

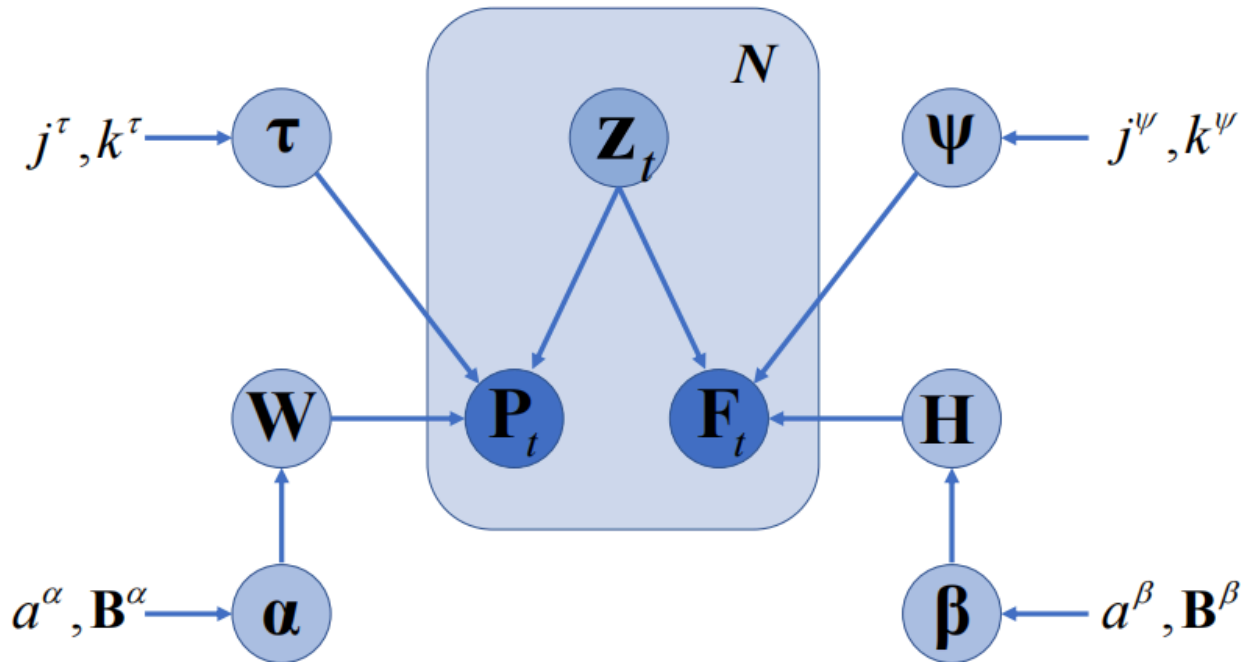
VI-CVA

VI-CVA here denotes the CVA model trained by VI, and the whole training process is mainly established according to the reference:

Yu J, Ye L, Zhou L, et al. Dynamic process monitoring based on variational Bayesian canonical variate analysis[J]. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2021, 52(4): 2412-2422.

Wishart distribution \mathcal{W} is used as the prior of the precision in Multivariate Gaussian.

Model Structure



input :

$$\mathbf{x}_t \in \mathbb{R}^{M \times 1}$$

$$\mathbf{P}^t = [\mathbf{x}_{t-1}^T, \mathbf{x}_{t-2}^T, \dots, \mathbf{x}_{t-l}^T]^T \in \mathbb{R}^{Ml \times 1}$$

$$\mathbf{F}^t = [\mathbf{x}_t^T, \mathbf{x}_{t+1}^T, \dots, \mathbf{x}_{t+s}^T]^T \in \mathbb{R}^{M(s+1) \times 1}$$

$$\mathbf{P}^t = [p_1^t, p_2^t, \dots, p_M^t]^T \in \mathbb{R}^{M \times 1}$$

$$\mathbf{F}^t = [f_1^t, f_2^t, \dots, f_{M(s+1)}^t]^T \in \mathbb{R}^{M(s+1) \times 1}$$

model mapping :

$$\mathbf{P} = \mathbf{W}^T \mathbf{z} + \epsilon$$

$$\mathbf{F} = \mathbf{H}^T \mathbf{z} + \delta$$

$$\mathbf{z}_t \in \mathbb{R}^{D \times 1}$$

$$\mathbf{W}^T \in \mathbb{R}^{M \times D}$$

$$\mathbf{H}^T \in \mathbb{R}^{M(s+1) \times D}$$

$$\mathbf{W}^T = \begin{pmatrix} -\mathbf{W}_1^T - \\ -\mathbf{W}_2^T - \\ \vdots \\ -\mathbf{W}_M^T - \end{pmatrix} = (\mathbf{W}_1, \mathbf{W}_2, \dots, \mathbf{W}_M), \mathbf{W}_m \in \mathbb{R}^{D \times 1}$$

$$\mathbf{H}^T = \begin{pmatrix} -\mathbf{H}_1^T - \\ -\mathbf{H}_2^T - \\ \vdots \\ -\mathbf{H}_{M(s+1)}^T - \end{pmatrix} = (\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_{M(s+1)}), \mathbf{H}_m \in \mathbb{R}^{D \times 1}$$

prior distributions :

$$\mathcal{P}(\mathbf{z}) = \prod_{t=1}^N \mathcal{N}(\mathbf{z}_t | 0, \mathbf{I})$$

$$\mathcal{P}(\epsilon) = \prod_{m=1}^M \mathcal{N}(\epsilon_m | 0, \tau_m^{-1}); \mathcal{P}(\delta) = \prod_{m=1}^{M(s+1)} \mathcal{N}(\delta_m | 0, \psi_m^{-1})$$

$$\mathcal{P}(\tau) = \prod_{m=1}^M \mathcal{G}(\tau_m | j_m^\tau, k_m^\tau); \mathcal{P}(\psi) = \prod_{m=1}^{M(s+1)} \mathcal{G}(\psi_m | j_m^\psi, k_m^\psi)$$

$$\mathcal{P}(\alpha) = \prod_{m=1}^{Ml} \mathcal{W}(\alpha_m | a_m^\alpha, \mathbf{B}_m^\alpha); \mathcal{P}(\beta) = \prod_{m=1}^{M(s+1)} \mathcal{W}(\beta_m | a_m^\beta, \mathbf{B}_m^\beta)$$

$$\mathcal{P}(\mathbf{W} | \alpha) = \prod_{m=1}^{Ml} \mathcal{N}(\mathbf{W}_m | 0, \alpha_m^{-1}); \mathcal{P}(\mathbf{H} | \beta) = \prod_{m=1}^{M(s+1)} \mathcal{N}(\mathbf{H}_m | 0, \beta_m^{-1})$$

$$\mathcal{P}(\mathbf{P} | \mathbf{z}, \mathbf{W}, \alpha, \tau) = \prod_{t=1}^N \prod_{m=1}^{Ml} \mathcal{N}(p_m^t | \mathbf{W}_m^T \mathbf{z}_t, \tau_m^{-1})$$

$$\mathcal{P}(\mathbf{F} | \mathbf{z}, \mathbf{H}, \beta, \psi) = \prod_{t=1}^N \prod_{m=1}^{M(s+1)} \mathcal{N}(f_m^t | \mathbf{H}_m^T \mathbf{z}_t, \psi_m^{-1})$$

variational distributions :

$$\mathcal{Q}(\mathbf{z}) = \prod_{t=1}^N \mathcal{N}(\mathbf{z}_t | \mu_t^z, (\Lambda_t^z)^{-1})$$

$$\mathcal{Q}(\mathbf{W}) = \prod_{m=1}^{Ml} \mathcal{N}(\mathbf{W}_m | \mu_m^W, (\Lambda_m^W)^{-1}); \mathcal{Q}(\mathbf{H}) = \prod_{m=1}^{M(s+1)} \mathcal{N}(\mathbf{H}_m | \mu_m^H, (\Lambda_m^H)^{-1})$$

$$\mathcal{Q}(\tau) = \prod_{m=1}^{Ml} \mathcal{G}(\tau_m | \lambda_m^\tau, \nu_m^\tau); \mathcal{Q}(\psi) = \prod_{m=1}^{M(s+1)} \mathcal{G}(\psi_m | \lambda_m^\psi, \nu_m^\psi)$$

$$\mathcal{Q}(\alpha) = \prod_{m=1}^{Ml} \mathcal{W}(\alpha_m | \nu_m^\alpha, \mathbf{V}_m^\alpha); \mathcal{Q}(\beta) = \prod_{m=1}^{M(s+1)} \mathcal{W}(\beta_m | \nu_m^\beta, \mathbf{V}_m^\beta)$$

Udate Strategy

joint distribution

$$\begin{aligned} & \mathcal{P}(\mathbf{P}, \mathbf{F}, \mathbf{H}, \mathbf{W}, \mathbf{z}, \tau, \psi, \alpha, \beta) \\ &= \mathcal{P}(\mathbf{P} | \mathbf{W}, \mathbf{z}, \tau) \mathcal{P}(\mathbf{W} | \alpha) \mathcal{P}(\alpha) \mathcal{P}(\tau) \\ & \mathcal{P}(\mathbf{F} | \mathbf{H}, \mathbf{z}, \psi) \mathcal{P}(\mathbf{H} | \beta) \mathcal{P}(\beta) \mathcal{P}(\psi) \mathcal{P}(\mathbf{z}) \end{aligned}$$

mean-field distribution

$$\mathcal{Q}(\mathbf{H}, \mathbf{W}, \mathbf{z}, \tau, \psi, \alpha, \beta)$$

$$= \mathcal{Q}(\mathbf{W})\mathcal{Q}(\alpha)\mathcal{Q}(\tau)\mathcal{Q}(\mathbf{H})\mathcal{Q}(\beta)\mathcal{Q}(\psi)\mathcal{Q}(\mathbf{z})$$

Update $\mathcal{Q}(\mathbf{z}_t)$

$$\Lambda_t^z = \sum_{m=1}^M \mathbb{E}_{Q(\tau_m)}(\tau_m) \mathbb{E}_{Q(\mathbf{W}_m)}(\mathbf{W}_m \mathbf{W}_m^T) + \sum_{m=1}^{M(s+1)} \mathbb{E}_{Q(\psi_m)}(\psi_m) \mathbb{E}_{Q(\mathbf{H}_m)}(\mathbf{H}_m \mathbf{H}_m^T) + \mathbf{I}$$

$$\mu_t^z = (\Lambda_t^z)^{-1} \{ \sum_{m=1}^M \mathbb{E}_{Q(\tau_m)}(\tau_m) \mathbb{E}_{Q(\mathbf{W}_m)}(\mathbf{W}_m) p_m^t + \sum_{m=1}^{M(s+1)} \mathbb{E}_{Q(\psi_m)}(\psi_m) \mathbb{E}_{Q(\mathbf{H}_m)}(\mathbf{H}_m) f_m^t \}$$

$$\mathbb{E}_{Q(\tau_m)}(\tau_m) = \frac{\lambda_m^\tau}{\nu_m^\tau}$$

$$\mathbb{E}_{Q(\mathbf{W}_m)}(\mathbf{W}_m \mathbf{W}_m^T) = (\Lambda_m^W)^{-1} + \mu_m \mu_m^T$$

update $\mathcal{Q}(\mathbf{H})$, $\mathcal{Q}(\mathbf{W})$

$$\Lambda_m^W = \sum_{t=1}^N \mathbb{E}_{Q(\tau_m)}(\tau_m) \mathbb{E}_{Q(\mathbf{z}_t)}(\mathbf{z}_t \mathbf{z}_t^T) + \mathbb{E}_{Q(\alpha_m)}(\alpha_m)$$

$$\mu_m^W = (\Lambda_m^W)^{-1} \mathbb{E}_{Q(\tau_m)}(\tau_m) \sum_{t=1}^N \mathbb{E}_{Q(\mathbf{z}_t)}(\mathbf{z}_t) p_m^t$$

$$\Lambda_m^H = \sum_{t=1}^N \mathbb{E}_{Q(\psi_m)}(\psi_m) \mathbb{E}_{Q(\mathbf{z}_t)}(\mathbf{z}_t \mathbf{z}_t^T) + \mathbb{E}_{Q(\beta_m)}(\beta_m)$$

$$\mu_m^H = (\Lambda_m^H)^{-1} \mathbb{E}_{Q(\psi_m)}(\psi_m) \sum_{t=1}^N \mathbb{E}_{Q(\mathbf{z}_t)}(\mathbf{z}_t) f_m^t$$

$$\mathbb{E}_{Q(\alpha_m)}(\alpha_m) = \nu_m^\alpha \mathbf{V}_m^\alpha$$

$$\mathbb{E}_{Q(\mathbf{z}_t)}(\mathbf{z}_t \mathbf{z}_t^T) = (\Lambda_t^z)^{-1} + \mu_t^z (\mu_t^z)^T$$

update $\mathcal{Q}(\tau)$, $\mathcal{Q}(\psi)$

$$\lambda_m^\tau = j_m^\tau + \frac{1}{2}N$$

$$\nu_m^\tau = k_m^\tau + \frac{1}{2} \sum_{t=1}^N \mathbb{E}_{Q(\mathbf{z}_t)Q(\mathbf{W}_m)} \{ (p_m^t - \mathbf{W}_m^T \mathbf{z}_t)^T (p_m^t - \mathbf{W}_m^T \mathbf{z}_t) \}$$

$$\lambda_m^\psi = j_m^\psi + \frac{1}{2}N$$

$$\nu_m^\psi = k_m^\psi + \frac{1}{2} \sum_{t=1}^N \mathbb{E}_{Q(\mathbf{z}_t)Q(\mathbf{H}_m)} \{ (f_m^t - \mathbf{H}_m^T \mathbf{z}_t)^T (f_m^t - \mathbf{H}_m^T \mathbf{z}_t) \}$$

$$\mathbb{E}_{Q(\mathbf{z})}(\mathbf{z}^T \mathbf{z}) = \text{Tr}(\Lambda_z^{-1}) + \mu_z^T \mu_z$$

update $Q(\alpha)$, $Q(\beta)$

$$\begin{aligned}\nu_m^\alpha &= a_m^\alpha + 1 \\ (\mathbf{V}_m^\alpha)^{-1} &= (\mathbf{B}_m^\alpha)^{-1} + \mathbb{E}_{Q(\mathbf{W}_m)}\{\mathbf{W}_m \mathbf{W}_m^T\}\end{aligned}$$

$$\begin{aligned}\nu_m^\beta &= a_m^\beta + 1 \\ (\mathbf{V}_m^\beta)^{-1} &= (\mathbf{B}_m^\beta)^{-1} + \mathbb{E}_{Q(\mathbf{H}_m)}\{\mathbf{H}_m \mathbf{H}_m^T\}\end{aligned}$$

Monitoring Strategy

Monitoring Strategy is completely established according to the aforementioned reference, where SPE and T^2 are constructed. The specific calculations are given as follows:

index T^2

$$T^2 = \mathbf{z}_{new}^T \mathbf{z}_{new}$$

where

$$\mathbf{z}_{new} = \Lambda_t^z (\mathbf{z}_{new}^p + \mathbf{z}_{new}^f)$$

$$\mathbf{z}_{new}^p = (\mu^W)^T \tau \mathbf{p}_{new}$$

$$\mathbf{z}_{new}^f = (\mu^H)^T \psi \mathbf{f}_{new}$$

$$\tau = \text{diag}\{[\mathbb{E}_{Q(\tau_m)}(\tau_m)]_{m=1}^{Ml}\}$$

$$\psi = \text{diag}\{[\mathbb{E}_{Q(\psi_m)}(\psi_m)]_{m=1}^{M(s+1)}\}$$

index SPE

$$SPE = \delta_{new}^T \psi \delta_{new}$$

$$\delta_{new} = \mathbf{f}_{new} - \mu^H \mathbf{z}_{new}$$