https://boazturya.github.io/cs435ALGO/BOAZ_LAB12.pdf

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LAB12

PROBLEM 1

No, not every dense graph has a hamiltonian cycle
If we have a disconnected graph G whose components each have a complete subgraph of G, then G is dense but we cant form a cycle to reach every edge.

~~Assuming the graph is connected~~
~~$G = (V,E)$ it is dense if~~

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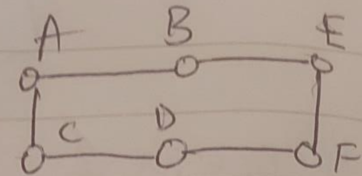
PROBLEM 2



a) Yes it is hamiltonian because there exists a simple cycle that connects every vertice in G.

b) deg (A) + deg (E) < 6

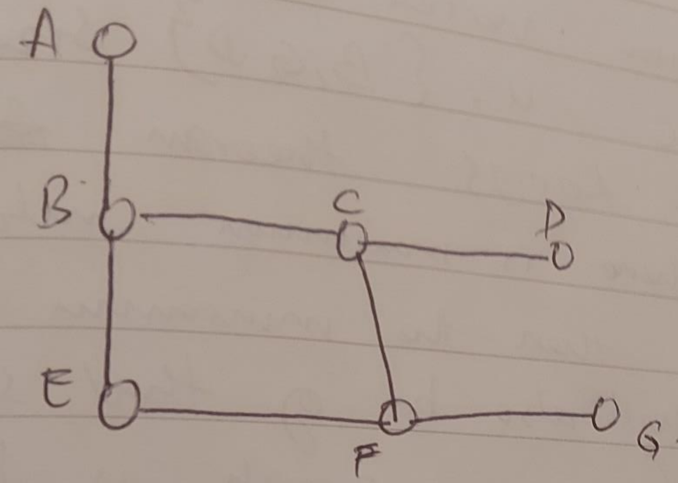
$2 + 2 < 6$

$4 < 6$ true.

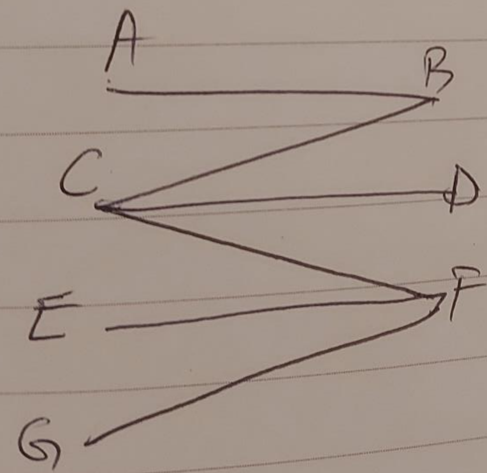
c) ore's theorem: $\deg(u) + \deg(v) \geq n$ where u and v are non-adjacent, then the graph has a hamiltonian cycle

- No, it doesnot contradict ore's theorem. because ore's theorem does not suggest that there is no other criteria for a graph to have a hamiltonian cycle

PROBLEM 3:



a) Yes, G is bipartite.



b) Maximum matching in G = AB, CD, FG

Size = 3

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c) Minimum vertex Cover

MVC = U = {B, G, D} Size = 3.

So, Konig's theorem ~~states~~ that if there is maximum matching of size n then the minimum vertex cover will also be of that same size holds (given a graph is bipartite)

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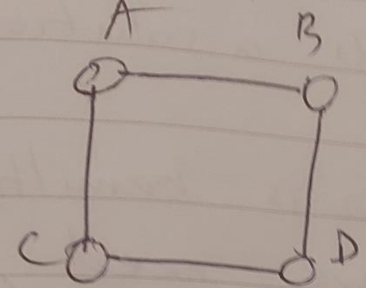
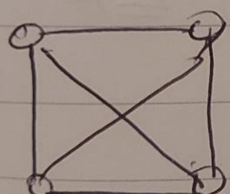
PROBLEM 4



Given $G = (V, E)$ with n vertices
we obtain a TSP problem.

We need a complete graph $H = K_n$.



$H = K_4$, G is a subgraph of H.
$H = (V_H, E_H)$

define k and cost function c.

let $k = 0$

$c : E_H \rightarrow \mathbb{N}$

$$c(e) = \begin{cases} 0 & \text{if } e \in E \\ 1 & \text{if } e \notin E \end{cases}$$

It takes $O(n^2)$ time to do this

Now, verify that a solution to HC problem ~~ex~~ yields a solution to TSP problem.

- Suppose G is hamiltonian cycle in G, we need
  - check C is Hc in H (H → new graph for TSP)

Since $V = V_H$, is C spanning? YES

- If C is still simple? YES.

check $\sum_{e \in C} c(e) \leq k$

Since each e in C belongs to E, $c(e) = 0$

$$\sum c(e) = 0 = k$$

We have verified that C is the solution to the TSP problem we ~~created~~ created.

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Then, lets verify that the solution to the TSP problem yields a solution to the HC problem

Suppose C is a solution to the TSP problem

$$\sum_{e \in C} c(e) \leq K = 0$$ by the mapping done earlier

Summation of integers (positive) can only be zero if each edge weight is valued to be zero.

$\Rightarrow$ e in C belongs to E $\Rightarrow C \subseteq E$

So C is a hamiltonian cycle that creates a graph G from H.

$\therefore$ We have a solution to the Hamiltonian Cycle problem

Question No. 5

Given: Hamitonian cycle problem is NP-complete.

proof: TSP is NP complete

sol^n:

Since we have HC as a NP-complete problem

$$SAT \xrightarrow{poly} HC \quad \text{——} \quad (1)$$

we proofed in question 2,

$$HC \xrightarrow{poly} TSP$$

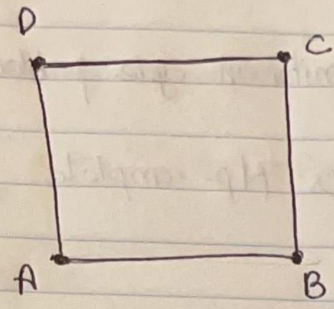
By using (Transitivity of Reducibility):

$$SAT \xrightarrow{poly} HC \xrightarrow{poly} TSP.$$

$$\therefore SAT \xrightarrow{Poly} TSP$$

$\therefore$ we can say that TSP is also NP-complete problem.

Question 6



Give S = size of vortex

(a) from above graph,

min vertex Cover $=$

$|U| = \{A, C\}$

(b) From vertexCoverApprox outputs size $= 2 * S$

from graph $E = \{AB, BC, CD, AD\}$

Using VertexCoverApprox

$C = \{A, B, C, D\}$

$|C| = 4$

$= 2 * |U|$

$= 2 * S$

It is clear that vertexCoverApprox outputs size is $2S$ using above approach.