



## Signal processing and analysis

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Major: Photogrammetry and Remote Sensing	Assignment Index: I
Course: Signal processing and analysis	Professor: LUO Bin TA: 韩承熙 Sigma

### I. Proving a function to be a periodic function and expression function\_(QN1&2)

SIGNAL PROCESSING AND ANALYSIS

① The Fourier transformation:

$$\Delta_p(x):$$

$$\rightarrow \text{I} \quad F_{\Delta p}(j\omega) = \frac{2\pi}{P} \sum_{n=-\infty}^{+\infty} \delta(\omega - n \frac{2\pi}{P})$$

By using convolution and Fourier transformation,  $f(x) = F(j\omega)$

$$\rightarrow \text{II} \quad F_p(j\omega) = F(j\omega) F_{\Delta p}(j\omega)$$

$$= \frac{2\pi}{P} F(j\omega) \sum_{n=-\infty}^{+\infty} \delta(\omega - n \frac{2\pi}{P})$$

The Inverse of Fourier transformation of  $F_p(j\omega)$ :

$$\rightarrow \text{III} \quad f_p(x) = \int_{-\infty}^{+\infty} \frac{2\pi}{P} F(j\omega) \sum_{n=-\infty}^{+\infty} \delta(\omega - n \frac{2\pi}{P}) e^{j\omega x} d\omega$$

$$= \sum_{n=-\infty}^{+\infty} f(x + np)$$

this is function is periodic

② at  $x < -2N$ ,  $t_N(x) = 0$

at  $-2N < x < 0$ ,  $t_N = \int_{-N}^{N+x} dt = 2N + x$

at  $0 < x < 2N$ ,  $t_N = \int_{-N}^{N+x} dt = 2N - x$

at  $x > 2N$ ,  $t_N(x) = 0$

$$\Rightarrow t_N(x) = \begin{cases} 2N - |x|, & \text{for } |x| \leq 2N \\ 0, & \text{Others} \end{cases}$$

but if we assume that:  $N > 1$ ,  $t_p(x)$  is to be convolution of  $t_N(x)$  and  $r_N(x)$ ,  $t_p(x) = t_N(x) \cdot r_N(x) = \int_{-\infty}^{+\infty} t_N(t) r_N(x-t) dt$

$$\begin{cases} \Rightarrow \text{for } -4N \leq x \leq 4N, & t_p(x) = \frac{1}{2} (4N + x)^2 \\ \Rightarrow \text{for } 1-4N \leq x \leq -1, & \frac{1}{2} (4N + x - 1)(4N + x) \end{cases}$$

### III. Result

⑧ - Fourier transformation

$$F_{rN}(j\omega) = \int_{-N}^N r_N(x) e^{-j\omega x} dx$$

$$= \frac{e^{j\omega N} - e^{-j\omega N}}{j\omega} = \frac{2 \sin \omega N}{\omega}$$

- For Fourier transformation, using convolution property

$$F_{tN}(j\omega) = F_{rN}(j\omega) F_{rN}(j\omega)$$

$$= \frac{4 \sin^2(\omega N)}{\omega^2} \text{ and transformation of } P_r(x)$$

Can be demonstrated into this way:

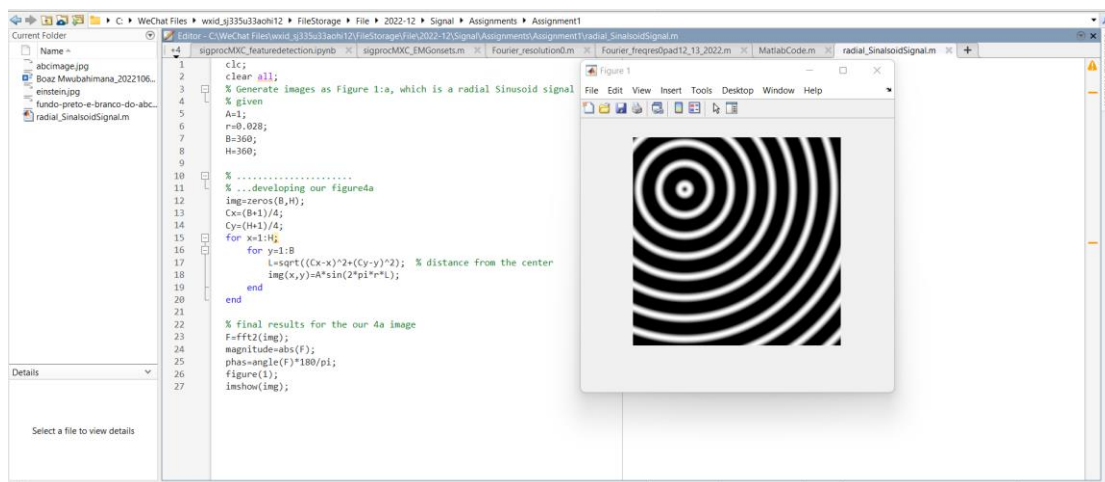
$$F_{tN}(j\omega) = F_{tN}(j\omega) F_{rN}(j\omega)$$

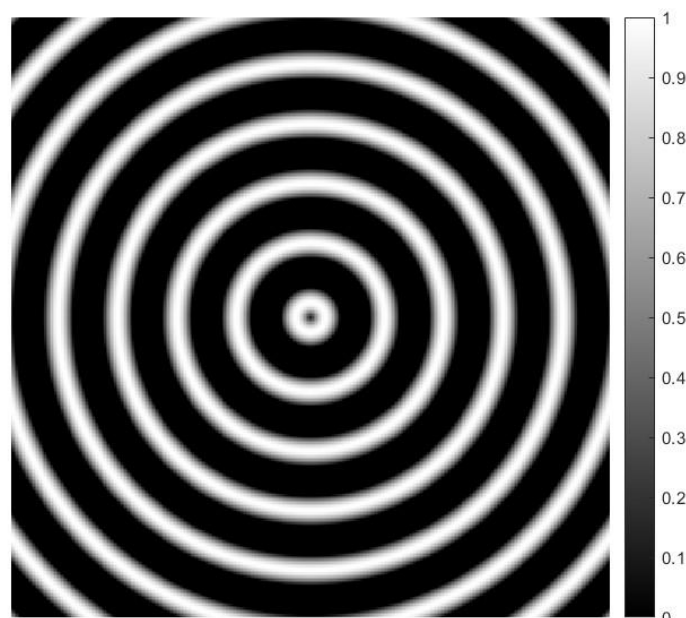
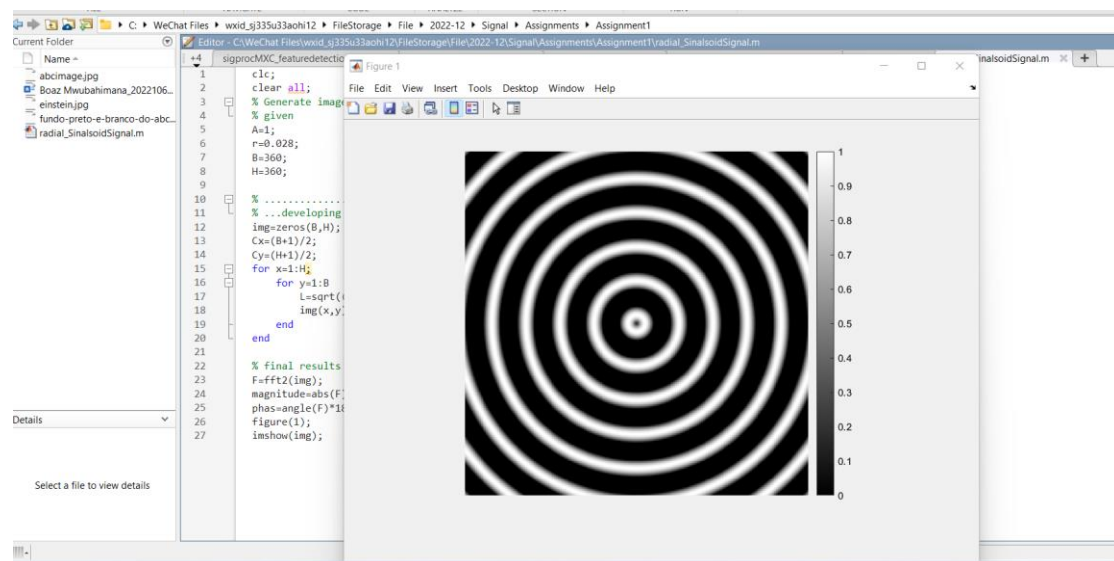
$$= \frac{4 \sin^2(\omega N)}{\omega^2} \frac{2 \sin \omega N}{\omega} = \frac{8 \sin^3(\omega N)}{\omega^3}$$

$$\Rightarrow \frac{8 \sin^3(\omega N)}{\omega^3}$$

### IV. Generate an image as Figure 4a, which is a radial Sinusoid signal.

a.

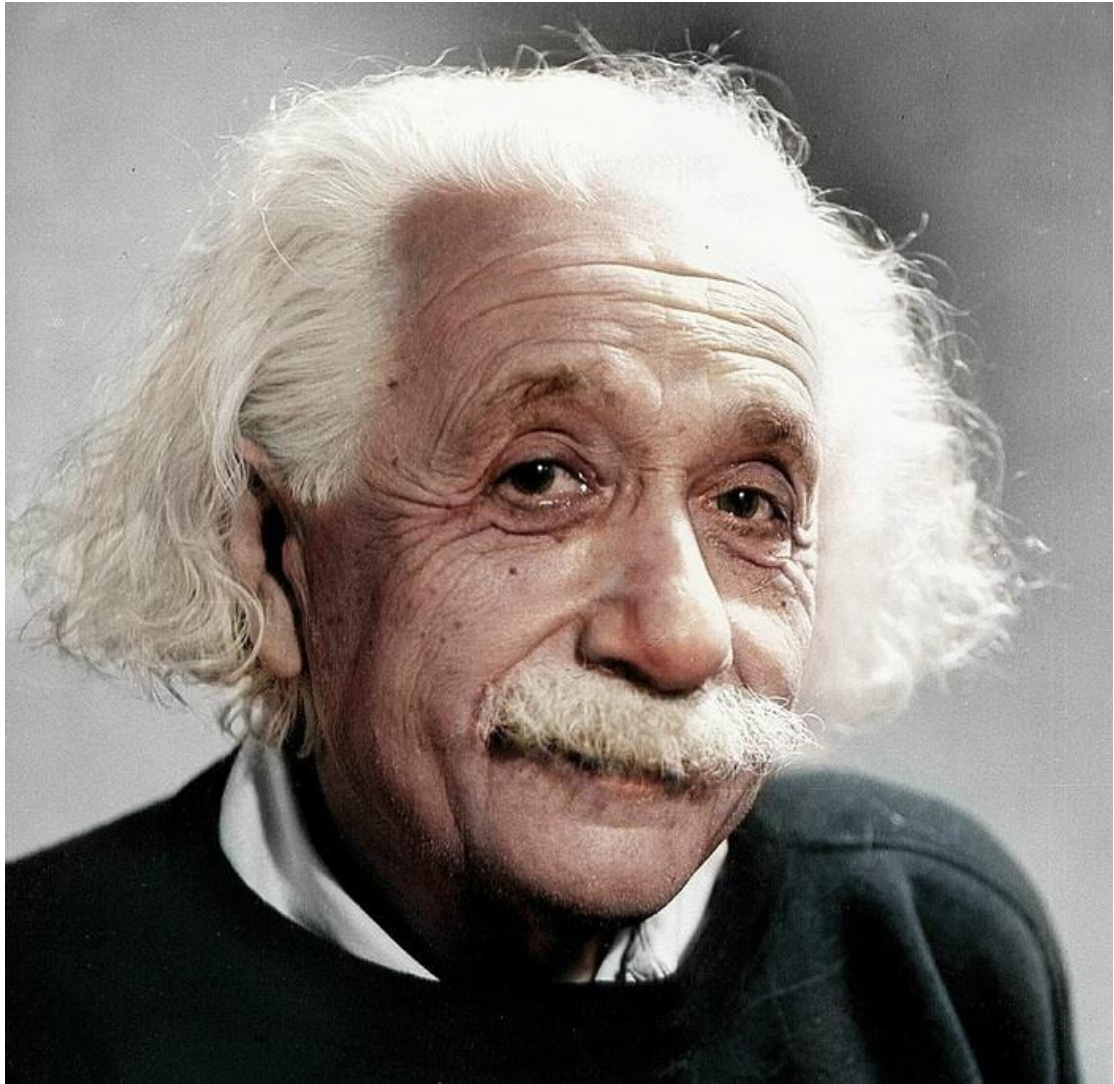




V.

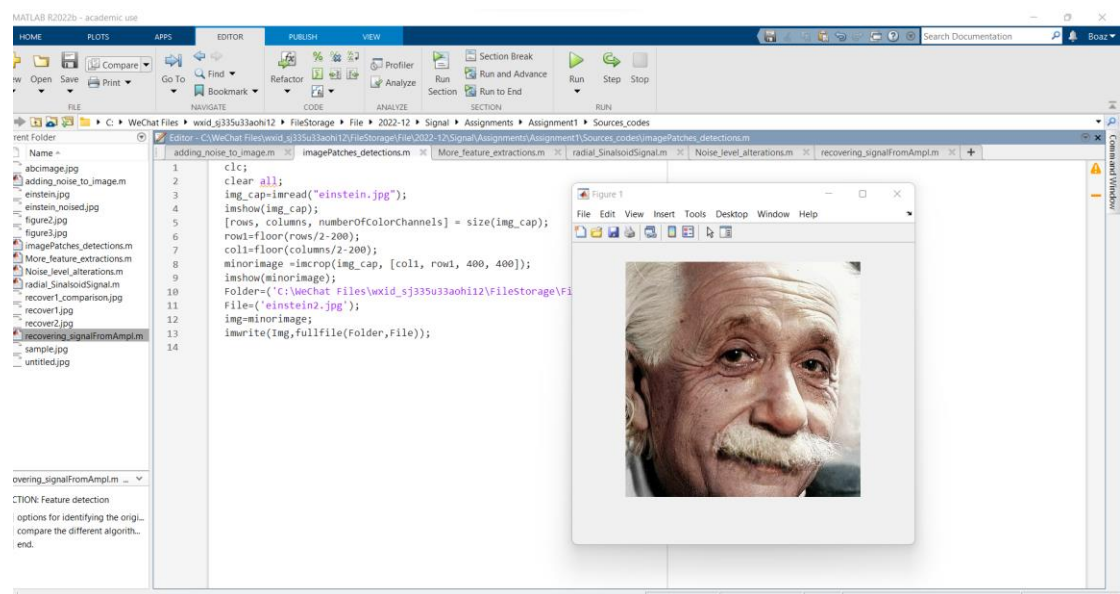
a.

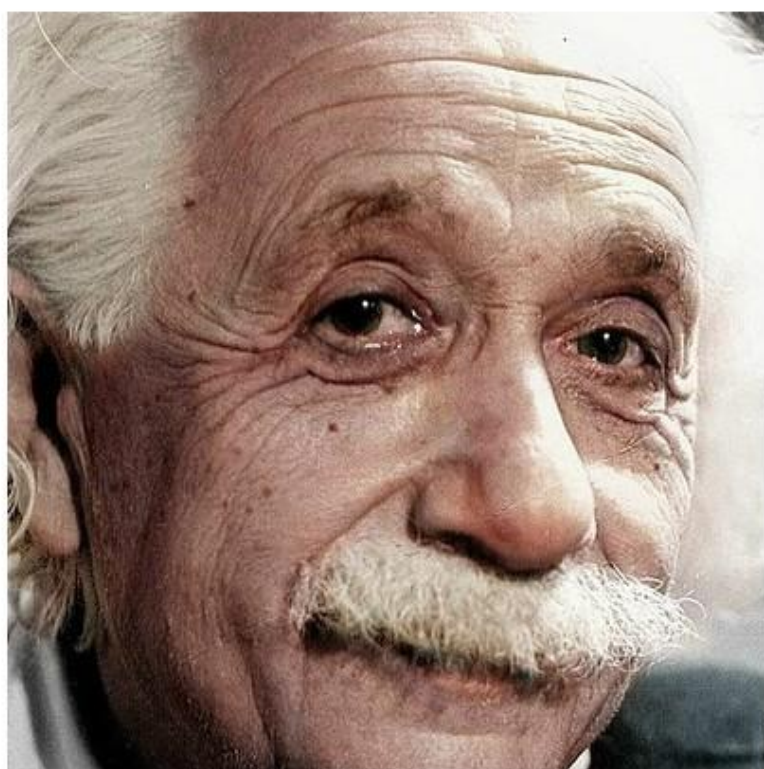




The original image (source: Wikipedia)

b.

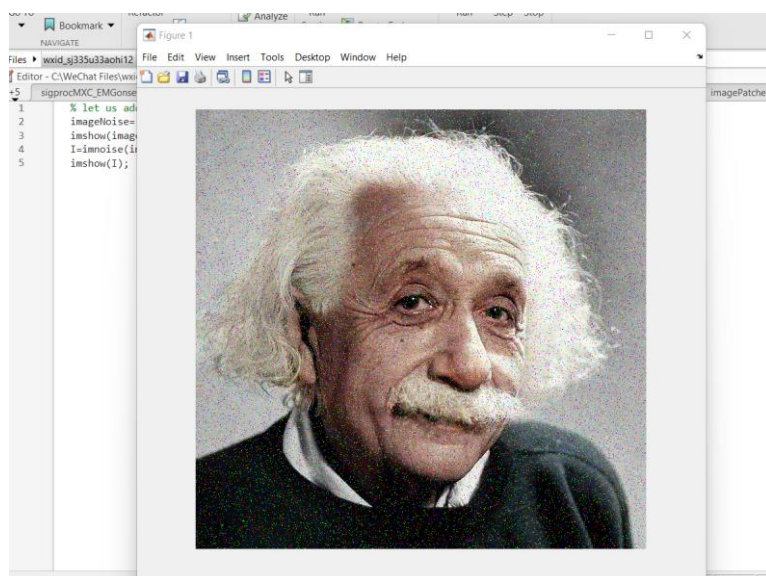




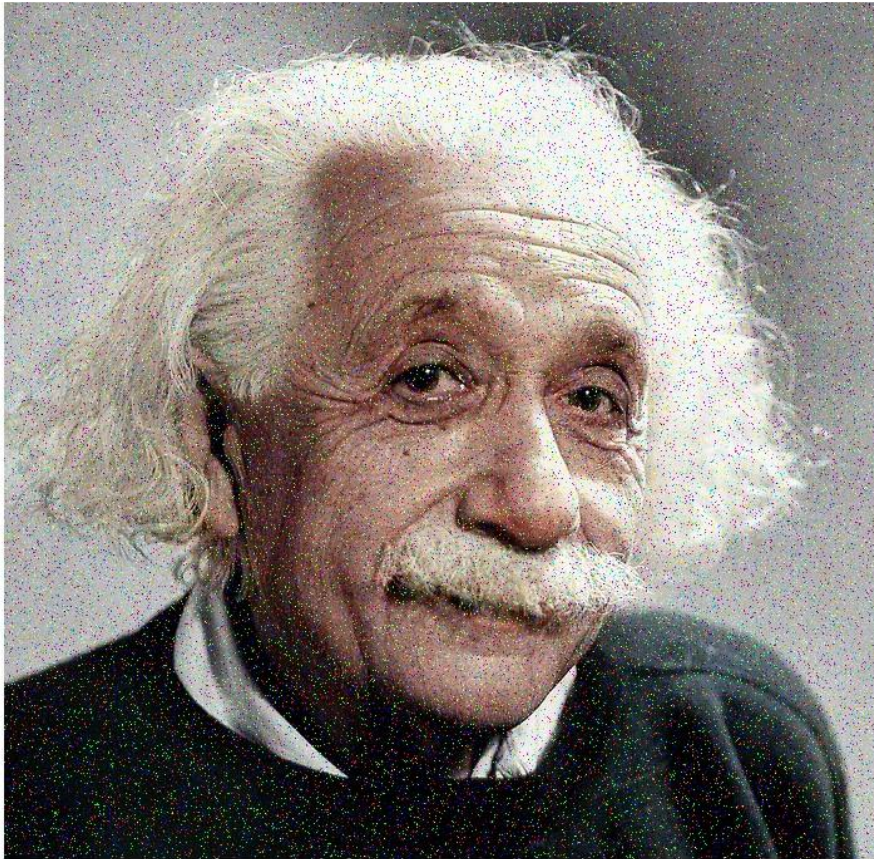


Subsisted original image and extractions of semple img

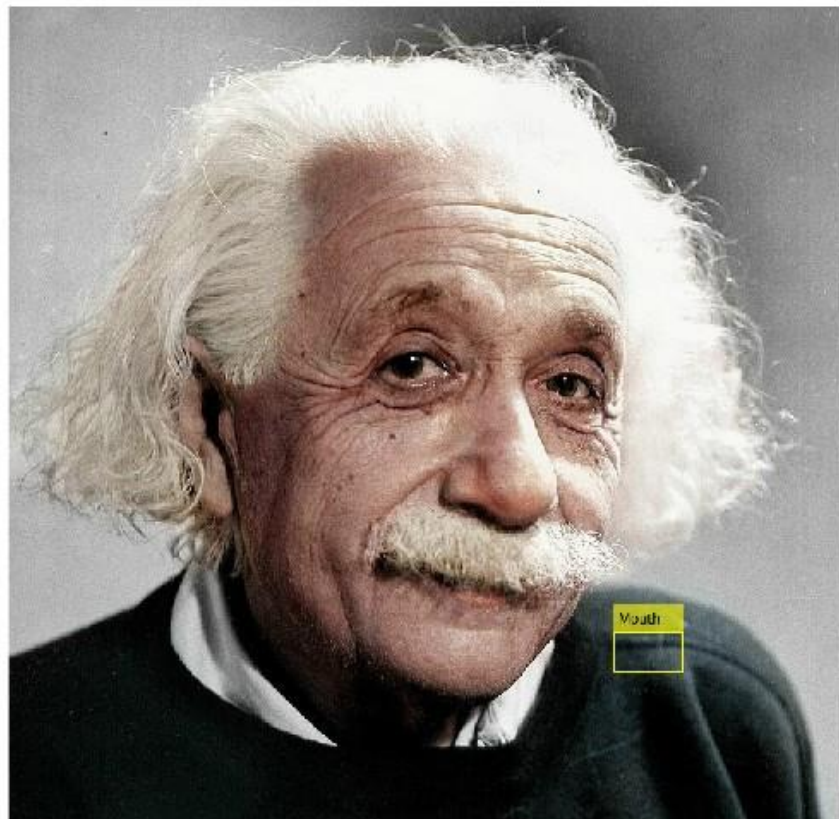
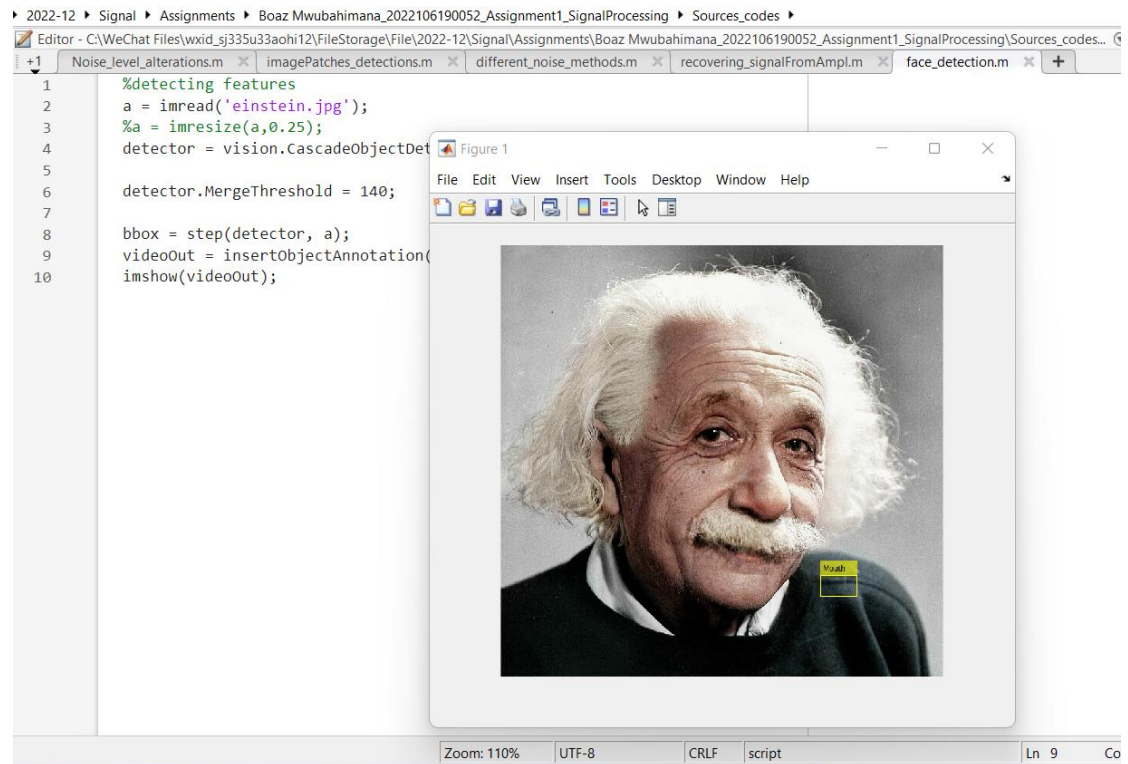
**c. some noise added to the original image.**





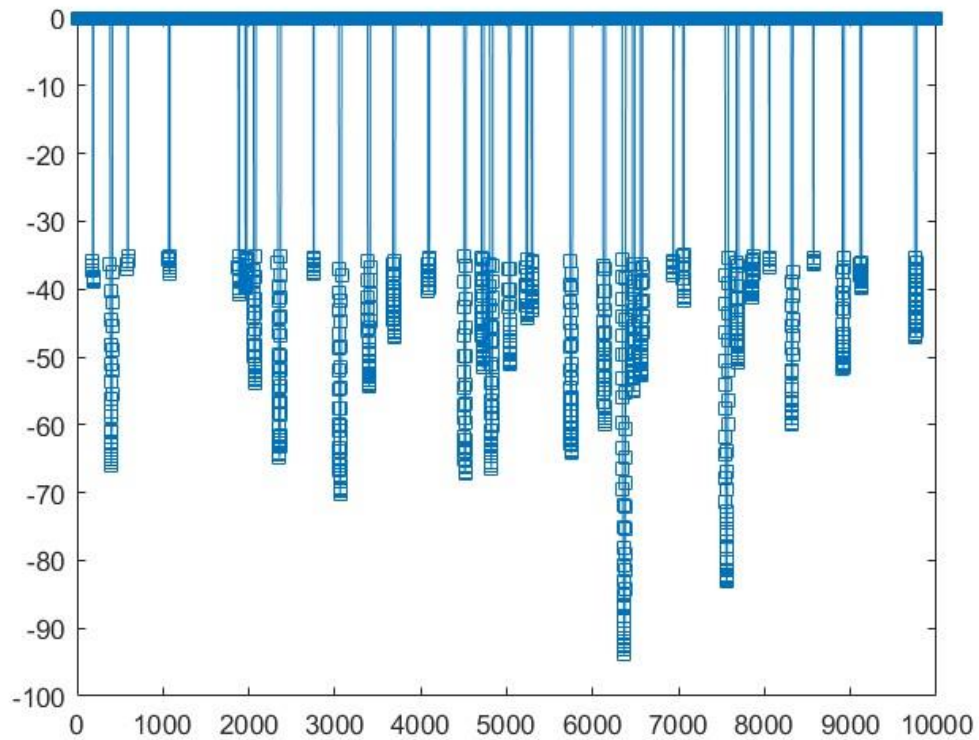
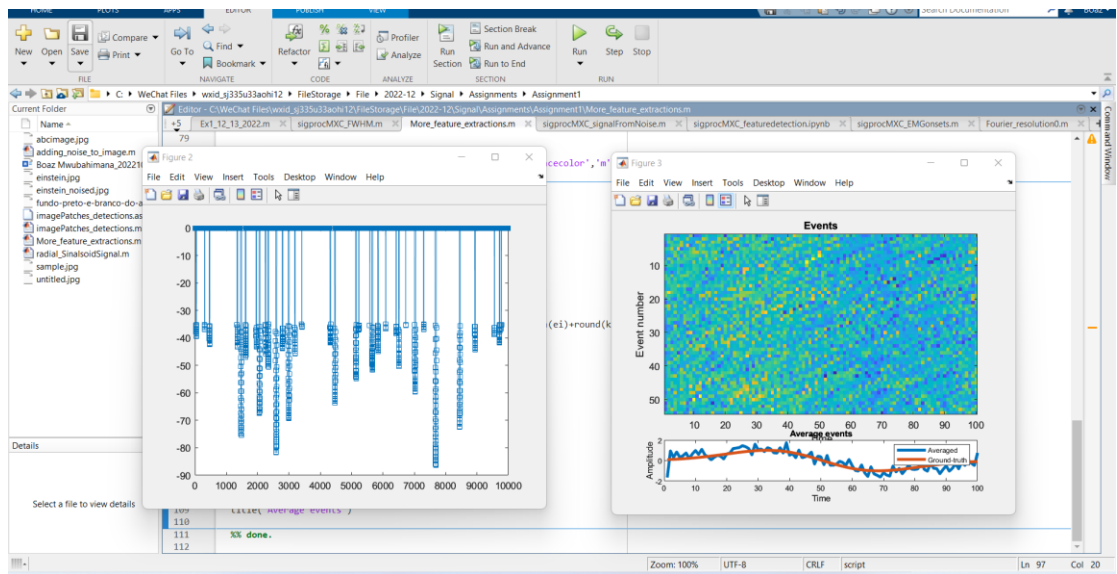


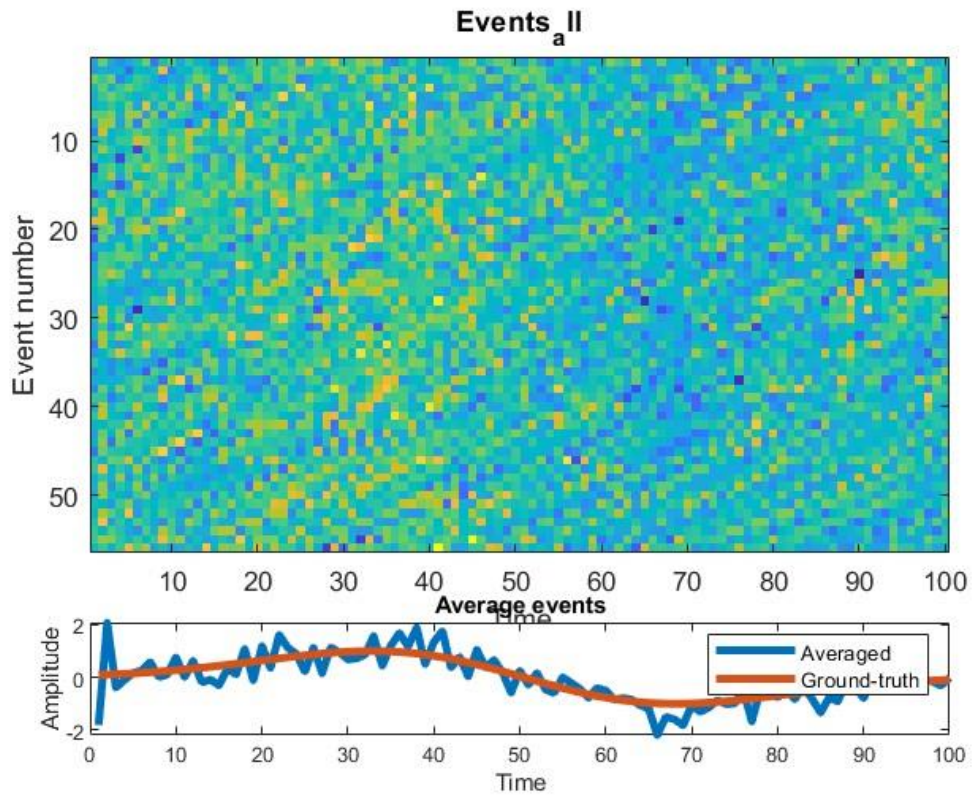
d. adding more detection to the image



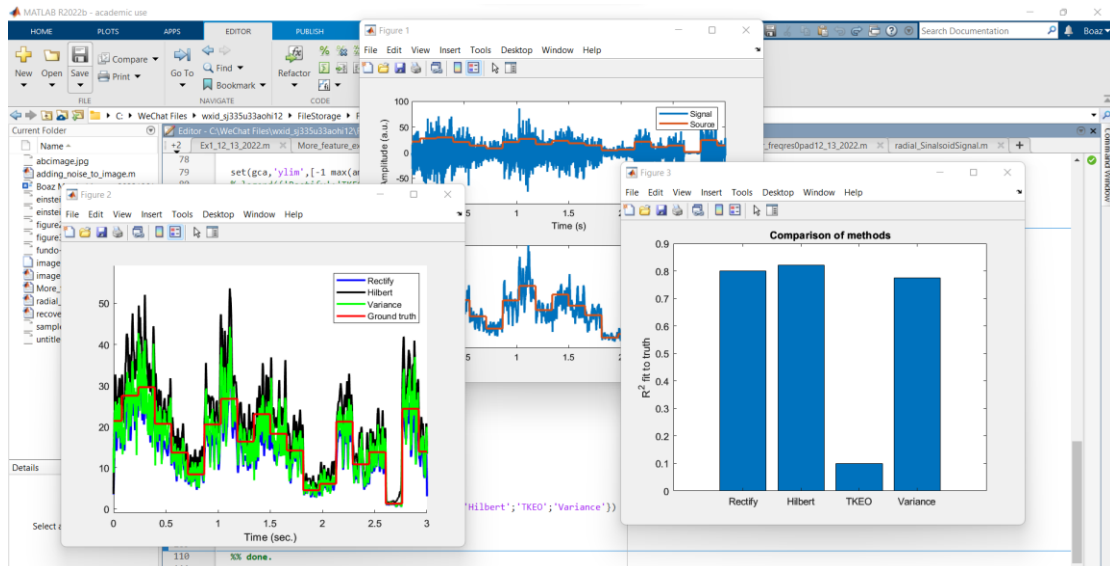
trying to detect near the mouth of Einstein



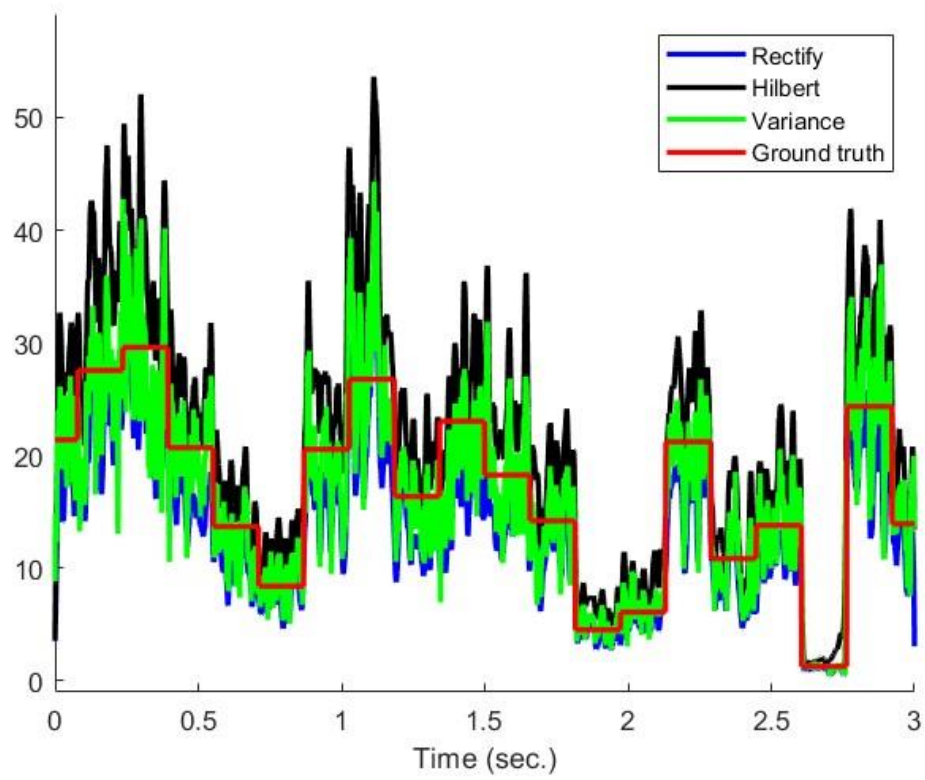
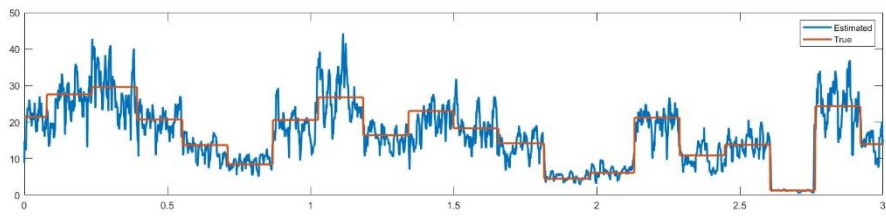
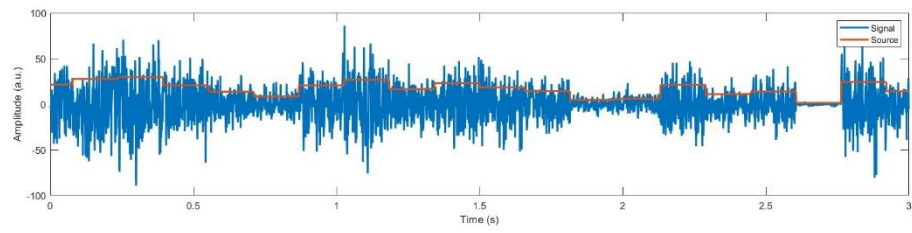




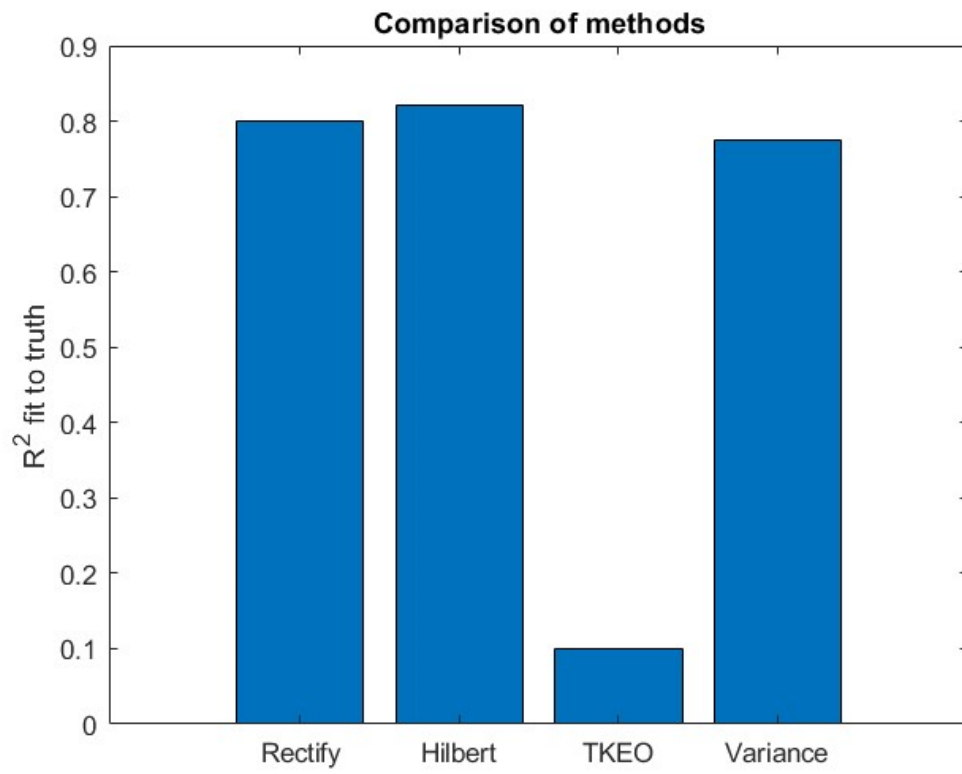
More: Recover signal from noise amplitude



Recovering signal from amplitude







comparing the two methods