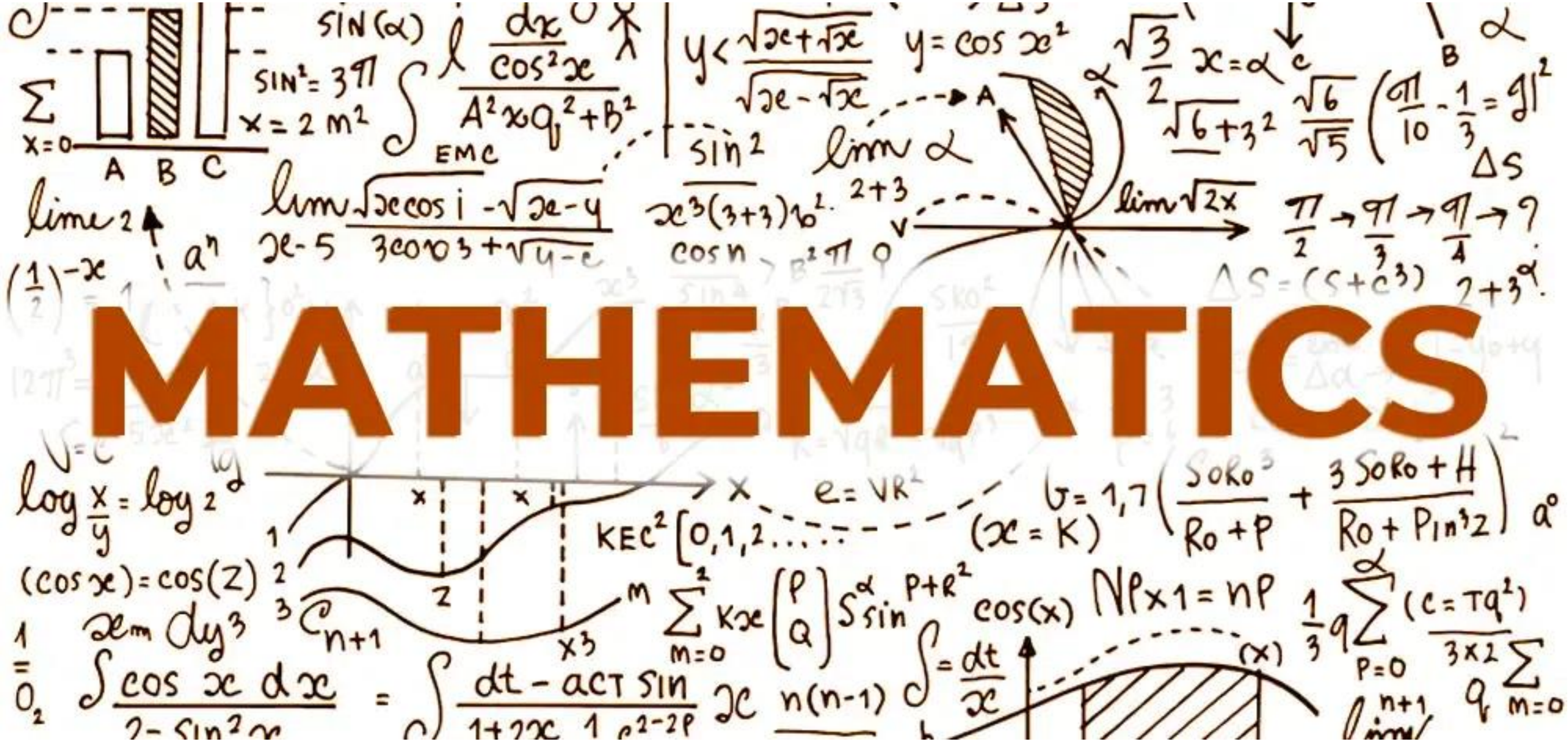
MATHEMATICS

Last lecture reminder

We learned about:

- Introduction to linear algebra
- Introduction to linear equations
- The slope-intercept form of linear equation
- Introduction to linear systems
- Solving linear systems with 2 unknowns
- Introduction to matrix and represent linear systems as matrix

Matrix - Basic Operations

Once we understand that we can represent multiple linear equations as matrix we now can learn how to apply basic operators on those matrices.

Matrix Addition/Subtraction → Two matrices can be added or subtracted if and only if their dimensions are the same. Meaning they must have the same number of rows and the same number of columns. The resulting matrix's element in the i-th row and j-th column is the sum/difference of the elements in the i-th row and j-th column of the two given matrices.

For example → Let’s say we want to add the following 2 matrices:

A → [1 3]   B → [5, 7]

      [2 4]   [6, 8]

Matrix - Basic Operations

Now let's perform the addition between the two matrix, the result matrix will be with the same dimensions as the original matrics.

Matrix A: Matrix B: Matrix A + B:

[1 3] [5 7] [(1+5) (3+7)] [6 10]

[2 4] + [6 8] = [(2+6) (4+8)] = [8 12]

So the result matrix of A + B will be → [6 10]

[8 12]

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Matrix - Basic Operations

Now let's say how we perform subtraction between those 2 matrices:

Matrix A: Matrix B: Matrix A - B:

[1 3]   [5 7]   [(1-5) (3-7)]   [-4 -4]

[2 4]   - [6 8]   = [(2-6) (4-8)] = [-4 -4]

So the result matrix of A - B will be → [-4 -4]

[-4 -4]

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Matrix - Basic Operations

Scalar → In mathematics, a scalar is a simple, single numerical quantity. Scalars are often used in linear algebra, physics, engineering, computer science, and many other fields. They can either be a real number or a complex number, which includes an angle as well as magnitude (but in everyday scenarios, they are often real numbers).

Scalar Multiplication → A matrix can be multiplied by a scalar (a real or complex number). This is performed by multiplying each element of the matrix by the scalar.

For example → Let's multiply the matrix A with the scalar 3:

Matrix A:

[2 3]

[4 5] X 3

Matrix - Basic Operations

The scalar multiplication of the matrix A by the scalar is performed by multiplying every element of the matrix A by the scalar.

So, the resulting matrix B (B = scalar * A) will be as follows:

Matrix B:

[3*2 3*3]   [6 9]

[3*4 3*5] = [12 15]

So, the result of scalar multiplication of matrix A by 3 is another matrix:

[6 9]

[12 15]

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Matrix - Basic Operations

Matrix Multiplication → Two matrices can be multiplied if the number of columns in the first matrix is equal to the number of rows in the second matrix. The resulting matrix has a size that is equal to the number of rows of the first matrix by the number of columns of the second matrix. The element in the i-th row and j-th column of the resulting matrix is the sum of the products of elements in the i-th row of the first matrix and the j-th column of the second matrix.

Let’s first understand what is the result of matrix multiplication regarding the matrices dimensions:

Let’s say for example we want to multiply 2x3 matrix with 3x3 matrix.
This is a valid multiplication because the number of columns in the first matrix (3) is equal to number of rows in the second matrix (3)

Matrix - Basic Operations

The result matrix will be the size of the rows from the first matrix X the size of the columns of the second matrix.

So [2 X 3] X [3 X 3] = [2 X 3]

The result matrix will be with 2 X 3 dimensions.

Now let's take a look on a real number example →

Matrix A (2x3 matrix):

[2 1 3]

[1 0 2]

Matrix B (3x3 matrix):

[3 1 4]

[1 2 3]

[0 1 2]

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Matrix - Basic Operations

To multiply these matrices, each element in each row of the first matrix must be multiplied by each element in each column of the second matrix, and then the results must be added together.

Matrix C (Resulting 2x3 matrix):

[(2*3 + 1*1 + 3*0) (2*1 + 1*2 + 3*1) (2*4 + 1*3 + 3*2)] = [7 7 17]

[(1*3 + 0*1 + 2*0) (1*1 + 0*2 + 2*1) (1*4 + 0*3 + 2*2)]   [3 3 8]

So, the multiplied matrix C is:

[7 7 17]

[3 3 8]

We can see that the result matrix dimensions are 2 X 3 as expected.

Class Exercise - Matrix Basic Operations

Instructions:

Perform the mathematical operation of each of the prevented matrices

- [2 3]   [1 2]
  [5 6] + [3 4]

- [7 6]   [3 2]
  [5 4] - [1 0]

- [2 3 5]
  [4 3 1] * Scalar 3

- [2 3 5]   [1 2]
  [4 3 1] * [6 5]
               [4 2]

Class Exercise Solution - Matrix Basic Operations

Introduction To Vectors

Vector → In mathematics, a vector is a quantity having direction as well as magnitude, especially as determining the position of one point in space relative to another. They are used in physics, engineering, computer science, and various other fields. Vectors can be represented in the form of an array of numbers that indicate direction and magnitude.

We consider a matrix as an array of vectors while each column in the matrix is a different vector.

For example → Matrix A (2 X 3 Matrix)

[4, 1, 3]

[2, 5, 7]

In this matrix, each column can be considered as a separate vector.

The first vector is [4, 2], the second vector is [1, 5], and the third vector is [3, 7].

Introduction To Vectors

Vectors Vs Scalars:

Working with vectors is different than working with scalars because unlike scalars vectors has a direction and they can represent as an array of numbers. Scalars on the other hand has only magnitude and are represented as a single number. The operation rules are also different as we previously saw in this lecture.

How vector is been represented:

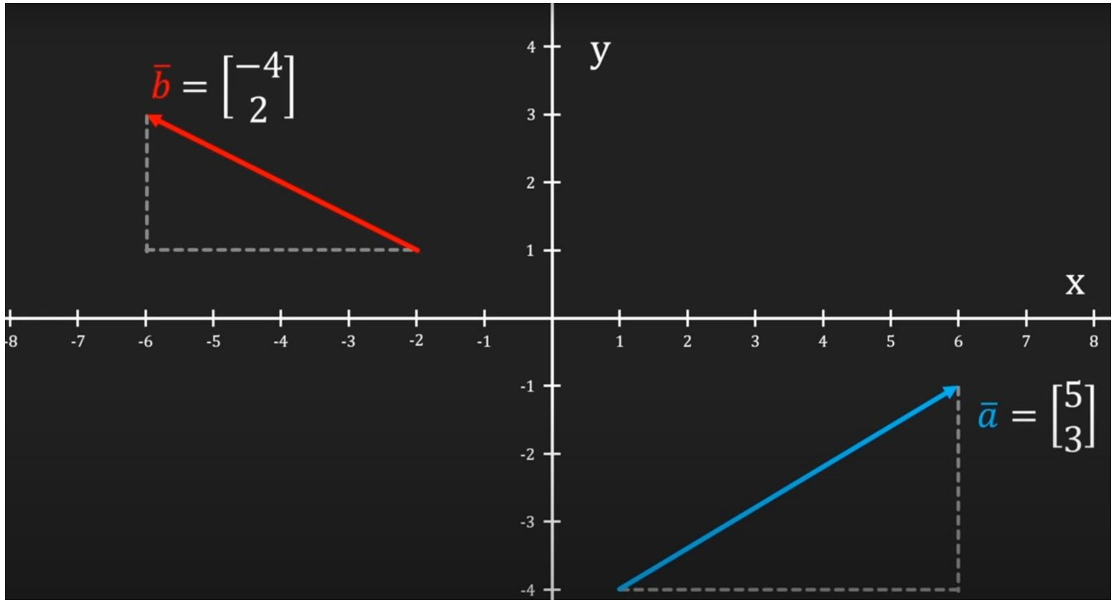
We can think on vector as a point in space, where each element of the vector signifies a coordinate along

a certain dimension.

Vectors doesn't have a starting point, they can start from anywhere in the space but only represent a magnitude and direction.

Introduction To Vectors

For example → Here we can see the vectors [-4, 2] and [5, 3]

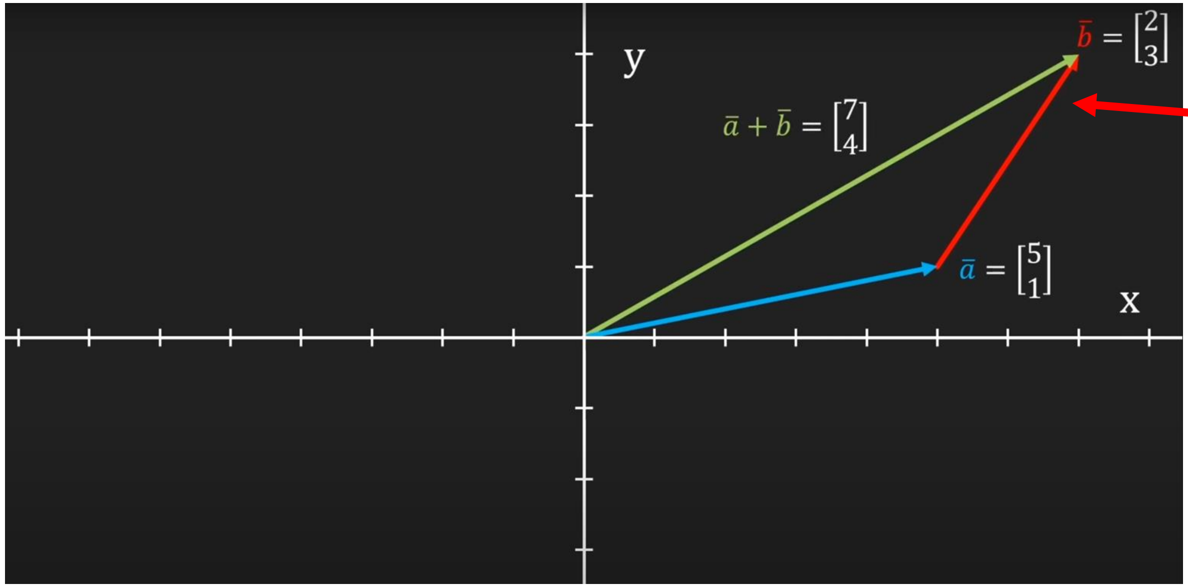


Those vectors can be putted anywhere in the space as long as they are pointing to the same direction and their magnitude is the same.

Introduction To Vectors

Adding vectors together → As we saw we can add matrices together but what we actually did was adding different vectors from the matrices together.

By adding 2 vectors together we simply combine the magnitude and the direction of both vectors and getting a new vector that represent the combination of the vectors that were added together:



A new vector [7,4] was created from the combination of [5,1] and [2,3]

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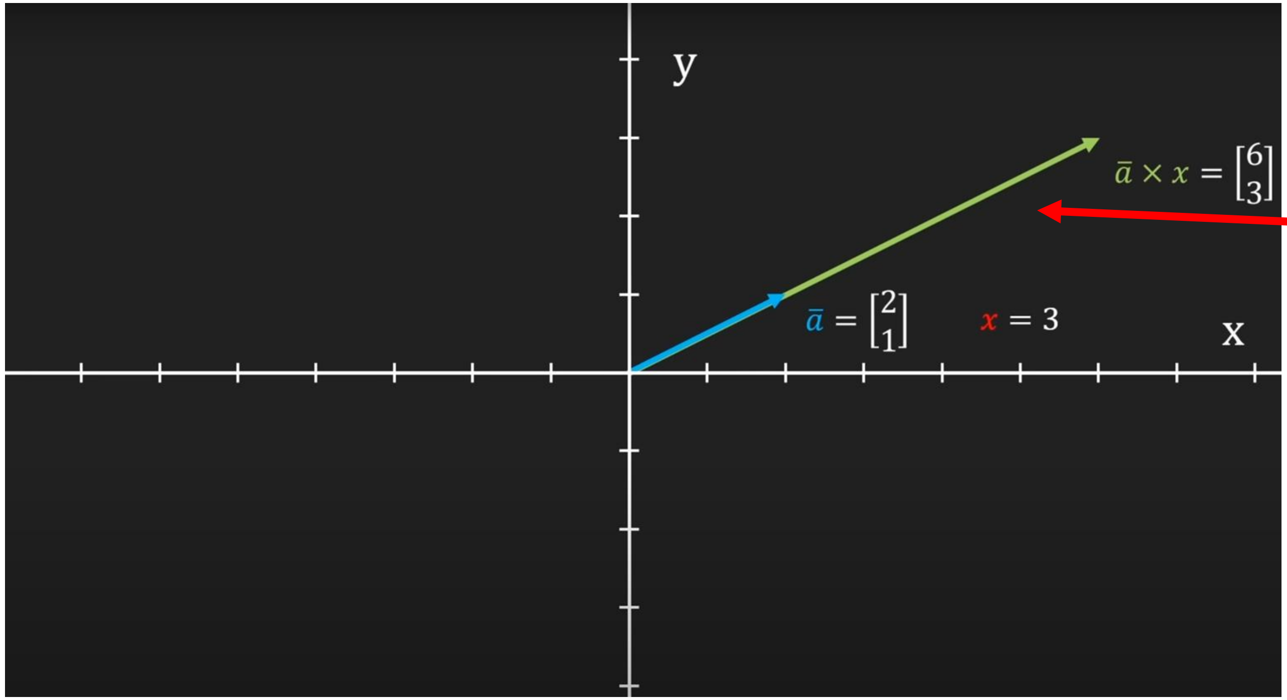
Introduction To Vectors

Multiply vector with scalar → As we learned scalar is different from vector because it doesn't have a direction only a magnitude so it's expected that multiply vector with scalar will change the magnitude of the vector but not it's direction.



Multiply the vector [2,1] with the scalar 3 didn't change the direction of the result vector but increase its size by 3

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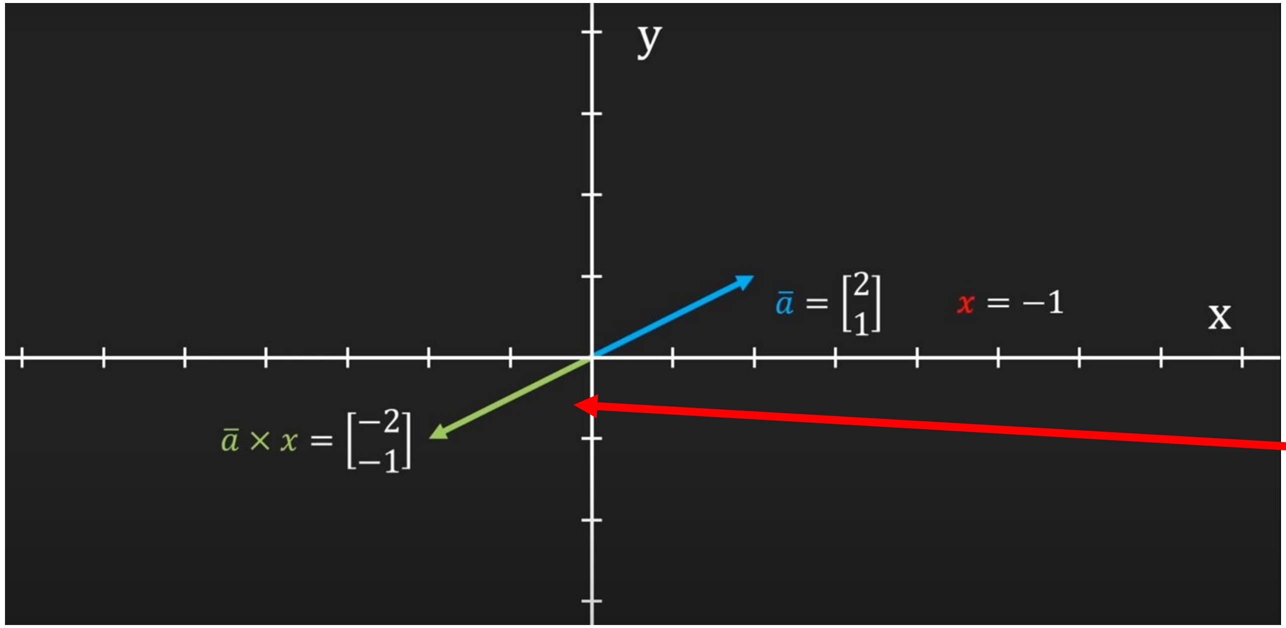
Introduction To Vectors

However, the only exception is that when the scalar is negative so in that case the direction of the vector will be to the opposite direction of the original. We can see that the result direction is a mirror of the original direction of the vector.

For example → Multiply the vector [2,1] with the scalar -1 will result an opposite vector direction



The result vector is in the opposite direction from its original vector

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Matrix - Additional Operations

Transpose → The transpose of a matrix is formed by interchanging its rows and columns. The first row becomes the first column, the second row becomes the second column and so on. The transpose operator also switch the matrix dimensions so if we apply transpose on [3 x 2] matrix we will get as a result [2 x 3] matrix.

Transpose often denoted as A^T or A' while A is the matrix itself.

For example → Consider the following 2x3 matrix:

Matrix A (2 x 3):

[1 2 3]

[4 5 6]

The transposed matrix, A^T, is a 3x2 matrix, and it would look like this:

[1 4]

[2 5]

[3 6]

Matrix - Additional Operations

Identity Matrix → This is a special square matrix (matrix with identical number of rows and columns) that has ones on the main diagonal and zeros everywhere else. When a matrix is multiplied by an identity matrix, the original matrix remains unchanged.

Identity matrix often denoted as (I)

For example → let's consider a 3x3 identity matrix. By definition, a 3x3 identity matrix has 1's along the diagonal and 0's elsewhere. It looks like this:

Identity Matrix (I):

[1 0 0]

[0 1 0]

[0 0 1]

Matrix - Additional Operations

The identity matrix is a very special matrix because if we will take it and multiply another matrix by it we will get the same result (just like multiply a number by 1)

For example → Let's multiply the matrix A with the identity 3 x 3 matrix:

Matrix A:	Identity matrix (3 x 3):
[2 3 4]	[1 0 0]
[5 6 7]	[0 1 0]
[8 9 10]	[0 0 1]

Matrix A * Identity Matrix:

[(2*1 + 3*0 + 4*0) (2*0 + 3*1 + 4*0) (2*0 + 3*0 + 4*1)]	=	[2 3 4]
[(5*1 + 6*0 + 7*0) (5*0 + 6*1 + 7*0) (5*0 + 6*0 + 7*1)]		[5 6 7]
[(8*1 + 9*0 + 10*0) (8*0 + 9*1 + 10*0) (8*0 + 9*0 + 10*1)]		[8 9 10]

We got that the multiplication result is the original A matrix