termvar, x	term variable		
typevar, X	type variable		
natural, n	natural number		
k	index		
prog, P	::=		program
	| $decl_1 \,..\, decl_k$		
decl, D	::=		declaration
	| $x : T;$		type
	| $x = t;$		definition
uop	::=		unary operator
	| $-$		negation
bop	::=		binary operator
	| $+$		addition
	| $-$		subtraction
	| $*$		multiplication
	| $/$		division
	| $\equiv$		equality
	| $<$		less than
	| &&		boolean and
	| $\Vert$		boolean or
term, t	::=		term
	| x		variable
	| $()$		unit value
	| **true**		true
	| **false**		false
	| $x \mapsto t$	bind x in t	abstraction
	| $t\ t'$		application
	| (t_1, t_2)		pair
	| **inl** t_1		left introduction
	| **inr** t_2		right introduction
	| n		natural literal
	| *uop* t		unary operator
	| t_1 *bop* t_2		binary operator
	| **let** $x = t_1$ **in** t_2	bind x in t_1 bind x in t_2	letrec
	| **case** *branches*		case analysis
	| $t : T$		type ascription
	| (t)	S	
value, v	::=		value
	| n		natural number literal
	| $x \mapsto t$		abstraction
	| (v_1, v_2)		pair
	| **inl** v		left introduction
	| **inr** v		right introduction
type, T	::=		type

	$\mid X$		variable
	$\mid \mathbb{0}$		void type
	$\mid \mathbb{1}$		unit type
	$\mid \mathbb{B}$		boolean type
	$\mid T_1 \to T_2$		function type
	$\mid T_1 \times T_2$		product type
	$\mid T_1 \uplus T_2$		sum type
	$\mid \mathbb{N}$		natural type
	$\mid \mathbb{Z}$		integer type
	$\mid \mathbb{Q}$		rational type
	$\mid (T)$	S	
ν	$::=$		numeric type
	$\mid \mathbb{N}$		natural type
	$\mid \mathbb{Z}$		integer type
	$\mid \mathbb{Q}$		rational type
Γ	$::=$		type context
	$\mid \varnothing$		empty context
	$\mid \Gamma, x : T$		cons
$\mathit{pattern},\ p$	$::=$		pattern
	$\mid x$	$\text{binders} = x$	variable
	$\mid \_$	$\text{binders} = \{\}$	wildcard
	$\mid ()$	$\text{binders} = \{\}$	unit
	$\mid \mathbf{true}$	$\text{binders} = \{\}$	true
	$\mid \mathbf{false}$	$\text{binders} = \{\}$	false
	$\mid (p_1, p_2)$	$\text{binders} = \text{binders}(p_1) \cup \text{binders}(p_2)$	pair
	$\mid \mathbf{inl}\, p$	$\text{binders} = \text{binders}(p)$	left
	$\mid \mathbf{inr}\, p$	$\text{binders} = \text{binders}(p)$	right
	$\mid n$	$\text{binders} = \{\}$	natural
	$\mid \mathbf{S}\, p$	$\text{binders} = \text{binders}(p)$	successor
	$\mid (p)$	S	
$\mathit{branches},\ b$	$::=$		
	$\mid \varnothing$		
	$\mid \{\mathit{branches}$		
	$\mid \{\mathit{term}\ \mathit{guard}\ \mathit{branches}$	bind $\text{binders}(\mathit{guard})$ in term	
$\mathit{aguard},\ \mathit{ag}$	$::=$		atomic guard
	$\mid \mathbf{if}\, t$	$\text{binders} = \{\}$	boolean guard
	$\mid \mathbf{when}\, t = p$	$\text{binders} = \text{binders}(p)$	pattern guard
$\mathit{guard},\ g$	$::=$		guard
	$\mid \top$	$\text{binders} = \{\}$	trivial guard
	$\mid \mathit{ag}\ g$	$\text{binders} = \text{binders}(\mathit{ag}) \cup \text{binders}(g)$	cons atomic

$$
\begin{array}{lll}
\mathit{subst},\ \theta & ::= & \\
 & | & \varnothing \\
 & | & \{t/x\}\mathit{subst}
\end{array}
$$

$\boxed{T_1 <: T_2}$ T_1 is a subtype of T_2

$$\frac{}{T <: T}\ \textsc{sub\_refl}$$

$$\frac{T_1 <: T_2 \quad T_2 <: T_3}{T_1 <: T_3}\ \textsc{sub\_trans}$$

$$\frac{}{\mathbb{0} <: T}\ \textsc{sub\_ex\_falso}$$

$$\frac{}{\mathbb{N} <: \mathbb{Z}}\ \textsc{sub\_n\_z}$$

$$\frac{}{\mathbb{Z} <: \mathbb{Q}}\ \textsc{sub\_z\_q}$$

$$\frac{T_1' <: T_1 \quad T_2 <: T_2'}{T_1 \to T_2 <: T_1' \to T_2'}\ \textsc{sub\_funty}$$

$$\frac{T_1 <: T_1' \quad T_2 <: T_2'}{T_1 \times T_2 <: T_1' \times T_2'}\ \textsc{sub\_prod}$$

$$\frac{T_1 <: T_1' \quad T_2 <: T_2'}{T_1 \uplus T_2 <: T_1' \uplus T_2'}\ \textsc{sub\_sum}$$

$\boxed{T_1 = T_2 \sqcup T_3}$ T_1 is the lub of T_2 and T_3

$$\frac{T_2 <: T_1}{T_1 = T_1 \sqcup T_2}\ \textsc{lub\_subl}$$

$$\frac{T_1 <: T_2}{T_2 = T_1 \sqcup T_2}\ \textsc{lub\_subr}$$

$$\frac{T_{21} = T_{22} \sqcup T_{23}}{T_1 \to T_{21} = T_1 \to T_{22} \sqcup T_1 \to T_{23}}\ \textsc{lub\_funty}$$

$$\frac{T_{11} = T_{12} \sqcup T_{13} \quad T_{21} = T_{22} \sqcup T_{23}}{T_{11} \times T_{21} = T_{12} \times T_{22} \sqcup T_{13} \times T_{23}}\ \textsc{lub\_prod}$$

$$\frac{T_{11} = T_{12} \sqcup T_{13} \quad T_{21} = T_{22} \sqcup T_{23}}{T_{11} \uplus T_{21} = T_{12} \uplus T_{22} \sqcup T_{13} \uplus T_{23}}\ \textsc{lub\_sum}$$

$\boxed{\mathbf{Finite}\ T}$ T is finite

$$\frac{}{\mathbf{Finite}\ \mathbb{0}}\ \textsc{fin\_Void}$$

$$\frac{}{\mathbf{Finite}\ \mathbb{1}}\ \textsc{fin\_Unit}$$

$$\frac{}{\mathbf{Finite}\ \mathbb{B}}\ \textsc{fin\_Bool}$$

$$\frac{\mathbf{Finite}\ T_1 \quad \mathbf{Finite}\ T_2}{\mathbf{Finite}\ T_1 \times T_2}\ \textsc{fin\_prod}$$

$$\frac{\mathbf{Finite}\ T_1 \quad \mathbf{Finite}\ T_2}{\mathbf{Finite}\ T_1 \uplus T_2}\ \textsc{fin\_sum}$$

$$\frac{\textbf{Finite}\ T_1 \quad \textbf{Finite}\ T_2}{\textbf{Finite}\ T_1 \to T_2}\ \textsc{fin\_funty}$$

$\boxed{\textbf{Decidable}\ T}$ $\quad T$ has decidable equality

$$\frac{}{\textbf{Decidable}\ \mathbb{0}}\ \textsc{dec\_Void}$$

$$\frac{}{\textbf{Decidable}\ \mathbb{1}}\ \textsc{dec\_Unit}$$

$$\frac{}{\textbf{Decidable}\ \mathbb{B}}\ \textsc{dec\_Bool}$$

$$\frac{}{\textbf{Decidable}\ \mathbb{N}}\ \textsc{dec\_N}$$

$$\frac{}{\textbf{Decidable}\ \mathbb{Z}}\ \textsc{dec\_Z}$$

$$\frac{}{\textbf{Decidable}\ \mathbb{Q}}\ \textsc{dec\_Q}$$

$$\frac{\textbf{Decidable}\ T_1 \quad \textbf{Decidable}\ T_2}{\textbf{Decidable}\ T_1 \times T_2}\ \textsc{dec\_prod}$$

$$\frac{\textbf{Decidable}\ T_1 \quad \textbf{Decidable}\ T_2}{\textbf{Decidable}\ T_1 \uplus T_2}\ \textsc{dec\_sum}$$

$$\frac{\textbf{Finite}\ T_1 \quad \textbf{Decidable}\ T_2}{\textbf{Decidable}\ T_1 \to T_2}\ \textsc{dec\_funty}$$

$\boxed{\textbf{Ordered}\ T}$ $\quad T$ is totally ordered

$$\frac{}{\textbf{Ordered}\ \mathbb{0}}\ \textsc{ord\_Void}$$

$$\frac{}{\textbf{Ordered}\ \mathbb{1}}\ \textsc{ord\_Unit}$$

$$\frac{}{\textbf{Ordered}\ \mathbb{B}}\ \textsc{ord\_Bool}$$

$$\frac{}{\textbf{Ordered}\ \mathbb{N}}\ \textsc{ord\_N}$$

$$\frac{}{\textbf{Ordered}\ \mathbb{Z}}\ \textsc{ord\_Z}$$

$$\frac{}{\textbf{Ordered}\ \mathbb{Q}}\ \textsc{ord\_Q}$$

$$\frac{\textbf{Ordered}\ T_1 \quad \textbf{Ordered}\ T_2}{\textbf{Ordered}\ T_1 \times T_2}\ \textsc{ord\_prod}$$

$$\frac{\textbf{Ordered}\ T_1 \quad \textbf{Ordered}\ T_2}{\textbf{Ordered}\ T_1 \uplus T_2}\ \textsc{ord\_sum}$$

$$\frac{\textbf{Finite}\ T_1 \quad \textbf{Ordered}\ T_1 \quad \textbf{Ordered}\ T_2}{\textbf{Ordered}\ T_1 \to T_2}\ \textsc{ord\_funty}$$

$\boxed{\Gamma \vdash t : T}$ $\quad t$ has type T in context Γ

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T}\ \textsc{ty\_var}$$

$$\frac{}{\Gamma \vdash () : \mathbb{1}}\ \textsc{ty\_unit}$$

$$\frac{}{\Gamma \vdash \textbf{true} : \mathbb{B}}\ \textsc{ty\_true}$$

$$\frac{}{\Gamma \vdash \textbf{false} : \mathbb{B}} \quad \text{TY\_FALSE}$$

$$\frac{\Gamma, x_1 : T_1 \vdash t : T}{\Gamma \vdash x_1 \mapsto t : T_1 \to T} \quad \text{TY\_ABS}$$

$$\frac{\Gamma \vdash t : T_1 \to T_2 \quad \Gamma \vdash t' : T_1}{\Gamma \vdash t\, t' : T_2} \quad \text{TY\_APPLY}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma \vdash t_2 : T_2}{\Gamma \vdash (t_1, t_2) : T_1 \times T_2} \quad \text{TY\_PAIR}$$

$$\frac{\Gamma \vdash t : T_1}{\Gamma \vdash \textbf{inl}\, t : T_1 \uplus T_2} \quad \text{TY\_INL}$$

$$\frac{\Gamma \vdash t : T_2}{\Gamma \vdash \textbf{inr}\, t : T_1 \uplus T_2} \quad \text{TY\_INR}$$

$$\frac{}{\Gamma \vdash n : \mathbb{N}} \quad \text{TY\_NAT}$$

$$\frac{\Gamma \vdash t_1 : \nu \quad \Gamma \vdash t_2 : \nu}{\Gamma \vdash t_1 \ + \ t_2 : \nu} \quad \text{TY\_ADD}$$

$$\frac{\Gamma \vdash t_1 : \nu \quad \Gamma \vdash t_2 : \nu}{\Gamma \vdash t_1 \ * \ t_2 : \nu} \quad \text{TY\_MUL}$$

$$\frac{\Gamma \vdash t_1 : \nu \quad \Gamma \vdash t_2 : \nu \quad \mathbb{Z} <: \nu}{\Gamma \vdash t_1 \ - \ t_2 : \nu} \quad \text{TY\_SUB}$$

$$\frac{\Gamma \vdash t : \nu \quad \mathbb{Z} <: \nu}{\Gamma \vdash -t : \nu} \quad \text{TY\_NEG}$$

$$\frac{\Gamma \vdash t_1 : \nu \quad \Gamma \vdash t_2 : \nu \quad \mathbb{Q} <: \nu}{\Gamma \vdash t_1 \,/\, t_2 : \nu} \quad \text{TY\_DIV}$$

$$\frac{\Gamma \vdash t : \nu \quad \Gamma \vdash t' : \nu}{\Gamma \vdash t\, t' : \nu} \quad \text{TY\_MUL\_NUM}$$

$$\frac{\Gamma \vdash t_1 : T \quad \Gamma \vdash t_2 : T \quad \textbf{Decidable}\ T}{\Gamma \vdash t_1 \ \equiv \ t_2 : \mathbb{B}} \quad \text{TY\_EQ}$$

$$\frac{\Gamma \vdash t_1 : T \quad \Gamma \vdash t_2 : T \quad \textbf{Ordered}\ T}{\Gamma \vdash t_1 \ < \ t_2 : \mathbb{B}} \quad \text{TY\_LT}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 : \mathbb{B}}{\Gamma \vdash t_1 \ \&\& \ t_2 : \mathbb{B}} \quad \text{TY\_AND}$$

$$\frac{\Gamma \vdash t_1 : \mathbb{B} \quad \Gamma \vdash t_2 : \mathbb{B}}{\Gamma \vdash t_1 \ \| \ t_2 : \mathbb{B}} \quad \text{TY\_OR}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash \textbf{let}\, x = t_1\, \textbf{in}\, t_2 : T_2} \quad \text{TY\_LET}$$

$$\frac{\Gamma \vdash_b b : T}{\Gamma \vdash \textbf{case}\, b : T} \quad \text{TY\_CASE}$$

$$\frac{\Gamma \vdash t : T}{\Gamma \vdash (t : T) : T} \quad \text{TY\_ASCRIBE}$$

$\boxed{\Gamma \vdash_b b : T}$ branches b have type T in context Γ

$$\frac{}{\Gamma \vdash_b \varnothing : T} \quad \text{BTY\_EMPTY}$$

$$\frac{\Gamma \vdash_b \mathit{branches} : T}{\Gamma \vdash_b \{\mathit{branches} : T} \quad \text{BTY\_SKIP}$$

$$\frac{\Gamma \vdash_g t, \mathit{guard} : T \quad \Gamma \vdash_b \mathit{branches} : T}{\Gamma \vdash_b \{t\,\mathit{guard}\,\mathit{branches} : T} \quad \text{BTY\_CONS}$$

$\boxed{\Gamma \vdash_g t, g : T}$ t guarded by g has type T in context Γ

$$\frac{\Gamma \vdash t : T}{\Gamma \vdash_g t, \top : T} \quad \text{GTY\_TOP}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_2 : \mathbb{B} \\ \Gamma \vdash_g t_1, g : T\end{array}}{\Gamma \vdash_g t_1, \mathbf{if}\, t_2\, g : T} \quad \text{GTY\_IF}$$

$$\frac{\begin{array}{l}\Gamma \vdash_p p : T_2 \rightsquigarrow \Gamma_2 \quad \Gamma \vdash t_2 : T_2 \\ \Gamma, \Gamma_2 \vdash_g t_1, g : T\end{array}}{\Gamma \vdash_g t_1, \mathbf{when}\, t_2 = p\, g : T} \quad \text{GTY\_WHEN}$$

$\boxed{\Gamma \vdash_p p : T \rightsquigarrow \Gamma'}$ p has type T in context Γ, and produces bindings Γ'

$$\frac{}{\Gamma \vdash_p x : T \rightsquigarrow \varnothing, x : T} \quad \text{P\_VAR}$$

$$\frac{}{\Gamma \vdash_p \_ : T \rightsquigarrow \varnothing} \quad \text{P\_WILD}$$

$$\frac{}{\Gamma \vdash_p () : \mathbb{1} \rightsquigarrow \varnothing} \quad \text{P\_UNIT}$$

$$\frac{}{\Gamma \vdash_p \mathbf{true} : \mathbb{B} \rightsquigarrow \varnothing} \quad \text{P\_TRUE}$$

$$\frac{}{\Gamma \vdash_p \mathbf{false} : \mathbb{B} \rightsquigarrow \varnothing} \quad \text{P\_FALSE}$$

$$\frac{\Gamma \vdash_p p_1 : T_1 \rightsquigarrow \Gamma_1 \quad \Gamma \vdash_p p_2 : T_2 \rightsquigarrow \Gamma_2}{\Gamma \vdash_p (p_1, p_2) : T_1 \times T_2 \rightsquigarrow \Gamma_1, \Gamma_2} \quad \text{P\_PAIR}$$

$$\frac{\Gamma \vdash_p p : T_1 \rightsquigarrow \Gamma_1}{\Gamma \vdash_p \mathbf{inl}\, p : T_1 \uplus T_2 \rightsquigarrow \Gamma_1} \quad \text{P\_INL}$$

$$\frac{\Gamma \vdash_p p : T_2 \rightsquigarrow \Gamma_2}{\Gamma \vdash_p \mathbf{inr}\, p : T_1 \uplus T_2 \rightsquigarrow \Gamma_2} \quad \text{P\_INR}$$

$$\frac{}{\Gamma \vdash_p n : \mathbb{N} \rightsquigarrow \varnothing} \quad \text{P\_NAT}$$

$$\frac{\Gamma \vdash_p p : \mathbb{N} \rightsquigarrow \Gamma'}{\Gamma \vdash_p \mathbf{S}\, p : \mathbb{N} \rightsquigarrow \Gamma'} \quad \text{P\_SUCC}$$

$\boxed{t_1 \longrightarrow t_2}$ t_1 reduces to t_2

$$\frac{\varnothing \vdash v_2 : \nu}{(x \mapsto t_{12})\, v_2 \longrightarrow \{v_2/x\}\, t_{12}} \quad \text{BETA\_NUM}$$

$$\frac{\neg\, (\varnothing \vdash t_2 : \nu)}{(x \mapsto t_{12})\, t_2 \longrightarrow \{t_2/x\}\, t_{12}} \quad \text{BETA}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ t \longrightarrow t_1'\ t} \quad \text{CONG\_APP\_FUN}$$

$$\frac{\varnothing \vdash t_1 : \nu \quad t_1 \longrightarrow t_1'}{v\ t_1 \longrightarrow v\ t_1'} \quad \text{CONG\_APP\_NUM}$$

$$\frac{t_1 \longrightarrow t_1'}{(t_1, t_2) \longrightarrow (t_1', t_2)} \quad \text{CONG\_FST}$$

$$\frac{t_2 \longrightarrow t_2'}{(t_1, t_2) \longrightarrow (t_1, t_2')} \quad \text{CONG\_SND}$$

$$\frac{t \longrightarrow t'}{\mathbf{inl}\, t \longrightarrow \mathbf{inl}\, t'} \quad \text{CONG\_INL}$$

$$\frac{t \longrightarrow t'}{\mathbf{inr}\, t \longrightarrow \mathbf{inr}\, t'} \quad \text{CONG\_INR}$$

$$\frac{t_1 \longrightarrow t_1'}{uop\ t_1 \longrightarrow uop\ t_1'} \quad \text{CONG\_UOP}$$

$$\frac{}{uop\ v \longrightarrow [\![uop\ v]\!]} \quad \text{UOP}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ bop\ t_2 \longrightarrow t_1'\ bop\ t_2} \quad \text{CONG\_BOP\_L}$$

$$\frac{t_2 \longrightarrow t_2'}{t_1\ bop\ t_2 \longrightarrow t_1\ bop\ t_2'} \quad \text{CONG\_BOP\_R}$$

$$\frac{}{v_1\ bop\ v_2 \longrightarrow [\![v_1\ bop\ v_2]\!]} \quad \text{BOP}$$

$$\frac{\varnothing \vdash t_1 : \nu \quad t_1 \longrightarrow t_1'}{\mathbf{let}\, x = t_1\, \mathbf{in}\, t_2 \longrightarrow \mathbf{let}\, x = t_1'\, \mathbf{in}\, t_2} \quad \text{CONG\_LET\_NUM}$$

$$\frac{\varnothing \vdash v_1 : \nu}{\mathbf{let}\, x = v_1\, \mathbf{in}\, t_2 \longrightarrow \{v_1/x\}\, t_2} \quad \text{LET\_NUM}$$

$$\frac{\neg\,(\varnothing \vdash t_1 : \nu)}{\mathbf{let}\, x = t_1\, \mathbf{in}\, t_2 \longrightarrow \{t_1/x\}\, t_2} \quad \text{LET}$$

$$\frac{}{\mathbf{case}\,\{\,branches \longrightarrow \mathbf{case}\,branches} \quad \text{CASE\_SKIP}$$

$$\frac{g \longrightarrow g'}{\mathbf{case}\,\{\,t\ g\ branches \longrightarrow \mathbf{case}\,\{\,t\ g'\ branches} \quad \text{CONG\_CASE}$$

$$\frac{g \rightsquigarrow \theta}{\mathbf{case}\,\{\,t\ g\ branches \longrightarrow \theta\ t} \quad \text{CASE\_SUCCESS}$$

$$\frac{g!}{\mathbf{case}\,\{\,t\ g\ branches \longrightarrow \mathbf{case}\,branches} \quad \text{CASE\_FAILURE}$$

$\boxed{g \longrightarrow g'}$ guard g reduces to g'

$$\frac{t \longrightarrow t'}{\mathbf{if}\ t\ g \longrightarrow \mathbf{if}\ t'\ g} \quad \text{GRED\_IF}$$

$$\frac{t \longrightarrow t'}{\mathbf{when}\ t = p\ g \longrightarrow \mathbf{when}\ t' = p\ g} \quad \text{GRED\_WHEN}$$

$\boxed{\mathit{guard} \rightsquigarrow \theta}$ *guard* succeeds, producing substitution θ

$$\frac{}{\top \rightsquigarrow \varnothing} \quad \text{SUCCESS\_TOP}$$

$$\frac{g \rightsquigarrow \theta}{\textbf{if true}\; g \rightsquigarrow \theta} \quad \text{SUCCESS\_TRUE}$$

$$\frac{\begin{array}{l} t \sim p \rightsquigarrow \theta_1 \\ g \rightsquigarrow \theta_2 \end{array}}{\textbf{when}\; t = p\; g \rightsquigarrow \theta_2\, \theta_1} \quad \text{SUCCESS\_MATCH}$$

$\boxed{\mathit{guard}!}$ *guard* fails

$$\frac{}{\textbf{if false}\; g!} \quad \text{FAIL\_FALSE}$$

$$\frac{\neg\,(t \sim p \rightsquigarrow \theta)}{\textbf{when}\; t = p\; g!} \quad \text{FAIL\_MATCH}$$

$$\frac{g!}{\mathit{aguard}\; g!} \quad \text{FAIL\_REST}$$

$\boxed{t \sim p \rightsquigarrow \theta}$ term t matches pattern p, producing substitution θ

$$\frac{}{t \sim x \rightsquigarrow \{t/x\}} \quad \text{MATCH\_VAR}$$

$$\frac{}{t \sim \_ \rightsquigarrow \varnothing} \quad \text{MATCH\_WILD}$$

$$\frac{}{t \sim () \rightsquigarrow \varnothing} \quad \text{MATCH\_UNIT}$$

$$\frac{}{\textbf{true} \sim \textbf{true} \rightsquigarrow \varnothing} \quad \text{MATCH\_TRUE}$$

$$\frac{}{\textbf{false} \sim \textbf{false} \rightsquigarrow \varnothing} \quad \text{MATCH\_FALSE}$$

$$\frac{t_1 \sim p_1 \rightsquigarrow \theta_1 \quad t_2 \sim p_2 \rightsquigarrow \theta_2}{(t_1, t_2) \sim (p_1, p_2) \rightsquigarrow \theta_1\, \theta_2} \quad \text{MATCH\_PAIR}$$

$$\frac{t \sim p \rightsquigarrow \theta}{\textbf{inl}\; t \sim \textbf{inl}\; p \rightsquigarrow \theta} \quad \text{MATCH\_INL}$$

$$\frac{t \sim p \rightsquigarrow \theta}{\textbf{inr}\; t \sim \textbf{inr}\; p \rightsquigarrow \theta} \quad \text{MATCH\_INR}$$

$$\frac{}{n \sim n \rightsquigarrow \varnothing} \quad \text{MATCH\_NAT}$$

$$\frac{n >= 1 \quad (n\, -\, 1) \sim p \rightsquigarrow \theta}{n \sim \mathbf{S}\, p \rightsquigarrow \theta} \quad \text{MATCH\_SUCC}$$

$\boxed{\Gamma \vdash t \Rightarrow T}$ t synthesizes type T in context Γ

$$\frac{x : T \in \Gamma}{\Gamma \vdash x \Rightarrow T} \quad \text{INF\_VAR}$$

$$\frac{}{\Gamma \vdash () \Rightarrow \mathbb{1}} \quad \text{INF\_UNIT}$$

$$\frac{}{\Gamma \vdash \textbf{true} \Rightarrow \mathbb{B}} \quad \text{INF\_TRUE}$$

$$\frac{}{\Gamma \vdash \textbf{false} \Rightarrow \mathbb{B}} \quad \text{INF\_FALSE}$$

$$\frac{\Gamma \vdash t \Rightarrow T_1 \to T_2 \quad \Gamma \vdash t' \Leftarrow T_1}{\Gamma \vdash t\, t' \Rightarrow T_2} \quad \text{INF\_APPLY}$$

$$\frac{\begin{array}{l}\Gamma \vdash t \Rightarrow \nu_1 \quad \Gamma \vdash t' \Rightarrow \nu_2 \\ \nu_3 = \nu_1 \sqcup \nu_2\end{array}}{\Gamma \vdash t\, t' \Rightarrow \nu_3} \quad \text{INF\_MUL\_NUM}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow T_1 \quad \Gamma \vdash t_2 \Rightarrow T_2}{\Gamma \vdash (t_1, t_2) \Rightarrow T_1 \times T_2} \quad \text{INF\_PAIR}$$

$$\frac{}{\Gamma \vdash n \Rightarrow \mathbb{N}} \quad \text{INF\_NAT}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 \Rightarrow \nu_1 \quad \Gamma \vdash t_2 \Rightarrow \nu_2 \\ \nu_3 = \nu_1 \sqcup \nu_2\end{array}}{\Gamma \vdash t_1 \ + \ t_2 \Rightarrow \nu_3} \quad \text{INF\_ADD}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 \Rightarrow \nu_1 \quad \Gamma \vdash t_2 \Rightarrow \nu_2 \\ \nu_3 = \nu_1 \sqcup \nu_2\end{array}}{\Gamma \vdash t_1 \ * \ t_2 \Rightarrow \nu_3} \quad \text{INF\_MUL}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 \Rightarrow \nu_1 \quad \Gamma \vdash t_2 \Rightarrow \nu_2 \\ \nu_3 = \nu_1 \sqcup \nu_2 \quad \nu_4 = \nu_3 \sqcup \mathbb{Z}\end{array}}{\Gamma \vdash t_1 \ - \ t_2 \Rightarrow \nu_3} \quad \text{INF\_SUB}$$

$$\frac{\Gamma \vdash t \Rightarrow \nu \quad \nu_2 = \nu \sqcup \mathbb{Z}}{\Gamma \vdash -t \Rightarrow \nu_2} \quad \text{INF\_NEG}$$

$$\frac{\Gamma \vdash t_1 \Leftarrow \mathbb{Q} \quad \Gamma \vdash t_2 \Leftarrow \mathbb{Q}}{\Gamma \vdash t_1 \,/\, t_2 \Rightarrow \mathbb{Q}} \quad \text{INF\_DIV}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 \Rightarrow T_1 \quad \Gamma \vdash t_2 \Rightarrow T_2 \\ T_3 = T_1 \sqcup T_2 \quad \textbf{Decidable}\ T_3\end{array}}{\Gamma \vdash t_1 \ \equiv \ t_2 \Rightarrow \mathbb{B}} \quad \text{INF\_EQ}$$

$$\frac{\begin{array}{l}\Gamma \vdash t_1 \Rightarrow T_1 \quad \Gamma \vdash t_2 \Rightarrow T_2 \\ T_3 = T_1 \sqcup T_2 \quad \textbf{Ordered}\ T_2\end{array}}{\Gamma \vdash t_1 \ < \ t_2 \Rightarrow \mathbb{B}} \quad \text{INF\_LT}$$

$$\frac{\Gamma \vdash t_1 \Leftarrow \mathbb{B} \quad \Gamma \vdash t_2 \Leftarrow \mathbb{B}}{\Gamma \vdash t_1 \ \&\& \ t_2 \Rightarrow \mathbb{B}} \quad \text{INF\_AND}$$

$$\frac{\Gamma \vdash t_1 \Leftarrow \mathbb{B} \quad \Gamma \vdash t_2 \Leftarrow \mathbb{B}}{\Gamma \vdash t_1 \ \| \ t_2 \Rightarrow \mathbb{B}} \quad \text{INF\_OR}$$

$$\frac{\Gamma \vdash t_1 \Rightarrow T_1 \quad \Gamma, x : T_1 \vdash t_2 \Rightarrow T_2}{\Gamma \vdash \textbf{let}\ x = t_1\ \textbf{in}\ t_2 \Rightarrow T_1} \quad \text{INF\_LET}$$

$$\frac{\Gamma \vdash t \Leftarrow T}{\Gamma \vdash (t : T) \Rightarrow T} \quad \text{INF\_ASCRIBE}$$

$\boxed{\Gamma \vdash t \Leftarrow T}$ t checks at type T in context Γ

$$\frac{\Gamma, x_1 : T_1 \vdash t \Leftarrow T}{\Gamma \vdash x_1 \mapsto t \Leftarrow T_1 \to T} \quad \text{CHK\_ABS}$$

$$\frac{\Gamma \vdash t \Leftarrow T_1}{\Gamma \vdash \textbf{inl}\, t \Leftarrow T_1 \uplus T_2} \quad \text{CHK\_INL}$$

$$\frac{\Gamma \vdash t \Leftarrow T_2}{\Gamma \vdash \mathbf{inr}\, t \Leftarrow T_1 \uplus T_2} \quad \text{CHK\_INR}$$

$$\frac{\Gamma \vdash t \Rightarrow T' \quad T' <: T}{\Gamma \vdash t \Leftarrow T} \quad \text{CHK\_FLIP}$$

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Definition rules:        137 good    0 bad
Definition rule clauses: 238 good    0 bad
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