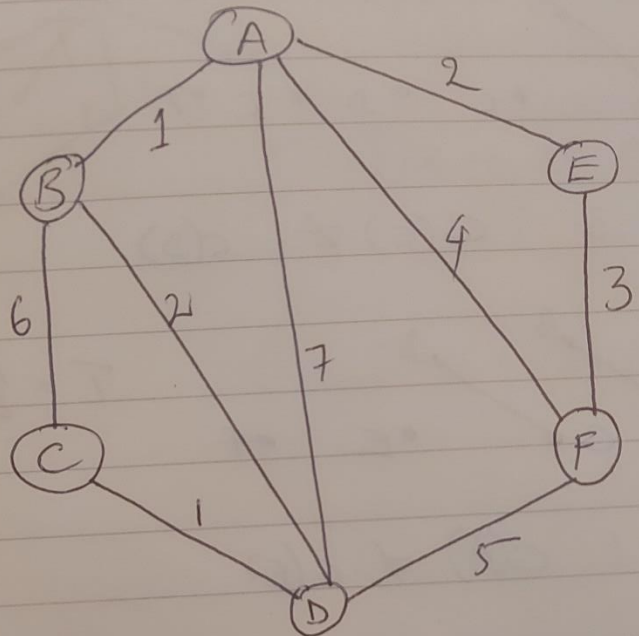


LAB.11.B

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# PROBLEM 1: KRUSKAL'S ALGORITHM



Sorted Edges = { AB, CD, AE, BD, EF, AF, DF, BC, AD }

$T = \{\}$

Step 1: Initialisation

A   B   C   D   E   F

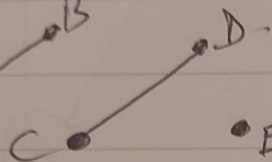
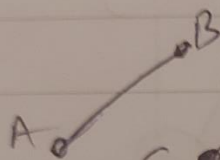
Step 2:  $C(A) \neq C(B)$



• C • D • E • F

$$T = \{AB\}$$

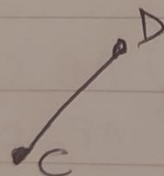
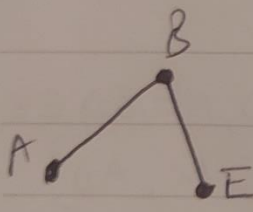
Step 3:  $C(C) \neq C(D)$



• E • F

$$T = \{AB, CD\}$$

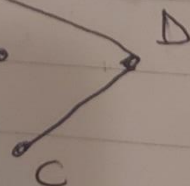
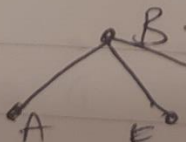
Step 4:  $C(A) \neq C(E)$



• F

$$T = \{AB, CD, AE\}$$

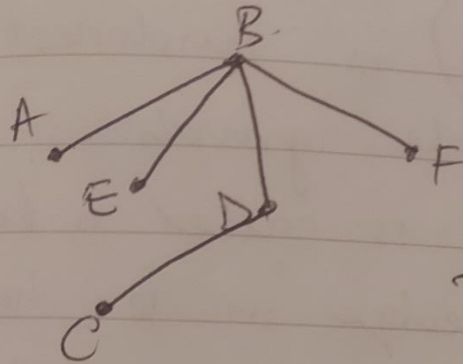
Step 5:  $C(B) \neq C(D)$



• F

$$T = \{AB, CD, AE, BD\}$$

Step 6:  $C(E) \neq C(F)$



$$T = \{AB, CD, AE, BD, EF\}$$

## Problem 2

Suppose  $G = (V, E)$  is undirected (un-weighted) simple graph.

A subset  $U$  of  $V$  is called a base for  $G$  if every edge in  $G$  has at least one endpoint in  $U$ .

a) Given  $G = (V, E)$

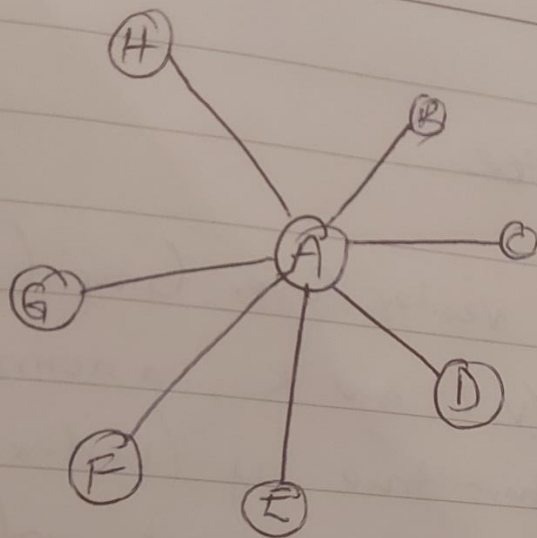
Yes, by definition of  $E$ , every  $e$  in  $E$  has an endpoint in  $V$ .

b) Yes any graph with one or more vertices and no edges is an example of this.

○ ○

○ ○

c)

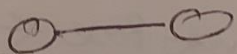
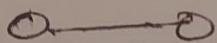


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If  $U = \{A\}$

then every edge in  $G$  has one end point in  $U$ .

d)



If  $n$  of the given edges are in  $U$ , then each ~~edge~~ <sup>edge</sup> in  $G$  has one end point in the graph  $U$ .

~~Q)~~

## E) Brute Force

ALGORITHM vertex Cover Graph( $G, k$ ).  
<sup>Integer</sup>

INPUT  $G=(V, E)$  and  $k$ , a nonnegative

OUTPUT return true if (Vertex Cover) base  
of  $U$  has size  $k$ , otherwise  
false.

$P \leftarrow$  Powerset of  $V$ .

~~$U \leftarrow V$~~  current Min  $\leftarrow |V|$

~~$S \leftarrow V$~~  current Base  $\leftarrow V$

for  $u$  in  $P$  do

for  $e$  in  $E$  do,

$a \leftarrow$  left endpoint of  $e$ .

$b \leftarrow$  right endpoint of  $e$

if  $a$  in  $u$  or  $b$  in  $u$ , then

current Min  $\leftarrow |u|$

current Base  $\leftarrow u$

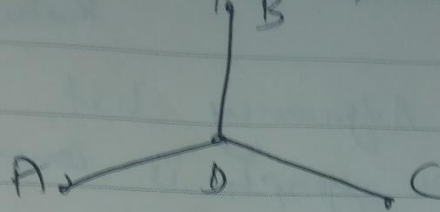
return  $U$

Running time  $O(m2^n)$ .



Problem 3

a) Imagine Graph  $G$  with  $V = \{A, B, C, D\}$



The graph above has no spanning cycle.

b) ALGORITHM spanningCycleDecision( $G(V, E)$ ).

INPUT Graph  $G$  with  $V$  vertices,  $E$  edges

OUTPUT True, if  $G$  contains a spanning cycle, otherwise False.

Perform BFS starting anywhere and keep a counter for number of components by incrementing it after the single component loop if there are still more unvisited vertices in the graph.

If graph has more than one component,  
then it is not connected, then  
Return False.

Get the Adjacency list which is  
formed when graph is constructed.

For Each key  $V$  in the adjacency list, get  
size  $S$  of its list of adjacent vertices

if  $S \neq 2$ , then

Return False.

Return true

---

Running time

BFS  $\rightarrow O(n+m)$ .

checking adjacency List  $\rightarrow O(n)$ .

Total time  $O(n+m)$