(f_1(t)) + k_2 H(f_2(t))$.

<u>*Time Invariant System* vs. *Time Varying System:*</u>

A time invariant system is one that does not depend on when it occurs: the shape of the output does not change with a delay of the input. That is to say that for a system H where $H(f(t)) = y(t)$, H is time invariant if for all T, one gets $H(f(t - T)) = y(t - T)$.

When this property does not hold for a system, then it is said to be time variant, or time-varying.

Causal System vs. *Non-causal System:*

A causal system is one that is *nonanticipative* ; that is, the output may depend on current and past inputs, but not future inputs. All “real-time” systems must be causal, since they can not have future inputs available to them.

One may think the idea of future inputs does not seem to make much physical sense; however, we have only been dealing with time as our dependent variable so far, which is not always the case. Imagine rather that we wanted to do image processing. Then the dependent variable might represent pixels to the left and right (the “future”) of the current position on the image, and we would have a non-causal system.

Stable System vs. *Unstable System:*

A stable system is one where the output does not diverge as long as the input does not diverge. A bounded input produces a bounded output. It is from this property that this type of system is referred to as bounded input-bounded output (BIBO) stable.

Representing this in a mathematical way, a stable system must have the following property, where $x(t)$ is the input and $y(t)$ is the output.

The output must satisfy the condition $|y(t)| \leq M_y < \infty$, when we have an input to the system that can be described as $|x(t)| \leq M_x < \infty$, M_x and M_y both represent a set of finite positive numbers and these relationships hold for all of t.

If these conditions are not met, i.e. a system’s output grows without limit (diverges) from a bounded input, then the system is unstable.

Properties of Systems:

Memoryless:

A system is memoryless if the output at time t (or n) depends only on the input at t (or n).

Example: 1. $y(t) = (2x(t) - x^2(t))^2$ is memoryless, because $y(t)$ depends on $x(t)$ only since there is no $x(t+1)$, or $x(t+1)$ terms.

2. $y[n] = x[n]$ is memoryless. In fact this system is passing the input to output directly.

3. $y[n] = x[n-1]$ is not memoryless, since the n-th output depends on $(n-1)$-th input.

4. $y[n] = x[n] + y[n-1]$ is not memoryless.

Since, $y[n] = x[n] + (x[n-1] + y[n-2]) = x[n] + x[n-1] + x[n-2] + ... = \sum_{k=-\infty}^{\infty} x[k]$

Invertible:

A system is invertible if distinct input signals produce distinct output signals. In other words, a system is invertible if there exists an one-to-one mapping from the set of input signals to the set of output signals.

Example: 1. The system $y(t) = (cos(t) + 2)x(t)$ is invertible and one gets, $x(t)= y(t)/(cos(t)+2)$.

2. The system $y[n] = x[n] + y[n-1]$ is invertible. Rearranging one gets $x[n] = y[n] - y[n-1]$.

3. The system $y(t) = x^2(t)$ is not invertible. Let us consider two signals $x_1(t) = 1$ and $x_2(t) = -1$. Clearly $x_1(t) \neq x2(t)$, but $(x_1(t))^2 = (x_2(t))^2$. Therefore, two different inputs give the same output. Hence the system is not invertible.

<u>Causal</u>

A system is causal if the output at time t (or n) depends only on inputs at time $< t$ (i.e., the present and past).

Examples: 1. $y[n] = x[n-1]$ is causal, because $y[n]$ depends on the past sample $x[n-1]$.

2. $y[n] = x[n] + x[n+1]$ is not causal, because $x[n+1]$ is a future sample.

3. $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$ is causal, as the integral evaluates from $-\infty$ to t (all in the past).

4. $y[n] = x[-n]$ is not causal, because $y[-1] = x[1]$, which means the output at $n = -1$ depends an input in the future.

5. $y(t) = x(t)\cos(t+1)$ causal (and memoryless), because $\cos(t+1)$ is a constant with respect to $x(t)$.

<u>Stable</u>

To describe a stable system, we first need to define the boundedness of a signal.

A signal $x(t)$ (and $x[n]$) is bounded if there exists $B < \infty$, such that $|x(t)| < B$ for all t.

A system is stable if a bounded input always produces a bounded output signal. That is, if for $|x(t)| \leq B$ for some B < ∞, then $|y(t)| < \infty$.

Example: 1. The system $y(t) = 2x^2(t-1) + x(3t)$ is stable.

Consider a bounded signal $x(t)$, that is, $|x(t)| < B$ for some $B < \infty$.

Then $|y(t)| = |2x^2(t-1) + x(3t)| \leq |2x^2(t-1)| + |x(3t)| \leq 2B^2 + B < \infty$

Therefore, for any bounded input $x(t)$, the output $y(t)$ is always bounded.

2. The system $y[n] = \sum_{k=-\infty}^{n} x[k]$ is not stable. Let $x[n] = u[n]$, with $|x[n]| \leq 1$ (i.e., bounded). Consequently, $|y[n]| = \left|\sum_{k=-\infty}^{n} u[k]\right| = \sum_{k=0}^{n} u[k] \leq \sum_{k=0}^{n} 1 = n+1$, which approaches ∞ as $n \to \infty$. Therefore, $|y[n]|$ is not bounded.

<u>Time-invariant</u>

A system is time-invariant if a time-shift of the input signal results in the same time-shift of the output signal.

If $x(t) \xrightarrow{yields} y(t)$ then the system is time-invariant if $x(t-t_0) \xrightarrow{yields} y(t-t_0)$ for any $t_0 \in \mathbb{R}$.

Example: 1. The system $y(t) = \sin[x(t)]$ is time-invariant.

Consider a time-shifted signal $x_1(t) = x(t-t_0)$. Correspondingly, we let $y_1(t)$ be the output of $x_1(t)$, thus, $y_1(t) = \sin[x_1(t)] = \sin[x(t-t_0)]$.

Now, we have to check whether $y_1(t) = y(t-t_0)$. Note that $y(t-t_0) = \sin[x(t-t_0)]$; which is the same as $y_1(t)$. Therefore, the system is time-invariant.

2. The system $y[n] = n\, x[n]$ is not time-invariant.

Let $x[n] = \delta[n]$, then $y[n] = n\, \delta[n] = 0$, for all n. Now, let $x_1[n] = x[n-1] = \delta[n-1]$. If $y_1[n]$ is the output produced by $x_1[n]$, it is easy to show that $y_1[n] = nx_1[n] = n\, \delta[n-1]= \delta[n-1]$.

However, $y[n-1] = (n-1)\, x[n-1] = (n-1)\, \delta[n-1] = 0$ for all n.

So $y_1[n] \neq y[n-1]$. Thus, $y[n-1]$ is not the output of $x[n-1]$.

Linear

A system is linear if it is additive and scalable. That is, $a\, x_1(t) + b\, x_2(t) \xrightarrow{yields} a\, y_1(t) + b\, y_2(t)$, for all a, b ∈ ℂ.

Example: 1. The system $y(t) = 2\pi\, x(t)$ is linear.

Consider a signal $x(t) = a\, x_1(t) + b\, x_2(t)$, where $y_1(t) = 2\pi\, x_1(t)$ and $y_2(t) = 2\pi\, x_2(t)$.

Then $a\, y_1(t) + b\, y_2(t) = a\, (2\pi\, x_1(t)) + b\, (2\pi\, x_2(t)) = 2\pi\, [a\, x_1(t) + b\, x_2(t)] = 2\pi\, x(t) = y(t)$:

2. The system $y[n] = (x[2n])^2$ is not linear.

Consider the signal $x[n] = a\, x_1[n] + b\, x_2[n]$; where $y_1[n] = (x_1[2n])^2$ and $y_2[n] = (x_2[2n])^2$.

We want to see whether $y[n] = a\, y_1[n] + b\, y_2[n]$.

It holds that $a\, y_1[n] + b\, y_2[n] = a\, (x_1[2n])^2 + b\, (x_2[2n])^2$:

However, $y[n]=(x[2n])^2=(a\, x_1[2n] + b\, x_2[2n])^2 = a^2(x_1[2n])^2 + b_2(x_2[2n])^2 + 2ab\, x_1[n]\, x_2[n]$.