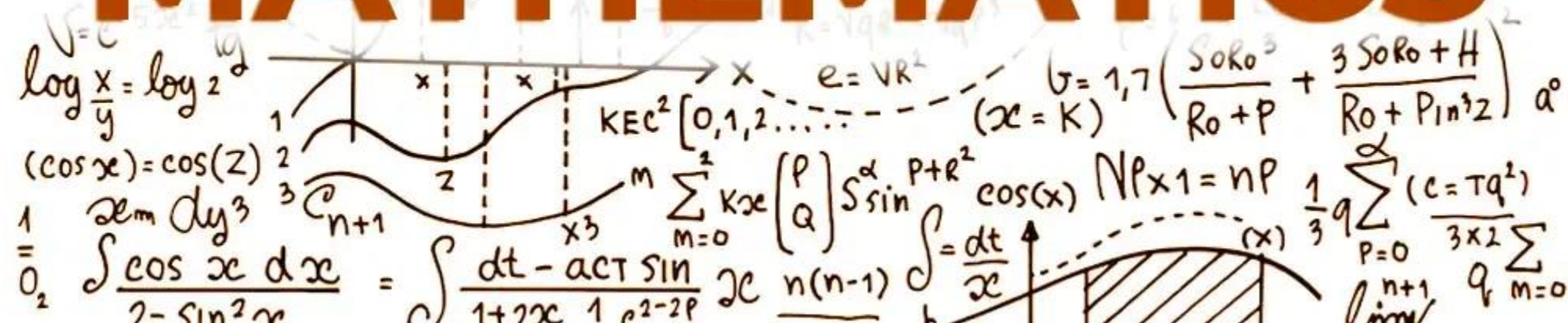


# MATHEMATICS



**ECOM SCHOOL**

המכללה למקצועות הדיגיטל וההייטק

# Why Should We Learn Basic Math In AI Course?

In order to understand why we need to learn basic math concepts before diving into AI & machine learning algorithms we first need to understand what machine learning is all about.

Machine learning is the ability to teach computer how to forecast a result given a specific data.

This leaning is been done by providing large data set and a model which the computer been processing the given data according to the given model.

Once the learning been done the computer will be able to provide a forecast result according to a new unseen data.

# Why Should We Learn Basic Math In AI Course?

Most of the models in machine learning are based on mathematics principles and the result the computer will provide is based on statistic.

This is why it's important for data scientists to understand the basic math and basic statistics that been used behind the scenes in those machine learning models.

In addition, learning basic math will allow us to:

- Develop a deeper understanding of the model - How it works, its limitations and potentials.
- Optimize models and problem-solving strategies more effectively.
- Evaluate the accuracy and efficiency of the model through statistics.

Without such understanding, we may be able to use machine learning tools but we won't be able to fully leverage their power or grasp why a particular model failed or succeeded.

$$\begin{bmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{bmatrix} \quad x = \sum_{i=1}^n x_i v_i = x_1 v_1 + x_2 v_2 + \dots + x_n v_n \quad v_k = y_k - \sum_{i=1}^{k-1} \frac{(v_i, y_k)}{(v_i, v_i)} v_i$$

$$\frac{p^T \nabla^2 F(x) p}{\|p\|^2}$$

$$F(x) = F(x^*) + \nabla F(x)^T|_{x=x^*} (x - x^*) + \frac{1}{2} (x - x^*)^T \nabla^2 F(x)^T|_{x=x^*} (x - x^*) + \dots$$

$$\nabla F(x) = \left[ \frac{\partial}{\partial x_1} F(x) \quad \frac{\partial}{\partial x_2} F(x) \quad \dots \quad \frac{\partial}{\partial x_n} F(x) \right]^T$$

$$\begin{bmatrix} p_1^T \\ p_2^T \\ \vdots \\ p_Q^T \end{bmatrix}$$

# LINEAR ALGEBRA

$$W^{new} = (1 - \gamma) W^{old} + \alpha t_q p_q^T$$

$$W^{new} = W^{old} + \alpha (t_q - a_q) p_q^T$$

$$W^{new} = W^{old} + \alpha a_q p_q^T$$

$$\begin{bmatrix} \frac{\partial}{\partial x_1^2} F(x) & \frac{\partial}{\partial x_1 \partial x_2} F(x) \dots & \frac{\partial}{\partial x_1 \partial x_n} F(x) \\ \frac{\partial}{\partial x_2 \partial x_1} F(x) & \frac{\partial}{\partial x_2^2} F(x) \dots & \frac{\partial}{\partial x_2 \partial x_n} F(x) \\ \vdots & \vdots & \vdots \\ \frac{\partial}{\partial x_n \partial x_1} F(x) & \frac{\partial}{\partial x_n \partial x_2} F(x) \dots & \frac{\partial}{\partial x_n^2} F(x) \end{bmatrix}$$





# Introduction To Linear Algebra

Linear algebra is a branch of mathematics that involves the study of lines, planes, and subspaces. It deals with concepts like vectors, spaces, matrices, and operations.

Linear algebra is widely used in most sciences and fields of [engineering](#), because it allows modeling many natural phenomena, and computing efficiently with such models.

For nonlinear systems, which cannot be modeled with linear algebra, it is often used for dealing with first-order approximations, using the fact that the [differential](#) of a multivariate function at a point is the linear map that best approximates the function near that point.

One of the most basic concepts in linear algebra is the **linear equation**.

# Introduction To Linear Equation

A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and a single variable. It is called 'linear' because it represents a straight line when plotted on a graph.

The standard form for a linear equation in two variables (x and y) is:

$$Ax + By = C$$

Where A, B, and C are constants and A and B are not both zero.

**For example** →  $2x + 3y = 6$  is a linear equation.

A system of linear equations is a collection of one or more linear equations involving the same variables. These are commonly used in fields like engineering, physics, and computer graphics.

# Introduction To Linear Equation

It's commonly to present a simple linear equation by the form of:

$$y = mx + b$$

This form is called the **slope-intercept form** and it allow us to see the linear equation as a linear line graph which for each x value there is the corresponding y value.

In this form each letter represent different element in the linear line:

- **x** → The x value of the equation
- **y** → The corresponding y value of the equation
- **m** → The linear line slope of the equation
- **b** → The intercept of the linear line with the y axis

**Note:** When the slope is positive the graph line will be upwards, and when it negative the graph line will be downwards.

# Linear Equation - Example

Let's take for example the simple linear equation from the previous slide:  $2x + 3y = 6$

By applying simple linear algebra we can get the equation to the slope-intercept form.

In order to do it let's do the following steps:

- First, subtract  $2x$  from both the sides  $\rightarrow 3y = -2x + 6$
- Next, in order to solve for  $y$  we need to divide all terms by  $3 \rightarrow y = (-2/3)x + 6/3$
- The final result is  $\rightarrow y = (-2/3)x + 2$

So we got that for this equation the slope ( $m$ ) is  $-2/3$  and the  $y$ -intercept ( $b$ ) is  $2$ .



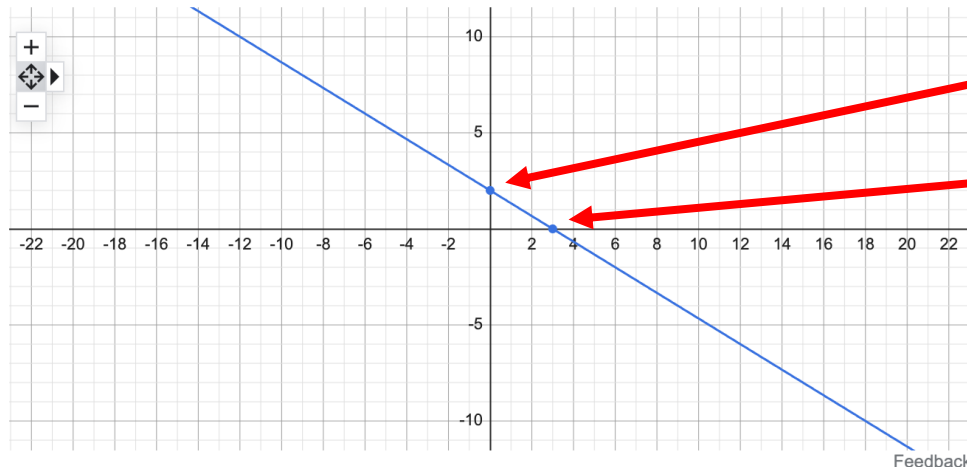
# Linear Equation - Example

We can now get values for (x,y) different points according to this equation for example:

- $x = 0 \rightarrow y = (-2/3)(0) + 2 = 0 + 2 = 2$ , so we are getting the point **(0, 2)**
- $x = 1 \rightarrow y = (-2/3)(1) + 2 = 4/3$ , so we are getting the point **(1, 4/3)**
- $x = -1 \rightarrow y = (-2/3)(-1) + 2 = 8/3$ , so we are getting the point **(-1, 8/3)**
- $x = 2 \rightarrow y = (-2/3)(2) + 2 = 2/3$ , so we are getting the point **(2, 2/3)**

We can also represent this linear equation as a linear line graph:

Graph for  $-2/3x + 2$



The point (0,2) when  $x = 0$

The point (3,0) when  $y = 0$

# Class Exercise - Linear Equations

## Instructions:

- For each of those simple linear equations, convert each equation to slope-intercept form and find the slope and y-intercept.
- Draw the linear line graph for each equation.
  - $4x - 5y = 10$
  - $3x - 2y = 12$
  - $5x + 7y = 32$

# Class Exercise Solution - Linear Equations



# Introduction To Linear Systems

Until now we saw a single linear equation that represent a linear line.

When we have multiple linear equations we are getting **linear systems**.

The most simple linear system is linear system with two unknowns ( $x, y$ ) but we can have linear systems with multiple unknowns.

As the number of unknowns in a system of equations increases, the solution process becomes more complex. As there are more variables to solve for, the calculations can become more involved.

Solving a linear system means to find all sets of values of the variables that make all

equations in the system true simultaneously.



# Linear Systems - Example

Let's take a simple linear system with two unknown  $\rightarrow$

1.  $x + y = 5$
2.  $2x - y = 1$

In order to solve those equations we need to find the value of the unknowns  $(x, y)$  that solve both equations.

We can do the following to solve this linear system:

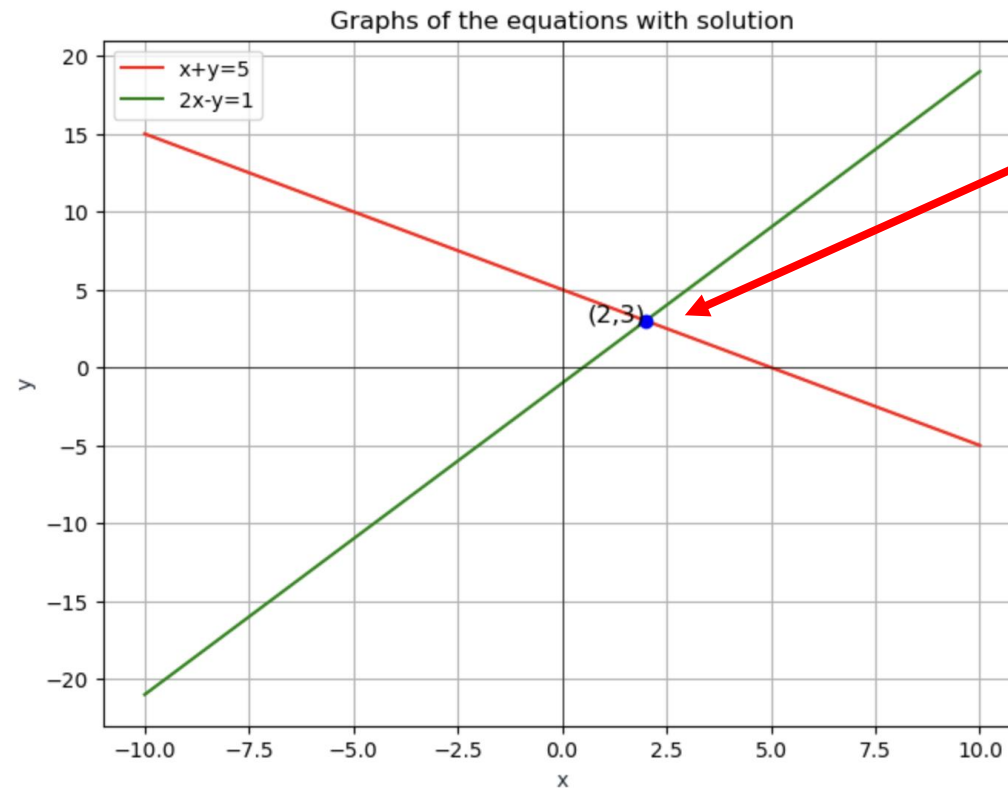
- First, let's get the slope-intercept form of the first equation and get  $\rightarrow y = -x + 5$
- Then, let's replace the  $y$  value from the second equation with  $-x + 5$  and get this new equation:  
$$2x - 1 * (-x + 5) = 1$$
- Then, let's solve this equation and get the  $x$  value  $\rightarrow 3x = 6 \rightarrow x = 2$
- Finally, let's get the corresponding  $y$  value of  $x = 2 \rightarrow y = -2 + 5 \rightarrow y = 3$

We got that the solution for this linear system is **(2,3)**

# Linear Systems - Example

We can also represent this linear equations system solution in a graph:

1.  $x + y = 5$
2.  $2x - y = 1$



The solution to the system represents the coordinates of the point where the lines intersect



# Linear Systems Solution

When solving linear systems with two unknowns there are three possibilities for solution:

1. **One Unique Solution** → This happens when the lines representing the equations intersect at exactly one point. The coordinates of this point are the solution of the system.
2. **No Solution** → This occurs when the lines representing the equations are parallel and never intersect. In this case, there's no point that satisfies all equations, meaning the system has no solution.
3. **Infinitely Many Solutions** → This happens when the lines representing the equations coincide - they're actually the same line. In this case, there are infinitely many points that satisfy all of the equations, meaning the system has an infinite number of solutions.



# Linear Systems Solution - Example

Let's see an example for each of those solution options:

1. **One Unique Solution Example** → The single solution is (2,3)

$$x + y = 5$$

$$x - y = 1$$

2. **No Solution Example** → The lines are parallel and will never meet each other so there is no solution for this linear system.

$$x + y = 5$$

$$x + y = 7$$

3. **Infinitely Many Solutions** → The lines represent the same line so they meet each other on every point, creating an infinitely amount of possible solutions.

$$x + y = 5$$

$$2x + 2y = 10$$



# Class Exercise - Linear Systems

## Instructions:

- For each of those linear systems, solve and find if the systems have single solution, no solution or infinitely solutions.

- $x + y = 12$

- $x - y = 4$

- $5x + 3y = 13$

- $4x - 2y = 6$

- $2x + 4y = 8$

- $x + 2y = 4$



# Class Exercise Solution - Linear Systems



# Introduction To Matrix

When dealing with multiple linear equations we can convert those equations to **matrix** that is represent the same linear system. This is a common action in linear algebra allowing us to handle those equation in a simpler and more understandable way.

**For example** → The following system of equations:

$$2x + 3y = 8$$

$$5x - 2y = 1$$

Can be represented as the following augmented matrix:

$$[ 2 \ 3 \mid 8 ]$$

$$[ 5 \ -2 \mid 1 ]$$

# Introduction To Matrix

In the same way we can represent linear systems with 3 unknowns:

**For example** → The following system of equations:

$$x + y + z = 6$$

$$2y + 3z = 7$$

$$x + z = 4$$

Can be represented as the following augmented matrix:

$$[ 1 \ 1 \ 1 \ | \ 6 ]$$

$$[ 0 \ 2 \ 3 \ | \ 7 ]$$

$$[ 1 \ 0 \ 1 \ | \ 4 ]$$



# Introduction To Matrix

The same logic can be applied on multiple linear equations (n linear equations) and for each equation we have multiple unknowns (n unknown):

**For example** → The following system of equations:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

.....

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

Can be represented as the following augmented matrix:

$$[ a_{11} \ a_{12} \ \dots \ a_{1n} \ | \ b_1 ]$$

$$[ a_{21} \ a_{22} \ \dots \ a_{2n} \ | \ b_2 ]$$

$$[ \dots \ \dots \ \dots \ \dots \ | \ \dots ]$$

$$[ a_{n1} \ a_{n2} \ \dots \ a_{nn} \ | \ b_n ]$$

# Matrix Dimensions

The dimension of a matrix refers to the number of rows and columns it has. The dimension is typically written as "rows  $\times$  columns".

It's important to rightly recognize the matrix dimension because certain mathematical operations with matrices can only be performed under specific conditions, some of which are dictated by the dimensions of the matrices involved.

**For example**  $\rightarrow$  The following system of equations:

$$2x + 3y = 8$$

$$5x - 2y = 1$$

Can be represented as the following augmented matrix:

$$[ 2 \ 3 \mid 8 ]$$

$$[ 5 \ -2 \mid 1 ]$$

So the unknown coefficient matrix dimensions are  $2 \times 2 \rightarrow$  we have 2 rows and 2 columns