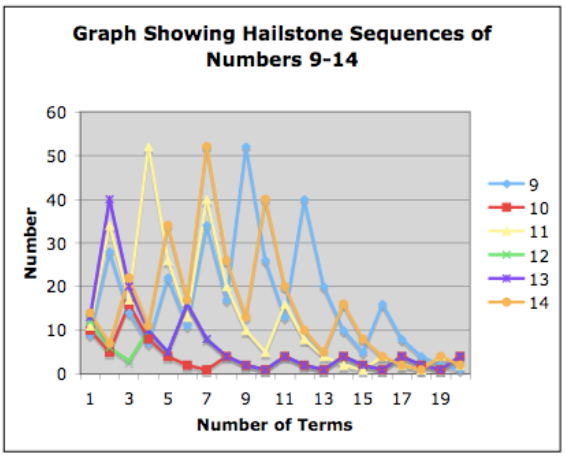
Hailstone numbers



Prerequisites

You need to understand the basics of 4bit Adders (7483) particularly with respect to the carry in (Ci) and carry out (Co). Also, we will be using 74157 Quadruple Multiplexers that have four 2-1 devices. The LogicWorks diagrams of each device is shown below.

Objectives

The objective of this laboratory assignment is to design and build and test circuitry that will perform the operations to generate a Hailstone number sequence. It is limited to 5 bits of input. Your constraint is that you only have 4 chips: two SN7483s and two SN74157s to complete the design.

Theory

Hailstone numbers are so called "because they go up and down just like a hailstone in a cloud before crashing to Earth." They are sequences of numbers that are obtained with the following rules.

1. ***If the current number is even, divide it by two; else if odd, multiply it by three and add one***
2. ***Repeat.***

For example, starting with n=7 gives the following
7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, ...

The (unproven) Collatz conjecture is that the Hailstone sequence for any starting number always terminates in 4, 2, 1 – see example above..

Procedure – 1st part

We will work through this design in steps. Firstly, we will test a simple 4-bit circuit that will multiply by 3 and add one. We have already done some multiplication using combinational circuits so think of it as adding a binary number to a shifted left version and add 1. Viz.

DCBA + DCBA0 + 1

or

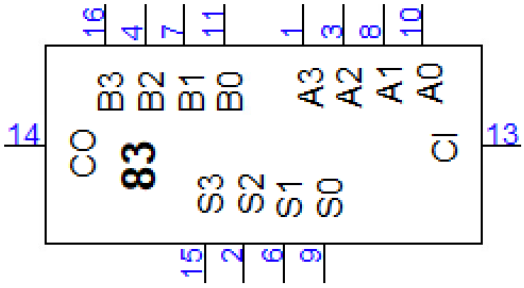
```
  DBCA
DCBA
    1
```

If DCBA were 3_{10} = 0011 then the addition would be

```
  0011
0011
    1
01010
```

hence (decimal) 3x3+1=10

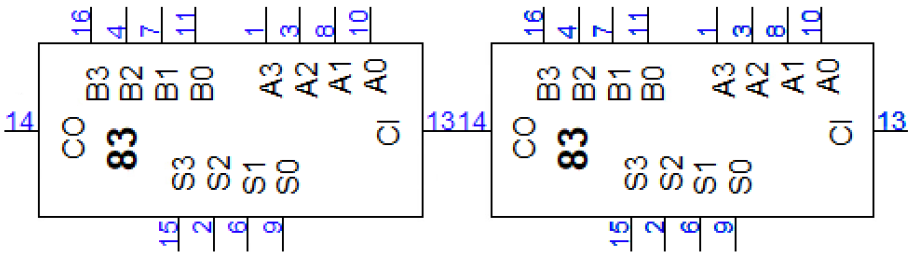
Implementation



Set up a table as below and test your circuit for the values of DCBA from 1 through 7 (0001 – 0111). Ci will be a 1 to complete the 3xN+1 term.

Input B				Input A				Carry in	Output S				Carry out
B3	B2	B1	B0	A3	A2	A1	A0	Ci	S3	S2	S1	S0	C0
0	0	1	0	0	0	0	1	1	0	1	0	0	0
0	1	0	0	0	0	1	0	1	0	1	1	1	0
0	1	1	0	0	0	1	1	1					
1	0	0	0	0	1	0	0	1					
1	0	1	0	0	1	0	1	1					
1	1	0	0	0	1	1	0	1					
1	1	1	0	0	1	1	1	1					

To increase the range of numbers (up to EDCBA) to be explored we need to add another 7483.



The right hand 83 will have the values as below (already tested)

B3B2B1B0 is CBA0, A3A2A1A0 is DCBA, Ci =1

The left hand 83 will have the (extended) values

B3B2B1B0 is 00ED, A3A2A1A0 is 000E, Ci =Co (from right hand 83)

Testing

You should test your circuit for EDCBA from 00000 to 11111 but not exhaustively! Choose a set of values at, and between, the extremes to convince yourself that the circuit is working correctly. Complete a table of your values.

Procedure – 2nd part

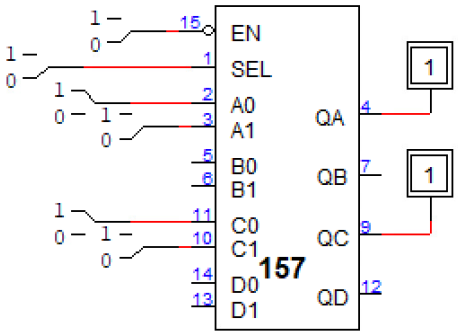
You have created a circuit that gives you the seven 3N+1 output bits. These are available for all values of the 5-bit input. We only need them for **odd** numbers, that is, where A=1.

To be able to get the Hailstone numbers for **even** number inputs we need to consider 5-bit inputs where A=0. Hence the input EDCBA has to result in an output of 0EDCB. You already have these values available.

So, if we apply the 3N+1 outputs to the MUX (pins A1B1C1D1) and select **SEL=1** the outputs QAQBQCQD will equal the **lower 4 bits** of the 3N+1 values.

If we apply the EDCB values to the MUX (pins A0B0C0D0) and select **SEL=0** the outputs QAQBQCQD will equal the **lower 4 bits** of the divided by two 5-bit input.

Hence we can switch between the correct outputs of an input **odd** number and an input **even** number. The diagram below indicates the 157 function.

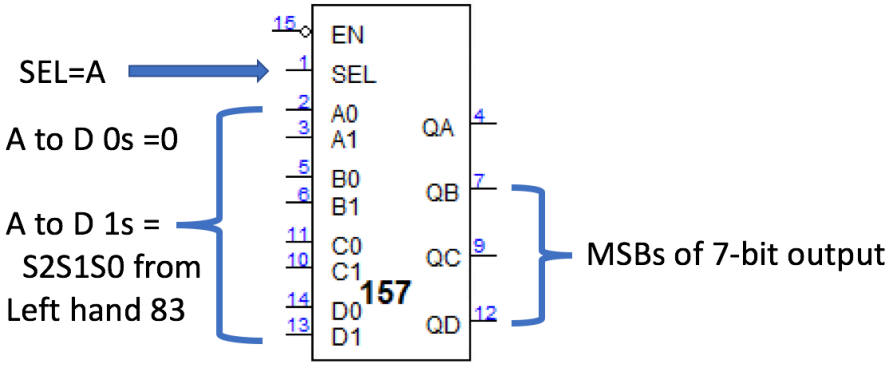


To complete the whole circuitry we need another MUX to provide the alternative three MSB bits. For example,

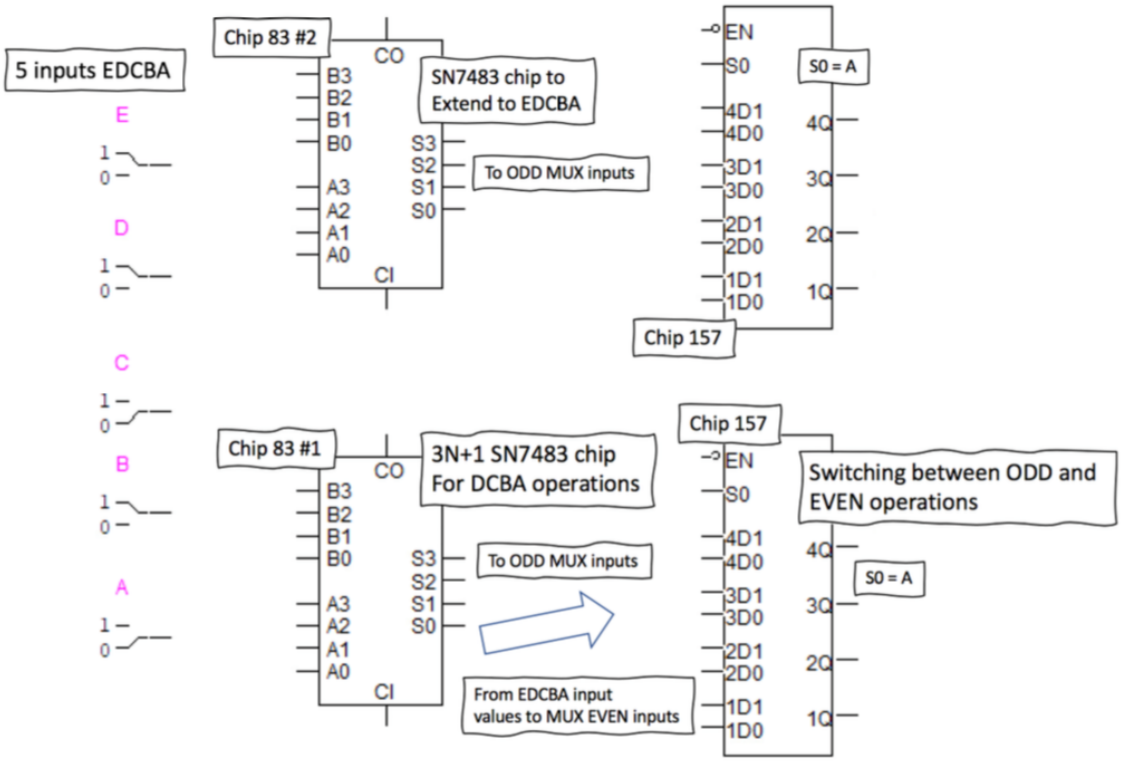
If EDCBA = 11011, the output will be **101**0010 SEL=1

If EDCBA = 11010, the output will be **000**1101 SEL=0

Hence the second MUX will use the SEL inputs to choose the appropriate MSB digits.



Schematic diagram



Question

What circuit/s would you add to your design to enable the Hailstone sequence to be continuously generated?

Deliverables

Your circuit must work correctly and your documentation should agree with your circuit outputs. A report is required for this lab. Your report should contain the theory and test results.

Reference

https://en.wikipedia.org/wiki/Collatz_conjecture

Implemented concept by Keelin Becker-Wheeler