Formal Verification and Synthesis

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https://github.com/BoazGur/SokobanVerficiation

Part 1:

1. The FDS is the group: $D = \{V, \theta, \rho, J, C\}$

$V = \{board,\ x,\ y,\ turn, possible\_up,\ possible\_down,\ possible\_right,\ possible\_left\}$
When the variables are defined as:
board: array representation of the map, $n \times m$ matrix (XSB format) ***in the nuXmv it's defined as a set of n*m variables (v_ij) which represent board[i][j], the nuXmv didn't work for us with a 2D array, also in the nuXmv we translated the XSB format to words:**
{@, +, $, *, #, ., -} -> {shtrudel, plus, dollar, star, solamit, dot, minus}
x: int, x-position of the keeper on board $s.t\ 0 \le x < m$
y: int, y-position of the keeper on board $s.t\ 0 \le y < n$
possible_u: boolean, true if the keeper can go up
possible_d: boolean, true if the keeper can go down
possible_r: boolean, true if the keeper can go right
possible_l: boolean, true if the keeper can go left
turn: enum, representation of action taken, $\{u, d, l, r, none\}$
***in the nuXmv we also defined 3 constants, n: # of rows, m: # of columns, done: boolean, true if there are no dollars (boxes).**

$\theta(starting\ board) = (turn = None) \wedge (board = input) \wedge (x = input.findcol(@ \vee +)) \wedge$
$\wedge (y = input.findrow(@ \vee +)) \wedge (possible\_u = up(x,\ y,\ board)) \wedge$
$\wedge (possible\_d = down(x,\ y,\ board)) \wedge (possible\_r = right(x,\ y,\ board)) \wedge$
$\wedge (possible\_l = left(x,\ y,\ board))$

Notice the board is given as an input to the nuXmv. The turn starts as None.
x and y are determined by the input where the keeper was found.
Let's define the functions up, down, right, and left. Which determine the behavior of possible_u, possible_d, possible_r, and possible_l respectively.

$possible\_u = !((y = 0) \vee ((y > 1) \wedge (input[y-1][x] = \#)) \vee$
$\vee ((y > 1) \wedge (input[y-1][x] \in \{\$, *\}) \wedge (input[y-2][x] \in \{\$, *, \#\})))$

$possible\_d = !((y = n-1) \vee ((y < n-1) \wedge (input[y+1][x] = \#)) \vee$
$\vee ((y < n-2) \wedge (input[y+1][x] \in \{\$, *\}) \wedge (input[y+2][x] \in \{\$, *, \#\})))$

$possible\_l = !((x = 0) \vee ((x > 1) \wedge (input[y][x-1] = \#)) \vee$
$\vee ((x > 1) \wedge (input[y][x-2] \in \{\$, *\}) \wedge (input[y][x-2] \in \{\$, *, \#\})))$

$possible\_r = !((x = m-1) \vee ((x < m-1) \wedge (input[y][x+1] = \#)) \vee$
$\vee ((x < m-2) \wedge (input[y][x+1] \in \{\$, *\}) \wedge (input[y][x+2] \in \{\$, *, \#\})))$

ρ consist of the transition of all the variables so we will defined each one here:

$possible\_u' = !((y = 0) \lor ((y > 1) \land (input[y-1][x] = \#)) \lor$
$\lor ((y > 1) \land (input[y-1][x] \in \{\$, *\}) \land (input[y-2][x] \in \{\$, *, \#\})))$

$possible\_d' = !((y = n-1) \lor ((y < n-1) \land (input[y+1][x] = \#)) \lor$
$\lor ((y < n-2) \land (input[y+1][x] \in \{\$, *\}) \land (input[y+2][x] \in \{\$, *, \#\})))$

$possible\_l' = !((x = 0) \lor ((x > 1) \land (input[y][x-1] = \#)) \lor$
$\lor ((x > 1) \land (input[y][x-2] \in \{\$, *\}) \land (input[y][x-2] \in \{\$, *, \#\})))$

$possible\_r' = !((x = m-1) \lor ((x < m-1) \land (input[y][x+1] = \#)) \lor$
$\lor ((x < m-2) \land (input[y][x+1] \in \{\$, *\}) \land (input[y][x+2] \in \{\$, *, \#\})))$

$\rho_{turn} = ((done = true) \land (turn' = None)) \lor$
$\lor ((possible\_u' = true) \land (possible\_d' = true) \land (possible\_r' = true) \land (possible\_l' = true) \land$
$\lor (turn = \{none, u, d, r, l\})) \lor$
$\lor ((possible\_u' = true) \land (possible\_d' = true) \land (possible\_l' = true) \land (turn' = \{none, u, d, l\})) \lor$
$\lor ((possible\_u' = true) \land (possible\_d' = true) \land (possible\_r' = true) \land (turn' = \{none, u, d, r\})) \lor$
$\lor ((possible\_u' = true) \land (possible\_r' = true) \land (possible\_l' = true) \land (turn' = \{none, u, r, l\})) \lor$
$\lor ((possible\_d' = true) \land (possible\_r' = true) \land (possible\_l' = true) \land (turn' = \{none, d, r, l\})) \lor$
$\lor ((possible\_d' = true) \land (possible\_r' = true) \land (turn' = \{none, d, r\})) \lor$
$\lor ((possible\_d' = true) \land (possible\_l' = true) \land (turn' = \{none, d, l\})) \lor$
$\lor ((possible\_u' = true) \land (possible\_r' = true) \land (turn' = \{none, u, r\})) \lor$
$\lor ((possible\_u' = true) \land (possible\_l' = true) \land (turn' = \{none, u, l\})) \lor$
$\lor ((possible\_d' = true) \land (possible\_u' = true) \land (turn' = \{none, d, u\})) \lor$
$\lor ((possible\_l' = true) \land (possible\_r' = true) \land (turn' = \{none, l, r\})) \lor$
$\lor ((possible\_u' = true) \land (turn' = \{none, u\})) \lor$
$\lor ((possible\_d' = true) \land (turn' = \{none, d\})) \lor$
$\lor ((possible\_r' = true) \land (turn' = \{none, r\})) \lor$
$\lor ((possible\_l' = true) \land (turn' = \{none, l\})) \lor$
$\lor ((possible\_u' = false) \land (possible\_d' = false) \land (possible\_l' = false) \land (turn' = none))$

The idea of turn transition is that if done so turn is None, but if not done then we check cases:
If $possible\_u' = true$ then $u \in turn$’, If $possible\_d' = true$ then $d \in turn$’
If $possible\_r' = true$ then r’, If $possible\_l' = true$ then $l \in turn$’
And always $None \in turn'$

$\rho_x = ((turn' = r) \wedge (x < m - 1) \wedge (x' = x + 1)) \vee ((turn' = l) \wedge (x > 0) \wedge (x' = x - 1))$

$\rho_y = ((turn' = d) \wedge (y < n - 1) \wedge (y' = y + 1)) \vee ((turn' = u) \wedge (y > 0) \wedge (y' = y - 1))$

$\rho_{board[i][j]} = ((y = i) \wedge (x = j) \wedge (board[i][j] = @) \wedge (turn' != None) \wedge (board[i][j]' =-)) \vee$
$((y = i) \wedge (x = j) \wedge (board[i][j] =+) \wedge (turn' != None) \wedge (board[i][j]' =.)) \vee$
$\vee ((y = i) \wedge (x = j - 1 >- 1) \wedge (board[i][j] \in \{-, \$\}) \wedge (turn' = r) \wedge (board[i][j]' = @)) \vee$
$\vee ((y = i) \wedge (x = j - 1 >- 1) \wedge (board[i][j] \in \{., *\}) \wedge (turn' = r) \wedge (board[i][j]' =+)) \vee$
$\vee ((y = i) \wedge (x = j - 2 >- 2) \wedge (board[i][j - 1] \in \{*, \$\}) \wedge (turn' = r) \wedge (board[i][j] =-) \wedge$
$\wedge (board[i][j]' = \$)) \vee$
$\vee ((y = i) \wedge (x = j - 2 >- 2) \wedge (board[i][j - 1] \in \{*, \$\}) \wedge (turn' = r) \wedge (board[i][j] =.) \wedge$
$\wedge (board[i][j]' =*)) \vee$
$\vee ((y = i) \wedge (x = j + 1 < m) \wedge (board[i][j] \in \{-, \$\}) \wedge (turn' = l) \wedge (board[i][j]' = @)) \vee$
$\vee ((y = i) \wedge (x = j + 1 < m) \wedge (board[i][j] \in \{., *\}) \wedge (turn' = l) \wedge (board[i][j]' =+)) \vee$
$\vee ((y = i) \wedge (x = j + 2 > m) \wedge (board[i][j + 1] \in \{*, \$\}) \wedge (turn' = l) \wedge (board[i][j] =-) \wedge$
$\wedge (board[i][j]' = \$)) \vee$
$\vee ((y = i) \wedge (x = j + 2 > m) \wedge (board[i][j + 1] \in \{*, \$\}) \wedge (turn' = l) \wedge (board[i][j] =.) \wedge$
$\wedge (board[i][j]' =*)) \vee$
$\vee ((y = i - 1 >- 1) \wedge (x = j) \wedge (board[i][j] \in \{-, \$\}) \wedge (turn' = d) \wedge (board[i][j]' = @)) \vee$
$\vee ((y = i - 1 >- 1) \wedge (x = j) \wedge (board[i][j] \in \{., *\}) \wedge (turn' = d) \wedge (board[i][j]' =+)) \vee$
$\vee ((y = i - 2 >- 2) \wedge (x = j) \wedge (board[i - 1][j] \in \{*, \$\}) \wedge (turn' = d) \wedge (board[i][j] =-) \wedge$
$\wedge (board[i][j]' = \$)) \vee$
$\vee ((y = i - 2 >- 2) \wedge (x = j) \wedge (board[i - 1][j] \in \{*, \$\}) \wedge (turn' = d) \wedge (board[i][j] =.) \wedge$
$\wedge (board[i][j]' =*)) \vee$
$\vee ((y = i + 1 < n) \wedge (x = j) \wedge (board[i][j] \in \{-, \$\}) \wedge (turn' = u) \wedge (board[i][j]' = @)) \vee$
$\vee ((y = i + 1 < n) \wedge (x = j) \wedge (board[i][j] \in \{., *\}) \wedge (turn' = u) \wedge (board[i][j]' =+)) \vee$
$\vee ((y = i + 2 < n) \wedge (x = j) \wedge (board[i + 1][j] \in \{*, \$\}) \wedge (turn' = u) \wedge (board[i][j] =-) \wedge$
$\wedge (board[i][j]' = \$)) \vee$
$\vee ((y = i + 2 < n) \wedge (x = j) \wedge (board[i + 1][j] \in \{*, \$\}) \wedge (turn' = u) \wedge (board[i][j] =.) \wedge$
$\wedge (board[i][j]' =*))$

J, we don’t have states that need to repeat an infinite amount of times, but we do know that if the board is solvable we get that turn=None repeats an infinite amount of times.

C, in our problem, we do know that if turn=right an infinite amount of times then turn=left an infinite amount of times, and the opposite of course. that is true also for up and down.

Part 1:

2. The LTLSPEC we defined for our problem is this: !F(done), which means that for a win eventually done will be true. We did the not-operator, to get the path to a win.

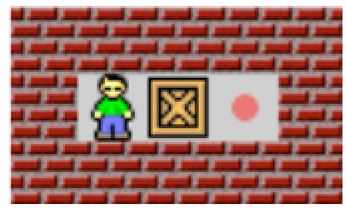
Part 2:

1. The code and the nuXmv's and the output files can be found in this GitHub repo: https://github.com/BoazGur/SokobanVerficiation.

2. We converted all the board examples that was provided in the last page of the exercise to XSB format and run the Python script on them.
   Most of them worked great with just the command "nuXmv <file name>".
   Just the last board (board7) took too long so we ran it with BMC manually with the commands "nuXmv -int <file name>" and then "go_bmc" and then "check_ltlspec_bmc -k 15"

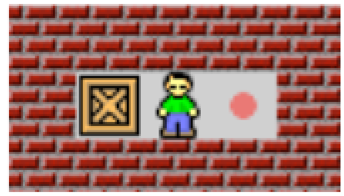
Board1.out:

```
-- specification !( F done)  is false
-- as demonstrated by the following execution sequence
Trace Description: LTL Counterexample
Trace Type: Counterexample
  -> State: 1.1 <-
    turn = none
    possible_up = FALSE
    possible_down = FALSE
    possible_right = TRUE
    possible_left = FALSE
    y = 1
    x = 1
    v_00 = solamit
    v_01 = solamit
    v_02 = solamit
    v_03 = solamit
    v_04 = solamit
    v_10 = solamit
    v_11 = shtrudel
    v_12 = dollar
    v_13 = dot
    v_14 = solamit
    v_20 = solamit
    v_21 = solamit
    v_22 = solamit
    v_23 = solamit
    v_24 = solamit
    done = FALSE
    m = 5
    n = 3
  -> State: 1.2 <-
    turn = r
    x = 2
    v_11 = minus
    v_12 = shtrudel
    v_13 = star
    done = TRUE
  -- Loop starts here
  -> State: 1.3 <-
    turn = none
    possible_right = FALSE
    possible_left = TRUE
  -> State: 1.4 <-
```



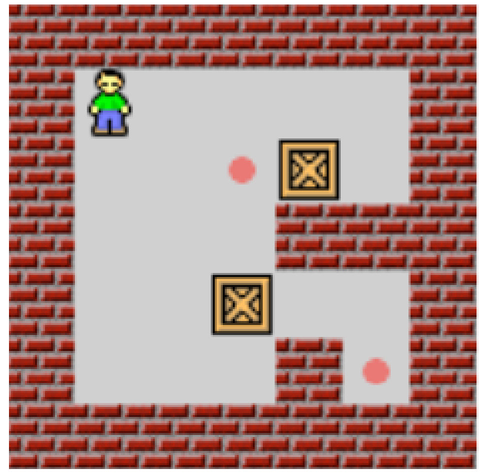
Board2.out:

```
-- specification !( F done)  is true
```



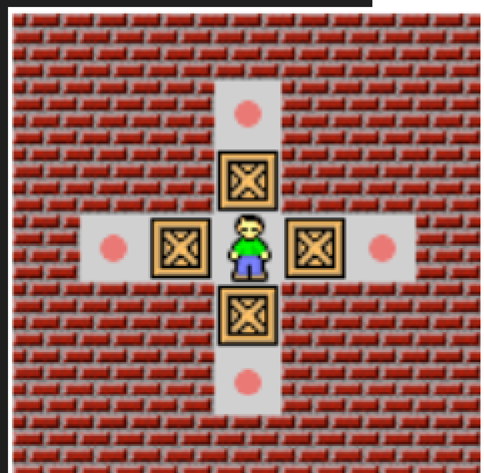
Board3.out:

```
-- specification !( F done)  is true
```



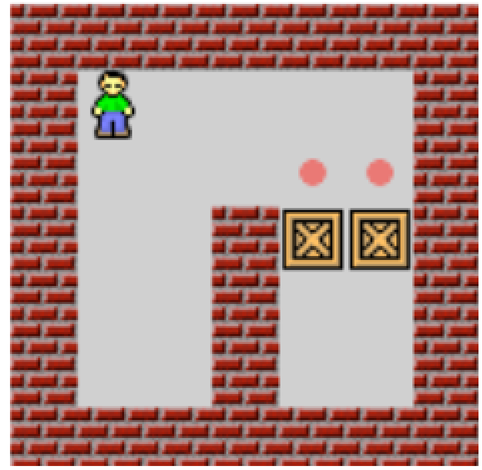
Board4.out:

```
-- specification !( F done)  is false
-- as demonstrated by the following ex
Trace Description: LTL Counterexample
Trace Type: Counterexample
  -> State: 1.1 <-
    turn = none
    possible_up = TRUE
    possible_down = TRUE
    possible_right = TRUE
    possible_left = TRUE
    y = 3
    x = 3
    v_00 = solamit
    v_01 = solamit
    v_02 = solamit
    v_03 = solamit
    v_04 = solamit
    v_05 = solamit
    v_06 = solamit
    v_10 = solamit
    v_11 = solamit
    v_12 = solamit
    v_13 = dot
    v_14 = solamit
    v_15 = solamit
    v_16 = solamit
    v_20 = solamit
    v_21 = solamit
    v_22 = solamit
    v_23 = dollar
    v_24 = solamit
    v_25 = solamit
    v_26 = solamit
    v_30 = solamit
    v_31 = dot
    v_32 = dollar
    v_33 = shtrudel
    v_34 = dollar
    v_35 = dot
    v_36 = solamit
    v_40 = solamit
    v_41 = solamit
    v_42 = solamit
    v_43 = dollar
    v_44 = solamit
    v_45 = solamit
    v_46 = solamit
    v_50 = solamit
    v_51 = solamit
    v_52 = solamit
    v_53 = dot
    v_54 = solamit
    v_55 = solamit
    v_56 = solamit
    v_60 = solamit
    v_61 = solamit
    v_62 = solamit
    v_63 = solamit
    v_64 = solamit
    v_65 = solamit
    v_66 = solamit
    done = FALSE
    m = 7
    n = 7
  -> State: 1.2 <-
    turn = d
    y = 4
    v_33 = minus
    v_43 = shtrudel
    v_53 = star
  -> State: 1.3 <-
    turn = u
    possible_down = FALSE
    possible_right = FALSE
    possible_left = FALSE
    y = 3
    v_33 = shtrudel
    v_43 = minus
  -> State: 1.4 <-
    possible_down = TRUE
    possible_right = TRUE
    possible_left = TRUE
    y = 2
    v_13 = star
    v_23 = shtrudel
    v_33 = minus
  -> State: 1.5 <-
    turn = d
    possible_up = FALSE
    possible_right = FALSE
    possible_left = FALSE
    y = 3
    v_23 = minus
    v_33 = shtrudel
  -> State: 1.6 <-
    turn = r
    possible_up = TRUE
    possible_right = TRUE
    possible_left = TRUE
    x = 4
    v_33 = minus
    v_34 = shtrudel
    v_35 = star
  -> State: 1.7 <-
    turn = l
    possible_up = FALSE
    possible_down = FALSE
    possible_right = FALSE
    x = 3
    v_33 = shtrudel
    v_34 = minus
  -> State: 1.8 <-
    possible_up = TRUE
    possible_down = TRUE
    possible_right = TRUE
    x = 2
    v_31 = star
    v_32 = shtrudel
    v_33 = minus
    done = TRUE
  -- Loop starts here
  -> State: 1.9 <-
    turn = none
    possible_up = FALSE
    possible_down = FALSE
    possible_left = FALSE
  -> State: 1.10 <-
```



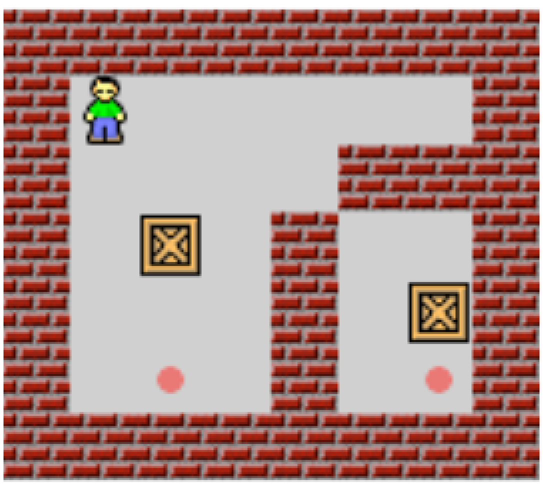
Board5.out:

```
-- specification !( F done)  is true
```



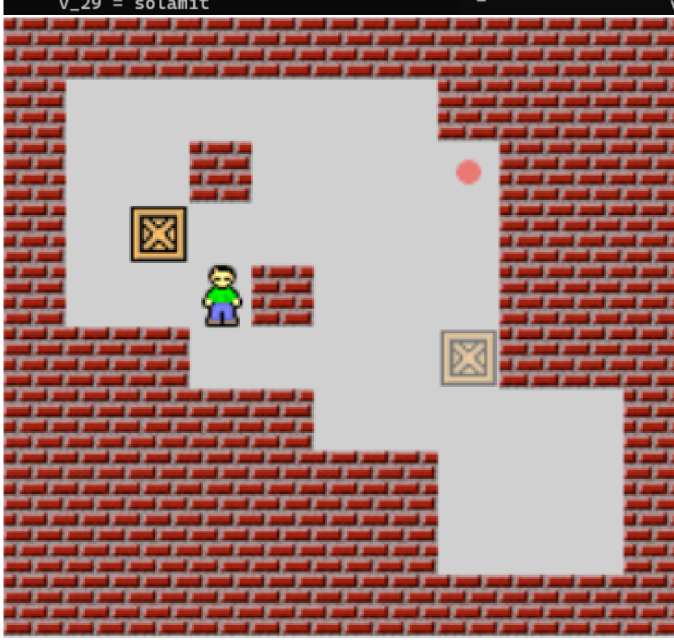
Board6.out:

```
-- specification !( F done)  is true
```



Board7.out:

```
nuXmv > check_ltlspec_bmc -k 15
-- no counterexample found with bound 0
-- no counterexample found with bound 1
-- no counterexample found with bound 2
-- no counterexample found with bound 3
-- no counterexample found with bound 4
-- no counterexample found with bound 5
-- no counterexample found with bound 6
-- no counterexample found with bound 7
-- no counterexample found with bound 8
-- no counterexample found with bound 9
-- no counterexample found with bound 10
-- specification !( F done)    is false
-- as demonstrated by the following execu
Trace Description: BMC Counterexample
Trace Type: Counterexample
  -> State: 1.1 <-
    turn = none
    possible_up = TRUE
    possible_down = TRUE
    possible_right = FALSE
    possible_left = TRUE
    y = 4
    x = 3
    v_00 = solamit
    v_01 = solamit
    v_02 = solamit
    v_03 = solamit
    v_04 = solamit
    v_05 = solamit
    v_06 = solamit
    v_07 = solamit
    v_08 = solamit
    v_09 = solamit
    v_010 = solamit
    v_10 = solamit
    v_11 = minus
    v_12 = minus
    v_13 = minus
    v_14 = minus
    v_15 = minus
    v_16 = minus
    v_17 = solamit
    v_18 = solamit
    v_19 = solamit
    v_110 = solamit
    v_20 = solamit
    v_21 = minus
    v_22 = minus
    v_23 = solamit
    v_24 = minus
    v_25 = minus
    v_26 = minus
    v_27 = dot
    v_28 = solamit
    v_29 = solamit
    v_210 = solamit
    v_30 = solamit
    v_31 = minus
    v_32 = dollar
    v_33 = minus
    v_34 = minus
    v_35 = minus
    v_36 = minus
    v_37 = minus
    v_38 = solamit
    v_39 = solamit
    v_310 = solamit
    v_40 = solamit
    v_41 = minus
    v_42 = minus
    v_43 = shtrudel
    v_44 = solamit
    v_45 = minus
    v_46 = minus
    v_47 = minus
    v_48 = solamit
    v_49 = solamit
    v_410 = solamit
    v_50 = solamit
    v_51 = solamit
    v_52 = solamit
    v_53 = minus
    v_54 = minus
    v_55 = minus
    v_56 = minus
    v_57 = star
    v_58 = solamit
    v_59 = solamit
    v_510 = solamit
    v_60 = solamit
    v_61 = solamit
    v_62 = solamit
    v_63 = solamit
    v_64 = solamit
    v_65 = minus
    v_66 = minus
    v_67 = minus
    v_68 = minus
    v_69 = minus
    v_610 = solamit
    v_70 = solamit
    v_71 = solamit
    v_72 = solamit
    v_73 = solamit
    v_74 = solamit
    v_75 = solamit
    v_76 = solamit
    v_77 = minus
    v_78 = minus
    v_79 = minus
    v_710 = solamit
    v_80 = solamit
    v_81 = solamit
    v_82 = solamit
    v_83 = solamit
    v_84 = solamit
    v_85 = solamit
    v_86 = solamit
    v_87 = minus
    v_88 = minus
    v_89 = minus
    v_810 = solamit
    v_90 = solamit
    v_91 = solamit
    v_92 = solamit
    v_93 = solamit
    v_94 = solamit
    v_95 = solamit
    v_96 = solamit
    v_97 = solamit
    v_98 = solamit
    v_99 = solamit
    v_910 = solamit
    done = FALSE
    m = 11
    n = 10
  -> State: 1.2 <-
    turn = l
    x = 2
    v_42 = shtrudel
    v_43 = minus
  -> State: 1.3 <-
    possible_down = FALSE
    possible_right = TRUE
    x = 1
    v_41 = shtrudel
    v_42 = minus
  -> State: 1.4 <-
    turn = u
    possible_left = FALSE
    y = 3
    v_31 = shtrudel
    v_41 = minus
  -> State: 1.5 <-
    turn = r
    possible_down = TRUE
    x = 2
    v_31 = minus
    v_32 = shtrudel
    v_33 = dollar
  -> State: 1.6 <-
    possible_left = TRUE
    x = 3
    v_32 = minus
    v_33 = shtrudel
    v_34 = dollar
  -> State: 1.7 <-
    possible_up = FALSE
    x = 4
    v_33 = minus
    v_34 = shtrudel
    v_35 = dollar
  -> State: 1.8 <-
    possible_up = TRUE
    possible_down = FALSE
    x = 5
    v_34 = minus
    v_35 = shtrudel
    v_36 = dollar
  -> State: 1.9 <-
    possible_down = TRUE
    x = 6
    v_35 = minus
    v_36 = shtrudel
    v_37 = dollar
  -> State: 1.10 <-
    turn = d
    possible_right = FALSE
    y = 4
    v_36 = minus
    v_46 = shtrudel
  -> State: 1.11 <-
    turn = r
    possible_right = TRUE
    x = 7
    v_46 = minus
    v_47 = shtrudel
  -> State: 1.12 <-
    turn = u
    possible_right = FALSE
    y = 3
    v_27 = star
    v_37 = shtrudel
    v_47 = minus
    done = TRUE
nuXmv >
```



Part 2:

3. For each board we found was solvable we'll define the winning moves:

   Board1: r
   Board4: d, u, u, d, r, l, l
   Board7: l, l, u, r, r, r, r, r, d, r, u

Part 3:

Board	**SAT**[sec]	**BDD**[sec]
board1	0.07	0.07
board2	0.58	0.07
board3	8.54	3.92
board4	0.66	3.51
board5	5.83	1.02
board6	3.66	0.9
board7	12.94	1023

When running the SVM's we used interactive mode and different commands for BDD and SAT:
BDD: commands = go -> check_ltlspec -> quit
SAT: commands = go_bmc -> check_ltlspec_bmc -k 15 -> quit

From those tests, we can understand that BDD solved faster for boards which isn't solvable probably because we forced the SAT to continue and stop after 15 moves. But we noticed that for solvable boards (1, 4, 7) the SAT solved them faster, and for board7 that is bigger than the others the SAT solved it much faster than the BDD.
To conclude, in our opinion the SAT is better than BDD, especially in solvable boards.

Part 4:

For this part, we added some new functionality to the SMVWriter class using the SMVWriterIterative class which inherits from SMVwriter. In this class, we also follow the position of one specific box in each iteration. For that we had modified 'done' to check if the chosen box had reached a target. At the end of each iteration we try to get our new board state. To do that, first of all, we check if our specification is true. If so, we know our board is unsolvable and we return None. otherwise, we inspect all the last changes of each place of the board (aka. the board state) and then return the board after the iteration.

For each iteration, we run the nuXmv in SAT mode, using these commands:
SAT: commands = go_bmc -> check_ltlspec_bmc -k 15 -> quit
For each board, we saved the total time that took the nuXmv to run, and these are the results:

Board	**ITERATIVE_SAT**[sec]	**ITERATIONS**	**TIME PER INTERATION**[sec]
board1	0.08	1	[0.08]
board2	0.81	1	[0.81]
board3	1.71	2	[0.63, 1.08]
board4	0.75	4	[0.17, 0.19, 0.19, 0.2]
board5	4.84	2	[1.17, 3.67]
board6	4.37	2	[0.35, 4.01]
board7	11.94	1	[11.94]

As for the times of the iterative algorithm, we got pretty similar results to the regular SAT mode. We can see a dramatic change in runtime in the board3 solution which is caused by the error we will explain in the next paragraph.

In some cases, two boxes can satisfy the specification although the board isn't solved. What's happening is that the new box pushed the first from the target and now the box that was on the target is not solved. This makes the program run another iteration as there is now 'another' box (which we solved in previous iterations). To solve that we found the number of boxes in the beginning and then we checked if our iteration number surpassed the number of boxes in the beginning. This solution doesn't really solve the problem, it just indicates that if our algorithm can get in a loop then it will stop it and define the board as unsolvable, which of course isn't always the true assumption.

Another problem we encountered when we tried to solve bigger, more complex boards, was the fact that when we solved for a specific box we could accidentally push another box to a corner or someplace it couldn’t reach a target, to solve this problem we just avoided it:)

We created a new bigger solvable board, especially for this part which in XSB format looks like this:

```
###########
#--$--.####
##-#---.###
#--$----###
#--@#---###
###----$###
#####-----#
#######---#
#######.--#
###########
```

Which is like board7, but with more boxes to make it harder to solve. We checked the times in the SAT model, and our iterative_SAT, and got these results:

Regular SAT: 121.58 sec, and solved in 23 moves (The full output file can be seen in the GitHub repo under outputSAT/board8.out).
Iterative_SAT: 13.3 sec, and 3 iterations (The full output file can be seen in the GitHub repo under outputIterative/board8_time.out,board8_box_iteration1-3.out).
The iteration times are: [2.51, 9.84, 0.95]

As we can see, using our Iterative solution on big and complex boards yields way better results in time, as expected.