The Irrationality of $\sqrt{2}$

Benny Chen

September 2, 2022

1 Introduction

In this short paper we will show that $\sqrt{2}$ is an irrational number. In Section 2, we will introduce the concepts of rational and irrational numbers. In Section 3, we will provide a proof of the fact that $\sqrt{2}$ is irrational. Finally, in Section 4, we will discuss what we know about other numbers that are known to be irrational.

2 Preliminaries

First, we define the rational numbers.

Definition 1. *Let $\mathbb{Q}$ be all rational numbers. A rational number is any real number that can be written as a ratio such as a quotient or a fraction of two integers.*

And now we can define the cocept of irrationality.

Definition 2. *A rational number is any real number that is not a quotient or a fraction of two integers. The number would be infinitely continuous decimal without repeating numbers.*

3 The Proof

We are finally ready to show our main result

Theorem 1. *The real number $\sqrt{2}$ is irrational*

Proof. *Assume that $\alpha = \sqrt{2}$ and $\alpha^2 = 2$. Now suppose that $\alpha = \frac{m}{n}$ where $m, n \in \mathbb{Z}$ and are in their lowest terms. Since m and n are in their lowest terms, one or both of m and n must be odd. We can now rearrange $(\frac{m}{n})^2 = 2$ so that m can take the form of an even integer, $a = 2k$. The same logic can be applied for n so it must be even. Since we have shown that m and n are both even, $\frac{m}{n}$ is not in its lowest terms. This is a contradiction to our assumption and therefore α must be irrational.*

4 Irrational Number

There are many example of numbers that deviate from being rational, making them irrational. More famous examples are π and Euler's Number.

A good example of a number is π. It is one of the most famous and well known numbers of all time. π is the ratio of the circumference of any circle to the diameter of that circle. π was first realized by Greek mathematician Archimedes of Syracuse around 250 B.C who created a algorithm to approximate the number called Archimedes' constant. The number approximated by Archimedes, π, is also irrational as the decimal continuously goes to infinity. There have been many proofs done on the subject that show that π is irrational from Lambert's proof to Hermite's proof.

Euler's Number is also a famous irrational number. Euler's number is a important mathematical constant that is the base of the natural logarithm. Euler's number was first discovered by Jacob Bernoulli in the late 1600's but was not known until the 17th century which when it was named after Swiss mathematician Leonhard Euler. Euler's number is similar to π in that it is irrational and has a continuous decimal expansion. There have been many proofs done that support that it is irrational like Euler's proof and Fourier's proof.

References

[1] Britannica, The Editors of Encyclopaedia. "pi". Encyclopedia Britannica, 14 Mar. 2022, https://www.britannica.com/science/pi-mathematics.

[2] https://mathshistory.st-andrews.ac.uk/HistTopics/e/