ANALOG SIMULATION:

- Analog simulation is used to solve various types of problems.

- A typical simulation of a physical system involves a mathematical model of the system consisting of one or more differential equations & initial conditions on the variables of interest.

- Most general purpose analog simulation employs an active, ~~elec~~ electric circuit — because it has no moving parts, has a high speed of operation, good accuracy & a high degree of versatility.

- Active electrical networks consisting of resistors, capacitors & Op-amps are capable of simulating any linear system — since the forward voltage transfer characteristics of these networks are analogous to the basic linear mathematical operations encountered in modeling the dynamics of a physical system.

- The input & output voltages of an analog simulation ~~is~~ are analogous to corresponding mathematical variables.

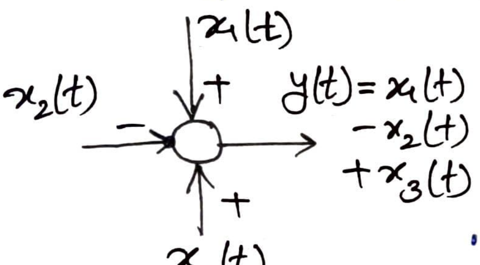
- Due to limitations in computer/associated equipments, it becomes necessary to change the scale of the variables (generally to keep them within limits) suitably — however, the time dependency of the variables remains the same.

- The normal procedure for simulating:
  - Determine the mathematical model that describes the physical quantities of interest;
  - Develop an analog block diagram to relate the sequence of mathematical operations;
  - Connect the electrical components accordingly;
  - Operate the simulation & observe/record the output variable (which need to be converted to original system variable).

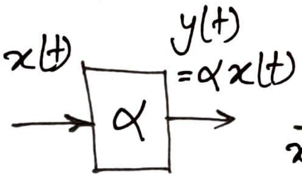
Solving Differential Equations:

- For a linear system, the differential equations are also linear, therefore the basic operations required are: ① Summation, ② Sign inversion; ③ Multiplication by a constant, ④ Integration & ⑤ Differentiation.

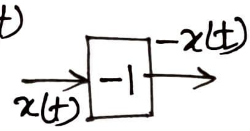
- From practical point of view integration is easier to implement & also more preferable than differentiation because — the signals involved are all real signals (say voltages, currents etc.) & therefore are corrupted by noise to certain extent — such noise will be averaged out by integration while the differentiation will accentuate it. — thus, integration yields more precise solution.



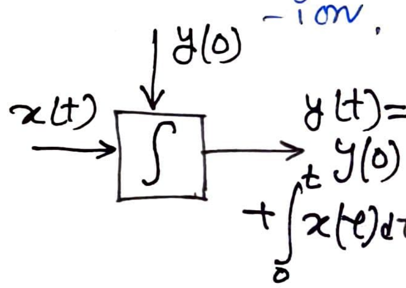
• Summation



• Multiplication by a constant



• Sign inversion

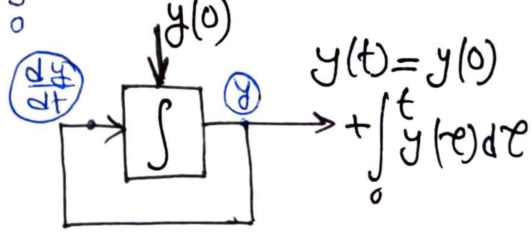


• Integration

Example: $\frac{dy}{dt} = y$, $y(0) = 1$.

- Integrating both sides, $y(t) = y(0) + \int_0^t y(\tau)d\tau$

- It is not necessary to know the input of the integrator to solve the equation — rather it is necessary to ensure the input to be equal to the output at all instants.



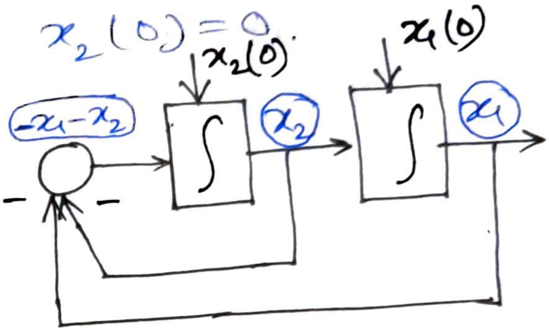
- The basis of analog simulation for solving differential equation is to feed the unknown output back to the input to generate the solution.

Example: $\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 0$, $y(0) = 1$, $\dot{y}(0) = 0$.

- Any higher order linear differential equation can be handled by reducing it to a set of first-order equations
- Let, $x_1 = y$ & $x_2 = \dot{x}_1 = \dot{y}$

$\therefore$ set of equations: $\dot{x}_1 = x_2$, $x_1(0) = 1$

$\dot{x}_2 = -x_1 - x_2$, $x_2(0) = 0$.



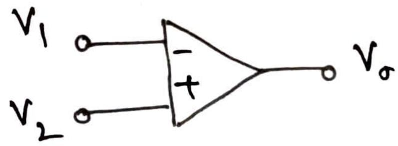
$\therefore$ Equivalent integral forms:

$$x_1(t) = x_1(0) + \int_0^t x_2(\tau)\, d\tau$$

$$\& \quad x_2(t) = x_2(0) - \int_0^t [x_1(\tau) + x_2(\tau)]\, d\tau$$

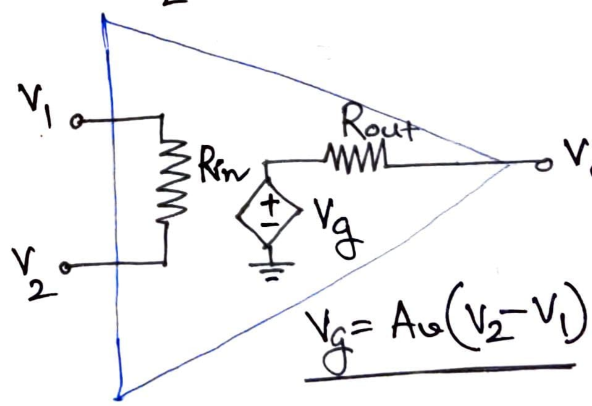
Physical Realization of Linear Operations:

- The basic linear operations, such as, summation, multiplication by a constant, integration will be realized with physical (electrical) components.



Operational Amplifier:

- Usually used as an high gain amplifier & described in terms of gain, input & output impedance, bandwidth & offset characteristics.



$$V_g = A_v(V_2 - V_1)$$

- It has two input terminals — one non-inverting (+) & an inverting (−) input.

- Equivalent op-amp circuit

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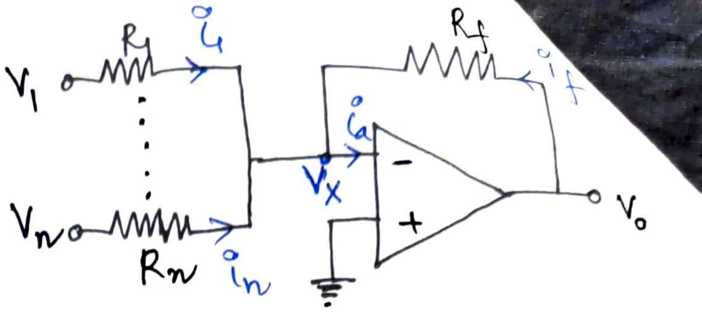
Summers & inverters:



- Applying KCL at summing junction:

$$i_1 + i_2 + \dots + i_n + i_f = i_a$$

or, in terms of voltage,

$$\frac{V_1 - V_x}{R_1} + \dots + \frac{V_n - V_x}{R_n} + \frac{V_o - V_x}{R_f} = \frac{V_x}{R_{in}}, \quad \text{with, } V_o = -A_v V_x$$

$$\Rightarrow \frac{V_1}{R_1} + \dots + \frac{V_n}{R_n} + \frac{V_o}{R_f} = \frac{-V_o}{A_v R_{in}} - \frac{V_o}{A_v R_1} - \dots - \frac{V_o}{A_v R_n} - \frac{V_o}{A_v R_f}$$

$$= \frac{-V_o}{A_v}\left(\frac{1}{R_{in}} + \frac{1}{R_1} + \dots + \frac{1}{R_n} + \frac{1}{R_f}\right)$$

$$= \frac{-V_o}{A_v R}; \quad \text{with, } \frac{1}{R} = \frac{1}{R_{in}} + \frac{1}{R_f} + \left(\frac{1}{R_1} + \dots + \frac{1}{R_n}\right)$$

$$\Rightarrow -V_o\left(\frac{1}{R_f} + \frac{1}{A_v R}\right) = \frac{V_1}{R_1} + \dots + \frac{V_n}{R_n}$$

$$\Rightarrow -\frac{V_o}{R_f}\left(1 + \frac{R_f}{A_v R}\right) = \frac{V_1}{R_1} + \dots + \frac{V_n}{R_n}$$

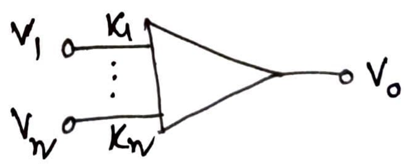
$$\Rightarrow V_o = \frac{-R_f}{R_1\left(1 + \frac{R_f}{A_v R}\right)} V_1 - \dots - \frac{R_f}{R_n\left(1 + \frac{R_f}{A_v R}\right)} V_n = \underline{-\frac{R_f}{R_1} V_1 - \dots - \frac{R_f}{R_n} V_n}$$

(since, $A_v > 10^5$, or $A_v \to \infty$).

- Usually analog diagrams for this weighted sum is given as,

with, $V_o = -K_1 V_1 - \dots - K_n V_n$

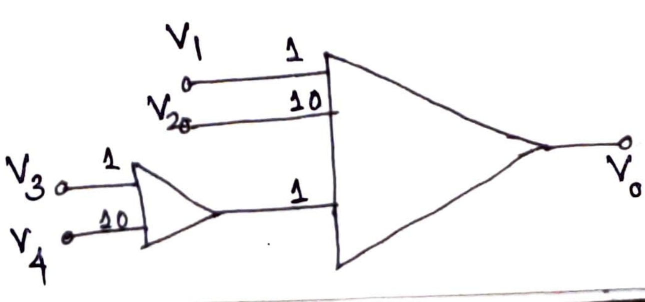
with, $K_i = \frac{R_f}{R_i}$, $i = 1, 2, \dots, n$.



- The inherent sign inversion is a result of the negative voltage gain of the op-amp.
- For a special case of summer having only one input & $K_1 = 1$ one gets an inverter.

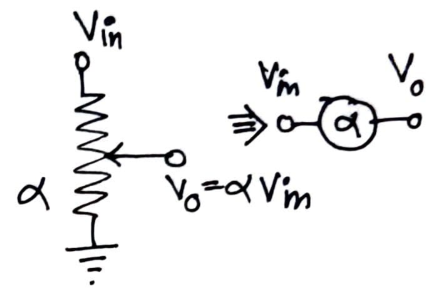
Example:

$$V_o = -V_1 - 10V_2 + V_3 + 10V_4$$



(ii) Multiplication by a constant:

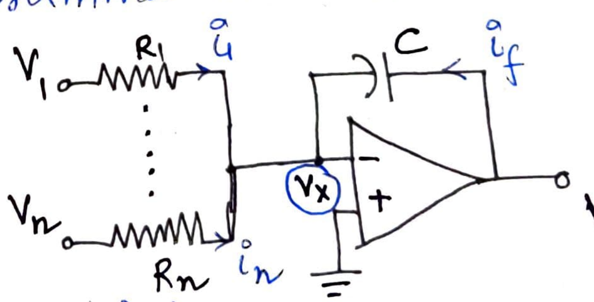
- Op-amps are generally accompanied by fixed resistors, then only a few gains can be realized.
- To realize any arbitrary gain a coefficient potentiometer (pot), a voltage divider that allows the output voltage to be a fraction of the input voltage, is employed.
- A pot has a gain of less than unity.



- $V_0 = \alpha V_{in}, \quad 0 \lesssim \alpha \lesssim 1$
- A constant term in the simulation of a differential equation is obtained by using a dc reference voltage at the input of the pot & suitably adjusting the gain.

(iii) Integrators:

- Integration is the most important operation available in analog computer.
- In fact analog computers owe their existence to their ability to integrate rapidly.
- Integration is different from summation or inversion because it is time dependent.



- Transfer characteristics of the circuit,

$$V_o(t) = V_o(0) - \int_0^t \left[\frac{V_1(\tau)}{R_1 C} + \dots + \frac{V_n(\tau)}{R_n C} \right] d\tau$$

- There is a sign inversion in integration operation.
- Summation & integration can be performed with a single amplifier.

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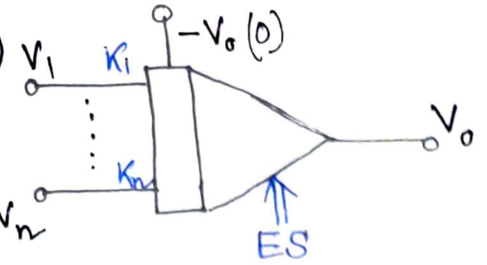
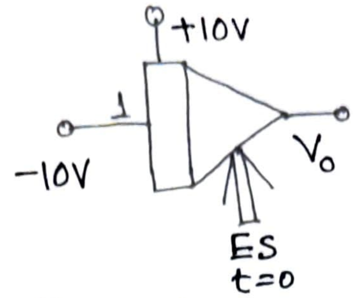
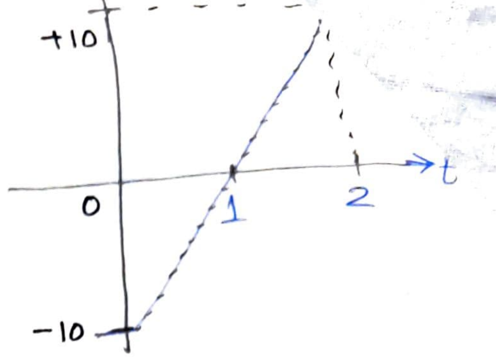






- E.S.: Electronic switch is used to control the operating modes of the integrator.
- The normal modes of operation are: initial condition (IC) & operate (OP) mode.

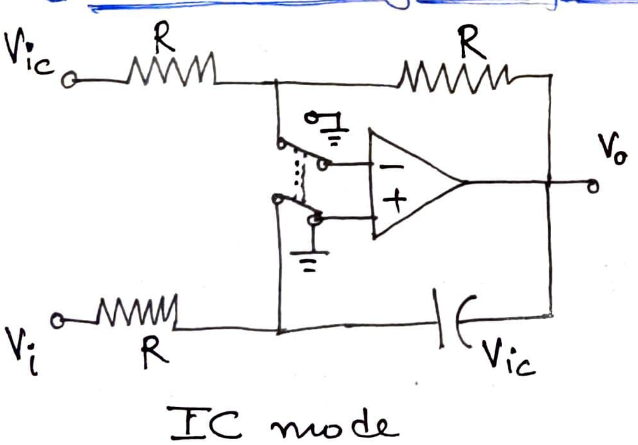
- IC mode allows integrator capacitor to be charged to the initial values.

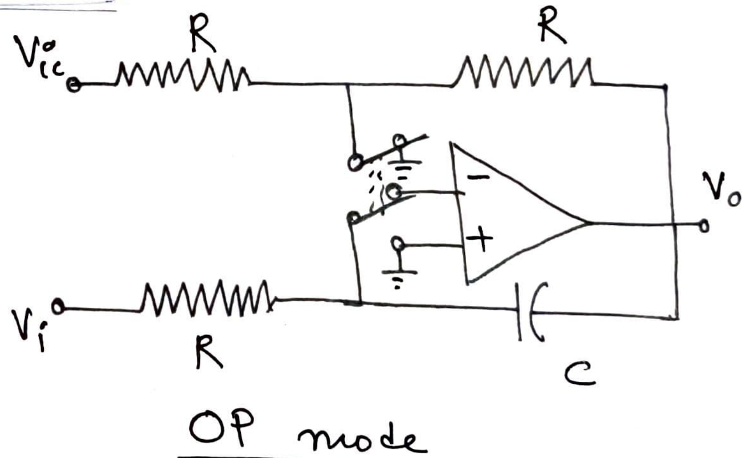
- OP mode causes the solution to occur.

- ES: nothing more than a SPDT switch with one pole grounded.

Note: • $-V_o(0)$ must be applied to the IC terminal to get $+V_o(0)$ at the output.

Circuit Diagram for Integrator:





IC mode   OP mode

- In IC mode the capacitor charges to $-V_{ic}$, which can be used to represent an initial condition.
- In OP mode the output of the system is the negative of the integral of the input starting from the initial condition (in this case initial condition part of the circuit is grounded)

A systematic procedure for developing Analog Simulation:

- This procedure does not require a first order integral equation to be considered for each integrator, but requires an equation for the highest order derivative.

- Consider the equation

$$\frac{d^2x}{dt^2} + 0.5\frac{dx}{dt} + x = 4, \quad x(0) = 0, \quad \dot{x}(0) = 1$$

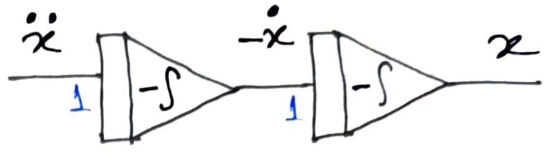
- Now, if any derivative of a variable is known then it may be integrated to obtain the variable.
- In case of above equation if $\frac{d^2x}{dt^2}$ is known then it could be integrated once to get $\frac{dx}{dt}$ & a second time to get x.
- **Note:** since each integration has a sign inversion associated with it, the output of every odd number of integration is negative.
- The equation gives 2nd order derivative in terms of lower order ones. Thus,
  - if $\frac{d^2x}{dt^2}$ is known then $-\frac{dx}{dt}$ & then x may be obtained
  - if $\frac{dx}{dt}$ is known & x is known, $\frac{d^2x}{dt^2}$ may be obtained.

- This circular argument states that only the relationship between the inputs & outputs of the integrators are known not their actual values.
- In fact a differential equation ~~of~~ represents only the class of solutions — the boundary values or the initial conditions are necessary to determine a particular solution.

- So one needs to consider the initial conditions in deriving the analog simulation diagram to obtain the solution.

Step I: Assume that the highest order derivative ($\frac{d^2x}{dt^2}$ in this case) is known & generate all lower order derivatives. (Note that the output should always be x), not $-x$.

Step II: Solve the differential equation for the input to the integrator string and form the indicated sum.

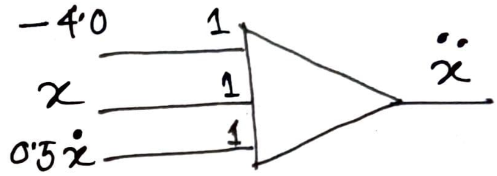


Step I

- One can write,

$$\frac{d^2x}{dt^2} = -0.5\frac{dx}{dt} - x + 4$$

(Note: summer also inverts)



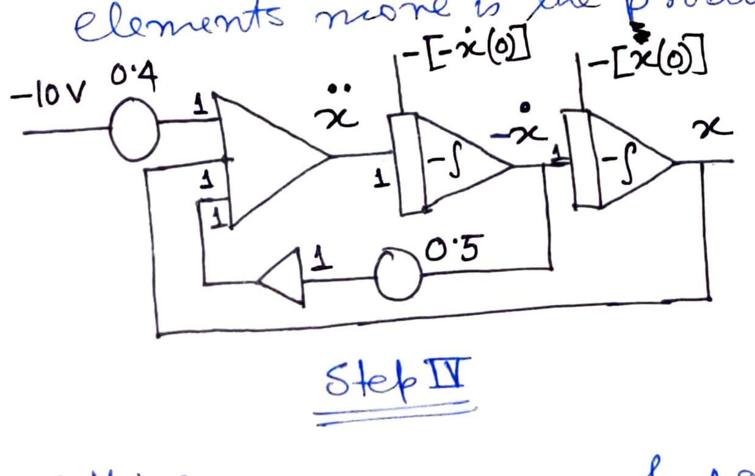
Step II

Step III: Combine the results of step I & II using pots, summers & inverters.

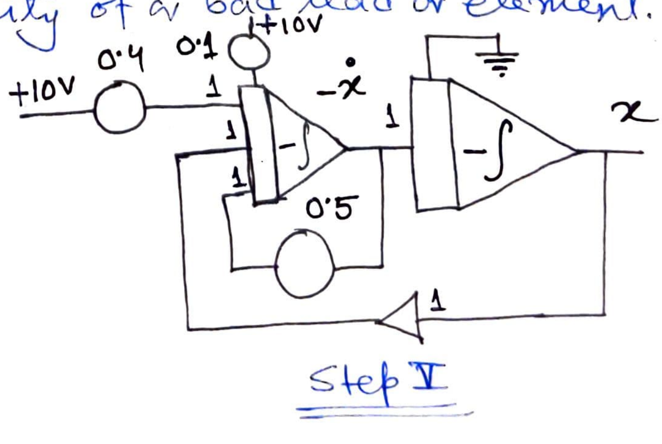
Step IV: Add all the initial conditions. (Note that the applied initial conditions should be the negative of the integrator output at t=0)

Step V: It is possible to remove the summer, as integrator can serve the purpose. (However, inverters must be inserted or removed from each of the feedback loop to take care of sign inversion.)

- **Note:** Usually the form which utilizes least number of elements is preferred, since more the number of elements more is the probability of a bad lead or element.



Step IV



Step V

- **Note:** For an odd value of highest order of differentiation one should start from $-\frac{d^n x}{dt^n}$ (n = odd) so that finally one reaches to x as final output.