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Question 1

Monomonial:

Looking for $p_m(x) = a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 1 & 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 8 & -2 \\ 1 & 4 & 16 & 64 & 350 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to \frac{1}{2}(R_3 - R_1) \\ R_4 \to \frac{1}{4}(R_4 - R_1) \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 1 & 4 & 16 & 88 \end{array}\right]$$

$$\begin{array}{l} R_3 \to R_3 - R_2 \\ R_4 \to R_4 - R_2 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 3 & 15 & 87 \end{array}\right]$$

$$R_4 \to R_4 - 3R_3 \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 6 & 90 \end{array}\right]$$

Thus: $a_0 = -2$

$a_3 = 15$

$a_2 = 1 - 3(15)$

$= -46$

$a_1 = 1 + 46 - 15$

$= 32$

Hence $p_m(x) = -2 + 32x - 46x^2 + 15x^3$

Lagrange:

Looking for: $p_L(x) = \sum_{i=0}^{3} y_i L_i(x)$

where $L_j(x) = \prod_{\substack{i=0 \\ i \neq j}}^{3} \left(\frac{x - x_i}{x_j - x_i}\right)$

Now: $L_0(x) = \frac{(x-1)(x-2)(x-4)}{-8}$

$L_1(x) = \frac{x(x-2)(x-4)}{3}$

$L_2(x) = \frac{x(x-1)(x-4)}{-4}$

$L_3(x) = \frac{x(x-1)(x-2)}{24}$

Thus $p_L(x) = -2L_0(x) - L_1(x) - 2L_2(x) + 350L_3(x)$

for functions $L_i(x)$, $i \in [0,3]$ above.

Newton:

$p_0(x) = -2$

$p_1(x) = p_0(x) + C_1(x-0)$

$= -2 + C_1 x$

$p_1(1) = -1 = -2 + C_1$

$\Rightarrow C_1 = 1$

$p_1(x) = -2 + x$

$p_2(x) = p_1(x) + C_2(x)(x-1)$

$= -2 + x + C_2 x(x-1)$

$p_2(2) = -2 = -2 + 2 + 2C_2$

$\Rightarrow C_2 = -1$

$p_2(x) = -2 + x - (x)(x-1)$

$p_3(x) = p_2(x) + C_3(x)(x-1)(x-2)$

$= -2 + x - (x)(x-1) + C_3(x)(x-1)(x-2)$

$p_3(4) = -2 + 4 - (4)(3) + C_3(4)(3)(2) = 350$

$\Rightarrow 360 = 24C_3$

$\Rightarrow C_3 = 15$

Hence $p_n(x) = -2 + x - (x)(x-1) + 15(x)(x-1)(x-2)$

Now: $p_n(x) = 15x^3 - 46x^2 + 32x - 2$

And: $P_L(x) = \frac{2}{8}(x-3)(x-2)(x-4)$

$+\frac{1}{3}(x)(x-2)(x-4)$

$+\frac{2}{4}(x)(x-1)(x-4)$

$+\frac{380}{24}(x)(x-1)(x-2)$

$= \frac{1}{4}(x^3-7x^2+14x-8)$

$+\frac{1}{3}(x^3-6x^2+8x)$

$+\frac{1}{2}(x^3-5x^2+4x)$

$+\frac{380}{24}(x^3-3x^2+2x)$

$= 15x^3-46x^2+32x-2$

$= P_M(x)$

And: $P_N(x) = -2+x-(x)(x-1)+15(x)(x-1)(x-2)$

$= 15(x^3-3x^2+2x)$

$-(x^2-x)$

$+x-2$

$= 15x^3-46x^2+32x-2$

$= P_M(x) = P_L(x)$

$\Rightarrow$ all forms of interpolation are equal.

Question 2:

a) using Lagrange form:

$$L_0(x) = \frac{(x-1)(x-2)(x-4)}{-8}$$

$$L_1(x) = \frac{x(x-2)(x-4)}{3}$$

$$L_2(x) = \frac{x(x-1)(x-4)}{-4}$$

$$L_3(x) = \frac{x(x-1)(x-2)}{24} \quad \text{(same as above)}$$

But: $p(x) = 4L_0(x) - L_1(x) - 2L_2(x) + 850\,L_3(x)$.

b) using Newton's form:

$$P_3(x) = -2 + x - (x)(x-1) + 5(x)(x-1)(x-2)$$

$$P_4(x) = P_3(x) + C_4(x)(x-1)(x-2)(x-4)$$
$$= -2 + x - x(x-1) + 5(x)(x-1)(x-2) + C_4(x)(x-$$

$$P_4(3) = -2 + 3 - 3(2) + 5(3)(2)(1) + C_4(3)(2)$$

$$\Rightarrow 85 - 6C_4 = 43$$

$$\Rightarrow C_4 = 7$$

$$\Rightarrow p(x) = -2 + x - (x)(x-1) + 5(x)(x-1)(x-2) + 7(x$$