termvar, x	term variable
const, c	constant
typevar, α	type variable
natural, n	natural number
k	index

term, t	$::=$			term
	$\|$	n		natural literal
	$\|$	x		variable
	$\|$	c		constant
	$\|$	$\lambda x.t$	bind x in t	abstraction
	$\|$	$\lambda x : \tau.t$	bind x in t	typed abstraction
	$\|$	$t_1\ t_2$		application
	$\|$	$\mathbf{let}\ x = t_1\ \mathbf{in}\ t_2$	bind x in t_2	let
	$\|$	$\mathbf{let}\ x : \sigma = t_1\ \mathbf{in}\ t_2$	bind x in t_2	typed let
	$\|$	$t : \sigma$		type ascription
	$\|$	(t)	S	
value, v	$::=$			value
	$\|$	n		natural number literal
	$\|$	$\lambda x.t$		abstraction
	$\|$	$\lambda x : \tau.t$		typed abstraction
tycon, K	$::=$			type constructor
	$\|$	$\mathbb{N}$		natural numbers
	$\|$	$\mathbb{Z}$		integers
	$\|$	Prod		product type
type, τ	$::=$			monotype
	$\|$	α		variable
	$\|$	$\mathsf{K}\,\tau_1\,..\,\tau_k$		type constructor
	$\|$	$\tau_1 \to \tau_2$		function type
	$\|$	(τ)	S	
qualifier, Q	$::=$			qualifier
	$\|$	add		additive
	$\|$	sub		subtractive
Δ	$::=$			qualifier context
	$\|$	$\varnothing$		empty context
	$\|$	$\Delta, Q\,\tau$		cons
polytype, σ	$::=$			polytype
	$\|$	τ		monotype
	$\|$	$\forall\,\alpha_1\,..\,\alpha_k[\Delta].\tau$		forall
Γ	$::=$			type context
	$\|$	$\varnothing$		empty context
	$\|$	$\Gamma, x : \sigma$		cons
constraint, C	$::=$			constraint
	$\|$	$\tau_1 \equiv \tau_2$		unification
	$\|$	$\tau_1 \leq \tau_2$		subtype

$$
\begin{array}{lll}
\mid & Q\,\tau & \text{qualifier} \\
\mid & \mathbf{true} & \text{trivial} \\
\mid & C_1 \wedge C_2 & \text{conjunction} \\
\mid & [\![\Delta]\!] & \text{qualifiers} \\
\mid & \Delta \Longrightarrow C & \text{entailment} \\
\mid & \forall\,\alpha_1\,..\,\alpha_k.C \quad \textsf{bind } \alpha_1..\alpha_k \textsf{ in } C & \text{universal}
\end{array}
$$

$\boxed{[\![\Delta]\!]}$ A function to convert qualifier contexts to constraints

$$
\begin{aligned}
[\![\varnothing]\!] &\equiv \mathbf{true} \\
[\![\Delta, Q\,\tau]\!] &\equiv [\![\Delta]\!] \wedge Q\,\tau
\end{aligned}
$$

$\boxed{\Gamma \vdash t \rhd \tau \rightsquigarrow C}$ t has inferred type τ in context Γ, generating constraints C

$$\frac{}{\Gamma \vdash n \rhd \mathbb{N} \rightsquigarrow \mathbf{true}} \quad \text{INF\_NAT}$$

$$\frac{x : \sigma \in \Gamma \quad \sigma \sqsubseteq^{\rhd} \tau \rightsquigarrow C}{\Gamma \vdash x \rhd \tau \rightsquigarrow C} \quad \text{INF\_VAR}$$

$$\frac{\Gamma, x : \tau_1 \vdash t \rhd \tau_2 \rightsquigarrow C}{\Gamma \vdash \lambda x : \tau_1.t \rhd \tau_1 \to \tau_2 \rightsquigarrow C} \quad \text{INF\_TABS}$$

$$\frac{\begin{array}{c}\alpha_1 \text{ fresh} \quad \alpha_2 \text{ fresh} \\ \Gamma \vdash t_1 \rhd \tau \rightsquigarrow C_1 \quad \Gamma \vdash t_2 \lhd \alpha_1 \rightsquigarrow C_2\end{array}}{\Gamma \vdash t_1\ t_2 \rhd \alpha_2 \rightsquigarrow C_1 \wedge C_2 \wedge \tau \equiv \alpha_1 \to \alpha_2} \quad \text{INF\_APP}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_1 \rhd \tau_1 \rightsquigarrow C_1 \\ \Gamma, x : \tau_1 \vdash t_2 \rhd \tau_2 \rightsquigarrow C_2\end{array}}{\Gamma \vdash \mathbf{let}\, x = t_1\, \mathbf{in}\, t_2 \rhd \tau_2 \rightsquigarrow C_1 \wedge C_2} \quad \text{INF\_LET}$$

$$\frac{\begin{array}{c}\Gamma \vdash^{\text{poly}} t_1 \lhd \sigma \rightsquigarrow C_1 \\ \Gamma, x : \sigma \vdash t_2 \rhd \tau \rightsquigarrow C_2\end{array}}{\Gamma \vdash \mathbf{let}\, x : \sigma = t_1\, \mathbf{in}\, t_2 \rhd \tau \rightsquigarrow C_1 \wedge C_2} \quad \text{INF\_TLET}$$

$$\frac{\Gamma \vdash^{\text{poly}} t \lhd \sigma \rightsquigarrow C_1 \quad \sigma \sqsubseteq^{\rhd} \tau \rightsquigarrow C_2}{\Gamma \vdash t : \sigma \rhd \tau \rightsquigarrow C_1 \wedge C_2} \quad \text{INF\_ASCRIBE}$$

$\boxed{\Gamma \vdash^{\text{poly}} t \lhd \sigma \rightsquigarrow C}$ t has checked polytype σ in context Γ, generating constraints C

$$\frac{\alpha_1\,..\,\alpha_k \text{ fresh} \quad \Gamma \vdash t \lhd \tau \rightsquigarrow C}{\Gamma \vdash^{\text{poly}} t \lhd \forall\,\alpha_1\,..\,\alpha_k[\Delta].\tau \rightsquigarrow \forall\,\alpha_1\,..\,\alpha_k.\Delta \Longrightarrow C} \quad \text{CHKP\_OPEN}$$

$\boxed{\Gamma \vdash t \lhd \tau \rightsquigarrow C}$ t has checked type τ in context Γ, generating constraints C

$$\frac{x : \sigma \in \Gamma \quad \sigma \sqsubseteq^{\lhd} \tau \rightsquigarrow C}{\Gamma \vdash x \lhd \tau \rightsquigarrow C} \quad \text{CHK\_VAR}$$

$$\frac{\begin{array}{c}\alpha_1 \text{ fresh} \quad \alpha_2 \text{ fresh} \\ \Gamma, x : \alpha_1 \vdash t \lhd \alpha_2 \rightsquigarrow C\end{array}}{\Gamma \vdash \lambda x.t \lhd \tau \rightsquigarrow C \wedge \tau \equiv \alpha_1 \to \alpha_2} \quad \text{CHK\_ABS}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_1 \rhd \tau_1 \rightsquigarrow C_1 \\ \Gamma, x : \tau_1 \vdash t_2 \lhd \tau_2 \rightsquigarrow C_2\end{array}}{\Gamma \vdash \mathbf{let}\, x = t_1\, \mathbf{in}\, t_2 \lhd \tau_2 \rightsquigarrow C_1 \wedge C_2} \quad \text{CHK\_LET}$$

$$\frac{\begin{array}{c}\Gamma \vdash^{\text{poly}} t_1 \lhd \sigma \rightsquigarrow C_1 \\ \Gamma, x : \sigma \vdash t_2 \lhd \tau \rightsquigarrow C_2\end{array}}{\Gamma \vdash \mathbf{let}\, x : \sigma = t_1\, \mathbf{in}\, t_2 \lhd \tau \rightsquigarrow C_1 \wedge C_2} \quad \text{CHK\_TLET}$$

$$\frac{\Gamma \vdash^{\text{poly}} t \lhd \sigma \rightsquigarrow C_1 \quad \sigma \sqsubseteq^{\lhd} \tau \rightsquigarrow C_2}{\Gamma \vdash t : \sigma \lhd \tau \rightsquigarrow C_1 \wedge C_2} \quad \text{CHK\_ASCRIBE}$$

$$\frac{\Gamma \vdash t \rhd \tau_1 \rightsquigarrow C}{\Gamma \vdash t \lhd \tau_2 \rightsquigarrow C \wedge \tau_1 \leq \tau_2} \quad \text{CHK\_INFER}$$

$\boxed{\sigma \sqsubseteq^{\rhd} \tau \rightsquigarrow C}$ τ is more specific than σ (inference mode), generating constraints C

$$\frac{\alpha_1 \mathinner{..} \alpha_k \text{ fresh}}{\forall\, \alpha_1 \mathinner{..} \alpha_k[\Delta].\tau \sqsubseteq^{\rhd} \tau \rightsquigarrow [\![\Delta]\!]} \quad \text{SS\_INST}$$

$\boxed{\sigma \sqsubseteq^{\lhd} \tau \rightsquigarrow C}$ τ is more specific than σ (checking mode), generating constraints C

$$\frac{\alpha_1 \mathinner{..} \alpha_k \text{ fresh}}{\forall\, \alpha_1 \mathinner{..} \alpha_k[\Delta].\tau_1 \sqsubseteq^{\lhd} \tau_2 \rightsquigarrow [\![\Delta]\!] \wedge \tau_1 \leq \tau_2} \quad \text{SC\_INST}$$

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Definition rules:        16 good    0 bad
Definition rule clauses: 37 good    0 bad
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