

<sub>1</sub> Robust lane markings detection and road  
<sub>2</sub> geometry computation

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## Abstract

Detection of lane markings based on a camera sensor can be a low cost solution to lane departure and curve over speed warning. A number of methods and implementations have been reported in the literature. However, reliable detection is still an issue due to cast shadows, wearied and occluded markings, variable ambient lighting conditions etc. We focus on increasing the reliability of detection in two ways. Firstly, we employ a different image feature other than the commonly used edges: ridges, which we claim is better suited to this problem. Secondly, we have adapted RANSAC, a generic robust estimation method, to fit a parametric model of a pair of lane lines to the image features, based on both ridgeness and ridge orientation. In addition this fitting is performed for the left and right lane lines simultaneously, thus enforcing a consistent result. Four measures of interest with regard several driver assistance applications are directly computed from the fitted parametric model at each frame: vehicle yaw angle and lateral offset with regard the lane medial axis, and lane width and curvature. We have qualitatively assessed our method in video sequences captured on several road types and under very different lighting conditions. Also, we have quantitatively assessed it on synthetic but realistic video sequences for which road geometry and vehicle trajectory ground truth are known.

**Keywords:** driving assistance system, lane line, ridge, robust fitting

# 1 Introduction

A present challenge of the automotive industry is to develop low cost advanced driver assistance systems (ADAS) able to increase traffic safety and driving comfort. Since vision is the most used human sense for driving, some ADAS features rely on visual sensors (Bertozzi, Broggi and Fascioli, 2000). Specifically, lane departure warning and lateral control can be addressed by detecting the lane markings on the road by means of a forward-facing camera and computer vision techniques. In this paper we focus on this problem, which is one of the first addressed in the field of ADAS. It is a difficult and not yet completely solved problem due to shadows, large contrast variations, vehicles occluding the marks, wearied markings, vehicle ego-motion etc. Recent reviews of detection methods can be found in (Jung and Kelber, 2005; Bertozzi et al., 2000).

Many of the proposed methods share the following three steps. First, collect cues on where the lane markings can be, typically in the form of image points labeled as lane markings candidates. Second, fit a certain lane model to them, commonly straight lines or some smooth parametric curve. Third, perform some sort of tracking in order to impose temporal continuity, yield a smooth response along time and facilitate real-time (the results in the present frame guide the search in the next one).

Ideally, lane markings are white lines on a dark pavement. Thus, the first step is usually based on image edges, defined as extrema of the gradient magnitude along the gradient direction. The gradient magnitude is an *edge-ness* measure and the gradient direction can be used to filter out edge points

51 having an orientation inconsistent with the expected orientation of a lane  
52 line. However, the gradient magnitude can be misleading: cast shadows and  
53 vehicles may give rise to high gradient values, while wearied marks and poor  
54 lighting conditions (e.g. in tunnels) reduce lane markings contrast. Also,  
55 the gradient orientation tends to be noisy because of its local nature. There-  
56 fore, methods based on edge detection algorithms must devise strategies to  
57 cope with these problems (*e.g.* local adaptive and hysteresis thresholding).  
58 Otherwise, lane lines model fitting would fail or be much more difficult.

59 The main contributions of this paper are three. The first one is to employ  
60 a different low-level image feature, namely, *ridgeness*, to obtain a more re-  
61 liable lane marking points detection under poor contrast conditions (section  
62 2). Aside from this practical consideration, conceptually, a ridge describes  
63 better than an edge what a lane line is: the medial axis of a thick, brighter  
64 elongated structure. Secondly, we have adapted RANSAC, a generic robust  
65 estimation method, to fit a parametric model to the candidate lane mark-  
66 ing points, using as input data both ridgeness and ridge orientation (section  
67 3). Our model consists in a pair of hyperbolas sharing a common horizontal  
68 asymptote, which are constrained to be parallel on the road plane. We claim  
69 that a better suited feature (ridges) combined with a robust fitting method  
70 contribute to improve lane lines detection reliability. We have intentionally  
71 avoided any kind of result post processing, tracking or lane line prediction,  
72 for example through a Kalman filtering. Instead, each frame is processed in-  
73 dependently of the others. This way we can better design the detection and  
74 fitting steps. Our aim has been to build a 'baseline' system to which later  
75 we can add filtering and data fusion to improve its performance. Thirdly, we

quantitatively assess the method with regard to four geometrically meaningful quantities derived from the segmented lane markings: vehicle yaw angle and lateral offset, lane curvature and width. This is possible on synthetic sequences, for which we know exactly the value for these parameters since they are provided as input to a simulator which generates the sequences (section 4). Qualitative (visual) evaluation is also performed on a number of frames from real sequences exhibiting challenging lighting and occlusion conditions. In addition, video results are also provided in a companion material web page. Section 5 draws the main conclusions and comments future work.

## 2 Lane Markings as Ridges

Ridges of a grey-level image are the center lines of elongated, bright structures. In the case of a lane line is its longitudinal center. This terminology comes from considering an image as a landscape, being the intensity the  $z$  axis or height, since then these center lines correspond to the landscape's ridges (figure 1). Accordingly, ridgeness stands for a measure of how much a pixel neighborhood resembles a ridge. Therefore, a ridgeness measure must have high values along the center of the line and decrease as the boundary is approached. A binary ridge image, corresponding to the centerline, can be obtained by simple thresholding, provided we have a well-contrasted and homogeneous ridgeness measure.

This notion of ridge or medial axis is a simpler and, as we will see in short, computationally better characterization of lane lines than that provided by edges. Instead of defining (and trying to find out) a lane line as points

99 between two parallel edge segments with opposite gradient direction, a ridge  
 100 is the center of the line itself, once a certain amount of smoothing has been  
 101 performed. And this amount is chosen as the scale at which ridges are sought.

102 There are different mathematical characterizations of ridges. In (López,  
 103 Lloret, Serrat and Villanueva, 2000) a new one is proposed which compares  
 104 favorably to others and that we have adapted for the problem at hand. Let  
 105  $G_\sigma(\mathbf{x})$  be a 2D Gaussian of standard deviation  $\sigma$  and  $L(\mathbf{x})$  be the grey-  
 106 level image, with  $\mathbf{x} = (u, v)$  the spatial coordinates ( $u$  columns,  $v$  rows).  
 107 Then, ridgeness is calculated as follows ( $*$  and  $\cdot$  stand for convolution and  
 108 the Hadamard product, respectively):

1. Compute a smoothed version of the image, namely

$$L_{\sigma_d}(\mathbf{x}) = G_{\sigma_d}(\mathbf{x}) * L(\mathbf{x}) \ , \quad (1)$$

2. Compute the gradient vector field

$$\mathbf{w}_{\sigma_d}(\mathbf{x}) = (\partial_u L_{\sigma_d}(\mathbf{x}), \partial_v L_{\sigma_d}(\mathbf{x}))^\top \ . \quad (2)$$

3. Compute the structure tensor field

$$\mathbf{S}_{\sigma_d, \sigma_i}(\mathbf{x}) = G_{\sigma_i}(\mathbf{x}) * \mathbf{s}_{\sigma_d}(\mathbf{x}) \ , \quad (3)$$

being

$$\mathbf{s}_{\sigma_d}(\mathbf{x}) = \mathbf{w}_{\sigma_d}(\mathbf{x}) \cdot \mathbf{w}_{\sigma_d}^\top(\mathbf{x}) \ . \quad (4)$$

- 109 4. Obtain the eigenvector corresponding to the highest eigenvalue of  $\mathbf{S}_{\sigma_d, \sigma_i}(\mathbf{x})$ ,

110 namely  $\mathbf{w}'_{\sigma_d, \sigma_i}(\mathbf{x})$ . It is known that  $\mathbf{w}'_{\sigma_d, \sigma_i}(\mathbf{x})$  yields the *dominant gra-*  
111 *dient orientation* of the original image at  $\mathbf{x}$  and is perpendicular to the  
112 *dominant image orientation* at  $\mathbf{x}$  (if  $\mathbf{x}$  is from a lane marking then the  
113 dominant image orientation is along it). Therefore, its is a more robust  
114 orientation measure than the image gradient itself,  $\mathbf{w}_{\sigma_d}(\mathbf{x})$ .

It is worth to notice that  $\mathbf{w}'_{\sigma_d, \sigma_i}(\mathbf{x})$  defines an orientation field in the image but for the next step we need a vector field. For this reason we project  $\mathbf{w}'_{\sigma_d, \sigma_i}(\mathbf{x})$  into  $\mathbf{w}_{\sigma_d}(\mathbf{x})$  as:

$$p_{\sigma_d, \sigma_i}(\mathbf{x}) = \mathbf{w}'_{\sigma_d, \sigma_i}(\mathbf{x}) \cdot \mathbf{w}_{\sigma_d}(\mathbf{x}) , \quad (5)$$

and define the following vector field:

$$\tilde{\mathbf{w}}_{\sigma_d, \sigma_i}(\mathbf{x}) = \text{sign}(p_{\sigma_d, \sigma_i}(\mathbf{x})) \mathbf{w}'_{\sigma_d, \sigma_i}(\mathbf{x}) . \quad (6)$$

5. Finally, the *ridgeness* measure is defined as the positive values of

$$\tilde{\kappa}_{\sigma_d, \sigma_i}(\mathbf{x}) = -\text{div}(\tilde{\mathbf{w}}_{\sigma_d, \sigma_i}(\mathbf{x})) , \quad (7)$$

115 where  $\text{div}()$  denotes divergence of a vector field.

116

117 The parameter  $\sigma_d$  is the *differentiation scale*, in opposition to  $\sigma_i$  which  
118 is the *integration scale*. The former must be tuned to the size of the target  
119 structures, while the later determines the size of the neighborhood we want  
120 to use in order to compute the dominant orientation.

121 Positive values of  $\tilde{\kappa}_{\sigma_d, \sigma_i}(\mathbf{x})$  measure the similarity of a neighborhood to  
 122 a ridge structure. In fact, it has been shown (López et al., 2000) that these  
 123 values lie in the range  $[0, 2.0]$ , where 0 means not at all ridge, around 1.0 quite  
 124 and 2.0 perfect local maximum. Besides, these values are homogeneously  
 125 distributed along the center lines, thus facilitating thresholding. We only  
 126 take into account those pixels  $\mathbf{x}$  for which  $\tilde{\kappa}_{\sigma_d, \sigma_i}(\mathbf{x}) > 0.25$ , a value fixed  
 127 experimentally but with a large margin before the selected pixels change  
 128 significantly.

129 Due to perspective, the imaged lane lines width decreases with distance.  
 130 In order not to miss them when computing  $L_{\sigma_d}$  in Eq. (1), we want the  
 131 upper rows to be less smoothed than lower rows, but just along the horizon-  
 132 tal direction. This is achieved by an anisotropic Gaussian smoothing, with  
 133 covariance matrix  $\Sigma = \text{diag}(\sigma_{dx}, \sigma_{dy})$  where  $\sigma_{dy}$  is constant and  $\sigma_{dx}$  increases  
 134 with the row number. Table 1 contains the actual values for all the detection  
 135 parameters.

136 Since the dominant orientation of a lane marking is perpendicular to the  
 137 dominant gradient orientation, and therefore perpendicular to  $\tilde{\mathbf{w}}_{\sigma_d, \sigma_i}(\mathbf{x})$ , this  
 138 vector field allows to discard pixels whose associated orientation is inconsis-  
 139 tent with that expected by a lane markings model instantiation. We discard  
 140 a ridge point  $\mathbf{x}_r$  if  $\tilde{\mathbf{w}}_{\sigma_d, \sigma_i}(\mathbf{x}_r)$  is in  $[3\pi/4, 5\pi/4]$ , which happens for lane mark-  
 141 ings with a large horizontal component. Of course, this could be a problem  
 142 for curves with a very high curvature. However, we have experimentally  
 143 checked that this criterion performs well even for curves that can not be  
 144 driven safely at more than 40 km/h.

145 Other interesting properties of  $\tilde{\kappa}_{\sigma_d, \sigma_i}(\mathbf{x})$  are invariance to image trans-



146 lation and rotation, as one would expect, but also to monotonic grey-level  
 147 transforms. The later greatly helps in lane detection in presence of shadows  
 148 and low contrast conditions, opposite to gradient-based measures. However,  
 149 this means that ridgeness also enhances some bright and elongated irregu-  
 150 larities in the pavement. Fortunately, this can be solved up to a large extent  
 151 by discarding those ridges surrounded by a very low gradient magnitude  
 152 neighborhood, less than a certain threshold  $t_{\text{grad}}$ . We want to remark that  
 153 the threshold value for selecting relevant ridge points has been fixed once  
 154 and used in all the sequences, even though the cameras, lenses, vehicles and  
 155 lighting conditions varied. Figure 2 shows the resulting candidate lane line  
 156 points in some specially difficult situations like worn off paint, shadows, high  
 157 contrast variation and tire marks.

## 158 **3 Lane model and fitting**

### 159 **3.1 Lane lines model**

160 A number of geometrical models for the projected lane lines have been pro-  
 161 posed, from simple straight lines to quadratic, spline and other polynomial  
 162 curves, with the aim of performing a good image segmentation. However,  
 163 few are built on a sound geometrical base like in (Guiducci, 1999). There it  
 164 is shown that, under the assumptions of flat road and constant curvature,  
 165 a lane line is projected onto the image plane as an hyperbola. Admittedly,  
 166 this is not a new model, but what that work reveals are the relationships  
 167 among model parameters and meaningful and interesting geometrical enti-

168 ties such as lane width, curvature and the vehicle's lateral position, which we  
 169 want to compute in order to validate our method, aside of their own evident  
 170 applicability for driver assistance.

171 Assume the road is on a plane, that is, there is not vertical curvature nei-  
 172 ther torsion. Furthermore, the curvature is either constant or varies linearly  
 173 with the arc length  $s$ :

$$C = \frac{1}{R} = C_0 + C_1 s \quad . \quad (8)$$

174 This is consistent with a road formed by segments of constant curvature  
 175 connected by clothoids (Dickmanns and Mysliwetz, 1992). Assume too that  
 176 changes in the road direction are smooth, being  $C_1$  small enough with regard  
 177 to  $s$  so that approximations  $C_1/C_0 \ll 1$  and  $C_1 \ll 1$  hold.

178 World and camera coordinate systems share a common origin but have  
 179 different orientation (figure 3). For the world coordinate system the  $Z$  axis is  
 180 parallel to the road tangent,  $Y$  axis points downwards and is orthogonal to the  
 181 road plane, whereas the  $X$  axis is parallel to the road plane and orthogonal  
 182 to the road tangent and therefore to the lane lines. The camera coordinate  
 183 system has  $Y$  axis coincident with the vehicle's direction and sustains an  
 184 angle  $\theta \ll 1$  radians with the road tangent line (also referred as yaw angle).  
 185 It also forms an angle  $\varphi$  with the road plane (pitch angle). The lane has  
 186 width  $L$  and the camera is located at a horizontal distance of  $d_r$  meters from  
 187 the right border and at height  $H$  above the ground. Of course,  $L, d_r, \theta$  and  
 188  $\varphi$  may vary over time, but  $H$  is supposed constant. Finally, let  $E_u$  and  $E_v$   
 189 be the focal lengths in pixels/meter along the horizontal and vertical camera

axes, and the image origin centered in the principal point (intersection of  
the optical axis with the image plane). Then, the following equation relates  
( $u_r, v_r$ ), the pixel coordinates where the right lane line is imaged, to the road  
parameters it belongs to (Guiducci, 1999):

$$u_r = E_u \left( \frac{\theta}{\cos \varphi} + \frac{d_r \cos \varphi}{H E_v} (v_r + E_v \tan \varphi) + \frac{E_v H C_0 / \cos^3 \varphi}{4(v_r + E_v \tan \varphi)} + \frac{E_v C_1 H^2 / \cos^5 \varphi}{6(v_r + E_v \tan \varphi)} \right) . \quad (9)$$

Let's make a final simplifying assumption, namely that the linear term  
of the curvature is negligible,  $C_1 \simeq 0$ . Hence, our road model is simply a  
succession of segments of constant curvature. The former equation clearly  
follows the formulation of a hyperbola with a horizontal asymptote at  $v = v_0$ :

$$u - u_0 = a(v - v_0) + \frac{b}{(v - v_0)} . \quad (10)$$

In order to enforce parallelism of lane borders, we introduce a new vari-  
able  $x_c$ , which is the signed distance along the  $X$  axis between the camera  
projection on the road plane and the central axis of the left lane line (figure  
3). It follows that  $d_r = x_c - L$ ,  $d_l = x_c$  and we have the following couple of  
equations, for points ( $u_l, v_l$ ), ( $u_r, v_r$ ) on the left and right border, respectively:

$$\begin{aligned} u_l &= E_u \left( \frac{\theta}{\cos \varphi} + \frac{\cos \varphi}{H E_v} x_c (v_l + E_v \tan \varphi) + \frac{E_v H C_0 / \cos^3 \varphi}{4(v_l + E_v \tan \varphi)} \right) , \\ u_r &= E_u \left( \frac{\theta}{\cos \varphi} + \frac{\cos \varphi}{H E_v} (x_c - L) (v_r + E_v \tan \varphi) + \frac{E_v H C_0 / \cos^3 \varphi}{4(v_r + E_v \tan \varphi)} \right) . \end{aligned} \quad (11)$$

203 Since parameters  $E_u, E_v, H$  and  $\varphi$  can be estimated through a camera  
 204 calibration process (Zhang, 2000; Bouguet, 2008), Eq. (12) is linear with  
 205 respect to the four unknowns  $\theta, x_c, L$  and  $C_0$ . It can be compactly rewritten  
 206 as:

$$\begin{bmatrix} 1 & 0 & v'_l & 1/v'_l \\ 1 & -v'_r & v'_r & 1/v'_r \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} u_l \\ u_r \end{bmatrix}, \quad (12)$$

207 with  $v'_r = v_r/E_v + \tan \varphi, v'_l = v_l/E_v + \tan \varphi$  and

$$\theta = \frac{\cos \varphi}{E_u} a_1, \quad L = \frac{H}{E_u \cos \varphi} a_2, \quad x_c = \frac{H}{E_u \cos \varphi} a_3, \quad C_0 = \frac{4 \cos^3 \varphi}{E_u H} a_4. \quad (13)$$

208 Note that according to this model, four points, not all on the same line, de-  
 209 fine *a pair* of hyperbolas sharing the same horizontal asymptote. In addition,  
 210 they correspond to two parallel curves  $L$  meters apart, when backprojected  
 211 to the road plane. This implies that we are going to fit *both left and right*  
 212 lane lines at the same time and enforcing parallelism, that is, consistency  
 213 in the solution. Besides, the sparsity of candidates in one lane side due to  
 214 occlusions or dashed lane marks can be compensated by those in the other  
 215 side. The parallelism constraint, however, is a potential drawback at places  
 216 where the present lane bifurcates, like in highway exits and lane splitting, as  
 217 shown in figure 6.

## 3.2 Model fitting

We would like to separate ridge points on each side of the lane in order to adjust the corresponding curve only to them. Since the camera is located at the center of the windshield screen and forward-facing, we can make a guess based on the horizontal coordinate with respect to a fixed image column  $u_{\text{vanish}}$  (figure 4). This column corresponds to the  $u$ -coordinate of the image vanishing point when the vehicle is centered ( $x_c = 0$ ,  $\theta = 0$ ) in a straight lane ( $C_0 = 0$ ). In curves, of course, this simple criterion is reliable only near the vehicle. Therefore, from rows  $v_{\text{min}}$  to  $v_{\text{common}}$  we cannot tell which side a ridge point belongs to, and we assume both are possible. It is only below row  $v_{\text{common}}$  that the image is safely divided by  $u_{\text{vanish}}$  into left and right lane regions.

A minimum of four points are necessary in order to solve Eq. (12), provided there is at least one point on each curve. If more points are known, we get an over constrained system that is solved in the least-squares sense. The problem, of course, is the selection of the right points among all candidates from the previous detection step. We need a robust technique in the sense of, simultaneously, classify candidate points into lane points (inliers) and not lane points (outliers, at least for that side), and perform the fitting only to the former ones. RANSAC, which stands for Random Sample Consensus (Fischler and Bolles, 1981), is a general estimation technique for such purpose based on the principle of hypotheses generation and verification.

Let be a parametric model  $M$  needing at least  $n$  points to be instantiated and a set  $\mathcal{P}$  of more than  $n$  points, possibly containing outliers with regard to

242 the right model instantiation. Also, let be  $n_{\text{trials}}$  a predetermined maximum  
 243 number of trials,  $d(p, M)$  a distance function between a given point  $p = (u, v)$   
 244 and an instantiated model  $M$ , and  $t_{\text{dist}}$  a distance threshold. The idea is  
 245 to randomly select  $n$  points from  $\mathcal{P}$ , instantiate a model  $M_i$  with them and  
 246 measure its support: how many points  $p \in \mathcal{P}$  lie in a close vicinity of  $M_i$ ,  
 247 that is,  $d(p, M_i) < t_{\text{dist}}$ . The instantiation with the largest support, or the  
 248 first one which exceeds a predetermined high support threshold  $t_{\text{consensus}}$ ,  
 249 defines the set of inliers from which to re-instantiate the model and produce  
 250 the final result.

251 In our particular case,  $n = 4$ ,  $p$  are candidate ridge points, models  $M_i$  are  
 252 pairs of hyperbolae parametrized by  $a_1 \dots a_4$  and easily instantiated from a  
 253 set  $\mathcal{S}_i$  of four points by solving the linear system of Eq. (12). As for the  
 254 needed number of trials  $n_{\text{trials}}$ , let  $p_w$  be the a priori probability that a given  
 255 data point is an inlier. If we want to ensure with probability  $p_z$  that at  
 256 least one of the  $\mathcal{S}_i, i = 1 \dots n_{\text{trials}}$  will only contain inliers, then we need  
 257 to make at least  $\log(1 - p_z) / \log(1 - p_w^n)$  trials (Fischler and Bolles, 1981).  
 258 We have estimated  $p_w$  from thousands of frames captured from sequences  
 259 corresponding to different lighting conditions (at day and night) and roads  
 260 (including highways, motorways and local roads). Table 1 lists the value for  
 261 this and all other fitting parameters which we describe in the following.

262 Before proceeding to assess the support of  $M_i$ , we check whether the lane  
 263 width obtained from Eq. (13) is a reasonable quantity, that is, within an  
 264 interval  $[L_{\min}, L_{\max}]$ . Otherwise,  $M_i$  is discarded and a new iteration begins.

265 The aim of the distance function  $d(p, M)$  is to classify points  $p$  into inliers  
 266 and outliers. To do so, we combine two factors: distance to the instantiate

267 lane line and orientation similarity. There is not a simple expression for the  
 268 geometric distance between a point and a hyperbola since there are up to  
 269 four lines through a point which are perpendicular to a conic. Instead, we  
 270 are going to use the Sampson's distance (Sampson, 1982), which lies between  
 271 the algebraic and the geometric distances in terms of complexity but which  
 272 gives a close approximation to the later. Let be  $\mathbf{p} = (u, v, 1)^\top$  a point  $p$  in  
 273 homogeneous coordinates and  $\mathbf{C}$  a symmetric  $3 \times 3$  matrix. Then,  $\mathbf{p}^\top \mathbf{C} \mathbf{p} = 0$   
 274 is the equation of a conic section. The Sampson's distance is defined by:

$$d_S(p, \mathbf{C}) = \frac{(\mathbf{p}^\top \mathbf{C} \mathbf{p})^2}{4((\mathbf{C} \mathbf{p})_1^2 + (\mathbf{C} \mathbf{p})_2^2)} \quad , \quad (14)$$

275 where  $(\mathbf{C} \mathbf{p})_i$  denotes the  $i$ -th component of the vector  $\mathbf{C} \mathbf{p}$ . For those ridge  
 276 points satisfying  $d_S(p, \mathbf{C}) < t_{\text{dist}}$ , a second test is applied before being ad-  
 277 mitted as inliers: at the point  $p'$  on  $\mathbf{C}$  closest to  $p$ , the line  $\mathbf{l} = \mathbf{C} \mathbf{p}'$  tangent to  
 278 the conic must be parallel to the dominant image orientation at  $p$ , or equiva-  
 279 lently, perpendicular to the dominant gradient orientation field  $\mathbf{w}'_{\sigma_d, \sigma_i}$  at that  
 280 point. A maximum deviation of  $\alpha$  is allowed. For the sake of computational  
 281 simplicity  $p'$  is taken as the point of  $\mathbf{C}$  in the same row as  $p$ .

282 A final observation must be made concerning the lane model of Eq. (12).  
 283 In it we supposed the pitch angle  $\varphi$  to be known from the calibration process,  
 284 but it actually suffers variations around its nominal value due to non-planar  
 285 roads, acceleration, brake actioning etc. To account for this fact, quite influ-  
 286 ential in the instantiated model because it changes its horizontal asymptote,  
 287 we test several possible values for  $\varphi$ , taking  $n_\varphi$  equispaced samples within  
 288  $\varphi \pm \Delta\varphi$ , for a certain margin value  $\Delta\varphi$  (again, refer to table 1 for the actual

parameters values).

## 4 Validation and results

As pointed out in a recent survey on video-based lane departure warning (McCall and Trivedi, 2006), results in the literature are often presented only in the form of several frames, where the reader can check the correspondence between detected lane lines and real lane markings. We also present results in this qualitative way, but just to show examples of challenging situations due to occlusions, shadows, reflections and poor lighting conditions, both in daytime and nighttime, where our method succeeds, at least by visual comparison (figure 5 and 6). Complete sequences from which these frames have been extracted can be viewed at

<http://www.cvc.uab.es/adas/projects/lanemarkings/IJAT/videos.html>.

However, since our fitted model has a direct relation to geometrically meaningful parameters of interest in the context of ADAS, we base the evaluation on the comparison of these computed values with the actual ones. And here we face the main difficulty in obtaining quantitative results for this kind of work: the lack of ground truth, that is, the precise knowledge of the road shape, the camera position and the viewing direction at each frame. True, they can be approximated by means of additional sensors like differential or high precision GPS, accelerometers etc. (Wang, Schroedl, Mezger, Ortloff, Joos and Passegger, 2005), but the construction of digital road maps at lane line resolution is still a research issue in itself and out of the scope of this paper. Therefore, we have resorted to build a simulator which generates



312 sequences of synthetic but realistic images from exactly known road geom-  
313 etry and camera dynamics. This has the evident advantage of controlling  
314 every possible factor, from the 3D road shape and contrast to the camera  
315 trajectory and pose along time.

316 Specifically, the simulator, implemented in Matlab, is based on four mod-  
317 els:

- 318 1. Road photometry. The road is composed of two lanes and thus it has  
319 two border and one central lane lines. Each of them can be continuous  
320 or dashed. Variable lighting conditions are simulated by sudden gray  
321 level changes of long road patches (figure 7b).
- 322 2. Road geometry. The road is divided into segments of varying length  
323 and constant but random curvature, and a linear interpolation of cur-  
324 vature is performed at their ends to get smooth transitions. Likewise,  
325 it is also divided into segments of fixed random slope, independently of  
326 curvature. Slope is later smoothed to avoid sudden unrealistic changes.
- 327 3. Camera model (intrinsic parameters). The camera is simulated by a  
328 central projection, according to the pin-hole model. The principal  
329 point is located at the image center. No radial distortions have been  
330 considered for the sake of simplicity. A focal length of 1200 pixels for  
331 both axes and a resolution of  $640 \times 480$  pixels yield a field of view  
332 very similar to that of real cameras we have used. However, images are  
333 sampled to half resolution to speed up processing (40 ms/frame in a  
334 2.0 GHz Pentium IV).

335 4. Camera dynamics (extrinsic parameters). Camera location changes  
 336 due to the simulated vehicle motion which is longitudinally 1 meter  
 337 per frame (thus, at 30 fps the simulated vehicle speed is 108 Km/h).  
 338 In order to determine the lateral displacement with respect the lane  
 339 central axis, the whole road is divided into segments of varying length.  
 340 Each is assigned a constant lateral offset, and then a Gaussian smooth-  
 341 ing is performed to avoid sudden vehicle direction changes. With regard  
 342 to camera pose, we have fixed the roll angle to zero (the horizon line is  
 343 parallel to the image horizontal axis) and set at each frame the yaw an-  
 344 gle  $\theta$  such that the camera is always forward facing, that is, the optical  
 345 axis is parallel to the vehicle trajectory tangent (figure 3). Finally, the  
 346 pitch angle  $\varphi$  has not been fixed, since it is responsible for the horizon  
 347 line vertical motion, which is not static in real sequences. Besides, it  
 348 turns out that this parameter is quite influential on the results. Thus,  
 349 we have randomly varied the pitch angle so as to mimic the effects of  
 350 an uneven road surface and of acceleration and brake actioning, both  
 351 observed in real sequences. Specifically, the pitch variation is gener-  
 352 ated by adding two random signals: the first one of high frequency  
 353 and small amplitude ( $\leq 0.2^\circ$ ) and the second one of low frequency but  
 354 larger amplitude (between  $0.5^\circ$  and  $1^\circ$ ), which account respectively for  
 355 the two former pitch variation sources (figure 7d).

356 Table 2 contains the specific values for the most relevant parameters of  
 357 the simulator, some of which correspond to a real road. We have performed  
 358 several tests on synthetic sequences in order to calculate the error in the  
 359 estimation of  $C_0, L, \theta, x_c$  and also  $\varphi$ . Figure 7a shows the whole 5 Km long

360 synthetic road from which quantitative results have been drawn at each me-  
 361 ter. Figure 7b shows a typical frame. Error computation has not been the  
 362 unique goal of testing, but we wanted also to assess the error contribution  
 363 due to the departure from the assumed road model and to errors in im-  
 364 age lane line detection. Specifically, we have conducted the following tests,  
 365 increasingly approaching the real testing scenario:

- 366 • Non-ideal road. We have generated a sequence of a synthetic road with  
 367 piecewise constant curvature and slope. Camera pitch has been fixed  
 368 to a known nominal value. Lane lines detection is ideal, since we obtain  
 369 one point per row points from the central projection of the generated  
 370 lane lines world coordinates, instead of the road image itself.
- 371 • Non-ideal camera. Like the former case but now the camera pitch is  
 372 allowed to vary from its nominal value, in the way explained above.
- 373 • Non-ideal detection. The only difference with the previous case is that  
 374 detection is performed on actual images of the sequence. This test  
 375 assesses the influence of detection, performed as described in sections  
 376 2 and 3.2.
- 377 • Best pitch search. Like the former case of non-ideal detection, but now  
 378 we do not rely on the nominal pitch. Instead, we test a fixed number of  
 379 values  $n_\varphi$  equispaced around the nominal value  $\varphi$ . This is equivalent to  
 380 look for the best horizon line near that produced by the nominal pitch  
 381 angle.

382 Figs. 8 to 11 (top) show that the difference between computed parameters

$x_c, C_0, \theta, L$  and their corresponding ground truth is very small if the road follows the ideal model of constant (but also linearly varying) curvature, flat surface and known camera pitch angle, thus confirming the suitability of the proposed method. At the same time, deviations from this ideal case due to sudden slope change introduce large errors, though logically localized in time. The larger the slope variation, the larger the error, but the sign of change does matter. The slope variation at times  $t = 450$  and  $850$  is almost equal (figure 7c) but the error is much smaller in the first case, for all four parameters. The reason is that, when the camera approaches a negative slope change (the vehicle goes uphill and almost reaches the 'top'), the number of image rows depicting road surface is reduced (figure 12). In principle, this should not cause any problem, since the detection of the lane line points is ideal, that is, the  $(u_l, v_l), (u_r, v_r)$  are exact. However, these points are taken only from the road visible region, one per row and side. If they have similar  $v$  coordinate, the over constrained linear system built by stacking pairs of equations (12) becomes ill-conditioned. When the slope change is positive (the camera faces a 'ramp', like in figure 7b), the lane line points do not fit into the flat road model and, consequently, there is some error, but the system is well conditioned.

The second row of Figs. 8 to 11 shows the error introduced by variations in the camera pitch (figure 7d). At frames where the pitch variation has its largest peaks, the error is small for  $x_c$ , moderate for  $L$  but large for  $C_0$  and  $\theta$ . The reason is that  $x_c$  and  $L$  are local road measures very close to the camera position and thus not affected by the global lane line shape, specially its shape at a large distance, close to the horizon line. On the contrary,  $C_0$  and

408  $\theta$  do depend on the global shape (according to the road model, the curvature  
 409 is supposed to be constant) which is in turn dependent on the shared lane  
 410 lines horizontal asymptote.

411 When the lane line points  $(u_l, v_l), (u_r, v_r)$  are extracted from the images, in  
 412 the non-ideal detection scenario, the former two types of errors are somehow  
 413 amplified for all four parameters (third row of Figs. 8 to 11). In addition, a  
 414 small amplitude noise appears everywhere. This latter could be attributed  
 415 to the detection process, but experiments have shown that it is mainly due  
 416 to the small amplitude pitch variation. We have tried to minimize the effect  
 417 of pitch changes (both large and small) by considering  $\varphi$  another parameter  
 418 to estimate, as explained in the next paragraph. Recall that the last testing  
 419 scenario consists of looking for the best pitch angle (in terms of RANSAC,  
 420 maximize the size of the consensus set) among  $n_\varphi$  possible values within the  
 421 nominal pitch  $\pm\Delta\varphi$ . However, the estimated  $\varphi$  is *not* used to recompute the  
 422 four parameters, it is just a way to check the success of the best pitch search  
 423 procedure when comparing the estimated  $\varphi$  to the ground truth (figure 13).

424 The fourth row of Figs. 8 to 11 shows the result for  $n_\varphi = 7$  and  $\Delta\varphi = 1^\circ$ .  
 425 From figure 13 we conclude that the best pitch search is often able to cor-  
 426 rectly estimate the pitch (the four largest pitch variations are well detected),  
 427 but not always. The most prominent errors are localized around the four  
 428 slope changes, where this simple approach of guessing the best pitch fails.  
 429 Elsewhere, a sort of impulsive error is observed, caused by a small number  
 430 of inliers. In addition, depending on the value of  $n_\varphi$ , the estimated  $\varphi$  suf-  
 431 fers from a quantization noise: the  $2^\circ$  interval is too wide for just 7 possible  
 432 values. Increasing  $n_\varphi$  yields a better estimation but the computational cost

precludes a high processing rate. In spite of it, a causal median filter (median of a number of pitch estimations before the current frame) produces an acceptable result, even for  $n_\varphi = 7$ . Likewise, the causal median filtering of  $x_c, C_0, \theta$  and  $L$  (bottom row of Figs. 8 to 11) produces more accurate values due to the impulsive and zero mean nature of the error induced by the pitch estimation. Finally, figure 14 shows the root-mean square error (RMSE) between computed and ground truth for  $n_\varphi = 1, 3, 7, 41$  and also for their median filtered versions. Whereas there is only a slight improvement, or even no improvement at all, when  $n_\varphi$  increases, the error of the filtered parameters clearly decreases. Therefore, it seems that it does not pay to look for the best pitch if no filtering is performed afterward. But the important thing to note is that even in the simplest case of  $n_\varphi = 1$ , the RMSE of  $x_c, \theta, C_0$  and  $L$  is only 25 cm,  $1.1^\circ$ ,  $0.0027 \text{ m}^{-1}$  and 22cm, respectively.

## 5 Conclusions

We have developed a new method for the extraction of lane lines from video sequences, with robustness and quantitative evaluation as the main considerations. Robustness is achieved both in the feature detection phase, where we employ an image feature well suited to this problem, and in the model fitting phase, which we have addressed with a RANSAC approach. This method relies just on images, that is, we do not take into account data from other vehicle sensors, like the steering angle, yaw rate or vehicle speed. In addition, each frame is processed independently of the others, since our goal has been to build a 'baseline' system on which to later add filtering, to enforce

456 temporal consistency, and data fusion, to improve reliability.

457 Our lane line extraction method has the advantage of computing four  
458 road and vehicle trajectory parameters which are of interest in the context  
459 of ADAS: road curvature, lane width, lateral vehicle offset and heading angle  
460 with respect the road medial axis. We have compared the computed values  
461 with ground truth from a synthetic but realistic road, in several testing sce-  
462 narios, increasingly closer to a real test. From the experiments we conclude  
463 that it is possible to compute reasonable estimations of these parameters,  
464 even in the case where the road does not exactly follow the assumed model  
465 of flatness, constant curvature and known camera pitch. We have also per-  
466 formed extensive visual testing on real sequences from different roads, with  
467 varying traffic density and lighting conditions (day, night, tunnels, cast shad-  
468 ows), and also recorded with several cameras (CCD, CMOS). Results have  
469 shown the robustness of our method to these factors.

470 The weak point of our method, with regard the correct computation of  
471 the road geometric parameters, is the estimation of the pitch angle. Its pre-  
472 diction on the basis of previous frames seems a promising way. Future work  
473 also includes the design of a post-processing phase which incorporates the  
474 temporal continuity of detected lane lines and parameters. Some promising  
475 results have already been obtained with a Kalman filter.

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detection	$\sigma_{\text{dx}}(u), u = u_{\text{vanish}} \dots M$	0.5 ... 6.0
	$\sigma_{\text{dy}}$	0.5
	$\sigma_i$	0.5
	$t_{\text{gradient}}$	2.0
fitting	$u_{\text{vanish}}$	160
	$v_{\text{min}}$	137
	$v_{\text{common}}$	187
	$n_{\text{trials}}$	<1000 for $p_Z = .90$
	$[L_{\text{min}}, L_{\text{max}}]$	[220, 300] pixels
	$\alpha$	15°
	$t_{\text{dist}}$	2 pixels
	$t_{\text{consensus}}$	8 points
	$\pm\Delta\varphi$	$\pm 1^\circ$ (approximately $\pm 10$ rows)

Table 1: Parameter values of the feature detection and model fitting phases. Original images of  $640 \times 480$  pixels are sampled to half resolution in order to speed up processing. Row  $v_{\text{min}}$  is at 35 to 40 meters from the camera.

camera model	image dimensions (cols×rows)	640 × 480
	bits/pixel	8
	focal lengths $E_u, E_v$	1200 pixels
	camera height $H$	1.6 m
	nominal camera pitch $\varphi$	1.6°
camera (vehicle) motion	maximum amplitude of low frequency pitch noise	1.0°
	idem, high frequency	0.2°
	roll angle	0°
	maximum lateral offset in % of lane width	80%
road geometry and photometry	length	5 Km
	length of segments of constant curvature and slope	300–600 m
	maximum slope (magnitude)	7%
	minimum radius of curvature	50.0 m
	lane width	3.65 m
	road border line	width 0.2 m
		length 20 m
		gap 4 m
	inter-lanes line	width 0.15 m
		length 4 m
		gap 7 m
	mean road surface intensity (1=white,0=black)	0.2
	idem, lane line	0.9

Table 2: Main simulator parameters.

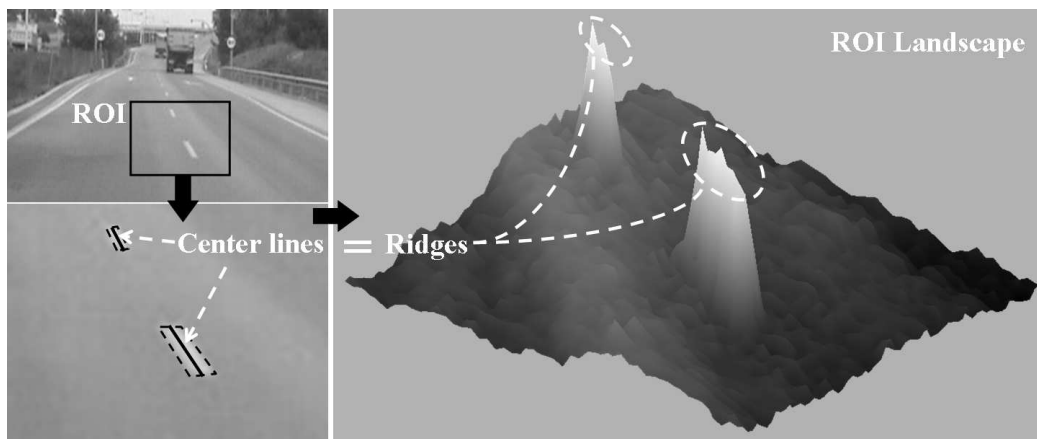


Figure 1: Left: road image with a region of interest (ROI) outlined. Right: intensity of ROI seen as a landscape, where lane markings resemble mountains and ridges correspond to the center of the lane markings.

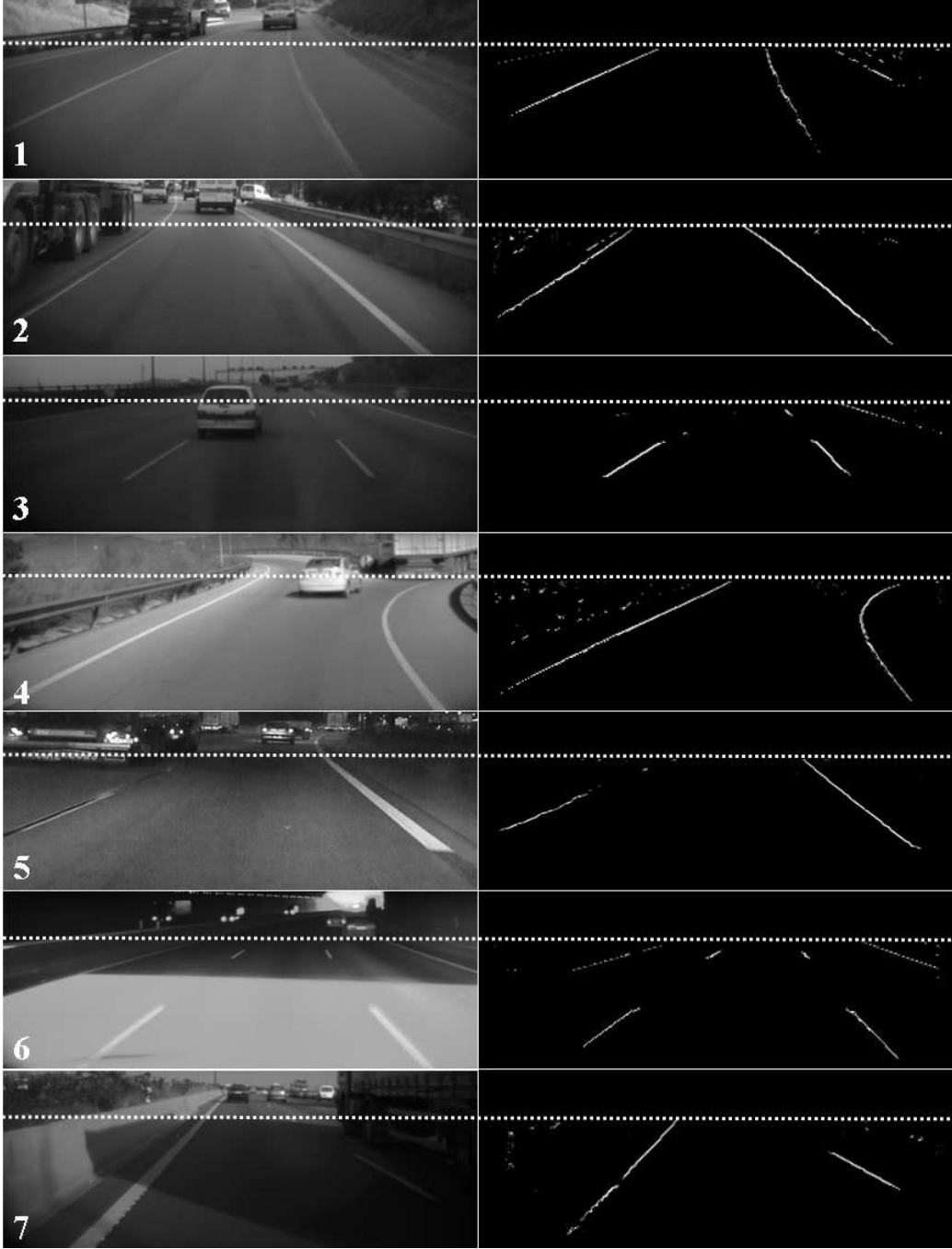


Figure 2: Detection in challenging conditions: (1) worn off paint, (2) tire marks, (3) white vehicle, (4) high curvature, (5) at night, (6) entering a tunnel, (7) shadow cast by a truck. Right column: ridgeness of points selected as candidate lane markings. The dotted line is the fixed initial row under which candidate points are sought ( $v_{\min}$  in figure 4)

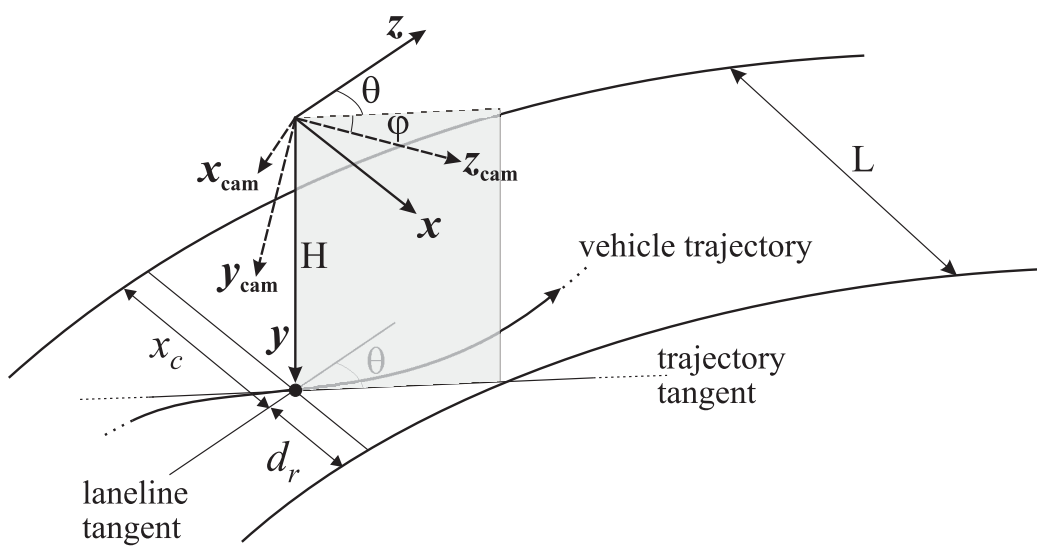


Figure 3: Image acquisition geometry.

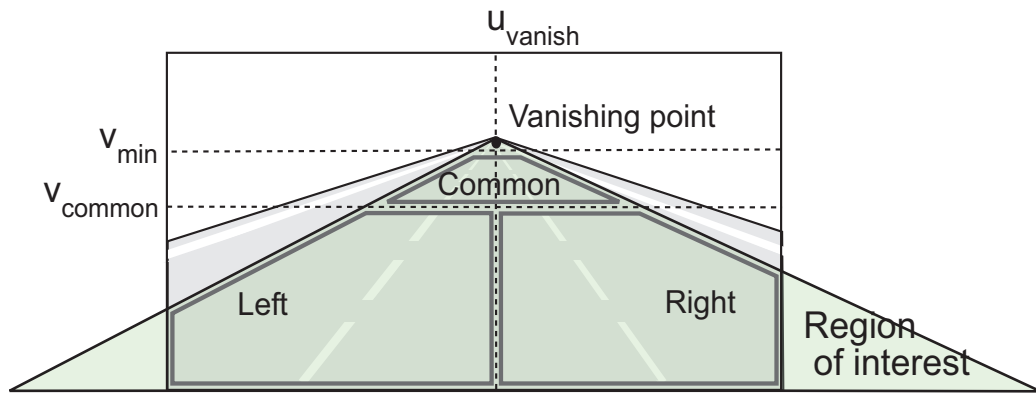


Figure 4: The detected features are divided into three groups, depending on their position in the image. Ridgeness is computed from row ( $v_{\text{min}}$ ) downwards.



Figure 5: Segmented lane curves on frames acquired by different cameras: (1–3) dashed lines, (4) occlusion. (5,6) tunnel exit and entrance, (7) horizontal marks, (8) cones, (9,10) special road marks, (11,12) shadows, (13,14) night images with low contrast and reflections.





Figure 6: Several frames of a sequence at 10 fps. The vehicle leaves a highway, finds a bifurcation and turns to the right. Notice that in spite of the high curvature of the lane lines the method is able to detect them very well, except in the frames where the two lines are not parallel ( $f_{68}$  and  $f_{75}$ ) due to the approaching bifurcation.

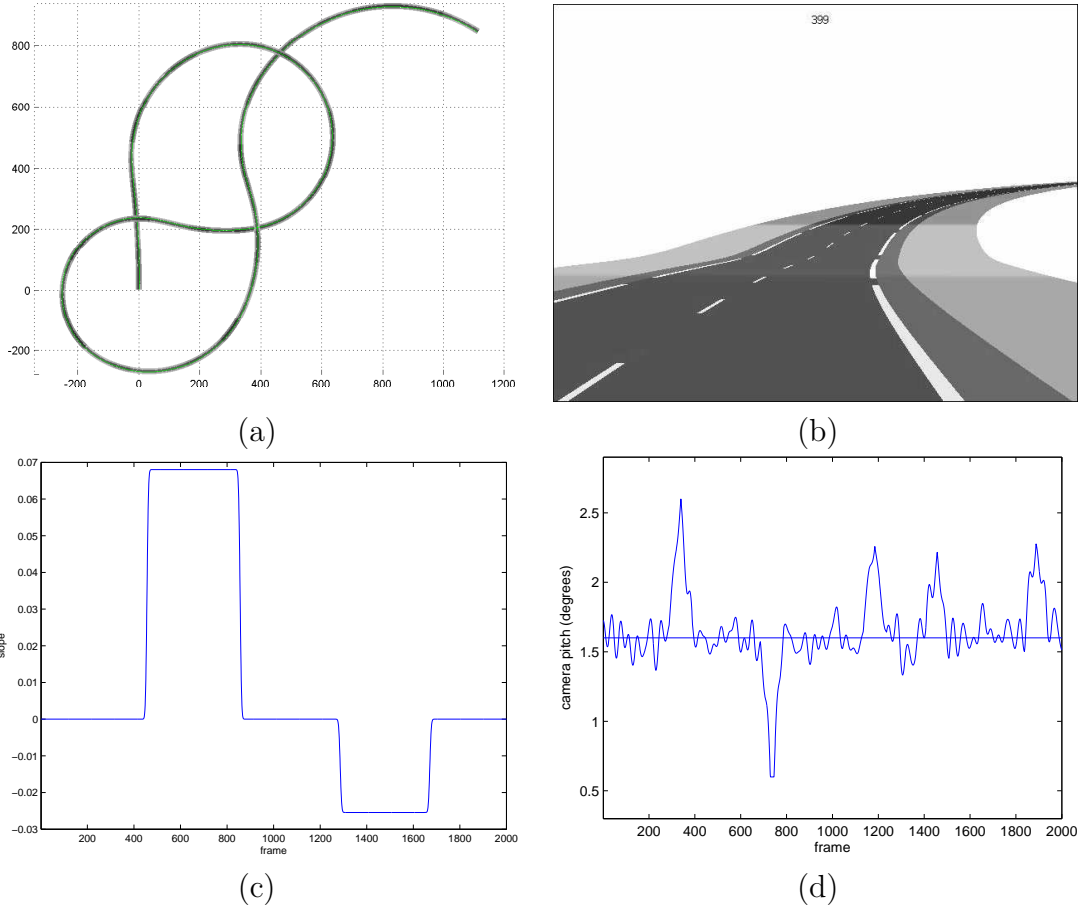


Figure 7: (a) Planar projection of the synthetic 3D road, (b) sample frame of a place close to changes in slope, curvature and road contrast, (c) road slope, (d) nominal and real pitch angle  $\varphi$ . For viewing the complete video sequence, see <http://www.cvc.uab.es/adas/projects/lanemarkings/IJAT/videos.html>.

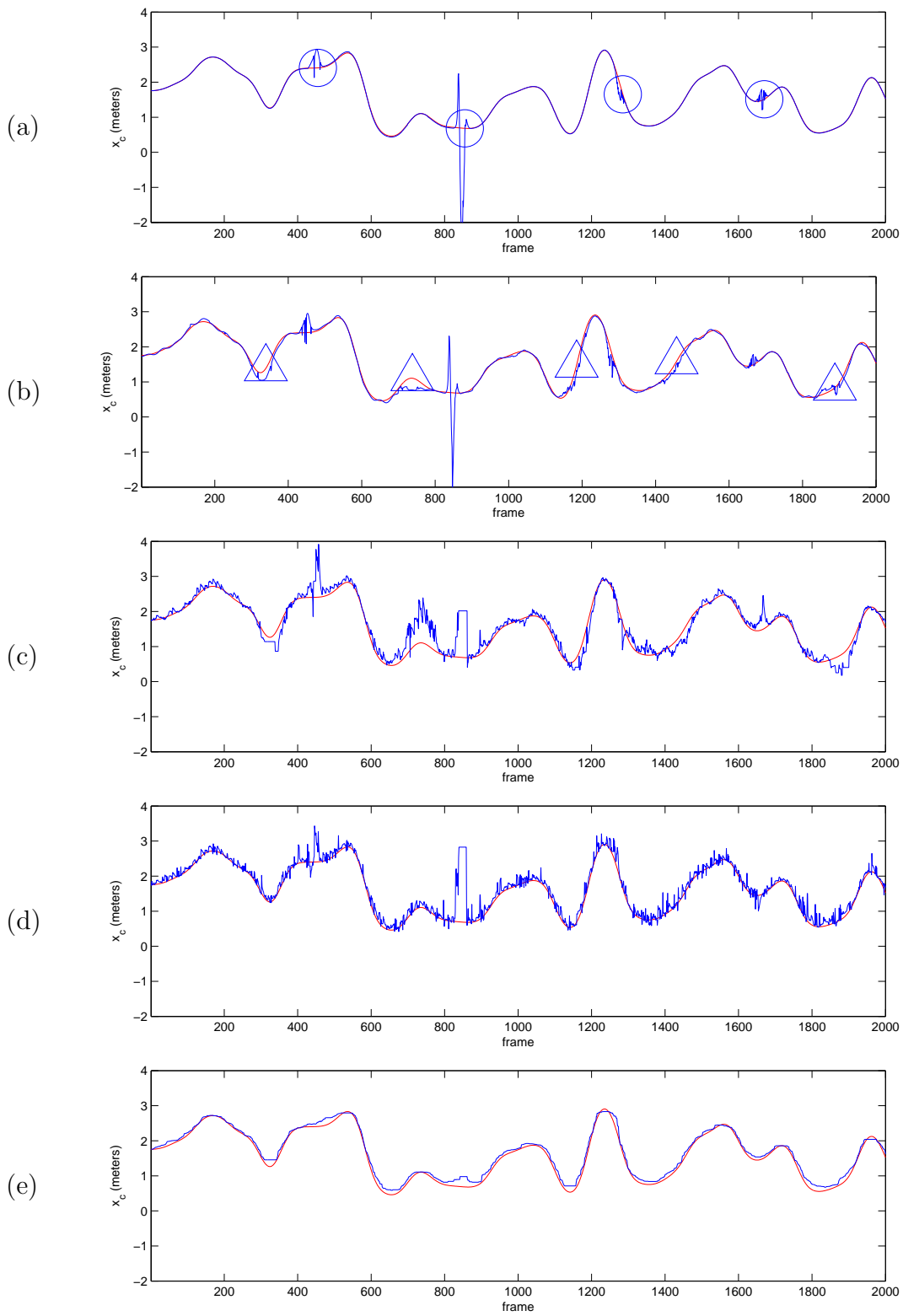


Figure 8: Ground truth and computed  $x_c$  in meters for (top to bottom) : a) non-ideal road, b) non-ideal camera, c) non-ideal detection with nominal pitch ( $n_\varphi=1$ ), d) non-ideal detection trying  $n_\varphi=7$  pitch angles around nominal camera pitch and median filtering of this latter result, e) causal median filtering of (d). To achieve a proper zoom only the first 2 Km are shown. Circles and triangles mark the locations of slope and large pitch variation, respectively.

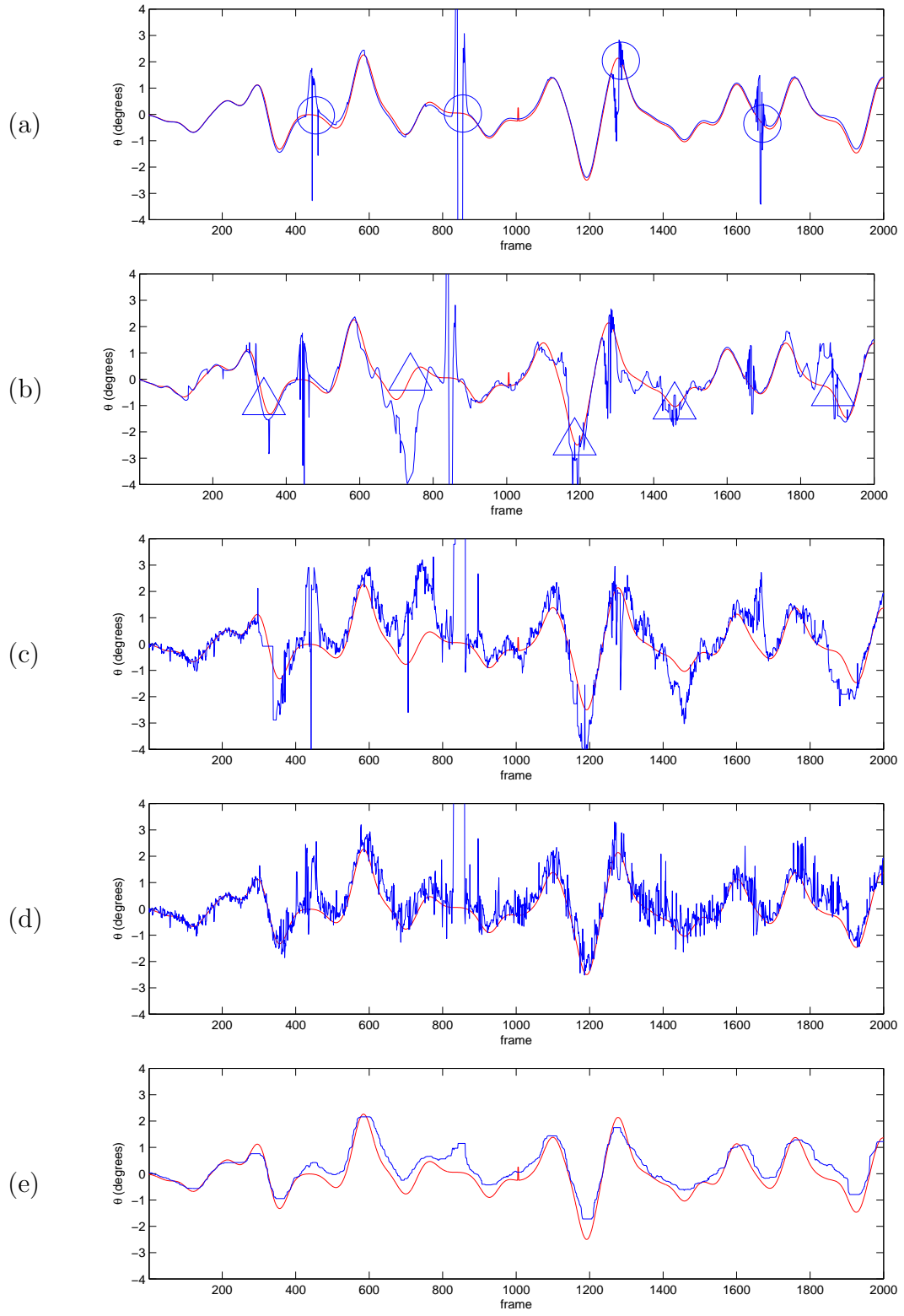


Figure 9: Ground truth and computed  $\theta$  in degrees, same cases as figure 8.

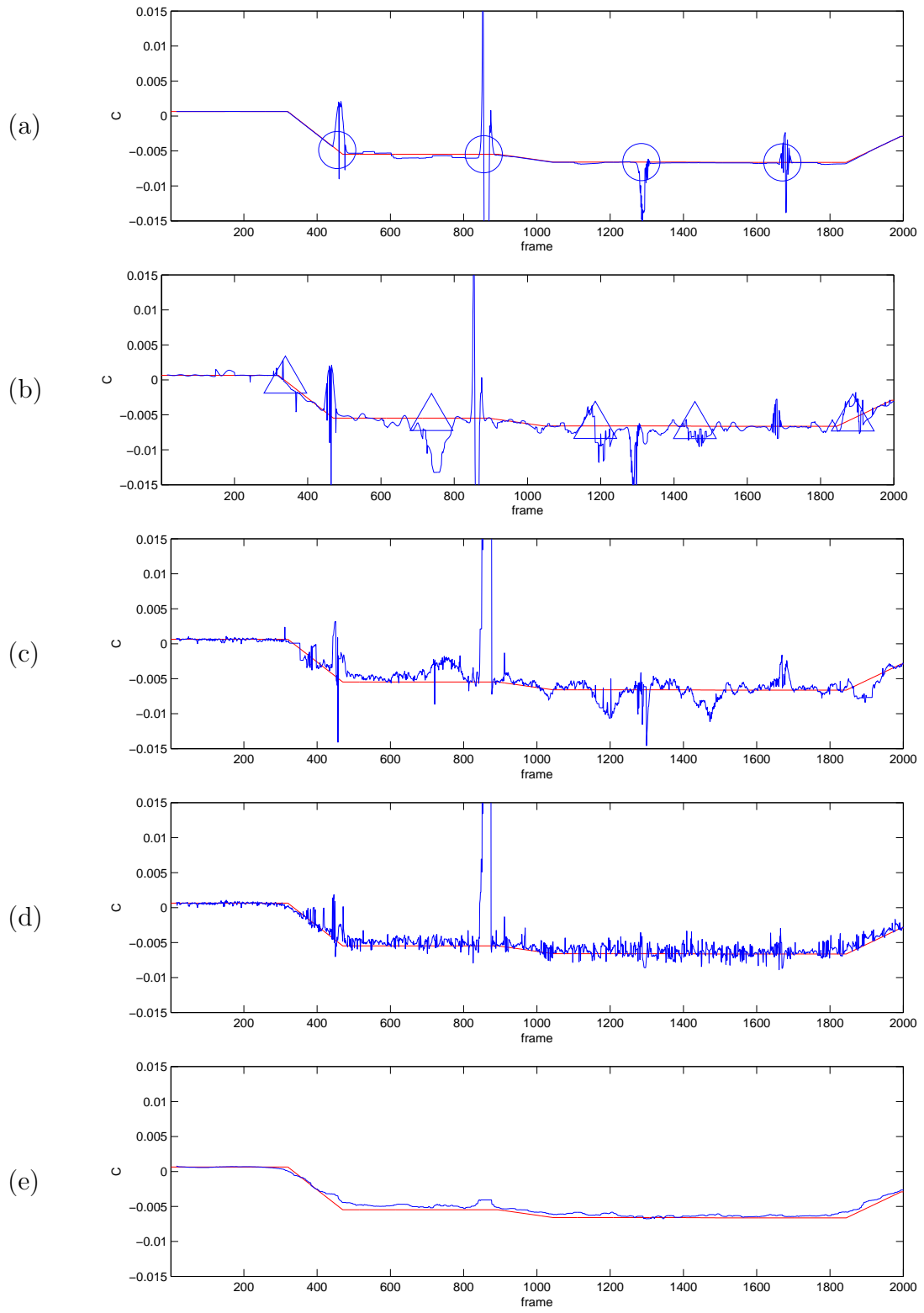


Figure 10: Ground truth and computed  $C_0$  in  $\text{m}^{-1}$ , same cases as figure 8.

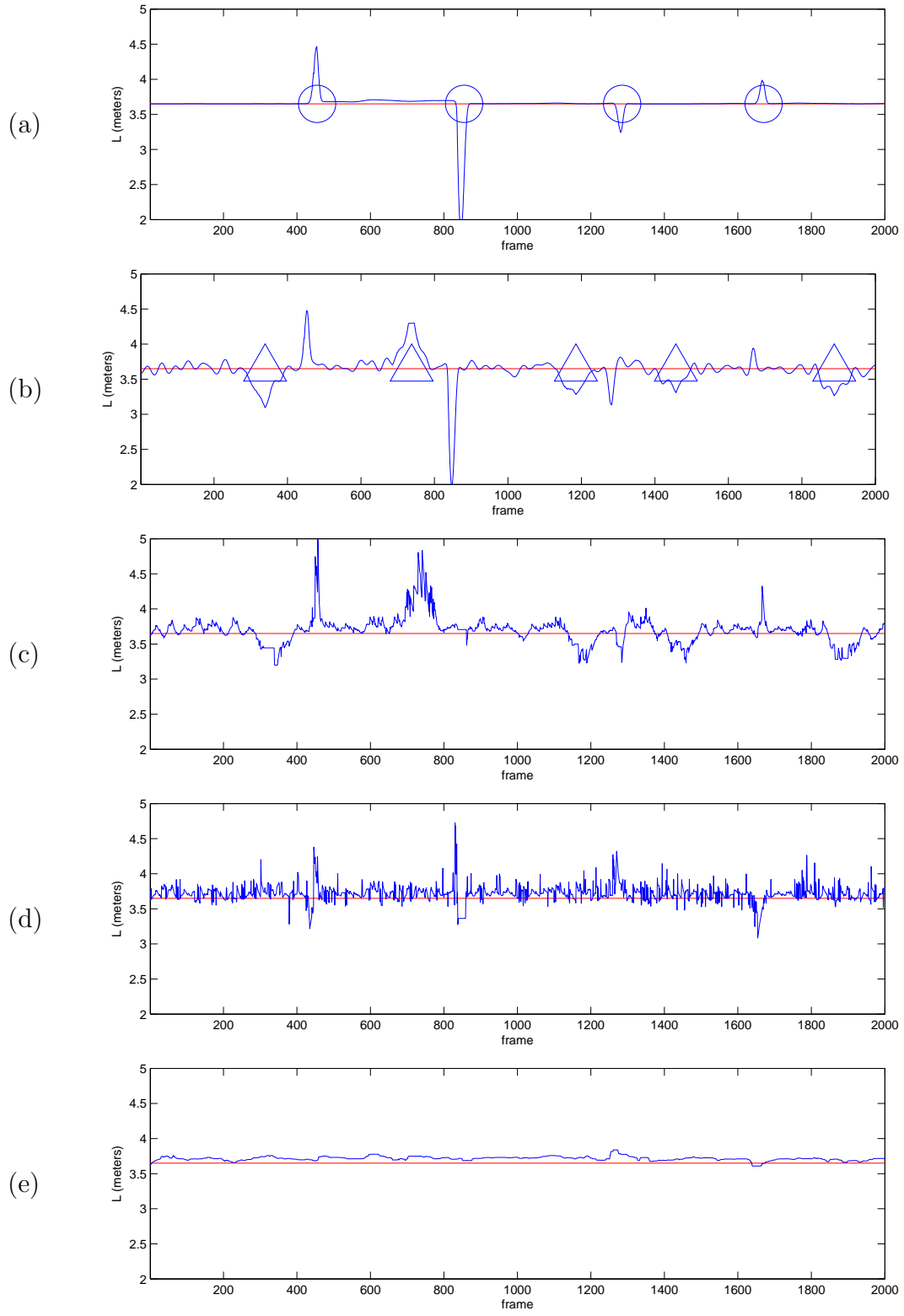


Figure 11: Ground truth and computed  $L$  in meters, same cases as figure 8.

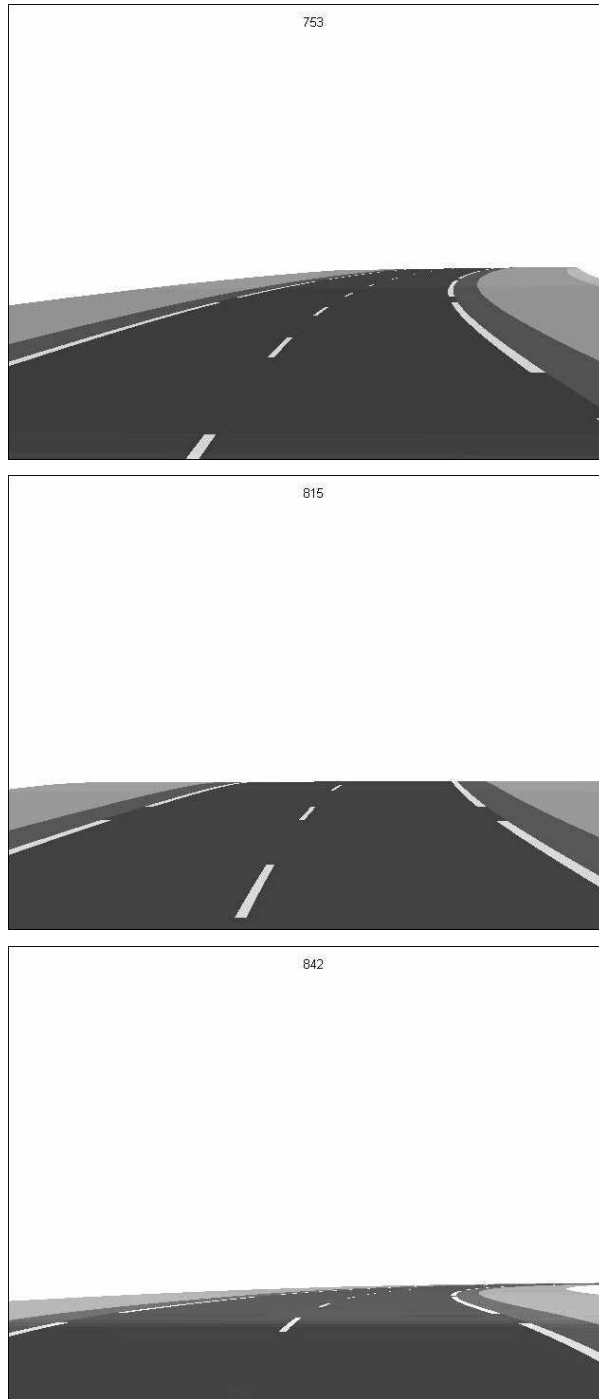


Figure 12: A negative slope change shrinks the road region from the image, causing large errors in the computed parameters.

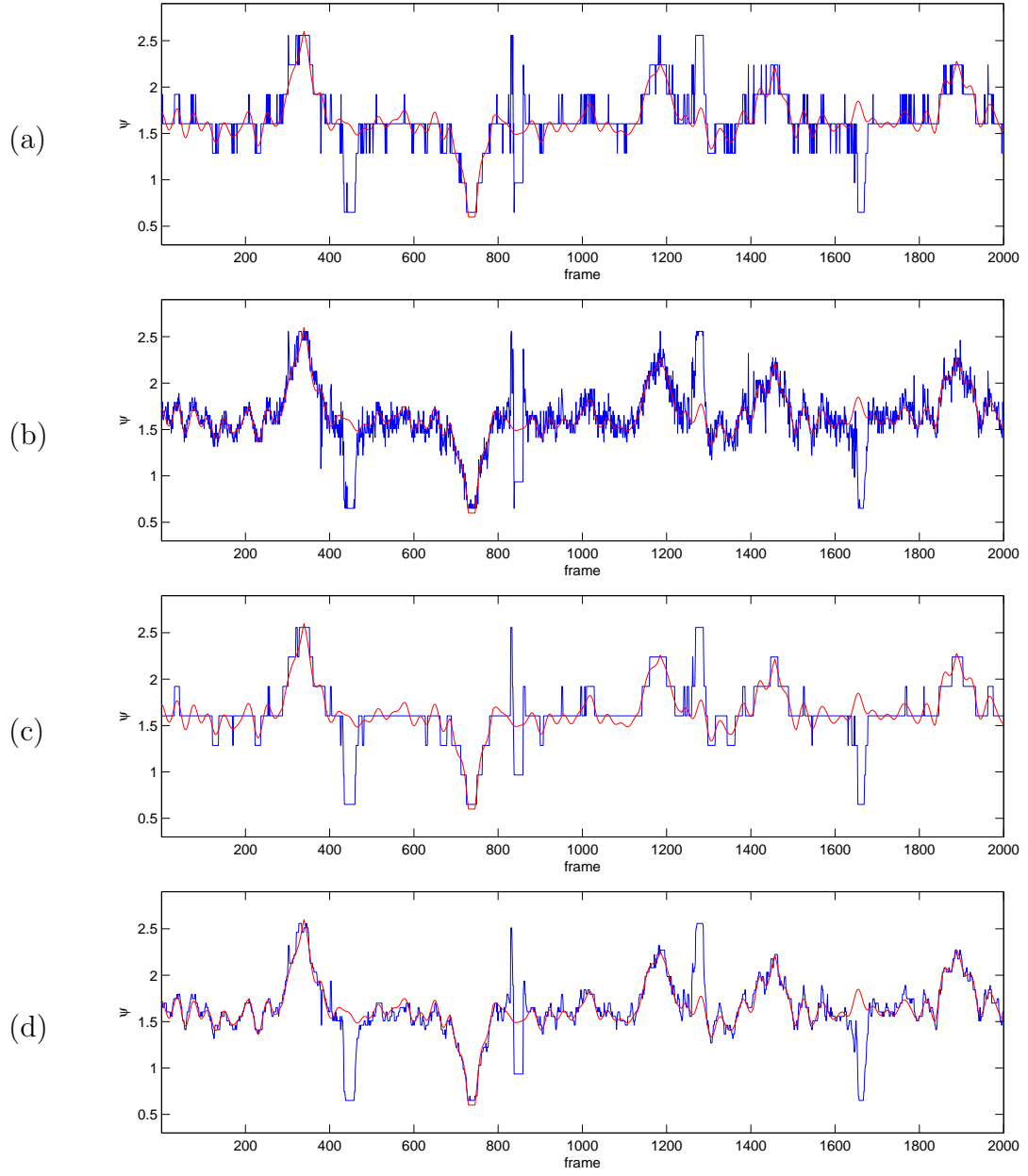


Figure 13: Ground truth and computed  $\varphi$  in degrees, for (a)  $n_\varphi=7$ , (b)  $n_\varphi=41$ , (c), (d) causal median filtering of (a) and (b).



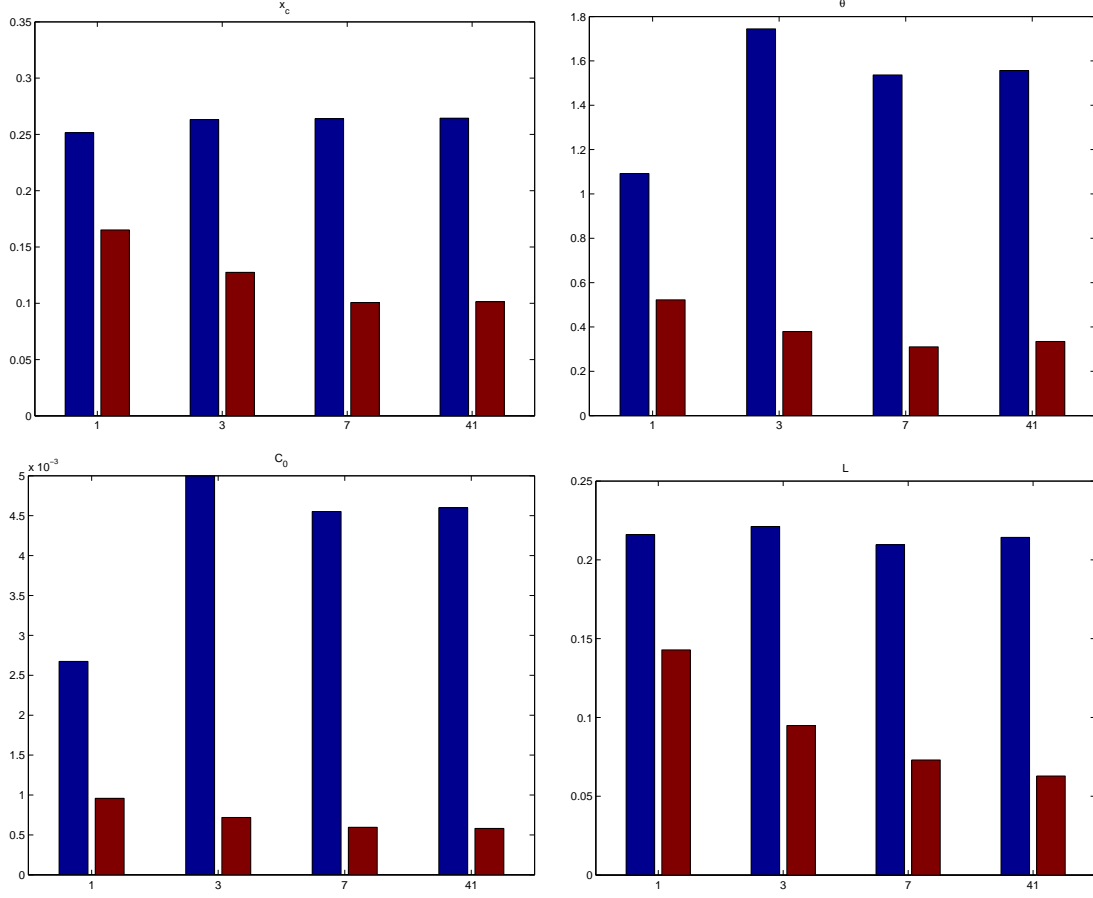


Figure 14: Root-mean square error of computed  $x_c, \theta, C_0$  and  $L$  (angles in degrees) for the case of non-ideal detection and  $n_\varphi = 1$  (non-ideal detection), 3, 7 and 41 (best pitch search). In each pair of bars, left bar corresponds to the computed value and right bar to its median filtered version.