(92)

MAGNITUDE & TIME SCALING TECHNIQUES

- In the process of solving the problems no consideration has so far been given to the magnitudes of the problem variables or the time required for solving each problem.

- Equipment constraints usually restrict the maximum voltage output of each ~~apt~~ amplifier (to 10 volts).
- Thus, amplifier overload would occur when the problem dictates a large voltage output — thus, the problem must be magnitude scaled to fit the computer range.
- It would be necessary to scale up some parts of a problem & scale down other parts.
- For example: if the problem of calculating rocket speed is solved then it would be impossible to relate 1 volt on computer to 1 ft/sec in real problem. However, it may be solved by relating 1 volt on computer to 1 mile/sec in real problem — by magnitude scaling up.
  - Again, in the same problem an acceleration term might have a maximum value of 1 ft/sec² — which on basic unit of mile for length would give a voltage of approximately 0.002 volt — which is in the noise level of signal & would yield erroneous results. — so it would be required to be scaled-up.

\# Thus, one basic change of units for the entire problem may not work.

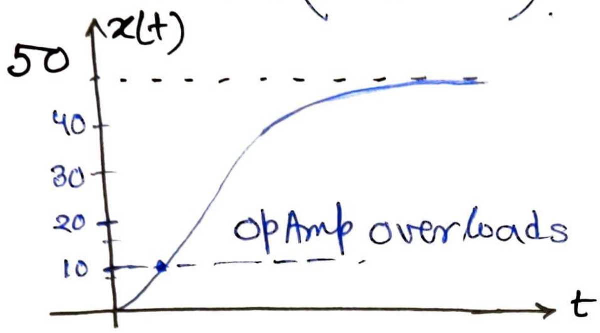
- Time scaling is needed when one intends to study a physical phenomenon that ~~els~~ evolves much faster or slower than is convenient for the computer operator.
  - For example: If one wishes to study planetary motion then he may not have the time to wait for a complete period of revolution about the sun — in fact an actual time of one year may be represented as 10 seconds (say) ~~so~~ of computer time.

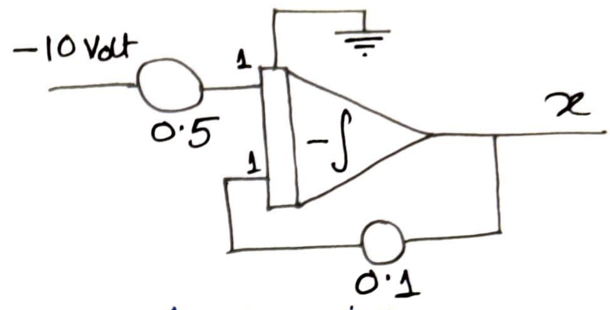
Magnitude Scaling – Basic Principles:

Consider the differential equation: $\frac{dx}{dt} + 0{\cdot}1x = 5;\ x(0) = 0$

- **Analytical Solution:**

$$x(t) = 50\left(1 - e^{-0{\cdot}1t}\right).$$





Analog Diagram

- The amplifier overloads – as it cannot supply 50 volts.
- To avoid this difficulty a different correspondence must be setup between the problem variable (x) & computer voltage.
- Since maximum value of V is 10 volt, let 5 units of x to correspond to 1 volt of V, i.e., $V = x/5$ — thus, amplifier won't be overloaded.
- The analogous equation is: $\frac{dV}{dt} + 0{\cdot}1V = 1$ & $V(0) = 0$

& the analytic solution is: $V(t) = 10\left(1 - e^{-0{\cdot}1t}\right)$

The basic idea is that the analog simulation does not solve the actual equation but an analogous equation.

Analog voltages normalized to units — it is common practice to redefine the magnitudes of reference voltages & all associated input & output voltages on an analog computer in terms of a quantity called the "Analog Unit" (or "Unit").

- By definition the maximum voltage obtainable on the computer is one unit — thus for a +10 volt computer, 1 unit = 10 volts
- Scaling problems in terms of units makes the scaled analog diagram machine-independent., i.e., good for 100 volt as well as 10 volt machines.

94 A systematic approach to Magnitude Scaling

— Objective: To construct a scaled analog diagram for $\frac{dx}{dt} + 0.1x = 5,\ x(0) = 0$ so that the maximum range of the amplifier is utilized but will not cause any overloaded.

Step 1: Draw the unscaled analog diagram. Note that +10 volt is assumed to be the maximum reference voltage available on the computer.

Step 2: Estimate the maximum value of each variable appearing as an amplifier output only (since all other elements have a gain less than unity— their output need not be considered).

In present case $x(t)$ is the only amplifier output which has a maximum value of 50.

Step 3: Define a scaling relationship between each problem variable & the corresponding computer variable.

In present case, $|x|_{max} = 50$, which may be represented as a voltage V_x on the analog computer.

Now, V_x can vary from −10 volt to 0 volt to +10 volt, whereas x can vary from 0 to 50. Therefore, +10 volt would represent 50.

• Thus, one may write, $V_x = \frac{x}{5} = L_x x$, with, L_x: level of $x = \frac{1}{5}$

• In general, one may choose, $L_x = \frac{10}{|x|_{max}}$

• For any variable having variation from −100 to 100, one may use $L_x = 1/10$ & +10 & −10 volt would represent 100 & −100, respectively.

• Scaling Table:

Problem Variable	Estimated Maximum	Level	Computer Variable	I.C.	Scaled I.C.
x	50	1/5	$[x/5]$	0	[0]

Step 4: Write down the scaled analog equations for each amplifier in terms of the computer variables (amplifier outputs) defined in step 3. No other variable should appear in these equations.

- For the present example, solving for the derivative, one can write, $\frac{dx}{dt} = 5 - 0.1x$, $x(0) = 0$.
- To get this equation in terms of the computer variable, divide & multiply each term by $L_x = 1/5$ to get,

$$\frac{d}{dt}\left(\frac{5x}{5}\right) = 5 - 0.1\left(\frac{5x}{5}\right), \quad x(0) = 0$$

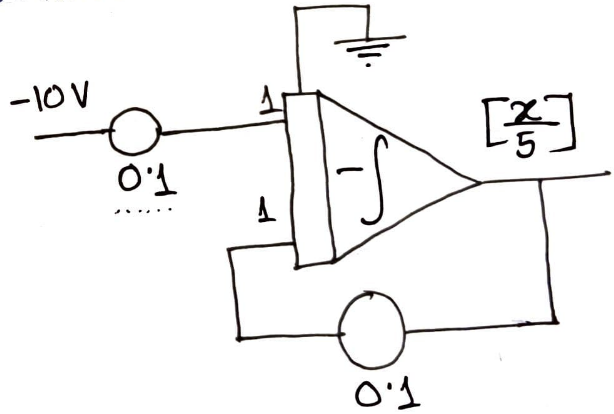
- Now, regrouping the constants, so that all variables are computer variables, one gets,

$$5 \cdot \frac{d}{dt}\left[\frac{x}{5}\right] = 5 - 0.5\left[\frac{x}{5}\right], \quad \left[\frac{x}{5}\right](0) = 0.$$

$$\text{or, } \underline{\frac{d}{dt}\left[\frac{x}{5}\right] = 1 - 0.1\left[\frac{x}{5}\right]}, \quad \underline{\left[\frac{x}{5}\right](0) = 0}$$

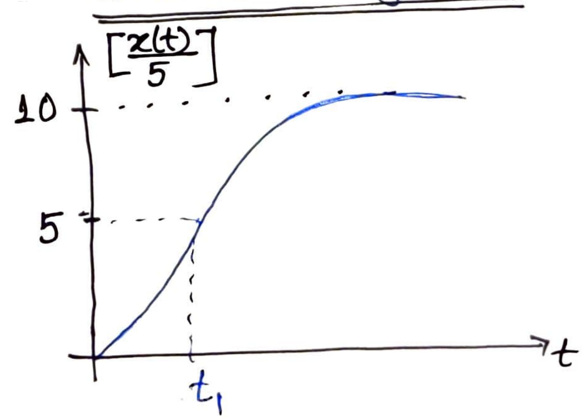
Step 5: Draw the scaled analog diagram using equations obtained in step 4. & label all the computer variables on the diagram.

- The scaled diagram is of the same form as the original one, but will have different pot values & amplifier gains.



Scaled Diagram

- The output of the analog diagram shows the response of $\left[\frac{x}{5}\right]$, i.e., the analog variable in volts.
- To find actual value of x (say at time t_1) one needs to use the value of L_x.
- Thus, actual value of x at $t = t_1$, $5 \times 5 = 25$.



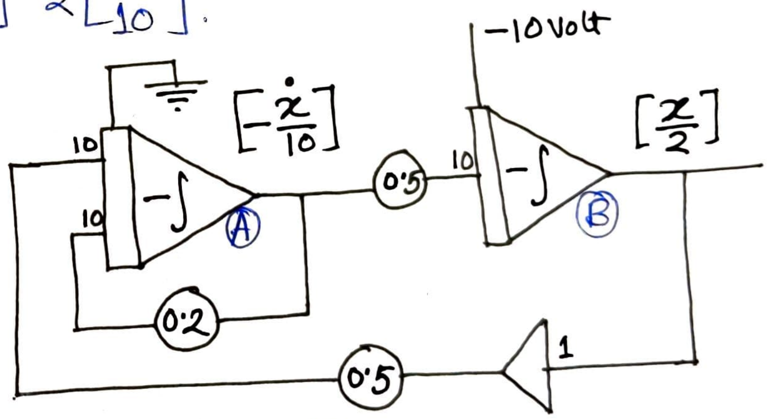
Time response

- Note: Constants are realized by patching a +10 volt reference to some pot set at a particular value. To be consistent with the magnitude scaling method the pot setting must be found using some formula.
- Initial condition, is the value of variable at $t=0$, & it should be similarly scaled to obtain the exact value of pot setting considering an ±10 volt voltage level.

Example: Magnitude-scale the following problem so that no over-load occurs & maximum range is used.

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+25x=0;\ x(0)=20;\ \dot{x}(0)=0;\ |x|\leq 20\ \&\ |\dot{x}|\leq 100.$$

- Solving for $\ddot{x}$, one gets, $\ddot{x}=-25x-2\dot{x}$
- The level constants are: $L_x=\frac{10}{20}=\frac{1}{2}$ & $L_{\dot{x}}=\frac{10}{100}=\frac{1}{10}$
- Thus, from the given problem, $\ddot{x}=-25\times 2\left[\frac{x}{2}\right]-2\times 10\left[\frac{\dot{x}}{10}\right]$, with, $\left[\frac{x}{2}\right]$ & $\left[\frac{\dot{x}}{10}\right]$ are computer variables.
- Similarly for initial conditions, $\left[\frac{x}{2}\right](0)=10$ & $\left[\frac{\dot{x}}{10}\right](0)=0$.
- In order to get $-[\dot{x}/10]$ to be the output of an amplifier, $\left[\frac{\ddot{x}}{10}\right]$ must be the input to the amplifier (integrator).
- Realizing the above fact, one needs to divide the equation by 10 to get, $\frac{\ddot{x}}{10}=-5\left[\frac{x}{2}\right]-2\left[\frac{\dot{x}}{10}\right]$.
- The initial condition for amplifier (A) should be grounded to emulate $\left[\frac{\dot{x}}{10}\right](0)=0$ & that of (B) should be connected to a -10 volt to emulate $\left[\frac{x}{2}\right](0)=10$.



Scaled diagram

Example: Simulate $y=ax$, $|x|_{max}=20$, $|y|_{max}=40$ & $a=-3$.

- Let, $V_x=L_x x$ & $V_y=L_y y$
- Now, $V_y=L_y y=-KGV_x=-KGL_x x$

$\Rightarrow y=\frac{-KGL_x}{L_y}x \Rightarrow a=\frac{-KGL_x}{L_y}$, or $\boxed{K=\frac{-aL_y}{GL_x}}$

Scaling diagram

- In our case, $L_x=\frac{1}{2}$ & $L_y=1/4$
- Thus, $K=\frac{-1/4}{1/2}\cdot\frac{-3}{G}=1.5/G$ [for, $G=1$, $K=1.5$ which is impossible; for $G=10$, $K=0.15$ which is feasible]

$$\boxed{\#\ K=\frac{\text{Level out}}{\text{Level in}}\times\frac{|\text{Co-efficient}|}{\text{Gain}}}$$

- **Level out:** level of variable immediately following the pot to be set;
- **Level in:** level of variable preceeding the pot
- Co-eff: relates to the equation to
- Gain: gain of the amplifier following pot.