

A1

Saturday, 25 February 2023 2:42 PM

Question 1

Mononomial :

Looking for $p_m(x) = a_0 + a_1 x^1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 1 & 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 8 & -2 \\ 1 & 4 & 16 & 64 & 350 \end{array} \right] \begin{matrix} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow \frac{1}{2}(R_3 - R_1) \\ R_4 \rightarrow \frac{1}{4}(R_4 - R_1) \end{matrix}$$

$$\begin{matrix} R_3 \rightarrow R_3 - R_2 \\ \hline R_4 \rightarrow R_4 - R_2 \end{matrix}$$

$$\underline{R_4 \rightarrow R_4 - 3R_2}$$

$$+ a_2 x^2 + a_3 x^3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 1 & 4 & 16 & 38 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 3 & 15 & 87 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 3 & 97 \end{array} \right]$$

$$\text{Thus: } a_0 = -2$$

$$a_3 = 15$$

$$a_2 = 1 - 3(15)$$

$$= -46$$

$$a_1 = 1 + 46 -$$

$$= 32$$

$$\text{Hence } p_n(x) = -2 + 32x - 46$$

Lagrange:

$$\text{Looking for: } p_L(x) = \sum_{i=0}^3 y_i L_i(x)$$

$$\text{where } L_i(x) = \prod_{j=0, j \neq i}^3 \left(\frac{x - x_j}{x_i - x_j} \right)$$

L U O C S . W J

15

$$x^2 + 15x^3$$

i)

$$\sum_{\substack{i=0 \\ i \neq j}}^{\infty} -r_i$$

$$-j$$

(x-4)

(x-4)

0

2)

$$x) -2L_2(x) + 350L_3(x)$$

$L_i(x)$, $i \in \{0, 3\}$ above.

$$= -2 + C_1 x$$

$$P_1(1) = -1 = -2 + C_1 \\ \Rightarrow C_1 = 1$$

$$P_1(x) = -2 + x$$

$$P_2(x) = P_1(x) + C_2(x)(x-1)$$

$$= -2 + x + C_2 x(x-1)$$

$$P_2(2) = -2 = -2 + 2 + 2C_2 \\ \Rightarrow C_2 = -1$$

$$P_2(x) = -2 + x - (x)(x-1)$$

$$P_3(x) = P_2(x) + C_3(x)(x-1)(x-2)$$

$$= -2 + x - (x)(x-1)(x-2)$$

$$P_3(4) = -2 + 4 - (4)(3) \\ \Rightarrow 360 = 24C_3$$

$$\Rightarrow C_3 = 15$$

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-1)

-1)(x-2)

+C_3(x)(x-1)(x-2)

+C_3(4)(3)(2) = 350

∴ 15x^3 - 20x^2 + 12x - 1

$$\text{Hence } P_n(x) = -2 + x - (x)$$

$$\text{Now: } P_n(x) = 15x^3 - 46x^2 + 32x - 2$$

$$\begin{aligned} \text{And: } P_L(x) &= \frac{1}{8} (x-1)(x-2)(x-4) \\ &\quad + \frac{1}{3} (x)(x-2)(x-4) \\ &\quad + \frac{2}{4} (x)(x-1)(x-4) \\ &\quad + \frac{350}{24} (x)(x-1)(x-2) \end{aligned}$$

$$\begin{aligned} &= \frac{1}{4} (x^3 - 7x^2 + 14x - \\ &\quad + \frac{1}{3} (x^3 - 6x^2 + 8x) \\ &\quad + \frac{1}{2} (x^3 - 5x^2 + 4x) \\ &\quad + \frac{350}{24} (x^3 - 3x^2 + 2x) \end{aligned}$$

$$x^2 + 10(x-1)(x-2)$$

8)

$$= 15x^3 - 46x^2 + 32x - 2$$

$$= P_m(x)$$

And: $P_n(x) = -2 + x - (x)(x-1) + 1$

$$= 15 \left(x^3 - 3x^2 + 2x \right)$$

$$- \left(x^2 - x \right)$$

$$+ x$$

$$= 15x^3 - 46x^2 + 32x$$

$$= P_m(x) = P_L(x)$$

\Rightarrow all forms of

$$5(x)(x-1)(x-2)$$

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)

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interpolation are equal.

Question 2:

a) using Lagrange form:

$$L_0(x) = \frac{(x-1)(x-2)(x-4)}{-8}$$

$$L_1(x) = \frac{x(x-2)(x-4)}{3}$$

$$L_2(x) = \frac{x(x-1)(x-4)}{-4}$$

$$L_3(x) = \frac{x(x-1)(x-2)}{24}$$

But: $p(x) = 4L_0(x) - L_1(x)$

b) using Newton's form:

$$P_3(x) = -2 + x - (x)(x-1) +$$

- same as above)

$$(x) - 2L_2(x) + 350 L_3(x).$$

$$- 15 (x)(x-1)(x-2)$$

$$P_4(x) = P_3(x) + C_4(x)$$
$$= -2+x - x(x-1) +$$

$$P_4(3) = -2+3 - 3(2) +$$

$$\Rightarrow 85 - 6C_4 =$$

$$\Rightarrow C_4 = 7$$

$$\Rightarrow P(x) = -2+x - (x)(x)$$

$$(x-1)(x-2)(x-4)$$
$$-5(x)(x-1)(x-2) + 4(x)(x-1)(x-2)$$

$$5(3)(2)(1) + 4(3)(2)(1)(-1)$$

43

$$(-1) + 5(x)(x-1)(x-2) + 7(x)(x-1)(x-2)$$

$$)(x-4)$$

$$) = 43$$

$$(x-2)(x-4)$$