# ACSL
# American Computer Science League

**2014 - 2015** **Contest #4**

Quine-McCluskyAlgorithm
Intermediate Division

PROBLEM: Use a part of the Quine-McClusky Algorithm method to simplify Boolean functions.

As an example $\mathbf{ABC\overline{D} + ABCD}$ is simplified by using DeMorgan's Theorem as follows
$\mathbf{ABC(\overline{D} + D)} = \mathbf{ABC(1)} = \mathbf{ABC}$.

If instead we are given which of the 16 possible ordered binary quadruples make the expression true (1110 and 1111 which are 14 and 15 in decimal) and we also note that they only differ in one place value. The two quadruples can be combined and one digit can be eliminated.

$$\begin{array}{c} 1\,1\,1\,0 \\ \underline{1\,1\,1\,1} \\ 1\,1\,1\,x \end{array}$$

Converting 111x to its Boolean expression representation gives ABC as above.

The above can be expressed mathematically as $f(\text{A,B,C,D}) = \sum m(14, 15) = \sum m(1110, 1111) = \text{ABC}$.

$f(\text{A,B,C,D}) = \sum m(4, 8, 9, 10, 11, 12, 14, 15)$ shows where the function evaluates to 1. That is shown in the $f$ column in the chart on the left. The chart on the right groups those binary representations by the number of 1's (index) in the binary representation. Combining takes place with values that have an index of n and n + 1.

| # | A | B | C | D | $f$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 1 |
| 5 | 0 | 1 | 0 | 1 | 0 |
| 6 | 0 | 1 | 1 | 0 | 0 |
| 7 | 0 | 1 | 1 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 1 |
| 9 | 1 | 0 | 0 | 1 | 1 |
| 10 | 1 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 | 1 |
| 12 | 1 | 1 | 0 | 0 | 1 |
| 13 | 1 | 1 | 0 | 1 | 0 |
| 14 | 1 | 1 | 1 | 0 | 1 |
| 15 | 1 | 1 | 1 | 1 | 1 |

| Index | Term number | Binary | Simplified pairs |
|---|---|---|---|
| 1 | 4<br>8 | 0100<br>1000 | $m$(4,12) = x100<br>$m$(8, 9) = 100x<br>$m$(8,10) = 10x0<br>$m$(8,12) = 1x00 |
| 2 | 9<br>10<br>12 | 1001<br>1010<br>1100 | $m$(9,11) = 10x1<br>$m$(10,11) = 101x<br>$m$(10,14) = 1x10<br>$m$(12, 14) = 11x0 |
| 3 | 11<br>14 | 1011<br>1110 | $m$(11,15) = 1x11<br>$m$(14,15) = 111x |
| 4 | 15 | 1111 | |

Note that $m$(4,12) can be combined because 4 has an index of 1 and 12 has an index of 2. In addition they differ by one digit in the same place.

$$\begin{array}{r} 0100 \\ \underline{1100} \\ x100 = Bcd \end{array}$$

Note that $m(4,9)$ can't be combined because they differ by 3 digits. Also $m(4,0)$ can't be combined because the 0 does not make the function true.

The process of combining continues by trying to combine 2 of the simplified pair values. Combining again takes place with values that have an index of n and n + 1. The pairs selected must be the same except for one digit and the x must be at the same place value. Lower case characters will be used to show negation.

$$\begin{array}{rl} m(10,11) = & 101x \\ m(8,9) = & \underline{100x} \\ m(8,9,10,11) = & 10xx = Ab \end{array}$$

| Index | Term number | Binary | Simplified pairs | Extended simplification |
|---|---|---|---|---|
| 1 | 4<br>8 | 0100<br>1000 | $m(4,12)$ = x100<br>$m(8, 9)$ = 100x<br>$m(8,10)$ = 10x0<br>$m(8,12)$ = 1x00 | $m(8,9,10,11)$ = 10xx<br>$m(8,10,12,14)$ = 1xx0 |
| 2 | 9<br>10<br>12 | 1001<br>1010<br>1100 | $m(9,11)$ = 10x1<br>$m(10,11)$ = 101x<br>$m(10,14)$ = 1x10<br>$m(12, 14)$ = 11x0 | $m(10,11,14, 15)$ = 1x1x |
| 3 | 11<br>14 | 1011<br>1110 | $m(11,15)$ = 1x11<br>$m(14,15)$ = 111x | |
| 4 | 15 | 1111 | | |

INPUT: There will be 6 lines of input. The first line will contain a listing of the term numbers of the function. That list will end with a -1. The next 2 lines will contain two term numbers that make the function true (0 to 15). The last 3 lines will contain 4 term numbers.

OUTPUT: Combine the binary representations, if possible, according to the rules above and print the result of the 4-character string with the deleted bit(s) represented by an "x" and then in Boolean expression form. Lower case characters will be used to show negation. Both outputs must be correct to be awarded the point. If the two terms can't be combined, print NONE.

SAMPLE INPUT

1. 4, 8, 9, 10, 11, 12, 14, 15, -1
2 4, 12
3. 9, 11
4. 8, 9, 10, 11
5. 8, 10, 12, 14
6. 10,11,14, 15

SAMPLE OUTPUT

1. x100, Bcd
2. 10x1, AbD
3. 10xx, Ab
4. 1xx0, Ad
5. 1x1x, AC

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TEST DATA

| TEST INPUT | TEST OUTPUT |
|---|---|
| 1. 1, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, -1 | 1. 10x0, Abd |
| 2. 8, 10 | 2. 101x, AbC |
| 3. 10, 11 | 3. x0x1, bD |
| 4. 1, 3, 9, 11 | 4. 10xx, Ab |
| 5. 8, 10, 9, 11 | 5. xx11, CD |
| 6. 3, 11, 7, 15 | |