

# DSP in VLSI

## HW5

電子所 ICS 組, R13943015, 張根齊

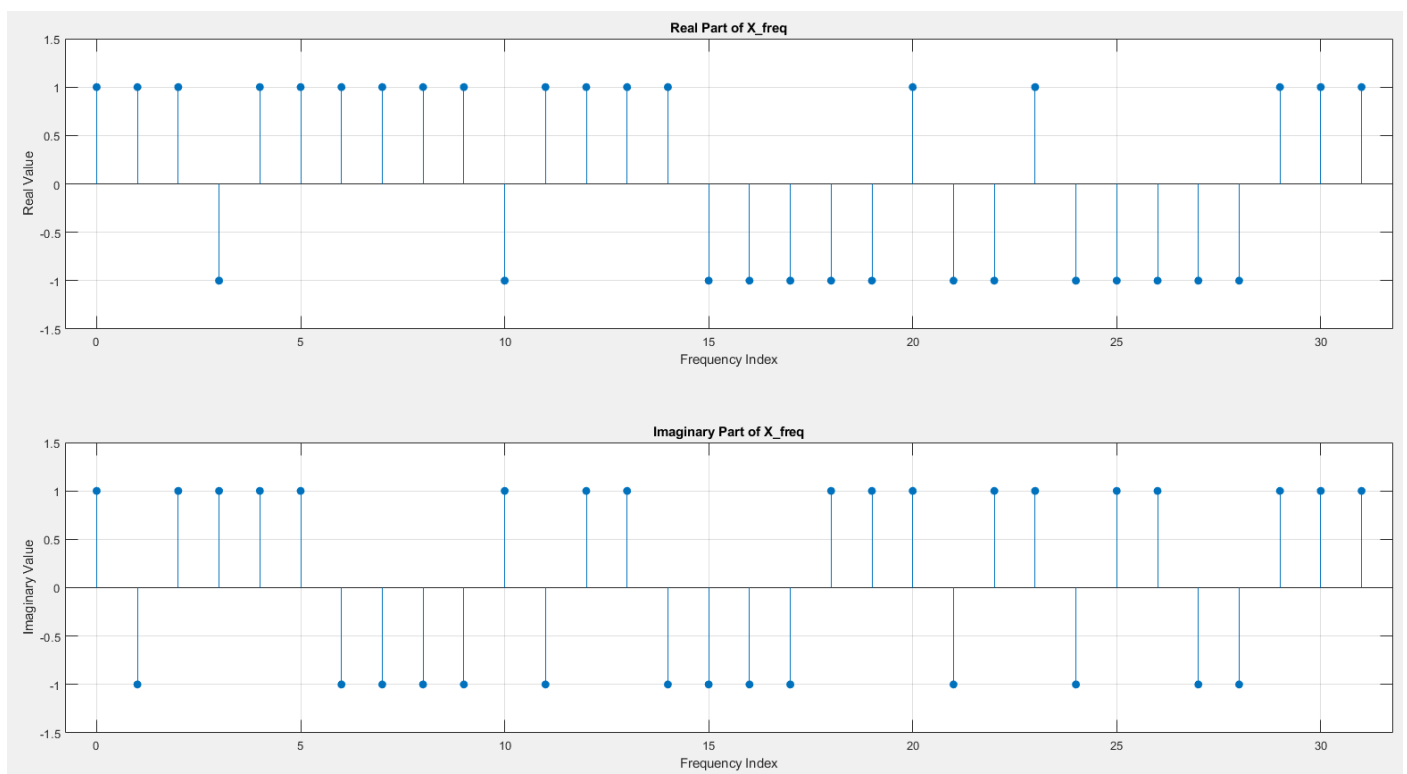
1. (Step 1) Please use FFTInput32.mat as the inputs. Use your MDC FFT Matlab/Python program to generate the 32 FFT outputs. The program outputs should be in a  $2 \times 16$  array. Because they are in bit-reversed order, write a program to generate their associated frequency domain indices. Now, you have an  $4 \times 16$  array containing the frequency domain indices and the MDC outputs. List the array. (Maybe you can copy the results in Work Space and paste onto your report.) (10%)

row 1 and row 2 are 32 FFT outputs (complexed number). And row3, row4 are corresponding frequency domain indices. (integer number)

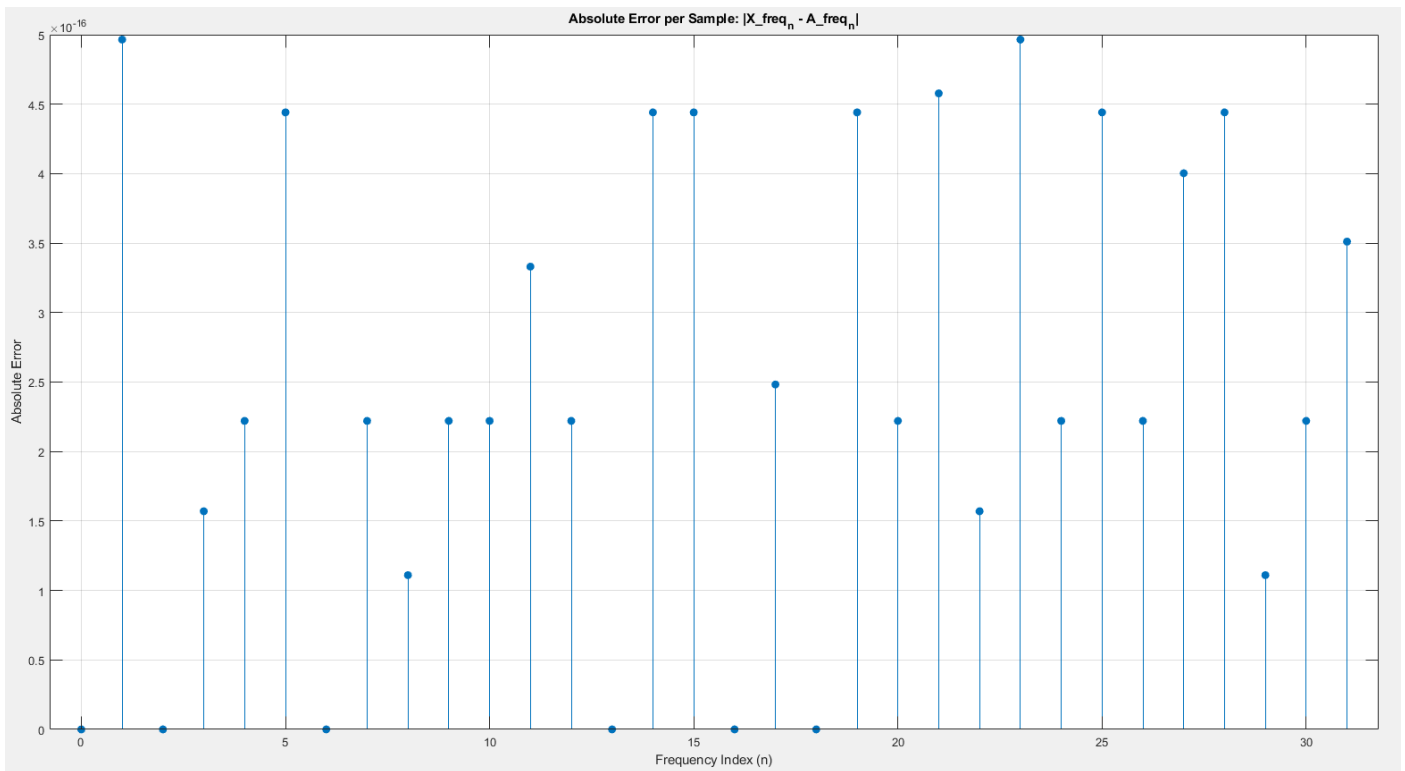
MDC_output_with_freq_indice															
4x16 complex double															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1.0000 + 1.0000i	1.0000 - 1.0000i	1.0000 + 1.0000i	1.0000 + 1.0000i	1.0000 + 1.0000i	-1.0000 + 1.0000i	1.0000 - 1.0000i	1.0000 - 1.0000i	1.0000 - 1.0000i	1.0000 - 1.0000i	1.0000 + 1.0000i	1.0000 + 1.0000i	-1.0000 + 1.0000i	1.0000 - 1.0000i	1.0000 - 1.0000i
2	-1.0000 - 1.0000i	-1.0000 - 1.0000i	1.0000 + 1.0000i	-1.0000 - 1.0000i	-1.0000 + 1.0000i	-1.0000 + 1.0000i	-1.0000 + 1.0000i	1.0000 + 1.0000i	-1.0000 - 1.0000i	-1.0000 + 1.0000i	-1.0000 - 1.0000i	1.0000 + 1.0000i	-1.0000 + 1.0000i	-1.0000 - 1.0000i	1.0000 + 1.0000i
3	0.0000 + 0.0000i	8.0000 + 0.0000i	4.0000 + 0.0000i	12.0000 + 0.0000i	2.0000 + 0.0000i	10.0000 + 0.0000i	6.0000 + 0.0000i	14.0000 + 0.0000i	1.0000 + 0.0000i	9.0000 + 0.0000i	5.0000 + 0.0000i	13.0000 + 0.0000i	3.0000 + 0.0000i	11.0000 + 0.0000i	7.0000 + 0.0000i
4	16.0000 + 0.0000i	24.0000 + 0.0000i	20.0000 + 0.0000i	28.0000 + 0.0000i	18.0000 + 0.0000i	26.0000 + 0.0000i	22.0000 + 0.0000i	30.0000 + 0.0000i	17.0000 + 0.0000i	25.0000 + 0.0000i	21.0000 + 0.0000i	29.0000 + 0.0000i	19.0000 + 0.0000i	27.0000 + 0.0000i	23.0000 + 0.0000i

2. (Step 2) Please use FFTInput32.mat as the inputs. Show that your program for output re-ordering is correct. Draw the real part and imaginary part of  $X_0 \sim X_{31}$ . (10%) Compare the results of  $X_0 \sim X_{31}$  to  $A_0 \sim A_{31}$ . Draw the absolute error of each sample. The error should be small than  $10^{-10}$  because of double-precision floating-point operations. If the error is not acceptable, the first 20 points will be deducted, too. (10%)

2.1 Draw the real part and imaginary part of  $X_0 \sim X_{31}$



2.2 Draw the absolute error of each sample



Average error ( $< 10^{-10}$ ):

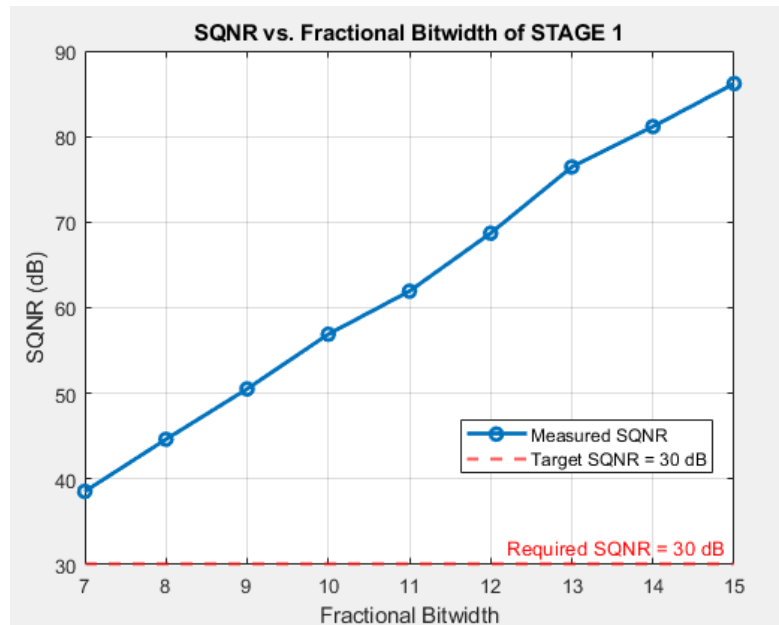
```
>> main
Average Absolute Error: 2.4945e-16
```

3. (Step 3) Generate your own inputs of 96 samples. Draw the SQNR versus fractional part word-length N stage by stage. (30%)

(a) Quantize the first stage. Evaluate the FFT output SQNR versus fractional part word-length N for N=7, 8, 9, 10, ..., 13, 14, 15 for stage 1.

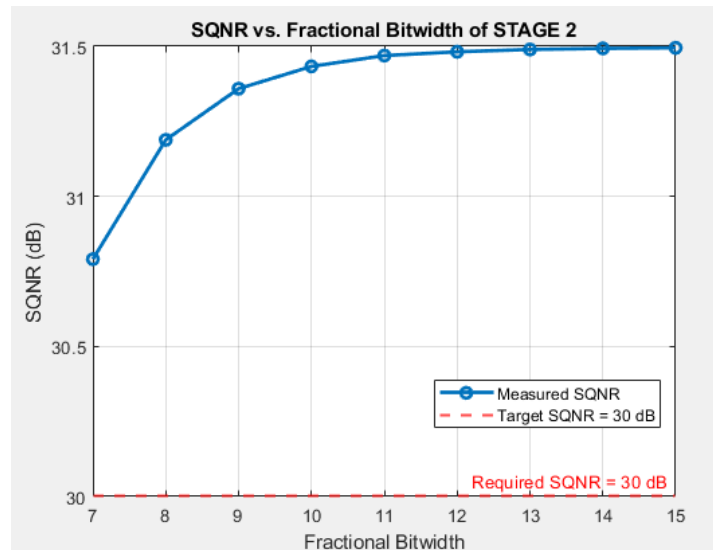
Draw the figure.

Choose stage 1 fractional bit-width = 10 (reserved margin).



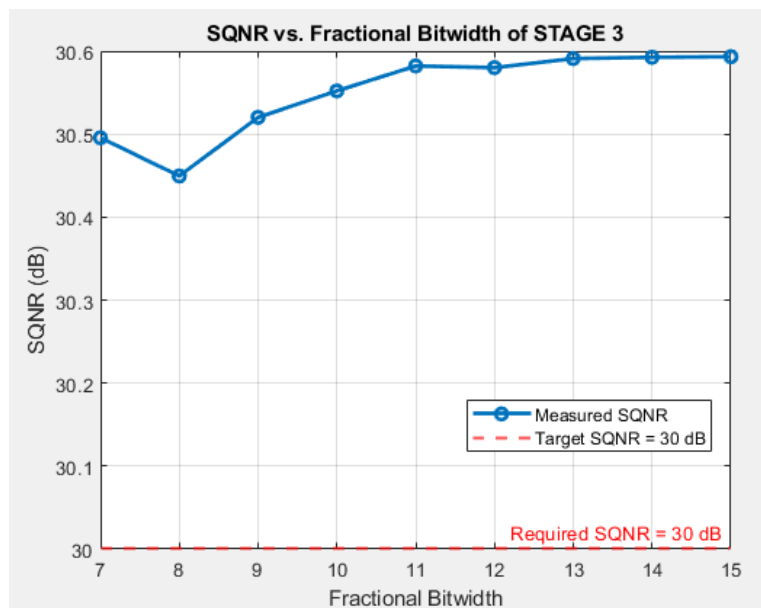
(b) Fix the setting for stage 1. Quantize the second stage. Evaluate the FFT output SQNR versus fractional part word-length  $N$  for  $N=7, 8, 9, 10, \dots, 13, 14, 15$  for stage 2. Draw the figure.

Choose stage 2 fractional bit-width = 10 (reserved margin):



(c) Fix the setting for stage 1 and 2. Quantize the third stage. Evaluate the FFT output SQNR versus fractional part word-length  $N$  for  $N=7, 8, 9, 10, \dots, 13, 14, 15$  for stage 3 and so on. Draw the figure

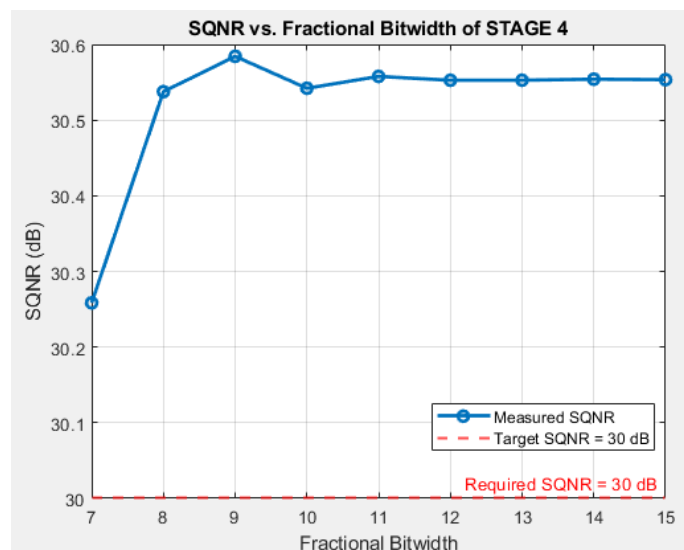
Choose stage 3 fractional bit-width = 10 (reserved margin):



(d) Repeat until stage 5. So, you have five figures.

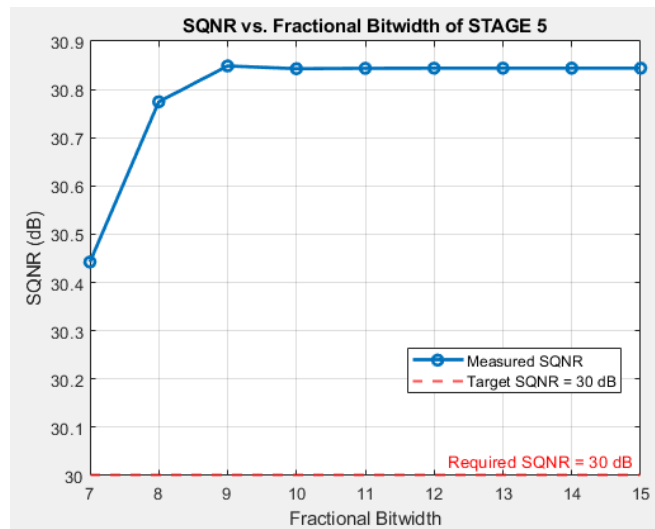
d-1: stage 4:

Choose stage 4 fractional bit-width = 10 (reserved margin):



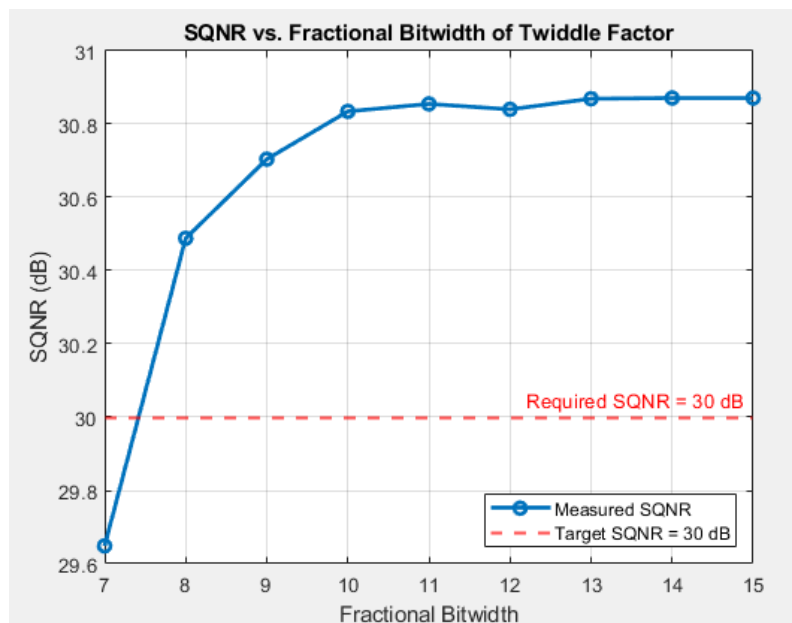
d-2: stage 5:

Choose stage 5 fractional bit-width = 10 (reserved margin):



(e) Then, decide the fractional part word-length for twiddle factors of all the stages. Draw the FFT output SQNR versus fractional part word-length of twiddle factors.

Choose twiddle factors fractional bit-width = 10:



The overall schedule is shown below (The file is included in the submitted assignment package). The yellow sections indicate that the control signal for the commutator should be set to switch mode; otherwise, it should be in bypass mode. The green sections indicate that the control signal for the butterfly should be set to computation mode; otherwise, it should be in bypass mode.

### 4.1. Stage 1

```
Counter = 0 ~ 15: switch mode
else: bypass mode
```

```
Counter = 16 ~ 31: computation mode
else: bypass mode
```

2	counter	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
3	commutator LI	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30	x31
4	commutator UO	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15																
5	commutator LO																	x16	x17	x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30	x31
6	butterfly UI																	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15
7	butterfly LI																	x16	x17	x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30	x31
8	butterfly UO (UI + LI)																	g0	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10	g11	g12	g13	g14	g15
9	butterfly LO (UI - LI)																	h0	h1	h2	h3	h4	h5	h6	h7	h8	h9	h10	h11	h12	h13	h14	h15
10	twiddle (LO = LO . * W)																	w0	w1	w2	w3	w4	w5	w6	w7	w8	w9	w10	w11	w12	w13	w14	w15

## 4.2. Stage 2

### 4.2.1 Commutator

Counter = 24 ~ 31: switch mode

else: bypass mode

### 4.2.2 Butterfly

Counter = 24 ~ 31, 0 ~ 7: computation mode

else: bypass mode

counter	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	0	1	2	3	4	5	6	7
commutator U1																	g0	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10	g11	g12	g13	g14	g15								
commutator L1																									h0	h1	h2	h3	h4	h5	h6	h7	h8	h9	h10	h11	h12	h13	h14	h15
commutator U/O																	g0	g1	g2	g3	g4	g5	g6	g7	h0	h1	h2	h3	h4	h5	h6	h7								
commutator L/O																								g8	g9	g10	g11	g12	g13	g14	g15	h8	h9	h10	h11	h12	h13	h14	h15	
butterfly U1																	g0	g1	g2	g3	g4	g5	g6	g7	h0	h1	h2	h3	h4	h5	h6	h7								
butterfly L1																								g8	g9	g10	g11	g12	g13	g14	g15	h8	h9	h10	h11	h12	h13	h14	h15	
butterfly UO (U1 + L1)																								p0	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12	p13	p14	p15	
butterfly LO (U1 - L1)																								q0	q1	q2	q3	q4	q5	q6	q7	q8	q9	q10	q11	q12	q13	q14	q15	
twiddle (LO = LO * W)																	W0	W2	W4	W6	W8	W10	W12	W14	W0	W2	W4	W6	W8	W10	W12	W14	W0	W2	W4	W6	W8	W10	W12	W14

### 4.3. Stage 3

### 4.3.1 Commutator

Counter = 28 ~ 31, 4 ~ 7: switch mode

else: bypass mode

### 4.3.2 Butterfly

Counter = 28 ~ 31, 0 ~ 11: computation mode

else: bypass mode

21	counter	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	0	1	2	3	4	5	6	7	8	9	10	11	
22	commutator UI																									p0	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12	p13	p14	p15					
23	commutator LI																												q0	q1	q2	q3	q4	q5	q6	q7	q8	q9	q10	q11	q12	q13	q14	q15		
24	commutator UO																									p0	p1	p2	p3	q0	q1	q2	q3	p8	p9	p10	p11	q8	q9	q10	q11					
25	commutator LO																												p4	p5	p6	p7	q4	q5	q6	q7	p12	p13	p14	p15	q12	q13	q14	q15		
26	butterfly UI																												p0	p1	p2	p3	q0	q1	q2	q3	p8	p9	p10	p11	q8	q9	q10	q11		
27	butterfly LI																												p4	p5	p6	p7	q4	q5	q6	q7	p12	p13	p14	p15	q12	q13	q14	q15		
28	butterfly UO (UI + LI)																												m0	m1	m2	m3	m4	m5	m6	m7	m8	m9	m10	m11	m12	m13	m14	m15		
29	butterfly LO (UI - LI)																												w0	w1	w2	w3	w4	w5	w6	w7	w8	w9	w10	w11	w12	w13	w14	w15		
30	twiddle (LO = LO.*W)																												a0	a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11	a12	a13	a14	a15		

#### 4.4. Stage 4

#### 4.4.1 Commutator

Counter = 30 ~ 31, 2 ~ 3, 6 ~ 7, 10 ~ 11: switch mode

else: bypass mode

### 4.4.2 Butterfly

Counter = 30 ~ 31, 0 ~ 13: computation mode

```
else: bypass mode
```

[illegible]



## 4.5. Stage 5

### 4.5.1 Commutator

Counter = 31, 1, 3, 5, 7, 9, 11, 13: switch mode

else: bypass mode

### 4.5.2 Butterfly

Counter = 31, 0 ~ 14: computation mode

else: bypass mode

[illegible]

5. (Step 5) Generate the control signals of complex multipliers for each stage. Show how you generate them and why? (10%)

### 5.1. Stage 1

### 5.1.1 Multipliers

Counter = 16 ~ 31: Multiplication mode

else: bypass mode

2	counter	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
3	commutator LI	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30	x31
4	commutator UO	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15																
5	commutator LO																																
6	butterfly UI																	x16	x17	x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30	x31
7	butterfly LI																	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15
8	butterfly UO (UI + LI)																	x16	x17	x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30	x31
9	butterfly LO (UI - LI)																	g0	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10	g11	g12	g13	g14	g15
10	twiddle (LO = LO.* W)																	h0	h1	h2	h3	h4	h5	h6	h7	h8	h9	h10	h11	h12	h13	h14	h15
																		w0	w1	w2	w3	w4	w5	w6	w7	w8	w9	w10	w11	w12	w13	w14	w15

## 5.2. Stage 2

### 5.2.1 Multipliers

Counter = 24 ~ 31, 0 ~ 7: Multiplication mode

else: bypass mode

[illegible]

### 5.3. Stage 3

### 5.3.1 Multipliers

Counter = 28 ~ 31, 0 ~ 11: Multiplication mode

else: bypass mode

21	counter	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	0	1	2	3	4	5	6	7	8	9	10	11	
22	commutator UI																									p0	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12	p13	p14	p15					
23	commutator LI																												q0	q1	q2	q3	q4	q5	q6	q7	q8	q9	q10	q11	q12	q13	q14	q15		
24	commutator UO																									p0	p1	p2	p3	q0	q1	q2	q3	p8	p9	p10	p11	q8	q9	q10	q11					
25	commutator LO																												p4	p5	p6	p7	q4	q5	q6	q7	p12	p13	p14	p15	q12	q13	q14	q15		
26	butterfly UI																												p0	p1	p2	p3	q0	q1	q2	q3	p8	p9	p10	p11	q8	q9	q10	q11		
27	butterfly LI																												p4	p5	p6	p7	q4	q5	q6	q7	p12	p13	p14	p15	q12	q13	q14	q15		
28	butterfly UO (UI + LI)																												m0	m1	m2	m3	m4	m5	m6	m7	m8	m9	m10	m11	m12	m13	m14	m15		
29	butterfly LO (UI - LI)																												n0	n1	n2	n3	n4	n5	n6	n7	n8	n9	n10	n11	n12	n13	n14	n15		
30	twiddle (LO = LO.*W)																												w0	w1	w2	w3	w4	w5	w6	w7	w8	w9	w10	w11	w12	w13	w14	w15		

## 5.4. Stage 4

### 5.4.1 Multipliers

Counter = 30 ~ 31, 0 ~ 13: Multiplication mode

else: bypass mode

[illegible]

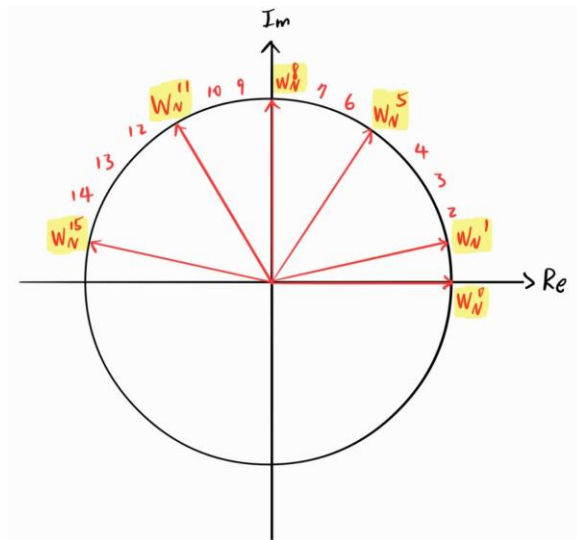
## 4.5. Stage 5

#### 4.5.1 Multipliers: bypass mode (no need to use a multiplier)

1	counter	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
42	commutator UI																																r0	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	
43	commutator LI																																r0	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10	s11	s12	s13	s14	s15	
44	commutator UO																																r0	r2	r2	r4	r4	r6	r6	r8	r8	r10	s10	r12	s12	r14	s14		
45	commutator LO																																r1	s1	r3	s3	s5	r5	r7	s7	r9	s9	r11	s11	r13	s13	r15	s15	
46	butterfly UI																																r0	r0	r2	s2	r4	s4	r6	s6	r8	s8	r10	s10	r12	s12	r14	s14	
47	butterfly LI																																r1	s1	r3	s3	r5	s5	r7	s7	r9	s9	r11	s11	r13	s13	r15	s15	
	butterfly UO (UI + LI)																																																
48	(bit-reverse)																																X0	X8	X4	X12	X2	X10	X6	X14	X1	X9	X5	X13	X3	X11	X7	X15	
	butterfly LO (UI · LI)																																																
49	(bit-reverse)																																	X16	X24	X20	X28	X18	X26	X22	X30	X17	X25	X21	X29	X19	X27	X23	X31

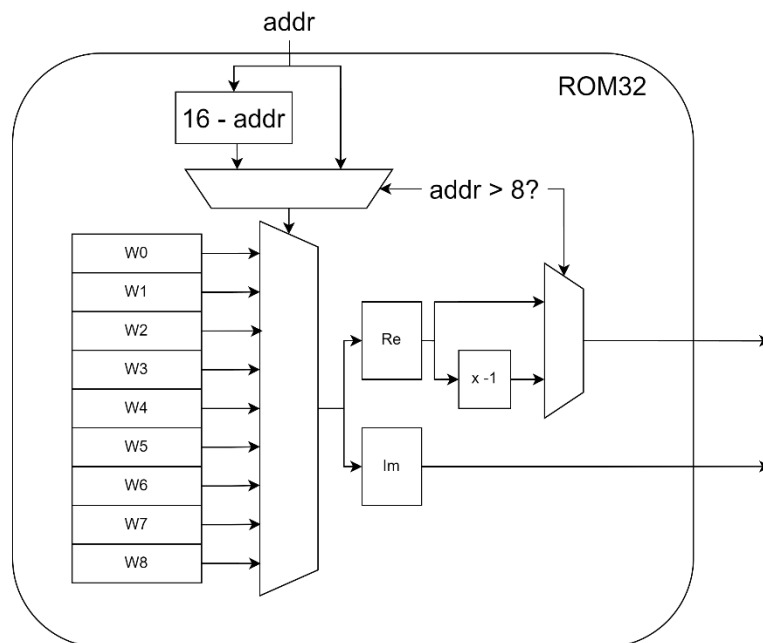
6. (Step 6) Explain the way that you use the sine/cosine values in the first quadrant to generate the required 32 phases for ROM 32. Draw the block diagram (10%)

As shown in the figure below, we can observe that the values in the first quadrant (W1 to W7) are the same as those in the second quadrant (W15 to W9), except for the sign of the real part. Ex:  $Im\{W_N^5\} = Im\{W_N^{11}\}$ ,  $Re\{W_N^5\} = -Re\{W_N^{11}\}$ . So, we only need to save the values in the first quadrant.



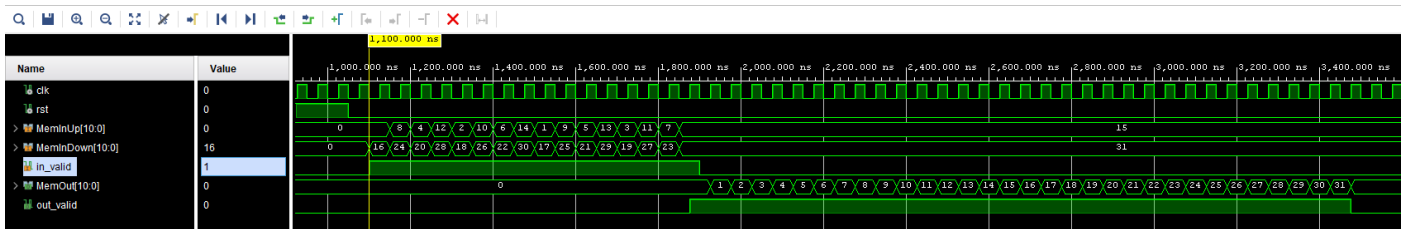
```
>> main
W0 = 1.000000+0.000000j
W1 = 0.980785-0.195090j
W2 = 0.923880-0.382683j
W3 = 0.831470-0.555570j
W4 = 0.707107-0.707107j
W5 = 0.555570-0.831470j
W6 = 0.382683-0.923880j
W7 = 0.195090-0.980785j
W8 = 0.000000-1.000000j
W9 = -0.195090-0.980785j
W10 = -0.382683-0.923880j
W11 = -0.555570-0.831470j
W12 = -0.707107-0.707107j
W13 = -0.831470-0.555570j
W14 = -0.923880-0.382683j
W15 = -0.980785-0.195090j
```

## 6.1. Block diagram



7. (Step 7) Show the timing diagram of behavior simulation for bit-reverse re-ordering with ping-pong accessed bit-reversal buffer as in Fig. 9. (15%)

## 7.1 timing diagram:



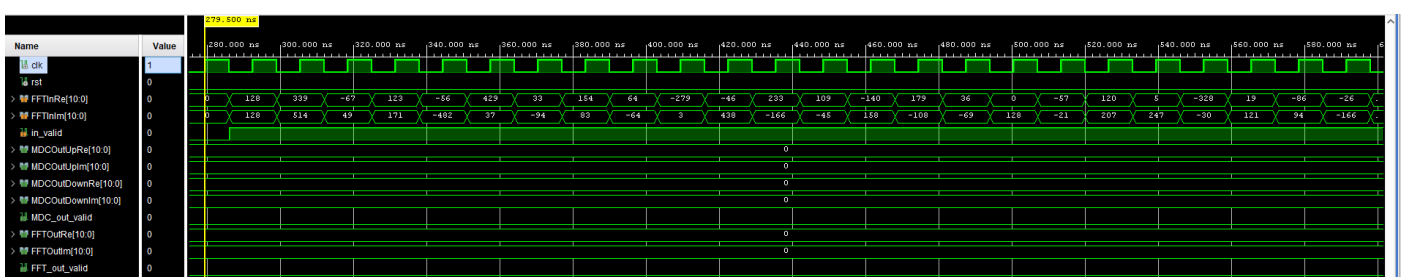
## 7.2 test pattern: MemInUp & MemInDown

INPUT_UP_I.dat	INPUT_DOWN_I.dat
1 00000	1 10000
2 01000	2 11000
3 00100	3 10100
4 01100	4 11100
5 00010	5 10010
6 01010	6 11010
7 00110	7 10110
8 01110	8 11110
9 00001	9 10001
10 01001	10 11001
11 00101	11 10101
12 01101	12 11101
13 00011	13 10011
14 01011	14 11011
15 00111	15 10111
16 01111	16 11111

8. (Step 8) Please use FFTInput32.mat as the inputs. Quantize them using the word-length selected by yourself. Show the timing diagram of behavior simulation for MDC FFT. Compared the results to your answer in Q1(Step 1). Draw the error for 32 samples of real part and imaginary part. (25%)

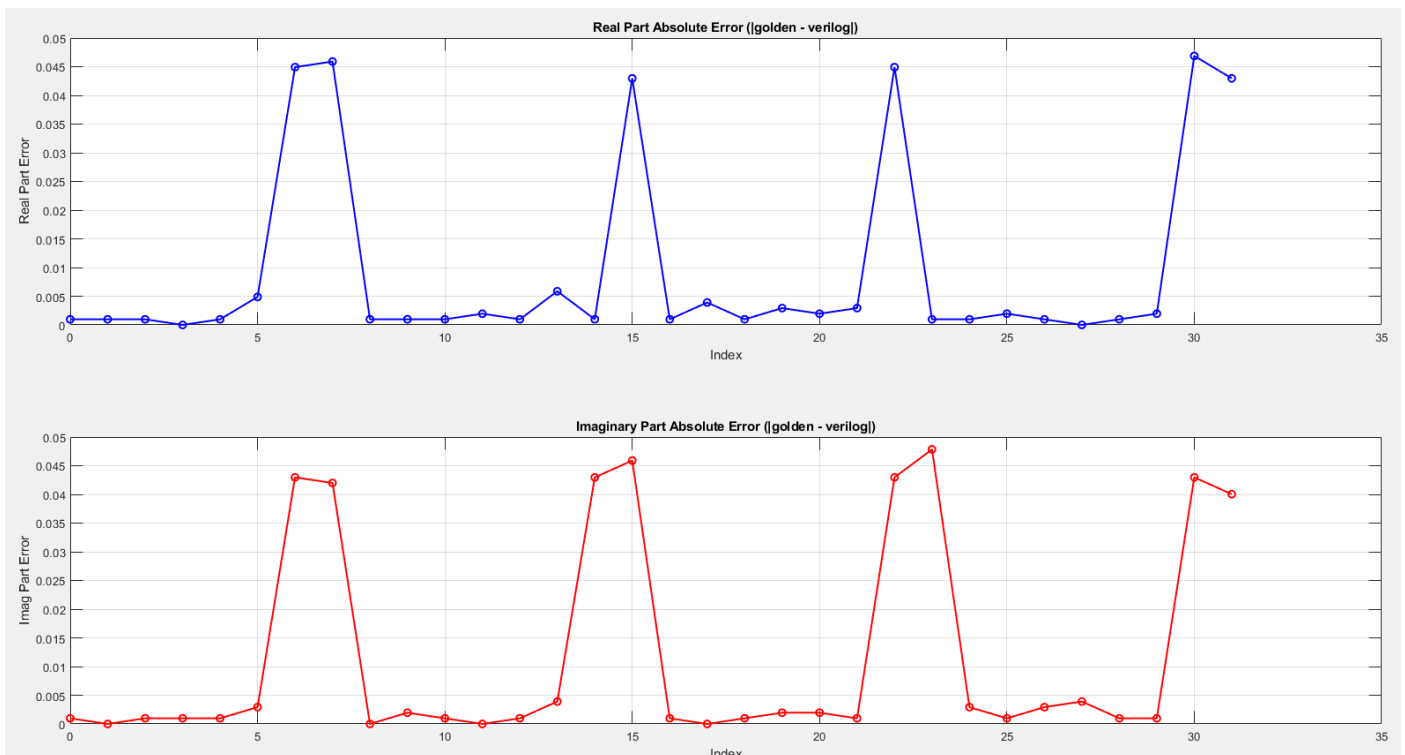
### 8.1. timing diagram of behavior simulation

The values are shown in signed decimal with 10 fractional bit-width, so divided it by 1024 is the real value.





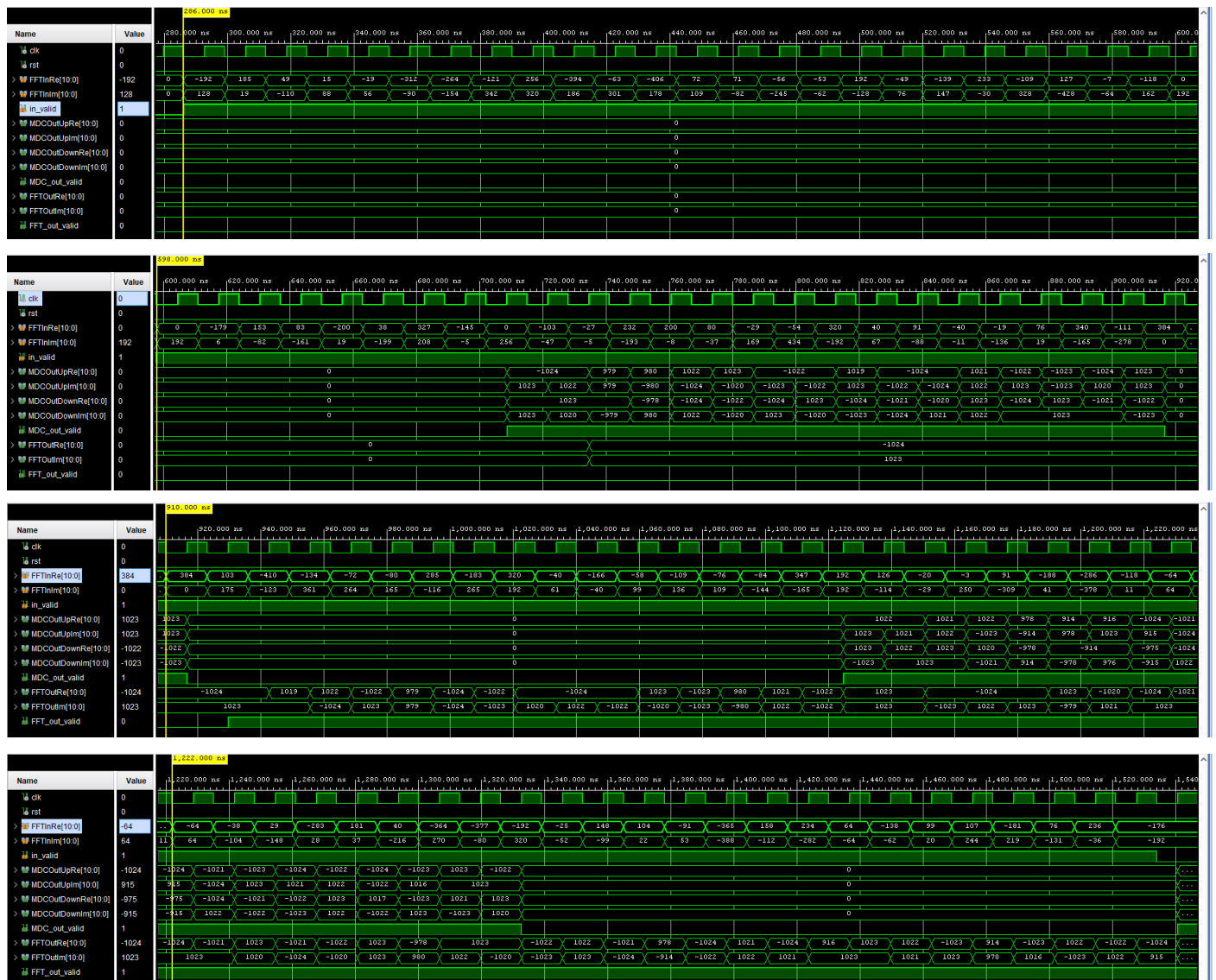
## 8.2. The error for 32 samples of real part and imaginary part

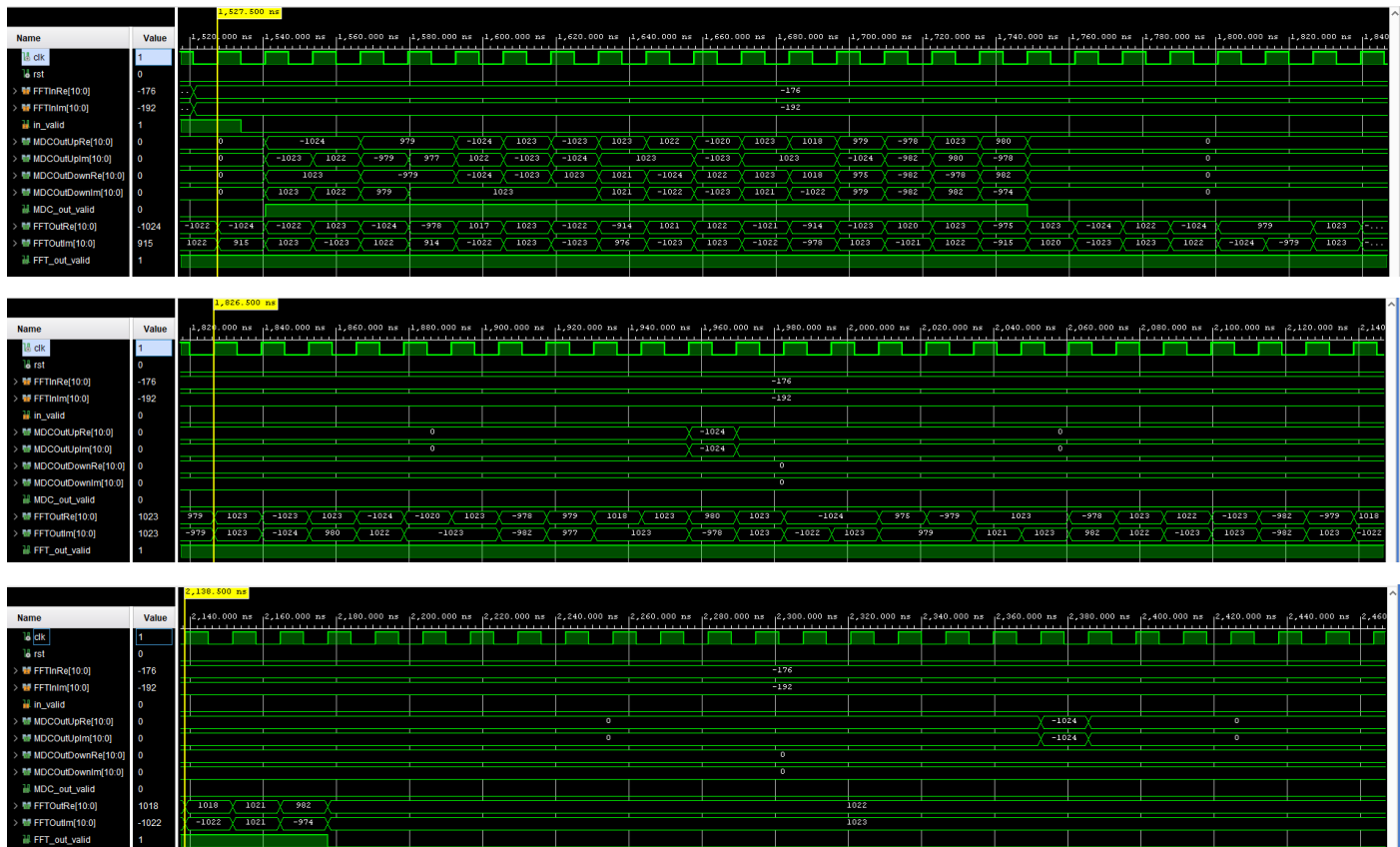


9. (Step 9) Use your own inputs of 96 samples in Q3 (Step3). Show the timing diagram of behavior simulation with streaming-input and streaming-output. Draw the error for 96 samples of real part and imaginary part. (25%) Calculate the SQNR of 96 samples.

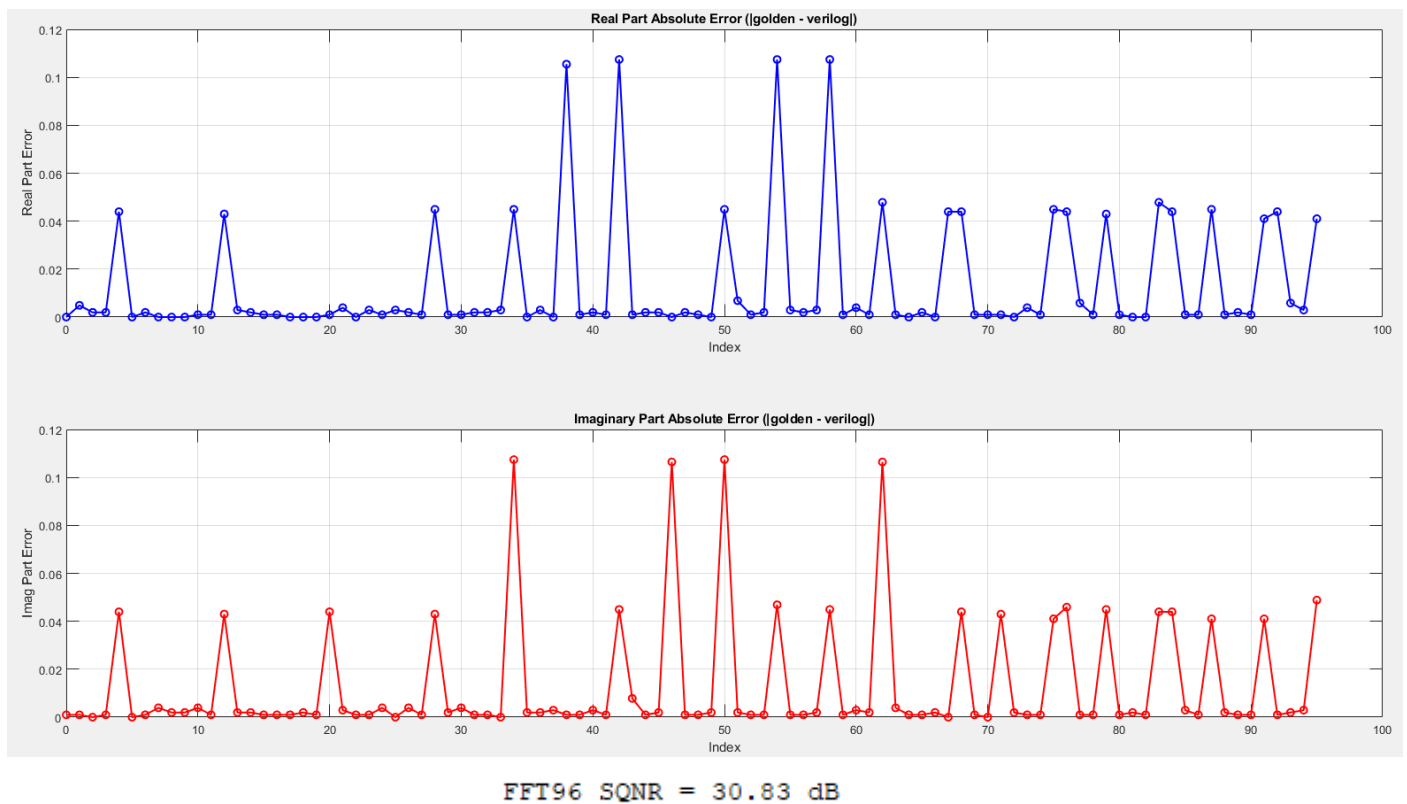
### 9.1 timing diagram of behavior simulation

The values are shown in signed decimal with 10 fractional bit-width, so divided it by 1024 is the real value.





## 9.2 the error for 96 samples of real part and imaginary part and SQNR



FFT96 SQNR = 30.83 dB

10. (Step 10) Insert D flip-flop at the input and output.  
Synthesize your design. Provide the report of max delay.  
Note that if you did not insert internal pipeline registers,  
the critical path is long and the operating frequency is not  
high. (10%)

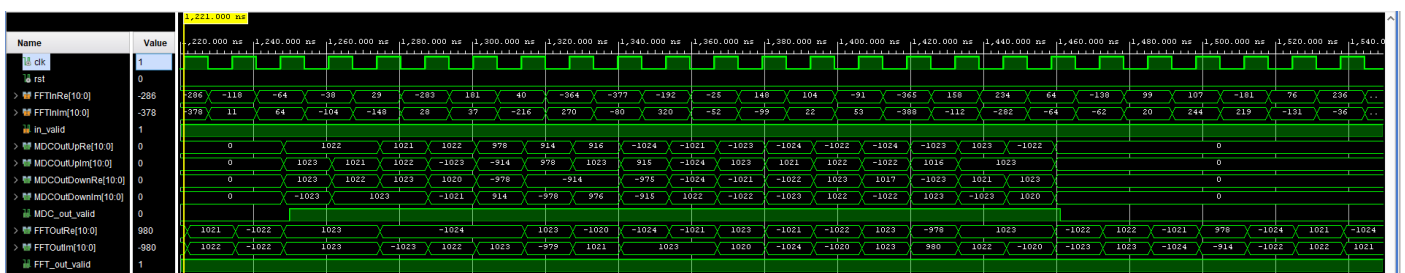
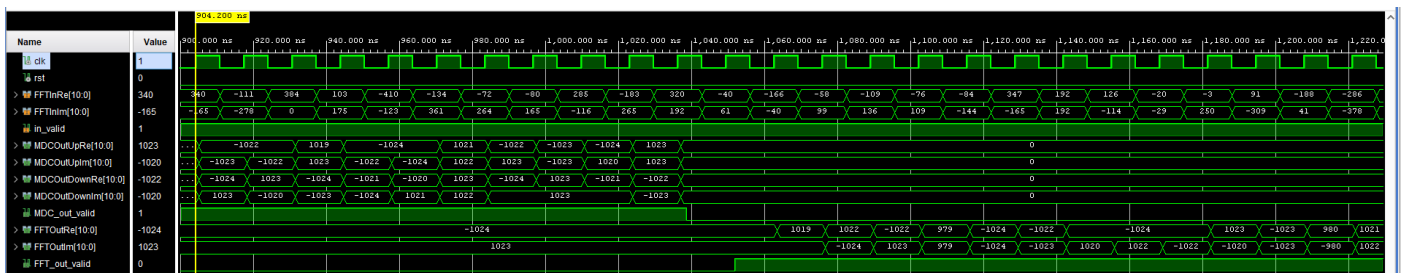
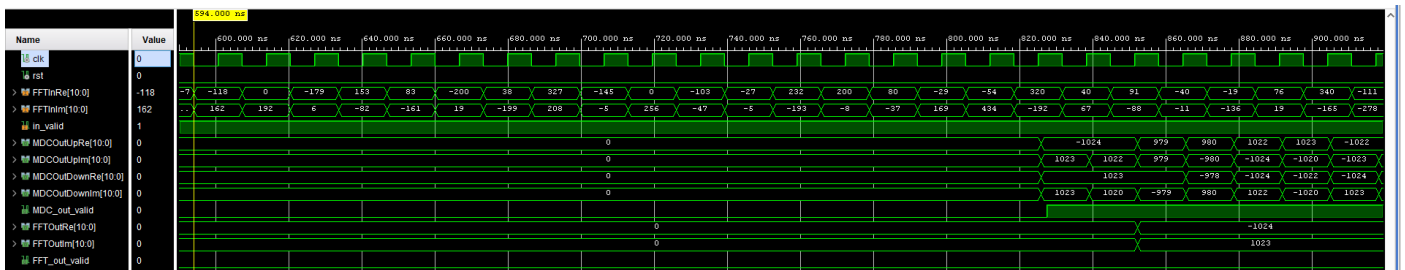
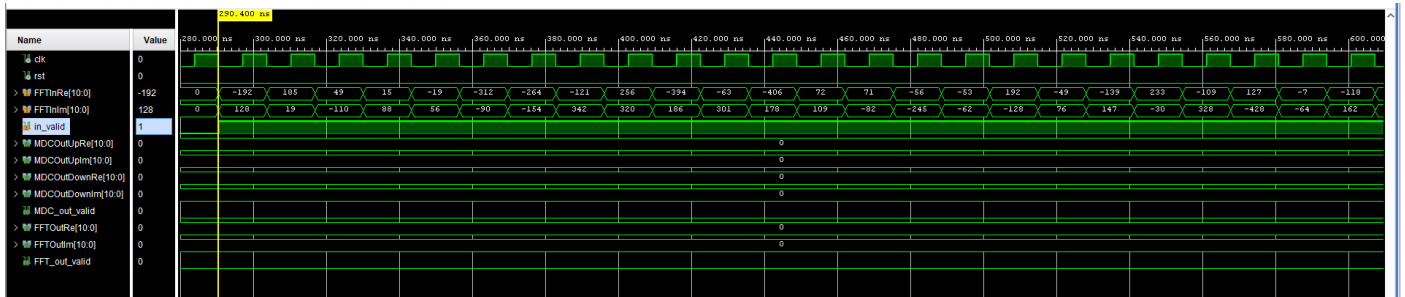
max delay = 19.866ns (operating frequency = 50.3MHz)

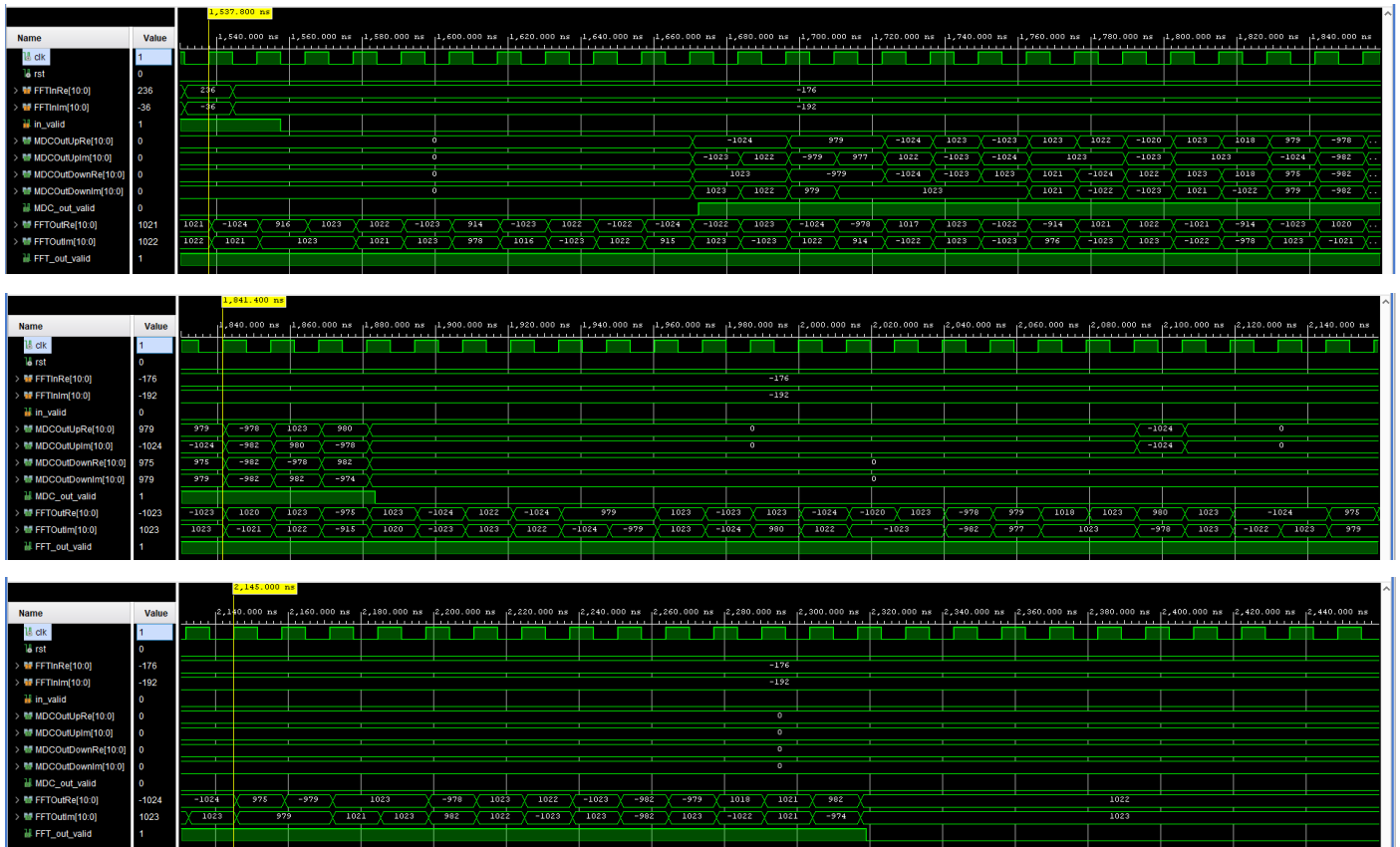
Summary	
Name	Path 1
Slack	1.982ns
Source	delay_element_U_stage1/DFF_re_r_reg[15][1]/C (rising edge-triggered cell FDRE clocked by clk {rise@0.000ns fall@11.000ns period=22.000ns})
Destination	delay_element_L_stage5/DFF_re_r_reg[0][10]/D (rising edge-triggered cell FDRE clocked by clk {rise@0.000ns fall@11.000ns period=22.000ns})
Path Group	clk
Path Type	Setup (Max at Slow Process Corner)
Requirement	22.000ns (clk rise@22.000ns - clk rise@0.000ns)
Data Path Delay	19.866ns (logic 12.661ns (63.731%) route 7.205ns (36.269%))
Logic Levels	24 (CARRY4=16 DSP48E1=2 LUT3=3 LUT6=3)
Clock Path Skew	-0.145ns
Clock Un...rtainty	0.035ns



11. (Step 11) Insert pipeline registers to accelerate your design. For FPGA flow, the target operating clock frequency ( $f_s$ ) is 75MHz. For cell-based design flow, the target operating frequency ( $f_s$ ) is 130MHz. Show the timing diagram of post-synthesis simulation results with proper clock period settings ( $1/f_s$ ). Draw the error for 96 samples of real part and imaginary part. (25%)

11.1 Clk period = 13.2 ns (operating frequency = 75.75MHz):





## 11.2 the error for 96 samples of real part and imaginary part

