$termvar, x$	term variable			
$typevar, X$	type variable			
$natural, n$	natural number			
k	index			

$prog,\ P$	$::=$			program
	$\vert$	$decl_1\ ..\ decl_k$		
$decl,\ D$	$::=$			declaration
	$\vert$	$x : T;$		type
	$\vert$	$x = t;$		definition
uop	$::=$			unary operator
	$\vert$	$-$		negation
bop	$::=$			binary operator
	$\vert$	$+$		addition
	$\vert$	$-$		subtraction
	$\vert$	$*$		multiplication
	$\vert$	$/$		division
	$\vert$	$\equiv$		equality
	$\vert$	$<$		less than
	$\vert$	$\mathbf{and}$		boolean and
	$\vert$	$\mathbf{or}$		boolean or
$term,\ t$	$::=$			term
	$\vert$	x		variable
	$\vert$	$()$		unit value
	$\vert$	$\mathbf{true}$		true
	$\vert$	$\mathbf{false}$		false
	$\vert$	$x \mapsto t$	bind x in t	abstraction
	$\vert$	$t\ t'$		application
	$\vert$	(t_1, t_2)		pair
	$\vert$	$\mathbf{inl}\ t_1$		left introduction
	$\vert$	$\mathbf{inr}\ t_2$		right introduction
	$\vert$	n		natural literal
	$\vert$	$uop\ t$		unary operator
	$\vert$	$t_1\ bop\ t_2$		binary operator
	$\vert$	$\mathbf{let}\ x = t_1\ \mathbf{in}\ t_2$	bind x in t_2	let
	$\vert$	$\mathbf{case}\ branches$		case analysis
	$\vert$	$t : T$		type ascription
	$\vert$	(t)	S	
$value,\ v$	$::=$			value
	$\vert$	n		natural number
	$\vert$	$x \mapsto t$		abstraction
	$\vert$	(v_1, v_2)		pair
	$\vert$	$\mathbf{inl}\ v$		left introduction
	$\vert$	$\mathbf{inr}\ v$		right introduction
$stype,\ S$	$::=$			simple type
	$\vert$	X		variable

	|	$\mathbb{0}$	void type
	|	$\mathbb{1}$	unit type
	|	$\mathbb{B}$	boolean type
	|	$T_1 \to T_2$	function type
	|	$T_1 \times T_2$	product type
	|	$T_1 \uplus T_2$	sum type
	|	$\mathbb{N}$	natural type
	|	$\mathbb{Z}$	integer type
	|	$\mathbb{Q}$	rational type
	|	(T) S	
qual	::=		qualifier
	|	$\forall X$	universal
	|	$S_1 <: S_2$	subtype
	|	**finite** S	finite
	|	**decidable** S	decidable
	|	**ordered** S	ordered
	|	**numeric** S	numeric
constraint, C	::=		constraint
	|	$\top$	trivial
	|	$\bot$	impossible
	|	$C_1 \wedge C_2$	conjunction
	|	$C_1 \vee C_2$	disjunction
	|	$S_1 <: S_2$	subtype
	|	$S_1 = S_2 \sqcup S_3$	lub
	|	$T_1 \ll: S_2$	instantiation + subtype
	|	**finite** S	finite
	|	**decidable** S	decidable
	|	**ordered** S	ordered
	|	**numeric** S	numeric
	|	(C) S	
type, T	::=		type
	|	S	simple type
	|	*qual*, T	qualified type
ν	::=		numeric type
	|	$\mathbb{N}$	natural type
	|	$\mathbb{Z}$	integer type
	|	$\mathbb{Q}$	rational type
Γ	::=		type context
	|	$\varnothing$	empty context
	|	$\Gamma, x : T$	cons
pattern, p	::=		pattern

$\vert$	x	binders $= x$	variable
$\vert$	$\_$	binders $= \{\}$	wildcard
$\vert$	$()$	binders $= \{\}$	unit
$\vert$	**true**	binders $= \{\}$	true
$\vert$	**false**	binders $= \{\}$	false
$\vert$	(p_1, p_2)	binders $=$ binders$(p_1) \cup$ binders(p_2)	pair
$\vert$	**inl** p	binders $=$ binders(p)	left
$\vert$	**inr** p	binders $=$ binders(p)	right
$\vert$	n	binders $= \{\}$	natural
$\vert$	$S\ p$	binders $=$ binders(p)	successor
$\vert$	(p)	S	

branches, *b*	::=		
	$\vert$	$\varnothing$	
	$\vert$	$\{$*branches*	
	$\vert$	$\{$*term guard branches*	bind binders(*guard*) in *term*

aguard, *ag*	::=			atomic guard
	$\vert$	**if** t	binders $= \{\}$	boolean guard
	$\vert$	**when** $t = p$	binders $=$ binders(p)	pattern guard

guard, *g*	::=			guard
	$\vert$	$\top$	binders $= \{\}$	trivial guard
	$\vert$	$ag\ g$	binders $=$ binders$(ag) \cup$ binders(g)	cons atomic

subst, θ	::=	
	$\vert$	$\varnothing$
	$\vert$	$\{t/x\}$*subst*

$\boxed{S_1 <: S_2}$ S_1 is a subtype of S_2

$$\frac{}{S <: S}\ \textsc{sub\_refl}$$

$$\frac{S_1 <: S_2 \quad S_2 <: S_3}{S_1 <: S_3}\ \textsc{sub\_trans}$$

$$\frac{}{\mathbb{N} <: \mathbb{Z}}\ \textsc{sub\_n\_z}$$

$$\frac{}{\mathbb{Z} <: \mathbb{Q}}\ \textsc{sub\_z\_q}$$

$$\frac{S_1' <: S_1 \quad S_2 <: S_2'}{S_1 \to S_2 <: S_1' \to S_2'}\ \textsc{sub\_funty}$$

$$\frac{S_1 <: S_1' \quad S_2 <: S_2'}{S_1 \times S_2 <: S_1' \times S_2'}\ \textsc{sub\_prod}$$

$$\frac{S_1 <: S_1' \quad S_2 <: S_2'}{S_1 \uplus S_2 <: S_1' \uplus S_2'}\ \textsc{sub\_sum}$$

$\boxed{S_1 = S_2 \sqcup S_3}$ S_1 is the lub of S_2 and S_3

$$\frac{S_2 <: S_1}{S_1 = S_1 \sqcup S_2}\ \textsc{lub\_subl}$$

$$\frac{S_1 <: S_2}{S_2 = S_1 \sqcup S_2}\ \textsc{lub\_subr}$$

$$\frac{S_{21} = S_{22} \sqcup S_{23}}{S_1 \to S_{21} = S_1 \to S_{22} \sqcup S_1 \to S_{23}}\ \textsc{lub\_funty}$$

$$\frac{S_{11} = S_{12} \sqcup S_{13} \quad S_{21} = S_{22} \sqcup S_{23}}{S_{11} \times S_{21} = S_{12} \times S_{22} \sqcup S_{13} \times S_{23}}\ \textsc{lub\_prod}$$

$$\frac{S_{11} = S_{12} \sqcup S_{13} \quad S_{21} = S_{22} \sqcup S_{23}}{S_{11} \uplus S_{21} = S_{12} \uplus S_{22} \sqcup S_{13} \uplus S_{23}}\ \textsc{lub\_sum}$$

$\boxed{\textbf{Finite}\, S}$ S is finite

$$\frac{}{\textbf{Finite}\, \mathbb{0}}\ \textsc{fin\_Void}$$

$$\frac{}{\textbf{Finite}\, \mathbb{1}}\ \textsc{fin\_Unit}$$

$$\frac{}{\textbf{Finite}\, \mathbb{B}}\ \textsc{fin\_Bool}$$

$$\frac{\textbf{Finite}\, S_1 \quad \textbf{Finite}\, S_2}{\textbf{Finite}\, S_1 \times S_2}\ \textsc{fin\_prod}$$

$$\frac{\textbf{Finite}\, S_1 \quad \textbf{Finite}\, S_2}{\textbf{Finite}\, S_1 \uplus S_2}\ \textsc{fin\_sum}$$

$$\frac{\textbf{Finite}\, S_1 \quad \textbf{Finite}\, S_2}{\textbf{Finite}\, S_1 \to S_2}\ \textsc{fin\_funty}$$

$\boxed{\textbf{Decidable}\, S}$ S has decidable equality

$$\frac{}{\textbf{Decidable}\, \mathbb{0}}\ \textsc{dec\_Void}$$

$$\frac{}{\textbf{Decidable}\, \mathbb{1}}\ \textsc{dec\_Unit}$$

$$\frac{}{\textbf{Decidable}\, \mathbb{B}}\ \textsc{dec\_Bool}$$

$$\frac{}{\textbf{Decidable}\, \mathbb{N}}\ \textsc{dec\_N}$$

$$\frac{}{\textbf{Decidable}\, \mathbb{Z}}\ \textsc{dec\_Z}$$

$$\frac{}{\textbf{Decidable}\, \mathbb{Q}}\ \textsc{dec\_Q}$$

$$\frac{\textbf{Decidable}\, S_1 \quad \textbf{Decidable}\, S_2}{\textbf{Decidable}\, S_1 \times S_2}\ \textsc{dec\_prod}$$

$$\frac{\textbf{Decidable}\, S_1 \quad \textbf{Decidable}\, S_2}{\textbf{Decidable}\, S_1 \uplus S_2}\ \textsc{dec\_sum}$$

$$\frac{\textbf{Finite}\, S_1 \quad \textbf{Decidable}\, S_2}{\textbf{Decidable}\, S_1 \to S_2}\ \textsc{dec\_funty}$$

$\boxed{\textbf{Ordered}\, S}$ S is totally ordered

$$\frac{}{\textbf{Ordered}\, \mathbb{0}}\ \textsc{ord\_Void}$$

$$\frac{}{\textbf{Ordered}\, \mathbb{1}}\ \textsc{ord\_Unit}$$

$$\frac{}{\textbf{Ordered}\,\mathbb{B}}\ \textsc{ord\_Bool}$$

$$\frac{}{\textbf{Ordered}\,\mathbb{N}}\ \textsc{ord\_N}$$

$$\frac{}{\textbf{Ordered}\,\mathbb{Z}}\ \textsc{ord\_Z}$$

$$\frac{}{\textbf{Ordered}\,\mathbb{Q}}\ \textsc{ord\_Q}$$

$$\frac{\textbf{Ordered}\,S_1 \quad \textbf{Ordered}\,S_2}{\textbf{Ordered}\,S_1 \times S_2}\ \textsc{ord\_prod}$$

$$\frac{\textbf{Ordered}\,S_1 \quad \textbf{Ordered}\,S_2}{\textbf{Ordered}\,S_1 \uplus S_2}\ \textsc{ord\_sum}$$

$$\frac{\textbf{Finite}\,S_1 \quad \textbf{Ordered}\,S_1 \quad \textbf{Ordered}\,S_2}{\textbf{Ordered}\,S_1 \to S_2}\ \textsc{ord\_funty}$$

$\boxed{\Gamma \vdash t : T}$ t has type T in context Γ

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T}\ \textsc{ty\_var}$$

$$\frac{}{\Gamma \vdash () : \mathbb{1}}\ \textsc{ty\_unit}$$

$$\frac{}{\Gamma \vdash \textbf{true} : \mathbb{B}}\ \textsc{ty\_true}$$

$$\frac{}{\Gamma \vdash \textbf{false} : \mathbb{B}}\ \textsc{ty\_false}$$

$$\frac{\Gamma, x_1 : T_1 \vdash t : T}{\Gamma \vdash x_1 \mapsto t : T_1 \to T}\ \textsc{ty\_abs}$$

$$\frac{\Gamma \vdash t : T_1 \to T_2 \quad \Gamma \vdash t' : T_1}{\Gamma \vdash t\,t' : T_2}\ \textsc{ty\_apply}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma \vdash t_2 : T_2}{\Gamma \vdash (t_1, t_2) : T_1 \times T_2}\ \textsc{ty\_pair}$$

$$\frac{\Gamma \vdash t : T_1}{\Gamma \vdash \textbf{inl}\,t : T_1 \uplus T_2}\ \textsc{ty\_inl}$$

$$\frac{\Gamma \vdash t : T_2}{\Gamma \vdash \textbf{inr}\,t : T_1 \uplus T_2}\ \textsc{ty\_inr}$$

$$\frac{}{\Gamma \vdash n : \mathbb{N}}\ \textsc{ty\_nat}$$

$$\frac{\Gamma \vdash t_1 : \nu \quad \Gamma \vdash t_2 : \nu}{\Gamma \vdash t_1 + t_2 : \nu}\ \textsc{ty\_add}$$

$$\frac{\Gamma \vdash t_1 : \nu \quad \Gamma \vdash t_2 : \nu}{\Gamma \vdash t_1 * t_2 : \nu}\ \textsc{ty\_mul}$$

$$\frac{\Gamma \vdash t_1 : \nu \quad \Gamma \vdash t_2 : \nu \quad \mathbb{Z} <: \nu}{\Gamma \vdash t_1 - t_2 : \nu}\ \textsc{ty\_sub}$$

$$\frac{\Gamma \vdash t : \nu \quad \mathbb{Z} <: \nu}{\Gamma \vdash -t : \nu}\ \textsc{ty\_neg}$$

$$\frac{\Gamma \vdash t_1 : \nu \quad \Gamma \vdash t_2 : \nu \quad \mathbb{Q} <: \nu}{\Gamma \vdash t_1 \,/\, t_2 : \nu}\ \textsc{ty\_div}$$

$$\frac{\Gamma \vdash t : \nu \quad \Gamma \vdash t' : \nu}{\Gamma \vdash t\, t' : \nu} \quad \text{TY\_MUL\_NUM}$$

$$\frac{\Gamma \vdash t_1 : S \quad \Gamma \vdash t_2 : S \quad \textbf{Decidable}\, S}{\Gamma \vdash t_1 \equiv t_2 : \mathbb{B}} \quad \text{TY\_EQ}$$

$$\frac{\Gamma \vdash t_1 : S \quad \Gamma \vdash t_2 : S \quad \textbf{Ordered}\, S}{\Gamma \vdash t_1 < t_2 : \mathbb{B}} \quad \text{TY\_LT}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 : \mathbb{B}}{\Gamma \vdash t_1\, \textbf{and}\, t_2 : \mathbb{B}} \quad \text{TY\_AND}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 : \mathbb{B}}{\Gamma \vdash t_1\, \textbf{or}\, t_2 : \mathbb{B}} \quad \text{TY\_OR}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash \textbf{let}\, x = t_1\, \textbf{in}\, t_2 : T_2} \quad \text{TY\_LET}$$

$$\frac{\Gamma \vdash_b b : T}{\Gamma \vdash \textbf{case}\, b : T} \quad \text{TY\_CASE}$$

$$\frac{\Gamma \vdash t : T}{\Gamma \vdash (t : T) : T} \quad \text{TY\_ASCRIBE}$$

$\boxed{\Gamma \vdash_b b : T}$ branches b have type T in context Γ

$$\frac{}{\Gamma \vdash_b \varnothing : T} \quad \text{BTY\_EMPTY}$$

$$\frac{\Gamma \vdash_b \mathit{branches} : T}{\Gamma \vdash_b \{\mathit{branches} : T} \quad \text{BTY\_SKIP}$$

$$\frac{\Gamma \vdash_g t, \mathit{guard} : T \quad \Gamma \vdash_b \mathit{branches} : T}{\Gamma \vdash_b \{t\; \mathit{guard}\; \mathit{branches} : T} \quad \text{BTY\_CONS}$$

$\boxed{\Gamma \vdash_g t, g : T}$ t guarded by g has type T in context Γ

$$\frac{\Gamma \vdash t : T}{\Gamma \vdash_g t, \top : T} \quad \text{GTY\_TOP}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_2 : \mathbb{B} \\ \Gamma \vdash_g t_1, g : T\end{array}}{\Gamma \vdash_g t_1, \textbf{if}\, t_2\, g : T} \quad \text{GTY\_IF}$$

$$\frac{\begin{array}{l}\Gamma \vdash_p p : T_2 \rightsquigarrow \Gamma_2 \quad \Gamma \vdash t_2 : T_2 \\ \Gamma, \Gamma_2 \vdash_g t_1, g : T\end{array}}{\Gamma \vdash_g t_1, \textbf{when}\, t_2 = p\; g : T} \quad \text{GTY\_WHEN}$$

$\boxed{\Gamma \vdash_p p : T \rightsquigarrow \Gamma'}$ p has type T in context Γ, and produces bindings Γ'

$$\frac{}{\Gamma \vdash_p x : T \rightsquigarrow \varnothing, x : T} \quad \text{P\_VAR}$$

$$\frac{}{\Gamma \vdash_p \_ : T \rightsquigarrow \varnothing} \quad \text{P\_WILD}$$

$$\frac{}{\Gamma \vdash_p () : \mathbb{1} \rightsquigarrow \varnothing} \quad \text{P\_UNIT}$$

$$\frac{}{\Gamma \vdash_p \textbf{true} : \mathbb{B} \rightsquigarrow \varnothing} \quad \text{P\_TRUE}$$

$$\frac{}{\Gamma \vdash_p \mathbf{false} : \mathbb{B} \rightsquigarrow \varnothing} \quad \text{P\_FALSE}$$

$$\frac{\Gamma \vdash_p p_1 : T_1 \rightsquigarrow \Gamma_1 \quad \Gamma \vdash_p p_2 : T_2 \rightsquigarrow \Gamma_2}{\Gamma \vdash_p (p_1, p_2) : T_1 \times T_2 \rightsquigarrow \Gamma_1, \Gamma_2} \quad \text{P\_PAIR}$$

$$\frac{\Gamma \vdash_p p : T_1 \rightsquigarrow \Gamma_1}{\Gamma \vdash_p \mathbf{inl}\, p : T_1 \uplus T_2 \rightsquigarrow \Gamma_1} \quad \text{P\_INL}$$

$$\frac{\Gamma \vdash_p p : T_2 \rightsquigarrow \Gamma_2}{\Gamma \vdash_p \mathbf{inr}\, p : T_1 \uplus T_2 \rightsquigarrow \Gamma_2} \quad \text{P\_INR}$$

$$\frac{}{\Gamma \vdash_p n : \mathbb{N} \rightsquigarrow \varnothing} \quad \text{P\_NAT}$$

$$\frac{\Gamma \vdash_p p : \mathbb{N} \rightsquigarrow \Gamma'}{\Gamma \vdash_p S\, p : \mathbb{N} \rightsquigarrow \Gamma'} \quad \text{P\_SUCC}$$

$\boxed{t_1 \longrightarrow t_2}$ t_1 reduces to t_2

$$\frac{\varnothing \vdash v_2 : \nu}{(x \mapsto t_{12})\, v_2 \longrightarrow \{v_2/x\}\, t_{12}} \quad \text{BETA\_NUM}$$

$$\frac{\neg\, (\varnothing \vdash t_2 : \nu)}{(x \mapsto t_{12})\, t_2 \longrightarrow \{t_2/x\}\, t_{12}} \quad \text{BETA}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ t \longrightarrow t_1'\ t} \quad \text{CONG\_APP\_FUN}$$

$$\frac{\varnothing \vdash t_1 : \nu \quad t_1 \longrightarrow t_1'}{v\ t_1 \longrightarrow v\ t_1'} \quad \text{CONG\_APP\_NUM}$$

$$\frac{t_1 \longrightarrow t_1'}{(t_1, t_2) \longrightarrow (t_1', t_2)} \quad \text{CONG\_FST}$$

$$\frac{t_2 \longrightarrow t_2'}{(t_1, t_2) \longrightarrow (t_1, t_2')} \quad \text{CONG\_SND}$$

$$\frac{t \longrightarrow t'}{\mathbf{inl}\, t \longrightarrow \mathbf{inl}\, t'} \quad \text{CONG\_INL}$$

$$\frac{t \longrightarrow t'}{\mathbf{inr}\, t \longrightarrow \mathbf{inr}\, t'} \quad \text{CONG\_INR}$$

$$\frac{t_1 \longrightarrow t_1'}{uop\ t_1 \longrightarrow uop\ t_1'} \quad \text{CONG\_UOP}$$

$$\frac{}{uop\ v \longrightarrow [\![uop\ v]\!]} \quad \text{UOP}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ bop\ t_2 \longrightarrow t_1'\ bop\ t_2} \quad \text{CONG\_BOP\_L}$$

$$\frac{t_2 \longrightarrow t_2'}{t_1\ bop\ t_2 \longrightarrow t_1\ bop\ t_2'} \quad \text{CONG\_BOP\_R}$$

$$\frac{}{v_1\ bop\ v_2 \longrightarrow [\![v_1\ bop\ v_2]\!]} \quad \text{BOP}$$

$$\frac{\varnothing \vdash t_1 : \nu \quad t_1 \longrightarrow t_1'}{\mathbf{let}\ x = t_1\ \mathbf{in}\ t_2 \longrightarrow \mathbf{let}\ x = t_1'\ \mathbf{in}\ t_2} \quad \text{CONG\_LET\_NUM}$$

$$\frac{\varnothing \vdash v_1 : \nu}{\textbf{let}\, x = v_1 \,\textbf{in}\, t_2 \longrightarrow \{v_1/x\}\, t_2} \quad \text{LET\_NUM}$$

$$\frac{\neg\,(\varnothing \vdash t_1 : \nu)}{\textbf{let}\, x = t_1 \,\textbf{in}\, t_2 \longrightarrow \{t_1/x\}\, t_2} \quad \text{LET}$$

$$\frac{}{\textbf{case}\,\{\,\mathit{branches} \longrightarrow \textbf{case}\,\mathit{branches}} \quad \text{CASE\_SKIP}$$

$$\frac{g \longrightarrow g'}{\textbf{case}\,\{\,t\; g\; \mathit{branches} \longrightarrow \textbf{case}\,\{\,t\; g'\; \mathit{branches}} \quad \text{CONG\_CASE}$$

$$\frac{g \rightsquigarrow \theta}{\textbf{case}\,\{\,t\; g\; \mathit{branches} \longrightarrow \theta\, t} \quad \text{CASE\_SUCCESS}$$

$$\frac{g!}{\textbf{case}\,\{\,t\; g\; \mathit{branches} \longrightarrow \textbf{case}\,\mathit{branches}} \quad \text{CASE\_FAILURE}$$

$\boxed{g \longrightarrow g'}$ guard g reduces to g'

$$\frac{t \longrightarrow t'}{\textbf{if}\; t\; g \longrightarrow \textbf{if}\; t'\; g} \quad \text{GRED\_IF}$$

$$\frac{t \longrightarrow t'}{\textbf{when}\; t = p\; g \longrightarrow \textbf{when}\; t' = p\; g} \quad \text{GRED\_WHEN}$$

$\boxed{\mathit{guard} \rightsquigarrow \theta}$ *guard* succeeds, producing substitution θ

$$\frac{}{\top \rightsquigarrow \varnothing} \quad \text{SUCCESS\_TOP}$$

$$\frac{g \rightsquigarrow \theta}{\textbf{if true}\; g \rightsquigarrow \theta} \quad \text{SUCCESS\_TRUE}$$

$$\frac{\begin{array}{l} t \sim p \rightsquigarrow \theta_1 \\ g \rightsquigarrow \theta_2 \end{array}}{\textbf{when}\; t = p\; g \rightsquigarrow \theta_2\, \theta_1} \quad \text{SUCCESS\_MATCH}$$

$\boxed{\mathit{guard}!}$ *guard* fails

$$\frac{}{\textbf{if false}\; g!} \quad \text{FAIL\_FALSE}$$

$$\frac{\neg\,(t \sim p \rightsquigarrow \theta)}{\textbf{when}\; t = p\; g!} \quad \text{FAIL\_MATCH}$$

$$\frac{g!}{\mathit{aguard}\; g!} \quad \text{FAIL\_REST}$$

$\boxed{t \sim p \rightsquigarrow \theta}$ term t matches pattern p, producing substitution θ

$$\frac{}{t \sim x \rightsquigarrow \{t/x\}} \quad \text{MATCH\_VAR}$$

$$\frac{}{t \sim \_ \rightsquigarrow \varnothing} \quad \text{MATCH\_WILD}$$

$$\frac{}{t \sim () \rightsquigarrow \varnothing} \quad \text{MATCH\_UNIT}$$

$$\frac{}{\textbf{true} \sim \textbf{true} \rightsquigarrow \varnothing} \quad \text{MATCH\_TRUE}$$

$$\frac{}{\mathbf{false} \sim \mathbf{false} \rightsquigarrow \varnothing} \quad \text{MATCH\_FALSE}$$

$$\frac{t_1 \sim p_1 \rightsquigarrow \theta_1 \quad t_2 \sim p_2 \rightsquigarrow \theta_2}{(t_1, t_2) \sim (p_1, p_2) \rightsquigarrow \theta_1\, \theta_2} \quad \text{MATCH\_PAIR}$$

$$\frac{t \sim p \rightsquigarrow \theta}{\mathbf{inl}\, t \sim \mathbf{inl}\, p \rightsquigarrow \theta} \quad \text{MATCH\_INL}$$

$$\frac{t \sim p \rightsquigarrow \theta}{\mathbf{inr}\, t \sim \mathbf{inr}\, p \rightsquigarrow \theta} \quad \text{MATCH\_INR}$$

$$\frac{}{n \sim n \rightsquigarrow \varnothing} \quad \text{MATCH\_NAT}$$

$$\frac{n >= 1 \quad (n\ -\ 1) \sim p \rightsquigarrow \theta}{n \sim S\, p \rightsquigarrow \theta} \quad \text{MATCH\_SUCC}$$

$\boxed{\Gamma \vdash t : T \triangleright C}$ t has type T in context Γ, producing constraints C

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T \triangleright \top} \quad \text{CTY\_VAR}$$

$$\frac{}{\Gamma \vdash () : \mathbb{1} \triangleright \top} \quad \text{CTY\_UNIT}$$

$$\frac{}{\Gamma \vdash \mathbf{true} : \mathbb{B} \triangleright \top} \quad \text{CTY\_TRUE}$$

$$\frac{}{\Gamma \vdash \mathbf{false} : \mathbb{B} \triangleright \top} \quad \text{CTY\_FALSE}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 : T_1 \triangleright C_1 \quad \Gamma \vdash t_2 : T_2 \triangleright C_2 \\ C_3 = T_1 \ll: (S_2 \rightarrow S_3) \wedge T_2 \ll: S_2 \\ C_4 = T_1 \ll: S_1 \wedge T_2 \ll: S_2 \wedge S_3 = S_1 \sqcup S_2 \wedge \mathbf{numeric}\, S_3\end{array}}{\Gamma \vdash t_1\, t_2 : S_3 \triangleright C_1 \wedge C_2 \wedge (C_3 \vee C_4)} \quad \text{CTY\_JUXT}$$

$$\frac{\Gamma \vdash t_1 : S_1 \triangleright C_1 \quad \Gamma \vdash t_2 : S_2 \triangleright C_2}{\Gamma \vdash (t_1, t_2) : S_1 \times S_2 \triangleright C_1 \wedge C_2} \quad \text{CTY\_PAIR}$$

$$\frac{}{\Gamma \vdash n : \mathbb{N} \triangleright \top} \quad \text{CTY\_NAT}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 : S_1 \triangleright C_1 \quad \Gamma \vdash t_2 : S_2 \triangleright C_2 \\ C_3 = (S_3 = S_1 \sqcup S_2 \wedge \mathbf{numeric}\, S_3)\end{array}}{\Gamma \vdash t_1\ +\ t_2 : S_3 \triangleright C_1 \wedge C_2 \wedge C_3} \quad \text{CTY\_ADD}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 : S_1 \triangleright C_1 \quad \Gamma \vdash t_2 : S_2 \triangleright C_2 \\ C_3 = (S_3 = S_1 \sqcup S_2 \wedge \mathbf{numeric}\, S_3)\end{array}}{\Gamma \vdash t_1\ *\ t_2 : S_3 \triangleright C_1 \wedge C_2 \wedge C_3} \quad \text{CTY\_MUL}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 : S_1 \triangleright C_1 \quad \Gamma \vdash t_2 : S_2 \triangleright C_2 \\ C_3 = (S_3 = S_1 \sqcup S_2 \wedge \mathbf{numeric}\, S_3 \wedge S_4 = S_3 \sqcup \mathbb{Z})\end{array}}{\Gamma \vdash t_1\ -\ t_2 : S_4 \triangleright C_1 \wedge C_2 \wedge C_3} \quad \text{CTY\_SUB}$$

$$\frac{\Gamma \vdash t : S_1 \triangleright C_1}{\Gamma \vdash -t : S_2 \triangleright C_1 \wedge \mathbf{numeric}\, S_1 \wedge S_2 = S_1 \sqcup \mathbb{Z}} \quad \text{CTY\_NEG}$$

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Definition rules:        124 good    0 bad
Definition rule clauses: 212 good    0 bad
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