termvar, x	term variable		
natural, n	natural number		
$t,\ T$	$::=$		term
	$\mid$ x		variable
	$\mid$ n		natural literal
	$\mid$ $x \mapsto t$	$\mathsf{bind}\ x\ \mathsf{in}\ t$	abstraction
	$\mid$ $t\ t'$		application
	$\mid$ $t + t'$		addition
	$\mid$ $t - t'$		subtraction
	$\mid$ $t * t'$		multiplication
	$\mid$ t/t'		division
	$\mid$ $\mathbb{N}$		natural number type
	$\mid$ $\mathbb{Z}$		integer type
	$\mid$ $\mathbb{Q}$		rational number type
	$\mid$ $t \to t'$		function type
	$\mid$ (t)	S	

v	$::=$		value
	$\mid$ n		natural number literal
	$\mid$ $x \mapsto t$		abstraction

ν	$::=$		numeric type
	$\mid$ $\mathbb{N}$		natural number type
	$\mid$ $\mathbb{Z}$		integer type
	$\mid$ $\mathbb{Q}$		rational number type

Γ	$::=$		type context
	$\mid$ $\varnothing$		
	$\mid$ $\Gamma, x : T$		

$\boxed{T_1 <: T_2}$ T_1 is a subtype of T_2

$$\frac{}{T <: T}\ \text{SUB\_REFL}$$

$$\frac{T_1 <: T_2 \qquad T_2 <: T_3}{T_1 <: T_3}\ \text{SUB\_TRANS}$$

$$\frac{}{\mathbb{N} <: \mathbb{Z}}\ \text{SUB\_N\_Z}$$

$$\frac{}{\mathbb{Z} <: \mathbb{Q}}\ \text{SUB\_Z\_Q}$$

$$\frac{T_1' <: T_1 \qquad T_2 <: T_2'}{T_1 \to T_2 <: T_1' \to T_2'}\ \text{SUB\_FUNTY}$$

$\boxed{\Gamma \vdash t : T}$ t has type T in context Γ

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T}\ \text{TY\_VAR}$$

$$\frac{\Gamma, x_1 : T_1 \vdash t : T}{\Gamma \vdash x_1 \mapsto t : T_1 \to T}\ \text{TY\_ABS}$$

$$\frac{\Gamma \vdash t : T_1 \to T_2 \qquad \Gamma \vdash t' : T_1}{\Gamma \vdash t\ t' : T_2}\ \text{TY\_APPLY}$$

$$\frac{}{\Gamma \vdash n : \mathbb{N}}\ \text{TY\_NAT}$$

$$\frac{\Gamma \vdash t_1 : \nu \qquad \Gamma \vdash t_2 : \nu}{\Gamma \vdash t_1 + t_2 : \nu}\ \text{TY\_ADD}$$

$$\frac{\Gamma \vdash t_1 : \nu \qquad \Gamma \vdash t_2 : \nu}{\Gamma \vdash t_1 * t_2 : \nu}\ \text{TY\_MUL}$$

$$\frac{\Gamma \vdash t_1 : \nu \qquad \Gamma \vdash t_2 : \nu \qquad \mathbb{Z} <: \nu}{\Gamma \vdash t_1 - t_2 : \nu}\ \text{TY\_SUB}$$

$$\frac{\Gamma \vdash t_1 : \nu \qquad \Gamma \vdash t_2 : \nu \qquad \mathbb{Q} <: \nu}{\Gamma \vdash t_1 / t_2 : \nu}\ \text{TY\_DIV}$$

$$\frac{\Gamma \vdash t : \nu \qquad \Gamma \vdash t' : \nu}{\Gamma \vdash t\ t' : \nu}\ \text{TY\_MUL\_NUM}$$

$\boxed{t_1 \longrightarrow t_2}$ t_1 reduces to t_2

$$\frac{}{(x \mapsto t_{12})\, v_2 \longrightarrow \{v_2/x\}t_{12}}\ \text{AX\_APP}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ t \longrightarrow t_1'\ t}\ \text{CTX\_APP\_FUN}$$

$$\frac{t_1 \longrightarrow t_1'}{v\ t_1 \longrightarrow v\ t_1'}\ \text{CTX\_APP\_ARG}$$

```
Definition rules:          17 good    0 bad
Definition rule clauses:   29 good    0 bad
```